

Portfolio Analysis and Equilibrium Asset Pricing with Heterogeneous Beliefs

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Certificate

I certify that this thesis has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Date

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Abstract

The representative agent paradigm with homogeneous expectations has been the dominant framework for the development of theories in portfolio analysis, equilibrium asset pricing and derivative pricing. Homogeneous expectations is the major assumption underlining the most widely used financial models including the *Capital Asset Pricing Model* (CAPM), Lucas's general equilibrium model and the Black-Scholes option-pricing formula. These models are popular because of their clear economic intuition and simplicity. However, this paradigm fails to take into account the heterogeneity, bounded rationality and speculative behaviour of different agents in the economy, which results in models with predictions that lack empirical support. There exist several empirical inconsistencies; (i) The CAPM predicts all investors hold an efficient portfolio in equilibrium and every asset's expected return is linearly related to the market portfolio by the asset's beta. However, it has been found that investors underdiversify in some situations; furthermore, other factors including value, size, momentum and dispersion of analyst forecast also predict future returns, which contradicts CAPM. (ii) Lucas' model predicts that the average equity premium should be proportional to relative risk aversion and covariance between equity return and aggregate consumption; however, the observed equity premium implies an implausibly high relative risk aversion. This is termed the *equity premium puzzle*. (iii) The Black-Scholes model of option pricing predicts that the implied volatility of option prices is independent of time to maturity and strike prices, but the implied volatility in real markets is observed to be skewed. This feature of option prices is called the volatility skew. Although the postulate of unbounded rationality has dominated economic modelling for several decades, empirical evidence, unconvincing justification of the assumption of unbounded rationality and investor psychology have led to the incorporation of heterogeneity in beliefs and bounded rationality into financial modelling. Heterogeneity can have profound consequences for the interpretation of empirical evidence and the formulation of economic policy. Heckman (2001), the 2001 Nobel Laureate in economics, said, "*The most important discovery was the evidence on pervasiveness of heterogeneity and*

diversity in economic life. When a full analysis was made of heterogeneity in response, a variety of candidate averages emerged to describe the “average” person, and the long-standing edifice of the representative consumer was shown to lack empirical support.” The aim of this thesis is to use a framework of heterogeneous agents to examine the impact of heterogeneous beliefs on portfolio analysis and asset pricing and explore the potential to explain the observed phenomenon mentioned above. The agents have heterogeneous beliefs regarding future outcomes in the market and the belief of the “average” agent is characterised by the *consensus belief*. The thesis consists of three main components:

- The impact of heterogeneous beliefs on the cross section of asset returns, the geometric tangency relation between the portfolio frontier and the capital market line, and the portfolio efficiency of investors’ subjectively optimal portfolios. This is the focus of Chapters 2 and 3. In Chapter 2, investors are assumed to have heterogeneous beliefs about asset payoffs while in Chapter 3 is based on the assumption of heterogeneous beliefs about the rates of return. We find that the tangency relation in the standard portfolio analysis does not hold in general and that adding a riskless asset in zero net supply can increase the marginal utility of the market in some situations. Subjectively optimal portfolios are mean-variance (MV) inefficient in general, depending on the various aspects of heterogeneity amongst investors.
- The relationship between heterogeneity and market risk premium and risk-free rate. This is the focus of Chapters 4 and 5. Chapter 4 considers a multi-asset economy in a static mean-variance framework and Chapter 5 considers a Lucas-type continuous-time general equilibrium model with one risky asset and one riskless asset. In a multi-asset economy, we find that various combinations of heterogeneity can increase the market risk premium and reduce the risk-free rate. The effect is significant in some cases and insignificant in others. In a pure exchange economy, we find that the impact of heterogeneity on the equity premium and interest rate can be magnified under a relative consumption framework.
- The pricing of options under heterogeneous beliefs. Chapter 6 develops a binomial lattice to model investors’ subjective beliefs in a multi-period discrete time setting and provides an option-pricing formula under heterogeneous beliefs. The framework is able to replicate various patterns of the implied volatilities observed in the market and provides some economic intuitions and explanations.

The three components together contribute to the growing literature of asset pricing under heterogeneous beliefs by improving the understanding of the impact of heterogeneity in preferences and beliefs on portfolio analysis and equilibrium asset prices.