The Evaluation of Early Exercise Exotic Options

A Thesis Submitted for the Degree of
Doctor of Philosophy

by

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate ....

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Contents

Abstract

Research on the pricing of multifactor American options has been growing at a slow pace due to the curse of dimensionality. If we start to consider the pricing of American option contracts written on more than one underlying asset or relax the constant volatility assumption of the Black and Scholes (1973) model, the computational burden increases as more computing power is required to handle the increasing number of dimensions.

This thesis deals with the problem of pricing multifactor American options under both constant and stochastic volatility. The main focus of the thesis is to extend the representation results of Kim (1990) and Carr, Jarrow and Myneni (1992) and to devise higher dimensional numerical techniques for pricing multifactor American options. We present numerical examples for two and three factor models. The pricing problems are formulated using the well known hedging arguments. We adopt two main approaches; the first involves deriving integral expressions for the American option prices with the aid of Jamshidian's (1992) transformation of the associated partial differential equation from a homogeneous problem on a restricted domain to an inhomogeneous problem on an unrestricted domain, Duhamel's principle and integral transform methods. The second technique involves implementing the method of lines algorithm for American exotic options, with the spread call option under stochastic volatility being the main example – this approach tackles directly the pricing partial differential equation. Chapter 1 contains an overview of the American option pricing problem from the viewpoint of the applications in this thesis. The chapter concludes with some technical results used in the rest of the thesis. The main contributions of the thesis are contained in the subsequent chapters.

Chapter 2 extends the integral transform approach of McKean (1965) and Chiarella and Ziogas (2005) to the pricing of American options written on two underlying assets under Geometric Brownian motion. A bivariate transition density function of the two underlying stochastic processes is derived by solving the associated backward Kolmogorov partial differential equation. Fourier transform techniques are used to transform the partial differential equation to a corresponding ordinary differential equation whose solution can be readily found by using the integrating factor method. An integral expression of the American option written on any two assets is then obtained by applying Duhamel's principle. A numerical algorithm for calculating American spread call option prices is given as an example, with the corresponding early exercise boundaries approximated by linear functions. Numerical results are presented and comparisons made with other alternative approaches.

Chapter 3 considers the pricing of an American call option whose underlying asset evolves under the influence of two independent stochastic variance processes of the Heston (1993) type. We derive the associated partial differential equation (PDE) for the option price using standard hedging arguments. An integral expression for the general solution of the PDE is derived using Duhamel's principle, which is expressed in terms of the yet to be determined trivariate transition density function for the driving stochastic processes. We solve the backward Kolmogorov PDE satisfied by the transition density function by first transforming it to the corresponding characteristic PDE using a combination of Fourier and Laplace transforms. The characteristic PDE is solved by the method of characteristics. Having determined the density function, we provide a full representation of the American call option price. By approximating the early exercise surface with a bivariate log-linear function, we develop a numerical algorithm to calculate the pricing function. Numerical results are compared with those from the method of lines algorithm. The approach is generalised in Chapter 4 to the case when the underlying asset evolves under the influence of more than two stochastic variance processes by using a combination of induction proofs and some lengthy derivations.

A numerical technique for the evaluation of American exotic options is developed in Chapter 5, with the American spread call option whose underlying assets dynamics evolve under the influence of a single stochastic variance process being presented as an example. The numerical algorithm involves extending the method of lines approach first presented in Meyer and van der Hoek (1997) when pricing the standard American put option to the multi-dimensional setting. We transform the pricing partial differential equation to a corresponding system of ordinary differential equations with the aid of the Riccati transformation. We use the implicit trapezoidal rule to solve the resulting Riccati equations. Numerical results are presented outlining the effectiveness of the algorithm. The effects of stochastic volatility are explored by making comparisons with the geometric Brownian motion results.

We summarise all thesis findings in Chapter 6. Concluding remarks and directions for future work are also presented.