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Model Predictive Control and Stabilisation of Interconnected Systems

by

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Certificate of Authorship/Originality

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for a degree except as fully acknowledged within the text.

I also certify that this thesis has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature quoted are indicated in the thesis.

TRI TRAN-CAO

ABSTRACT

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The attraction of having higher efficiency and quality, as well as increasing reliability and flexibility for industrial plants and network systems has created opportunities for new research in the control and optimisation fields. Among various design methods, model predictive control (MPC) strategies have proved to be effective in industrial applications. Whilst found widespread used with stand-alone controllers in the refining and many other industries, the field of orchestrating non-centralised MPCs and distributed MPCs is evaluated as still in its infancy.

The work in this thesis is concerned with stabilising methods for the control of complex interconnected systems with mixed connection configurations employing distributed and decentralised model predictive control schemes. Inheriting the advantage of the MPC strategy, the control and state constraints are naturally dealt with by the employed methods. As a result, the novel concept of asymptotically positive realness constraint (APRC) and the segregation and integration constructive methods for the constrained stabilisation of interconnected systems are introduced and developed. The MPC is formulated with state space models and stabilising constraints within the open-loop paradigm in this thesis. By having the control inputs entirely decoupled between subsystems and no additional constraints imposed on the interactive variables rather than the coupling constraint itself, the proposed approaches outreach various types of systems and applications. For parallel connections that emulate parallel redundant structures and have unknown splitting ratios, a fully decentralised

control strategy is developed as an alternative to the hybrid approaches. For the semi-automatic control systems, which is involved with both closed-loop and human-in-the-loop regulatory controls, the stability-guaranteed method of decentralised stabilising agents which are interoperable with different control algorithms is germinated and implemented for each single subsystem.

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Nomenclature and Notation

Capital letters denote matrices. Lower-case alphabet and Greek letters denote column vectors and scalars, respectively.

$\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{N}$ denote the field of real numbers, the set of non-negative reals, the set of integers, the set of non-negative integers and the set of natural numbers, respectively.

The notation $\Omega \subseteq \Phi$ is used to denote that Ω is a subset of Φ and $\Omega \subset \Phi$ denotes that Ω is a proper subset of Φ. For two arbitrary sets $\mathcal{P}_1 \subseteq \mathbb{R}^n$ and $\mathcal{P}_2 \subseteq \mathbb{R}^n$, $\mathcal{P}_1 \backslash \mathcal{P}_2$ denotes the relative complement of $\mathcal{P}_2$ in $\mathcal{P}_1$, i.e. their set difference.

$(.)^T$ denotes the transpose operation.

I_n is the identity matrix of dimension $n \times n$. 0_n is the zero matrix of dimension $n \times n$.

$\text{diag}[A_i]_1^N$ is the block diagonal matrix with diagonal entries $A_i, i = 1, 2, ..., N$, while $[M_{ij}]_{i,j=1,2,...,h}$ is the matrix of sub-blocks M_{ij}.

$\text{diag}[Q_{ji}]_{j=1...h,\, i=1...g_j}$ is the block diagonal matrix with diagonal entries Q_{ji}, where $j = 1, 2, ..., h;\ i = 1, 2, ..., g_j$.

$\|u_i\|_2$ and $\|u_i\|$ denote the ℓ_2-norm of the vector u_i.

$\|Q\|_2$ is the induced 2-norm of the matrix Q, which is defined as

$$\|Q\|_2 = \max \{\|Qv\|_2 : v \in \mathbb{R}^n, \|v\|_2 \leq 1\}.$$

$Q \preccurlyeq 0$ means Q is positive semi-definite i.e. $x^T Q x \leq 0\ \forall x$. $Q \prec 0$ means Q is positive definite i.e. $x^T Q x < 0\ \forall x \neq 0$.

A function $\gamma :\ \mathbb{R}^+ \rightarrow \mathbb{R}^+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$.

A function $\alpha : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{KL}$ if for each fixed $\tau \in \mathbb{R}^+$, $\alpha(.,\tau) \in \mathcal{K}$ and for each fixed $s \in \mathbb{R}^+$, $\alpha(s,.)$ is decreasing and $\lim_{\tau\to\infty} \alpha(s,\tau) = 0$.

In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for terms that are induced by symmetry, e.g.,

$$(*)\begin{bmatrix} (*)+R & S \\ * & Q \end{bmatrix} K = K^T \begin{bmatrix} R^T+R & S \\ S^T & Q \end{bmatrix} K, \text{ or}$$
$$K^T \begin{bmatrix} R^T+(*) & * \\ S^T & Q \end{bmatrix} (*) = K^T \begin{bmatrix} R^T+R & S \\ S^T & Q \end{bmatrix} K.$$

$\lambda_{\min}(.)$ and $\lambda_{\max}(.)$ indicate the minimum and maximum eigenvalues of the argument, respectively.

The function $\mathsf{sgn}(.)$ is defined by

$$\mathsf{sgn}(\xi) = \begin{cases} 1, \text{ if } \xi > 0, \\ 0, \text{ if } \xi = 0, \\ -1, \text{ if } \xi < 0. \end{cases}$$

In a proof, when the time index k is omitted for conciseness, $v(-\tau)$ denotes the vector $v(k-\tau)$.

$\hat{u}$ denotes a sequence of predictive vectors of $u(j)$ starting from the current time step. $\breve{u}$ denotes a sequence of $u(-j)$ representing historical data of u.

The ℓ^{th} element of a vector $u_i(k)$ is denoted as $u_i^{(\ell)}(k)$.

In the discrete-time domain, the time index is denoted by $k, k \in \mathbb{Z}$. For signals belonging to $\mathcal{L}_2$ space, $k \geq 0$.

The boldface style for letters is used in optimisation formulations to emphasise that they are variables.

Abbreviation

APRC - Asymptotically Positive Realness Constraint.

ASPRC - Asymptotically Surely Positive Realness Constraint.

DeMPC - Decentralised Model Predictive Control.

DMPC - Distributed Model Predictive Control.

DRD - Delay-Robust Dissipativity.

IQC - Integral Quadratic Constraint.

LMI - Linear Matrix Inequality.

MATI - Maximum Allowable Transmission Interval.

MPC - Model Predictive Control.

NMPC - Nonlinear Model Predictive Control.

NCS - Networked Control System.

ODE - Ordinary Differential Equation.

PRC - Positive Real (or Realness) Constraint.

QP - Quadratic Programming.

SDP - Semi-Definite Programming.