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Model Predictive Control and Stabilisation of Interconnected Systems

by

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Certificate of Authorship/Originality

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for a degree except as fully acknowledged within the text.

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TRI TRAN-CAO

ABSTRACT

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The attraction of having higher efficiency and quality, as well as increasing reliability and flexibility for industrial plants and network systems has created opportunities for new research in the control and optimisation fields. Among various design methods, model predictive control (MPC) strategies have proved to be effective in industrial applications. Whilst found widespread used with stand-alone controllers in the refining and many other industries, the field of orchestrating non-centralised MPCs and distributed MPCs is evaluated as still in its infancy.

The work in this thesis is concerned with stabilising methods for the control of complex interconnected systems with mixed connection configurations employing distributed and decentralised model predictive control schemes. Inheriting the advantage of the MPC strategy, the control and state constraints are naturally dealt with by the employed methods. As a result, the novel concept of asymptotically positive realness constraint (APRC) and the segregation and integration constructive methods for the constrained stabilisation of interconnected systems are introduced and developed. The MPC is formulated with state space models and stabilising constraints within the open-loop paradigm in this thesis. By having the control inputs entirely decoupled between subsystems and no additional constraints imposed on the interactive variables rather than the coupling constraint itself, the proposed approaches outreach various types of systems and applications. For parallel connections that emulate parallel redundant structures and have unknown splitting ratios, a fully decentralised

control strategy is developed as an alternative to the hybrid approaches. For the semi-automatic control systems, which is involved with both closed-loop and human-in-the-loop regulatory controls, the stability-guaranteed method of decentralised stabilising agents which are interoperable with different control algorithms is germinated and implemented for each single subsystem.

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This work is dedicated to my dear Mother who has taught her children about the importance of studies in life, and sacrificed for us.

Tri Tran-Cao
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Nomenclature and Notation

Capital letters denote matrices. Lower-case alphabet and Greek letters denote column vectors and scalars, respectively.

$\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{N}$ denote the field of real numbers, the set of non-negative reals, the set of integers, the set of non-negative integers and the set of natural numbers, respectively.

The notation $\Omega \subseteq \Phi$ is used to denote that Ω is a subset of Φ and $\Omega \subset \Phi$ denotes that Ω is a proper subset of Φ. For two arbitrary sets $\mathcal{P}_1 \subseteq \mathbb{R}^n$ and $\mathcal{P}_2 \subseteq \mathbb{R}^n$, $\mathcal{P}_1 \backslash \mathcal{P}_2$ denotes the relative complement of $\mathcal{P}_2$ in $\mathcal{P}_1$, i.e. their set difference.

$(.)^T$ denotes the transpose operation.

I_n is the identity matrix of dimension $n \times n$. 0_n is the zero matrix of dimension $n \times n$.

$\text{diag}[A_i]_1^N$ is the block diagonal matrix with diagonal entries $A_i, i = 1, 2, ..., N$, while $[M_{ij}]_{i,j=1,2,...,h}$ is the matrix of sub-blocks M_{ij}.

$\text{diag}[Q_{ji}]_{j=1...h,\, i=1...g_j}$ is the block diagonal matrix with diagonal entries Q_{ji}, where $j = 1, 2, ..., h;\; i = 1, 2, ..., g_j$.

$\|u_i\|_2$ and $\|u_i\|$ denote the ℓ_2-norm of the vector u_i.

$\|Q\|_2$ is the induced 2-norm of the matrix Q, which is defined as

$$\|Q\|_2 = \max\left\{\|Qv\|_2 : v \in \mathbb{R}^n, \|v\|_2 \leq 1\right\}.$$

$Q \preccurlyeq 0$ means Q is positive semi-definite i.e. $x^T Q x \leq 0 \; \forall x$. $Q \prec 0$ means Q is positive definite i.e. $x^T Q x < 0 \; \forall x \neq 0$.

A function $\gamma : \; \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$.

A function $\alpha : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{KL}$ if for each fixed $\tau \in \mathbb{R}^+$, $\alpha(., \tau) \in \mathcal{K}$ and for each fixed $s \in \mathbb{R}^+$, $\alpha(s, .)$ is decreasing and $\lim_{\tau \to \infty} \alpha(s, \tau) = 0$.

In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for terms that are induced by symmetry, e.g.,

$$(*) \begin{bmatrix} (*) + R & S \\ * & Q \end{bmatrix} K = K^T \begin{bmatrix} R^T + R & S \\ S^T & Q \end{bmatrix} K, \text{ or}$$

$$K^T \begin{bmatrix} R^T + (*) & * \\ S^T & Q \end{bmatrix} (*) = K^T \begin{bmatrix} R^T + R & S \\ S^T & Q \end{bmatrix} K.$$

$\lambda_{\min}(.)$ and $\lambda_{\max}(.)$ indicate the minimum and maximum eigenvalues of the argument, respectively.

The function $\mathsf{sgn}(.)$ is defined by

$$\mathsf{sgn}(\xi) = \begin{cases} 1, \text{ if } \xi > 0, \\ 0, \text{ if } \xi = 0, \\ -1, \text{ if } \xi < 0. \end{cases}$$

In a proof, when the time index k is omitted for conciseness, $v(-\tau)$ denotes the vector $v(k - \tau)$.

$\hat{u}$ denotes a sequence of predictive vectors of $u(j)$ starting from the current time step. $\breve{u}$ denotes a sequence of $u(-j)$ representing historical data of u.

The ℓ^{th} element of a vector $u_i(k)$ is denoted as $u_i^{(\ell)}(k)$.

In the discrete-time domain, the time index is denoted by $k, k \in \mathbb{Z}$. For signals belonging to $\mathcal{L}_2$ space, $k \geq 0$.

The boldface style for letters is used in optimisation formulations to emphasise that they are variables.

Abbreviation

APRC - Asymptotically Positive Realness Constraint.

ASPRC - Asymptotically Surely Positive Realness Constraint.

DeMPC - Decentralised Model Predictive Control.

DMPC - Distributed Model Predictive Control.

DRD - Delay-Robust Dissipativity.

IQC - Integral Quadratic Constraint.

LMI - Linear Matrix Inequality.

MATI - Maximum Allowable Transmission Interval.

MPC - Model Predictive Control.

NMPC - Nonlinear Model Predictive Control.

NCS - Networked Control System.

ODE - Ordinary Differential Equation.

PRC - Positive Real (or Realness) Constraint.

QP - Quadratic Programming.

SDP - Semi-Definite Programming.

Chapter 1

Introduction

1.1 Background

Automatic and semi-automatic control of large-scale interconnected systems, also known as network systems [107, 9], remains a challenging problem due to the increasing interactions between multivariable subsystems with mixed structures connected in parallel, serial and cycle configurations. In a broad sense, a network system can be modeled as a large set of interconnected nodes, in which a node is a fundamental unit with specific contents [82], linking to each other in an arbitrary connection topology. The network system notion is extended over many application domains such as, but is not limited to, chemical and mineral processes [128], network robotics [37], power networks [106], infrastructure systems [32], and supply chain management [169].

Figure 1.1 depicts a conceptual block diagram of such network systems with mixed

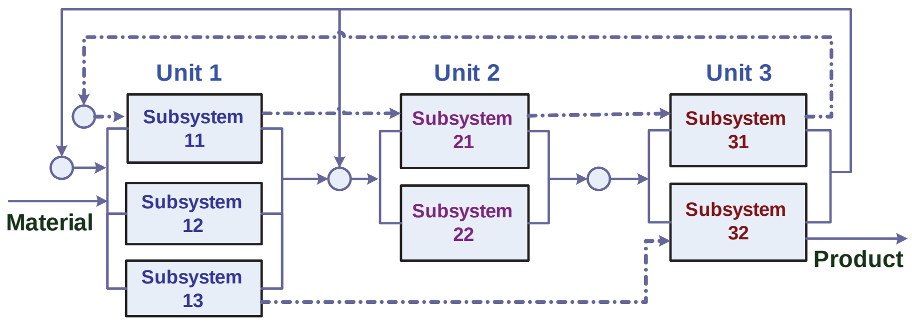


Figure 1.1 : Mixed Connection Topology of a Network System.

parallel and serial connections. Feedback control of large-scale systems is a classical topic in the control theory; see, e.g., [87, 144]. And the coordination methods for the control of interconnected systems and various decomposition techniques have been popular since the 70's [146]. It is not, however, trivial to apply those approaches directly to the modern control designs with mathematical programming methods and flexible features of on-the-fly adjustments in a real-time computing environment. While the control theory for non-interactive systems is mature in its own right, the development for the distributed and decentralised model predictive control of large-scale interconnected systems is still in its infancy, as indicated by, e.g., Rawlings et al. in [128]. In this Thesis, the communication between subsystems is not employed in a decentralised approach. Details on definitions of distributed and decentralised control methods are given in [125].

The stability guaranteed problem of MPC and systems has long been a research topic that draws the interests from both academia and industry [119, 94, 4, 73, 89, 134, 124, 18, 79, 3, 90, 127]. The most favorite approaches for guaranteeing the closed-loop stability are based on the terminal constraint sets [25] and contractive state regions [72]. Those approaches adopt the Lyapunov-based stability criterion. A thorough review on the stability and optimality of different MPC formulations can be found in a renowned paper by Mayne et al. [94]. These methods, with the emphasis on the optimality, may become conservative due to the resultant small region of attraction. They may also provide conservative feasible regions in distributed applications where the terminal regions are asynchronously overlapped [20].

The dissipative systems theory is attractive to the control design problems for decentralised architectures, wherein the interactions between coupling subsystems are to be orchestrated to attain the global stabilisation and performance, owing to its extensive employment of input and output properties. It is well known that, combining subsystems whose energy dissipating rate (abstracted energy only) is greater than the energy yield will results in dissipative systems [101], which under mild conditions, are stable systems. The mathematical theory of dissipative systems [164] has been applying to the stability analysis and control design of linear and nonlinear systems with robustness to disturbances and model uncertainties since the 70's; see, e.g., [160, 16, 54]. Within this framework, the supplied energy is measured by an inner product of input and output signals ($u(t)$ and $y(t)$, respectively). A subsystem is then said to be passive if it is dissipative with respect to the supply rate of $y^T(t)u(t)$.

Inspired by the (Q,S,R)-dissipativity in [164, 57, 139], the quadratic supply rate has been extensively employed in the developments for stabilisability conditions in this Thesis. We focused our works on the open-loop approach, as we believe it is suitable to the decentralised open-loop model predictive control schemes. Under this open-loop realm, the quadratic supply rates are developed into quadratic constraints on the initial control vectors for the model predictive controller. The multiplier matrices Q, S and R are dynamic (online updated) for guaranteeing the constrained and recursive feasibility.

The dissipation inequalities have been successfully employed in the robust model predictive control schemes for nonlinear systems [77, 90, 118, 78] with closed-loop approaches. To this end, the asymptotic gain with respect to input disturbances and outputs or states is mainly discussed for robust stabilisations, but not for the control constructive method. Therefore, the quadratic supply rate (with the passivity element, $y^T u$) has not been employed in the development which normally a closed-loop formulation with Lyapunov functions. The quadratic supply rate has been employed in a related framework of IQC (integral quadratic constraint), which proved effective in robustness analysis and robust control synthesis [96, 71]. The IQC-based control synthesis method has been, nonetheless, essentially introduced and developed in the frequency domain with recent developments presented in [140]. For deriving the control constraints in an open-loop approach, such as those in this Thesis, it is necessary to extend the supply rate to include also the passivity component $y^T u$. In the dissipativity-based output feedback control synthesizes, the quadratic supply rates have always been used in the developments [138, 143, 31]. With the same spirit, the passivity-based method has been successfully deployed for the teleoperation applications [112] as a real-time constructive method. Inspired by all these existing developments, we have introduced the notions of Asymptotically Positive Realness Constraint (APRC) and its application to the stabilisation of interconnection systems associated with decentralised and distributed model predictive control strategies. The basic APRC inequality, which is a constraint on the supply rate $\xi_i(y_i(k), u_i(k))$ — $\xi_i(.,.)$ is a quadratic function (supply rate) with respect to the output and input pair of y_i and u_i, is initially expressed as a quadratic constraint in $u_i(k)$, when $y_i(k)$ and $\xi_i(k-1)$ are known ($y_i(k)$ is the output at the current time step k; $\xi_i(k-1)$ is a historical datum at step $(k-1)$. This APRC-based constraint is subsequently recast and incorporated into the online optimisation problem of MPC for $\boldsymbol{u}_i(k)$. This open-loop approach is nonconservative to the decentralised control problems by virtue of its non-monotonic state norm profile (instead of the monotonic Lyapunov function in closed-loop approaches), which also implies the convergence of the state vector towards the zero equilibrium point. By adopting the robust nonlinear control theory in the developments, the APRC is directly extendible to C^1 continuous nonlinear input-affine interconnected systems [152].

1.2 Thesis Objectives

Methodologically, online centric stabilisations of linear interconnected systems and their stabilisability with decentralised and distributed model predictive controllers will be the main focus of this research. Unstable and non-minimum phase subsystems are accommodated for within the scope of this project. Centralised, decentralised and distributed architectures are three alternatives for the control design of large-scale interconnected systems. In distributed and decentralised control schemes, the multivariable controllers are installed on a network of computer systems consisting of several independent platforms or nodes. Each platform uses sensors to gather measurement data, and industrial actuators to manipulate field devices, such as control valves. Depending on the implemented control strategy, the system computational capacity and the reliability requirements, the amount of data exchanged between the controllers via the inter-communication links of the computerised control system will vary to fulfill the demands.

From the theoretical perspective, the work in this Thesis project applies the fundamental results of control theory and numerical optimisations to the problems of predictive control of interconnected systems. By virtue of the real-time nature of an MPC problem, convex programming numerical methods with their elegant solutions [14, 109] will be considered as the right choice for this research. Linear matrix inequalities (LMIs) [7] are comprehensively employed in the derivations of dissipativity criteria and stabilisability conditions for interconnected systems in this Thesis. In addition to the MPC control performance and constrained stabilisability requirements, the solutions to recursive feasibility and procedural developments will be another facets of this research project.

1.3 Thesis Contributions

The LMI-based stabilisability conditions for interconnected systems in the discrete-time domain have been derived in this Thesis based on the newly introduced asymptotically positive realness constraint (APRC). From the control theoretical perspective, this work introduces the employment of quadratic supply rates, popularly used in the dissipative systems theory and integral quadratic constrain (IQC), in the stabilisability conditions and their application to the online stabilisation of constrained interconnected systems.

Two constructive approaches of stabilising agent and MPC stabilising constraint have been developed for implementations. They are designated as segregation and integration methods, respectively. The distributed and decentralised control strategies are cutting-edge by employing only decoupled control vectors and precluding artificial constraints on the coupling variables. The former improves the system reliability. The latter mitigates any conservativeness of limiting the energy or material flows between dynamically-coupled subsystems.

A continuous and decentralised control strategy is proposed for parallel redundant systems as an alternative to the hybrid approaches (with Boolean variables in the system models), which effectively stabilise the plant-wide process in all operational scenarios, including concurrently parallel and duty-standby modes. The special requirement in parallelism comes from the splitting ratios of parallel subsystems, which are unknown. Albeit being able to treat the parallel splitting signals with a robust control perspective, it is preferable to maintain the splitting connection structures for linear and nonlinear systems in a decentralised control scheme.

The APRC-based stabilisability conditions and their extensions have been derived for linear systems. Centralised and non-conservative decentralised control strategies are facilitated for by the developed stabilisability conditions. The summarised contributions are provided in the next subsections.

1.3.1 Asymptotically Positive Realness Constraint (APRC) and Novel Stabilisability Conditions - An Open-Loop Paradigm

Contribution: *A novel stabilisability condition for constrained interconnected systems in the discrete-time domain, inspired by the* $(Q, S, R)-$*dissipativity* [164, 57, 138, 143] *and the integral quadratic constraint* (IQC) [96].

It is well known that if a system is quadratically dissipative with respect to its inputs and outputs then the positive real constraint is sufficient for its stabilisability (see, e.g., [16, 17]). It is, however, difficult to employ these conditions in the constrained control applications, due to its lack of correlations between the recursive control states. In other words, the fundamental result of positive real constraint and passivity is not always realisable, due to the fact that an infinitely power control may be required. It is natural that, if a passive system does not satisfy the positive real constraint, one cannot make it bounded immediately because the control is also power constrained. Therefore, an intermediary steps that manipulate the system towards a stabilising state is more realistic and desirable. And this is where the APRC concept come to exist for use with constrained control problems.

It is noted here that the APRC is fundamentally different to an another approach called incremental quadratic constraint introduced in [2], or to the incremental passivity approaches presented in [34, 136, 44, 114], which imply the usages of signal increments Δu_k and Δy_k in the supply rate $\xi\big(\Delta u_k, \Delta y_k\big)$. In this thesis, the APRC deals with $\Delta\,\xi\big(u_k, y_k\big) := \xi\big(u_k, y_k\big) - \gamma\,\xi\big(u_{k-1}, y_{k-1}\big)$ and the likes.

1.3.2 Parallel Connections with Unknown Splitting Ratios - A Fully Decentralised Approach

Contribution: *Parallel splitting subsystems with unknown splitting ratios in a complex interconnected system are introduced. A decentralised MPC strategy for interconnected systems applying the APRC as decoupled stability constraints is developed.*

A decentralised control strategy is proposed for parallel redundant systems as an alternative to hybrid approaches, which effectively stabilise the plant-wide process in all operational scenarios, including concurrent and duty-standby modes. The parallel connection of parallel redundancies is universal in process system designs for increasing the reliability, availability and business continuity. In parallel redundant operations both duty and standby subsystems are designed to run concurrently to share a common production rate. These parallel configurations, which may destroy the guaranteed-stability property of an existing strategy, are explicitly addressed in our developed interconnection stabilisability conditions. The special requirement in parallelism comes from the splitting ratios of parallel subsystems, which are unknown. The control of complex interconnection structures which are involved with both serial and parallel configurations have been designed using different hybrid approaches. Whilst being effective with duty-standby operations, wherein a standby subsystem is switched on or off, the hybrid solution cannot be adopted for parallel operations. Furthermore, a centralised controller for parallel connections systems will not be viable by virtue of the reliability and maintainability requirements. Decentralised controllers are always preferable and desirable. A continuous control strategy must therefore be employed instead of a hybrid strategy. In a continuous control strategy, the changing-overs between duty and standby parallel subsystems are assumed being smooth, i.e. state jumps will not incur due to the changing-over operations.

The mixed connection configuration is dealt with by explicitly incorporating the two types of connections, serial and parallel, into the state space models. The input and output attributes are then capitalised to derive two new interconnection stabilisability conditions in the time domain. In this chapter, the issue of unknown splitting ratios in parallel subsystems is tackled by having a combined solution, wherein the quadratic constraint is developed for each individual subsystem while the dissipativity criteria are derived for the whole bulk of subsystems in parallel. Albeit being able to treat the parallel splitting signals with a robust control perspective, it is preferable to maintain the splitting connection structures for linear and nonlinear systems in a decentralised control scheme. Thanks to this parallel masking approach, the framework is extendible directly to decentralised control of C^1 continuous nonlinear input-affine systems using the parameterised dissipativity criteria.

1.3.3 Stabilising Constraints for Distributed and Decentralised MPCs - Integrated Constructive Method

Contribution: *APRC-based stabilising constraints for model predictive controllers.*

A progressing approach to the constructive method for stabilising controllers based on the asymptotically positive realness constraint is introduced and applied to the distributed and decentralised MPC problems. The formulation of MPC is developed in conjunction with the APRC-based quadratic stabilising constraints, which can be solved by any convex quadratic programming (QP) numerical methods. The stabilisability and feasibility conditions are rendered by the linear matrix inequalities (LMI). Parts or all of the multiplier matrices of quadratic stability constraints are redesigned in this approach to guarantee the constraint and recursive feasibility of MPC optimisation problems.

Receding-Horizon Stabilising Constraints

The APRC is developed into a *receding-horizon stabilising constraint* for the model predictive controller. The APRC-based receding-horizon stability constraint is an accumulative terminal constraint. This receding-horizon stabilising constraint is a structured constraint on the entire predictive control vector. Since it is formulated with state predictions towards the end of the predictive horizon, the control trajectory relaxes the APRC at the current time step. This is much more flexible and easier to solve than the one-time-step constraint on the initial control vector as described above.

This approach has been critisised as lacking of robustness to large model uncertainties. The predictive horizon will, therefore, need to be gradually decreased back to one step to achieve the system stability, yet being able to eliminate the conservativeness of an immediate one-time-step stabilising constraint. The term receding-horizon stabilising constraint appropriately describes the developed method.

1.3.4 Stabilising Agents for Semi-Automatic Control Systems - Segregated Constructive Method

Contribution: *A stabilising strategy for the semi-automatic control of interconnected systems, inspired by the segregation principle of dependable system designs* [141].

The method of stabilising interconnected systems with external stabilising agents stems from the idea of segregating the stabilising mechanism apart from the control algorithm. This strategy is inspired by the segregation principle in the design of dependable systems [141] and critical control systems. The control algorithm will initiate an immediate overriding action to steer the system into a stabilisable trajectory upon any deviations from its evaluated APRC-based region. The physical isolation of stabilising agents from the automatic controllers ensures that the overall system is stabilisable despite of operational modes.

The International Electrotechnical Commission IEC-61508/61511 safety standard for process systems and guideline to engineer safety instrumented systems (SIS) for the process industry sector recommends the segregation between the process control system (such as the distributed control system - DCS) and the SIS (or safety shutdown/protection systems). The SIS will automatically initiate an immediate safety shutdown action to put the system into a safe state when one or more process variables have been steered into an unsafe region or out of the allowable limits. The physical segregation with the DCS ensures that the SIS achieves the assigned quantitative reliability and safety integrity level. The DCS design and implementation should have been such that to minimize the escalation of unsafe process upsets into a plant-wide shutdown that may disrupt the operation. Inexperienced operators, human errors, disturbances, and controllers whose performance degrades over time without regular adjustments, may, however, cause unexpected system instability or constraint violations or both. An automatic tool that prevents these problems from occurring will improve safety, efficiency and business continuity. An independent stability assurance measure that continuously monitors the stabilisability of control systems, as well as limits their trajectories within a stabilisable boundary will be desirable for this purpose. Such measure and tool have not been reported in the literature of control theory and engineering to date.

1.3.5 Semi-Automatic Control Method for Networked-Control-System

Contribution: *The constructive method of stabilising agents is extended to systems having intermittent data losses on local device networks. Effective compensations for the intermittent losses of output data are commensurate on-the-fly right after every single interrupted incident.*

The stability and performance of systems operating in an imperfect shared communication network for the control applications has been attracting a great deal of research since the 90's. In such a non-deterministic networking environment, the output measurements or the control commands, or both, are subject to occasional delays and random data losses due to external interferences and limited bandwidths which may cause congestions and transmission errors. A new constrained stabilisation strategy for modular systems in semi-automatic control modes that suffer from sporadic data losses of device networks (e.g. wireless mesh networks) has been developed with stabilising agents. The APRC-based quadratic constraint is employed in the algorithm of stabilising agents to derive the stabilisable bounds for each subsystem at every successive time steps of the NCS. The intermittent data losses of output measurements are effectively compensated for right after each dropout occurrence. The last values of control vectors are used during these disruptive periods. The APRC is characterised such that the stabilising bounds are assuredly feasible in the presences of data losses. In this development, the data losses are assumed being identical to all variables by hardware and firmware configurations.

1.3.6 Decentralised MPC for Coupling-Delayed Interconnected Systems with State and Input Constraints

Contribution: *The stabilisation problem of constrained interconnected systems having a coupling delay element is solved by the developed decentralised MPC approach.*

A decentralised model predictive control strategy for interconnected systems that suffer from an arbitrary coupling delay has been developed employing the APRC ($\tau-$APRC). Sectoral constraints are introduced and applied to the coupling delay problem. The asymptotic property of APRC eliminates any conservativeness of constraints on the delay operator. Global state constraints are dealt with by the reachable set approach. A single random coupling delay is considered in this development, i.e. the coupling delays between network nodes are assumed identical. The storage function of the Lyapunov-Krasovskii functional form (in discrete time) is deployed for the delay-dependent dissipativity criteria. The interconnection stabilisability conditions are subsequently derived in association with the $\tau-$APRC.

1.4 Significance and Innovation

According to the literature survey, the interfaces with academia around the world and extensive practical experience of the candidate, especially with Honeywell Automation and Control, the research is up-to-date with the current research themes, and future trends in the control theory and engineering field as well as in industrial best practices. It is a research for enabling technologies that reply to the future demands from various arising real-life applications from different industries, such as the control for energy saving of power plants, minimising greenhouse gas emission while maximising yields in manufacturing industries, optimising dynamic operations, coordinating energy usages in automotive systems, integrating renewable energy into power networks, swarming unmanned vehicles, automating space rendezvous in space exploration, and last but not least, the control of biomedical and health systems. These listed applications are extracted from the literature survey for distributed MPC. In this thesis, a novel real-time control framework based on the mathematical programming methods that are conceivably adopted to the stabilisation of large-scale systems is formed and developed with innovative implementation strategies. The framework covers different types of systems, including Hurwitz stable, non-minimum phase and unstable linear systems, as well as directly extendible to Lipschitz continuous nonlinear input-affine systems.

Achieving the system stability with decentralised MPC is a challenging task if the interactions between subsystems are not negligible. The well-known methods of terminal constraint sets and control invariant sets for open-loop and closed-loop MPC may face major difficulties [20, 37, 21, 105, 161, 128] for such strongly coupled systems. An alternative solution for stabilising the large-scale system with a decentralised architecture, that is compatible with, and suitable for, the complexity of interconnections comes alive as a result of this thesis. It opens a new direction for effectively stabilising constrained systems with decentralised and distributed MPCs in various arising applications as mentioned herein. In other words, this thesis lays a new foundation for the decentralised stabilisation of constrained network systems employing convex quadratic constraints.

1.5 Outline of the Thesis

This thesis is organised as follows:

- *Chapter 1*: This chapter is the introduction to Thesis. The contributions are summarised in this first chapter.

- *Chapter 2*: A survey on distributed model predictive control constitutes the main content of this chapter.

- *Chapter 3*: The novel notion of asymptotically positive realness constraint (APRC) is introduced and applied for deriving the interconnection stabilisability condition in the time domain. The APRC prescribes an approaching characteristic towards the stabilisable regions of the dissipative system under control, albeit not leading to a positive real property. The multipliers of the APRC quadratic supply rate are updated online to ensure that the constraint satisfaction is recursively feasible.

  A constructive method of quadratic constraints for decentralised model predictive controllers (MPC) based on the APRC is introduced in this chapter. These constraints are additionally imposed on the constrained optimisation problem of each local MPC. The recursive feasibility is guaranteed by characterising the APRC-based stability constraints with dynamic multiplier matrices. By virtue of adjustable asymptotic attributes, the APRC-based stability constraint is able to smooth out wind-up control actions.

  Handling parallel redundant subsystems that have unknown splitting ratios with the parallel masking technique is another essential part of this development.

- *Chapter 4*: This chapter presents two extended implementation methods for applying APRC to the control of interconnected systems. In the first extended implementation method, a new control design strategy for stabilising the large-scale interconnected systems operating in the semi-automatic control mode is introduced. The APRC is employed in the so-called stabilising agent to accommodate the coexistence of feedback and human-in-the-loop modes in semi-automatic control systems. The stabilising agents are developed independently from the control law under the same auspice controller. Due to this independence, operational errors from the manual control adjustments, that may destabilise the control systems, is avoidable. The decentralised agents render stabilising bounds for the manipulated variables in the automatic control mode, and at the same time, provide warning signals and manipulation guidance for the operators to prevent possible plant-wide destabilisation in the manual control mode.

The development of APRC-based receding-horizon stabilising constraint for use with model predictive controllers is presented as the second extended implementation method in this chapter. The APRC-based receding-horizon stability constraint is an asymptotic terminal constraint on the control vector. A constraint on the receding vector within the predictive control sequence, starting from the last vector, is derived from the receding-horizon stability constraint and imposed on the local MPC optimisation. The simulation results have shown the effectiveness of the APRC-based receding-horizon stability constraint for decentralised and distributed MPCs.

- *Chapter 5*: This chapter presents a control procedure for dynamically-coupled systems operating in the imperfect data environment of a mesh device network. Intermittent data losses are compensated for on-the-fly using the APRC. Taking the control constraint into consideration, the feasibility of APRC-based stabilising bounds is guaranteed for the consecutive data-lost periods of device networks. It is, therefore, called data-lost robust APRC (d-APRC). The innovative aspect of the proposed approach rests on the method of remedying data losses right after the incidents.

The development for multiple communication topologies of the interconnected system formed by subsystems is delineated in another part of this chapter. The intermittent data losses of local measurements and states also occur to each single subsystem. Herein, demands on the communication links and online data are limited with a virtual perturbed state feedback (PSF) strategy which consists of the static state-feedbacks and a perturbation ($u_i = w_i + \sum_j K_j x_j$). The static state-feedback gains (K_j) are determined in accordance with the communication network topologies and for constraint satisfactions, while the perturbations (w_i) are calculated by the online optimisation. Only a virtual PSF form is considered herein. It is not applied directly to the control calculations in real time, but only used to shape the quadratic stability constraint with respect to u_i for the online calculations which determine the control u_i directly.

- *Chapter 6*: The coupling delays of interconnected systems are incorporated into the problem formulation in this chapter. Multiple bounded delays are considered in the problem formulations. Two developments are presented in this chapter. The notion of delay-robust dissipativity is introduced and employed in the APRC-based stabilisability condition. The notions of accumulatively quadratic dissipativity and the sectorial quadratic constraint based on APRC are introduced and applied to the derivation for another stabilisability condition.

- *Chapter 7 - Conclusion*: A brief summary of the thesis contents and its contributions are given in this last chapter. Details of the novelty, contributions and outcomes of this thesis have already been outlined in this first chapter. Possible future works and extensions are discussed and recommended therein.

Chapter 2

Model Predictive Control - A Survey

2.1 General

Model Predictive Control (MPC), also known as moving or receding horizon control, has been originated in the refining industry as a practical control algorithm for multivariable systems that have constraints and time delays [18]. It has received much attention and is receiving growing interests for applications in the process control and many other fields [124, 73, 90]. The applications of MPC in process industries predominant the field of advanced process control due to their unique proven-in-use advantages. In the last decade, they have shown a tremendous growth. If only around 300 systems had been recorded at the beginning of the 1990's [80], more than 2,200 installed systems were found a few years later [122], and the number keeps increasing (e.g. at approximately 18% annual rate [33]) concurrently with the enhancements over nonlinearities which are typical in many process industries [73]. More than 4,000 systems have been implemented as of 2004 [19] and the number has exceeded 10,000 in 2007 [129]. Another surveys on the control techniques for industrial applications in Japan [150, 62] have provided similar trends.

Several industrial MPC packages from different vendors have been successfully implemented in the oil refinering industry [13], where the linear models (e.g., for separation processes with distillation columns) are sufficiently good for the early versions of MPCs [73]. For linear systems having unmeasured disturbances, a simple Kalman

Filter [61] is suitable for these MPCs [103, 170]. Such advantages had not been, however, convincing to the plant-wide control community [88] initially. MPCs have not proved to be effective in the Tennessee Eastman challenge in 1996 [132]. Nevertheless, with the widespread use of MPCs as shown above, and more commercialised tools supporting the implementations become available, the perception is gradually changing [129]. Once the techniques are further improved to become beneficial to different applications and industries, especially for processes characterised with severe nonlinearities, we may see MPC installations in almost all industries in a foreseeable future.

How different levels of nonlinearities are addressed by a common process model type, and how the optimisation algorithms will adopt such nonlinear models, yet produce stablising and optimising performance in real time, are the two impediments that need appropriate answers [73] when deploying nonlinear models in MPC schemes. Developing adequate nonlinear empirical models is not always trivial as there are not any universal models representing different nonlinear processes [42, 123, 4]. If the step and impulse responses or low order transfer functions are comfortably obtained for linear systems, they pose substantial challenges to nonlinear systems due to lacking of the superposition principle. As a result, the identification of nonlinear models from the input and output data becomes a daunting task, as the step tests for nonlinear systems demands a higher amount of data than that for linear systems to span the operational range [19]. First-principle and grey-box models (a combination of first-principle model and empirical model) are thus a promising candidate, yet to be exploited, for severely nonlinear systems [19, 115].

In terms of optimising performances, MPC for linear systems has proved an effective solution in industrial applications [18, 124]. The linear MPC problem induces a sequence of convex optimisation problems which have been well studied; see, e.g. [167]. For nonlinear processes, a linear control law is not normally effective, so nonlinear programming methods will be essential for improving the control performance. The MPC strategies that employ nonlinear models and nonlinear numerical methods are designated as NMPC. As the computation cost is higher, NMPC does not, however, guarantee an optimising solution within a prescribed time period for fast responding systems. As a result of research on NMPC the last decade, the new generation of NMPCs that works with all types of Lipschitz continuous nonlinear systems have been developed successfully [4, 90]. The explicit MPC paradigm which combines the offline computation of multi-parametric programming and the efficient online calculations is now available [11]. MPC strategies with mix-integer programming have also been developed for hybrid systems [10], whereby multiple models and digital logics are employed to encapsulate the lumped time-varying dynamics of nonlinear processes. Robust NMPC [74] with the remarkable tube-based approach [127] have been attracting much research lately. Effective solutions for assuring its recursive feasibility have been proposed in recent publications; see, e.g., [23]. Similar progress for stochastic NMPC [24] has also been envisaged. Parameterised approaches have appeared as the current research interests in this field; see, e.g., [126, 78, 53].

2.2 Industrial Model Predictive Control

If three-term PID control techniques traditionally dominate the process control industry, it is the Model (Based) Predictive Control (MPC/MBPC) that represents the advanced control techniques for the process industry. It should be noted here that, only the process control industry is mentioned here. While the applications of artificial intelligence in control engineering such as neural control and fuzzy control show some advantageous features in the robotic industries, they are not always applicable in the industrial process control, except for the system identification purpose. This is partly reasoned by the fact that the computational capacity and data storage of the industrial computerised systems for a large-scale plant with several hundred PID controllers are limited [47].

The use of process models appeared early in Coales and Noton's pioneering approach of on-line identifications in 1956 [29], and in an internal model controller introduced by Smith in 1957 [147]. But Richalet is actually the first to introduce MPC to industry with the receding (predictive) horizon idea in 1972 [130]. The technique has evolved a long way since. Almost all process control vendors nowadays offer MPC to their customers, either as a functional block for unit operation in a distributed control system (DCS), or as a separate software package for plant-wide control. It is notably that DCS is a specific name designated to the proprietary computer systems tailored made for large industrial plants, similarly to PLC (programmable logic controller) for discrete logic control applications. Predictive control, as a matter of fact, assimilates basic human activities [79]. A particular human action is usually undertaken based on assumptions and expectations about the future outcome. Model predictive control uses the past information of states and the process model to predict future control actions. The applications of MPC techniques are apparently more popular in refineries and petrochemical plants than in other industries, because the linearised system models are adequate for processes in those industries. The typical installations of MPC in industry are hierarchical. The MPCs provide set points to the local controllers with traditional PI/PID controllers finely tuned. If a predictive controller starts to mislead the system behavior, the operator simply isolates it and lets the local loops hold the plant using the last received set points [89]. The process plant should be stable in this condition, so in the worst case it is safe to run the plant as it is without predictive control layer.

It is interesting to refer back to the debate for optimal control and optimisation necessary to engineers in the 1960s [86]. While it was found to be very successful in the aerospace industry (there were more than 300 references in a survey about theory and practice of optimal control at that time, as quoted in [86]), the optimal control techniques initially failed to provide a convincing solution to industrial process industries due to inherent constraints, model uncertainty, unmeasured disturbance, as well as computer capability at that time [81]. In the industrial process control field, it had acquired a reputation for being out of touch with reality for quite a long time as indicated by Leigh in [79]. Fortunately, there has been a resurgence of the method that widespread in industry due to strategic changes. The new strategy is as follows: "... for large complex problems, it may be better to encode optimality criteria in more vague but more realistic terms, than to force unwilling problems into an ill-fitting straitjacket to allow rigorous optimisation" [79]. MPC is among successful stories using this strategy, turning optimal control theory into a success story.

There are many good references on industrial MPC. The original work on DMC (Dynamic Matrix Control) is referenced to in [30, 119]. The original IDCOM (Identification and Command) is referenced to in [130]. Other well known commercialised packages such as QDMC Plus (Quadratic Dynamic Matrix Control)[46], RMPCT (Robust Model Predictive Control Technology), Connoisseur^{TM}, PFC (Predictive Functional Control), HIECON (Hierarchical Constraint Control), OPC (Optimum Predictive Control), NOVA-NLC predominant the industrial MPC and NMPC market. NMPC is an active field of research in the advanced process control area. The aim is to expand the applications of MPC to various process industries, where nonlinear systems are dominant. In parallel with that, rigorous control theory is being further developed to provide inherently stable and robust NMPC for harsh environments.

For NMPC, the Sequential Quadratic Programming (SQP) [109] is the well-known chosen numerical method, wherein the objective function is quadratically approximated and the nonlinear constraints are linearised before solving at each iteration step. They are the extension of Newton-type methods for converging to the solution of the Karush-Kuhn-Tucker (KKT) conditions of the constrained optimisation. The solutions do not, however, guarantee the system stability. To this end, the analytical methods that reveal the closed-loop stability based on the knowledge of plant models, objective functions and horizon lengths are not always transparent [4]. The applications also require that the system stability should be obtained regardless of the choices of horizon lengths and weighting coefficients in the cost function. In this field, the stability of the closed-loop system is usually achievable by adding a suitable equality or inequality terminal constraint to the setup. The additional constraints do not come from physical restrictions or control performance requirements but have the sole purpose of enforcing stability. They are termed stability constraints. The nominal stability of infinite horizon problems is described in [95]. The formal proof for the closed-loop stability for the receding-horizon control (finite horizon) of Lipschitz continuous nonlinear systems using the equality terminal constraints was provided by Keerthi and Gilbert in 1988 [64]. Adding the terminal constraints may, nonetheless, cause extra computation cost or even infeasibility to the optimisation algorithm. Moreover, if the chosen predictive horizon is short, the problem may not have any solutions. The region of attraction corresponding to such terminal constraints is normally conservative [4]. The mature results on the stabilisation problem of NMPC consist of those employing the inequality terminal state constraints. The most imminent approach of inequality constraints is dual-mode control [93, 99, 25], which defines a neighborhood around the desired steady state, within which the system can be steered to, by a linear state feedback controller. If the current state lies outside this region, then the NMPC algorithm is solved with the above inequality constraint. Once inside the predetermined region, the control switches to the previously computed linear state feedback control law. Discrete time versions of dual-mode NMPC with fixed horizons are developed in [142]. Extending the dual-mode approach, the finite horizon cost function is approximated by the infinite horizon in the quasi-infinite horizon approach [25]. And the control action is determined by solving a finite horizon problem without switching to the linear controller even inside the terminal region. Several research papers have proposed methods on how to enlarge the region of attraction for NMPC algorithms since then. The other approaches for guaranteeing stability are the contractive state MPC [72], the Control Lyapunov Function (CLF) MPC [120, 98] and the converse Lyapunov-based MPC [38]. The perturbed state-feedback [49] with the control invariant set approach [70, 12] has proved to be effective for robust NMPCs and attracted much research during the last decade [27, 65, 22, 50, 111, 59]. The tube-based approach is becoming more and more widespread within the MPC research community lately [127]. The stabilising methods presented in the above solutions has been achieved based on the availability of state measurements at every recursive step. For output feedbacks, the most common approach is to estimate states using an observer. The closed-loop stability is not guaranteed, nonetheless, even when both the state based NMPC and the nonlinear observer are stable. Findeisen et al. [42] shown that an additional conditions must be considered to guarantee the stability for the augmented system. The stabilisation of discrete-time output feedback NMPCs is addressed by Elaiw, 2010 in [39]. The early NMPC software packages available to the industrial users consist of NOVA-NLC of Pavilion Technologies (using first principles models), Process Perfecter (using input-output Hammerstein models) and some other partial solutions for nonlinear systems such as Aspen Target or Invensys Connoisseur using artificial intelligent techniques. There are two types of state constraints, hard and soft, in these industrial MPCs. Soft constraints allow temporary and short-term violations of respective state bounds. Hard constraints imply strict containments of state trajectories within given bounds. The soft constraints in Honeywell RMPCT are achieved by using a 'frustum' method that permits a larger control error in the beginning of the horizon than in the end.

2.3 Stabilising Methods for Distributed and Decentralised Model Predictive Control

The MPC strategies are prevalently used in the process control field for the multivariable control problems owing to their natural ability to address the input and state constraints. Handling constraints with optimisations is a much better method to avoid wind-up actions than other high-gain compensation techniques [124]. A thorough review on the stability and optimality of different MPC formulations for linear and nonlinear systems can be found in a renowned paper by Mayne et al. in 2000 [94] (2nd most cited paper from Automatica). The terminal constraint set and contractive state beside other methodologies are mostly employed in the guaranteed-stability MPC schemes which exercise open-loop optimisations. So are the control invariant set approaches for state-feedback setups [27]. These approaches adopt the Lyapunov stability criterion in their algorithms wherein the value function treated as a Lyapunov function monotonically decreases over time to achieve the stability.

Having been recently extended to large-scale and interconnected systems in various distributed and decentralised MPC schemes as indicated in [125], the field of distributed MPC is still in its infancy [128]. Distributed model predictive control (DMPC) has started drawing interests since Acar [1] discussed the importance of coupling constraints in a decentralised MPC scheme. Various strategies for DMPC and decentralised MPC, a non-communication version of DMPC, have been proposed for both dynamically-coupled and -decoupled systems, see, e.g., [20, 37, 105, 161]. It is noteworthy that the notions of decentralised and distributed controllers used here are referenced to the paper of Scattolini, 2009 [125]. In a decentralised control scheme, there is not any communication link between subsystems among the population, otherwise it is a distributed scheme.

For distributed model predictive control of dynamically-coupled systems, the solutions to the closed-loop stability problem usually require an uninterrupted communication between subsystems, or to impose an additional constraint on the coupling vectors and/or the objective function. The former requirement may lead to an unreliable system that cannot be stabilised due to the losses of communication channels. The latter may deliver a conservative control performance due to the constraint problem of MPC.

Acar, 1995 [1] discussed the importance of coupling constraints in a decentralised MPC scheme. In that work, a dynamically decentralised state-feedback approach for linear systems has been employed under a condition of weak interactions between coupled systems defined by the so-called ε-coupling constraint. The closed-loop stability is ensured if the systems are weakly ε-coupled.

Decentralised MPCs which are the non-communication versions of DMPC had been initiated previously by scholars from the field of decentralised control of large-scale systems [144] appeared in the works of, e.g., Xu, Xi and Zhang, 1988 [168] and Tzafestas, Kyriannakis and Kapsiotis, 1997 [159]. These decentralised MPCs, which applied the equality terminal constraints, are limited to weakly-coupled systems.

Distributed GPC (Generalised Predictive Control) [28] with state space models using decentralised Kalman filters has been addressed in a work of Katebi and Johnson, 1997 [63], whereby a two-level optimisation algorithm applying distributed optimisation techniques from the mathematical programming literature was introduced and applied to achieve the global stability. The necessary condition for the optimality (on the Hamiltonian of the global system) is imposed to the upper-level optimisation problem as equality constraints and subsequently distributed to the decomposed optimisation problems in the lower level by a coordinator. The convergence is achievable accordingly.

A condition for converging to the Nash equilibrium for DMPC of linear systems fully based on system models has been derived by Li, Zhang, and Zhu, 2005 [83]. The inequality terminal condition associated with a convergent condition inferred from the necessary condition of Nash optimality is employed in this approach. The maintenance of nominal stability under communication failures in a single time step is discussed therein.

A DMPC algorithm based on the decentralised estimation and state feedback control (DDEC) framework [104] for linear systems has been proposed by Mercangöz and Doyle III, 2007 in [97]. The mechanism of exchanging state information between local systems in the original DDEC methodology is enhanced to include the communication of interactive MPC controllers. A DMPC network is formed in this approach based on the model decomposition structure.

A DMPC algorithm called coordination MPC that assimilates the real-time optimisation (RTO) scheme for multiple linear systems in a hierarchical architecture has been introduced by Cheng, Forbes, and Yip, 2008 in [26], and Aske, Strand, and Skogestad, 2008 in [8]. An LMI approach has been developed by Ding, Xie, and Cai, 2010 [35] in which the weighting matrices need to be synthesised, subject to the feasibility of the LMIs derived from a dissipation inequality taking one-step objective function as the chosen supply rate.

The magazine paper by Componogara, Jia, Kroh and Talukdar, 2002 [20] celebrating the new millennium was perhaps a turning point for DMPC of linear and nonlinear systems, after which important results have been presented in control conferences one after another. It was, in fact, two years after the formal review paper for MPC [94] had been published in 2000. The contractive state [72] approach is considered as a less difficult-to-use approach for DMPC in that paper.

The work of Dunbar and Murray [37] systematically applied the results from dual-mode [99] and quasi-infinite horizon [25] approaches to DMPC in association with a compatibility constraint that requires fast update period to each local controllers. The upper bounds on the update period are determined recursively to assure the convergence of closed-loop cost functions, which are proportional to the size of terminal regions. The nature of robot coordination problems ultimately requires distributed terminal controllers in this work.

In a similar approach of determining terminal regions using distributed terminal controllers, Keviczky, Borrelli, and Balas, 2006 [67] proposed to predict the neighbour data locally, and to exchange the measurement data recursively for the prediction purposes. A constraint on the predictive discrepancies between neighbours are sufficiently imposed on the objective functions for achieving the global closed-loop stability. Based on simulation results the authors summarised that the prediction discrepancies tend to vanish near the equilibrium due to the fast decays of prediction mismatches. The couplings between neighbours in this work are defined as inequality constraints for the local optimisation problems.

Richards and How, 2007 [131] have proposed to apply a sequential iteration scheme to their DMPC algorithms for the output coupling systems (consensus problem) in a robotic lead-follower formation control strategy. A guaranteed feasibility DMPC employing the tightening constraint sets [27] to calculate the dynamic state feedback gains is presented. The nominal state feedback gain is determined from the nilpotent condition in N time steps of the predictive horizon. The tightening constraint sets in DMPC are calculated locally using the exchanged information of state and constraint sets from the corresponding neighbours in a predefined iteration structure. The algorithm of dynamic invariant sets by Kerrigan and Maciejowski, 2004 [66] using Pontryagin difference operator introduced by Kolmanovsky and Gilbert, 1998 [70] has been employed.

A DMPC for linear systems have been proposed by Venkat, Rawlings and Wright, 2005 [162] wherein the decomposed global objective functions are used by local optimisers in a distributed algorithm that guarantees the convexity of local cost functions in order to achieve to the Pareto global optimality. In this work, the open-loop optimisation with terminal equality constraints is developed for stable (Hurwitz) systems while the closed-loop optimisation with distributed state feedback terminal controllers is employed for unstable or other types of systems.

A decentralised approach applying contractive state MPC for dynamically-coupled systems has been proposed by Magi and Scattolini, 2006 [91]. The convergences are assured by the coupling constraints between subsystems and the contractive state conditions. To eliminate the conservativeness of coupling constraints in this approach, the authors have suggested employing distributed controllers instead of decentralised controllers.

A decentralised MPC that achieves ISS stability has been introduced by Raimondo, Magni and Scattolini, 2007 in [91]. The ISS stability criteria using asymptotic gains (without the passivity term) may, nonetheless, lead to conservative solutions and sub-optimal performances for the decentralised control of interconnected systems. The feasibility condition for decentralised controllers has not been delivered in this work.

Negenborn, 2007 in [105] has applied the Lagrange multipliers to combine the coupling constraints with the objective functions in a DMPC scheme for transportation and power networks. Decomposed constraints and objective functions require multi-layer optimisation with serial or parallel iterations. The conditions for global stability in this approach are similar to those in the contractive state approach used by Camponogara et al. in [20]. A constraint on the increments of Lagrange multipliers is artificially added for the convergence of the outer loop optimisation problem. This constraint may, however, slow down the convergent time and result in infeasibility.

Lavaei, Momeni, and A. Aghdam, 2008 in [76] has combined decentralised ODE controllers with distributed predictive controllers for unconstrained problems. An emergency controller is used for collision avoidances due to lacking of coupling constraints in the optimisation problem. The developments are based on the continuous-time models without deriving conditions on updating time for avoiding state jumps which may consequently lead to the system instability.

An equality terminal constraint approach employed in a distributed consensus negotiation scheme for robot formation is proposed by Johansson et al., 2008 [60] in which the primal decomposition is applied to the distributed optimisation problem to achieve the global convergence. Beside the main optimisation problem, this approach requires the local optimisers to estimate the terminal states using the incremental sub-gradient method, and passing the result around between them. Due to the use of equality constraints, the conservativeness is unavoidable.

A DMPC scheme for systems operating in the presences of intermittent disruptions and time-varying delays of intersystem communication have recently been presented in [85] by Liu, de la Pena, and Christofides, 2009. The stability guaranteed method in this work is based on the converse Lyapunov theorems. A Lyapunov-based nonlinear state feedback controller is used as a contractive constraint for the optimisation problem to achieve the closed-loop stability. The sufficient condition on maximum updating time period is provided herein. The initial states are supposed to belong to an attractive region without showing how to compute such regions. Only the input constraints and a simple coupling structure of two systems have been addressed therein. A similar approach of quasi-decentralised MPC has also been introduced by Sun and El-Farra, 2008 in [149].

It can be summarised from this literature review on the existing stabilising methods for DMPC that, the open-loop optimisation has only been applied to stable (Hurwitz) linear systems. For systems that are not Hurwitz linear or nonlinear systems, the distributed state feedback controllers, either static or dynamic, are predominantly used as terminal controllers for guaranteeing the closed-loop stability. Additional conditions for feasibility and requirements on inter-system communication are always cohered with the stability conditions in DMPC. Some of DMPC schemes that are developed based on the multi-layer distributed optimisations also requires extra feasible conditions for the algorithm convergency.

Chapter 3

Asymptotically Positive Realness Constraint and Stabilising Constraint for Model Predictive Control

3.1 Introduction

In processing and manufacturing industries, a complex plant can be viewed as a large-scale system formed by various subsystems interconnected to each other in an arbitrary connection topology. By virtue of intricate interactions between state variables within a subsystem, the computerised control system is normally designed such that a dedicated multivariable controller is installed for each of its single subsystem. The multiple installations translate into a new requirement on the orchestration for these multivariable controllers to avoid destabilisations owing to the coupling effects between subsystems [128, 125]. An approach of stabilising-bounds for the control of interconnected systems in a decentralised architecture is developed in this chapter. A stabilisability condition is derived from the newly introduced *asymptotically positive realness constraint* (APRC). The APRC encapsulates the evolution of inputs and outputs, as well as their correlations over the passage of time. The proposed stabilisability condition is established based on the open-loop dissipativity property [16] and the APRC-based quadratic constraint of respective input and output trajectories.

The contributions of this chapter are threefold. *Firstly*, the asymptotically positive realness constraint is introduced as an effective tool for designing smooth controllers in the discrete-time domain. *Secondly*, the APRC-based stabilisability condition is derived for the interconnected system based on the dissipativity of its subsystems and the connection topology. This condition is suitable for the decentralised control of large-scale interconnected systems that prohibits artificial constraints on the coupling vectors. For parallel subsystems that have unknown splitting ratios, the dissipativity of a unit consisting of those parallel connected subsystems is employed instead of the dissipativity of individual subsystems. It is designated as parallel masking technique. *Thirdly*, a convex quadratic constraint on the initial control vector is derived from the APRC and applied to the model predictive controller as a stabilising constraint. The local stabilising constraints for DeMPC are fully decoupled in this approach.

This chapter is organised as follows. The system and control models are given in Section 2. The APRC, its constrained feasibility and the APRC-based stabilisability conditions in the time domain for interconnected systems are derived in Section 3. Parallel connections with unknown splitting ratios and the APRC-based stabilisability condition with the parallel masking technique are delineated in Section 4. The procedural development for APRC-based decentralised model predictive controllers is the main focus of Section 5. A conclusion is then drawn in Section 6.

3.2 Control and System Models

The interaction oriented model [87] is employed in this chapter as it is well suited to the input and output nature of interconnected systems. Consider an interconnected system Σ consisting of h subsystems, each denoted as $\mathcal{S}_i, i = 1, \ldots, h$ and has a discrete time state space model of the form:

$$\mathcal{S}_i : \begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) + E_i v_i(k), \\ y_i(k) = C_i x_i(k), \quad w_i(k) = F_i x_i(k), \end{cases} \tag{3.1}$$

where $u_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}, y_i \in \mathbb{Y}_i \subset \mathbb{R}^{q_i}, x_i \in \mathbb{X}_i \subset \mathbb{R}^{n_i}, v_i \in \mathbb{V}_i \subset \mathbb{R}^{m_{v_i}}, w_i \in \mathbb{W}_i \subset \mathbb{R}^{q_{w_i}}$; x_i, u_i, y_i, v_i, and w_i are, respectively, state, control, measurement output, interaction input and interaction output vectors.

In this chapter, the following control constraint is considered

$$\mathbb{U}_i \quad = \quad \{u_i : \ \|u_i\|_2^2 \leq \eta_i\}, \tag{3.2}$$

where η_i is the upper bound deduced from the physical limits of the actuators corresponding to the control vector u_i.

The input $v_\xi(k)$ of subsystem $\mathcal{S}_\xi$ and the output $w_\zeta(k)$ of subsystem $\mathcal{S}_\zeta$, $\xi \neq \zeta$, are connected to each other in an arbitrary topology. The large scale system Σ is represented by a state space model of the block diagonal system $\mathcal{S}$ formed by h subsystems $\mathcal{S}_i$ and the interconnection process H, as follows:

$$\Sigma : \begin{cases} x(k+1) = A\, x(k) + B\, u(k) + E\, v(k), \\ y(k) = C\, x(k), \quad w(k) = F\, x(k), \\ v(k) = H\, w(k), \end{cases} \tag{3.3}$$

where $A = \text{diag}[A_i]_1^h\,, B = \text{diag}[B_i]_1^h\,, C = \text{diag}[C_i]_1^h\,, E = \text{diag}[E_i]_1^h\,, F = \text{diag}[F_i]_1^h$ $x = [{x_1}^T ... {x_h}^T]^T,\ u = [{u_1}^T ... {u_h}^T]^T,\ y = [{y_1}^T ... {y_h}^T]^T,\ v = [{v_1}^T ... {v_h}^T]^T,\ w = [{w_1}^T ... {w_h}^T]^T$, and H is the global coupling matrix. The elements of H is either 1 or 0 only.

From a centralised point of view, the system Σ can be rewritten as

$$\Sigma : \begin{cases} x(k+1) & = (A + EHF)\, x(k) + B\, u(k) = A_{\Sigma}\, x(k) + B\, u(k), \\ y(k) & = C\, x(k). \end{cases} \tag{3.4}$$

In a decentralised control scheme, the models of stand-alone subsystems are requisite. When all the coupling inputs and outputs vanish (i.e. $v_i = 0$ and $w_i = 0$), a stand-alone subsystem $\mathcal{S}_i$, denoted as $\mathcal{S}_i^s$, has the state space model of the form:

$$\mathcal{S}_i^s : \begin{cases} x_i(k+1) &= A_i x_i(k) + B_i u_i(k), \\ y_i(k) &= C_i x_i(k). \end{cases} \tag{3.5}$$

Assumption 1 (i) (A, C) is observable. (ii) The updating instants are synchronised between all subsystems and their controllers.

Problem Description:

In this chapter, the conventional model predictive control (MPC) algorithm using the open-loop model in predictions (called open-loop MPC) is implemented for the multivariable controller of each single subsystem. The large-scale system may be open-loop unstable. Our problem is to design these MPC controllers to achieve the overall control performance and the plant-wide stabilisation. In brief, we are concerned with the design of h decoupled stabilising constraints for the local MPCs of h associated subsystems $\mathcal{S}_i$ in decentralised architectures.

3.3 Interconnection Stabilisability Condition

This section presents the main results of this chapter on establishing a new stabilisability condition for the large-scale interconnected systems. The open-loop approach will be employed throughout this Thesis.

3.3.1 Asymptotically Positive Realness Constraint

Define a parameterised quadratic supply rate for $\mathcal{S}_i$ in the presence of MPC, as follows:

$$\xi_{ic}^k\Big(u_i(k), y_i(k)|_{\Gamma_i^k}\Big) \triangleq \begin{bmatrix} u_i(k)^T & y_i(k)^T \end{bmatrix} \begin{bmatrix} Q_{ic}^k & S_{ic}^k \\ S_{ic}^{kT} & R_{ic}^k \end{bmatrix} \begin{bmatrix} u_i(k) \\ y_i(k) \end{bmatrix}, \tag{3.6}$$

where $Q_{ic}^k, R_{ic}^k, S_{ic}^k$ are multiplier matrices with symmetric Q_{ic}^k and R_{ic}^k, $\Gamma_i^k := \{Q_{ic}^k, R_{ic}^k, S_{ic}^k\}$. For conciseness, $\xi_{ic}^k\Big(u_i(k), y_i(k)|_{\Gamma_i^k}\Big)$ is denoted as ξ_{ic}^k and the time index k is represented by a superscript typeface without brackets where appropriate.

Definition 1 [53] *An input-output pair $(u_i(k), y_i(k))$ of $\mathcal{S}_i$ is said to satisfy the asymptotically positive realness constraint* (APRC) *if*

$$\xi_{ic}^k \geq o_i(k) \quad as \quad k \to +\infty, \tag{3.7}$$

for

$$\lim_{k\to+\infty} o_i(k) = 0. \tag{3.8}$$

One can see that the system positiveness constraint expressed by

$$\xi_{ic}^k \geq 0 \quad \forall k > k_0 \tag{3.9}$$

is a particular case of APRC. The following result shows that the APRC (3.7) is more flexible and less conservative than this positiveness constraint.

Lemma 1 *The input-output pairs $\big(u_i(k), y_i(k)\big)$ of $\mathcal{S}_i$, $i = 1, 2, \ldots, h$, satisfy APRC if there are $k_0 \in \mathbb{Z}^+$ and $0 \leq \gamma_i(k) < 1 - \epsilon_i$ $(0 < \epsilon_i < 1)$ such that*

$$\xi_{ic}^k \geq \frac{1 - \mathrm{sgn}(\xi_{ic}^{k-1})}{2} \gamma_i(k)\, \xi_{ic}^{k-1} \quad \forall\ k > k_0,\ for\ i = 1, 2, \ldots, h. \tag{3.10}$$

Proof: There are two possibilities:

- $\xi_{ic}^{k_0} \geq 0$, then (3.10) means $\xi_{ic}^{k} \geq 0\ \forall k > k_0$, which verifies (3.8) for $o_i(k) \equiv 0$.

- $\xi_{ic}^{k_0} < 0$, then (3.10) and $0 \leq \gamma_i(k) < 1 - \epsilon_i$ results in $\gamma_i(k) \neq 0\ \forall k > k_0$:

$$\begin{aligned} 0 > \xi_{ic}^{k} &\geq \xi_{ic}^{k_0} \prod_{\ell=k_0}^{k-1} \gamma_i(\ell) \\ &\geq \xi_{ic}^{k_0}(1-\epsilon_i)^{k-k_0} \\ &\rightarrow 0 \text{ as } k \rightarrow +\infty, \end{aligned}$$

  which verifies (3.8) for $o_i(k) = \xi_{ic}^{k_0}(1-\epsilon_i)^{k-k_0}$.

The proof is complete. ■

To our best knowledge, condition (3.10) has never been considered in the control literature hitherto, whereas the asymptotic approximations are frequently used in numerical methods [158]. It is well known that if a system is quadratically dissipative with respect to its inputs and outputs then the positive real constraint (PRC) derived from the integral quadratic constraint (IQC) in the time domain (see, e.g., [6, 71, 96]), is sufficient for its stabilisability. The stabilising control can then be found from those satisfying the positive real condition. However, its lack of correlations between the recursive control states $u_i(k), u_i(k+1), \ldots$ causes unacceptably non-smooth behaviours to the control. As a consequence, it cannot be implemented in constrained control applications. The variations of asymptotic approximation parameters in APRC will make the recursive control states well correlated, thus leads to an efficiently implemented controllers. In other words, the fundamental result of a stabilisable positive real system is not always accompanied by a realisable controller, due to the fact that an infinitely power control may be required. This thesis addresses the issue by introducing the APRC concept for use with constrained control problems. The APRC represents an approaching characteristic towards the positive real property of the system under control.

It is worth noting here that, the passivity supply rate with $S_{ic} = -I$ (see, e.g., [16]) is not required in the APRC. Moreover, the generalised dissipativity inequality without a storage function, as defined by Willems in [165], is achieved by the APRC in an asymptotic manner.

The APRC will be cast as an inequality constraint in $u_i(k)$, when $y_i(k)$ and ξ_i^{k-1} are known. An existence condition for the constrained stabilisation of interconnected systems in the discrete time domain will be derived in this chapter based on the newly introduced APRC. The APRC-based stabilisability conditions are subsequently developed into stabilising constraints for the local MPCs in a decentralised configuration. The recursive feasibility is guaranteed by characterising the APRC with dynamic multiplier matrices. With the asymptotic attribute, the APRC-based stabilising constraint is able to smooth out significant wind-up control actions. The derivations are provided in the following sections.

3.3.2 Bounded Control Feasibility of Asymptotically Positive Realness Constraints

Rewrite (3.10) as the following constraint in the variable u_i^k,

$$\boldsymbol{u}_i^k Q_{ic}^k \boldsymbol{u}_i^k + 2y_i^{kT} S_{ic}^{kT} \boldsymbol{u}_i^k \geq \frac{1}{2}\gamma_i(k)\varphi_i^{k-1} - y_i^{kT} R_{ic}^k y_i^k,\ \varphi_i^k = \big(1 - \mathsf{sgn}(\xi_{ic}^k)\big)\xi_{ic}^k, \tag{3.11}$$

which is convex if and only if $Q_{ic}^k \prec 0$. Furthermore, by rewriting

$$\xi_{ic}^k(u_i, y_i) = (u_i^k + (Q_{ic}^k)^{-1} S_{ic}^k y_i^k)^T Q_{ic}^k (u_i^k + (Q_{ic}^k)^{-1} S_{ic}^k y_i^k) + y_i^{kT}(R_{ic}^k - S_{ic}^{kT}(Q_{ic}^k)^{-1} S_{ic}^k) y_i^k,$$

(3.11) becomes

$$(u_i^k + (Q_{ic}^k)^{-1} S_{ic}^k y_i^k)^T Q_{ic}^k (u_i^k + (Q_{ic}^k)^{-1} S_{ic}^k y_i^k) \geq \frac{1}{2}\gamma_i(k)\varphi_i^{k-1} - y_i^{kT}(R_{ic}^k - S_{ic}^{kT}(Q_{ic}^k)^{-1} S_{ic}^k) y_i^k,$$

The feasibility of (3.11) in u_i^k thus requires

$$y_i^{kT}\big(R_{ic}^k - S_{ic}^{kT}(Q_{ic}^k)^{-1}S_{ic}^k\big)y_i^k - \frac{1}{2}\gamma_i(k)\varphi_i^{k-1} \geq 0, \tag{3.12}$$

as the RHS is non-positive and $Q_{ic}^k \prec 0$, and thus, the constraint region does not become to a singleton; i.e. it is nonempty.

Remark 1 It is apparently that when $\varphi_i^{k-1} \leq 0$, $Q_{ic}^k \prec 0$ and $R_{ic}^k \succ 0$, the convex constraint region is always nonempty when $y_i(k) \neq 0$, as (3.12) always holds. In later sections, we will provide the stabilisability conditions that facilitate both negative and positive definite R_{ic}^k.

The multiplier matrices R_{ic}^k, S_{ic}^k and Q_{ic}^k can be determined online by solving LMIs governing the constrained stabilisability conditions given following. The APRC is constrained feasible if and only if the intersection of the two feasibility sets of (3.11) and (3.2) is nonempty.

Lemma 2 *For* $\tilde{Q}_{ic}^k := (Q_{ic}^k)^{-1}$, *suppose that*

$$y_i^{kT}(R_{ic}^k - S_{ic}^{kT}\tilde{Q}_{ic}^k S_{ic}^k)y_i^k - \frac{1}{2}\gamma_i(k)\varphi_i^{k-1} \geq 0, \quad \tilde{Q}_{ic}^k \prec 0, \quad \text{and} \tag{3.13}$$

$$\begin{bmatrix} -\eta_i & y_i^{kT}S_{ic}^{kT}\tilde{Q}_{ic}^k \\ \tilde{Q}_{ic}^k S_{ic}^k y_i^k & -I \end{bmatrix} \preccurlyeq 0. \tag{3.14}$$

Then, the intersection of the two convex feasibility sets of (3.11) *and* (3.2) *is nonempty at step* k.

Proof: The feasibility set of (3.11) at step k is an ellipsoid region having the center at $\tilde{u}_i(k) = (Q_{ic}^k)^{-1}S_{ic}^k y_i^k$. Obviously, the intersection of $\mathbb{U}_i$ and (3.11) will be nonempty if this center point lies inside $\mathbb{U}_i$, i.e. if

$$||(Q_{ic}^k)^{-1}S_{ic}^k y_i^k||_2^2 \leq \eta_i \Leftrightarrow y_i^{kT}S_{ic}^{kT}(Q_{ic}^k)^{-2}S_{ic}^k y_i^k - \eta_i \leq 0,$$

which is fulfilled by (3.14). The first condition (3.13) ensures that the ellipsoid does not shrink to a singleton ■

3.3.3 Dissipativity and APRC-Based Stabilisability Conditions

We are now ready to introduce the interconnection stabilisability condition. Define

$$\xi^i(w_i, y_i; v_i, u_i) = \begin{bmatrix} w_i \\ y_i \end{bmatrix}^T Q_i \begin{bmatrix} w_i \\ y_i \end{bmatrix} + 2 \begin{bmatrix} w_i \\ y_i \end{bmatrix}^T S_i \begin{bmatrix} v_i \\ u_i \end{bmatrix} + \begin{bmatrix} v_i \\ u_i \end{bmatrix}^T R_i \begin{bmatrix} v_i \\ u_i \end{bmatrix}, \tag{3.15}$$

whereby the dissipativity matrices Q_i, S_i, R_i are block diagonal or block structured of the form

$$Q_i = \mathrm{diag}\{Q_{i11}, Q_{i22}\}, S_i = \mathrm{diag}\{S_{i11}, S_{i22}\}, R_i = \mathrm{diag}\{R_{i11}, R_{i22}\}, \tag{3.16}$$

in which $Q_{i22}, R_{i22}, Q_{i11},$ and R_{i11} are symmetric matrices of dimensions $q_i \times q_i, m_i \times m_i, q_{w_i} \times q_{w_i},$ and $m_{v_i} \times m_{v_i}$ respectively, S_{i22} are rectangular matrices of $q_i \times m_i$ dimensions, and S_{i11} are rectangular matrices of $q_{w_i} \times m_{v_i}$ dimensions. Accordingly, a subsystem $\mathcal{S}_i$ is said to be *quadratically dissipative* with respect to the quadratic supply rate $\xi^i(w_i, y_i; v_i, u_i)$ if there is a nonnegative C^1 function $V_i(x_i(k))$, addressed as storage function, $V_i(0) = 0$, such that for $0 < \tau_i < 1$, all $(v_i(k), u_i(k))$ and $(w_i(k), y_i(k))$ and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied irrespectively of the initial value of the state $x_i(0)$:

$$V_i(x_i(k+1)) - \tau_i V_i(x_i(k)) \leq \xi^i(w_i(k), y_i(k); v_i(k), u_i(k)). \tag{3.17}$$

The square storage function of the form $V_i(x_i(k)) := x_i^T(k) P_i x_i(k),\ P_i = P_i^T \succ 0$ is considered in this chapter.

The stabilisability condition for Σ with offline multiplier matrices Q_{ic}, S_{ic}, R_{ic} is stated in below theorem.

Theorem 1 *Let $0 < \tau_i < 1$. Suppose that*

(*i*) *The following LMIs are feasible in $\boldsymbol{P}_i, \boldsymbol{Q}_{i22}, \boldsymbol{S}_{i22}, \boldsymbol{R}_{i22}, \boldsymbol{Q}_{i11}, \boldsymbol{S}_{i11}, \boldsymbol{R}_{i11}$:*

$$\mathsf{diag}[\boldsymbol{Q}_{i11}]_1^h + H^T\mathsf{diag}[\boldsymbol{R}_{i11}]_1^h H + H^T\mathsf{diag}[\boldsymbol{S}_{i11}^T]_1^h + \mathsf{diag}[\boldsymbol{S}_{i11}]_1^h H \prec 0, \tag{3.18a}$$

and

$$\begin{bmatrix} A_i{}^T\boldsymbol{P}_iA_i - \tau_i\boldsymbol{P}_i - C_i{}^T\boldsymbol{Q}_{i22}C_i - F_i{}^T\boldsymbol{Q}_{i11}F_i & A_i{}^T\boldsymbol{P}_iB_i - C_i^T\boldsymbol{S}_{i22} & A_i{}^T\boldsymbol{P}_iE_i - F_i^T\boldsymbol{S}_{i11} \\ * & B_i{}^T\boldsymbol{P}_iB_i - \boldsymbol{R}_{i22} & B_i{}^T\boldsymbol{P}_iE_i \\ * & * & E_i{}^T\boldsymbol{P}_iE_i - \boldsymbol{R}_{i11} \end{bmatrix} \prec 0,$$

$$\boldsymbol{P}_i \succ 0, \quad \boldsymbol{Q}_{i22} \prec 0, \quad \boldsymbol{R}_{i22} \succ 0, \quad \text{for} \quad i = 1, 2, \ldots, h; \tag{3.18b}$$

(*ii*) *With feasible solution $R_{i22}, S_{i22}, Q_{i22}$ of the above system of LMIs* (3.18), *$Q_{ic} \equiv -R_{i22},\ S_{ic} \equiv -S_{i22},\ R_{ic} \equiv -Q_{i22}$ and the following APRC-based constraint in $\boldsymbol{u}_i^k$ is recursively feasible for all $k \geq k_0$:*

$$\boldsymbol{u}_i^k Q_{ic}\boldsymbol{u}_i^k + 2y_i^{kT}S_{ic}^T\boldsymbol{u}_i^k \geq \frac{1}{2}\gamma_i(k)\varphi_i^{k-1} - y_i^{kT}R_{ic}y_i^k, \tag{3.19}$$

with $0 \leq \gamma_i(k) \leq 1 - \epsilon_0,\ 0 < \epsilon_0 < 1,$ for $i = 1, 2, \ldots, h$.

Then any h controls $\tilde{u}_i^k,\ i = 1, 2, \ldots, h$, feasible to (3.19) *stabilise Σ.*

Proof:

(a) *Quadratical dissipativity of subsystem $\mathcal{S}_i$:*

$\mathcal{S}_i$ is dissipative w.r.t. the supply rate $\xi^i(w_i, y_i; v_i, u_i)$ (3.15) if and only if LMI (3.18b) holds (see, e.g., [16]). Thus (3.17) is verified for all $(v_i(k), u_i(k))$ and $(w_i(k), y_i(k))$.

(b) *Constraint Feasibility*:

Due to $Q_{i22} \prec 0$, $R_{i22} \succ 0$ in (3.18b), $R_{i22} = -Q_{ic}$, $Q_{i22} = -R_{ic}$ in (*ii*), and $\varphi_i^k \leq 0$, the inequality (3.12) holds true for all $y_i \neq 0$. Therefore, there exists u_i^k feasible to (3.19), which is equivalent to (3.20) in the following :

$$-\boldsymbol{u}_i^k Q_{ic}\boldsymbol{u}_i^k - 2y_i^{kT}S_{ic}^T\boldsymbol{u}_i^k \leq -\frac{1}{2}\gamma_i(k)\varphi_i^{k-1} + y_i^{kT}R_{ic}y_i^k, \tag{3.20}$$

since $Q_{ic} \prec 0$ and the right hand side of (3.20) is greater than or equal to 0 with $R_{ic} \succ 0$. Moreover, there always exists $u_i^k \in \mathbb{U}_i$ feasible to (3.2) that is also feasible to (3.20), for example $u_i^k = 0$.

(c) *Stabilisability*:

Define $\gamma^k := \max_{i=1\ldots h} \gamma_i^k$, $\tau = \max_{i=1\ldots h} \tau_i$, $\varphi_i^{k-1} := \big(1 - \mathsf{sgn}(\xi_{ic}^{k-1})\big)\ \xi_{ic}^{k-1}$, $\varphi^k := \sum_{i=1}^{h} \varphi_i^k$,

$$\text{and} \quad V\big(x(k)\big) := \sum_{i=1}^{h} V_i\big(x_i(k)\big).$$

Using (3.17) and $v(k) = Hw(k)$, we obtain for

$$V\big(x(k+1)\big) - \tau V\big(x(k)\big) \leq \sigma^k,$$

where

$$\begin{aligned}
\sigma^k \quad &:= \quad \sum_{i=1}^{h} \xi^i(w_i(k), y_i(k); v_i(k), u_i(k)) + \xi_{ic}^k(u_i(k), y_i(k)|_{\ \Gamma_i^k}) \\
&= \quad w^T(k)\Big[\mathsf{diag}[Q_{i11}]_1^h + H^T\mathsf{diag}[R_{i11}]_1^h H + 2\mathsf{diag}[S_{i11}]_1^h H\Big]w(k) \\
&\quad\quad - \sum_{i=1}^{h} [y_i^T(k) \quad u_i^T(k)] \begin{bmatrix} R_{ic} & S_{ic}^T \\ S_{ic} & Q_{ic} \end{bmatrix} \begin{bmatrix} y_i(k) \\ u_i(k) \end{bmatrix},
\end{aligned}$$

which, due to (3.19), yields

$$V\big(x(k+1)\big) \leq \tau V\big(x(k)\big) + \frac{1}{2}\gamma^k|\varphi^{k-1}|.$$

Then, using the chain rule, which is solicited for by the proof item (b) above, we obtain for an initial time $k_0 \geq 0$ [77],

$$\begin{aligned}
V\big(x(k)\big) \quad &\leq \quad \tau^{k-k_0}V\big(x(k_0)\big) + \frac{1}{2}\gamma|\varphi^{k_0-1}|(1 + \tau + \cdots + \tau^{k-k_0-1}),\ \gamma := \sup_{k>\bar{k}} \gamma^k \\
&= \quad \tau^{k-k_0}V\big(x(k_0)\big) + \frac{1}{2}\gamma|\varphi^{k_0-1}|\frac{1-\tau^{k-k_0}}{1-\tau}.
\end{aligned}$$

The convergence of $V\big(x(k)\big)$ is then proved in the sequel [53]:

We need to show that

$$V(x(k)) \to 0 \quad \text{as} \quad k \to +\infty, \tag{3.21}$$

i.e. for each $\beta > 0$ there is $0 < k(\beta) < +\infty$ such that $V(x(k)) \leq \beta \quad \forall k \leq k(\beta)$.

Indeed, for each $\beta \geq 0$, there exist two time instants $\bar{k}$ and $\tilde{k}$ such that

$$|\varphi^{k-1}| \leq \beta\frac{1-\tau}{\gamma} \ \forall k > \bar{k}, \text{ due to Lemma 1, i.e. } \frac{1}{2}\gamma|\varphi^{\bar{k}-1}|\frac{1-\tau^{k-\bar{k}}}{1-\tau} \leq \frac{\beta}{2},$$

$$\text{and } \tau^{k-\tilde{k}}V\big(x(\tilde{k})\big) \leq \frac{\beta}{2} \ \forall k > \tilde{k}.$$

Therefore, there exists $\check{k} = \sup(\bar{k}, \tilde{k})$ such that for each $\beta \geq 0$,

$$V\big(x(k)\big) < \frac{\beta}{2} + \frac{\beta}{2} = \beta \ \forall k \geq \check{k}.$$

As $P_i \succ 0$, (3.21) implies $\|x(k)\|_2 \to 0$ as $k \to +\infty$.

Finally, the constraint (3.19) is convex in u_i^k due to $Q_{ic} \prec 0$, as $Q_{ic} = -R_{i22}$. ■

Remark 2 Under the stabilisability condition stated in Theorem 1 above, the trajectory of $\|x(k)\|_2$ is normally non-monotonic in certain initial time periods. This is reasoned by the fact that the convergence of $\|x\|_2$ is achieved by the sum of two convergence sequences, as indicated in the above proof.

Now, the following theorem re-states the stabilisability condition in Theorem 1 without the requirement on $Q_{i22} \prec 0$. It is worth reminding here that a negative definite Q_{i22} will normally be obtained from an LMI-based dissipativity criterion for a stable subsystem; see, e.g., [57]. If the condition $Q_{i22} \prec 0$ being eliminated, the present approach will be able to accommodate unstable subsystems, with which Q_{i22} may be an indefinite or positive definite matrix.

To provide more flexibilities in the choices of the multiplier matrices Q_{ic}, S_{ic}, R_{ic}, the online multiplier matrices $Q_{ic}^k, S_{ic}^k, R_{ic}^k$ are deployed in the next theorem [53]. In the following, assume that $\varphi_i^{k-1} = \varphi_i^k < 0$ for all $k \leq 0$.

Theorem 2 *Let $0 < \tau_i < 1$. Suppose that*

(i) The following offline LMIs are feasible in $\boldsymbol{P}_i, \boldsymbol{Q}_{i22}, \boldsymbol{S}_{i22}, \boldsymbol{R}_{i22}, \boldsymbol{Q}_{i11}, \boldsymbol{S}_{i11}, \boldsymbol{R}_{i11}$:

$$\mathsf{diag}[\boldsymbol{Q}_{i11}]_1^h + H^T \mathsf{diag}[\boldsymbol{R}_{i11}]_1^h H + H^T \mathsf{diag}[\boldsymbol{S}_{i11}^T]_1^h + \mathsf{diag}[\boldsymbol{S}_{i11}]_1^h H \prec 0, \ \textit{and} \tag{3.22a}$$

$$\begin{bmatrix} A_i{}^T \boldsymbol{P}_i A_i - \tau_i \boldsymbol{P}_i - C_i{}^T \boldsymbol{Q}_{i22} C_i - F_i{}^T \boldsymbol{Q}_{i11} F_i & A_i{}^T \boldsymbol{P}_i B_i - C_i^T \boldsymbol{S}_{i22} & A_i{}^T \boldsymbol{P}_i E_i - F_i^T \boldsymbol{S}_{i11} \\ * & B_i{}^T \boldsymbol{P}_i B_i - \boldsymbol{R}_{i22} & B_i{}^T \boldsymbol{P}_i E_i \\ * & * & E_i{}^T \boldsymbol{P}_i E_i - \boldsymbol{R}_{i11} \end{bmatrix} \prec 0,$$

$$\boldsymbol{P}_i \succ 0, \quad \boldsymbol{R}_{i22} \succ 0, \quad \text{for} \quad i = 1, 2, \ldots, h; \tag{3.22b}$$

(ii) Whenever $\varphi_i^{k-1} < 0$, the following online LMIs are feasible in $\boldsymbol{Q}_{ic}^k$ and $\boldsymbol{R}_{ic}^k$:

$$\begin{bmatrix} y_i^{kT}(Q_{i22} + \boldsymbol{R}_{ic}^k) y_i^k & y_i^{kT} S_{i22} \\ S_{i22}^T y_i^k & R_{i22} + \boldsymbol{Q}_{ic}^k \end{bmatrix} \preccurlyeq 0, \quad \boldsymbol{Q}_{ic}^k \prec 0, \tag{3.23a}$$

$$y_i^{kT} \boldsymbol{R}_{ic}^k y_i^k - \tfrac{1}{2}\gamma_i(k)\varphi_i^{k-1} > 0; \tag{3.23b}$$

and choosing S_{ic}^k as a null space of $y_i^T(k)$, i.e. $\boldsymbol{S}_{ic}^k y_i^T(k) = 0$;

Then there exist h controls $\tilde{u}_i(k)$, $i = 1, 2, \ldots, h$, satisfying the control constraint (3.2) *and the APRC-based constraint* (3.11)*, that stabilise Σ.*

The procedure of decentralised MPC with APRC stabilising constraint based on Theorem 2 is formulated in the next section.

3.4 APRC-Based Decentralised Model Predictive Control

Consider a conventional quadratic objective function with respect to the state and control input vectors and adequately chosen weighting matrices $\mathcal{Q}_i, \mathcal{R}_i$ over a predictive horizon N for subsystem $\mathcal{S}_i^s$:

$$\mathcal{J}_i^k = \sum_{\ell=1}^{N+1} x_i^T(k+\ell)\mathcal{Q}_i x_i(k+\ell) + \sum_{\ell=0}^{N} u_i^T(k+\ell)\mathcal{R}_i u_i(k+\ell). \tag{3.24}$$

According to the MPC literature, see, e.g. [89], the constrained optimisation problem for a stand-alone MPC at step k is as follows:

$$\min_{\hat{\boldsymbol{u}}_i(k)} \quad \hat{\boldsymbol{u}}_i^T(k)\varPhi_i\hat{\boldsymbol{u}}_i(k) + 2\varUpsilon_i(k)\hat{\boldsymbol{u}}_i(k) + \delta_i(k) \quad \text{subject to} \quad \hat{\boldsymbol{u}}_i(k) \in \hat{\mathbb{U}}_i, \tag{3.25}$$

where

$$\varPhi_i := \varGamma_i^T\tilde{\mathcal{Q}}_{iN}\varGamma_i + \tilde{\mathcal{R}}_{iN}, \varUpsilon_i(k) := \mathtt{r}_i^T(k)\tilde{\mathcal{Q}}_{iN}\varGamma_i, \delta_i(k) := \mathtt{r}_i^T(k)\tilde{\mathcal{Q}}_{iN}\mathtt{r}_i(k),$$

$$\mathtt{r}_i(k) := \varTheta_i x_i(k),\ \tilde{\mathcal{Q}}_{iN} := \mathrm{diag}[\mathcal{Q}_i]_1^{N_i}, \tilde{\mathcal{R}}_{iN} := \mathrm{diag}[\mathcal{R}_i]_1^{N_i},$$

$$\varGamma_i := \begin{bmatrix} B_i & \dots & 0 & 0 \\ A_iB_i & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ A_i^NB_i & \dots & A_iB_i & B_i \end{bmatrix}, \ \varTheta_i := \begin{bmatrix} A_i \\ A_i^2 \\ \dots \\ A_i^{N+1} \end{bmatrix},$$

and $\hat{\mathbb{U}}_i$ is the constraint set for the variable $\hat{\boldsymbol{u}}_i$, which is deduced from the constraint set $\mathbb{U}_i$ (3.61). Specifically,

$$\hat{\mathbb{U}}_i \triangleq \{\hat{\boldsymbol{u}}_i = [u_i^T(k), u_i^T(k+1), ..., u_i^T(k+N)]^T \ : \ \|u_i(.)\|_2^2 \le \eta_i\}. \tag{3.26}$$

Using the inequality of APRC (3.11), the decentralised MPC (DeMPC) with an APRC-based one-time-step control constraint is cast as:

$$\begin{aligned} \min_{\hat{\boldsymbol{u}}_i(k)} \quad & \hat{\boldsymbol{u}}_i^T(k)\varPhi_i\hat{\boldsymbol{u}}_i(k) + 2\varUpsilon_i(k)\hat{\boldsymbol{u}}_i(k) + \delta_i(k) \quad \text{subject to} \quad \hat{\boldsymbol{u}}_i(k) \in \hat{\mathbb{U}}_i \quad \text{and} \\ & \hat{\boldsymbol{u}}_i^k M_i^T Q_{ic}^k M_i \hat{\boldsymbol{u}}_i^k + 2y_i^{kT} S_{ic}^k M_i \hat{\boldsymbol{u}}_i^k + y_i^{kT} R_{ic}^k y_i^k - \gamma_i^k \xi_{ic}^{k-1} \ge 0, \\ & \text{for} \quad \mathcal{S}_i \in I^+(k) \end{aligned} \tag{3.27}$$

where $I^+(k) = \{\mathcal{S}_i, i \in (1,h) : \ \zeta_i^k \ge 0 \text{ and } \xi_{ic}^{k-1} < 0\}$, $M_i = [I_{m_i}\, 0_{m_i} ...\, 0_{m_i}]$, for $\zeta_i^k := y_i^{kT} R_{ic}^k y_i^k - \frac{1}{2}\gamma_i(k)\varphi_i^{k-1}$.

The problem (3.27) is conceivably solved by the local controller for the minimising vector sequence $\hat{\boldsymbol{u}}_i^*(k)$ which consists of N elements of $u_i^*(k+\ell)$, $\ell = 0, 1, \dots, N$.

Only the first element $u_i^*(k)$ of the sequence $\hat{\boldsymbol{u}}_i^*(k)$ is outputted to $\mathcal{S}_i$. This rolling process is repeated at the next time step, and continues thereon. This problem can be computationally solved by any optimisers using the convex quadratic programming (QP) numerical methods.

Based on Theorem 2, the conceptual algorithm of decentralised model predictive control is outlined in Procedures 3.1 and 3.2 below.

Procedure 3.1–*Offline Calculations:*

1. Initiate $\xi_i^0, y_i(0), \Upsilon_i(0), \delta_i(0), x_i(1),$, and select the coefficient γ_i (only time-invariant γ_i is used in this procedure).

2. Determine the offline matrices $Q_{i22}, S_{i22}, R_{i22}$ feasible to LMIs (3.34) in Theorem 2. And calculate the matrix Φ_i.

Procedure 3.2–*Online Calculations:*

At every synchronised step $k \geq k_0$ of the local controllers,

1. Using the measurement outputs $y_i(k)$ to determine the multiplier matrices $Q_{ic}^k, S_{ic}^k, R_{ic}^k$ as per Theorem 2.

2. Using $y_i(k)$ and the memorised ξ_{ic}^{k-1} to update $\Upsilon_i(k)$ and $\delta_i(k)$.

3. Calculating ξ_{ic}^k then verifying $\xi_{ic}^k \geq 0$. If true, set $\gamma_i = 0$.

4. Formulating then solving the optimisation problem (3.27) to yield the local control sequence $\hat{\boldsymbol{u}}_i(k)$. The first vector element $u_i(k)$ is applied to control $\mathcal{S}_i$.

Remark 3 The choice of γ_i^k is crucial to the convergence to zero of solutions. When γ_i^k approaches 1, the controllers will tend to be smoother. It is notably that, a time-invariant γ_i can be employed, as it is in this procedure, to simplify the implementation.

Remark 4 The system of LMIs (3.31) may become numerically infeasible when $y_i^k \simeq 0$ and $\varphi_i^k \simeq 0$ simultaneously. One of the solutions for the feasibility is to consider the convex duality problem. It is to ensure that the complement of (3.11) is merely feasible for $u_i^k \in \mathbb{R}\backslash\mathbb{U}_i$ while the right hand side of (3.11) is positive. The multiplier matrices Q_{ic}^k and S_{ic}^k will then need to be redesigned as as per Theorem 3 below [53]. Denote $\zeta_i^k := y_{ic}^T R_{ic}^k y_{ic} - \frac{1}{2}\gamma_i^k \varphi_i^{k-1}$.

Theorem 3 *Let* $0 < \tau < 1$ *and* $\zeta_i(0) < 0$. *Suppose that*

(i) The following offline LMI optimisation is feasible:

$$\min_{\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{S},\ \boldsymbol{R}} \quad \mathrm{trace}(\boldsymbol{Q}) \tag{3.28}$$

$$\textit{subject to} \qquad \begin{bmatrix} A^T\boldsymbol{P}A - \tau\boldsymbol{P} - C^T\boldsymbol{Q}C & A^T\boldsymbol{P}B - C^T\boldsymbol{S} \\ * & B^T\boldsymbol{P}B - \boldsymbol{R} \end{bmatrix} \prec 0, \quad \boldsymbol{P} \succ 0, \quad \boldsymbol{R} \succ 0, \tag{3.29}$$

for $Q := \mathsf{diag}[Q_i]_1^h,\ R := \mathsf{diag}[R_i]_1^h,\ S := \mathsf{diag}[S_i]_1^h$;

(ii) Whenever $\varphi_i^{k-1} < 0$,

(iia) The following LMI optimisation is feasible:

$$\min_{\boldsymbol{Q}_{ic}^k,\ \boldsymbol{S}_{ic}^k,\ \boldsymbol{R}_{ic}^k,\ \boldsymbol{\tau}_i} \quad \mathrm{trace}(\boldsymbol{R}_{ic}^k) \tag{3.30}$$

$$\textit{subject to} \qquad \begin{bmatrix} y_i^{kT}(Q_i + \boldsymbol{R}_{ic}^k)y_i^k & y_i^{kT}(S_i + \boldsymbol{S}_{ic}^k) \\ * & R_i + \boldsymbol{Q}_{ic}^k \end{bmatrix} \preccurlyeq 0, \quad \boldsymbol{Q}_{ic}^k \prec 0, \tag{3.31a}$$

$$y_i^{kT}\boldsymbol{R}_{ic}^k y_i^k - \tfrac{1}{2}\gamma_i(k)\varphi_i^{k-1} + \boldsymbol{\tau}_i \geq 0; \tag{3.31b}$$

(iib) If $\zeta_i^k < 0$ *with the resultant* R_{ic}^k *from (iia), the following LMIs are feasible in*

$\boldsymbol{Q}_{ic}^k$ *and* $\boldsymbol{S}_{ic}^k$*:*

$$\begin{bmatrix} -\boldsymbol{Q}_{ic}^k - \boldsymbol{\lambda}_i I_{m_i} & -\boldsymbol{S}_{ic}^k y_i^k \\ * & \boldsymbol{\lambda}_i \eta_i - \zeta_i^k \end{bmatrix} \succcurlyeq 0, \ \ \boldsymbol{\lambda}_i > 0, \tag{3.32a}$$

$$\begin{bmatrix} y_i^{kT}(Q_i + R_{ic}^k) y_i^k & y_i^{kT}(S_i + \boldsymbol{S}_{ic}^k) \\ * & R_i + \boldsymbol{Q}_{ic}^k \end{bmatrix} \preccurlyeq 0, \ \ \boldsymbol{Q}_{ic}^k \prec 0. \tag{3.32b}$$

Then there exist h *controls* $\tilde{u}_i(k), \ i = 1, 2, \ldots, h$*, satisfying the control constraint* (3.2) *and the APRC-based constraint* (3.11)*, that stabilise* Σ*.*

Remark 5 The above LMI (3.32a) is obtained from the following inequality:

$$\boldsymbol{u}_i^{kT} Q_{ic}^k \boldsymbol{u}_i^k + 2 y_i^{kT} S_{ic}^k \boldsymbol{u}_i^k \le -\lambda_i(\boldsymbol{u}_i^{kT}\boldsymbol{u}_i^k - \eta_i) - \zeta_i^k \le -\zeta_i^k,$$

which ensures that $\boldsymbol{u}_i^{kT} Q_{ic}^k \boldsymbol{u}_i^k + 2 y_i^{kT} S_{ic}^k \boldsymbol{u}_i^k \le -\zeta_i^k$ is only feasible for $u_i^k \in \mathbb{R}\backslash\mathbb{U}_i$ while the right hand side of (3.11) is positive (i.e. $\boldsymbol{u}_i^{kT} Q_{ic}^k \boldsymbol{u}_i^k + 2 y_i^{kT} S_{ic}^k \boldsymbol{u}_i^k \ge -\zeta_i^k, \zeta_i^k < 0$). The following LMI optimisation can be solved for Q_{ic}^k and S_{ic}^k instead of (3.32):

$$\max_{\boldsymbol{\lambda}_i > 0, \ \boldsymbol{Q}_{ic}^k \prec 0, \ \boldsymbol{S}_{ic}^k, \ \boldsymbol{\tau}_i} \{\boldsymbol{\tau}_i : \quad (3.32)\} \le \zeta_i^k. \tag{3.33}$$

An alternative and agile solution for the feasibility problem is given by another LMI-based condition stated in Theorem 4.

3.5 Flexible Stabilisability Conditions

3.5.1 With Non-Conservative Multiplier Matrices

A more flexible calculation of multiplier matrices $\boldsymbol{R}_{ic}^k, \boldsymbol{S}_{ic}^k, \boldsymbol{Q}_{ic}^k$ for non-parallel connection systems is given in the following theorem:

Theorem 4 *Let $0 < \tau_i < 1$. Suppose that*

(i) The following offline LMI optimisations are feasible:

$$\min_{\boldsymbol{P},\boldsymbol{Q},\boldsymbol{S},\boldsymbol{R}} \quad \mathrm{trace}(\boldsymbol{Q}) \qquad \textit{subject to} \tag{3.34a}$$

$$\begin{bmatrix} A^T\boldsymbol{P}A - \tau\boldsymbol{P} - C^T\boldsymbol{Q}C & A^T\boldsymbol{P}B - C^T\boldsymbol{S} \\ * & B^T\boldsymbol{P}B - \boldsymbol{R} \end{bmatrix} \prec 0, \tag{3.34b}$$

$$\boldsymbol{P} \succ 0, \quad \boldsymbol{R} \succ 0,$$

for $Q := \mathsf{diag}[Q_i]_1^h$, $R := \mathsf{diag}[R_i]_1^h$, $S := \mathsf{diag}[S_i]_1^h$;

(ii) With the offline matrices Q_i, S_i, R_i obtained from the above offline LMIs (3.34)*, the online conditions in the sequel are fulfilled recursively for all $k \geq k_0$:*

(iia) The following LMIs are feasible in $\boldsymbol{R}_{ic}^k, \boldsymbol{S}_{ic}^k$ with a chosen parameter ϵ_i:

$$\begin{bmatrix} y_i^{kT}(Q_i + \boldsymbol{R}_{ic}^k)y_i^k & y_i^{kT}(S_i + \boldsymbol{S}_{ic}^{kT}) \\ (S_i^T + \boldsymbol{S}_{ic}^k)y_i^k & -\epsilon_i I_{m_i} \end{bmatrix} \preccurlyeq 0, \tag{3.35a}$$

$$\begin{bmatrix} -\rho_i & y_i^{kT}\boldsymbol{S}_{ic}^{kT}(Q_{0ic})^{-1} \\ (Q_{0ic})^{-1}\boldsymbol{S}_{ic}^k y_i^k & -I_{m_i} \end{bmatrix} \preccurlyeq 0, \; Q_{0ic} = -R_i - \epsilon_i I_{m_i}, \tag{3.35b}$$

$$\text{for} \quad i = 1, 2, \ldots, h;$$

(iib) With the matrices S_{ic}^k, R_{ic}^k obtained from the above LMIs (3.35) *and the chosen Q_{0ic}, the following LMIs are feasible in $\boldsymbol{Q}_{ic}^k$, $\boldsymbol{Q}_{ic}^k := (\boldsymbol{Q}_{1ic}^k)^{-1}$:*

$$y_i^{kT}(R_{ic}^k - S_{ic}^{kT}\boldsymbol{Q}_{1ic}^k S_{ic}^k)y_i^k - \frac{1}{2}\gamma_i(k)\varphi_i^{k-1} > 0, \tag{3.36a}$$

$$\begin{bmatrix} -\rho_i & y_i^{kT}S_{ic}^{kT}\boldsymbol{Q}_{1ic}^k \\ \boldsymbol{Q}_{1ic}^k S_{ic}^k y_i^k & -I_{m_i} \end{bmatrix} \preccurlyeq 0, \quad I_{m_i} - Q_{0ic}\boldsymbol{Q}_{1ic}^k \succ 0, \quad \boldsymbol{Q}_{1ic}^k \prec 0, \tag{3.36b}$$

$$\text{for} \quad i = 1, 2, \ldots, h.$$

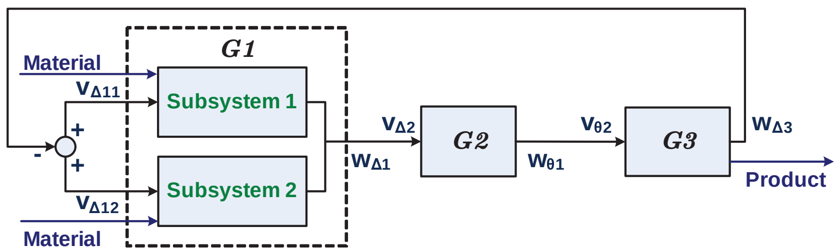


Figure 3.1 : Mixed Connection Structure of an Interconnected System.

Then there exist h controls $\tilde{u}_i(k)$, $i = 1, 2, \ldots, h$, *satisfying the constraints* (3.2) *and* (3.11), *that stabilise* Σ.

Proof: This is a result of Lemma 2, Theorem 1 and Theorem 2, as well as the flexible uses of Q_{0ic} in (3.35b), and Q_{1ic} and Q_{0ic} in (3.36b). ■

3.6 Parallel Splitting Subsystems

In this section, the coupling vector v_i and w_i in the above sections are split into two coupling vectors to explicitly describe the serial and parallel (splitting) connections. Consider a large-scale system Σ consisting of h units, denoted as $\mathcal{G}_j, j = 1, ...h$ like that described by Figure 3.1.

Each unit $\mathcal{G}_j$, $j = 1, 2, ..., h$ is represented by

$$\mathcal{G}_j : \begin{cases} x_j(k+1) = A_j\, x_j(k) + B_j\, u_j(k) + E_{\Theta j}\, v_{\Theta j}(k) + E_{\Delta j}\, v_{\Delta j}(k), \\ y_j(k) = C_j\, x_j(k), \quad w_{\Theta j}(k) = F_{\Theta j}\, x_j(k), \quad w_{\Delta j}(k) = F_{\Delta j}\, x_j(k) \end{cases} \tag{3.37}$$

where $x_j(k)$, $u_j(k)$ and $y_j(k)$ are the state, control input and measurement output of each unit.

Unit $\mathcal{G}_i$ is said to be *connected in serial* to unit $\mathcal{G}_j$ if there is a nonzero coupling matrix H_Θ^{ij} with zero or one entries such that $v_{\Theta j}(k) = H_\Theta^{ij} w_{\Theta i}$. If $\mathcal{G}_i$ is not serially connected to $\mathcal{G}_j$ set $H_{\Theta}^{ij} = 0$. Accordingly,

$$H_{\Theta} = [H_{\Theta}^{ij}]_{i,j=1,\ldots,h}$$

is regarded as the global serial coupling matrix for $v_{\Theta}(k) = H_{\Theta} w_{\Theta}(k)$, where $v_{\Theta}(k) := [v_{\Theta 1}^T(k) \ldots v_{\Theta h}^T(k)]^T$, $w_{\Theta}(k) := [w_{\Theta 1}^T(k) \ldots w_{\Theta h}^T(k)]^T$. For instance, in Figure 3.1 unit $\mathcal{G}_2$ is connected in serial to unit $\mathcal{G}_3$.

On the other hand, unit $\mathcal{G}_i$ is said to be *connected in parallel* to unit $\mathcal{G}_j$ if there is nonzero but singular matrix H_{Δ}^{ij} of zero or one entries such that $w_{\Delta i}(k) = H_{\Delta}^{ij} v_{\Delta j}$. If $\mathcal{G}_i$ is not connected in parallel to $\mathcal{G}_j$, simply set $H_{\Delta}^{ij} = 0$. Accordingly,

$$H_{\Delta} = [H_{\Delta}^{ij}]_{i,j=1,2,\ldots,h}$$

is regarded as the global parallel coupling matrix for $w_{\Delta}(k) = H_{\Delta} v_{\Delta}$, where $v_{\Delta}(k) := [v_{\Delta 1}^T(k) \ldots v_{\Delta h}^T(k)]^T$, $w_{\Delta}(k) := [w_{\Delta 1}^T(k) \ldots w_{\Delta h}^T(k)]^T$. For instance, in Figure 3.1 unit $\mathcal{G}_3$ is connected in parallel to unit $\mathcal{G}_1$:

$$w_{\Delta 3}(k) = v_{\Delta 11}(k) + v_{\Delta 12}(k) = [1 \quad 1] v_{\Delta 1}(k)$$

for $v_{\Delta 1}(k) = [v_{\Delta 11}^T(k)\ v_{\Delta 12}^T(k)]^T$.

In this development, the splitting ratios of $w_{\Delta 3}(k)$ to $v_{\Delta 11}(k)$ and $v_{\Delta 12}(k)$ are unknown. Now, with the definitions of

$$\begin{gathered} A := \mathsf{diag}\ [A_j]_1^h,\ B := \mathsf{diag}\ [B_j]_1^h,\ E_{\Theta} := \mathsf{diag}\ [E_{\Theta j}]_1^h, \\ E_{\Delta} := \mathsf{diag}\ [E_{\Delta j}]_1^h,\ C := \mathsf{diag}\ [C_j]_1^h, F_{\Theta} := \mathsf{diag}\ [F_{\Theta j}]_1^h,\ F_{\Delta} := \mathsf{diag}\ [F_{\Delta j}]_1^h, \\ x(k) := [x_1^T(k)\ \ldots\ x_h^T(k)]^T,\ u(k) := [u_1^T(k)\ \ldots\ u_h^T(k)]^T, \end{gathered} \tag{3.38}$$

the large-scale system has the state space model of the following:

$$\Sigma : \begin{cases} x(k+1) = (A + E_{\Theta} H_{\Theta} F_{\Theta}) x(k) + B u(k) + E_{\Delta}\, v_{\Delta}(k), \\ y(k) = C\, x(k), \quad w_{\Delta}(k) = F_{\Delta}\, x(k),\ w_{\Delta}(k) = H_{\Delta} v_{\Delta}(k). \end{cases} \tag{3.39}$$

The same control constraint (3.2) is considered for $u_j(k)$.

3.6.1 Interconnection Stabilisability Condition

Denote the null base of $\begin{bmatrix} F_\Delta & -H_\Delta \end{bmatrix}$ as $\mathcal{N}$. If $\boldsymbol{Q}_\Delta,\ \boldsymbol{S}_\Delta,\ \boldsymbol{R}_\Delta$ are such that

$$\mathcal{N}^T \begin{bmatrix} \boldsymbol{Q}_\Delta & \boldsymbol{S}_\Delta \\ \boldsymbol{S}_\Delta^T & \boldsymbol{R}_\Delta \end{bmatrix} \mathcal{N} \geq 0, \tag{3.40}$$

then

$$\xi_\Delta(k) := \begin{bmatrix} x(k) \\ v_\Delta(k) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{Q}_\Delta & \boldsymbol{S}_\Delta \\ \boldsymbol{S}_\Delta^T & \boldsymbol{R}_\Delta \end{bmatrix} \begin{bmatrix} x(k) \\ v_\Delta(k) \end{bmatrix} \geq 0 \tag{3.41}$$

for all $(x(k), v_\Delta(k))$ satisfying $H_\Delta v_\Delta(k) = F_\Delta\, x(k)$ from (5.24).

Define the quadratic function of

$$\xi_j(k) := \begin{bmatrix} y_j(k) \\ u_j(k) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{Q}_j & \boldsymbol{S}_j \\ \boldsymbol{S}_j^T & \boldsymbol{R}_j \end{bmatrix} \begin{bmatrix} y_j(k) \\ v_j(k) \end{bmatrix} \tag{3.42}$$

for symmetric matrices $\boldsymbol{Q}_j$ and $\boldsymbol{R}_j$, and matrix $\boldsymbol{S}_j$ of appropriate dimension.

Accordingly,

$$\xi(k) := \sum_{j=1}^{h} \xi_j = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & \boldsymbol{R} \end{bmatrix} \begin{bmatrix} y(k) \\ u(k) \end{bmatrix},$$

with $\boldsymbol{Q} := \mathsf{diag}[\boldsymbol{Q}_j]_1^h,\ \boldsymbol{S}_j := \mathsf{diag}[\boldsymbol{S}_j]_1^h,\ \boldsymbol{R} := \mathsf{diag}[\boldsymbol{R}_j]_1^h$.

Definition 2 *An input-output pair $\big(u_j(k), y_j(k)\big)$ of unit $\mathcal{G}_j$ (3.37) is said to satisfy the asymptotically surely positive realness constraint with respect to $\xi_j(k)$, or simply ASPRC, if*

$$\xi_j(k) \geq 0 \quad as \quad k \to +\infty. \tag{3.43}$$

One can see that ASPRC (3.43) is an enhanced version of the APRC (3.7).

Lemma 3 *The input-output pair* $\big(u_j(k), y_j(k)\big)$ *of unit* $\mathcal{G}_j$ *satisfies the ASPRC if there is* $k_0 \in \mathbb{Z}^+$ *and* $0 < \gamma_j(k) < 1-\rho$ *with* $0 < \rho < 1$ *and* $\epsilon_j(k) > \epsilon$ *with* $\epsilon > 0$ *such that*

$$\xi_j(k) \geq \frac{1-\mathsf{sgn}(\xi_j(k-1))}{2}\big[\gamma_j(k)\xi_j(k-1) - \epsilon_j(k)\mathsf{sgn}(\xi_j(k-1))\big] \quad \forall k > k_0. \quad (3.44)$$

Proof. Note that (3.44) implies $\xi_j(k) \geq 0 \ \forall k \geq \bar{k}$ whenever $\xi_j(\bar{k}) \geq 0$. Thus, it is sufficient to show that there is $\bar{k} < +\infty$ such that $\xi_j(\bar{k}) \geq 0$. Assume contrarily that

$$\xi_j(k) < 0 \quad \forall k > k_0, \quad (3.45)$$

so according to (3.44), $\quad \xi_j(k) \geq \gamma_j(k)\xi_j(k-1) + \epsilon \quad \forall k > k_0$. Particularly,

$$\begin{aligned}
\xi_j(k_0+2) \ &\geq \ \gamma_j(k_0+2)\xi_j(k_0+1) + \epsilon \\
&\geq \ \gamma_j(k_0+2)(\gamma_j(k_0+1)\xi_j(k_0) + \epsilon) + \epsilon \\
&> \ \gamma_j(k_0+2)\gamma_j(k_0+1)\xi_j(k_0) + \epsilon.
\end{aligned}$$

By mathematical induction, it can be easily shown that for all $k > k_0$,

$$\xi_j(k) > [\prod_{\ell=k_0+1}^{k} \gamma_j(\ell)]\xi_j(k_0) + \epsilon,$$

which leads to $\xi_j(k) > 0$ for sufficient large k because $[\prod_{\ell=k_0+1}^{k} \gamma_j(\ell)]\xi_j(k_0) \to 0$ as $k \to +\infty$. This contradicts to (3.45). ■

The stabilisability condition for Σ (5.24) is stated below.

Theorem 5 *For*

$$\mathcal{M}(\boldsymbol{P}, \boldsymbol{Q}_\Delta, \boldsymbol{S}_\Delta, \boldsymbol{R}_\Delta, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{R}) :=$$

$$\begin{bmatrix}
(A+E_\Theta H_\Theta F_\Theta)^T\boldsymbol{P}(A+E_\Theta H_\Theta F_\Theta) - \boldsymbol{P} + \boldsymbol{Q}_\Delta + C^T\boldsymbol{Q}C & * & * \\
B^T\boldsymbol{P}(A+E_\Theta H_\Theta F_\Theta) + \boldsymbol{S}^TC & B^T\boldsymbol{P}B + \boldsymbol{R} & * \\
E_\Delta^T\boldsymbol{P}(A+E_\Theta H_\Theta F_\Theta) + \boldsymbol{S}_\Delta^TF_\Delta & E_\Delta^T\boldsymbol{P}B & E_\Delta^T\boldsymbol{P}E_\Delta + \boldsymbol{R}_\Delta
\end{bmatrix}, \quad (3.46)$$

suppose that

(i) The system of LMIs (3.40) and

$$\mathcal{M}(\boldsymbol{P}, \boldsymbol{Q}_\Delta, \boldsymbol{S}_\Delta, \boldsymbol{R}_\Delta, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{R}) \prec 0, \ \boldsymbol{P} \succ 0, \tag{3.47}$$

is feasible in $\boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{R}, \boldsymbol{S}, \boldsymbol{Q}_\Delta, \boldsymbol{R}_\Delta, \boldsymbol{S}_\Delta$*;*

(ii) With feasible solution Q, S, R *of LMIs (3.47), (3.40) and* $0 < \gamma_j(k) < 1 - \rho$*,* $0 < \eta < 1$ *and* $\epsilon_j(k) > \epsilon > 0$*, the convex constraint (3.44) is recursively feasible in* $u_j(k)$*.*

Then any h *feasible* $u_j(k)$ *to (3.44),* $j = 1, 2, \ldots, h$*, stabilise* Σ *(5.24).*

Proof. Let $V(x) := x^T \boldsymbol{P} x$. By (3.47), there is $0 < \alpha < 1$ such that

$$\begin{array}{rcl} V(x(k+1)) - (1-\alpha)V(x(k)) + \xi_\Delta(k) + \xi(k) & = & \\ \begin{bmatrix} x(k) \\ u(k) \\ v_\Delta(k) \end{bmatrix}^T \mathcal{M}(\boldsymbol{P}, \boldsymbol{Q}_\Delta, \boldsymbol{S}_\Delta, \boldsymbol{R}_\Delta, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{R}) \begin{bmatrix} x(k) \\ u(k) \\ v_\Delta(k) \end{bmatrix} + \alpha x^T(k) \boldsymbol{P} x(k) & \leq & 0. \end{array} \tag{3.48}$$

As $\xi_\Delta(k) \geq 0 \ \forall k$ and $\xi(k) \geq 0$ as $k \to +\infty$ by Lemma 3,

$$V(x(k+1)) - (1-\alpha)V(x(k)) \leq 0 \quad \text{as} \quad k \to +\infty \tag{3.49}$$

so $V(x(k)) \to 0$ as $k \to +\infty$, which implies $x(k) \to 0$ as $k \to +\infty$. ■

Remark 6 Similarly to Theorem 1, the constraint (3.44) in $u_j(k)$ is convex iff $R_j \prec 0$ and feasible if, additionally, $Q_j \succ 0$. $R_j \prec 0$ is obviously obtained from LMI (3.46). On the contrary, $Q_j \succ 0$ is not a favour condition for an unstable system $\mathit{\Sigma}$ (5.24).

It is clear that Theorem 5 remains valid whenever the online condition (3.44) is replaced by the online feasibility of the following LMI in $\boldsymbol{Q}_j(k), \boldsymbol{S}_j(k)$ and $\boldsymbol{R}_j(k)$

$$\xi_j^k := \xi_j(y_j(k), u_j(k)) = \begin{bmatrix} y_j(k) \\ u_j(k) \end{bmatrix}^T \begin{bmatrix} Q_j & S_j \\ S_j^T & R_j \end{bmatrix} \begin{bmatrix} y_j(k) \\ u_j(k) \end{bmatrix} \quad \geq$$

$$\delta_j(u_j(k), y_j(k)) := \begin{bmatrix} y_j(k) \\ u_j(k) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{Q}_j(k) & \boldsymbol{S}_j(k) \\ \boldsymbol{S}_j^T(k) & \boldsymbol{R}_j(k) \end{bmatrix} \begin{bmatrix} y_j(k) \\ u_j(k) \end{bmatrix} \qquad \forall u_j(k)$$

$$\Leftrightarrow \quad \begin{bmatrix} y_j^T(k)(Q_j - \boldsymbol{Q}_j(k))y_j(k) & y_j^T(k)(S_j - \boldsymbol{S}_j(k)) \\ (S_j - \boldsymbol{S}_j(k))^T y_j(k) & R_j - \boldsymbol{R}_j(k) \end{bmatrix} \succcurlyeq 0 \tag{3.50}$$

and, then, the feasibility in $\boldsymbol{u}_j(k)$ of

$$\delta_j(\boldsymbol{u}_j(k), y_j(k)) \geq \gamma_j(k)\varphi_j(k-1) + \epsilon_j(k)\psi_j(k-1), \quad \forall k > k_0, \tag{3.51}$$

$$j = 1, 2, \ldots, h,$$

where

$\varphi_j(k) := \theta_j(k-1)\,\delta_j(u_j(k), y_j(k))$ and $\psi_j(k-1) := -\theta_j(k-1)\,\mathsf{sgn}\big(\delta_j(u_j(k-1), y_j(k-1))\big)$,

for $\theta_j(k) := \dfrac{1 - \mathsf{sgn}\big(\delta_j(u_j(k), y_j(k))\big)}{2}$ instead of (3.44).

3.6.2 Decentralised MPC for Parallel Splitting Systems

For implementations,

- Whenever $\xi_j(y_j(k-1), u_j(k-1)) < -\epsilon$ set

  $$\varphi_j(k-1) := \xi_j^{k-1}, \;\; \gamma_j(k) \equiv 1 - \rho > 0, \;\; S_j^T(k) \equiv \mathcal{N}_{y_j^T(k)}, \tag{3.52}$$

  where $\mathcal{N}_{y_j^T(k)}$ stands for the base of the zero space of $y_j^T(k)$. Then (3.50) reads

  $$\begin{bmatrix} y_j^T(k)(Q_j - \boldsymbol{Q}_j(k))y_j(k) & y_j^T(k)S_j \\ S_j^T y_j(k) & R_j - \boldsymbol{R}_j(k) \end{bmatrix} \succcurlyeq 0 \tag{3.53}$$

and (3.51) is the following quadratic constraint in $\boldsymbol{u}_j(k)$,

$$\boldsymbol{u}_j^T(k)R_j(k)\boldsymbol{u}_j(k) \geq -y_j^T(k)Q_j(k)y_j(k) + (1-\rho)\,\xi_j^{k-1} + \epsilon_j(k). \tag{3.54}$$

Obviously, the condition

$$y_j^T(k)\boldsymbol{Q}_j(k)y_j(k) - (1-\rho)\,\xi_j^{k-1} - \epsilon_j(k) \geq 0 \tag{3.55}$$

is needed to ensure that the intersection of the feasibility set of (3.54) and the control constraint (3.2) is nonempty.

- Whenever $\xi_j(y_j(k-1), u_j(k-1)) \geq -\epsilon$, (3.51) is replaced by the following convex constraint in $\boldsymbol{u}_j(k)$

$$\xi_j(y_j(k), \boldsymbol{u}_j(k)) := \begin{bmatrix} y_j(k) \\ \boldsymbol{u}_j(k) \end{bmatrix}^T \begin{bmatrix} Q_j & S_j \\ S_j^T & R_j \end{bmatrix} \begin{bmatrix} y_j(k) \\ \boldsymbol{u}_j(k) \end{bmatrix} \geq 0, \tag{3.56}$$

provided that $y_j^T(k)\big(Q_j - S_jR_j^{-1}S_j^T\big)y_j(k) \geq 0$.

- Whenever $\xi_j(y_j(k-1), u_j(k-1)) \geq 0$, the system will deems be stabilised.

Theorem 6 *Suppose that*

(i) The above LMIs (3.47), (3.40) *are feasible in* $\boldsymbol{P}, \boldsymbol{Q}_\Delta, \boldsymbol{R}_\Delta, \boldsymbol{S}_\Delta, \boldsymbol{Q}$, $\boldsymbol{R}$, *and* $\boldsymbol{S}$*;*

(ii) With feasible matrices $Q = \mathsf{diag}\ [Q_j]_1^h\ S = \mathsf{diag}\ [S_j]_1^h$ *and* $R = \mathsf{diag}\ [R_j]_1^h$ *of the above LMIs* (3.47), (3.40) *the following recursive conditions hold*

(iia) Whenever $\xi_j(y_j(k-1), u_j(k-1)) < -\epsilon$ *LMIs* (3.53) *and* (3.55) *are feasible in* $\boldsymbol{R}_j(k)$ *and* $\boldsymbol{Q}_j(k)$*;*

(iib) Whenever $\xi_j(y_j(k-1), u_j(k-1)) \geq -\epsilon$*, convex constraints* (3.56) *are feasible in* $\boldsymbol{u}_j(k)$*;*

Then any feasible $u_j(k)$ *to the convex constraints* (3.54) *and* (3.2) *for*

$\xi_j(y_j(k-1), u_j(k-1)) < -\epsilon$*, or to convex constraints* (3.56) *for*

$\xi_j(y_j(k-1), u_j(k-1)) \geq -\epsilon$*, stabilise* Σ (3.39).

With a prediction horizon N, the ASPRC-based Decentralised MPC is in the following:

$$\min_{\boldsymbol{u}_j(k+\ell),\ j=1,...,h} \sum_{\ell=1}^{N+1} x_j^T(k+\ell)\mathcal{Q}_j x_j(k+\ell) + \sum_{\ell=0}^{N} u_j^T(k+\ell)\mathcal{R}_j u_j(k+\ell) \tag{3.57}$$

subject to

$$x_j(k+\ell+1) = A_j x_j(k+\ell) + B_j u_j(k+\ell),\ \ell = 0,1,...,N, \tag{3.58}$$

$$(3.54) \quad \text{for} \quad j \in I^-(k) \quad \text{and} \quad (3.56) \quad \text{for} \quad j \in I^+(k) \tag{3.59}$$

where $I^-(k) := \{j \in \{1,2,...,h\} :\ \xi_j(y_j(k-1), u_j(k-1)) < -\epsilon\}$ and

$$I^+ := \{1,2,...,h\} \setminus I^-(k) \bigcap I^{++}(k),$$

in which $I^{++}(k) := \big\{j \in \{1,2,...,h\} :\ y_j^T(k)\big(Q_j - S_j R_j^{-1} S_j^T\big) y_j(k) \geq 0\big\}$.

For each subsystem, the optimisation problem (3.57)-(3.59) is solved by the local optimiser for the minimising sequence $\boldsymbol{u}_j(k+\ell), j = 0,1,...,N$ but only its first element $u_j(k)$ is applied to control the plant. This rolling or receding process is repeated at the next time step, and continues thereon.

3.6.3 With Parallel Masking Dissipativity Criterion

A fully decentralised dissipativity criterion up to the subsystem level for parallel connection systems is given in this section [157]. Consider a large-scale system Σ consisting of h units, denoted as $\mathcal{G}_j, j = 1 \ldots h$. Each unit $\mathcal{G}_j$ has g_j subsystems, denoted as $\mathcal{S}_i, i = 1 \ldots g_j$. Each subsystem $\mathcal{S}_i$ is represented by the discrete-time state space model of the form

$$\mathcal{S}_i : \begin{cases} x_i(k+1) = A_i\, x_i(k) + B_i\, u_i(k) + E_i\, v_i(k), \\ \\ y_i(k) = C_i\, x_i(k), \quad w_i(k) = F_i\, x_i(k), \end{cases} \tag{3.60}$$

where $E_i = [E_{\pi ji}\ E_{\sigma ji}]$, $v_i^T = [v_{\pi ji}^T\ v_{\sigma ji}^T]$, $F_i = [F_{\pi ji}\ F_{\sigma ji}]$, $w_i^T = [w_{\pi ji}^T\ w_{\sigma ji}^T]$, in which $v_{\sigma ji} \in \mathbb{V}_i \subset \mathbb{R}^{m_{v_{\sigma ji}}}$ and $w_{\sigma ji} \in \mathbb{W}_i \subset \mathbb{R}^{q_{w_{\sigma ji}}}$ are *serial coupling* input and output vectors respectively, while $v_{\pi ji} \in \mathbb{R}^{m_{v_{\pi j}}}$ and $w_{\pi ji} \in \mathbb{R}^{q_{w_{\pi j}}}$ are *parallel coupling* input and output vectors; $x_i \in \mathbb{X}_i \subset \mathbb{R}^{n_i}$ is the state vector, $y_i \in \mathbb{Y}_i \subset \mathbb{R}^{q_i}$ is the measurement output vector, $u_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}$ is the control input vector. It is notably that, there are $h_\Sigma := \sum_{j=1}^{h} g_j$ subsystems $\mathcal{S}_i$ in Σ.

In this section, the same control constraint is considered, i.e.

$$\mathbb{U}_i \triangleq \{u_i :\ \|u_i\|_2^2 \leq \rho_i\}. \tag{3.61}$$

Serial Connections: Two subsystems $\mathcal{S}_{\xi\mathsf{i}}$ and $\mathcal{S}_{\zeta\mathsf{o}}$ are said to be *serially connected* (SC) if the coupling input vector $v_{\sigma\xi\mathsf{i}}$ of $\mathcal{S}_{\xi\mathsf{i}}$ is the coupling output vector $w_{\sigma\zeta\mathsf{o}}$ of $\mathcal{S}_{\zeta\mathsf{o}}$, i.e.

$$(\mathsf{SC})\ v_{\sigma\xi\mathsf{i}}(k) = w_{\sigma\zeta\mathsf{o}}(k). \tag{3.62}$$

In the following, M_j (of unit $\mathcal{G}_j$) denotes the diagonal matrix $\mathrm{diag}[M_i]_1^{g_j}$, and v_{*j} denotes the stacking vector $[{v_{*j1}}^T \ldots {v_{*jg_j}}^T]^T$ (i.e. unit subscript j remains). A unit $\mathcal{G}_j$ is represented by the block diagonal system formed by g_j parallel subsystems $\mathcal{S}_i$ as

$$\mathcal{G}_j : \begin{cases} x_j(k+1) = A_j\, x_j(k) + B_j\, u_j(k) + E_j\, v_j(k), \\ \\ y_j(k) = C_j\, x_j(k), \quad w_j(k) = F_j\, x_j(k), \end{cases} \tag{3.63}$$

where $E_j = [E_{\pi j}\ E_{\sigma j}]$, $v_j^T = [v_{\pi j}^T\ v_{\sigma j}^T]$, $F_j = [F_{\pi j}\ F_{\sigma j}]$, $w_j^T = [w_{\pi j}^T\ w_{\sigma j}^T]$. Specifically, $A_j = \mathrm{diag}[A_i]_1^{g_j}$, $B_j = \mathrm{diag}[B_i]_1^{g_j}$, $C_j = \mathrm{diag}[C_i]_1^{g_j}$, $E_{\pi j} = \mathrm{diag}[E_{\pi ji}]_1^{g_j}$, $F_{\pi j} =$

$\text{diag}[F_{\pi ji}]_1^{g_j}$, $E_{\sigma j} = \text{diag}[E_{\sigma ji}]_1^{g_j}$, $F_{\sigma j} = \text{diag}[F_{\sigma ji}]_1^{g_j}$, $x_j^T = [x_{j1}^T \ldots x_{jg_j}^T]$, $u_j^T = [u_{j1}^T \ldots u_{jg_j}^T]$, $y_j^T = [y_{j1}^T \ldots y_{jg_j}^T]$, $v_{\sigma j}^T = [v_{\sigma j1}^T \ldots v_{\sigma jg_j}^T]$, $w_{\sigma j}^T = [w_{\sigma j1}^T \ldots w_{\sigma jg_j}^T]$, $v_{\pi j}^T = [v_{\pi j1}^T \ldots v_{\pi jg_j}^T]$, $w_{\pi j}^T = [w_{\pi j1}^T \ldots w_{\pi jg_j}^T]$.

Parallel Connections within One Unit: The parallel coupling vectors $v_{\pi ji}$ and $w_{\pi ji}$ of all subsystems $\mathcal{S}_i$ belonging to a unit $\mathcal{G}_j$ are assumed, without loss of generality, having the same size. If there is only one parallel signal, the block diagram of the parallel connections within a unit $\mathcal{G}_j$ having three subsystems ($g_j = 3$) is given in Figure 3.2. The divider operator at $v_{\pi\pi j}$ in this figure represents the splitting of $v_{\pi\pi j}$ into $v_{\pi j}^{(\ell)}$.

Two new signals, $v_{\pi\pi j}$ and $w_{\pi\pi j}$ are introduced here. While the outputs $w_{\pi j}$ are sum up to become $w_{\pi\pi j}$, the input $v_{\pi\pi j}$ is split up into $v_{\pi j}$. Their relationships are represented by two matrices Ψ_{v_j} and Ψ_{w_j}, which are defined as

$$\text{(PC)} \quad \begin{aligned} v_{\pi\pi j} &\triangleq v_{\pi j1} + \cdots + v_{\pi jg_j} := \Psi_{v_j} v_{\pi j}, \\ w_{\pi\pi j} &\triangleq w_{\pi j1} + \cdots + w_{\pi jg_j} := \Psi_{w_j} w_{\pi j}, \end{aligned} \tag{3.64}$$

where $v_{\pi j}^T = [v_{\pi j1}^T \ldots v_{\pi jg_j}^T], w_{\pi j}^T = [w_{\pi j1}^T \ldots w_{\pi jg_j}^T]$.

In the parallel redundant configuration, where the material or energy flows are split into smaller flows to each subsystems, it is impossible to have a constant splitting ratio at all time, but rather dynamic ratios. The splitting ratios between $v_{\pi ji}$ of $v_{\pi j}$ should, therefore, be *unknown*. Due to these unknown splitting ratios of parallel subsystems, the connectivity of the large-scale interconnection process can not be established on the basis of subsystems. In later Sections, we tackle this issue by establishing the dissipativity criteria for a unit consisting of parallel subsystems, i.e the open-loop subsystems are not necessarily dissipative, but only their auspice unit

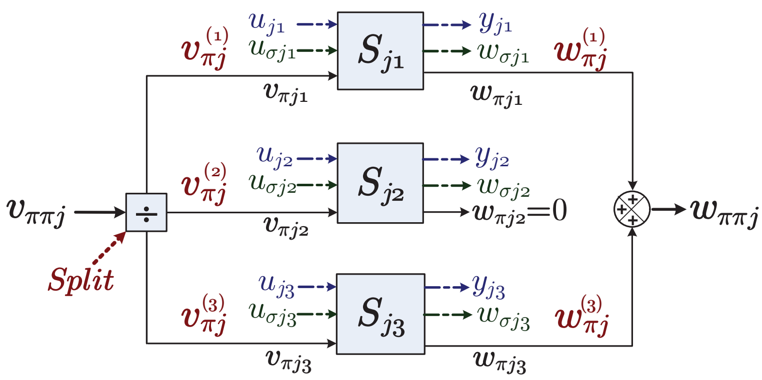


Figure 3.2 : Parallel Splitting of a Unit $\mathcal{G}_j$ having Three Subsystems.

is. It is noted here that the common input and output vectors, $v_{\pi\pi j}$ and $w_{\pi\pi j}$, of parallel connections are involved in the connection processes, instead of $v_{\pi j}$ and $w_{\pi j}$.

In the following, M (of Σ) denotes the diagonal matrix $\mathrm{diag}[M_j]_1^h$, and v_* denotes the stacking vector $[{v_{*1}}^T \dots {v_{*h}}^T]^T$ (unit subscripts j vanish). The large-scale system Σ is represented by the block diagonal system formed by h diagonal units $\mathcal{G}_j$ (or h_Σ subsystems $\mathcal{S}_i$), and the large-scale interconnection processes $H_\sigma, H_{\pi\pi}$ as

$$\Sigma : \begin{cases} x(k+1) = A\,x(k) + B\,u(k) + Ev(k) \\ y(k) = Cx(k),\ w(k) = Fx(k), \\ v_{\pi\pi}(k) = H_{\pi\pi}\,w_{\pi\pi}(k),\ v_\sigma(k) = H_\sigma\,w_\sigma(k). \end{cases} \tag{3.65}$$

where $E = [E_\pi\ E_\sigma],\ F = [F_\pi\ F_\sigma],\ v^T = [v_\pi^T\ v_\sigma^T],\ w^T = [w_\pi^T\ w_\sigma^T]$. Specifically, $A = \mathrm{diag}[A_j]_1^h,\ B = \mathrm{diag}[B_j]_1^h,\ C = \mathrm{diag}[C_j]_1^h,\ E_\pi = \mathrm{diag}[E_{\pi j}]_1^h,\ F_\pi = \mathrm{diag}[F_{\pi j}]_1^h,\ E_\sigma = \mathrm{diag}[E_{\sigma j}]_1^h,\ F_\sigma = \mathrm{diag}[F_{\sigma j}]_1^h,\ x^T = [{x_1}^T \dots x_h^T],\ u^T = [{u_1}^T \dots u_h^T],\ y^T = [y_1^T \dots y_h^T],$ $v_\pi^T = [v_{\pi 1}^T \dots v_{\pi h}^T],\ w_\pi^T = [w_{\pi 1}^T \dots w_{\pi h}^T],\ v_{\pi\pi}^T = [v_{\pi\pi 1}^T \dots v_{\pi\pi h}^T],\ w_{\pi\pi}^T = [w_{\pi\pi 1}^T \dots w_{\pi\pi h}^T],$ $v_\sigma^T = [v_{\sigma 1}^T \dots v_{\sigma h}^T],\ w_\sigma^T = [w_{\sigma 1}^T \dots w_{\sigma h}^T].$

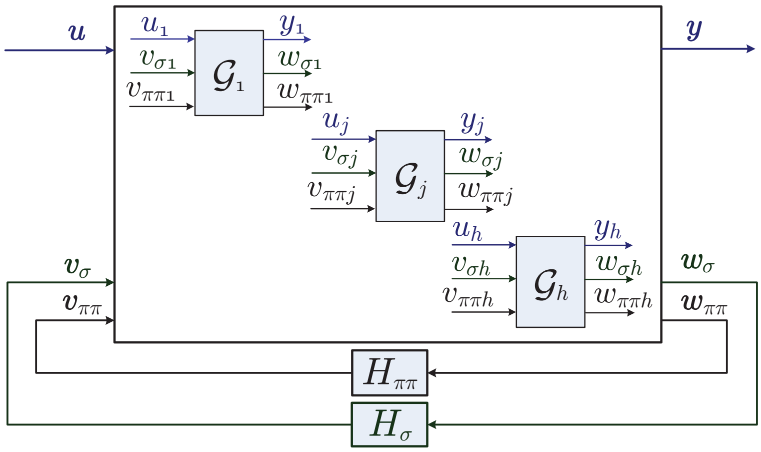


Figure 3.3 : Block Diagram of the Large-Scale System Σ on the Basis of Units $\mathcal{G}_j$.

The parallel connection process inside Σ is as follows:

$$v_{\pi\pi}(k) = \Psi_v \, v_\pi(k), \;\; w_{\pi\pi}(k) = \Psi_w \, w_\pi(k), \tag{3.66}$$

where $\Psi_v := \mathrm{diag}[\Psi_{v_j}]_1^h$, $\Psi_w := \mathrm{diag}[\Psi_{w_j}]_1^h$.

The interconnections between units and subsystems are specified by the interconnection matrices $H_{\pi\pi}$ and H_σ respectively. The elements of $H_{\pi\pi}$ and H_σ are zero or one only. The block diagrams of the large-scale system Σ from the unit perspective is depicted by Figure 3.3.

In a decentralised architecture, the stand-alone subsystem model (when all coupling inputs and outputs vanish: $v_i = 0$, $w_i = 0$) is also requisite, which is represented by

$$\mathcal{S}_i^s : \begin{cases} x_i(k+1) = A_i \, x_i(k) + B_i \, u_i(k), \\[2ex] y_i(k) = C_i \, x_i(k). \end{cases} \tag{3.67}$$

It is assumed that the changing-over between the duty and standby subsystems within one unit is smooth, i.e. state jumps will not incur due to the changing-over operations. The fault-tolerant controls for duty and standby subsystems, and the distributed optimisation schemes for parallel operations within one unit, if there are any, are regarded as internal activities of that unit.

As mentioned in the Introduction chapter, the issue of unknown splitting ratios in parallel subsystems is tackled by having a combined solution, wherein the APRC is developed for each individual subsystem $\mathcal{S}_i$ while the dissipativity criterion is derived for the unit $\mathcal{G}_j$ consisting of g_j subsystems $\mathcal{S}_i$ in parallel.

A unit $\mathcal{G}_j$ is said to be quadratically dissipative with respect to the quadratic supply rate $\xi_j(y_{j\mathrm{D}}, u_{j\mathrm{D}})$ defined as,

$$\xi_j\big(y_{j\mathrm{D}}(k), u_{j\mathrm{D}}(k)\big) \triangleq \begin{bmatrix} y_{j\mathrm{D}}(k) \\ u_{j\mathrm{D}}(k) \end{bmatrix}^T \begin{bmatrix} Q_j & S_j \\ S_j^T & R_j \end{bmatrix} \begin{bmatrix} y_{j\mathrm{D}}(k) \\ u_{j\mathrm{D}}(k) \end{bmatrix}, \tag{3.68}$$

where $y_{j\mathrm{D}}^T := [y_j^T w_{\pi\pi j}^T w_{\sigma j}^T]$, $u_{j\mathrm{D}}^T := [u_j^T v_{\pi\pi j}^T v_{\sigma j}^T]$, the multiplier matrices Q_j, S_j, R_j are block diagonal of the form

$$Q_j := \mathrm{diag}\{Q_{11j}, Q_{22j}, Q_{33j}\}, S_j := \mathrm{diag}\{S_{11j}, S_{22j}, S_{33j}\}, R_j := \mathrm{diag}\{R_{11j}, R_{22j}, R_{33j}\},$$

in which each block element has a block diagonal structure corresponding to subsystem dimensions, with symmetric Q_j and R_j, if there exists a nonnegative C^1 function, addressed as storage function, $V_j(x_j(k))$, $V_j(0) = 0$, such that for all $u_{j\mathrm{D}}$ and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied irrespectively of the initial value of the state $x_j(0)$:

$$V_j\big(x_j(k+1)\big) - \tau_j V_j\big(x_j(k)\big) \leq \xi_j\big(y_{j\mathrm{D}}(k), u_{j\mathrm{D}}(k)\big),\ 0 < \tau_j < 1. \tag{3.69}$$

The square storage function of the form $V_j\big(x(k)\big) = x_j(k)^T P_j x_j(k), P_j = P_j^T \succ 0$, is considered in this chapter. Define

- The dissipativity matrices of the large-scale system Σ as $Q_{11} := \mathrm{diag}[Q_{11j}]_1^h$, $S_{11} := \mathrm{diag}[S_{11j}]_1^h$, $R_{11} := \mathrm{diag}[R_{11j}]_1^h$, $Q_{22} := \mathrm{diag}[Q_{22j}]_1^h$, $S_{22} := \mathrm{diag}[S_{22j}]_1^h$, $R_{22} := \mathrm{diag}[R_{22j}]_1^h$, $Q_{33} := \mathrm{diag}[Q_{33j}]_1^h$, $S_{33} := \mathrm{diag}[S_{33j}]_1^h$, $R_{33} := \mathrm{diag}[R_{33j}]_1^h$;

- The dissipativity matrices of unit $\mathcal{G}_j$ as $Q_{11j} := \mathrm{diag}[Q_{11i}]_1^{g_j}$, $Q_{22j} := \mathrm{diag}[Q_{22i}]_1^{g_j}$, $Q_{33j} := \mathrm{diag}[Q_{33i}]_1^{g_j}$, $S_{11j} := \mathrm{diag}[S_{11i}]_1^{g_j}$, $S_{22j} := \mathrm{diag}[S_{22i}]_1^{g_j}$, $S_{33j} := \mathrm{diag}[S_{33i}]_1^{g_j}$, $R_{11j} := \mathrm{diag}[R_{11i}]_1^{g_j}$, $R_{22j} := \mathrm{diag}[R_{22i}]_1^{g_j}$, $R_{33j} := \mathrm{diag}[R_{33i}]_1^{g_j}$, where Q_{11i}, R_{11i} are symmetric matrices of dimensions $q_i \times q_i, m_i \times m_i$ respectively, Q_{22i}, R_{22i} are symmetric matrices of dimensions $q_{w_{\pi j}} \times q_{w_{\pi j}}, m_{v_{\pi j}} \times m_{v_{\pi j}}$ respectively, Q_{33i}, R_{33i} are symmetric matrices of dimensions $q_{w_{\sigma ji}} \times q_{w_{\sigma ji}}, m_{v_{\sigma ji}} \times m_{v_{\sigma ji}}$ respectively, S_{11i} are rectangular matrices of $q_i \times m_i$ dimensions, S_{22i} are rectangular matrices of $q_{w_{\pi j}} \times m_{v_{\pi j}}$ dimensions, S_{33i} are rectangular matrices of $q_{w_{\sigma ji}} \times m_{v_{\sigma ji}}$ dimensions;

- The multiplier matrices for the APRC of the large-scale diagonal controller as $Q_c^k := \mathrm{diag}[Q_{ic}^k]_{j=1\ldots h, i=1\ldots g_j}$, $S_c^k := \mathrm{diag}[S_{ic}^k]_{j=1\ldots h, i=1\ldots g_j}$, $R_c^k := \mathrm{diag}[R_{ic}^k]_{j=1\ldots h, i=1\ldots g_j}$.

The interconnection stabilisability condition for the large-scale system based on the APRC is stated next.

Theorem 7 *Let* $0 < \tau_j < 1$. *Suppose that*

(*i*) *The following LMIs are feasible in* $\boldsymbol{P}_j, \boldsymbol{Q}_{11i}, \boldsymbol{S}_{11i}, \boldsymbol{R}_{11i}, \boldsymbol{Q}_{22j}, \boldsymbol{S}_{22j}, \boldsymbol{R}_{22j}, \boldsymbol{Q}_{33j}, \boldsymbol{S}_{33j}, \boldsymbol{R}_{33j}$:

$$\boldsymbol{Q}_{\mathcal{L}\pi} \prec 0, \quad \boldsymbol{Q}_{\mathcal{L}\sigma} \prec 0, \tag{3.70a}$$

$$\begin{bmatrix} \boldsymbol{Q}_{j\mathcal{L}} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} & \boldsymbol{M}_{14} \\ * & \boldsymbol{M}_{22} & {B_j}^T \boldsymbol{P}_j E_{\pi j} & {B_j}^T \boldsymbol{P}_j E_{\sigma j} \\ * & * & \boldsymbol{M}_{33} & 0 \\ * & * & * & \boldsymbol{M}_{44} \end{bmatrix} \prec 0, \quad \boldsymbol{P}_j \succ 0,\ \boldsymbol{Q}_{11j} \prec 0,\ \boldsymbol{R}_{11j} \succ 0,$$

$$j = 1 \dots h, i = 1 \dots g_j, \tag{3.70b}$$

where

$$\boldsymbol{M}_{12} := {A_j}^T \boldsymbol{P}_j B_j - C_j^T \boldsymbol{S}_{11j},\ \boldsymbol{M}_{13} := {A_j}^T \boldsymbol{P}_j E_{\pi j} - F_{\pi j}^T \Psi_{w_j}^T \boldsymbol{S}_{22j} \Psi_{v_j},$$
$$\boldsymbol{M}_{14} := {A_j}^T \boldsymbol{P}_j E_{\sigma j} - F_{\sigma j}^T \boldsymbol{S}_{33j},\ \boldsymbol{M}_{22} := {B_j}^T \boldsymbol{P}_j B_j - \boldsymbol{R}_{11j},$$
$$\boldsymbol{M}_{33} := E_{\pi j}^T \boldsymbol{P}_j E_{\pi j} - {\Psi_{v_j}}^T \boldsymbol{R}_{22j} \Psi_{v_j},\ \boldsymbol{M}_{44} := E_{\sigma j}^T \boldsymbol{P}_j E_{\sigma j} - \boldsymbol{R}_{33j},$$
$$\boldsymbol{Q}_{j\mathcal{L}} := A_j^T \boldsymbol{P}_j A_j - \tau_j \boldsymbol{P}_j - C_j^T \boldsymbol{Q}_{11j} C_j - F_{\sigma j}^T \boldsymbol{Q}_{33j} F_{\sigma j} - \ F_{\pi j}^T {\Psi_{w_j}}^T \boldsymbol{Q}_{22j} \Psi_{w_j} F_{\pi j},$$

$$\boldsymbol{Q}_{\mathcal{L}\pi} := \Psi_w^T \boldsymbol{Q}_{22} \Psi_w + \Psi_w^T H_{\pi\pi}^T \boldsymbol{R}_{22} H_{\pi\pi} \Psi_w, + \Psi_w^T \boldsymbol{S}_{22} H_{\pi\pi} \Psi_w + \Psi_w^T H_{\pi\pi}^T \boldsymbol{S}_{22}^T \Psi_w,$$
$$\boldsymbol{Q}_{\mathcal{L}\sigma} := \boldsymbol{Q}_{33} + \boldsymbol{S}_{33} H_\sigma + H_\sigma^T \boldsymbol{S}_{33}^T + H_\sigma^T \boldsymbol{R}_{33} H_\sigma;$$

(ii) *With feasible solution* $R_{11j}, S_{11j}, Q_{11j}$ *of the above system of LMIs* (3.70), $Q_{ic} \equiv -R_{11i},\ S_{ic} \equiv -S_{11i},\ R_{ic} \equiv -Q_{11i}$ *and the following APRC-based constraint in* $\boldsymbol{u}_i^k$ *is recursively feasible for all* $k > k_0$*:*

$$\boldsymbol{u}_i^k Q_{ic} \boldsymbol{u}_i^k + 2 y_i^{kT} S_{ic}^T \boldsymbol{u}_i^k \geq \frac{1}{2} \gamma_i(k) \varphi_i^{k-1} - y_i^{kT} R_{ic} y_i^k, \tag{3.71}$$

with $0 \leq \gamma_i(k) < 1$, *for* $i = 1, 2, \dots, h$.

Then any h *controls* $\tilde{u}_i^k,\ i = 1, 2, \dots, h$, *feasible to* (3.71) *stabilise* Σ (3.60).

Proof:

(a) *Quadratical Dissipativity of Unit* $\mathcal{G}_j$:

By substituting the model of $\mathcal{G}_j$ (3.63) and the parallel connections of (3.64) into the dissipation inequality of (3.69), it follows that $\mathcal{G}_j$ is quadratically dissipative w.r.t the supply rate (3.68) (i.e. with respect to the output and input pair of $y_{j\mathrm{D}} = (y_j^T, (\Psi_{w_j} w_{\pi j})^T, w_{\sigma j}^T)^T$, $u_{j\mathrm{D}} = (u_j^T, (\Psi_{v_j} v_{\pi j})^T, v_{\sigma j}^T)^T$ if and only if LMI (3.70b) holds (see, e.g., [16]). Thus (3.69) is verified for all $(y_{j\mathrm{D}}, u_{j\mathrm{D}})$.

(b) *Stabilisability*: Define $\xi_c^k := u(k)^T Q_c^k u(k) + 2y(k)^T S_c^{kT} u(k) + y(k)^T R_c^k y(k)$. Under the condition (ii), there are unconstrained $u_i(k)$ feasible to (3.11).

Using (3.69) and the diagonality of $Q_{11}, Q_{22}, Q_{33}, S_{11}, S_{22}, S_{33}, R_{11}, R_{22}, R_{33}$, we obtain for $V\big(x(k)\big) := \sum_{j=1}^h V_j\big(x_j(k)\big)$,

$$V\big(x(k+1)\big) - \tau V\big(x(k)\big) \leq \xi\big((y^k, w_{\pi\pi}^k, w_\sigma^k), (u^k, v_{\pi\pi}^k, v_\sigma^k)\big),$$

where

$$\xi\big((y^k, w_{\pi\pi}^k, w_\sigma^k), (u^k, v_{\pi\pi}^k, v_\sigma^k)\big) = (y^{kT} Q_{11} y^k + 2y^{kT} S_{11} u^k + u^{kT} R_{11} u^k)$$
$$+(w_{\pi\pi}^{kT} Q_{22} w_{\pi\pi}^k + 2w_{\pi\pi}^{kT} S_{22} v_{\pi\pi}^k + v_{\pi\pi}^{kT} R_{22} v_{\pi\pi}^k) + (w_\sigma^{kT} Q_{33} w_\sigma^k + 2w_\sigma^{kT} S_{33} v_\sigma^k + v_\sigma^{kT} R_{33} v_\sigma^k).$$

Then, due to (3.66), (3.70a) and the APRC w.r.t. ξ_{ic} (refer to Theorem 1 proof), it follows that

$$V(x(k+1)) \leq \tau V(x(k)) + \frac{1}{2}\gamma^k |\varphi(k-1)|.$$

Similar to Theorem 1, $V(x(k)) \to 0$ as $k \to +\infty$. ■

We have shown that the issue of unknown splitting ratios have been solved by applying the common input and output vectors $(v_{\pi\pi j}(k), w_{\pi\pi j}(k))$ representing the sums of parallel coupling vectors in the supply rates of dissipative units. This is a parallel masking method. The present stabilisability condition is fully decentralised, yet naturally eliminates any conservativeness caused by the fixed splitting ratios of material or energy flows of dynamically-coupled subsystems.

3.7 Numerical Simulations

Numerical simulations with Matlab Yalmip toolbox and MPC toolbox have been conducted with different scenarios to investigate the effectiveness of the present approaches.

3.7.1 Illustrative Example 1

A. Without Parallel Splitting Connections

The following state space models with state feedbacks, but not output feedbacks with state estimations, are considered in this numerical example:

$$\dot{x}(t) = \begin{bmatrix} A_1 & & \\ & A_2 & \\ & & A_3 \end{bmatrix} x(t) + \begin{bmatrix} B_1 & & \\ & B_2 & \\ & & B_3 \end{bmatrix} u(t) + \begin{bmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{bmatrix} v(t), \tag{3.72}$$

$$w(t) = \begin{bmatrix} F_1 & & \\ & F_2 & \\ & & F_3 \end{bmatrix} x(t), \qquad v(t) = Hw(t), \tag{3.73}$$

$$y(t) = \begin{bmatrix} C_1 & & \\ & C_2 & \\ & & C_3 \end{bmatrix} u(t), \tag{3.74}$$

where

$$A_1 = \begin{bmatrix} -1.4 & 0.3 & 0 \\ 0 & -1.8 & 1.5 \\ 0.1 & -2.7 & 1.06 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, E_1 = \begin{bmatrix} 5.1 \\ 0 \\ 3.8 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.76 & 0 & 0.25 \\ .48 & -0.56 & 0 \\ 0 & 0.2 & -0.34 \end{bmatrix}, B_2 = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ E_2 = \begin{bmatrix} 4.2 \\ 0 \\ 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -4.3 & 5.9 \\ -1.8 & 2.7 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 6.95 \\ 0 \end{bmatrix},$$

$$C_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F_3 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \ H = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The predictive horizons $N_i \equiv 6, \ i = 1, 2, 3,$ have been chosen for all subsystems. The MPC weighting matrices are set as follows:

$$\mathcal{R}_1 = \mathsf{diag}\{0.2, 0.2\}, \ \mathcal{Q}_1 = \mathsf{diag}\{1, 1, 2\}, \ \mathcal{R}_2 = \mathsf{diag}\{0.2, 0.2, 0.2\},$$
$$\mathcal{Q}_2 = \mathsf{diag}\{1.5, 2.4, 1.6\}, \ \mathcal{R}_3 = \mathsf{diag}\{0.5, 0.5\}, \ \mathcal{Q}_3 = \mathsf{diag}\{1, 3\}.$$

The constraints on the control inputs are set up as

$$\|u_1\|_2 \leqslant 2, \|u_2\|_2 \leqslant 2, \|u_3\|_2 \leqslant 2.$$

The initial states are as follows:

$$x_1(0) = \begin{bmatrix} 8 \\ -7 \\ -8 \end{bmatrix}, \ x_2(0) = \begin{bmatrix} -6 \\ 8 \\ -1.5 \end{bmatrix}, \ x_3(0) = \begin{bmatrix} .55 \\ -1.4 \end{bmatrix}.$$

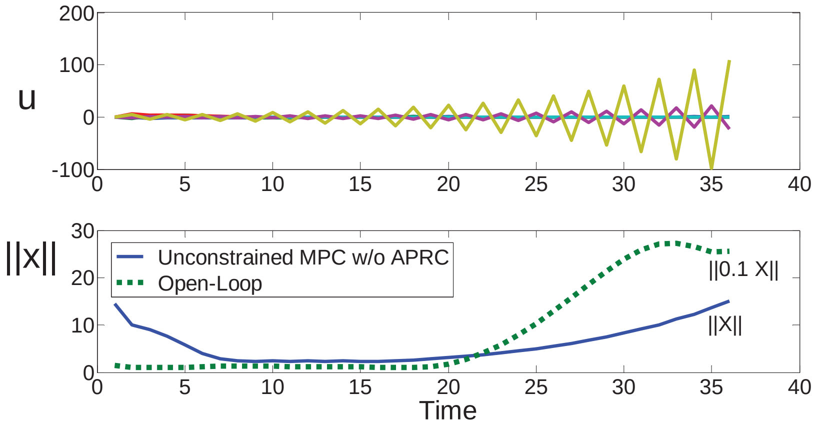


Figure 3.4 : Decentralised MPCs for the Pre-Desilication Unit Without both Stabilising Constraints and Control Bounds - Time Responses.

The unconstrained and decentralised MPCs are not able to stabilise the interconnected system, as shown in Figure 3.4. $\|x\|$ in this and the following figures denotes the ℓ_2-norm $\|x\|_2$. This system is open-loop unstable. Its discrete-time eigenvalues are as follows:

$$1.1634 + 0.2051i,\ 1.1634 - 0.2051i,\ 0.5411 + 0.1302i,$$
$$0.5411 - 0.1302i,\ 0.8628 + 0.3398i,\ 0.8628 - 0.3398i,$$
$$0.7623 + 0.3412i,\ \text{and}\ 0.7623 - 0.3412i.$$

Figure 3.5 depicts the plant-output time responses of three interconnection subsystems regulated by control constrained and decentralised MPCs. In this simulation, the stabilising constraints are not used. The system is clearly unstable, asymptotically.

Decentralised MPC with APRC-Based Stabilising Constraints:

When the APRC-based decentralised optimisation problem is employed, the time

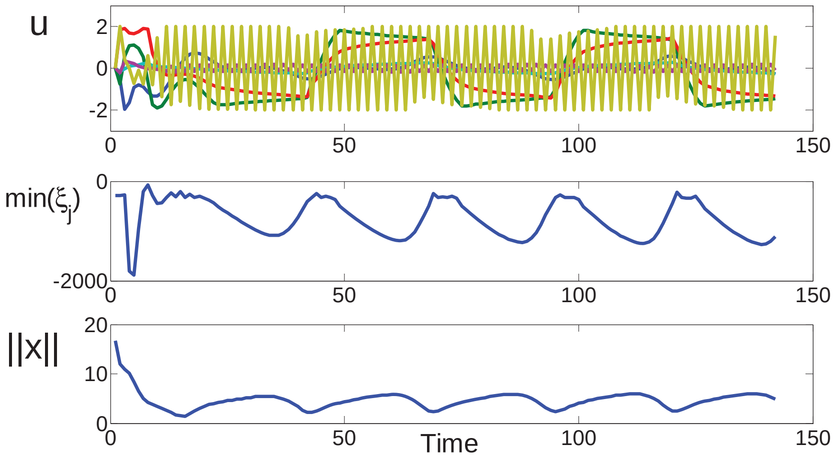


Figure 3.5 : Decentralised MPCs for the Pre-Desilication Unit Without Stabilising Constraints - Time Responses.

responses are shown in Figure 3.6, indicating clearly a stabilised interconnected system. The multiplier matrices Q_{ic}, S_{ic}, R_{ic} have been redesigned online using LMIs in Theorem 3 once $\xi_{ic}^k < 0$. Simulation results from Theorem 2 are quite similar to those from Theorem 3 in this particular example. The offline multiplier matrices are as follows:

$$Q_{1_{22}} = \begin{bmatrix} -11.7867 & -34.2257 & 0.3111 \\ -34.2257 & 21.5558 & -55.3959 \\ 0.3111 & -55.3959 & -72.5118 \end{bmatrix}, Q_{2_{22}} = \begin{bmatrix} -32.6274 & -11.8166 & -0.7111 \\ -11.8166 & -4.6037 & -3.8606 \\ -0.7111 & -3.8606 & -17.3838 \end{bmatrix},$$

$$Q_{3_{22}} = \begin{bmatrix} 4.6137 & -29.6772 \\ -29.6772 & -49.3944 \end{bmatrix},$$

$$S_{1_{22}} = \begin{bmatrix} 5.7491 \\ -0.5833 \\ -13.5016 \end{bmatrix}, S_{2_{22}} = \begin{bmatrix} 5.9834 & 0.0064 & -3.3183 \\ 0.0064 & 22.5096 & -0.5098 \\ -3.3183 & -0.5098 & 24.4831 \end{bmatrix}, S_{3_{22}} = \begin{bmatrix} -4.5162 & 1.0599 \\ 1.0599 & -11.0225 \end{bmatrix},$$

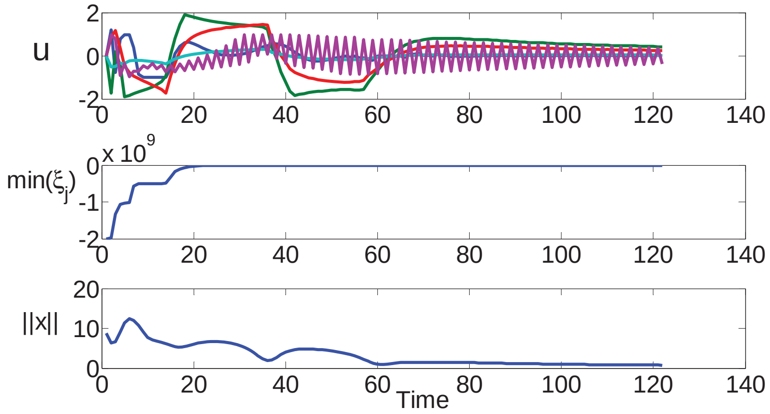


Figure 3.6 : APRC-Based Decentralised MPCs of the Pre-Desilication Unit with Online Updated Multiplier Matrices - Time Responses.

$$R_{1_{22}} = 77.0619, R_{2_{22}} = \begin{bmatrix} 75.6640 & 0.1853 & 0.5272 \\ 0.1853 & 80.2649 & -0.1551 \\ 0.5272 & -0.1551 & 83.0173 \end{bmatrix}, R_{3_{22}} = \begin{bmatrix} 76.3570 & -0.3167 \\ -0.3167 & 76.9403 \end{bmatrix}.$$

$$x_1(0) = \begin{bmatrix} -3 \\ 3 \\ -6 \end{bmatrix}, \ x_2(0) = \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix}, \ x_3(0) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

The flexible and decentralised multiplier matrices in Theorem 4 has produced stabilised trajectories as shown in Figure 3.7.

B. With Parallel Splitting Connections

Now, the simulations with parallel splitting units are provided and compared to those without parallel splitting connections. Considering $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_2$ being interconnected as per Figure 3.1. The state space models as as follows:

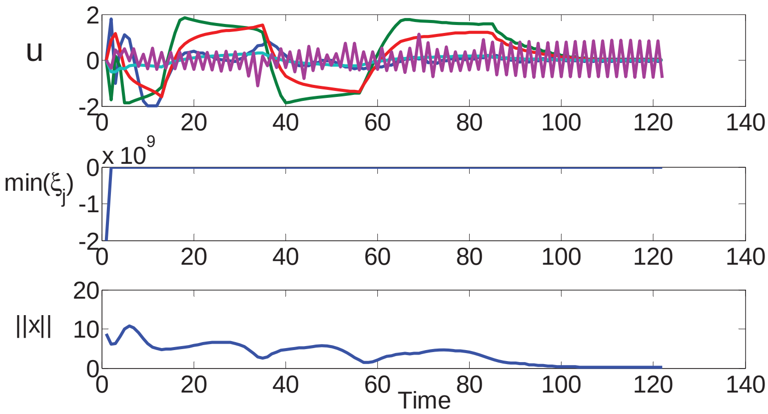


Figure 3.7 : APRC-Based Decentralised MPCs of the Pre-Desilication Unit with Flexible Multiplier Matrices - Time Responses.

$$
\mathcal{G}_1: \qquad A_1 = \mathsf{diag}\left(\begin{bmatrix} -1.4 & .3 & 0 \\ 0 & -1.8 & 1.5 \\ .1 & -2.7 & 1.06 \end{bmatrix}, \begin{bmatrix} -1.6 & .2 & 0 \\ 0 & -2.1 & 1.7 \\ .3 & -1.8 & .9 \end{bmatrix} \right),
$$

$$
B_1 = \mathsf{diag}[(0,0,1)^T, (0,0,1)^T], E_{\Delta 1} = \mathsf{diag}[(2,0,2)^T,\ (2,0,2)^T],\ E_{\Theta 1} = 0,
$$

$$
C_1 = I_6,\ F_{\Delta 1} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},\ F_{\Theta 1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},
$$

$$
\mathcal{G}_2: \qquad A_2 = \begin{bmatrix} -.76 & 0 & .25 \\ .48 & -.56 & 0 \\ 0 & .2 & -.34 \end{bmatrix}, B_2 = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ E_{\Delta 2} = 0,\ E_{\Theta 2} = \begin{bmatrix} 3.6 \\ 0 \\ 0 \end{bmatrix},
$$

$$
C_2 = I_3, F_{\Delta 2} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},\ F_{\Theta 2} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},
$$

$$
\mathcal{G}_3: \qquad A_3 = \begin{bmatrix} -4.3 & 5.9 \\ -1.8 & 2.7 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_{\Delta 3} = 0,\ E_{\Theta} = \begin{bmatrix} 6 \\ 0 \end{bmatrix},
$$

$$
C_3 = I_2,\ F_{\Delta 3} = \begin{bmatrix} 1 & 0 \end{bmatrix},\ F_{\Theta 3} = \begin{bmatrix} 0 & 0 \end{bmatrix},
$$

$$H_{\Delta} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \quad H_{\Theta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The predictive horizons are set with $N_1 = N_2 = N_3 = 6$. The weighting matrices $\mathcal{Q}_1 = \mathsf{diag}\{1, 1, 2; 1, 1, 1.2\}$, $\mathcal{R}_1 = \mathsf{diag}\{2, 0.6\}$, $\mathcal{Q}_2 = \mathsf{diag}\{1.5, 2.4, 1.6\}$, $\mathcal{R}_2 = \mathsf{diag}\{0.5, 0.5, 0.5\}$, $\mathcal{Q}_3 = \mathsf{diag}\{1, 3\}$, $\mathcal{R}_3 = \mathsf{diag}\{0.5, 1\}$, and the initial state vectors $x_1(0) = [2\ \ -2\ \ -3\ \ -2\ \ 1.2\ \ -0.5]^T$, $x_2(0) = [-3\ \ 2\ \ -1]^T$, $x_3(0) = [1\ \ -0.8]^T$ are chosen. The following bounds on the control inputs are considered in this simulation study: $\|u_{1_1}\|_2 \leqslant 2$, $\|u_{1_2}\|_2 \leqslant 2$, $\|u_2\|_2 \leqslant 2$, $\|u_3\|_2 \leqslant 2$.

Unconstrained Decentralised MPCs

In the first simulation, the decentralised MPCs alone, without the ASPRC-based stabilising constraints, are unable to stabilise the system when the control bounds are also absent, as shown in Figure 3.8.

Decentralised MPCs without ASPRC

The trends produced by the decentralised MPC with control bounds, but without the ASPRC-based stabilising constraints, are then given in Figure 3.9. The system is clearly not stabilised asymptotically. It also shows, in the same figure, that the large-scale system is open-loop unstable with $\|x\|_2$ multiplied by 10^{-4}. It is noted that $\|x\|$ is printed in this and the following figures, instead of $\|x\|_2$.

Now, a random variable is deployed to simulate the unknown splitting ratios, as follows:

$$E_{\pi 1} = \mathsf{diag}\{\alpha(k) * 2 * (2, 0, 2)^T,\ (1 - \alpha(k)) * 2 * (2, 0, 2)^T\},\ \alpha(k) \in (0, 1). \quad (3.75)$$

The trends for two different random sets of $\alpha(k)$ are given in Figures 3.10 and 3.11.

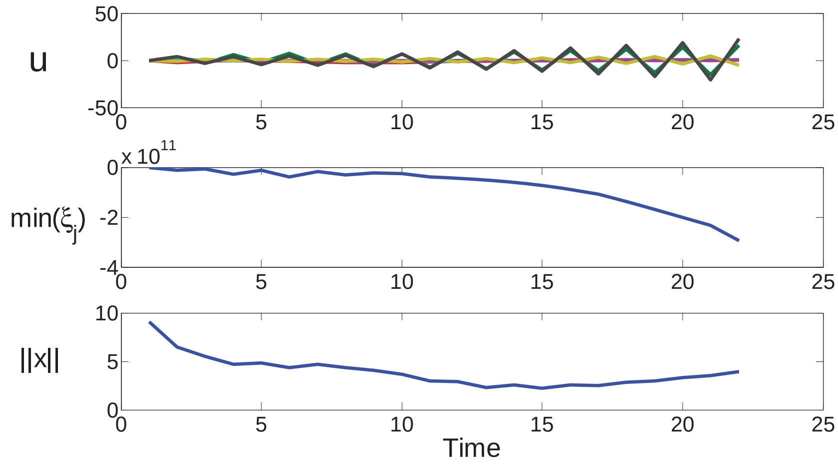


Figure 3.8 : Parallel Splitting Systems - Decentralised MPC without both Stabilising Constraints and Control Bounds - Time Responses.

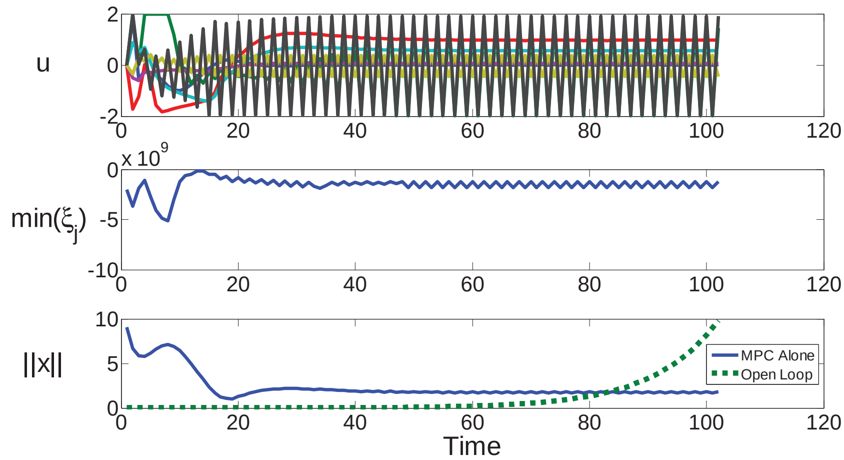


Figure 3.9 : Parallel Splitting Systems - Decentralised MPC without Stabilising Constraints - Time Responses.

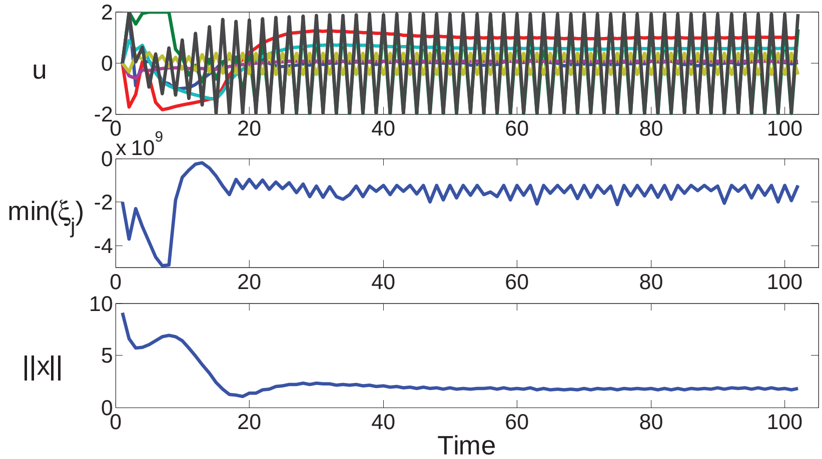


Figure 3.10 : Parallel Splitting Systems - Mixed Connection Structure - Decentralised MPC without Stabilising Constraints - Unknown Splitting Ratios.

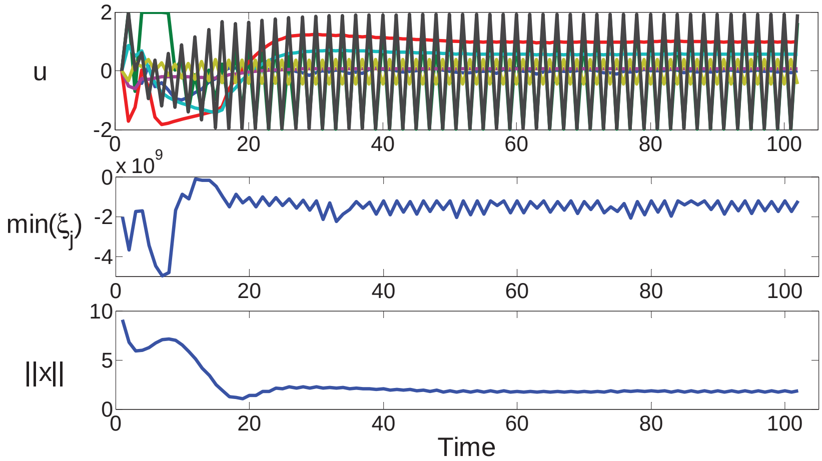


Figure 3.11 : Parallel Splitting Systems - Mixed Connection Structure - Decentralised MPC without Stabilising Constraints - Unknown Splitting Ratios.

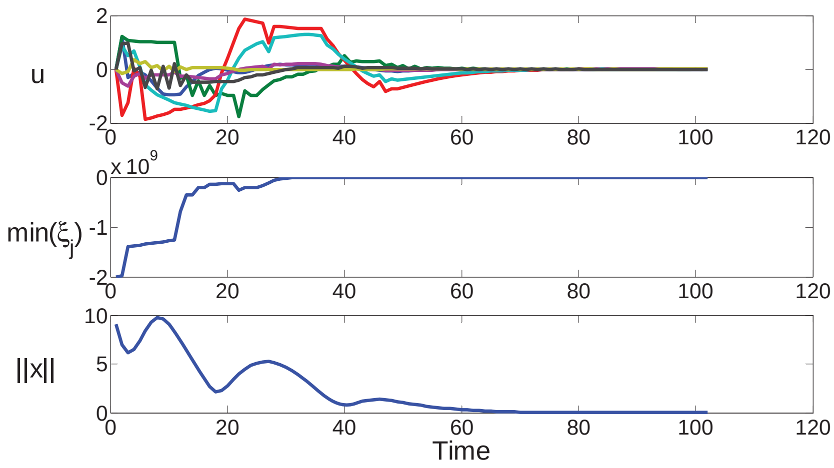


Figure 3.12 : Parallel Splitting Systems - Mixed Connection Structure - Equal Splitting Ratio - ASPRC-Based Decentralised MPC

B.1 *Case Study 1*

B.1.1 Mixed Connection Structure

When ASPRC-based Decentralised MPC is run by subsystems of units 1, 2 and 3, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.12.

The offline multiplier matrices Q_{ji}, S_{ji}, R_{ji} are listed below, with the denotations $R_j = \mathsf{diag}[R_{ji}]_{i=1}^{g_j}, S_j = \mathsf{diag}[S_{ji}]_{i=1}^{g_j}, Q_j = \mathsf{diag}[Q_{ji}]_{i=1}^{g_j}$ as per Theorem 5, in which g_j is the number of subsystems of the unit $\mathcal{G}_j$. The ASPRC coefficients $\gamma_1 = \gamma_2 = \gamma_3 = 0.99$ have been used in this simulation.

$$Q_{1_1} = 10^4 \begin{bmatrix} -1.2689 & -1.4172 & -0.0795 \\ -1.4172 & 9.7890 & -3.6063 \\ -0.0795 & -3.6063 & -3.8168 \end{bmatrix}, \; Q_2 = 10^4 \begin{bmatrix} -2.2824 & -0.2336 & -0.0036 \\ -0.2336 & -1.7564 & -0.1088 \\ -0.0036 & -0.1088 & 9.2822 \end{bmatrix},$$

$$Q_3 = 10^4 \begin{bmatrix} 6.8503 & 0.1993 \\ 0.1993 & -2.9051 \end{bmatrix},$$

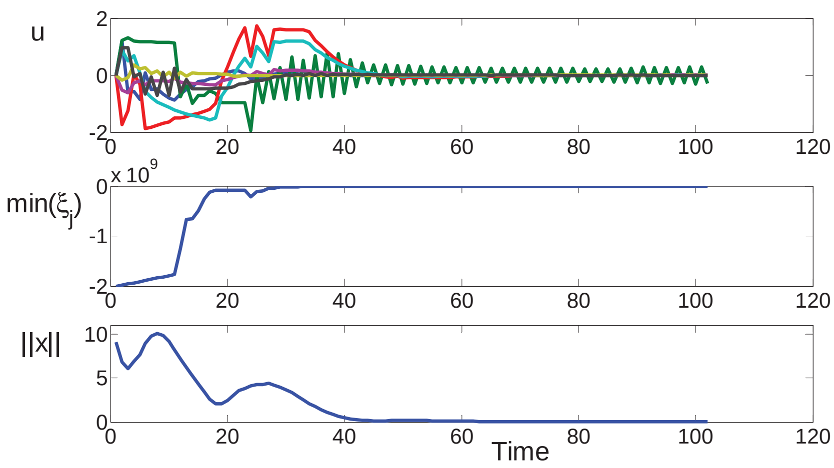


Figure 3.13 : Parallel Splitting Systems - Mixed Connection Structure - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 1.

$$S_{1_1} = 10^3 \begin{bmatrix} 2.4649 \\ 0.7294 \\ -5.0120 \end{bmatrix},\ S_2 = 10^3 \begin{bmatrix} 0.0012 & 0.1490 & 0.0146 \\ 0.1490 & 4.5902 & 0.2324 \\ 0.0146 & 0.2324 & 6.5412 \end{bmatrix},\ S_3 = 10^3 \begin{bmatrix} 0.0018 & 0.1259 \\ 0.1259 & -1.5170 \end{bmatrix},$$

$$R_{1_1} = -3.0399 \times 10^4,\ R_2 = 10^4 \begin{bmatrix} -2.2881 & 0.0003 & -0.0006 \\ 0.0003 & -2.4092 & -0.0030 \\ -0.0006 & -0.0030 & -2.4433 \end{bmatrix},\ R_3 = 10^4 \begin{bmatrix} -2.2744 & 0.0043 \\ 0.0043 & -2.3146 \end{bmatrix}.$$

Now, a random variable $\alpha(k) \in (0, 1)$ as per (3.75) is deployed to simulate the unknown splitting ratios, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.13 and 3.14.

Smooth Control Actions

In this simulation, the weighting matrices of $\mathcal{Q}_1 = \mathsf{diag}\{1, 1, 1; 1, 1, 1\}$, $\mathcal{R}_1 = \mathsf{diag}\{2.5, 2.5\}$, $\mathcal{Q}_2 = \mathsf{diag}\{1, 1, 1\}$, $\mathcal{R}_2 = \mathsf{diag}\{1, 1, 1\}$, $\mathcal{Q}_3 = \mathsf{diag}\{1, 1\}$, $\mathcal{R}_3 = \mathsf{diag}\{1.5, 2.5\}$, and the initial state vectors of $x_1(0) = [6.4\ -5.6\ -6.4,$ $1.6\ -1.4\ -1.6]^T$, $x_2(0) = [-6\ \ 8\ -1.5]^T$, $x_3(0) = [.55\ -1.4]^T$ are set up. The ASPRC coefficients of $\gamma_1 = \gamma_2 = \gamma_3 = 1 - 10^{-12}$ are deployed herein.

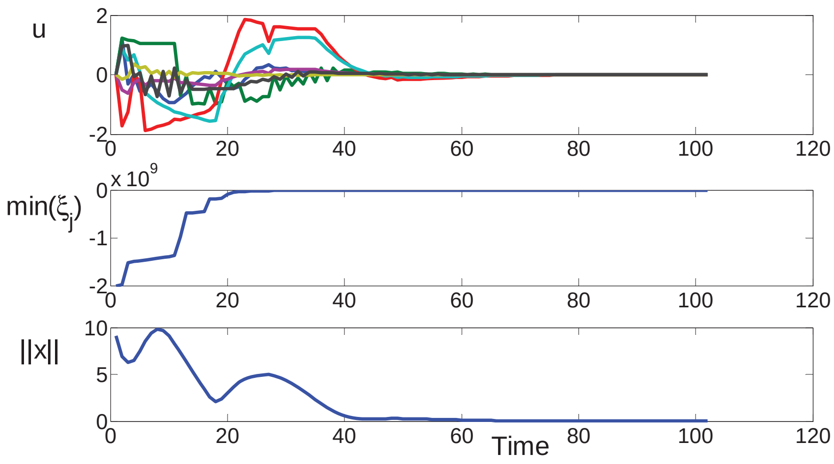


Figure 3.14 : Parallel Splitting Systems - Mixed Connection Structure - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 2.

When ASPRC-based Decentralised MPC is run by subsystems of units 1, 2 and 3, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.15.

Now, a random variable $\alpha(k) \in (0, 1)$ as per (3.75) is deployed to simulate the unknown splitting ratio, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.16 and 3.17. It is clearly that the smoother control actions are traded off by the longer convergent time of states towards the equilibrium.

B.1.2 Parallel Connection Structure Only

Next, consider $\mathcal{G}_2$ and $\mathcal{G}_3$ as one unit, which connects in parallel to unit $\mathcal{G}_1$, and there is no serial connection between them, so H_Δ remains unchanged, but $H_\theta = 0$ with $F_{\Delta 2} = [0\ 0\ 1]$, $F_{\Theta 2} = [0\ 0\ 0]$.

When the ASPRC-based Decentralised MPC is run by subsystems of units 1 and 2, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.18. The system is also stabilised by this parallel-only interconnection structure.

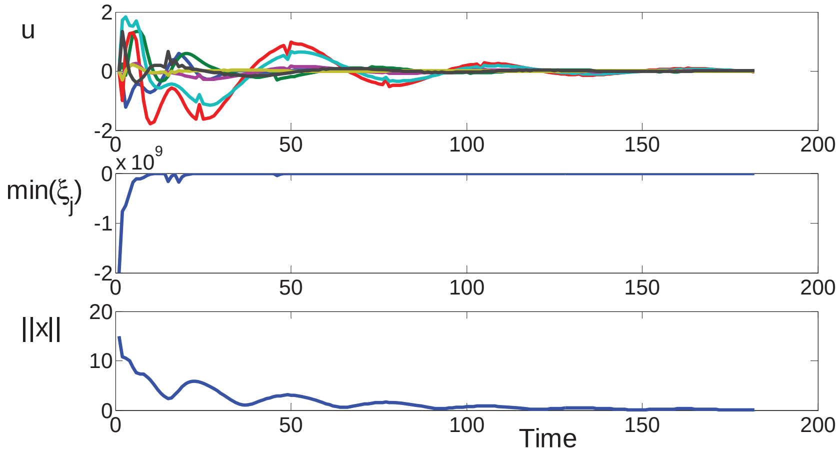


Figure 3.15 : Parallel Splitting Systems - Mixed Connection Structure - ASPRC-Based Decentralised MPC with an Equal Splitting Ratio - Smooth Control.

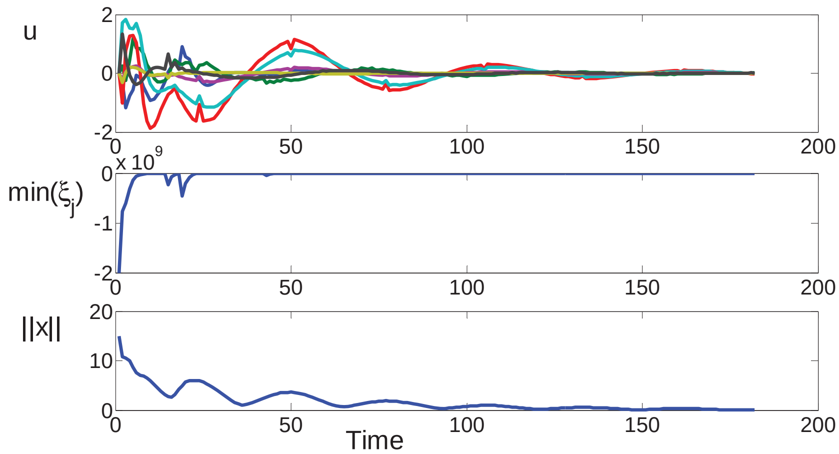


Figure 3.16 : Parallel Splitting Systems - Mixed Connection Structure - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Smooth Control - Simulation 1.

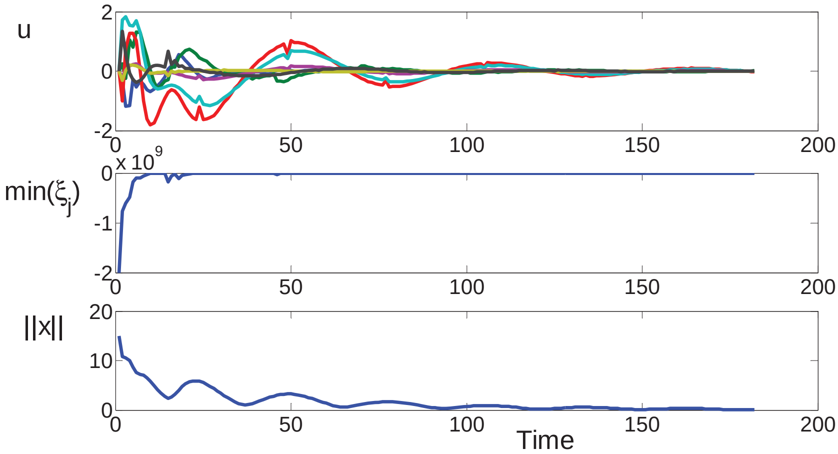


Figure 3.17 : Parallel Splitting Systems - Mixed Connection Structure - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Smooth Control - Simulation 2

Now, a random variable $\alpha(k) \in (0, 1)$ as per (3.75) is deployed to simulate the unknown splitting ratios, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.19 and 3.20.

B.1.3 Serial Connection Structure Only

Lastly in this section, to show the importance of the mixed connection structure models, the multiplier matrices of non-parallel systems are used in the following simulation. They are found when one subsystem inside unit $\mathcal{G}_1$ is absent. The multiplier matrices for two subsystems in this first unit are identical. $\mathcal{G}_1$ state space model is as follows:

$$A_1 = \mathsf{diag}\left(\begin{bmatrix} -1.4 & .3 & 0 \\ 0 & -1.8 & 1.5 \\ .1 & -2.7 & 1.06 \end{bmatrix}, \begin{bmatrix} -1.4 & .3 & 0 \\ 0 & -1.8 & 1.5 \\ .1 & -2.7 & 1.06 \end{bmatrix} \right),$$

$$B_1 = \mathsf{diag}[(0,0,1)^T, (0,0,1)^T],\ E_{\Theta 1} = \mathsf{diag}[(2,0,2)^T, (2,0,2)^T],\ E_{\Delta 1} = 0,\ C_1 = I_6,$$

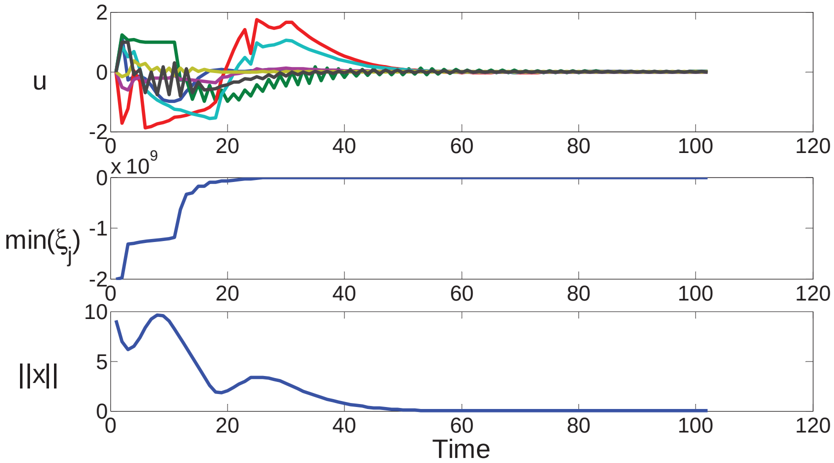


Figure 3.18 : Parallel Splitting Systems - Parallel Connection Structure Only - ASPRC-Based Decentralised MPC with an Equal Splitting Ratio.

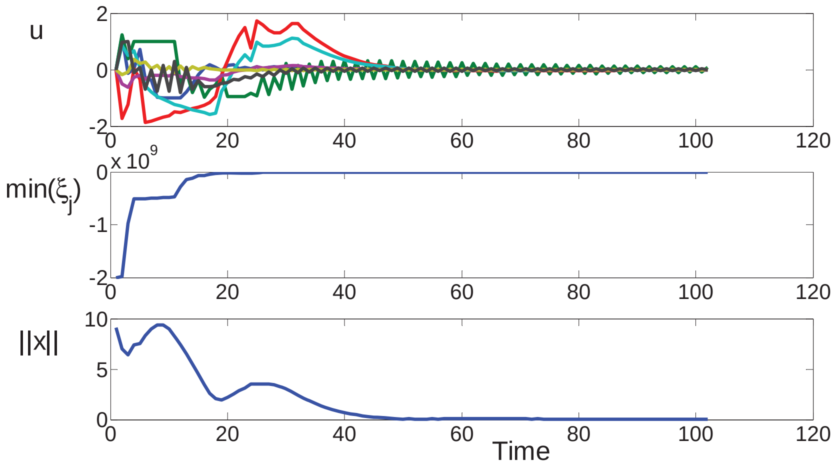


Figure 3.19 : Parallel Splitting Systems - Parallel Connection Structure Only - ASPRC-Based Decentralised MPC with Unknown Splitting Ratio - Simulation 1.

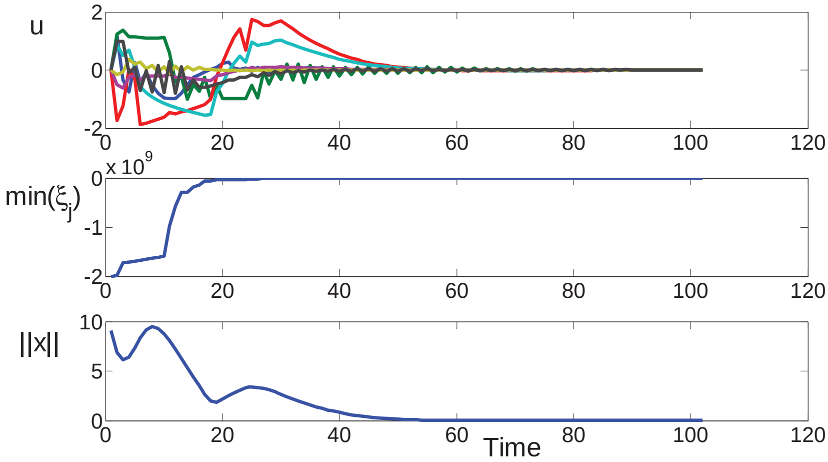


Figure 3.20 : Parallel Splitting Systems - Parallel Connection Structure Only - ASPRC-Based Decentralised MPC with Unknown Splitting Ratio - Simulation 2.

$$F_{\Theta 1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, F_{\Delta 1} = [0\ 0\ 0].$$

When the ASPRC-based Decentralised MPC is run by three individual subsystems, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.21. The system is also stabilised, but has much larger oscillations of control actions around the zero equilibrium point, compared to those in the mixed connection and parallel connection structures, presented above.

When the a random variable $\alpha(k) \in (0, 1)$ as per (3.75) is deployed to simulate the unknown splitting ratios, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.22 and 3.23.

B.2 *Case Study 2*

In the following, a different set of initial states are used. The weighting matrices of $\mathcal{Q}_1 = \mathsf{diag}\{1, 1, 2; 1.5, 1.5, 2.5\}$, $\mathcal{R}_1 = \mathsf{diag}\{2, 0.2\}$, $\mathcal{Q}_2 = \mathsf{diag}\{1.5, 2.4, 1.6\}$,

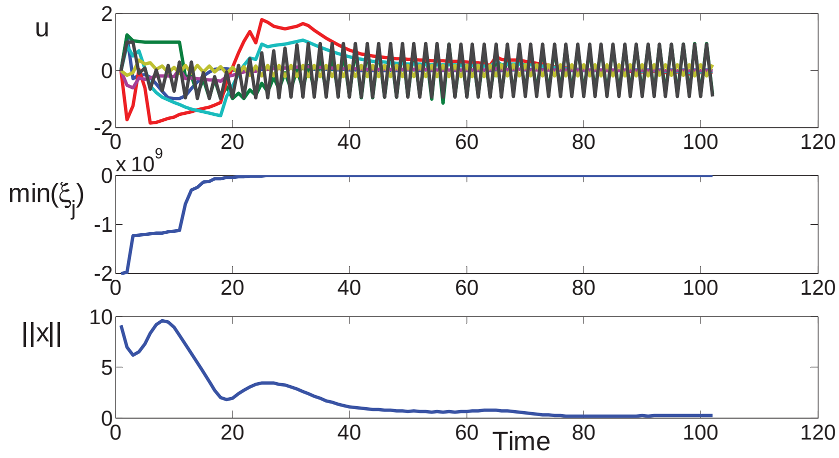


Figure 3.21 : Parallel Splitting Systems - Serial Connection Structure Only - ASPRC-Based Decentralised MPC with an Equal Splitting Ratio.

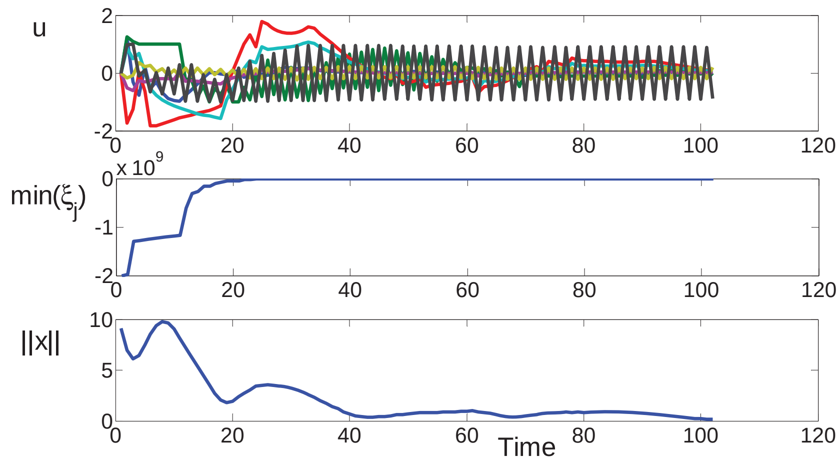


Figure 3.22 : Parallel Splitting Systems - Serial Connection Structure Only - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 1.

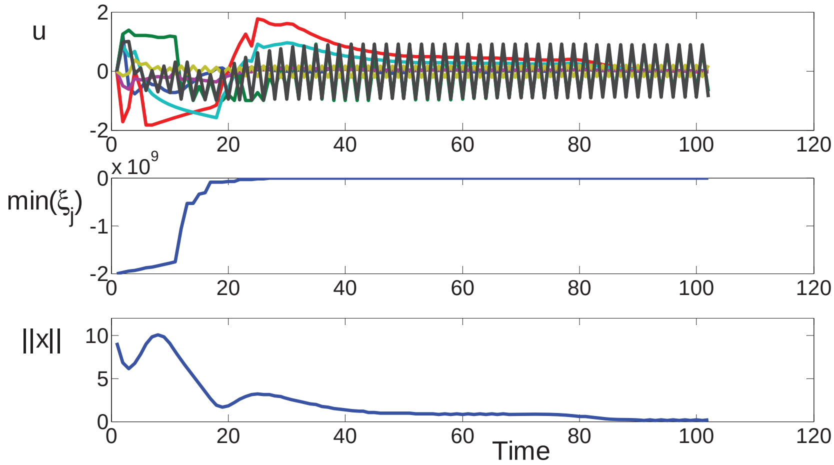


Figure 3.23 : Parallel Splitting Systems - Serial Connection Structure Only - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 2.

$\mathcal{R}_2 = \mathsf{diag}\{0.5, 0.5, 0.5\}$, $\mathcal{Q}_3 = \mathsf{diag}\{1, 3\}$, $\mathcal{R}_3 = \mathsf{diag}\{0.5, 0.5\}$, and the initial state vectors of $x_1(0) = [6.4\ -5.6\ -6.4,\ 1.6\ -1.4\ -1.6]^T$, $x_2(0) = [-6\ 8\ -1.5]^T$, $x_3(0) = [.55\ -1.4]^T$ are set up for the simulations in this section.

B.2.1 Mixed Connection Structure

When ASPRC-based Decentralised MPC is run by subsystems of units 1, 2 and 3, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.24.

Now, a random variable $\alpha(k) \in (0, 1)$ as per (3.75) is deployed to simulate the unknown splitting ratio, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.25 and 3.26.

B.2.2 Parallel Connection Structure Only

When the ASPRC-based Decentralised MPC is run by subsystems of units 1 and 2, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.27. The system

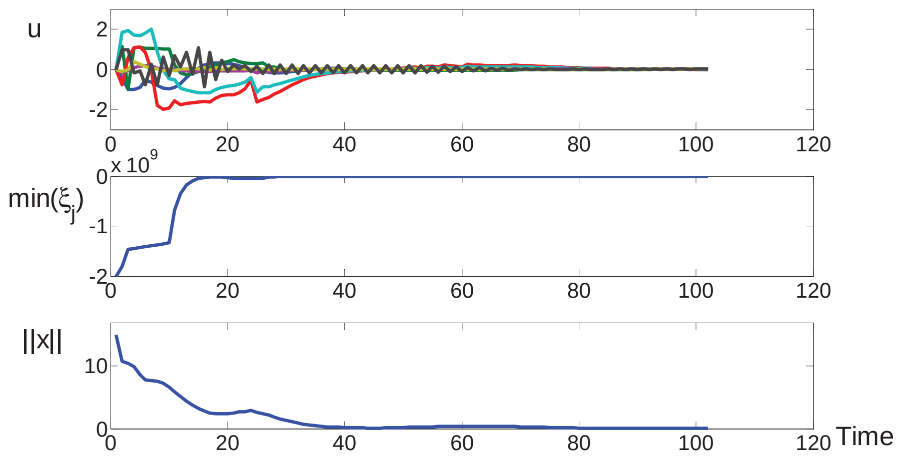


Figure 3.24 : Parallel Splitting Systems - Mixed Connection Structure Case 2 - ASPRC-Based Decentralised MPC with an Equal Splitting Ratio.

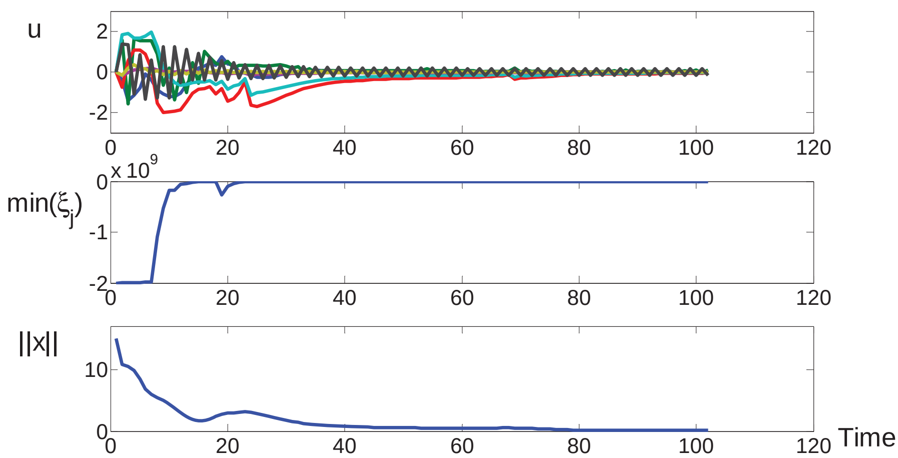


Figure 3.25 : Parallel Splitting Systems - Mixed Connection Structure Case 2 - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 1.

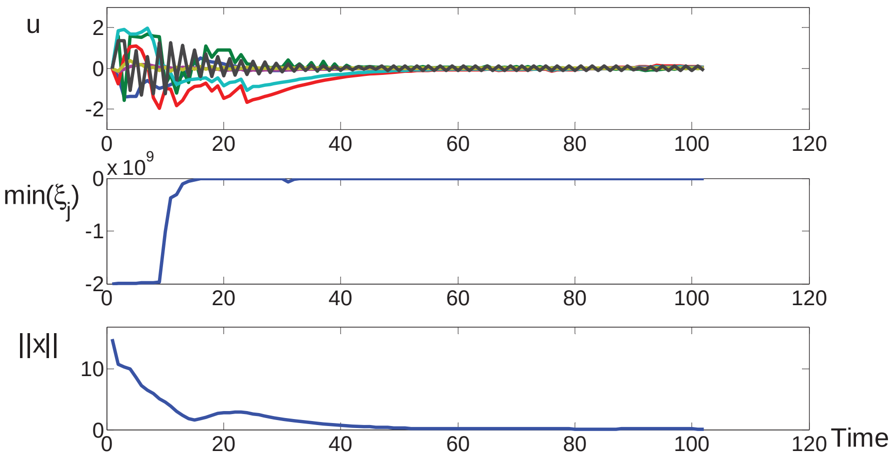


Figure 3.26 : Parallel Splitting Systems - Mixed Connection Structure Case 2 - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 2.

is also stabilised by this parallel-only interconnection structure.

Now, a random variable $\alpha(k) \in (0,1)$ as per (3.75) is deployed to simulate the unknown splitting ratios, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.28 and 3.29.

The system is also stabilised, but has much larger oscillations of control actions around the zero equilibrium point, compared to those in the mixed connection and parallel connection structures, presented above.

When the a random variable $\alpha(k) \in (0,1)$ as per (3.75) is deployed to simulate the unknown splitting ratios, the trends for two different random sets of $\alpha(k)$ are given in Figures 3.31 and 3.32. In these simulations, the constant ϵ_j between 10^{-3} to 10 has been chosen.

B.2.3 Serial Connection Structure Only

When the ASPRC-based Decentralised MPC Procedure is run by three individual

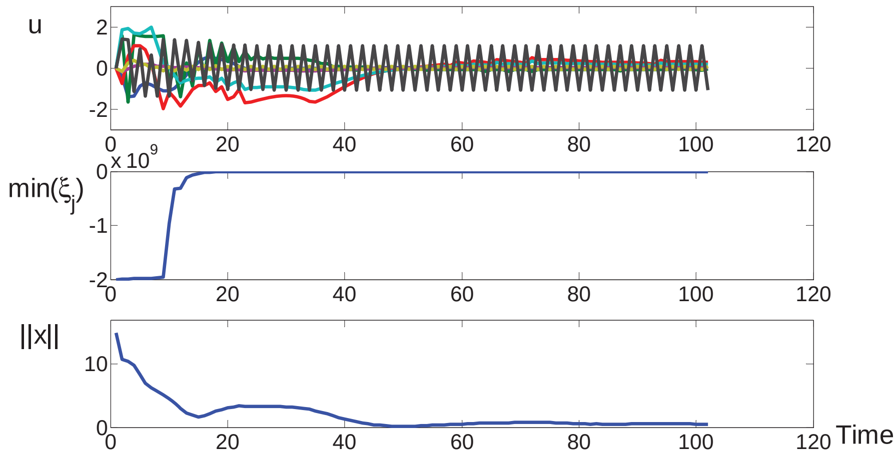


Figure 3.27 : Parallel Splitting Systems - Parallel Connection Structure Only Case 2 - ASPRC-Based Decentralised MPC with an Equal Splitting Ratio.

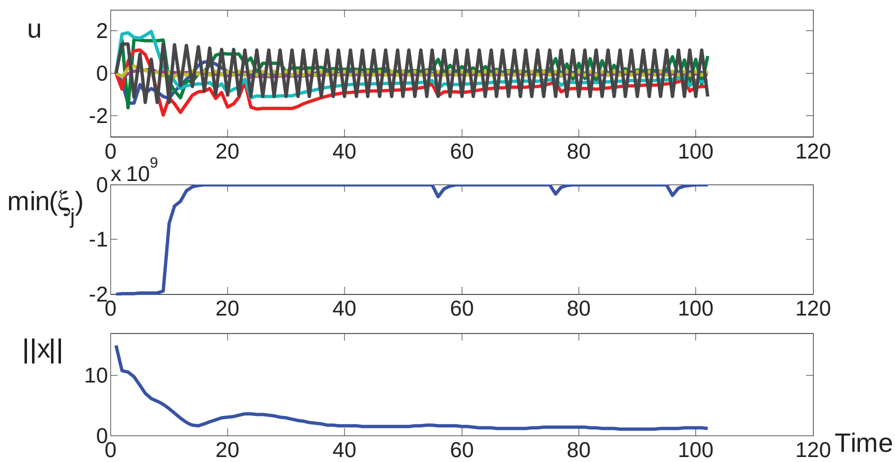


Figure 3.28 : Parallel Splitting Systems - Parallel Connection Structure Only Case 2 - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 1.

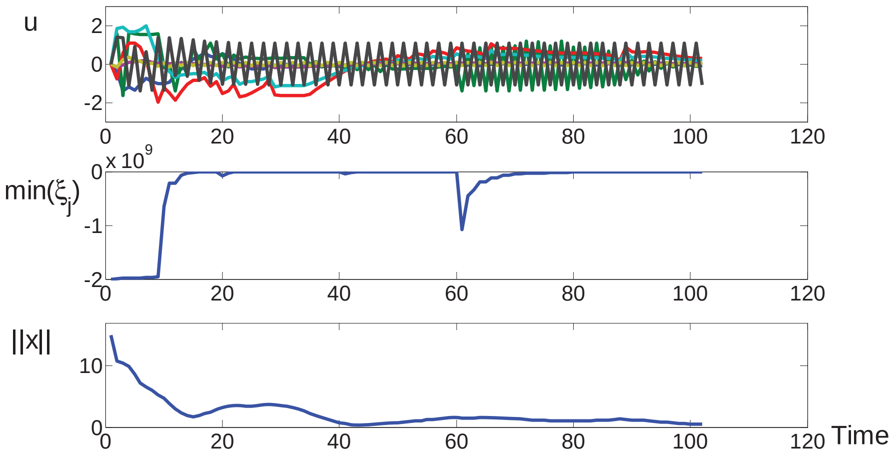


Figure 3.29 : Parallel Splitting Systems - Parallel Connection Structure Only Case 2 - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 2.

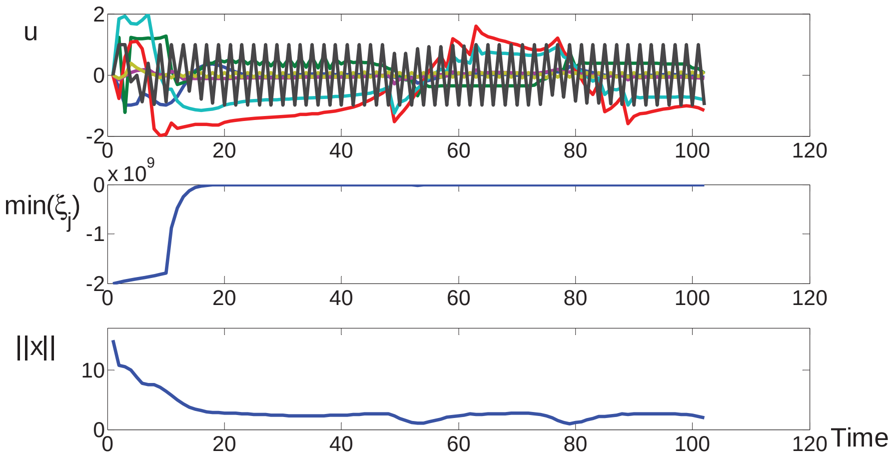


Figure 3.30 : Parallel Splitting Systems - Serial Connection Structure Only Case 2 - ASPRC-Based Decentralised MPC with an Equal Splitting Ratio.

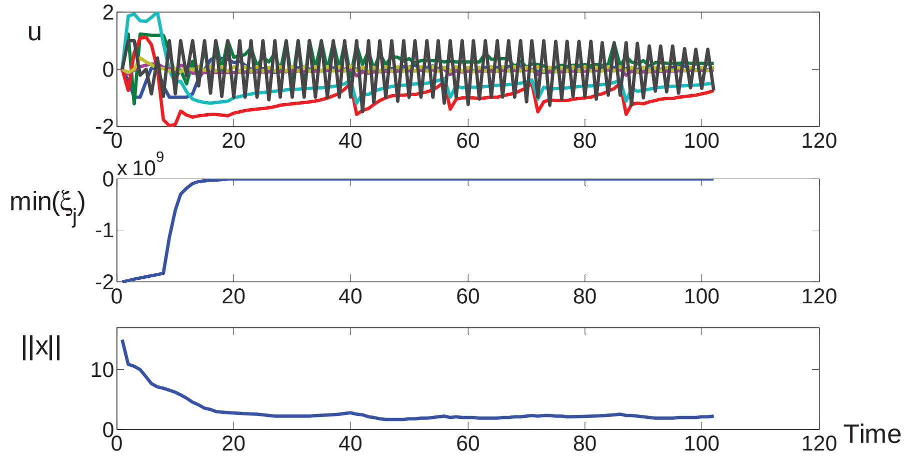


Figure 3.31 : Parallel Splitting Systems - Serial Connection Structure Only Case 2 - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 1.

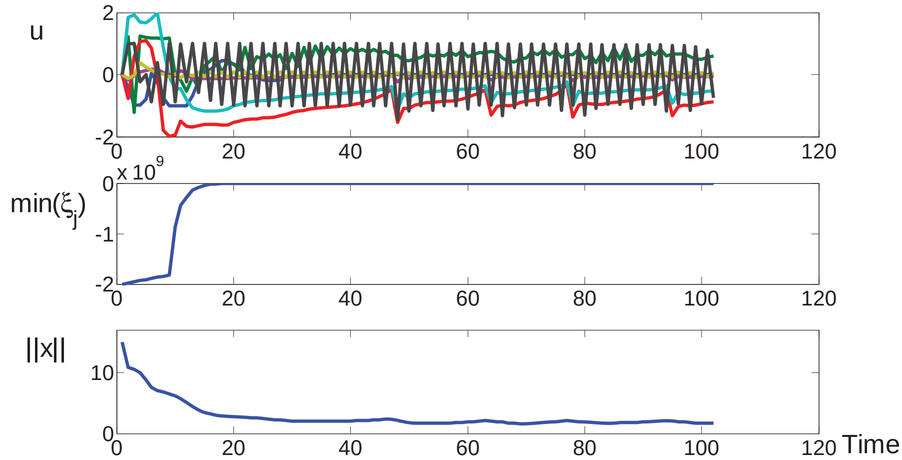


Figure 3.32 : Parallel Splitting Systems - Serial Connection Structure Only Case 2 - ASPRC-Based Decentralised MPC with Unknown Splitting Ratios - Simulation 2.

subsystems, the trends of $u_j(k)$, $\|x(k)\|_2$ and $\min_{j=1,2,3} \xi_j$ are given in Figure 3.30.

The simulations in this subsection have shown that the mixed connection structure is the only one that provides stabilised trends within the simulated time interval.

3.7.2 Illustrative Example 2

The following state space models and state feedback controllers are considered:

$$\dot{x}(t) = \begin{bmatrix} A_1 & & \\ & A_2 & \\ & & A_3 \end{bmatrix} x(t) + \begin{bmatrix} B_1 & & \\ & B_2 & \\ & & B_3 \end{bmatrix} u(t) + \begin{bmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{bmatrix} v(t),$$

$$w(t) = \begin{bmatrix} F_1 & & \\ & F_2 & \\ & & F_3 \end{bmatrix} x(t), \qquad v(t) = Hw(t), \quad y(t) = \begin{bmatrix} C_1 & & \\ & C_2 & \\ & & C_3 \end{bmatrix} u(t),$$

where

$$A_1 = \begin{bmatrix} -28.8 & 0 & 0 \\ 17.2 & -5.9 & 0 \\ -8.4 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -14.4 & 0 & 0 \\ 17.8 & -.6 & 0 \\ -3 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -16.3 & 0 & 0 \\ 11.9 & -.7 & 0 \\ -6.2 & 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} -.1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, C_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$E_1 = \begin{bmatrix} 0 & -9.21 \\ 0 & 14.57 \\ 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 18.44 & 0 & -12.35 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1.47 & 0 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 23.64 & 0 \\ 0 & 0 \\ 0 & 1.81 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_3 = \begin{bmatrix} 0 & 0.7 & 0 \end{bmatrix},$$

while

$$H = \left[\begin{array}{cc|cc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

State feedbacks are pragmatically applied in many industrial process control applications. The weighting matrices in (3.25) are set as follows:

$$\mathcal{R}_1 = \mathsf{diag}\{2, 1\},\ \mathcal{Q}_1 = \mathsf{diag}\{5, 1, 2\},\ \mathcal{R}_2 = 1,$$

$$\mathcal{Q}_2 = \mathsf{diag}\{1, 1, 2\},\ \mathcal{R}_3 = \mathsf{diag}\{0.5, 1\},\ \mathcal{Q}_3 = \mathsf{diag}\{4, 1, 5\}.$$

The constraints on the control inputs are set up as

$$\|u_1\|_2 \leqslant 2, \|u_2\|_2 \leqslant 10, \|u_3\|_2 \leqslant 10.$$

The initial states are as follows:

$$x_1(0) = \begin{bmatrix} 7 \\ 14 \\ -10 \end{bmatrix},\ x_2(0) = \begin{bmatrix} -12 \\ -55 \\ 16 \end{bmatrix},\ x_3(0) = \begin{bmatrix} -9 \\ 7 \\ -6 \end{bmatrix}.$$

Figure 3.33 depicts the time responses of three interconnection subsystems regulated by decentralised MPC controllers. In this simulation, the stabilising constraints are not used. The system is clearly unstable.

Decentralised MPC with APRC-Based Stabilising Constraints:

When the decentralised optimisation problem (3.25) is employed, the time responses are shown in Figure 3.34, indicating clearly a stabilised interconnected system. Theorem 2 has been used in this simulation. The offline multiplier matrices are as follows:

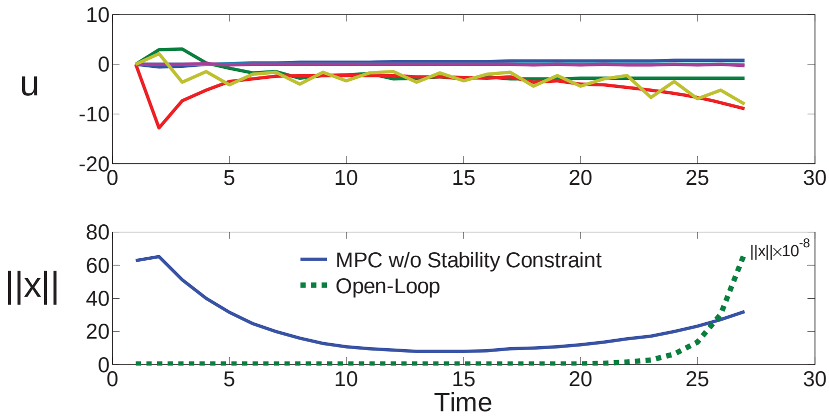


Figure 3.33 : Decentralised MPCs for the Washing Circuit Without Stabilising Constraints - Time Responses.

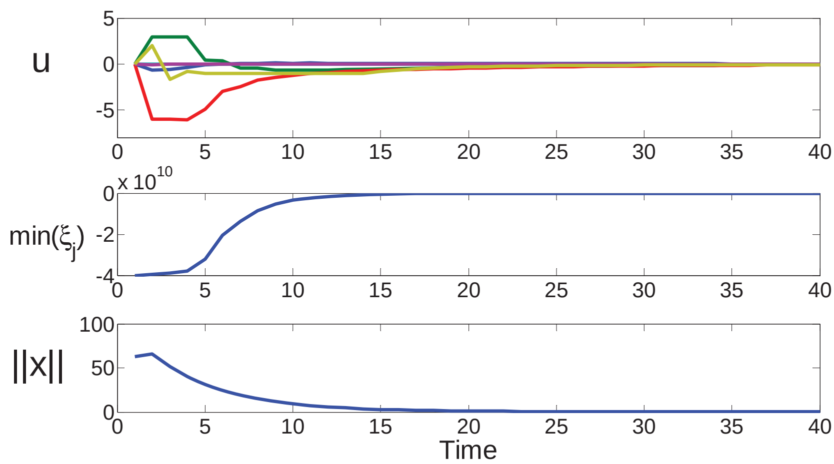


Figure 3.34 : APRC-Based Decentralised MPCs of the Counter-Current Washing Circuit with Online Updated Multiplier Matrices - Time Responses.

$$Q_1 = \begin{bmatrix} 4.1774 & 0.7683 & 1.3327 \\ 0.7683 & 0.5076 & -2.4361 \\ 1.3327 & -2.4361 & -0.8254 \end{bmatrix}, \; Q_2 = \begin{bmatrix} 3.3861 & -1.1387 & 0.1554 \\ -1.1387 & -1.9627 & -0.2457 \\ 0.1554 & -0.2457 & -3.7694 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 2.8385 & -1.5048 & 0.3944 \\ -1.5048 & 0.0517 & -0.1730 \\ 0.3944 & -0.1730 & -2.2238 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} -0.1220 & 0.3023 \\ -0.1395 & 0.1755 \\ 0.2939 & -0.8126 \end{bmatrix}, S_2 = \begin{bmatrix} 0.0295 & 0 \\ -0.1505 & 0 \\ -0.9662 & 0 \end{bmatrix}, \; S_3 = \begin{bmatrix} -0.7497 & 0.2595 \\ -0.8920 & -0.1391 \\ -0.1594 & -0.9415 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 3.4771 & 0.2019 \\ 0.2019 & 3.7955 \end{bmatrix}, \; R_2 = \begin{bmatrix} 3.6754 & 0 \\ 0 & 3.3593 \end{bmatrix}, \; R_3 = \begin{bmatrix} 3.8203 & 0.0483 \\ 0.0483 & 3.7358 \end{bmatrix}.$$

The time responses for a different set of initial states of the following:

$$x_1(0) = \begin{bmatrix} 1 \\ .7 \\ 3 \end{bmatrix}, \; x_2(0) = \begin{bmatrix} .9 \\ .6 \\ 2 \end{bmatrix}, \; x_3(0) = \begin{bmatrix} .2 \\ .8 \\ 4 \end{bmatrix}$$

are provided in Figure 3.35. The system is also stabilised, but the settling time is longer, compared to those in Figure 3.34.

Centralised MPC

The centralised MPC using the *mpcmove* function does not work due to unstable system models. Its open-loop discrete-time eigenvalues are as follows:

$$0.0907, \; 0.7866, \; 0.9608,$$

$$2.2349, \; 1.2941, \; 0.4623,$$

$$-0.0004 + 0.0014i, \; -0.0004 - 0.0014i, \text{ and } 0.$$

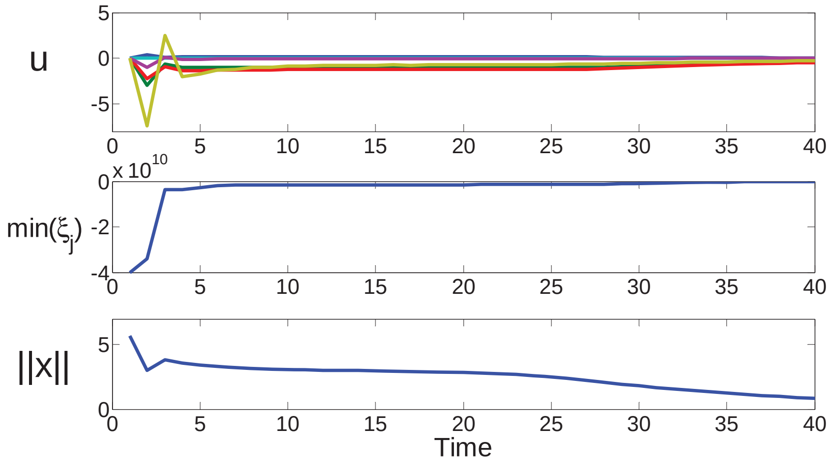


Figure 3.35 : APRC-Based Decentralised MPCs of the Counter-Current Washing Circuit with Online Updated Multiplier Matrices - Time Responses.

3.8 Concluding Remarks

A stabilisability conditions for interconnected systems in the discrete time domain have been derived in this chapter based on the newly introduced asymptotically positive realness constraint (APRC) and its enhancement version of asymptotically surely positive realness constraint (ASPRC). Parallel splitting connections are considered as an essential part of the theoretical developments. A constructive approach of MPC stabilising constraints has subsequently been presented for feasible implementations. The numerical simulations for the developed APRC-based and ASPRC-based decentralised MPC strategies with two interconnected systems have been investigated with a Matlab Yalmip toolbox.

Chapter 4

Extended Implementation Methods
Semi-Automatic Control with Stabilising Agent and Receding-Horizon Stabilising Constraint for Distributed Model Predictive Control

4.1 Introduction

4.1.1 Stabilising Agent for Semi-Automatic Control Systems

In processing and manufacturing industries, a complex plant can be viewed as a large-scale system formed by various subsystems interconnected to each other in an arbitrary connection topology. By virtue of intricate interactions between state variables within a subsystem, the computerised control system is normally designed such that a dedicated multivariable controller is installed for each of its single subsystem. The multiple installations translate into a new requirement on the orchestration for these multivariable controllers to avoid destabilisations owing to the coupling effects between subsystems [128, 125]. To assure the continuous operability and maintainability of the aggregated plant, all of these controllers operate in two modes, automatic and manual, which allow for human machine interfaces (HMI) and switching between the remote control-room operators and the automatic control algorithms. As a consequence, the large-scale system may operate in a semi-automatic control mode wherein feedback control and manual regulations coexist. The manual control tasks in a large-scale plant are usually executed via the remote HMI functions of the computerised system in a remote control room. The semi-automatic control operations that require both man machine interactions and automatic regulations may be classified as subsystem start-ups or warm-restarts after a shutdown, and subsystem operational mode changes. By virtue of these activities, the plant-wide operational performances are likely to be disturbed. Or even worse, they may cause a shutdown from the process upsets and unexpected instabilities. Confining the interconnected system within the stabilising boundaries in all operational modes will be essential for a safe and reliable operation.

An approach of stabilising-bounds for the control of interconnected systems in a decentralised architecture is developed in this chapter. Herein, the APRC is used by the *stabilising agent* to derive the stabilising bounds on-the-fly in a segregated and independent manner. It also steers the system into a stabilisable region, from which the controller takes over the stabilisation task by employing the APRC-based stability constraint in its optimisation formulation.

The method of stabilising interconnected systems with external agents developed in this chapter stems from the idea of splitting (implicitly) the stabilising mechanism with the control algorithm. We are inspired by the segregation principle in reliability designs [141] when developing this stabilising strategy. Roughly speaking, the stabilising agents look at the system inputs and outputs as an alias of the associated controllers and use their online and historical data to decide necessary intervening actions required for avoiding destabilisations. The stabilising mechanism will initiate an immediate overriding action to steer the system into a stablisable trajectory upon any deviations from its evaluated APRC region. The physical isolation of stabilising agents from the automatic controllers ensures that the system is stabilisable in various operational modes. These agents continuously monitor the controller input and output trajectories, as well as limit the manipulations within the stabilisable boundaries.

The problem set ups and stabilisability condition with non-zero set-point supply rate and the equilibrium independent dissipativity is developed at the end of this chapter. Time varying set points are widespread used in the industrial model predictive control schemes that have dynamic steady state calculations (se, e.g., [148]). In these schemes, the economic objective functions are primarily used to set the target for the underlying MPC algorithm. As a result, the calculated steady states are dynamic and updated at every time step for the MPC. The stabilisation with non-zero and dynamic set points are, therefore, crucial for the industrial MPC applications.

4.1.2 Receding-Horizon Stabilising Constraint

In the second part of this chapter, the APRC is developed into a receding-horizon stability constraint for the distributed model predictive controller. The APRC-based receding-horizon stability constraint is an asymptotic terminal constraint on the control vector. A one-time-step constraint on the last vector in the predictive control sequence is derived and imposed on the local MPC optimisation. Since the receding-horizon stability constraints are formulated with state predictions towards the end of the predictive horizon, the system trajectory is not required to satisfy the APRC at the current time step. This is much more flexible and easier to solve than the one-time-step constraint on the initial control vector presented in Chapter 3. This approach has, however, been critisised as lacking of robustness to large model uncertainties. The predictive horizon will, therefore, need to be gradually decreased back to the one step version to guarantee the closed-loop stability, yet being able to eliminate the conservativeness of a one-time-step stability constraint. The term receding-horizon stability constraint, therefore, well describes the development.

4.2 Stabilising Agent

Consider the system and control model having a control constraint in Chapter 3. In this section, the open-loop model predictive control (MPC) algorithm without any stabilising constraints is implemented for the multivariable controller of each single subsystem. As mentioned in Chapter 3, the given MPC vectors $\hat{u}_i(k)$ may destabilise the large-scale system. As a solution, the control inputs will be corrected on-the-fly in a segregated fashion by the stabilising agent so as the APRCs are always feasible. In other words, we are concerned with the design of h disparate laws to implement the stabilising agents segregated from the local MPCs of h associated subsystems $\mathcal{S}_i$ in a decentralised configuration. In the online operation, a stabilising agent accesses the input and output vectors of $\mathcal{S}_i$ and its controller $\mathcal{C}_i$ from an alias position. The online and historical data of $\mathcal{S}_i$ and $\mathcal{C}_i$ are used to deduce the APRC-based stabilising bounds for the outputs of $\mathcal{C}_i$ (i.e. control inputs of $\mathcal{S}_i$). An immediate overriding action will be initiated to steer the system back into a stabilisable state upon any deviations from the evaluated stabilising regions. The physical isolation of stabilising agents from their associated controllers ensures that the stabilising bounds will be derived despite of operational modes. The communication between local agents is not available for coordinations.

4.2.1 Stabilising Agent Operation

The mechanism of stabilising agents is developed from the stabilisability condition in Theorem 2 for linear systems. The block diagram shown in Figure 4.1 describes the activities of a stabilising agent, which are also listed in the following:

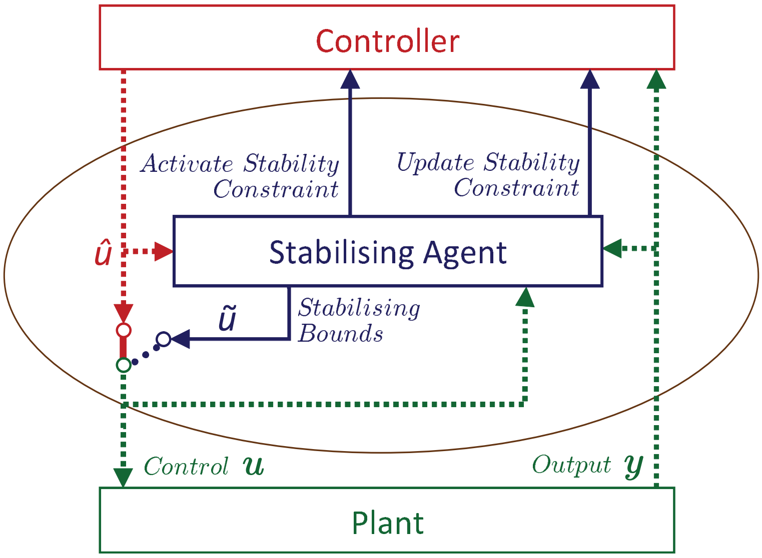


Figure 4.1 : Stabilising Agent Activities.

- Monitor the values of ξ_{ic}^k and check their negativeness. The controllers run their own control algorithms without any additional constraints until the first initial time $k_0(i)$, at which $\xi_{ic}^k < 0$, is detected.

- A tuning scalar ρ_i is used in our approach. It will prevent a conservative overridden action by having the stabilising agent activated only when $\xi_{ic}^k < -\rho_i^2$ instead of $\xi_{ic}^k < 0$.

- Calculate the stabilising bounds $\tilde{u}_i$ if $\xi_{ic}^k < -\rho_i^2$ based on the APRC (3.11) and the control constraint (3.2) with respect to the variable $\boldsymbol{u}_i$ using the value of ξ_{ic}^{k-1} calculated in the immediate previous step. Then override the predicted control $\hat{u}_i$ produced by the controller with $\tilde{u}_i$.

- Derive the stabilisable time $\bar{k}$ based on the positiveness of ξ_{ic}^k.

- Employ the inequality of APRC when $\gamma_i(k) = 0$ (i.e. $\xi_{ic}^k \geq 0$) as a stability constraint in the MPC optimisation problem from time $\bar{k}$.

In short, the stabilising agent initially remains inactive. It only calculates and monitors ξ_{ic}^k until the first time instant $k_0(i)$, at which $\xi_{ic}^k < -\rho_i^2$. Such negative values of ξ_{ic}^k will activate the corresponding stabilising agents. The time step $k_0(i)$ then becomes the initial time for activating the procedure that calculates the stabilising bounds. The stabilising agent will only remain active until $\bar{k}(i)$ at which ξ_{ic}^k changes its sign from negative to positive. We now have sufficient details to outline the algorithm of a stabilising agent in the next subsection.

4.2.2 Constructive Procedure for Stabilising Agents

Assuming that the sampling and updating time instants of all subsystems and their controllers are synchronised, the detailed algorithm of stabilising agents is given in the below procedure.

Procedure 4.1– Stabilising Agent for a Linear Subsystem

1. **Offline**:

   - The dissipativity matrices $Q_{i11}, S_{i11}, R_{i11}, Q_{i22}, S_{i22}, R_{i22}$ and P_i, and multiplier matrices Q_{ic}^0, S_{ic}^0 and R_{ic}^0 for all subsystems $\mathcal{S}_i$ are predetermined from LMIs (3.34) rendered in Theorem 2 and $0 \succ R_{ic}^0 \succ S_{ic}^{0T}(Q_{ic}^0)^{-1}S_{ic}^0$.
   - Select the tuning parameter ρ_i for each subsystem.

2. **Online**: Initially, there are not any additional constraints imposed on the controllers at $k = 0$ that generate the predicted control $\hat{u}_i(k)$. For each subsystem $\mathcal{S}_i, i = 1, ..., h$, declare two variables $k_0(i)$ and $\bar{k}(i)$. Select γ_i as a tuning pa-

rameter (only time-invariant γ_i is used in this procedure). At every time step $k > 0$,

- ***Step 1**: Initiating $k_0(i)$*
  - Iterate the predicted control $\hat{u}_i(k)$ generated by the controller and controlled outputs $y_i(k)$ to calculate ξ_{ic}^k;
  - Verify $\xi_{ic}^k < -\rho_i^2$. If true, assign k to $k_0(i)$, and go to Step 2.
- ***Step 2**: Stabilising Bounds*

  While $\xi_{ic}^k < -\rho_i^2$, activate the stability bound calculation for the local controller of $\mathcal{S}_i$ that already has $k_0(i)$ assigned to (i.e. $\xi_{ic}^{k_0} < -\rho_i^2$):
  - Update the multiplier matrices $Q_{ic}^k, S_{ic}^k, R_{ic}^k$ by solving LMIs in Theorem 2 or those in Theorem 4 using the offline dissipativity matrices $Q_{i22}, S_{i22}, R_{i22}$, the output y_i^k and ξ_{ic}^{k-1};
  - Calculate the stabilising bound vector $\tilde{u}_i(k)$ for the activated $\mathcal{S}_i$ by solving the optimisation problem of the following:

$$\min_{\tilde{\boldsymbol{u}}_i(k)} \|\tilde{\boldsymbol{u}}_i(k) - \hat{u}_i(k)\|_2^2,$$

$$\text{subject to} \quad \xi_{ic}\big(\tilde{\boldsymbol{u}}_i(k), y_i(k)\big) \geq \gamma_i \xi_{ic}\big(\hat{u}_i(k), y_i(k)\big), \text{ and } \| \tilde{\boldsymbol{u}}_i(k) \|_2^2 \leq \eta_i; \tag{4.1}$$

  - Replace $\hat{u}_i(k)$ with $\tilde{u}_i(k)$ and output to $u_i(k)$;
  - Calculate ξ_{ic}^k and verify $\xi_{ic}^k \geq 0$. If true, assign k to $\bar{k}(i)$, deactivate the stabilising agent and go to Step 3.
- ***Step 3**: Stability Constraint for MPC*

  While $\xi_{ic}^k \geq 0$ activate the stability constraint for the corresponding local controller that already has $\bar{k}(i)$ assigned to (i.e. $\xi_{ic}^{\bar{k}} \geq 0$):

– Update the multiplier matrices $Q_{ic}^k, S_{ic}^k, R_{ic}^k$ similarly to Step 2;

– Enforce the stability constraint of the form $\xi_{ic}^k \geq 0$ to the local controller;

– Calculate ξ_{ic}^k and verify $\xi_{ic}^k < 0$. If true, assign k to $k_0(i)$ and go back to Step 2.

4.2.3 Graphical Presentation

Consider a subsystem having two control inputs, i.e. u_i has two elements $u_i^{(1)}$ and $u_i^{(2)}$. The control plane plot with two axes of $u_i^{(1)}$ and $u_i^{(2)}$ is shown in Figure 4.2. The plot in this figure consists of the control trajectory of $u_i(j)$, ellipsoidal APRC regions, the predicted controls $\hat{u}_i$, the balls of $\|\tilde{\boldsymbol{u}}_i(k) - \hat{u}_i(k)\|_2^2$, and the stabilising bounds $\tilde{u}_i$ from the solutions to the optimisation problem (4.1) in Procedure 3.1. Two cases of incremental changes of the supply rate ξ_{ic}^k are included in this figure. Case (a) shows the convex region of $\xi_{ic}^k \geq \gamma_i^k \xi_{ic}^{k-1}$ which is the main focus of the stabilising agent in this chapter. Case (b) shows the region of $0 < \xi_{ic}^k < \xi_{ic}^{k-1}$ for illustration only.

After the point $u_i(k_0)$ (and before the point $u_i(\bar{k})$) along the control trajectory, $\xi_{ic} < 0$. The control u_i represents the control trajectory at time index $k-1$. At time k, the control algorithm generates the predicted control vector $\hat{u}_i(k)$ outside the ellipsoidal APRC region in Case(a). The stabilising agent then solves the problem (4.1) for $\tilde{u}_i(k)$ which is represented by a point inside a ball taking $\hat{u}_i(k)$ as the centre. The stabilising bound $\tilde{u}_i(k)$ is inside the ellipsoidal APRC region for the convex Case (a), and reversely for Case (b).

Three steps of Procedure 4.1 correspond to three sections of the control trajectory. The initial Step 1 takes place between the two points $u_i(0)$ and $u_i(k_0)$. The main activity of a stabilising agent is in Step 2 wherein the stabilising bound is found at

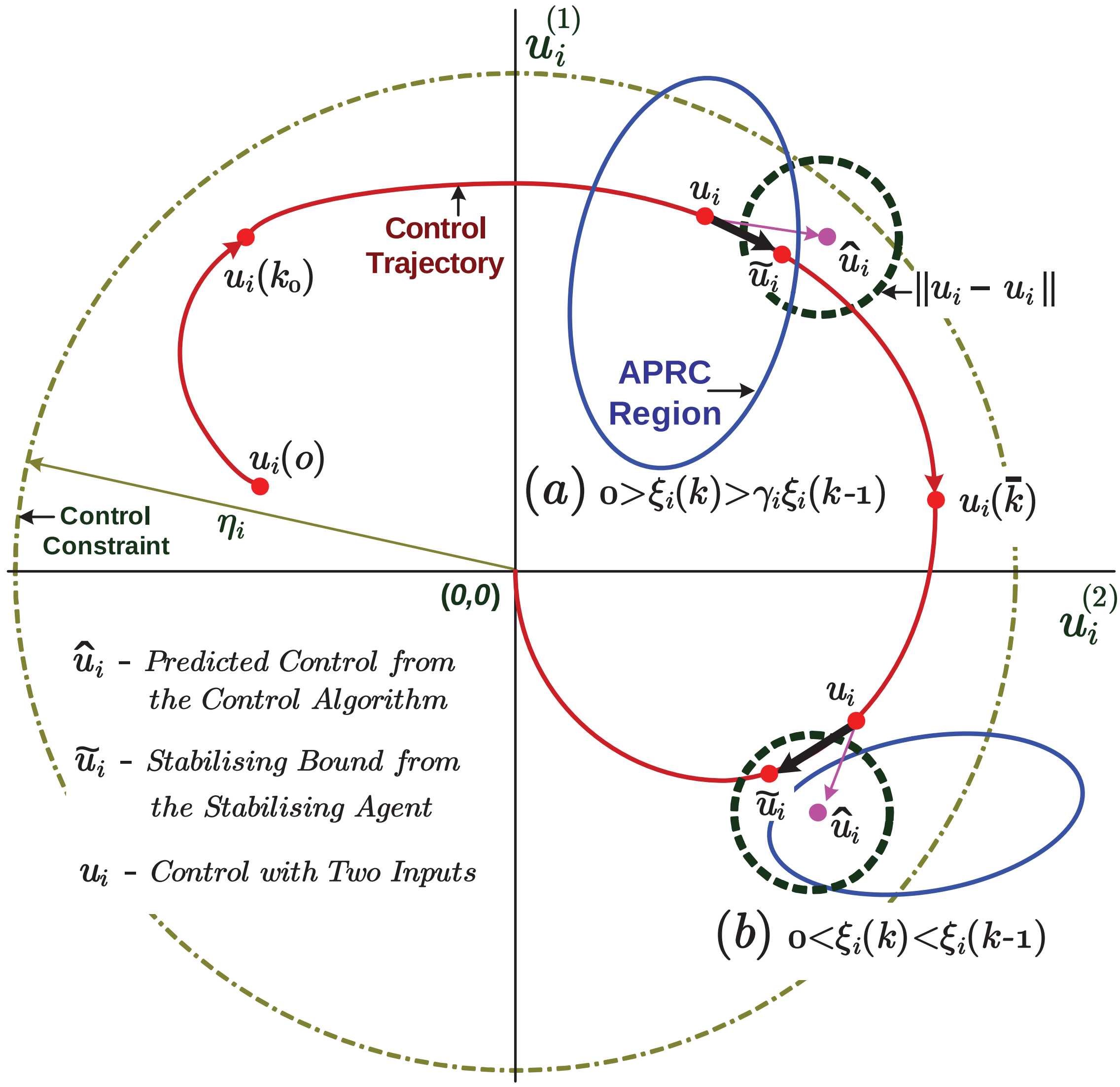


Figure 4.2 : The Control Plane Plot with APRC Ellipsoids and the Stabilising Bounds.

every time step. Step 2 is executed between the two points $u_i(k_0)$ and $u_i(\bar{k})$. Beyond the point $u_i(\bar{k})$, the stability constraint can be activated and applied to the automatic controllers which incurred in Step 3 of Procedure 4.1.

4.3 Illustrative Example

Consider the continuous-time state space models of a typical three-stage countercurrent washing circuit as follows:

$$A_1 = \begin{bmatrix} -29 & 0 & 0 \\ 17 & -6 & 0 \\ -8 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & -0.4 \\ 0 & 2 \\ 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -14 & 0 & 0 \\ 18 & -0.5 & 0 \\ -3 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -16 & 0 & 0 \\ 12 & -0.7 & 0 \\ -6 & 0 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -0.1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 2 & 0 \\ 0 & 0.8 \\ 0 & 0 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_3 = \begin{bmatrix} 0 & 0.7 & 0 \end{bmatrix},$$

$$H = \left[\begin{array}{cc|cc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

In this simulation, the MPC controllers have been set up and coded in Matlab using the *mpcmove* function. Short predictive and control horizons ($N_y = 3,\ N_u = 2$) have been chosen for the first two subsystems, while longer horizons ($N_y = 12,\ N_u = 4$)

are chosen for the third one. Three subsystems have the same sampling time of 0.4 steps; and all invocations are synchronised between all subsystems. The weighting coefficients for the control inputs, their increments, and plant outputs in the *mpcmove* functions are as follows:

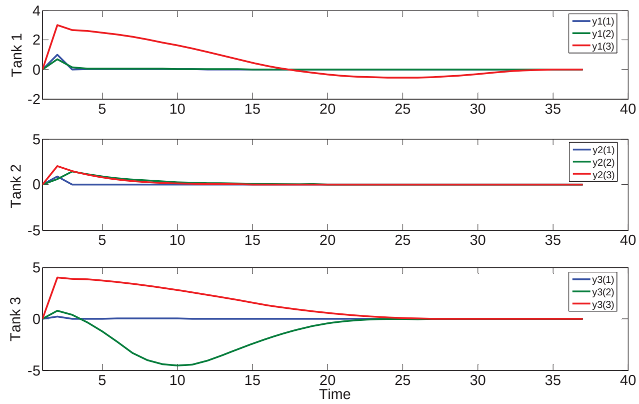
$W_{u_1} = \text{diag}\{2 \quad 1\}, W_{\Delta u_1} = \text{diag}\{0.5 \quad 1\}$, $W_{y_1} = \text{diag}\{5 \quad 1 \quad 2\}$,

$W_{u_2} = [1]$, $W_{\Delta u_2} = [0.3], W_{y_2} = \text{diag}\{1 \quad 1 \quad 2\}$,

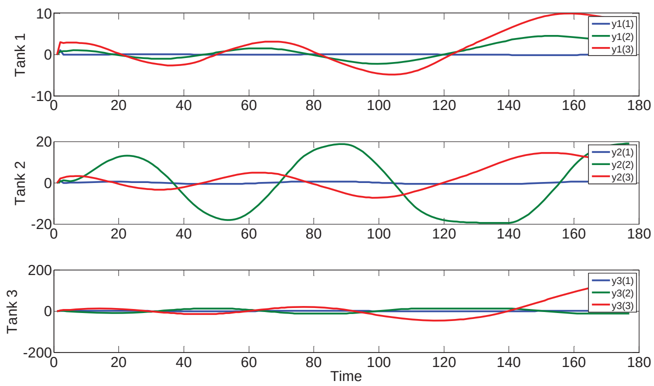
$W_{u_3} = \text{diag}\{0.5 \quad 1\}$, $W_{\Delta u_3} = \text{diag}\{0.2 \quad 0.7\}$, $W_{y_3} = \text{diag}\{4 \quad 1 \quad 5\}$.

The output constraints are ignored to emulate the control system instability. Figure 4.3(a) depicts the plant-output time responses of three stand-alone subsystems regulated by autonomous controllers applying MPC algorithms. When three tanks are connected into a circuit, the plant-output time responses of three interconnection subsystems regulated by MPC controllers are given in Figure 4.3(b). In this simulation, the stabilising agents are not used. The system is clearly unstable. To demonstrate the efficacy of Procedure 4.1, we firstly simulate the stabilising agent using only offline multiplier matrices for the APRC (i.e. the multiplier matrices are not updated online as required by Step 2 of Procedure 4.1). The algorithm developed in [153] has been applied to solve the problem (4.1) in this simulation. When the MPCs of subsystems are not imposed with stability constraints (Step 3 of Procedure 4.1 is ignored), the initial time $k_0(1)$ of subsystem 1 appears at steps 11, 58, 73, 105 and 185. The stability constraint time $\bar{k}(1)$ of subsystem 1 occurs at steps 28 , 70, 84 and 175. The initial time $k_0(3)$ of subsystem 3 occurs at steps 10, 30 and 71 the stability constraint time $\bar{k}(3)$ of subsystem 3 takes place at steps 19 and 67.

When the multiplier matrices of the APRC are updated online (i.e. the full Procedure 4.1 is employed), the plant-output and control time responses are shown in Figure 4.5, also indicating clearly a stabilised interconnected system. Moreover, they

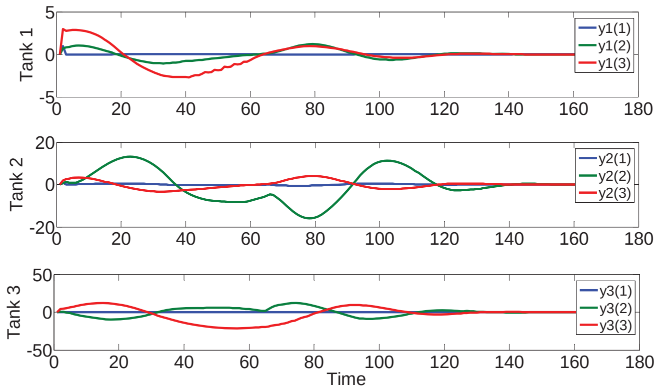


(a) Stand Alone MPCs - Plant Output Trends.

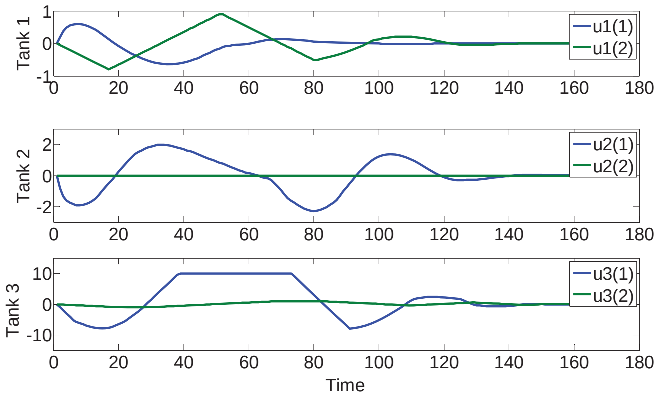


(b) Without Stabilising Agents - Plant Output Trends.

Figure 4.3 : Control of the Washing Circuit without Stabilising Agents - Plant-Output Time Responses.

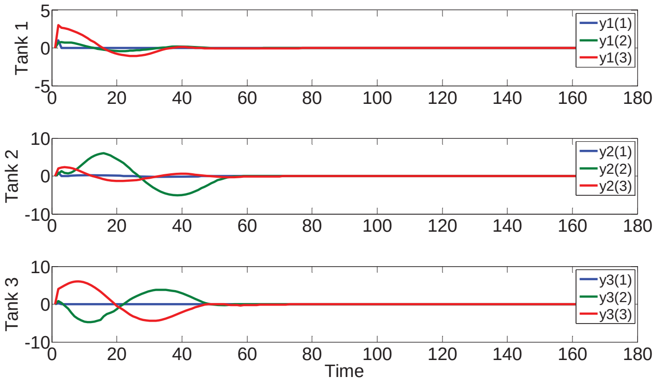


(a) Offline Multiplier Matrices - Plant Output Trends.

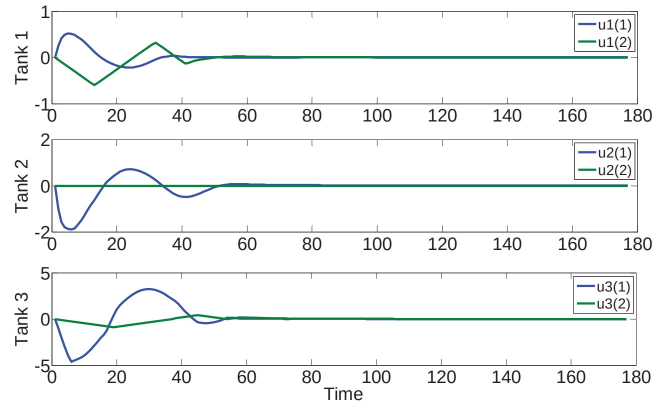


(b) Offline Multiplier Matrices - Control Input Trends.

Figure 4.4 : Decentralised Stabilising Agents for the Washing Circuit - Time Responses.



(a) Online Multiplier Matrices - Plant Outputs.



(b) Online Multiplier Matrices - Control Inputs.

Figure 4.5 : Decentralised Stabilising Agents for the Washing Circuit - Time Responses.

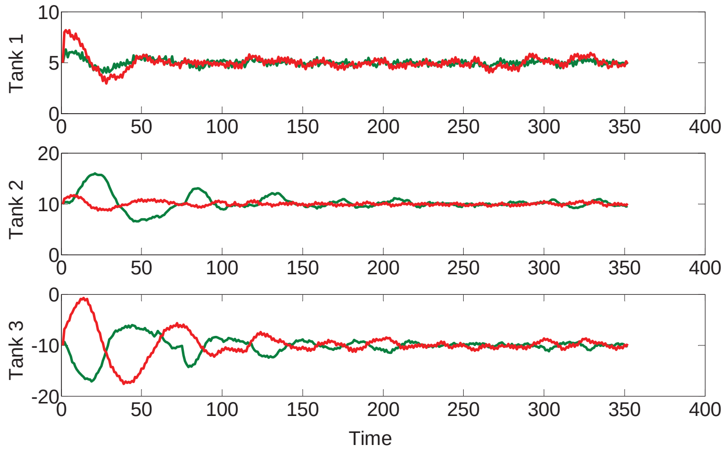


Figure 4.6 : Decentralised Stabilising Agents with a Non-zero Set Point and Unmeasured Disturbance - Plant-Output Time Responses.

exhibit significant performance improvements compared to the offline solution shown in Figure 4.3. There are not any wind-up actions in these control profiles. The online multiplier matrices are updated with a simplified version of Theorem 4, presented in [156]. The non-zero set-point case is demonstrated in Figure 4.6.

4.3.1 Concluding Remarks

A constructive approach of stabilising agents has been developed for the feasible implementation. The stabilising bounds on the immediate future value of the controller outputs are updated on-the-fly in this approach. The stabilising agents that operate independently to and segregated from the control algorithm will override the controller outputs if they are determined as beyond a tuned APRC boundary. Being disparate from the associated control laws, the decentralised stabilising agents are interoperable with different control algorithms while using the same stabilising mechanism.

4.4 Distributed Model Predictive Control Problem with Receding-Horizon Stabilising Constraint

The system and control models in Chapter 3 are re-used in this section. The control problem in this chapter is to design h dynamic stability constraints for the MPCs of h dynamically-coupled subsystems $\mathcal{S}_i$ (3.1) in distributed or decentralised architecture. The APRC-based constraint on a control vector in the predictive sequence are formed by employing state predictions towards the end of the MPC horizon. The inter-subsystem communication between the MPCs is allowed for performance enhancement purposes.

Consider a subsystem $\mathcal{S}_i$ (3.1) in a distributed setting of the large-scale system Σ (3.3), and the conventional MPC objective function of the following form:

$$\mathcal{J}^i(k) = \sum_{j=1}^{N_i+1} x_i^T(k+j)\mathcal{Q}_i x_i(k+j) + \sum_{j=0}^{N_i} u_i^T(k+j)\mathcal{R}_i u_i(k+j),$$

where $\mathcal{Q}_i, \mathcal{R}_i$ are weighting matrices and N_i is the predictive horizon. The constrained optimisation problem of minimising $\mathcal{J}^i(k)$ subject to the equality constraints of the open-loop subsystem (3.1) and the inequality constraints deduced from physical limits of u_i is solved for the control vector. The problem is reformulated by predicting the state vector x_i in N_i time steps, assuming that the current state vector $x_i(k)$ is known, and the predictive sequences of interactive inputs $v_i(k)$ are obtained from the corresponding neighbours via the intercommunication links. The state predictive sequence is then used in both the objective function and the APRC-based stability constraint, as detailed in the next subsections.

4.4.1 Optimisation Objective Function

The predictive state vector $\hat{x}_i(k, \hat{u}_i, N)$ is computed by

$$\hat{x}_i(k, \hat{u}_i, N_i) = \Theta_i x_i(k) + \Gamma_{id}\hat{u}_{id}(k, x_i^k, N_i), \tag{4.2}$$

where $\hat{u}_{id}^T(k, x_i^k, N_i) := [\hat{u}_i^T(k, x_i^k, N_i)\ \hat{v}_i^T(k, x_i^k, N_i)]$

$\hat{u}_i^T(k, x_i^k, N_i) := [u_i^T(k) \ldots u_i^T(k + N_i)]$,

$\hat{v}_i^T(k, x_i^k, N_i) := [v_i^T(k) \ldots v_i^T(k + N_i)]$,

$\hat{x}_i^T(k, \hat{u}_i, N_i) := [x_i^T(k + 1) \ldots x_i^T(k + N_i + 1)]$,

$$\Gamma_{id} = \begin{bmatrix} B_{id} & 0 & \ldots & 0 \\ A_iB_{id} & B_{id} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ (A_i)^{N_i}B_{id} & \ldots & A_iB_{id} & B_{id} \end{bmatrix}, \text{ and } \Theta_i = \begin{bmatrix} A_i \\ (A_i)^2 \\ \ldots \\ (A_i)^{N_i+1} \end{bmatrix},$$

in which $B_{id} = [B_i\ E_i]$.

By denoting $\hat{x}_i(k, \hat{u}_i, N_i)$ as $\hat{x}_i$, $\hat{u}_i(k, x_i^k, N_i)$ as $\hat{u}_i$, $\hat{v}_i(k, x_i^k, N_i)$ as $\hat{v}_i$, and $\hat{u}_{id}(k, x_i^k, N_i)$ as $\hat{u}_{id}$, we have:

$$\hat{x}_i = r_i^k + \Gamma_i\hat{u}_i,\ r_i^k = r_{ic}^k + \Gamma_{iv}\hat{v}_i, \tag{4.3}$$

where $\Gamma_i\hat{u}_i + \Gamma_{iv}\hat{v}_i := \Gamma_{id}\hat{u}_{id}$, and $r_{ic}^k := \Theta_i x_i(k)$.

The objective function is now rewritten as

$$\mathcal{J}^i(k) = [r_i^k + \Gamma_i\hat{u}_i^k]^T\mathcal{Q}_{iN}[r_i^k + \Gamma_i\hat{u}_i] + \hat{u}_i^T\mathcal{R}_{iN}\hat{u}_i, \tag{4.4}$$

where $\mathcal{Q}_{iN} = \text{diag}[\mathcal{Q}_i]_1^{N_i}, \mathcal{R}_{iN} = \text{diag}[\mathcal{R}_i]_1^{N_i}$, which can be expressed by

$$\mathcal{J}^i(k) = \hat{u}_i^T\Phi_i\hat{u}_i + 2\Upsilon_i^k\hat{u}_i + \delta_i^k, \tag{4.5}$$

where $\Phi_i = \Gamma_i^T\mathcal{Q}_{iN}\Gamma_i + \mathcal{R}_{iN}$, $\Upsilon_i^k = r_i^{kT}\mathcal{Q}_{iN}\Gamma_i$, $\delta_i^k = r_i^{kT}\mathcal{Q}_{iN}r_i^k$.

4.4.2 Constraint on Decision Variables

A bounded constraint on decision variables of the following form:

$$\|u_i\|_2^2 \leq \eta_i, \tag{4.6}$$

where η_i is the upper bound deduced from the physical limits of the control input vector u_i is considered in this chapter. The constraint set for the variable vector $\hat{u}_i$ will be as follows:

$$\hat{\mathbb{U}}_i = \{\hat{u}_i = [u_i^T(k) \ldots u_i^T(k+N_i)]^T : \|u_i(.)\|_2^2 \leq \eta_i\}. \tag{4.7}$$

4.4.3 APRC-Based Receding-Horizon Stabilising Constraint

To assure the system stability, a receding-horizon stability constraint derived from the predictive APRC, which is characterised in conjunction with the stabilisability condition, is incorporated into the MPC optimisation problem in this section. In the following, the state prediction toward the end of $k + N_{ic}$ will be employed in the inequality of predictive APRC with respect to $\xi_{ic}(k+N_{ic})$. The predictive horizon for the stability constraint is denoted by N_{ic} to distinguish it with the predictive horizon N_i of the MPC problem, $N_{ic} \leq N_i$. From (4.3), $x_i(k + N_{ic})$ can be expressed as

$$x_i(k+N_{ic}) = r_{iu}^{N_{ic}} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i,\ r_{iu}^{N_{ic}} = (A_i)^{N_{ic}+1}x_i(k) + \mathit{\Gamma}_{iv}^{N}\hat{v}_i,$$

where $\mathit{\Gamma}_{iu}^{N} := [(A_i)^{N_{ic}}B_i \quad \ldots \quad A_iB_i \quad B_i], \mathit{\Gamma}_{iv}^{N} := [(A_i)^{N_{ic}}E_i \quad \ldots \quad A_iE_i \quad E_i]$.
Therefore,

$$\begin{aligned}\xi_{ic}(k+N_{ic}) = [r_{iu}^{N} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i]^T C_i^T R_{ic} C_i [r_{iu}^{N_{ic}} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i] + 2[r_{iu}^{N_{ic}} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i]^T C_i^T S_{ic} u_i(k+N_i)\\ + u_i(k+N_{ic})^T Q_{ic} u_i(k+N_{ic}).\end{aligned}$$

$$\xi_{ic}(k+N_{ic}-1) = [r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{N-1}\hat{u}_{i-1}]^T C_i^T R_{ic} C_i [r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{N-1}\hat{u}_{i-1}]$$
$$+2[r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{N-1}\hat{u}_{i-1}]^T C_i^T S_{ic} u_i(k+N_{ic}-1)$$
$$+u_i(k+N_{ic}-1)^T Q_{ic} u_i(k+N_{ic}-1),$$

where $\hat{u}_{i-1} = [u_i^T(k)\ u_i^T(k+1)\dots u_i^T(k+N_{ic}-1)]^T$.

Now, by denoting

$$M_{iN} := [I_{m_i}\ m_i\ 0_{m_i}\ \dots\ 0_{m_i}]_{N_{ic}},\ M_{iN-1} := [0_{m_i}\ I_{m_i}\ 0_{m_i}\ \dots\ 0_{m_i}]_{N_{ic}},$$

$$\mathit{\Gamma}_{iu}^{0N} := [0_{m_i} \quad \mathit{\Gamma}_{iu}^{N}],\ \mathit{\Gamma}_{iv}^{0N} := [0_{m_i} \quad \mathit{\Gamma}_{iv}^{N}],$$

we obtain

$$\xi_{ic}(k+N_{ic}) = [r_{iu}^{N_{ic}} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i]^T C_i^T R_{ic} C_i [r_{iu}^{N_{ic}} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i] + 2[r_{iu}^{N_{ic}} + \mathit{\Gamma}_{iu}^{N}\hat{u}_i]^T C_i^T S_{ic} M_{iN}\hat{u}_i$$
$$+\hat{u}_i^T M_{iN}^T Q_{ic} M_{iN}\hat{u}_i$$

and

$$\xi_{ic}(k+N_{ic}-1) = [r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{0N}\hat{u}_i]^T C_i^T R_{ic} C_i [r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{0N}\hat{u}_i]$$
$$+2[r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{N-1}\hat{u}_{i-1}]^T C_i^T S_{ic} u_i(k+N_{ic}-1)$$
$$+u_i(k+N_{ic}-1)^T Q_{ic} u_i(k+N_{ic}-1)$$

$$= [r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{0N}\hat{u}_i]^T C_i^T R_{ic} C_i [r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{0N}\hat{u}_i] + 2[r_{iu}^{N_{ic}-1} + \mathit{\Gamma}_{iu}^{0N}\hat{u}_i]^T C_i^T S_{ic} M_{iN-1}\hat{u}_i$$
$$+\hat{u}_i^T M_{iN-1}^T Q_{ic} M_{iN-1}\hat{u}_i.$$

Then, the predictive APRC inequality of $\xi_{ic}(k+N_{ic}) \geq \gamma_i^{k+N_{ic}-1}\xi_{ic}(k+N_{ic}-1)$, is equivalent to

$$\hat{\boldsymbol{u}}_i^T(-\mathit{\Psi}_i)\hat{\boldsymbol{u}}_i - 2\mathit{\Upsilon}_{ic}^k\hat{\boldsymbol{u}}_i - \delta_{ic}^k \leq 0, \tag{4.8}$$

where

$$\Psi_i = (*)\begin{bmatrix} R_{ic} & 0 & S_{ic} & 0 \\ * & \gamma_i^{k+N_{ic}-1}R_{ic} & 0 & \gamma_i^{k+N_{ic}-1}S_{ic} \\ * & * & Q_{ic} & 0 \\ * & * & * & \gamma_i^{k+N_{ic}-1}Q_{ic} \end{bmatrix}\begin{bmatrix} C_i\Gamma_i^N \\ C_i\Gamma_i^{0N} \\ M_{iN} \\ M_{iN-1} \end{bmatrix}$$

$$= \Gamma_{iu}^{NT}C_i^T R_{ic}C_i\Gamma_{iu}^N + 2\Gamma_{iu}^{NT}C_i^T S_{ic}M_{iN} + M_{iN}^T Q_{ic}M_{iN} +$$

$$+\gamma_i^{k+N_{ic}-1}[\Gamma_{iu}^{0NT}C_i^T R_{ic}C_i\Gamma_{iu}^{0N} + 2\Gamma_{iu}^{0NT}C_i^T S_{ic}M_{iN-1} + M_{iN-1}^T Q_{ic}M_{iN-1}],$$

$$\Upsilon_{ic}^k = \begin{bmatrix} C_i r_{iu}^{N_{ic}} \\ C_i r_{iu}^{N_{ic}-1} \\ C_i r_{iu}^{N_{ic}} \\ C_i r_{iu}^{N_{ic}-1} \end{bmatrix}^T \begin{bmatrix} R_{ic} & 0 & 0 & 0 \\ * & \gamma_i^{k+N_{ic}-1}R_{ic} & 0 & 0 \\ * & * & S_{ic} & 0 \\ * & * & * & \gamma_i^{k+N_{ic}-1}S_{ic} \end{bmatrix}\begin{bmatrix} C_i\Gamma_i^N \\ C_i\Gamma_i^{0N} \\ M_{iN} \\ M_{iN-1} \end{bmatrix}$$

$$= {r_{iu}^{N_{ic}}}^T C_i^T(R_{ic}C_i\Gamma_i^N + S_{ic}M_{iN}) + \gamma_i^{k+N_{ic}-1}[{r_{iu}^{N_{ic}}}^T C_i^T(R_{ic}C_i\Gamma_i^{0N} + S_{ic}M_{iN-1})],$$

$$\delta_{ic}^k = (*)\begin{bmatrix} R_{ic} & 0 \\ * & \gamma_i^{k+N_{ic}-1}R_{ic} \end{bmatrix}\begin{bmatrix} C_i r_{iu}^{N_{ic}} \\ C_i r_{iu}^{N_{ic}-1} \end{bmatrix}$$

$$= {r_{iu}^{N_{ic}}}^T C_i^T R_{ic}C_i r_{iu}^{N_{ic}} + \gamma_i^{k+N_{ic}-1}[{r_{iu}^{N_{ic}-1}}^T C_i^T R_{ic}C_i r_{iu}^{N_{ic}-1}].$$

4.4.4 Distributed MPC with Receding-Horizon Stabilising Constraint

From (4.5) and (4.8), the distributed MPC optimisation problem is rendered by

$$\min_{\hat{\boldsymbol{u}}_i} \quad \hat{\boldsymbol{u}}_i^T \Phi_i \hat{\boldsymbol{u}}_i + 2\Upsilon_i^k \hat{\boldsymbol{u}}_i + \delta_i^k \tag{4.9}$$

$$\text{subject to } \hat{\boldsymbol{u}}_i^T(-\Psi_i)\hat{\boldsymbol{u}}_i - 2\Upsilon_{ic}^k\hat{\boldsymbol{u}}_i - \delta_{ic}^k \le 0 \text{ and } \hat{\boldsymbol{u}}_i \in \hat{\mathbb{U}}_i, \text{ for } \quad \mathcal{S}_i \in I^+(k),$$

where $\hat{\mathbb{U}}_i$ is defined in (4.7); $\Phi_i, \Upsilon_i^k, \delta_i^k$ are defined in (4.5); $\Psi_i, \Upsilon_{ic}^k, \delta_{ic}^k$ are defined in (4.8), $I^+(k) = \{\mathcal{S}_i, i \in (1,h) : \;\; \delta_{ic}^k \geq 0 \text{ and } \xi_{ic}^{k-1} < 0\}$. For each subsystem, the optimisation problem (4.9) is solved by the local optimiser for the minimising sequence $\hat{\boldsymbol{u}}_i$ comprising N_i elements of vector $u_i(k+j), j = 0, 1, ..., N_i$. Only the first element $u_i(k)$ of the sequence $\hat{\boldsymbol{u}}_i$ is applied to control the plant. This rolling or receding process is repeated at the next time step, and continues thereon.

4.4.5 Decentralised MPC with Receding-Horizon Stabilising Constraint

The distributed MPC formulation (4.9) will become decentralised when the interactive vectors v_i are not taken into account. The decentralised formulation is therefore obtained when r_i^k is replaced by r_{ic}^k in (4.5) and the predictive vector $\hat{v}_i = 0$. Specifically,

$$\min_{\hat{\boldsymbol{u}}_i} \quad \hat{\boldsymbol{u}}_i^T \Phi_i \hat{\boldsymbol{u}}_i + 2\Upsilon_{id}^k \hat{\boldsymbol{u}}_i + \delta_{id}^k \tag{4.10}$$
$$\text{subject to } \hat{\boldsymbol{u}}_i^T(-\Psi_i)\hat{\boldsymbol{u}}_i - 2\Upsilon_{ic}^k \hat{\boldsymbol{u}}_i - \delta_{ic}^k \leq 0 \text{ and } \hat{\boldsymbol{u}}_i \in \hat{\mathbb{U}}_i, \text{ for } \quad \mathcal{S}_i \in I^+(k),$$

where $\hat{\mathbb{U}}_i$ is defined in (4.7); Φ_i is defined in (4.5); $\Psi_i, \Upsilon_{ic}^k, \delta_{ic}^k$ are defined in (4.8) with $\hat{v}_i = 0$; $\Upsilon_{id}^k = r_{ic}^{kT} \mathcal{Q}_{iN} \Gamma_i,\; \delta_{id}^k = r_{ic}^{kT} \mathcal{Q}_{iN} r_{ic}^k$.

The numerical simulations in below sections show that the control performance of the distributed MPC is higher that that of the decentralised MPC, in exchanged for a higher demand on the inter-subsystem communication, nonetheless.

4.4.6 Optimising Feasibility

To guarantee the constrained feasibility of (4.9), it is necessary that its feasible region is non-empty at every step k. The feasibility condition is stated next.

Theorem 8 *Suppose that the following LMIs are feasible in* $\boldsymbol{Q}_{ic}, \boldsymbol{S}_{ic}, \boldsymbol{R}_{ic}$*:*

$$\Psi_i(\boldsymbol{Q}_{ic},\ \boldsymbol{S}_{ic},\ \boldsymbol{R}_{ic}) \prec 0, \qquad \delta_{ic}^k(\boldsymbol{R}_{ic}) \geq 0, \tag{4.11}$$

$$\begin{bmatrix} Q_{i22} + \boldsymbol{R}_{ic} & S_{i22} + \boldsymbol{S}_{ic} \\ * & R_{i22} + \boldsymbol{Q}_{ic} \end{bmatrix} \preccurlyeq 0, \tag{4.12}$$

where

$$\Psi_i = (*) \begin{bmatrix} \boldsymbol{R}_{ic} & 0 & \boldsymbol{S}_{ic} & 0 \\ * & \gamma_i^{k+N_{ic}-1}\boldsymbol{R}_{ic} & 0 & \gamma_i^{k+N_{ic}-1}\boldsymbol{S}_{ic} \\ * & * & \boldsymbol{Q}_{ic} & 0 \\ * & * & * & \gamma_i^{k+N_{ic}-1}\boldsymbol{Q}_{ic} \end{bmatrix} \begin{bmatrix} C_i \Gamma_i^N \\ C_i \Gamma_i^{0N} \\ M_{iN} \\ M_{iN-1} \end{bmatrix},$$

$$\delta_{ic}^k = (*) \begin{bmatrix} \boldsymbol{R}_{ic} & 0 \\ * & \gamma_i^{k+N_{ic}-1}\boldsymbol{R}_{ic} \end{bmatrix} \begin{bmatrix} C_i r_{iu}^{N_{ic}} \\ C_i r_{iu}^{N_{ic}-1} \end{bmatrix};$$

Then, the feasible region of (4.9) *w.r.t.* $\hat{\boldsymbol{u}}_i$ *is nonempty recursively in* k.

Proof: With $\Psi_{in} \prec 0$, the inequality constraint (4.8) is convex. (4.8) is feasible in $\hat{u}_i$ if additionally $\delta_{ic}^k \geq 0$ (one obvious solution is $\hat{u}_i = 0$). (4.12) assures the convergence of state vectors, as shown in Theorem 1. ■

By using feasible parameter matrices R_{ic}, S_{ic}, Q_{ic} from LMIs (4.11) with known values of $x_i(k)$ and $\hat{v}_i$ from respective neighbours (thus r_i^k and r_{ic}^k are known) , the optimisation problem (4.9) is guaranteed feasible recursively in k.

Remark 7 The above LMIs may become numerically infeasible when $r_{iu}^{N_{ic}} \simeq 0$. Similarly to Theorem 3, one of the solutions for this issue is to ensure that the predictive APRC is feasible for all $\hat{u}_i \in \hat{\mathbb{U}}_i$ while δ_{ic}^k is negative. The multiplier matrices Q_{ic}^k, R_{ic}^k and S_{ic}^k will then need to be redesigned as feasible solutions to the following LMI optimisation while $\delta_{ic}^k < 0$:

$$\max_{\boldsymbol{Q}_{ic}^k,\ \boldsymbol{R}_{ic}^k,\ \boldsymbol{S}_{ic}^k,\ \boldsymbol{\lambda}_i>0,\ \boldsymbol{\tau}_i<\delta_{ic}^k} \quad \boldsymbol{\tau}_i \qquad \text{subject to} \tag{4.13}$$

$$\begin{bmatrix} -\boldsymbol{\Psi}_i^k - \boldsymbol{\lambda}_i I & -\boldsymbol{\Upsilon}_{ic}^k \\ * & \boldsymbol{\lambda}_i N_{ic}\eta_i - \boldsymbol{\tau}_i \end{bmatrix} \succcurlyeq 0,\ \boldsymbol{\Psi}_i^k \prec 0, \tag{4.14a}$$

$$\begin{bmatrix} Q_{i22} + \boldsymbol{R}_{ic}^k & S_{i22} + \boldsymbol{S}_{ic}^k \\ * & R_{i22} + \boldsymbol{Q}_{ic}^k \end{bmatrix} \preccurlyeq 0, \tag{4.14b}$$

for

$$\boldsymbol{\Psi}_i^k = (*) \begin{bmatrix} \boldsymbol{R}_{ic}^k & 0 & \boldsymbol{S}_{ic}^k & 0 \\ * & \gamma_i^{k+N_{ic}-1}\boldsymbol{R}_{ic}^k & 0 & \gamma_i^{k+N_{ic}-1}\boldsymbol{S}_{ic}^k \\ * & * & \boldsymbol{Q}_{ic}^k & 0 \\ * & * & * & \gamma_i^{k+N_{ic}-1}\boldsymbol{Q}_{ic}^k \end{bmatrix} \begin{bmatrix} C_i\Gamma_i^N \\ C_i\Gamma_i^{0N} \\ M_{iN} \\ M_{iN-1} \end{bmatrix}$$

$$= \Gamma_{iu}^{NT} C_i^T \boldsymbol{R}_{ic}^k C_i \Gamma_{iu}^N + 2\Gamma_{iu}^{NT} C_i^T \boldsymbol{S}_{ic}^k M_{iN} + M_{iN}^T \boldsymbol{Q}_{ic}^k M_{iN} +$$

$$+\gamma_i^{k+N_{ic}-1}[\Gamma_{iu}^{0NT} C_i^T \boldsymbol{R}_{ic}^k C_i \Gamma_{iu}^{0N} + 2\Gamma_{iu}^{0NT} C_i^T \boldsymbol{S}_{ic}^k M_{iN-1} + M_{iN-1}^T \boldsymbol{Q}_{ic}^k M_{iN-1}],$$

$$\boldsymbol{\Upsilon}_{ic}^k = \begin{bmatrix} C_i r_{iu}^{N_{ic}} \\ C_i r_{iu}^{N_{ic}-1} \\ C_i r_{iu}^{N_{ic}} \\ C_i r_{iu}^{N_{ic}-1} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{R}_{ic}^k & 0 & 0 & 0 \\ * & \gamma_i^{k+N_{ic}-1}\boldsymbol{R}_{ic}^k & 0 & 0 \\ * & * & \boldsymbol{S}_{ic}^k & 0 \\ * & * & * & \gamma_i^{k+N_{ic}-1}\boldsymbol{S}_{ic}^k \end{bmatrix} \begin{bmatrix} C_i\Gamma_i^N \\ C_i\Gamma_i^{0N} \\ M_{iN} \\ M_{iN-1} \end{bmatrix}$$

$$= {r_{iu}^{N_{ic}}}^T C_i^T (\boldsymbol{R}_{ic}^k C_i \Gamma_i^N + \boldsymbol{S}_{ic}^k M_{iN}) + \gamma_i^{k+N_{ic}-1}[{r_{iu}^{N_{ic}}}^T C_i^T (\boldsymbol{R}_{ic}^k C_i \Gamma_i^{0N} + \boldsymbol{S}_{ic}^k M_{iN-1})],$$

An alternative online procedure derived from Theorem 4 in Chapter 3 together with the feasibility condition (4.11) has also provided stabilised trajectories as demonstrated in the simulation section.

Discussion: The main drawback of the proposed approach, that employs state predictions in the stability constraint, rests at the possible mismatches between the predictive and actual states. A robustness analysis is, therefore, required as an integrated step in the synthesising procedure of the control algorithm. A trivial alternative is to have a receding horizon for the stability constraint, which only requires the predictive horizon N_{ic} to be gradually decreased until $N_{ic} = 1$ in (4.11). The stabilising constraint will then become a one-time-step constraint beyond a certain transient time period. The stability constraint is, therefore, designated as receding-horizon. The computation for the stability-guaranteed MPC problem of each subsystem S_i (3.1) is summerised below.

Procedure 4.2. Distributed MPC with APRC-Based Receding-Horizon Stabilising Constraint.

1. The parameter matrices S_{ic}, Q_{ic}, and R_{ic} are obtained from the LMI optimisation in Theorem 2 (3.30) subject to (4.11) using the initial values of y_i and v_i at step $k = 0$.

2. At every synchronised step $k > 1$, apply a predefined serial iteration scheme for the member subsystems following the pre-determined logical order. For each subsystem,

    (a) Obtain the predictive vector sequence $\hat{v}_i(k)$ from respective neighbours, then whenever $\delta_i^k < 0$, redesign the parameter matrices $R_{ic}(k), S_{ic}(k)$ and $Q_{ic}(k)$ as solutions to online LMIs (3.35), (3.36) and (4.11).

(b) Update the stabilising constraint (4.8) and form the objective function (4.5) using the state prediction (4.2);

(c) Solve (4.9) and apply the first element $u_i(k)$ of the minimising sequence of $\hat{u}_i$ to the plant;

(d) If $N_{ic} > 1$, assign $N_{ic} \leftarrow N_{ic} - 1$;

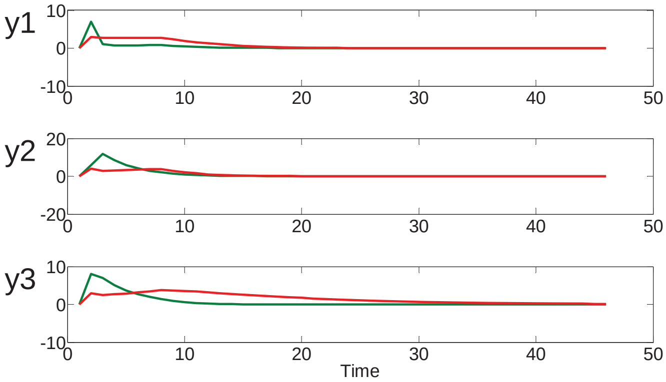
(e) Calculate the predictive vectors $\hat{w}_i$, then send to respective neighbours. □

4.4.7 Illustrative Examples

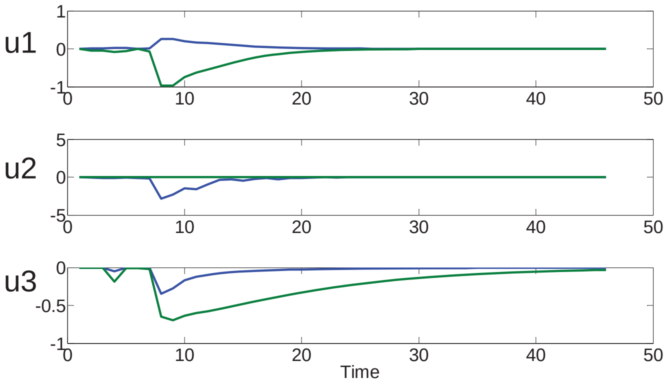
The above counter-current washing circuit is simulated with the presented APRC-based receding-horizon stability constraint for the distributed MPC strategy. In this simulation, the predictive and control horizons of $N_y = 12,\ N_u = 12$ have been chosen for all three subsystems. The weighting coefficients for the control inputs and plant outputs in the objective functions of three MPCs are as follows:
$\mathcal{R}_1 = \text{diag}\{2, 1\}, \mathcal{Q}_1 = \text{diag}\{5, 1, 2\}; \mathcal{R}_2 = [1], \mathcal{Q}_2 = \text{diag}\{1, 1, 2\}, \mathcal{R}_3 = \text{diag}\{0.5, 1\}$, $\mathcal{Q}_3 = \text{diag}\{4, 1, 5\}$.

The constraints on the control inputs are set up as $\eta_1 = 2, \eta_2 = 10, \eta_3 = 10$. The output constraints are ignored in this simulation. The initial states and controls are as follows: $x_1(0) = [1\ 7\ 3]^T, x_2(0) = [7\ 6\ 4]^T, x_3(0) = [2\ 8\ 3]^T$.

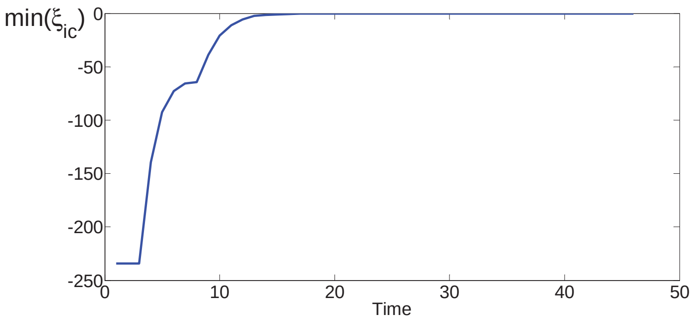
To demonstrate the efficacy of the receding-horizon stabilising constraint approach, $N_{ic} = N_i$ has been used in this simulation with the perfect model without any disturbances. The offline multiplier matrices are firstly employed (i.e. the first online step in Procedure 4.2 is omitted). The results from Procedure 4.2 for distributed MPC are given in Figure 4.7, indicating clearly a stabilised system. When the multiplier matrices are redesigned using the online step (iia) in Procedure 4.2, the distributed MPC produces the trends in Figure 4.8. In this particular example, the



(a) Plant Outputs - y_i.

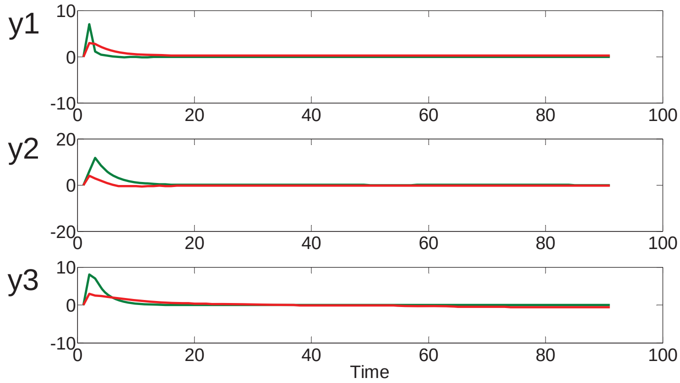


(b) Control Inputs - u_i.

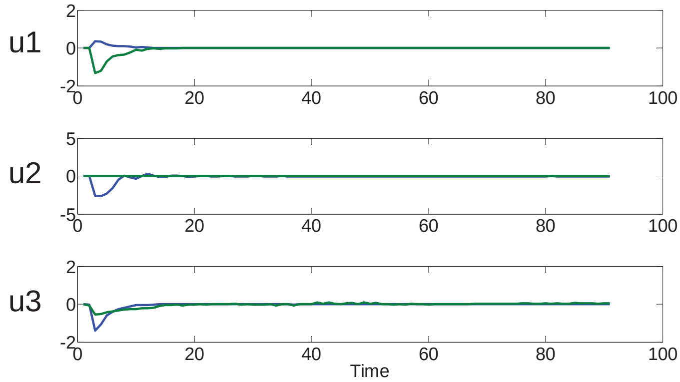


(c) Positive Realness Constraints - $\max_{i=1,2,3}(\xi_{ic})$.

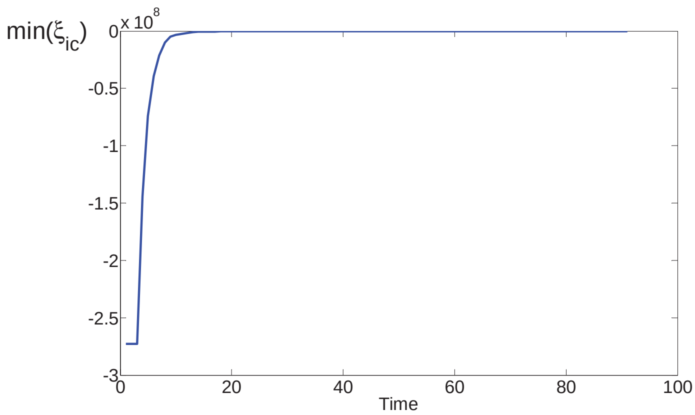
Figure 4.7 : Distributed MPCs with APRC-Based Receding-Horizon Stabilising Constraint - Offline Multiplier Matrices - Simulation 1.



(a) Plant Outputs - y_i.



(b) Control Inputs - u_i.



(c) Positive Realness Constraint - $\max_{i=1,2,3}(\xi_{ic})$.

Figure 4.8 : Distributed MPCs with APRC-Based Receding-Horizon Stabilising Constraint - Online Multiplier Matrices - Simulation 1.

online characterisation for the APRC multiplier matrices has, indeed, improved the overall performance, as observed from the trends in Figure 4.8 relatively to those in Figure 4.7. The settling times are shorter and the controls are smoother than those employed with offline multiplier matrices.

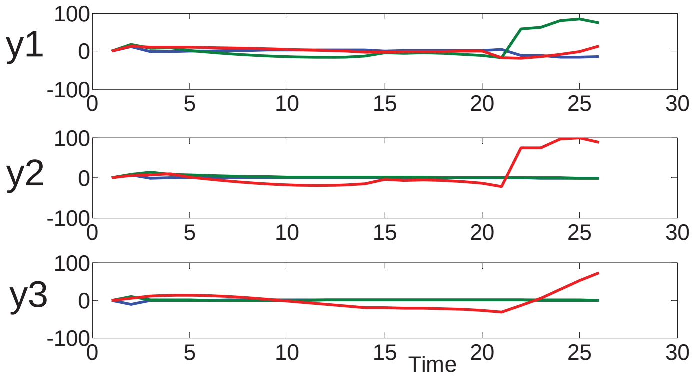
In the following, the washing circuit is emulated with higher subsystem interactions (e.g. flow rates between washing stages are higher using higher power pumps). The interaction space models are as follows:

$$E_1 = \begin{bmatrix} 0 & -1.6 \\ 0 & 8 \\ 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 6 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2.4 & 0 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 6 & 0 \\ 0 & 2.4 \\ 0 & 0 \end{bmatrix}$$

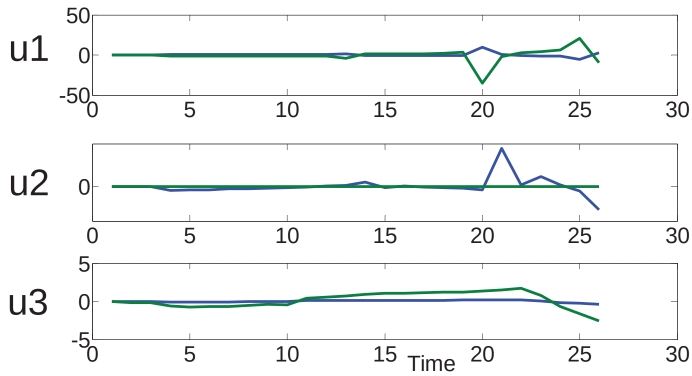
The intitial states have higher values of

$$x_1(0) = [11\ 17\ 13]^T, x_2(0) = [7\ 8\ 5]^T, x_3(0) = [-11\ 9\ 5]^T.$$

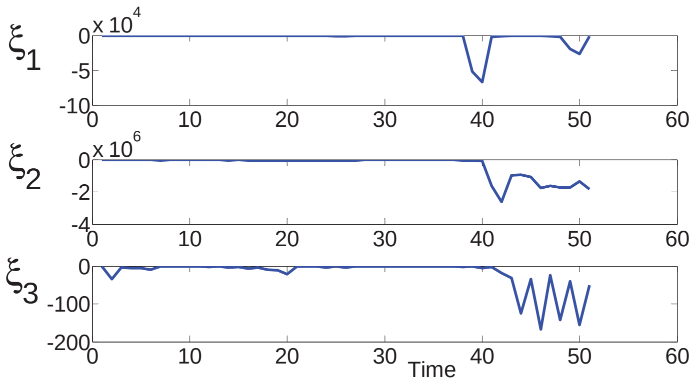
The offline parameter matrices are firstly employed. The results from Procedure 4.2 for distributed MPC are given in Figure 4.9, indicating clearly destabilised trajectories. When the parameter matrices are redesigned, the distributed MPC produces the trends in Figure 4.10. In this particular example, the online characterisation for the parameter matrices has, indeed, stabilised the overall system.



(a) Plant Outputs - y_i.

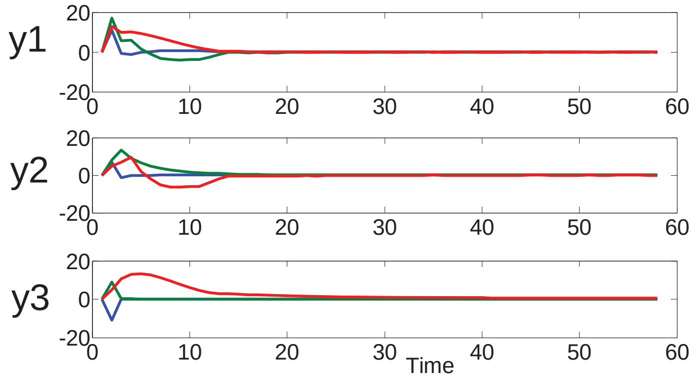


(b) Control Inputs - u_i.

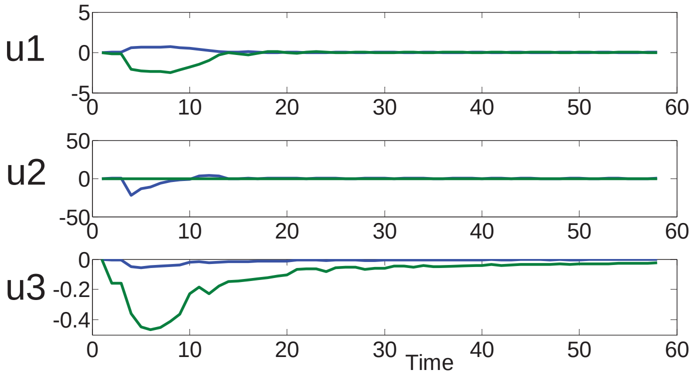


(c) Positive Realness Constraints - ξ_{ic}.

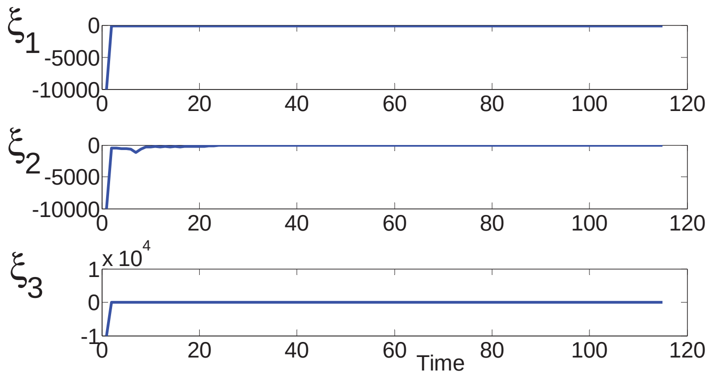
Figure 4.9 : Distributed MPCs with APRC-Based Receding-Horizon Stabilising Constraint - Offline Multiplier Matrices - Simulation 2.



(a) Plant Outputs - y_i.



(b) Control Inputs - u_i.



(c) Positive Realness Constraint - ξ_{ic}.

Figure 4.10 : Distributed MPCs with APRC-Based Receding-Horizon Stabilising Constraint - Online Multiplier Matrices - Simulation 2.

4.5 An Energy Dissipation View Point on the Equilibrium-Independent Dissipativity

In this section, an interpretation of dissipativity in the time domain is presented with an "energy dissipation" view point. The effectiveness of this time domain analysis is demonstrated with a predictive PID algorithm herein.

4.5.1 Preliminary

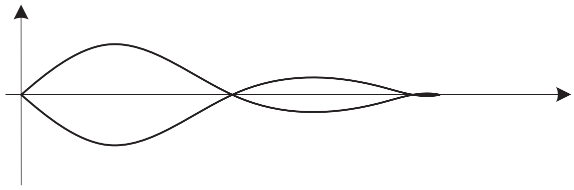
The discussion is started with the input and output trajectories of an *absolute passive* (AP) trend, such as input and output trajectories of a feedback control system, to illustrate the physical meaning of dissipating "abstracted energy" around the non-zero steady states.

Definition 3 *An input and output system is called absolute passive (AP) if and only if its input and output trajectories always pass through their steady states at the same time instants.*

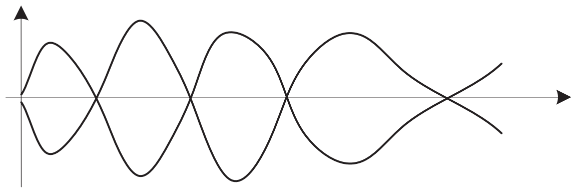
For convenience, assuming that the output set point and the input steady state are normalised to zero ($y_{ss} \equiv 0$, $u_{ss} \equiv 0$). A system that is AP is not necessarily stable or attractive. Figure 4.11 depicts the behaviour of an AP trajectories for both stable and non-attractive cases.

Proposition 1 *For systems having AP trajectories, the following discrete time inequality holds true for all $k \geq 0$:*

$$p_k := (y_k - y_{ss})^T (u_{ss} - u_k) \geq 0. \tag{4.15}$$



(a) AP Trajectory - Attractive.



(b) AP Trajectory - Non-Attractive.

Figure 4.11 : Absolute Passive (AP) responses.

It is clearly that AP is a property of systems having zero phase shift between its input and output. AP response is actually nothing else than a special case of a passive system. Inequality (4.15) is called instant absolute passive inequality (iAPi) herein.

The absolute passive trajectory is usually difficult to obtain in real life. In the feedback control and circuit theory, it is quite usual to deal with the *one-overshoot stable* (OOS) response instead. This would be an ideal response one would like to have when designing a controller as depicted in Figure 4.12. While AP is not necessarily equivalent to stable or attractive, OOS is a special case of stable and well performed systems.

Definition 4 *The critically damped response is one-overshoot stable or OOS if and only if it is AP and the output and input attract to their steady states after only one overshoot.*

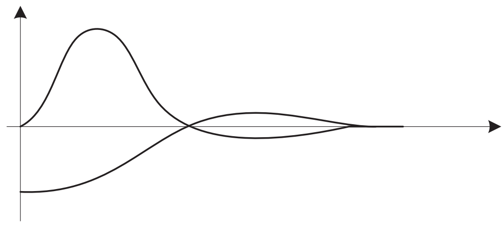


Figure 4.12 : One-Overshoot Response (OOS) Trajectory.

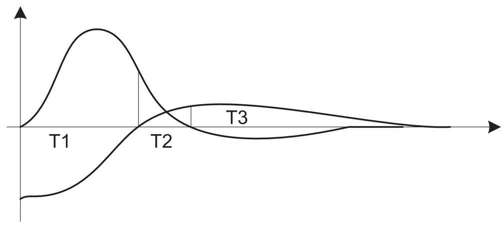


Figure 4.13 : OOS and Passive, but Not AP.

Similar to the AP response, OOS is quite difficult to achieve in a control synthesis problem. It is quite usual to have a stable response that may approximately has an overshoot, but does not have the AP property; i.e., the input and output trajectories do not across their steady states at the same time instants, as described in Figure 4.13. This is a typical critically-damped response. One easily verifies that the sum of p_k over time periods of $T1$ and $T3$ are greater than that for $T2$. This response will ultimately become AP when $T_2 \to 0$.

This typical case helped us conceptualise the response of a passive and stable system. The relationship between stable responses with passivity is generalised in definition 5 below.

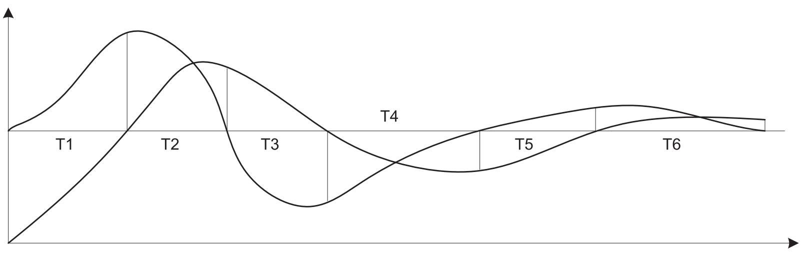


Figure 4.14 : A Stable Response.

Definition 5 *For a passive and stable system, the summation of p_k overtime is always nonnegative,*

$$w_k := \sum_{k=0}^{\kappa} (y_k - y_{ss})^T (u_{ss} - u_k) \geq 0. \tag{4.16}$$

Alternatively, it can be said that, for a passive and stable system, the sum s_k of p_k where $p_k \geq 0$, denoted as s_k^+, is greater than or equal to the sum s_k of p_k where $p_k < 0$, denoted as s_k^-, i.e.

$$s_k^+ \geq s_k^-. \tag{4.17}$$

The response of a more general case of stable responses can be graphically represented by Figure 4.14. In this figure, the sum of p_k in odd periods $T1,\ T3,\ T5,\ldots$ (S_k^+) is greater than the sum of p_k in even periods $T2,\ T4,\ T6,\ldots$ (S_k^-). As a result of that, the passive response is attractive. The even time periods represent the phase shift between the input and output trajectories. If all the even time periods $T_{\text{even}} \to 0$, the response will become AP or OOS. This type of response is much easier to achieve in a control design problem. The greater the difference between the two sums ($s_k^+ - s_k^-$), the closer to AP the system response will be. When the difference becomes zero, we have a lossless response. And it will be the boundary for a passive and stable response. It is easy to see that a 90^o phase shift is equivalent to a lossless

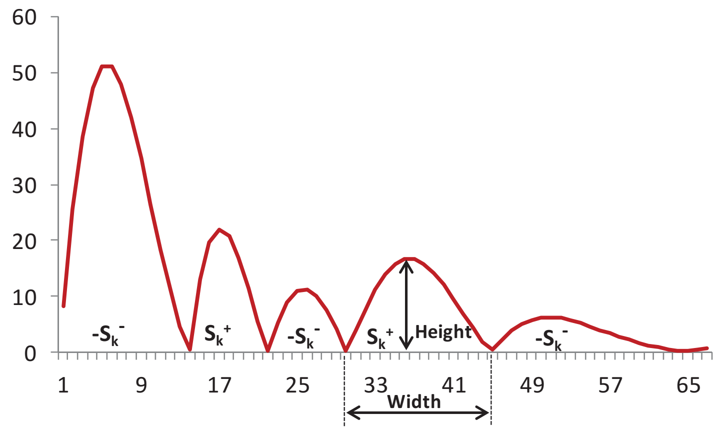


Figure 4.15 : An Energy Viewpoint for Stable and Passive Responses.

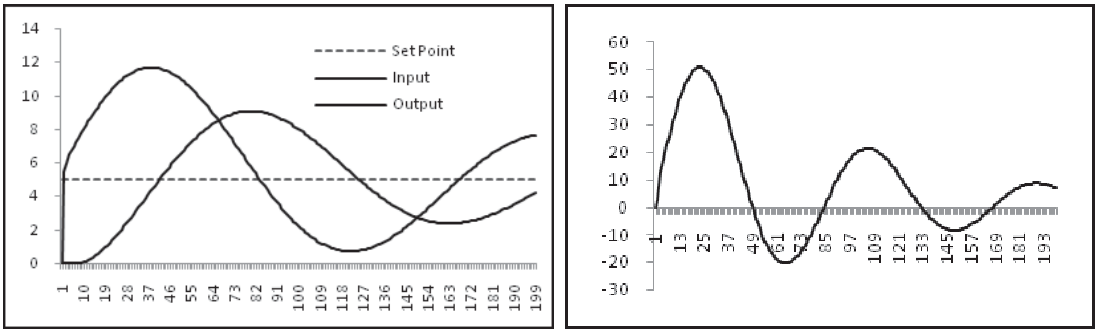
response. This gave us a visual interpretation of the passive response concept in the time domain and its relationship with attractive trajectories as well as between the phase property and passivity.

Now, under an energy dissipation perspective, if $p_k \equiv (y_k - y_{ss})^T(u_{ss} - u_k)$ is considered as an abstracted supply power, then s_k^+ is the consumed energy, while s_k^- is the generated energy. When (4.17) holds true, we say the system response is energy dissipative, i.e. energy is dissipated away. By virtue of this interpretation, the passivity is related to the energy balance of conservation law. Recalling the energy conservation equality in physics, it is obvious that the availability of a storage function (energy) will provide certain margins for the inequality (4.17) to be fulfilled, and so will the energy dissipativity. If we draw the trends of $|p_k|$ for the case represented by Figure 4.15, the sum s_k is nothing else than the cross sections of areas under the trend curve. It can, therefore, be viewed as an abstracted energy of p_k. The area width and height are the metrics for this abstracted energy quantity. As a result, the energy dissipation rate will vary following these metrics. The width is timing or phase related, while the height is magnitude related. They will definitely change the response behaviour when being adjusted. Talking about the phase and magnitude related quantities, we are introducing the concept of energy dissipativity. Dissipativity contains passivity components with phase related properties, as well as magnitude components with gain related properties. The dissipation inequality, that assimilates the energy balance process, is a more natural match for the virtual energy conservation laws for input and output trajectories. It is natural to say that, passivity is phase related content only, while dissipativity has both phase and amplitude related contents. The quadratic supply rate of the form

$$p_k := (y_k - y_{ss})^T Q(y_k - y_{ss}) + (y_k - y_{ss})^T S(u_{ss} - u_k) - (u_k - u_{ss})^T R(u_{ss} - u_k).$$

is therefore preferable for use in describing an energy dissipation system.

One may easily see that, artificially adjusting the signal u_k (manipulated variable) will instantly change the amplitude response, but may not have an immediate effect on the phase quantity, if the adjustments are bounded by the physical constraint of u_k. In other word, it will be difficult, if not say impossible, to change the phase shift in an immediate time frame around the overlap areas of the input and output trajectories (even periods T_{even} in Figure 4.14). Some forms of prediction will need to be applied well in advance if the adjustment aims to change the phase related quantity, i.e. the passivity, and so does the stability. Along this line, predictions are probably well suited to a synthesising strategy for the dissipativity and/or passivity based methods.



(a) Initial Fixed Gains.

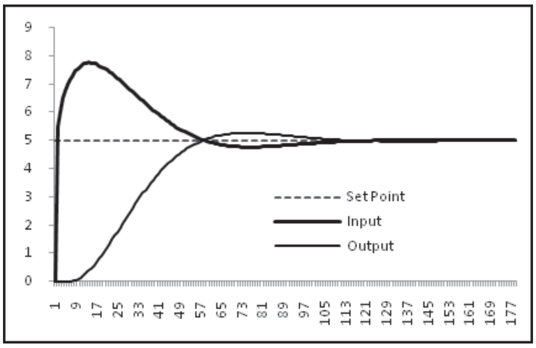
(b) The Supply Rate Trend.

Figure 4.16 : Illustrative Predictive PID based on Dissipativity.

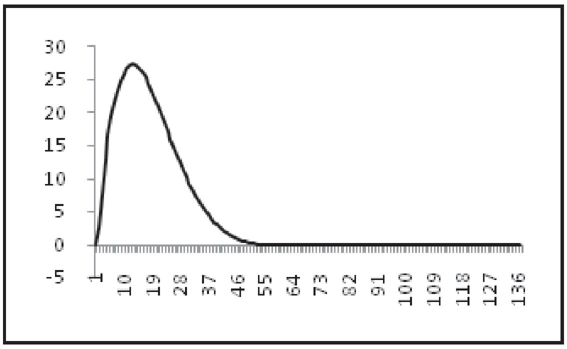
4.5.2 Experimental Predictive PID for Equilibrium Independent Dissipativity

To illustrate the analysis, we have developed a custom adjustment algorithm for the SISO LTI stable systems. Similarly to the so-called predictive PID control, wherein the P, I and D gains are adjusted on-the-fly to assure the system stability while achieving a higher control performance. Since it is only an illustrative algorithm, a simple Excel tabular sheet has been used for predicting a "second order plus dead time" system. The PID controller with high gains is initially implemented. It will definitely cause a limit-cycle alike response. The on-line adjustment strategy will change the PID gains so as the system response approaches a passive behaviour after some certain future moves. Similar to a predictive control strategy, only the first predictive step is used to manipulate the system. A system model is needed to predict the future response within a chosen finite horizon. The algorithm iterates the predictive values of a passivity supply rate p_k till the end of a chosen horizon N in the future. N shall properly be greater than the system time constant.

One possible strategy on choosing the horizon is that it must be greater than the



(a) With Online Gain Adjustments.



(b) The Supply Rate Trend.

Figure 4.17 : Illustrative Predictive PID based on Dissipativity - Smooth Adjustment for OOS.

time instant T_s when the output trajectory gets over its set point the first time. The horizon N is not a fixed number, but will vary. The second basis for choosing the new PID gains is to assure the increases of T_s after every prediction, i.e. $T_s|(k) > T_s|(k-1)$. The algorithm will initially reduce the PID gains in small increments within each prediction based on the inequality $p_N > -\epsilon^2$ and another two constraints. The first constraint is the physical constraint on u_k. The second constraint is the inequality $T_s|(k) > T_s|(k-1)$ described above. After some predictions, if reducing PID gains does not recover the passivity condition beyond N (i.e. $p_N > -\epsilon^2$), then start increasing them in small increments. This process will stop when the output stays around the set point steadily, without having any PID gain adjustments.

This simulation study has clearly shown us that the dissipativity supply rate will relax the constraint $p_N > -\epsilon^2$ and requires less calculations in the iterative predictions. The result is quite interesting, because the response is almost identical to OOS. Figure 4.16a shows the closed-loop response with large and fixed PID gains. Figure 4.17a indicates a tremendous improvement on the control performance when the above predictive PID adjustment is employed. Their supply rate trends are given in Figures 4.16b and 4.17b, respectively. The online PID gains are provided in bellow table. The transfer function in the simulation is as follows:

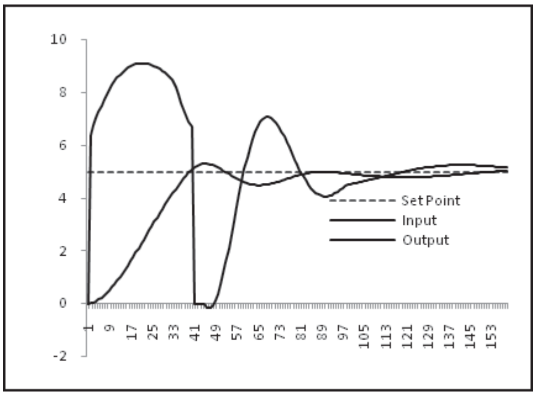
$$G(s) = \frac{0.25}{s^2 + 0.5s + 0.25} e^{-7s}.$$

Step No.	K_p, K_I	Horizon
1	0.1, 0.5	50
6	0.05, 0.2	50
8	0.3, 0.1	52
11	0.02, 0.1	52
14	0.0075, 0.001	55
17	0.002, 0.2	59
18	0.005, 0.5	60
19	0.005, 0.8	60

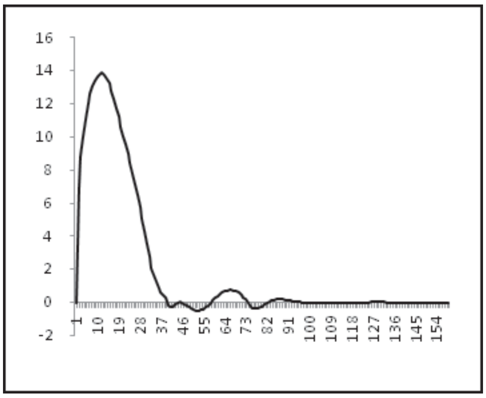
The following dissipativity inequality has been employed in the simulation study.

$$0.1(u_k - u_{ss})^T(u_k - u_{ss}) + 0.985(y_k - y_{ss})^T(u_{ss} - u_k) + 0.02(y_k - y_{ss})^T(y_k - y_{ss}) \geq 0.05.$$

The non-smooth adjustments have also been simulated without looking at the constraints on Δu_k. The resulting trends are given in Figures 4.18. The corresponding online PID gains are provided in bellow table.



(a) With Online Gain Adjustments.



(b) The Supply Rate Trend.

Figure 4.18 : Illustrative Predictive PID based on Dissipativity - Non-smooth Adjustments.

Step No.	K_p, K_I	Horizon	Over-Shoot
1	0.08, 1.2	35	6.55
5	0.06, 1	40	6.35
35	0.2, 3	40	5.65
38	0.1, 2	40	5.6
42	0.0001, 0.0001	40	5.2
47	0.2, 8	40	5.3
100	0.1, 1.2	40	0.3

This simulation gave us some insights into the behaviour of input and output trajectories against the online adjustments based on the information of a dissipativity supply rate. Looking at the time responses of p_k in the above Figures, we can see that it is crucial to maintain smooth p_k trajectories for perceiving a realisable control performance. And the dissipativity constraint presented in this section has accomplished our findings. It has introduced a novel notion of dissipative responses around a given non-zero set point. We have presented this result in an internal report in early 2008. An article published on Automatica in September 2011 (submitted in February 2010) has introduced a similar definition for the "equilibrium-independent passivity" [58]. The development for non-zero set points has been presented in [154].

4.5.3 Concluding Remarks

A constructive method using *stabilising agent* has been developed for the feasible implementation. The stabilising bound on the input vector is updated on-the-fly in this approach. The stabilising agent that operate independently to the controller will override the controller outputs if they are determined as beyond a tuned APRC boundary. Being disparate from the associated control laws, the decentralised stabilising agents are reliably interoperable with different control algorithms while using the same stabilising mechanism.

The predictive APRC has been developed into receding-horizon stabilising constraints for the model predictive controllers in distributed and decentralised configurations. The recursive feasibilities of these strategies are guaranteed by updating the APRC multiplier matrices online. The presented receding-horizon stability constraint has successfully stabilised the large-scale system with constrained controls without causing any wind-up actions. In order to achieve robustness, the predictive horizon of the stabilising constraint is made decreasing over time in this approach. By having the one-time-step stability constraint on the initial control vector only after a transient time period, the receding-horizon stabilising constraint is much more flexible and nonconservative.

An interpretation for dissipativity and passivity of input and output trajectories under an energy dissipation view point has been illustrated with a predictive PID simulation.

Chapter 5

Networked-Control Interconnected Systems with Asymptotically Positive Realness Constraint

5.1 Introduction

The Asymptotically Positive Realness Constraint (APRC) presented in Chapter 3 is extended in this chapter for networked control systems (NCS) and distributed model predictive control with multiple communication network topologies. The conceptual system architecture is given in Figure 5.1. For guaranteeing the system stability, the APRC-based stabilisability condition is established from the data-lost robust dissipativity of the open-loop subsystems [16] and the parameterised quadratic constraints of their controllers. The sporadic data losses of output measurements are effectively compensated for right after the dropout occurrences. The last values of control vectors are used during the disruptive periods. The APRC is characterised such that the stabilising bounds are assuredly feasible in the presences of data losses. To avoid any conservativeness, the interactive (or coupling) vectors between subsystems are not imposed by any additional constraints in this approach.

The stability of control systems operating in an imperfect shared communication network for real-time control has been attracting many researches since the problem was raised in [55, 15]. In such a non-deterministic networking environment, the measurements from transmitters as well as the output signals to actuators (or to lower layers in hierarchical architectures), are subject to occasional delays and random data

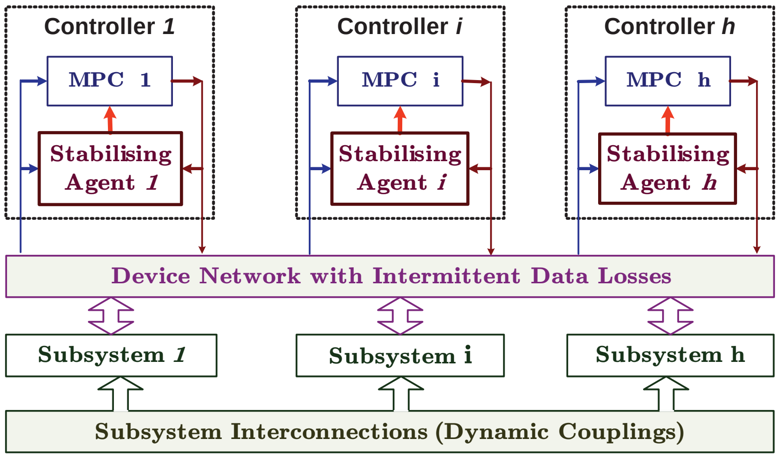


Figure 5.1 : Conceptual Block Diagram of the System Architecture.

losses, due to the limited bandwidths that may cause congestions, and transmission errors. For example, the checksum bits in the data-link layer of IP-based protocols are checked by the receiver for non-error data packets. If parity is not met, the packet will be dropped. The performance of networked control systems (NCS) is therefore easily degraded, and the stability is not ensured.

By modeling the network systems with data losses and transmission delays as asynchronous multi-rate sampled-data systems, whose controllers do not received the measured data from sensors at fixed time instants, the authors in [56] have referenced the problem back to the early 80's. Since then, the classical analysis and design of NCS using state feedback and output feedback to deal with either data losses and random delays, or both, in various optimal control schemes, have been presented in many papers, such as those in [166, 163, 100]. Within which, the closed-loop stability is ensured if the network inaccessible period is shorter than MATI (maximum allowable transmission interval). Both deterministic and stochastic (Markov chain) models have already been used in analysis and control designs. In the deterministic approaches, the packet drop process and the asynchronously sampling process are modeled as systems having continuous variables conform to continuous time ordinary differential equations (ODEs), while the unfolding of discrete variables is randomly determined [163]. An augmented model, which has bounded model uncertainties, is normally launched for the discrete time model. The problem turns out to be a robust control problem. The research in NCS is quite mature in its own right with recent results reported in [137, 51, 84] and references thereof.

Apart from the classical closed-formed methods, the online optimisation-based model predictive control framework is recognised as highly appropriate to the data lost problem when the future control sequence is buffered by smart actuators to overcome the transmission problems [151]. The minimum requirement for stability is, always that, the control horizon is not less than MATI. Different MPC strategies for robust stability, quantization errors, as well as for reducing the network traffic for linear systems have been proposed in [151, 48, 52]. For nonlinear systems, the Lyapunov-based and terminal constraint set methods of NMPC have been extended to NCS [43, 85]. Instead of sending the whole control sequence, the last received values are used in [116]. The constraints that define the optimisation problems and the implementation procedures have been modified to account for data losses in those approaches. The prediction inconsistency or trajectory drifts between the old and new predictions has been analysed in [43] to maintain the NMPC stability. A two-tier architecture for the distributed MPCs with NCS has been presented in [85], which proved to preserve the stability and improve the performance relatively to the centralised controller with asynchronous measurements. The input-to-state stability condition for NMPC has been developed with time-stamping information for NCS in [118].

While much research has been focused on the networked control systems with data packet dropouts and transmission delays, the problem of assuring stability for large scale interconnected systems with partial and total communication loss between the subsystems has not been addressed up until recently. The stability condition for distributed controllers in the failures of inter-system communication is derived in [75]. The graph theory applying to the coordination controls of autonomous vehicles has been discussed in [133]. The approach of dynamic graphs for interconnected systems with different coupling structures is introduced in [145]. The cooperative control schemes for multi-agent systems having time-varying communication topologies have been developed in [40, 110, 68, 41]. These approaches do not, nonetheless, accommodate the transmission packet dropouts in local sensor networks.

The developments in this Chapter address the data lost problem in local sensor/actuator network and the flexible communication links between subsystems that have multiple network topologies. Section 5.2 only considers local data losses, wherein the data-lost robust dissipativity criterion is developed for use with APRC. Decentralised stabilising agents are implemented for the simulated interconnected system to demonstrate the developed stabilisability condition. Both issues of local data losses and flexible communication links are resolved in Section 5.5 by a perturbed state feedback scheme for distributed model predictive controllers in the flexible communication environment.

5.2 System and Networked Control Models

The models in Chapter 3 are reused herein. For completeness, they are provided with new reference numbers in this chapter.

Consider an interconnected system Σ consisting of h subsystems, each denoted as

$\mathcal{S}_i, i = 1, \ldots, h$ and has a discrete time state space model of the form:

$$\mathcal{S}_i : \begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) + E_i v_i(k), \\ y_i(k) = C_i x_i(k), \quad w_i(k) = F_i x_i(k) + J_i u_i(k), \end{cases} \tag{5.1}$$

where $u_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}, y_i \in \mathbb{Y}_i \subset \mathbb{R}^{q_i}, x_i \in \mathbb{X}_i \subset \mathbb{R}^{n_i}, v_i \in \mathbb{V}_i \subset \mathbb{R}^{m_{v_i}}, w_i \in \mathbb{W}_i \subset \mathbb{R}^{q_{w_i}}$; x_i, u_i, y_i, v_i, and w_i are, respectively, state, control, measurement output, interactive input and interactive output vectors. In this chapter, the following bounded constraint is considered:

$$\mathbb{U}_i \triangleq \{u_i : \|u_i\|_2^2 \leq \eta_i\}, \tag{5.2}$$

where η_i is the upper bound deduced from the physical limits of the actuators connected to the control vector u_i.

The input $v_\xi(k)$ of subsystem $\mathcal{S}_\xi$ and the output $w_\zeta(k)$ of subsystem $\mathcal{S}_\zeta$, $\xi \neq \zeta$, are connected to each other in an arbitrary topology. The large-scale system Σ is represented by a state space model of the block diagonal system $\mathcal{S}$ formed by h subsystems $\mathcal{S}_i$ and the interconnection process H, as follows:

$$\Sigma : \begin{cases} x(k+1) = A\,x(k) + B\,u(k) + E\,v(k), \\ y(k) = C\,x(k), \quad w(k) = F\,x(k) + J\,u(k), \\ v(k) = H\,w(k), \end{cases} \tag{5.3}$$

where $A = \text{diag}[A_i]_1^h,\ B = \text{diag}[B_i]_1^h,\ C = \text{diag}[C_i]_1^h,\ E = \text{diag}[E_i]_1^h,\ F = \text{diag}[F_i]_1^h$, $x = [{x_1}^T ... {x_h}^T]^T$, $u = [{u_1}^T ... {u_h}^T]^T$, $y = [{y_1}^T ... {y_h}^T]^T$, $v = [{v_1}^T ... {v_h}^T]^T$, $w = [{w_1}^T ... {w_h}^T]^T$, and H is the global coupling matrix. The elements of H is either 1 or 0 only. From a centralised point of view, the system Σ can be rewritten as

$$\Sigma : \begin{cases} x(k+1) & = A_\Sigma\,x(k) + B_\Sigma\,u(k), \\ y(k) & = C\,x(k), \end{cases} \tag{5.4}$$

where $A_{\Sigma} = A + EHF,\ B_{\Sigma} = B + EHJ$.

When all the interactive inputs and outputs vanish (i.e. $v_i = 0$ and $w_i = 0$), a stand-alone subsystem $\mathcal{S}_i$, denoted as $\mathcal{S}_i^s$, has the state space model of the form:

$$\mathcal{S}_i^s : \begin{cases} x_i(k+1) & = A_i x_i(k) + B_i u_i(k), \\ y_i(k) & = C_i x_i(k). \end{cases} \tag{5.5}$$

Data-Lost Process: In this chapter, a mesh device network is installed for the large-scale system for updating output measurements and actuating control devices. The updating time instants are assumed synchronised between all controllers. The state vector x of Σ becomes $\tilde{x}$ beyond the network interface ports inside the controllers. It is assumed that the data losses are identical to all variables by hardware and firmware configurations. When the state vector $x(k)$ at the time instant k is transmitted successfully, we have $\tilde{x}(k) = x(k)$, otherwise $\tilde{x}(k) = x(k-1)$ (i.e. when the data is lost). For the outputs, the last received values of control vectors are applied to the plant (e.g. by industrial smart actuators) during the transmission disruptive periods. By representing the consecutive updating instants of $\tilde{x}(k)$ with a sequence of integer numbers $\mathcal{J} = \{j_1, ..., j_q, ..., j_\kappa, ...\}, \mathcal{J} \subset \mathbb{Z}^+$, the time interval between j_q and j_{q+1} is treated as one transmission period. If the communication data on device networks is perfect at time k, we have $j_\kappa = k$. The upper bound of the successful transmission periods is denoted as $\boldsymbol{\mu}$ (or MATI - maximum allowable transmission interval).

$$\boldsymbol{\mu} \triangleq \max_{j_q \in \mathcal{J}} \big(\tau(q)\big), \quad \tau(q) := j_q - j_{q-1}. \tag{5.6}$$

Assumption 2 (i) All stand-alone subsystems $\mathcal{S}_i^s$ (5.5) and the large-scale system Σ (5.4) are controllable and observable. (ii) The updating time instants are synchronised between all subsystems and their controllers.

5.3 Stabilisability Conditions for Networked Control Systems

Let us first introduce the dissipativity for Σ and APRC for each subsystem $\mathcal{S}_i^s$ (5.1). A large-scale system Σ (5.4) is said to be *data-lost robustly quadratically dissipative* with respect to the quadratic supply rate $\xi\big(y(k), u(k)\big)$ defined as,

$$\xi\big(y(k), u(k)\big) \triangleq [y(k)^T u(k)^T] \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} y(k) \\ u(k) \end{bmatrix}, \tag{5.7}$$

where Q, R, S are multiplier matrices with symmetric Q and R, if there exists a nonnegative C^1 function, addressed as storage function, $V(x(k))$, $V(0) = 0$, such that for all $u(k)$ and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied irrespectively of the initial value of the state $x(0)$:

$$\begin{gathered} V\big(x(k+\tau)\big) - \alpha V\big(x(k)\big) \leq \xi\big(y(k), u(k)\big), \\ \forall \tau = 1, \ \dots \ , \boldsymbol{\mu}, \quad 0 < \alpha < 1. \end{gathered} \tag{5.8}$$

The square storage function of the form $V\big(x(k)\big) = x(k)^T P x(k), P = P^T \succ 0$, is considered in this chapter. Here, define another quadratic function ξ_{ic}, addressed as supply rate of the controller $\mathcal{C}_i$ of a subsystem $\mathcal{S}_i$, as

$$\xi_{ic}\big(u_i(k), y_i(k)\big) \triangleq [u_i(k)^T y_i(k)^T] \begin{bmatrix} Q_{ic} & S_{ic} \\ * & R_{ic}(k) \end{bmatrix} \begin{bmatrix} u_i(k) \\ y_i(k) \end{bmatrix} \tag{5.9}$$

For conciseness, $\xi_{ic}\big(u_i(k), y_i(k)\big)$ is denoted as ξ_{ic}^k; $\xi\big(y(k), u(k)\big)$ is denoted as ξ^k; and the time index k and j_q (q is the index of the data lost process of MATI=$\boldsymbol{\mu}$) are represented by the superscript typefaces k and q, respectively, without brackets where appropriated.

Definition 6 *The controller $\mathcal{C}_i$ is said to satisfy the affirmative APRC with respect to $\xi_{ic}(u^q, y^q)$ (5.9), or simply aAPRC, in the presence of data-lost process of MATI= $\boldsymbol{\mu}$, if there is $j_{q0} \in \mathcal{J}$, $0 \leq \gamma_i^q < 1$ and $|\beta_i^q| \geq \sqrt{(1-\gamma_i^q)|\xi_i^{q-1}|}$, such that*

$$\begin{cases} \xi_{ic}^q \geq 0, & \text{if } \xi_{ic}^{q-1} \geq 0, \\ \xi_{ic}^q \geq \xi_{ic}^{q-1} + (\beta_i^q)^2, & \text{if } \xi_{ic}^{q-1} < 0, \end{cases} \quad \forall j_q > j_{q0}. \tag{5.10}$$

Remark 8 (5.10) can also be expressed as follows:

$$\xi_{ic}^q \geq \frac{1 - \operatorname{sgn}(\xi_{ic}^{q-1})}{2} \Big(\xi_{ic}^{q-1} - \operatorname{sgn}(\xi_{ic}^{q-1})(\beta_i^q)^2 \Big) \; \forall j_q > j_{q0}. \tag{5.11}$$

The bias number β_i^q may be different at every successful transmission instants j_q. The condition of aAPRC is dynamically established as there will be several j_{q_0} during the lifetime of a control subsystem at which $\xi_{ic}^{q_0} < 0$.

Now, by denoting $Q_c := \operatorname{diag}[Q_{ic}]_1^h, S_c := \operatorname{diag}[S_{ic}]_1^h$, $R_c := \operatorname{diag}[R_{ic}]_1^h$ and

$$\xi_c^k := u^{k^T} Q_c u^k + 2y^{k^T} S_c u^k + y^{k^T} R_c^k y^k$$

as the supply rate of $\varSigma$ ($u \in \mathbb{R}^m$) in the presence of local MPCs, the aAPRC-based stabilisability condition for the controls $\tilde{u}_i(j_q)$ subject to the data lost process (5.25) is stated in the next Theorem. Denote $\varphi_i^q := \frac{1}{2}(1 - \mathsf{sgn}(\xi_{ic}^q))\xi_{ic}^q$, $\theta_i^q := \sigma_i^q \operatorname{sgn}(\xi_{ic}^{q-1})(\beta_i^q)^2$.

Theorem 9 *Suppose that*

(i) The following LMIs are feasible in $\boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{R}$:

$$\begin{bmatrix} A_\varSigma^{\tau\,T} \boldsymbol{P} A_\varSigma^\tau - \alpha \boldsymbol{P} - C^T \boldsymbol{Q} C & A_\varSigma^{\tau\,T} \boldsymbol{P} \mathscr{B} - C^T \boldsymbol{S} M \\ * & \mathscr{B}^T \boldsymbol{P} \mathscr{B} - M^T \boldsymbol{R} M \end{bmatrix} \preccurlyeq 0, \;\; \boldsymbol{P} \succ 0, \tag{5.12}$$

$$\forall \tau = 1, ..., \boldsymbol{\mu},$$

where $M = [0 \; ... \; 0 \; I_m]$ and $\mathscr{B} = [A_\varSigma^{\tau-1} B_\varSigma \;\; A_\varSigma^{\tau-2} B_\varSigma \; ... \; A_\varSigma B_\varSigma \;\; B_\varSigma]$;

(ii) Whenever $\varphi_i^{q-1} < 0$, the following online LMIs are feasible in $\boldsymbol{Q}_{ic}^q$ and $\boldsymbol{R}_{ic}^q$:

$$\begin{bmatrix} y_i^{qT}(Q_i + \boldsymbol{R}_{ic}^q)y_q^k & y_i^{qT}S_i \\ S_i^T y_i^k & R_i + \boldsymbol{Q}_{ic}^q \end{bmatrix} \preccurlyeq 0, \quad \boldsymbol{Q}_{ic}^q \prec 0, \tag{5.13a}$$

$$y_i^{qT}\boldsymbol{R}_{ic}^q y_i^q - \varphi_i^{q-1} - \theta_i^q > 0; \tag{5.13b}$$

and choosing S_{ic}^q as a null space of $y_i^T(j_q)$, i.e. $\boldsymbol{S}_{ic}^q y_i^T(j_q) = 0$;

Then, there exist controls $\tilde{u}_i(j_q)$, $i = 1...h$, satisfying the aAPRC, that stabilise Σ.

Proof: (i) *Dissipativity*: The large-scale system Σ is data-lost robust dissipative if and only if LMI (5.12b) holds. This is the well-known dissipativity condition for Σ with respect to the dissipation inequality (5.8) [16] and the predictive state vector of the form $x(j+\tau+1) = A_\Sigma^\tau x(j) + \mathscr{B}\check{u}(j)$, where $\check{u}(j) = [u(j)^T u(j+1)^T ... u(j+\tau)^T]^T$. (ii) *Stabilisability*: The proof starts at some time instant when $\xi_c^q < 0$. It is apparently that ξ_c^q will eventually go to the vicinity of zero and become non-negative with $|\beta_i^q| \geq \sqrt{(1-\gamma_i^q)|\xi_i^{q-1}|}$, $0 < \gamma_i^q < 1$ (see Lemma 1) as the second inequalities of aAPRC in (5.10) of h subsystems are feasible for all $q > q_0$ which are given by LMIs (5.16) and proved in (iii) below. Similarly to Theorem 2,

$$V(x^{q+1}) - \alpha V(x^q) \leq \delta^q := (*) \begin{bmatrix} y^{qT}(\boldsymbol{Q} + \boldsymbol{R}_c^q)y^q & y^{qT}(\boldsymbol{S} + \boldsymbol{S}_c) \\ * & \boldsymbol{R} + \boldsymbol{Q}_c \end{bmatrix} \begin{bmatrix} I \\ u^q \end{bmatrix} + \gamma^q \varphi(j_q),$$

where $V(x^q) := x^{qT}\boldsymbol{P}x^q$.

(iii) *Feasibility condition for aAPRC with control constraints*: (5.16) is a direct result of (3.31) in Theorem 2. ■

In the following, the constraint on the control increment $\Delta u_i(k)$, ($\Delta u_i(k) \triangleq u_i(k) - u_i(k-1)$) are taken into account in the stabilisability condition. Consider the bounded constraint below,

$$\mathbb{U}_{\Delta i} \triangleq \{\Delta u_i : \ \|\Delta u_i\|_2^2 \leq \eta_{\Delta i}\}. \tag{5.14}$$

The stabilisability condition for the controls $\tilde{u}_i(j_q)$ subject to the data lost process (5.25) and constraints (5.2) and (5.14) is stated in the next Theorem.

Theorem 10 *Suppose that*

(i) The following LMIs are feasible in $\boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{R}$*:*

$$\begin{bmatrix} A_{\Sigma}^{\tau\,T}\boldsymbol{P}A_{\Sigma}^{\tau} - \alpha\boldsymbol{P} - C^T\boldsymbol{Q}C & A_{\Sigma}^{\tau\,T}\boldsymbol{P}\mathscr{B} - C^T\boldsymbol{S}M \\ * & \mathscr{B}^T\boldsymbol{P}\mathscr{B} - M^T\boldsymbol{R}M \end{bmatrix} \preccurlyeq 0, \ \ \boldsymbol{P} \succ 0, \tag{5.15}$$

$$\forall \tau = 1, ..., \boldsymbol{\mu};$$

(ii) Whenever $\varphi_i^{q-1} < 0$*, the following online LMIs are feasible in* $\boldsymbol{Q}_{ic}^q, \boldsymbol{S}_{ic}^q, \boldsymbol{R}_{ic}^q$*:*

$$\begin{bmatrix} y_i^{qT}(Q_i + \boldsymbol{R}_{ic}^q)y_q^k & y_i^{qT}(S_i + \boldsymbol{S}_{ic}^q) \\ * & R_i + \boldsymbol{Q}_{ic}^q \end{bmatrix} \preccurlyeq 0, \quad \boldsymbol{Q}_{ic}^q \prec 0, \tag{5.16a}$$

$$y_i^{qT}\boldsymbol{R}_{ic}^q y_i^q - \varphi_i^{q-1} - \theta_i^q > 0; \tag{5.16b}$$

$$\begin{bmatrix} -\boldsymbol{\alpha}_{d_i}^q \eta_{\Delta i} - \varphi_i^{q-1} - \theta_i^q + (*)\mathscr{R}_{di}\begin{bmatrix} x_i^{q-1} \\ u_i^{q-1} \end{bmatrix} & \begin{bmatrix} u_i^{(q-1)T}(B_{di}^T\boldsymbol{S}_{ic}^q + M_\tau^T\boldsymbol{Q}_{ic}^q) \\ +x_i^{(q-1)T}A_{di}{}^T\boldsymbol{S}_{ic}^q \end{bmatrix} \\ * & \boldsymbol{Q}_{ic}^q + \boldsymbol{\alpha}_{d_i}^q I \end{bmatrix} \succcurlyeq 0,$$

$$\boldsymbol{\alpha}_{di}^q > 0, \tag{5.16c}$$

where

$$\mathscr{R}_{di} := \begin{bmatrix} A_{di}{}^T\boldsymbol{R}_{ic}^q A_{di} & A_{di}{}^T\boldsymbol{R}_{ic}^q B_{di} + A_{di}{}^T\boldsymbol{S}_{ic}^q M_\tau \\ * & B_{di}^T\boldsymbol{R}_{ic}^q B_{di} + B_{di}^T\boldsymbol{S}_{ic}^q M_\tau + M_\tau^T\boldsymbol{Q}_{ic}^q M_\tau \end{bmatrix},$$

$A_{di} = C_i A_i^\mu, \quad B_{di} = C_i\mathscr{B}_{di},$

$\mathscr{B}_{di} := [A_i^{\mu-1}B_{di}\ A_i^{\mu-2}B_{di}\ ...\ A_iB_{di}\ \ B_{di}],$

$B_{di} := [B_i\ E_i],\ M_\tau = [0_{m_i}\ ...\ 0_{m_i}\ I_{m_i}\ 0_{m_{v_i}}\ ...\ 0_{m_{v_i}}]\ (u_i^{q-1} = M_\tau \breve{u}_{di},\ \breve{u}_{di} := [\breve{u}_i\ \breve{v}_i]);$

Then, there exist controls $\tilde{u}_i(j_q)$*,* $i = 1...h$*, satisfying the aAPRC, that stabilise* Σ *subject to the data-lost process* μ *(5.25).*

Proof: The proof of this theorem is identical to that of Theorem 9, except for the last LMI 5.16c which is derived as follows:

$$\begin{aligned}
& \xi_{ic}^q \geq \sigma_i^q \xi_{ic}^{q-1} + \sigma_i^q (\beta_i^q)^2 \\
\Leftrightarrow \quad & (x_i^{(q-1)T} A_i^{\tau T} + \breve{u}_{di}^{(q-1)T} \mathscr{B}_{di}^T) C_i^T \boldsymbol{R}_{ic}^q C_i (A_i^\tau x_i^{q-1} + \mathscr{B}_{di} \breve{u}_{di}^{q-1}) \\
+ \quad & (x_i^{(q-1)T} A_i^{\tau T} + \breve{u}_{di}^{(q-1)T} \mathscr{B}_{di}^T) C_i^T \boldsymbol{S}_{ic}^q (M_\tau \breve{u}_{di}^{q-1} + \Delta_{di}^q) \\
+ \quad & (\breve{u}_{di}^{(q-1)T} M_\tau^T + \Delta u_i^{qT}) \boldsymbol{Q}_{ic}^q (M_\tau \breve{u}_{di}^{q-1} + \Delta u_{di}^q) \geq \sigma_i^q \xi_{ic}^{q-1} + \sigma_i^q (\beta_i^q)^2,
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad & x_i^{(q-1)T} (A_{di}^T \boldsymbol{R}_{ic}^q A_{di}) x_i^{q-1} + \breve{u}_{di}^{(q-1)T} (B_{di}^T \boldsymbol{R}_{ic}^q B_{di} + B_{di}^T \boldsymbol{S}_{ic}^q M_\tau + M_\tau^T \boldsymbol{Q}_{ic}^q M_\tau) \breve{u}_{di}^{(q-1)} \\
+ \quad & 2 x_i^{(q-1)T} (A_{di}^T \boldsymbol{R}_{ic}^q B_{di} + A_{di}^T \boldsymbol{S}_{ic}^q M_\tau) \breve{u}_{di}^{(q-1)} + \Delta u_i^{qT} \boldsymbol{Q}_{ic}^q \Delta u_i^{qT} \\
+ \quad & 2 x_i^{(q-1)T} (A_{di}^T \boldsymbol{S}_{ic}^q) \Delta u_i^{qT} + 2 \breve{u}_i^{(q-1)T} (B_{di}^T \boldsymbol{S}_{ic}^q + M_\tau^T \boldsymbol{Q}_{ic}^q) \Delta u_i^{qT} \\
\geq \quad & \sigma_i^q \xi_{ic}^{q-1} + \sigma_i^q (\beta_i^q)^2, \text{ where } A_{di} = C_i A_i^\mu, \; B_{di} = C_i \mathscr{B}_{di}
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad & \Delta u_i^T(k) \boldsymbol{Q}_{ic}^q \Delta u_i(k) + 2[\breve{u}_i^{(q-1)T} (B_{di}^T \boldsymbol{S}_{ic}^q + M_\tau^T \boldsymbol{Q}_{ic}^q) + x_i^{(q-1)T} (A_i^T C_i^T \boldsymbol{S}_{ic}^q)] \Delta u_i(k) \\
+ \quad & \begin{bmatrix} x_i^{q-1} \\ \breve{u}_i^{(q-1)} \end{bmatrix}^T \begin{bmatrix} A_{di}^T \boldsymbol{R}_{ic}^q A_{di} & A_{di}^T \boldsymbol{R}_{ic}^q B_{di} + A_{di}^T \boldsymbol{S}_{ic}^q M_\tau \\ * & B_{di}^T \boldsymbol{R}_{ic}^q B_{di} + B_{di}^T \boldsymbol{S}_{ic}^q M_\tau + M_\tau^T \boldsymbol{Q}_{ic}^q M_\tau \end{bmatrix} \begin{bmatrix} x_i^{q-1} \\ \breve{u}_i^{(q-1)} \end{bmatrix} \\
- \quad & \sigma_i^q \xi_{ic}^{q-1} - \sigma_i^q (\beta_i^q)^2 \geq 0.
\end{aligned}$$

This derived constraint is fulfilled for all $\Delta u_i \in \mathbb{U}_{\Delta_i}$ (5.14) by the feasibility of LMI (5.16c). ■

5.4 Illustrative Example

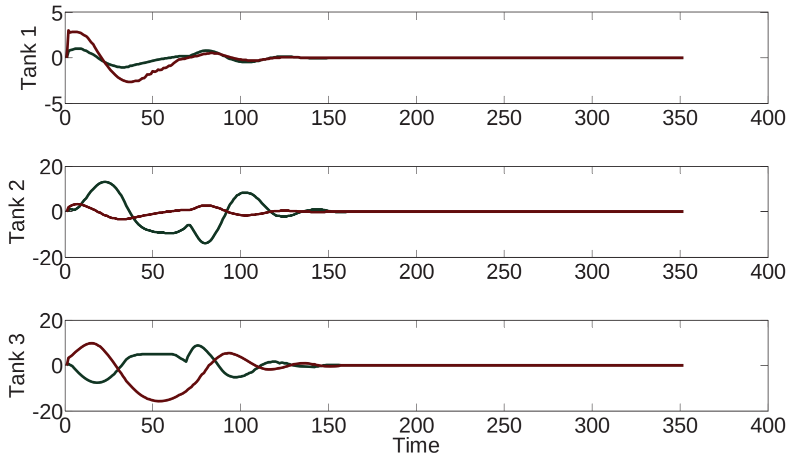
5.4.1 Case Study 1

Consider the continuous-time state space models of a typical three-stage countercurrent washing circuit in Chapter 4. In this simulation, the MPC controllers have also been set up and coded in Matlab with the *mpcmove* function. The APRC-based stabilising constraint is combined with this Matlab function by the algorithm delivered in [153]. The predictive and control horizons of $N_y = 3$ and $N_u = 2$ have been chosen for the first two subsystems, while longer horizons ($N_y = 12,\ N_u = 4$) are chosen for the third one. Three subsystems have the same sampling time of 0.4 steps. All state updates are synchronised between all subsystems. The MPC setups are the identical to those in Chapter 4.

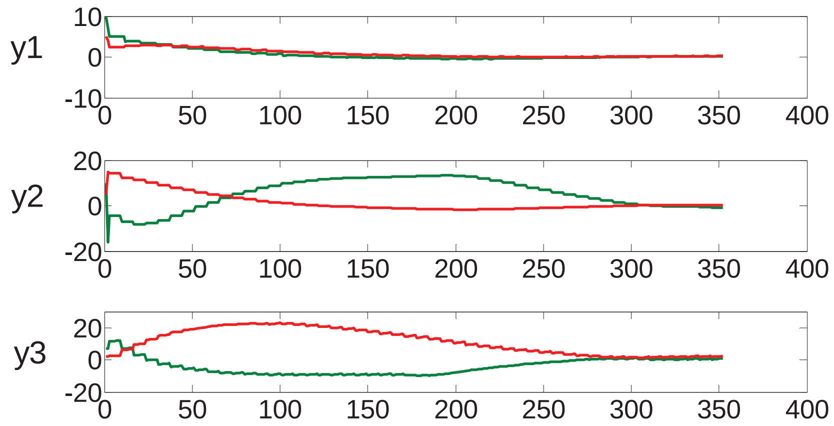
To demonstrate the efficacy of the APRC-based stabilising constraint, only offline multiplier matrices ($R_{ic} \succ 0,\ Q_{ic} \prec 0$) are used in this simulation study. When the stabilising agents for subsystems 3 and 1 are employed, the plant-output time responses are shown in Figure 5.2(a), indicating clearly a stabilised interconnected system.

Figure 5.2(b) shows the output responses in the case of MATI = 5 steps of sporadic data losses occurring to all subsystems being simulated with arbitrarily flatting inputs and outputs. The tuning numbers $\rho_i = 3$ is used for all three subsystems. A fixed bias number $\beta_i = 0.03$ or $\beta_i = 0$ is applied to all subsystems and at every time steps. The initial time $q_0(1)$ of subsystem 1 occurs at steps 11, 58, 73 and 105. The recurrences of $\xi_{1c} < \rho_1^2$ are at steps 28 , 70 and 84. The initial time $q_0(3)$ of subsystem 3 occurs at steps 10, 30 and 71. The recurrences of $\xi_{3c} < -\rho_3^2$ are at steps 19 and 67.

The plant-output trends with input disturbances and $\mu = 5$ steps of sporadic data losses occurring to all subsystems (being simulated with arbitrarily flatting inputs and outputs) are given in Figure 5.3(b). The output set points are $\bar{y}_1 = 5,\ \bar{y}_2 = 10$ and $\bar{y}_3 = -10$. The tuning numbers $\rho_i = 3$, $i = 1, 2, 3$, are used for all three subsystems. Time-invariant coefficients $\beta_i = 0.99$, $i = 1, 2, 3$, are applied in this simulation study.

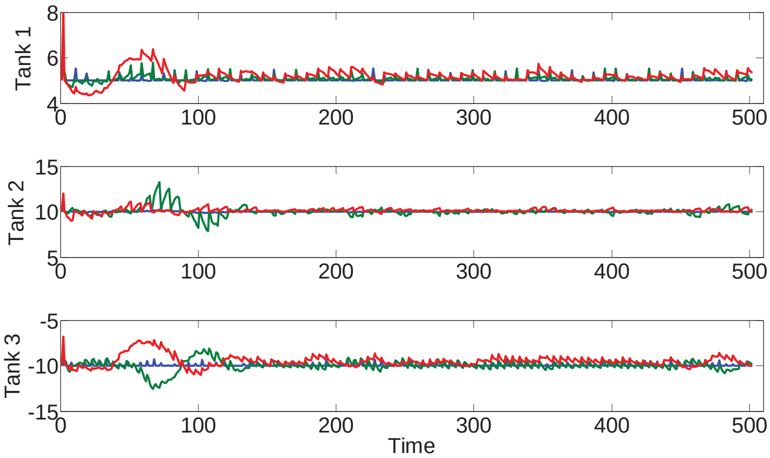


(a) APRC-based Stabilising Agents in a Perfect Data Environment.

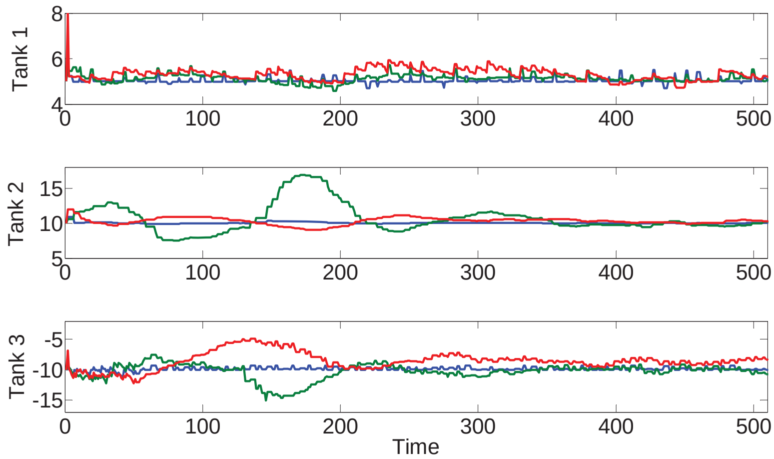


(b) APRC-based Stabilising Agents with the Presences of Data Losses.

Figure 5.2 : Decentralised Stabilising Agents for the Washing Circuit with Networked Control Systems having MATI = 5 steps - Plant-Output Time Responses.



(a) With APRC-based Stabilising Agents - With Bounded Input Disturbances.



(b) With APRC-based Stabilising Agents - With Bounded Input Disturbances and Intermittent Data Losses.

Figure 5.3 : Decentralised Stabilising Agents with Intermittent Data Losses - Plant-Output Time Responses, $y_i(k)$.

5.4.2 Case Study 2

In the second simulation study, the strong coupling subsystems of a feed distribution process in the second illustrative example in Chapter 3 are considered with randomly data losses within 7 time steps. The trajectory shapes of different executions are different due to the Matlab random function of the following:

```
State_Vector_Temp = State_Vector;
d = rand(1, 1);
if mod(Time_Step, round(5 × d)) = 0
   State_Vector = State_Vector_Temp;
else
   Calculate new state vector;
```

The weighting coefficients of $\mathcal{R}_1 = \text{diag}\{2,1\}, \mathcal{Q}_1 = \text{diag}\{5,1,2\}; \mathcal{R}_2 = [1], \mathcal{Q}_2 = \text{diag}\{1,1,2\}, \mathcal{R}_3 = \text{diag}\{0.5,1\}$, $\mathcal{Q}_3 = \text{diag}\{4,1,5\}$ have been set up for the MPC optimisations.The constraints on the control inputs are set up as $\eta_1 = 2, \eta_2 = 10, \eta_3 = 10$. The output constraints are ignored in this simulation. The initial states are as follows: $x_1(0) = [1\ 7\ 3]^T, x_2(0) = [7\ 6\ 4]^T, x_3(0) = [2\ 8\ 3]^T$.

The stabilised trajectories are depicted by Figure 5.4. The input and output responses are given in the next figure 5.5, showing clearly the flattering outputs due to data losses. We learnt from this simulation study that the overall system can become unstable when μ (MATI) is long than 6 time steps.

In the next two following sections, we present the developments for APRC-based stabilisability conditions for linear and nonlinear systems under the perspective of input-to-state stabilisation. We also address the problem of flexible network topologies for the communication links between subsystems. Taking into account the ex-

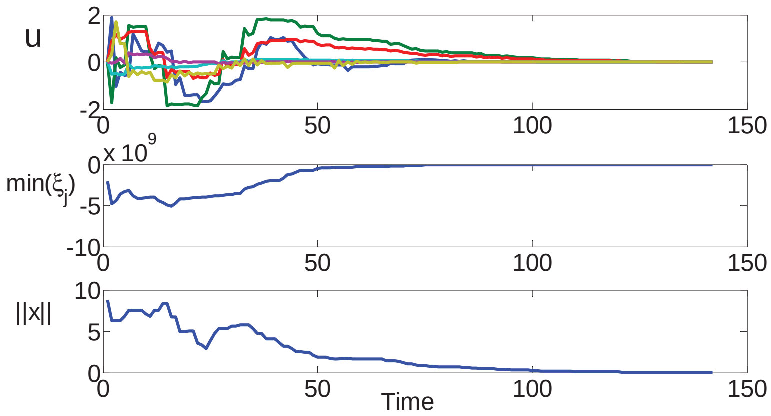


Figure 5.4 : APRC-Based Decentralised Model Predictive Control with Intermittent Data Losses - Strong Dynamic Coupling System - Time Responses.

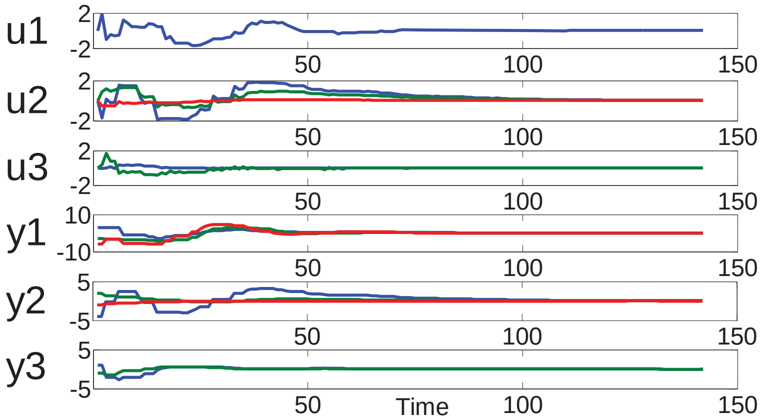


Figure 5.5 : APRC-Based Decentralised Model Predictive Control with Intermittent Data Losses - Strong Dynamic Coupling System - Input and Output Trajectories.

amining scope, only theoretical derivations are delineated herein.

5.5 Distributed Model Predictive Control with Perturbed State Feedback for Multiple Network Topologies

5.5.1 Introduction to DeMPC and PSF

A partially decentralised model predictive control (DeMPC) scheme engaging offline and online computations for constrained interconnected systems and making use of the available information collected via the communication channels between subsystems is subsequently developed with APRC-based stability constraints. The stability constraint for DeMPC in this approach is a quadratic constraint on the initial control vector. Herein, we consider the inter-subsystem communication network that has multiple connection topologies. Demands on communication links and online data are limited with a *perturbed state-feedback* strategy, which consists of a static state-feedback and a perturbation. The static state-feedback gains are determined in accordance with the communication network topologies and for constraint satisfactions, while perturbations are calculated by the online optimisations adopting only decoupled objective functions.

The DeMPC strategy overcomes the disadvantages exhibited by the fully decentralised or other distributed control schemes. In fully decentralised control, the communication between subsystems are totally disconnected, which will affect the overall control performance for strongly coupled subsystems. On the other hand, the continuous communication between subsystems is heavily required in distributed control. A communication disruption will break down its stability guarantee property. The totally disconnections and permanent fixed-connections are seldom the reality sce-

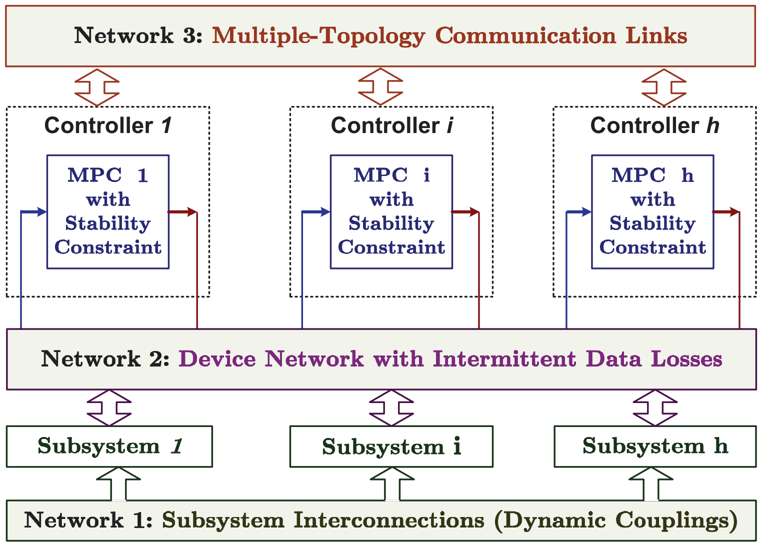


Figure 5.6 : Partially Decentralised Model Predictive Control with Two Communication Networks and a Subsystem Coupling Network.

narios. In practice, the connections between subsystems are usually available but are neither 100% continuous nor have a fixed connection topology. Moreover, the sensor network (or device network), such as the wireless field-bus in the lower levels, also exhibits intermittent data dropouts. This is the second network apart from the above mentioned inter-subsystem network. The block diagram of such interconnected systems having two communication networks on top of the subsystem dynamic-coupling network (subsystem interconnection) is sketched out in Figure 5.6 (i.e., there are three networks in this figure).

To tolerate both data dropouts in sensor networks and communication topology changes in the inter-subsystem networks in real time, a parameterised quadratic constraint on the control vector derived from the APRC is employed as a decoupled stability constraint for DeMPC in this chapter. Putting emphasis on the simplification aspect of the online solution, a *perturbed state-feedback* strategy (PSF) is introduced and applied to the asymptotically dissipative constraints for all subsystems to achieve the plant-wide stabilisation. Under this strategy, the control inputs have the virtual form of

$$u_i(k) = w_i(k) + \sum_j K_{ij}\, x_j(k),$$

where x_j are the known neighbours' states (via the available communication channels only), K_{ij} is the static state-feedback gain, and w_i is the perturbation. This is only a virtual PSF form, and will not be applied directly to calculate the control $\boldsymbol{u}_i$ in real time. It is used to shape the quadratic stability constraint with respect to $\boldsymbol{u}_i$ for the online calculations which determine the control $\boldsymbol{u}_i$ directly, as explained herein. In this development, the basic APRC inequality of the following form

$$\xi_i(x_i(k), w_i(k)) \geq \beta_i\, \xi_i(x_i(k-1), w_i(k-1)),\ 0 \leq \beta_i < 1,$$

where ξ_i is a quadratic function (supply rate) w.r.t. the state and input pair of x_i and w_i, if $\xi_i(x_i(k-1), w_i(k-1)) < 0$, is applied to the stabilisability condition in this chapter (x_i plays the role of an output vector herein).

The APRC-based inequality is initially expressed as an inequality in $w_i(k)$ assuming $x_i(k)$ and $\xi_i(x_i(k-1), w_i(k-1))$ are known, which will render a quadratic constraint on $w_i(k)$. It is noted here that $x_i(k)$ and $\xi_i(x_i(k-1), w_i(k-1))$ are measurement and historical data. From this APRC-based constraint w.r.t. $w_i(k)$ (with $w_i = \boldsymbol{u}_i - \sum_j K_{ij}x_j$), the quadratic constraint w.r.t. $\boldsymbol{u}_i(k)$ is subsequently recast and incorporated into the online optimisation problems for $\boldsymbol{u}_i(k)$. Herein, the state feedback gain K_{ij} is determined in accordance with the communication topologies (of the top network in Fig. 5.6) which are predefined and not necessarily identical to the fixed subsystem dynamic-coupling topology (the bottom network in Fig. 5.6). If more than one topology are predefined, a set of different gains will be computed offline for using with the corresponding topologies in real time. As mentioned above, this gain K_{ij} and neighbours' states $x_j(k)$ are only used to derive the APRC-based stability constraint w.r.t. $\boldsymbol{u}_i(k)$, but not to calculate $\boldsymbol{u}_i(k)$ directly.

Since neighbours' states are not involved in the objective function, the computational cost is less than those employing a fully coupled objective function and constraint simultaneously; see, e.g. [161, 68, 102]. In exchange for that, a set of small dimension linear matrix inequalities will need to be solved online to update the APRC-based stability constraint. Due to small dimensions, the computational cost is not a significant burden, especially for the modern computerised systems. By not employing the PSF directly in the real-time control law, but only in the derivation of a stability constraint for the control $\boldsymbol{u}_i$, the semi-automatic control scheme is also accommodated by the present strategy. Semi-automatic controllers are understood as those having two modes, automatic and manual, which allow for human machine interfaces (HMI) and switching between the remote control-room operators and the automatic control algorithms [156].

5.5.2 Communication Networks

The second facet of the control problem is to deal with the communication interruptions, inherently to interconnected systems installed with an internet-based sensor/device network such as wireless field-bus (the middle network in Fig. 5.6). Data losses from the communication abruptions in this network will ultimately cause deteriorations to the control performance and system stability. Effective solutions for this type of intermittent disruptions have been presented in the literature of the networked control systems (NCS). The research in NCS is quite mature in its own right with recent results reported in [137, 51, 118] and references thereof. An alternative *online solution* for using with the asymptotically dissipative constraint, is proposed herein. By acquiring the data-lost robust dissipativity for all periods of consecutive data dropouts, and applying the APRC-based quadratic constraints w.r.t actual states and controls from the received information, the sporadic data losses are effectively compensated for right after their occurrences. The recursive feasibility, that is affected by the described communication disruptions, is guaranteed by characterising on-the-fly the APRC-based stabilising constraints and applying a conservative bounded reachable set built upon the robust stabilisable sets [12].

The above sensor/device network is normally a mesh network for local state measurements and control without interactions between subsystems. For the other two networks, the unweighted Laplacian matrices [107] will be used in this development to model the time-varying communication topology (of the top network in Fig. 5.6) and the subsystem dynamic-coupling topology (of the bottom network in Fig. 5.6). While accepting the time-varying communication topology, it is viable that all topologies are predefined such that the static state-feedback gains K_{ij} will be determined offline accordingly. Directed graphs are employed in this work. The communication matrix with weighted graphs and the permanent loss of certain communication links, as usually be considered in the cooperative control schemes (see, e.g., [40, 110]), will not be considered in this development.

5.5.3 System and Control Model

Consider an interconnected system Σ consisting of h nodes representing h subsystems, each denoted as $\mathcal{S}_i$, indexed by the elements of the set $\mathcal{N} := \{1, \ldots, h\}$, and has a discrete-time state space model of the form:

$$\mathcal{S}_i : \quad x_i(k+1) = A_i x_i(k) + B_i u_i(k) + E_i v_i(k) + L_i d_i(k), \tag{5.17}$$

where $u_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}$ and $x_i \in \mathbb{X}_i \subset \mathbb{R}^{n_i}$ are local control and state vectors, respectively, $v_i \subset \mathbb{R}^{m_{v_i}}$ is the interactive, or dynamic coupling, vector. d_i represents the input disturbance, which is assumed a white noise sequence with zero mean and finite variances, $d_i \in \mathbb{V}_i \subset \mathbb{R}^{m_i}$. The following control constraint is considered herein:

$$\mathbb{U}_i \triangleq \{u_i : \ \|u_i\|_2^2 \leq \eta_i\}. \tag{5.18}$$

In this approach, d_i is unknown but norm bounded. v_i is also unknown and norm bounded.

A. Network Graph

A network system is identified by a graph $G = (\mathcal{N}, \mathcal{S}, \mathcal{E})$, where $\mathcal{S} = \{\mathcal{S}_1, ..., \mathcal{S}_n\}$ is the set of nodes, and $\mathcal{E} \subseteq \{(\mathcal{S}_i, \mathcal{S}_j) : \ \mathcal{S}_i, \mathcal{S}_j \in \mathcal{S}, \ i, j \in \mathcal{N} \mid i \neq j\}$ is the set of edges. The graph is undirected (or bidirectional) if for any $i, j \in \mathcal{N}$, $(\mathcal{S}_i, \mathcal{S}_j) \in \mathcal{E} \Leftrightarrow (\mathcal{S}_j, \mathcal{S}_i) \in \mathcal{E}$. A node $\mathcal{S}_i \in \mathcal{N}$ is connected to a node $\mathcal{S}_j \in \mathcal{N}$ if there is a path from $\mathcal{S}_i$ to $\mathcal{S}_j$ in the graph following the orientation of the arcs. The graph is strongly connected if, $\forall(\mathcal{S}_i, \mathcal{S}_j) \in \mathcal{S} \times \mathcal{S}$, $\mathcal{S}_i$ is connected to $\mathcal{S}_j$. A directed graph is said to have a spanning tree if and only if $\exists i \in \mathcal{N}$ such that there is a path from node $\mathcal{S}_i$ to any other node $\mathcal{S}_j \in \mathcal{S}$. If $(\mathcal{S}_i, \mathcal{S}_j) \in \mathcal{E}$, we say that $\mathcal{S}_j$ is neighbour to $\mathcal{S}_i$ and subsystem $\mathcal{S}_j$ transmits its state vector $x_j(k)$ to subsystem $\mathcal{S}_i$ at every step $k \geq 0$. The set of all neighbours to node $i \in \mathcal{N}$ is denoted as $\mathcal{N}_i(G) = \{j, j \in \mathcal{N} : (\mathcal{S}_i, \mathcal{S}_j) \in \mathcal{E}\}$ and $|\mathcal{N}_i|$ is the valency of node i.

In this chapter, the network of fixed subsystem interconnections (subsystem coupling network), denoted as G_s, and the time-varying communication network, denoted as $G_c(k)$, as depicted by the top network in Figure 5.6, are considered. We, nonetheless, limited ourselves to a predefined number of G_c in this development. By this limitation, the state feedback gains are determined offline using the unweighted Laplacian matrices of those pre-informed topologies. The requirement on spanning trees is not discussed herein. The sensor/device network, which is normally a mesh network, does not belong to this network class.

B. Perturbed State Feedback

For a given G_c, when the control is structured to form a perturbed state feedback of $u_i(k) = w_i(k) + z_i(k)$ [155], where

$$z_i(k) = \sum_{j \in \mathcal{N}_i(G_c)} K_{ij}\, x_j(k), \tag{5.19}$$

the subsystem model can be rewritten as

$$x_i(k+1) = A_i x_i(k) + B_i w_i(k) + B_i z_i(k) + E_i v_i(k) + L_i d_i(k), \tag{5.20}$$

where $w_i \in \mathbb{W}_i \subset \mathbb{R}^{m_i}$, $\|d_i\|_2^2 \leq \theta_i$, and the subsystem coupling being defined by:

$$v_i(k) = \sum_{j \in \mathcal{N}_i(G_s)} F_j x_j(k). \tag{5.21}$$

The subsystem coupling network G_s is associated with the global coupling matrix H_s. The elements of H_s is either 1 or 0 only (unweighted Laplacian matrix, see [107]). Similarly, the communication network G_c is associated with the communication matrix H_c. By denoting the i^{th} block-row of H as $H_{s[i]}$, it is apparently that

$$v_i = H_{s[i]} F x, \tag{5.22}$$

where $x := [{x_1}^T \ldots {x_h}^T]^T$ is the global state vector of Σ. And similarly, with a block-column matrix H_c,

$$z_i = [K_{ij}]_{[i]}\, H_{c[i]} x. \tag{5.23}$$

From a centralised point of view, Σ is expressed as

$$\Sigma: \; x(k+1) = A_{\Sigma}x(k) + Bw(k) + Ld(k), \tag{5.24}$$

where $A_{\Sigma} := A + EH + BKH_c$, in which

$$A := \mathsf{diag}[A_i]_i^h, \; B := \mathsf{diag}[B_i]_i^h, \; E := \mathsf{diag}[E_i]_i^h,$$
$$L := \mathsf{diag}[L_i]_i^h, \; K := [K_{ij}], \; d := [{d_1}^T \dots {d_h}^T]^T.$$

5.5.4 Data Loss Process

The updating time instants are assumed to be synchronised between all controllers and their state measurements. The state vector x_i of $\mathcal{S}_i$ becomes $\tilde{x}_i$ beyond the communication interface ports of the controllers. In this cooperative application, it is assumed that the data losses are identical and concurrent to all state and control variables. When the state vector $x_i(k)$ at time instant k is transmitted successfully, we have $\tilde{x}_i(k) = x_i(k)$, otherwise $\tilde{x}_i(k) = x_i(k-1)$ (i.e. when data are lost). Similarly, the last received value is applied when the communication is lost for the control u_i, i.e. $\tilde{u}_i(k) = u_i(k-1)$. By representing the consecutive updating instants of $\tilde{x}_i(k)$ with a sequence of integer numbers $\mathcal{J} \triangleq \{j_1, ..., j_q, ..., j_\kappa, ...\}, \mathcal{J} \subset \mathbb{Z}^+$, the time interval between j_q and j_{q+1} is treated as one transmission period. By applying online compensations, the problem is not conservative when the upper bound $\boldsymbol{\mu}$ of the successful transmission periods is assumed, as follows:

$$\boldsymbol{\mu} \triangleq \max_{j_q \in \mathcal{J}} \tau(q), \quad \tau(q) \triangleq j_q - j_{q-1}. \tag{5.25}$$

5.5.5 State Constraint

The following constraint on the state vector is considered herein:

$$\|x_i\|_2^2 \leq \; \rho_i, \; \rho_i > 0. \tag{5.26}$$

For any $\rho_i > 0$, the ball $\mathscr{B}_i(\rho_i)$, is defined by

$$\mathscr{B}_i(\rho_i) \triangleq \{x_i \in \mathbb{R}^{n_i}: \ \|x_i\|_2^2 \leq \rho_i\}. \tag{5.27}$$

To ensure the state constraint satisfaction in the presence of data losses, an LMI-based condition will be developed from an upper bound of the difference of $V_i(x_i(k+\tau)) - V_i(x_i(k))$ [45]. A reachable set, which is invariant (called robust stabilisable set, and robust positively invariant set when $u_i = 0$ and $v_i = 0$ [12]), is then obtained by having the ellipsoid in x_i governing the negative definiteness of this upper bound contained by the ball $\mathscr{B}_i(\rho_i)$ (5.27).

5.5.6 Asymptotic Quadratic Constraint

Define a *parameterised quadratic supply rate* in the perfect data environment for node $\mathcal{S}_i$, as follows:

$$\xi_{ik}\big(w_i, x_i|_{Q_{ik},S_{ik},R_{ik}}\big) \triangleq w_i^T R_{ik} w_i + 2x_i^T S_{ik} w_i + x_i^T Q_{ik} x_i, \tag{5.28}$$

where Q_{ik}, R_{ik}, S_{ik} are dynamic multiplier matrices with symmetric Q_{ik}, R_{ik}. In the data-lost environment of NCS, the supply rates w.r.t. the revived time step j_q are considered. For conciseness, $\xi_{ik}(.,.;.,.,.)$, Q_{ik}, R_{ik}, S_{ik} are denoted as $\xi_{i(q)}, Q_{iq}, R_{iq}, S_{iq}$, respectively. The supply rate (5.24) subject to the data-lost process $\boldsymbol{\mu}$ is then written as

$$\xi_{i(q)} = w_i^T R_{iq} w_i + 2x_i^T S_{iq} w_i + x_i^T Q_{iq} x_i.$$

The APRC is obtainable by employing a convex *parameterised quadratic constraint* w.r.t. $\boldsymbol{u}_i|_{Q_{iq},R_{iq},S_{iq}}$, when $R_{iq} \prec 0$:

$$\begin{aligned} &\boldsymbol{w}_i^T R_{iq} \boldsymbol{w}_i + 2x_i^T S_{iq} \boldsymbol{w}_i + x_i^T Q_{iq} x_i - \delta_{i(q-1)} \geq 0, \\ \Leftrightarrow\quad &\boldsymbol{u}_i^T R_{iq} \boldsymbol{u}_i + 2\mathit{\Gamma}_i \boldsymbol{u}_i + \psi_i \geq 0, \ \ \boldsymbol{u}_i = \boldsymbol{w}_i + z_i, \end{aligned} \tag{5.29}$$

where $\varGamma_i := x_i^T S_{iq} - z_i^T R_{iq}$,

$$\delta_{i(q-1)} := \beta_i[1 - \mathsf{sgn}(\xi_{i(q-1)})]\xi_{i(q-1)},\ 0 < \beta_i < \tfrac{1}{2},$$

$$\psi_i := z_i^T R_{iq} z_i - 2x_i^T S_{iq} z_i + x_i^T Q_{iq} x_i - \delta_{i(q-1)}.$$

The constraint (5.29) can be imposed on the local online calculations using MPC for $u_i(j_q)$. To satisfy the control constraint (5.18), the feedback gains K_{ij} need to be feasible to an LMI derived herein. Firstly, S_{iq} is chosen as a null space of x_i^T, as it is in [53]. Then, similarly to the development in [156], $z_i(j_q) = [K_{ij}]_{[i]}\, H_{c[i]} x(j_q)$ needs to be contained in $\mathbb{U}_i$. For every state vector $x_i(j_q) \in \mathbb{X}_i$ (5.26), this requirement is fulfilled by the following LMI:

$$\begin{bmatrix} -\frac{\eta_i}{\rho_{ij}} I & [\boldsymbol{K}_{ij}]_{[i]}^T \\ [\boldsymbol{K}_{ij}]_{[i]} & -I \end{bmatrix} \prec 0, \tag{5.30}$$

where $\rho_{ij} := \sum\limits_{j \in \mathcal{N}_i(G_c)} \rho_j$.

It is noted here that, $[K_{ij}]_{[i]} = [K_{i1} ... K_{ij} ...]_{j \in \mathcal{N}_i(G_c)}$. The gain matrix $[K_{ij}]$ is updated online in association with LMIs governing the stabilisability condition for $\varSigma$ in this present approach.

5.5.7 Subsystem Data-Lost Robust Dissipativity

For the robust stabilisation of $\varSigma$ with given directed graphs G_s and G_c, the perturbation input w_i are required to be bounded by the above APRC-based quadratic constraint, in association with the data-lost robust dissipativity of every subsystem (5.20) w.r.t. the supply rate $\breve{\xi}_{\triangle i}$ of the form:

$$\begin{aligned} \breve{\xi}_{\triangle i}(x_i; \breve{w}_i, \breve{z}_i, \breve{v}_i, \breve{d}_i) \triangleq\ & \breve{d}_i^T \breve{Z}_i \breve{d}_i + \breve{v}_i^T \breve{V}_i \breve{v}_i + \breve{z}_i^T \breve{X}_i \breve{z}_i + \breve{w}_i^T \breve{R}_i \breve{w}_i \\ & + 2x_i^T \breve{S}_i \breve{w}_i + x_i^T Q_i x_i, \end{aligned} \tag{5.31}$$

where $\breve{Z}_i, \breve{X}_i, \breve{V}_i, \breve{R}_i, \breve{S}_i, Q_i$ are dissipativity matrices; $\breve{Z}_i, \breve{X}_i, \breve{V}_i, \breve{R}_i, Q_i$ are symmetric defined as $\breve{R}_i := \mathsf{diag}[R_i]_1^{\tau}$, $\breve{Z}_i := \mathsf{diag}[Z_i]_1^{\tau}$, $\breve{V}_i := \mathsf{diag}[V_i]_1^{\tau}$, $\breve{X}_i := \mathsf{diag}[X_i]_1^{\tau}$; while $\breve{S}_i := [S_i\ S_i \dots S_i]_{\tau}$, and a vector $\breve{g}_i(j_q)$ is defined as

$$\breve{g}_i^T(j_q) := [g_i^T(j_{q-1})\ g_i^T(j_{q-1})\ \dots\ g_i^T(j_{q-1})]_{\tau}.$$

A subsystem $\mathcal{S}_i$ with PSF (5.20) is then said to be *data-lost robustly and quadratically dissipative* w.r.t the quadratic supply rate $\breve{\xi}_{\triangle i}$, if there exists a storage function $V_i(x_i) := x_i^T P_i x_i,\ P_i \succ 0$ such that for all $x_i(k)$ and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied irrespectively of the initial value of the state $x_i(0)$:

$$V_i(x_i(k+\tau)) - \sigma V_i(x_i(k)) \leq \breve{\xi}_{\triangle i}\big(x_i(k); \breve{w}_i, \breve{z}_i, \breve{v}_i, \breve{d}_i\big)\ \forall \tau = 1 \dots \boldsymbol{\mu},\ 0 < \sigma < 1. \quad (5.32)$$

5.5.8 Task Description

The control task is to drive h subsystems of Σ from their initial states $x_i(0)$ to their zero equilibrium without any state and control constraint violations. Each subsystem possesses communication channels to some or all of its neighbours or other subsystems, but not necessarily with all nodes in G_c. The link interruptions may occur intermittently in these communication channels (thus having time-varying topologies) and in the device network for local state measurements and control. To fulfill these tasks, the perturbed state feedback control $u_i(j_q) = w_i(j_q) + v_i(j_q)$ will be adopted. The static state-feedback gains K_{ij} are determined offline based on the communication topology of G_c prescribed in H_c, and the node models (5.20). Most of the time, $H_c = H$, but this formulation facilitates $H_c \neq H$. As a result, a new requirement arises in this approach, that is, all nodes must be informed with the communication topology in real time. This requirement is realisable with the current networking technology and a realistic scenario. The subsystems will synchronously apply the new gains K_{ij} corresponding to the new topology each time it is refreshingly formed.

For the automatic control mode, the control $u_i(j_q)$ will be calculated online by the local controllers using a model predictive control (MPC) algorithm that employs the local model (5.17) only. This MPC is imposed with the APRC-based quadratic constraint (5.29). By virtue of decoupled objective functions, the present control scheme is decentralised. It is worth repeating here that the control u_i, but not the perturbation w_i, is involved in the performance-centric calculations, accomplished by the finite-horizon optimisation of MPC herein. The conventional MPC objective function of the following form is considered for each single subsystem:

$$\mathcal{J}_i(j_q) = \sum_{\ell=1}^{N_i} \|x_i^T(j_q+\ell)\|_{\mathcal{Q}_i} + \|u_i^T(j_q+\ell)\|_{\mathcal{R}_i},$$

where $\mathcal{Q}_i, \mathcal{R}_i$ are weighting matrices, and N_i is the predictive horizon. The current local state $x_i(j_q)$ and neighbours' states $x_j(j_q),\ j \in \mathcal{N}_i(G_c)$, at the revived step j_q are known. The local optimisation problem of minimising $\mathcal{J}_i(k)$ subject to the local model (5.17), the control constraint $u_i \in \mathbb{U}_i$ (5.18), and the APRC-based stability constraint (5.29) w.r.t u_i, in the following:

$$\begin{aligned} &\min_{\hat{\boldsymbol{u}}_i} \mathcal{J}_i(k) \\ \text{subject to} \quad &(5.17), (5.18), \text{ and } (5.29), \end{aligned} \tag{5.33}$$

is then solved for the predictive vector sequence $\hat{\boldsymbol{u}}_i$. The minimising sequence $\hat{u}_i^*$ which consists of N_i elements of $u_i^*(j_q+\ell), \ell = 0, 1, ..., N_i$, will be obtained as a result of this online computation. Only the first element $u_i^*(j_q)$ is used to control $\mathcal{S}_i$. This rolling process is repeated at the next time step, and continues thereon.

5.6 Concluding Remarks

We have presented the fully and partially decentralised stabilisations scheme for interconnected systems suffering from intermittent data losses on device networks and having multiple inter-subsystem communication topologies. The stabilising bounds on the immediate future values of controller outputs are updated at the successive data communication instants. The data losses are effectively compensated for right after every dropout incident.

The effective and practical PSF (perturbed state-feedback) control reduces the demands on communication links and online data. Not only simplifying the optimisation with decoupled objective functions for the partially decentralised MPC and using less online data, the approach is also cutting edge by accommodating a fully-decentralised semi-automatic control paradigm in the imperfect communication network and data environment.

Chapter 6

Coupling Delayed Interconnected Systems with Asymptotically Positive Realness Constraint

This chapter addresses a decentralised model predictive control strategy (MPC) for network systems that suffer from an arbitrary coupling delay. The network system is subject to a global state constraint and local input constraints, as well as persistent input disturbances. The decentralised MPC strategy tackles the coupling delay problem by acquiring the property of *delay-robust dissipativity* (DRD) for all periods of time delays that are bounded by a maximum delay time. A single random coupling delay is considered in this development, i.e. the coupling delays between network nodes are assumed identical. The storage function of the Lyapunov-Krasovskii functional form is employed in the delay-dependent and delay-independent dissipativity criteria.

The interconnection stabilisability condition for the delay robust systems is subsequently derived in association with the APRC. In this approach, the storage function plays the role of a candidate Lyapunov-Krasovskii functional (in a discrete-time form). Alternatively, the stabilisability condition is established from the *accumulatively quadratic dissipativity* (AQD) of each single interactive subsystem and the corresponding *accumulative supply rate* (ASR) within the delay time period. The stability constraint applying ASR is designated as *sectoral stability constraint* in this chapter.

For state constraint satisfactions, an upper bound of the difference of the discrete-time Lyapunov-Krasovskii functional is derived to determine a robust stabilisable invariant set for the state trajectory. This derived condition on the upper bound is combined with the aforementioned dissipativity condition to ensure that the two targets of undisturbed asymptotic stability and set invariance for constraint satisfactions are achieved simultaneously. In this discrete-time approach, the storage function of a Lyapunov-Krasovskii functional form is allowed to increase in the first constructive phase of the controller, similarly to other approach of reachable sets in delayed systems (see, e.g. [5, 45, 113, 69]). Furthermore, we extend those approaches to combine the monotonically decreasing property of a stabilisation problem as in many other MPC approaches [94]. The state converges to a neighborhood of the origin in the second constructive phase of the controller upon the positivity of the partial supply rate, which is achieved by the employment of the APRCs in all local MPC problems during the first construction phase. For coupling delayed systems, the APRC is also extended to an accumulative version, or namely $\beta\tau-$APRC, in which the supply rate is replace by the accumulative supply rate (ASR) within the delay time period τ. If APRC considers the input and output vectors at two consecutively single time steps, $\beta\tau-$APRC considers two consecutive windows of τ time steps.

Time delay systems have emerged as an important research strand in the control literature [108, 92] and a plethora of research papers lately, see, e.g., [36, 117, 92] . Along with delay-independent and delay-dependent solutions employing various types of the Lyapunov-Krasovkii functionals, passivity-based approaches have been adopted for different time delay applications, see, e.g. [135] and references therein. Whilst laying firm foundations for the field, the solutions to decentralised control problems in these approaches usually require artificial or assumed constraints on the coupling (interactive) vectors, or on the objective functions. These constraints may, however, create conservativeness to dynamically-coupled subsystems. In process control applications where the couplings are material or energy flows, such constraints will add unnecessary limitations to, or even worse, diminish the overall control performance. In the present approach, the stability of a network system is achieved without having any artificial constraints on the coupling vectors, yet only decoupled controls are used.

6.1 Control System Models

The models in Chapter 3 are modified with input disturbances and the coupling delay between subsystems. For completeness, the models are reproduced with new reference numbers in this chapter. Consider a network system Σ consisting of h subsystems, each denoted as $\mathcal{S}_i,\ i = 1, \ldots, h$ and has a discrete-time state space model of the form:

$$\mathcal{S}_i : \begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) + E_i v_i(k) + L_i d_i(k), \\ y_i(k) = C_i x_i(k), \quad w_i(k) = F_i x_i(k), \end{cases} \tag{6.1}$$

where $x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}, y_i \in \mathbb{R}^{q_i}, v_i \in \mathbb{R}^{m_{v_i}}, w_i \in \mathbb{R}^{q_{w_i}}$; x_i, u_i, y_i, v_i, and w_i are respectively the state, control input, measurement output, interaction input and interaction output vectors. Vector d_i represents the unmeasured input disturbances, assumed to be norm-bounded, $\|d_i(k)\|_2 \leq 1$. $\mathbb{U}_i$ a bounded subset on $\mathbb{R}^{m_i}$.

The input $v_\xi(k)$ of subsystem $\mathcal{S}_\xi$ and the output $w_\zeta(k)$ of subsystem $\mathcal{S}_\zeta$, $\xi \neq \zeta$, are connected to each other in an arbitrary topology. Here, an arbitrary coupling delay between subsystem nodes of the network system is considered. The large-scale system Σ is then represented by a state space model of the block diagonal system $\mathcal{S}$

formed by h subsystems $\mathcal{S}_i$ and the interconnection process H, as follows:

$$\Sigma : \begin{cases} x(k+1) = A\,x(k) + B\,u(k) + E\,v(k) + L\,d(k), \\ y(k) = C\,x(k), \quad w(k) = F\,x(k), \\ v(k) = H\,w(k-\tau), \end{cases} \tag{6.2}$$

where $A = \mathrm{diag}[A_i]_1^h\,, B = \mathrm{diag}[B_i]_1^h\,, C = \mathrm{diag}[C_i]_1^h\,, E = \mathrm{diag}[E_i]_1^h\,, F = \mathrm{diag}[F_i]_1^h\,, L = \mathrm{diag}[L_i]_1^h\,, x = [{x_1}^T \ldots {x_h}^T]^T$, $u = [{u_1}^T \ldots {u_h}^T]^T$, $y = [{y_1}^T \ldots {y_h}^T]^T$, $v = [{v_1}^T \ldots {v_h}^T]^T$, $w = [{w_1}^T \ldots {w_h}^T]^T$, $d = [{d_1}^T \ldots {d_h}^T]^T$, τ is the number of delay time steps, $\tau \in \{1 \ldots \tau_{max}\}$, and H is the global coupling/interconnection matrix. The elements of H is either 1 or 0 only.

From a centralized point of view, the model of Σ can be rewritten as

$$\Sigma : \begin{cases} x(k+1) &= A\,x(k) + EHF\,x(k-\tau) + B\,u(k) + L\,d(k), \\ y(k) &= C\,x(k). \end{cases} \tag{6.3}$$

In a decentralised control scheme, the models of stand-alone subsystems are requisite. When all the interaction inputs and outputs vanish (i.e. $v_i = 0$ and $w_i = 0$), a stand-alone subsystem $\mathcal{S}_i$, denoted as $\mathcal{S}_i^s$, has the state space model of the form:

$$\mathcal{S}_i^s : \begin{cases} x_i(k+1) &= A_i x_i(k) + B_i u_i(k) + L_i d_i(k), \\ y_i(k) &= C_i x_i(k). \end{cases} \tag{6.4}$$

Assumptions: (i) All stand-alone subsystems $\mathcal{S}_i^s$ (6.1), and the large-scale system Σ (6.3) without state delays are controllable and observable. (ii) The updating time instants are synchronised between all subsystems and their controllers.

Problem Description: The problem of designing h disparate MPC controllers for h subsystems $\mathcal{S}_i$ in a decentralised configuration, that are subject to an arbitrary coupling delay is addressed in this chapter. The inter-subsystem communication between local controllers is not required for exchanging online data by the optimisation solver. In this approach, stability constraints on the initial input vector are used for guaranteeing the system stability.

6.2 Interconnection Stabilisability Conditions with Coupling Delay

This section presents the two interconnection stabilisability conditions in time domain. The *accumulative quadratic dissipativity* and the *delay-robust dissipativity* of an open-loop subsystem are firstly defined in the next subsections.

6.2.1 Accumulatively Quadratic Dissipativity

In this chapter, a subsystem $\mathcal{S}_i$ (6.1) is said to be accumulatively and quadratically dissipative if it is quadratically dissipative with respect to the following accumulative supply rate:

$$\xi^{si}(\breve{y}_{si}, \breve{u}_{si}) \triangleq \breve{y}_{si}^T \breve{Q}_{si} \breve{y}_{si} + 2\breve{y}_{si}^T \breve{S}_{si} \breve{u}_{si} + \breve{u}_{si}^T \breve{R}_{si} \breve{u}_{si}, \tag{6.5}$$

where $\quad \breve{u}_{si}^T(k) \triangleq [\breve{v}_i^T(k) \quad \breve{u}_i^T(k) \quad \breve{d}_i^T(k)], \quad \breve{y}_{si}^T(k) \triangleq [\breve{w}_i^T(k) \quad \breve{y}_i^T(k)],$

in which $\breve{z}(k)$ is defined by $\breve{z}(k) \triangleq [z^T(k-\tau)\ z^T(k-\tau+1)\dots z^T(k-1)\ z^T(k)]^T$, and

$$\breve{Q}_{si} := \begin{bmatrix} \breve{Q}_{\tau i} & 0 \\ 0 & \breve{Q}_i \end{bmatrix},\ \breve{R}_{si} := \begin{bmatrix} \breve{R}_{\tau i} & 0 \\ 0 & \begin{bmatrix} \breve{R}_i & 0 \\ 0 & \breve{R}_{di} \end{bmatrix} \end{bmatrix},\ \breve{S}_{si} := \begin{bmatrix} \breve{S}_{\tau i} & 0 \\ 0 & \begin{bmatrix} \breve{S}_i & 0 \end{bmatrix} \end{bmatrix}.$$

All block matrices are, in turn, block diagonal matrices of the following form:

$$\breve{M}_{*i} := \begin{bmatrix} M_{*i}(0) & 0 & \dots & 0 \\ 0 & M_{*i}(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & M_{*i}(\tau) \end{bmatrix}.$$

Now, define $\breve{x}_i(k) \triangleq [x_i^T(k-\tau)\ x_i^T(k-\tau+1)\dots x_i^T(k-1)\ x_i^T(k)]^T$. Assume that $x_i(j) \equiv 0 \quad \forall j = -\tau, \dots, -1, 0$; i.e. $\breve{x}_i(k) \equiv 0$. The accumulative storage function of a Lyapunov-Krasovskii functional form of the following will be considered with the above defined accumulative supply rate:

$$V_{si}(\breve{x}_i(k)) := x_i^T(k)P_i x_i(k) + \sum_{j=k-\tau}^{k-1} x_i^T(j)P_{\tau i}(j-\tau)x_i(j), \tag{6.6}$$

$$P_i = P_i^T \succ 0, \quad P_{\tau i}(\kappa) = P_{\tau i}^T(\kappa) \succ 0.$$

Alternatively,

$$V_{si}(\breve{x}_i(k)) = \breve{x}_i^T(k)\breve{P}_{\tau i}\breve{x}_i(k),\ \breve{P}_{\tau i} := \begin{bmatrix} P_i & 0 & \dots & 0 \\ 0 & P_{\tau i}(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{\tau i}(\tau) \end{bmatrix}. \tag{6.7}$$

A subsystem $\mathcal{S}_i$ is then said to be quadratically dissipative with respect to the quadratic supply rate $\xi^{si}(\breve{y}_{si}, \breve{u}_{si})$ if there exists a nonnegative C^1 function, addressed as storage function, $V_{si}(\breve{x}_i(k))$, $V_i(\breve{x}_i(0)) = 0$, such that for all u_{si} and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied:

$$V_{si}(\breve{x}_i(k+1)) - \alpha_i V_{si}(\breve{x}_i(k)) \leq \xi^{si}(\breve{y}_{si}, \breve{u}_{si}), \quad 0 < \alpha_i < 1. \tag{6.8}$$

It is noted that in the above supply rates, the passivity terms describing the phase property between $\breve{y}_i$ and $\breve{d}_i$ are not considered. Based on these definitions, we derive the delay-dependent accumulatively-quadratic-dissipativity condition for an open-loop interactive subsystem stated in the next Theorem.

Theorem 11 *Consider a subsystem* $\mathcal{S}_i$ (6.1) *and the accumulative storage function of the Lyapunov-Krasovskii functional form* (6.6)*. Subsystem* $\mathcal{S}_i$ *is delay-dependent accumulatively and quadratically dissipative with respect to the accumulative supply rate* $\xi^{si}(\breve{y}_{si}, \breve{u}_{si})$ (6.5)*, if and only if the following LMI is feasible in* $\boldsymbol{P}_i, \boldsymbol{P}_{\tau i}(j), \boldsymbol{Q}_i(j), \boldsymbol{S}_i(j), \boldsymbol{R}_i(j), \boldsymbol{R}_{di}(j), \boldsymbol{Q}_{\tau i}(j), \boldsymbol{S}_{\tau i}(j), \boldsymbol{R}_{\tau i}(j)$*, for* $j = 0 \ldots \tau_{\max}$*:*

$$\mathcal{Q}_i + \sum_{j=1}^{\tau} \mathcal{Q}_{\tau i}(j) \preccurlyeq 0, \tag{6.9}$$

$$\mathcal{Q}_i := \begin{bmatrix} \mathcal{P}_i & A_i^T \boldsymbol{P}_i E_i - F_i^T \boldsymbol{S}_{\tau i}(0) & A_i^T \boldsymbol{P}_i B_i - C_i^T \boldsymbol{S}_i(0) & A_i^T \boldsymbol{P}_i L_i \\ * & E_i^T \boldsymbol{P}_i E_i - \boldsymbol{R}_{\tau i}(0) & E_i^T \boldsymbol{P}_i B_i & E_i^T \boldsymbol{P}_i L_i \\ * & * & B_i^T \boldsymbol{P}_i B_i - \boldsymbol{R}_i(0) & B_i^T \boldsymbol{P}_i L_i \\ * & * & * & L_i^T \boldsymbol{P}_i L_i - \boldsymbol{R}_{di}(0) \end{bmatrix}, \tag{6.10}$$

in which $\mathcal{P}_i := A_i^T \boldsymbol{P}_i A_i - C_i^T \boldsymbol{Q}_i(0) C_i - F_i^T \boldsymbol{Q}_{\tau i}(0) F_i - \alpha_i \boldsymbol{P}_i, \quad 0 < \alpha_i < 1,$ *and*

$$\mathcal{Q}_{\tau i}(j) := \begin{bmatrix} \mathcal{P}_{\tau i}(j) & A_i^T \boldsymbol{P}_{\tau i}(j) E_i - F_i^T \boldsymbol{S}_{\tau i}(j) & A_i^T \boldsymbol{P}_{\tau i}(j) B_i - C_i^T \boldsymbol{S}_i(j) & A_i^T \boldsymbol{P}_{\tau i}(j) L_i \\ * & E_i^T \boldsymbol{P}_{\tau i}(j) E_i - \boldsymbol{R}_{\tau i}(j) & E_i^T \boldsymbol{P}_{\tau i}(j) B_i & E_i^T \boldsymbol{P}_{\tau i}(j) L_i \\ * & * & B_i^T \boldsymbol{P}_{\tau i}(j) B_i - \boldsymbol{R}_i(j) & B_i^T \boldsymbol{P}_{\tau i}(j) L_i \\ * & * & * & L_i^T \boldsymbol{P}_{\tau i}(j) L_i - \boldsymbol{R}_{di}(j) \end{bmatrix}, \tag{6.11}$$

in which $\mathcal{P}_{\tau i}(j) := A_i^T \boldsymbol{P}_{\tau i}(j) A_i - C_i^T \boldsymbol{Q}_i(j) C_i - F_i^T \boldsymbol{Q}_{\tau i}(j) F_i - \boldsymbol{P}_{\tau i}(j)$.

Proof: By expanding the dissipation inequality of the following form

$$V_{si}(\breve{x}_i(k+1)) - \alpha_i V_{si}(\breve{x}_i(k)) \leq \xi^{si}(\breve{y}_{si}, \breve{u}_{si}),$$

using the state space model (6.1) and the accumulative storage function of the Lyapunov-Krasovskii functional form (6.6), the resultant accumulatively quadratic dissipativity criterion cast in LMI will be obtained after some matrix manipulations (see, e.g., [16]). ■

6.2.2 Delay-Robust Quadratic Dissipativity

In this subsection, a subsystem $\mathcal{S}_i$ (6.1) is said to be delay-robust dissipative if it is quadratically dissipative with respect to the following quadratic supply rate:

$$\xi^i(y_{si}, u_{si}) = y_{si}^T Q_{si} y_{si} + 2 y_{si}^T S_{si} u_{si} + u_{si}^T R_{si} u_{si}, \tag{6.12}$$

where $\quad {u_{si}}^T(k) = [v_i^T(k) \quad u_i^T(k) \quad d_i^T(k)], \quad {y_{si}}^T(k) = [w_i^T(k-\tau) \quad y_i^T(k)],$

$$Q_{si} = \begin{bmatrix} Q_{\tau i} & 0 \\ 0 & Q_i \end{bmatrix}, \; R_{si} = \begin{bmatrix} R_{\tau i} & 0 \\ 0 & \begin{bmatrix} R_i & 0 \\ 0 & R_{di} \end{bmatrix} \end{bmatrix}, \; S_{si} = \begin{bmatrix} S_{\tau i} & 0 \\ 0 & \begin{bmatrix} S_i & 0 \end{bmatrix} \end{bmatrix}.$$

A subsystem $\mathcal{S}_i$ is said to be quadratically dissipative with respect to the quadratic supply rate $\xi^i(y_{si}, u_{si})$ if there exists a nonnegative C^1 function, addressed as storage function, $V_i(\breve{x}_i(k))$, $V_i(0) = 0$, such that for all u_{si} and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied [164]:

$$V_i(\breve{x}_i(k+1)) - \alpha_i V_i(\breve{x}_i(k)) \leq \xi^i(y_{si}, u_{si}), \quad 0 < \alpha_i < 1. \tag{6.13}$$

The storage function of a Lyapunov-Krasovskii functional form of the following will be considered in this subsection chapter:

$$V_i(\breve{x}_i(k)) = x_i^T(k)P_i x_i(k) + \sum_{j=k-\tau}^{k-1} x_i^T(j)P_{\tau i}x_i(j), \quad P_i = P_i^T \succ 0, \quad P_{\tau i} = P_{\tau i}^T \succ 0. \tag{6.14}$$

It is noted here that, only one constant $P_{\tau i}$ is considered in V_i instead of τ matrices $P_{\tau i}(\kappa)$ in V_{si}. The following Theorem provides a delay-independent dissipativity criterion.

Theorem 12 *Consider a subsystem* $\mathcal{S}_i$ (6.1) *and the storage function of the Lyapunov-Krasovskii functional form* (6.14). *Subsystem* $\mathcal{S}_i$ *is delay-independent delay-robust dissipative with respect to the supply rate* $\xi^i(y_{si}, u_{si})$ (6.12), *if and only if the following LMI is feasible in* $\boldsymbol{P}_i, \boldsymbol{P}_{\tau i}, \boldsymbol{Q}_i, \boldsymbol{S}_i, \boldsymbol{R}_i, \boldsymbol{R}_{di}, \boldsymbol{Q}_{\tau i}, \boldsymbol{S}_{\tau i}, \boldsymbol{R}_{\tau i}$*:*

$$\begin{bmatrix} \mathscr{P}_i & 0 & A_i^T\boldsymbol{P}_iE_i & A_i^T\boldsymbol{P}_iB_i - C_i^T\boldsymbol{S}_i & A_i^T\boldsymbol{P}_iL_i \\ * & -\alpha_i\boldsymbol{P}_{\tau i} - F_i^T\boldsymbol{Q}_{\tau i}F_i & -F_i^T\boldsymbol{S}_{\tau i} & 0 & 0 \\ * & * & E_i^T\boldsymbol{P}_iE_i - \boldsymbol{R}_{\tau i} & E_i^T\boldsymbol{P}_iB_i & E_i^T\boldsymbol{P}_iL_i \\ * & * & * & B_i^T\boldsymbol{P}_iB_i - \boldsymbol{R}_i & B_i^T\boldsymbol{P}_iL_i \\ * & * & * & * & L_i^T\boldsymbol{P}_iL_i - \boldsymbol{R}_{di} \end{bmatrix} \preccurlyeq 0, \tag{6.15}$$

where $\mathscr{P}_i = A_i^T\boldsymbol{P}_iA_i - \alpha_i\boldsymbol{P}_i + \boldsymbol{P}_{\tau i} - C_i^T\boldsymbol{Q}_iC_i, \quad 0 < \alpha_i < 1.$

Proof: By expanding the dissipation inequality of the following form

$$V_i(\breve{x}_i(k+1)) - \alpha_i V_i(\breve{x}_i(k)) \leq \xi^i(y_{si}(k), u_{si}(k)),$$

using the state space model (6.1) and the storage function of the Lyapunov-Krasovskii functional form (6.14), the resultant delay-robust dissipativity criterion cast in LMI will be obtained after some matrix manipulations, as detailed in the sequel. Substituting (6.14) to the above dissipation inequality gives

$$x_i(k+1)^T P_i x_i(k+1) - \alpha_i x_i(k)^T P_i x_i(k) - \alpha_i x_i(k-\tau)^T P_{\tau i} x_i(k-\tau) + x_i(k)^T P_{\tau i} x_i(k)$$
$$\leq y_i^T(k) Q_i y_i(k) + u_i^T(k) R_i u_i(k) + d_i^T(k) R_{di} d_i(k) + 2y_i^T(k) S_i u_i(k)$$
$$+ w_i^T(k-\tau) Q_{\tau i} w_i(k-\tau) + 2w_i^T(k-\tau) S_{\tau i} v_i(k) + v_i^T(k) R_{\tau i} v_i(k).$$

By omitting all time index k, for conciseness, and substituting (6.1) to the above, we have:

$$(x_i^T A_i^T + u_i^T B_i^T + v_i^T E_i^T + d_i^T L_i^T) P_i (A_i x_i + B_i u_i + E_i v_i + L_i d_i)$$
$$-x_i^T(\alpha_i P_i - P_{\tau i}) x_i - \alpha_i x_i^T(-\tau) P_i x_i(-\tau) \leq x_i^T C_i^T Q_i C_i x_i + u_i^T R_i u_i + d_i^T R_{di} d_i$$
$$+2y_i^T S_i u_i + x_i^T(-\tau) F_i^T Q_{\tau i} F_i x_i(-\tau) + 2x_i^T(-\tau) F_i^T S_{\tau i} v_i + v_i^T R_{\tau i} v_i \quad (6.16)$$

$$\Leftrightarrow x_i^T(A_i^T P_i A_i - \alpha_i P_i + P_{\tau i} - C_i^T Q_i C_i) x_i + u_i^T(B_i^T P_i B_i - R_i) u_i + v_i^T(E_i^T P_i E_i - R_{\tau i}) v_i$$
$$+ d_i^T(L_i^T P_i L_i - R_{di}) d_i + 2x_i^T(A_i^T P_i B_i - C_i^T S_i) u_i + 2x_i^T A_i^T P_i E_i v_i$$
$$+ 2x_i^T A_i^T P_i L_i d_i + 2u_i^T B_i^T P_i E_i v_i + 2u_i^T B_i^T P_i L_i d_i + 2v_i^T E_i^T P_i L_i d_i$$
$$+ x_i^T(-\tau)(-\alpha_i P_{\tau i} - F_i^T Q_{\tau i} F_i) x_i(-\tau) - 2x_i^T(-\tau) F_i^T S_i v_i \leq 0$$

which is fulfilled by (6.15).■

The following result provides a delay-dependent delay-robust dissipativity condition for $\mathcal{S}_i$. It will be seen from the proof that the delay-dependent condition is an extension of the delay-independent condition in Theorem 12.

Theorem 13 *Consider a subsystem $\mathcal{S}_i$ (6.1) and the storage function of the Lyapunov-Krasovskii functional form (6.14). Subsystem $\mathcal{S}_i$ is delay-dependent delay-robust dissipative with respect to the supply rate $\xi^i(y_{si}, u_{si})$ (6.12), if the following LMI is feasible* *in* $\boldsymbol{P}_i, \boldsymbol{P}_{\tau i}, \boldsymbol{Q}_i, \boldsymbol{S}_i, \boldsymbol{R}_i, \boldsymbol{R}_{di}, \boldsymbol{Q}_{\tau i}, \boldsymbol{S}_{\tau i}, \boldsymbol{R}_{\tau i}$:

$$\begin{bmatrix} \mathscr{P}_{\tau i} & {A_i^{\tau}}^T \mathscr{P}_i \mathcal{B}_i & {A_i^{\tau+1}}^T \boldsymbol{P}_i E_i - F_i^T \boldsymbol{S}_{\tau i} & {A_i^{\tau}}^T (A_i^T \boldsymbol{P}_i B_i - C_i^T \boldsymbol{S}_i) & {A_i^{\tau+1}}^T \boldsymbol{P}_i L_i \\ * & \mathcal{B}_i^T \mathscr{P}_i \mathcal{B}_i & \mathcal{B}_i^T A_i^T \boldsymbol{P}_i E_i & \mathcal{B}_i^T (A_i^T \boldsymbol{P}_i B_i - C_i^T \boldsymbol{S}_i) & \mathcal{B}_i^T A_i^T \boldsymbol{P}_i L_i \\ * & * & E_i^T \boldsymbol{P}_i E_i - \boldsymbol{R}_{\tau i} & E_i^T \boldsymbol{P}_i B_i & E_i^T \boldsymbol{P}_i L_i \\ * & * & * & B_i^T \boldsymbol{P}_i B_i - \boldsymbol{R}_i & B_i^T \boldsymbol{P}_i L_i \\ * & * & * & * & L_i^T \boldsymbol{P}_i L_i - \boldsymbol{R}_{di} \end{bmatrix} \preccurlyeq 0,$$

$$\tau = 1, \ldots, \tau_{max}, \tag{6.17}$$

where $\mathscr{P}_i = A_i^T \boldsymbol{P}_i A_i - \alpha_i \boldsymbol{P}_i + \boldsymbol{P}_{\tau i} - C_i^T \boldsymbol{Q}_i C_i$, $\mathscr{P}_{\tau i} = {A_i^{\tau}}^T \mathscr{P}_i A_i^{\tau} - \alpha_i \boldsymbol{P}_{\tau i} - F_i^T \boldsymbol{Q}_{\tau i} F_i$, $\mathcal{B}_i = [A_i^{\tau-1} B_{si} \ \ldots \ A_i B_{si} \quad B_{si}]$.

Proof: From the state space model (6.1), we obtain $x_i = A_i^{\tau} x_i(-\tau) + \mathcal{B}_i \ \breve{u}_{\tau i}$, where

$$\begin{aligned} \mathcal{B}_i &= [A_i^{\tau-1} B_{si} \ \ldots \ A_i B_{si} \quad B_{si}], \quad B_{si} = [B_i \quad E_i \quad L_i], \\ \breve{u}_{\tau i} &= [u_{si}(k-\tau) \ \ldots \ u_{si}(k-1)], \quad u_{si} = [u_i \quad d_i \quad v_i]. \end{aligned}$$

Substituting to the inequality (6.17) in the proof of Proposition 12 yields:

$$\begin{aligned} &[x_i^T(-\tau){A_i^{\tau}}^T + \breve{u}_{\tau i}^T \mathcal{B}_i^T] \mathscr{P}_i [A_i^{\tau} x_i(-\tau) + \mathcal{B}_i \breve{u}_{\tau i}] + u_i^T (B_i^T P_i B_i - R_i) u_i + v_i^T (E_i^T P_i E_i - R_i) v_i \\ &+2[x_i^T(-\tau){A_i^{\tau}}^T + \breve{u}_{\tau i}^T \mathcal{B}_i^T](A_i^T P_i B_i - C_i^T S_i) u_i + 2[x_i^T(-\tau){A_i^{\tau}}^T + \breve{u}_{\tau i}^T \mathcal{B}_i^T] A_i^T P_i E_i v_i \\ &+2[x_i^T(-\tau){A_i^{\tau}}^T + \breve{u}_{\tau i}^T \mathcal{B}_i^T] A_i^T P_i L_i d_i + 2u_i^T B_i^T P_i E_i v_i + 2u_i^T B_i^T P_i L_i d_i + 2v_i^T E_i^T P_i L_i d_i \\ &+x_i^T(-\tau)(-\alpha_i P_{\tau i} - F_i^T Q_{\tau i} F_i) x_i(-\tau) - 2x_i^T(-\tau) F_i^T S_{\tau i} v_i + d_i^T (L_i^T P_i L_i - R_{di}) d_i \leq 0, \end{aligned}$$

$$\Leftrightarrow \begin{bmatrix} x_i(-\tau) \\ \breve{u}_{\tau i} \\ v_i \\ u_i \\ d_i \end{bmatrix}^T \begin{bmatrix} {A_i}^{\tau T}\mathscr{P}_i{A_i}^{\tau} - \alpha_i P_{\tau i} - F_i^T Q_{\tau i} F_i & {A_i}^{\tau T}\mathscr{P}_i\mathcal{B}_i & {A_i}^{\tau+1^T} P_i E_i - F_i^T S_{\tau i} & {A_i}^{\tau T}(A_i^T P_i B_i - C_i^T S_i) & {A_i}^{\tau+1^T} P_i L_i \\ * & \mathcal{B}_i^T \mathscr{P}_i \mathcal{B}_i & \mathcal{B}_i^T A_i^T P_i E_i & \mathcal{B}_i^T (A_i^T P_i B_i - C_i^T S_i) & \mathcal{B}_i^T A_i^T P_i L_i \\ * & * & E_i^T P_i E_i - R_i & E_i^T P_i B_i & E_i^T P_i L_i \\ * & * & * & B_i^T P_i B_i - R_i & B_i^T P_i L_i \\ & * & * & * & L_i^T P_i L_i - R_{di} \end{bmatrix} \begin{bmatrix} x_i(-\tau) \\ \breve{u}_{\tau i} \\ v_i \\ u_i \\ d_i \end{bmatrix} \preccurlyeq 0,$$

which is fulfilled by (6.17). ■

6.2.3 Quadratic Constraint

The controller $\mathcal{C}_i$ is considered as an uncertain system connected with subsystem $\mathcal{S}_i^s$ (6.4) in the stand-alone mode or with subsystem $\mathcal{S}_i$ in the decentralised control scheme. Defining a quadratic function ξ^{ic} with respect to the output and input pair of $\Big((u_i, d_i); y_i\Big)$ by

$$\xi^{ic}\Big((u_i, d_i); y_i\Big) \triangleq u_i^T Q_{ic} u_i + 2u_i^T S_{ic}^T y_i + y_i^T R_{ic} y_i + d_i^T Q_{ic}^d d_i, \tag{6.18}$$

the trajectory of controller $\mathcal{C}_i$ is then said bounded by the $\tau-$PRC with respect to the supply rate ξ^{ic} (6.18), if the accumulation of $\xi^{ic}\Big(\big(u_i(k), d_i(k)\big), y_i(k)\Big)$ for the time period $(k-\tau)\dots k$, denoted as $\xi^{ic}_{\Sigma}(k,\tau)$, is nonnegative; i.e.

$$\xi^{ic}_{\Sigma}(k,\tau) \geq 0 \quad \forall\tau = 1,2,\dots\tau_{\max}, \quad \text{where} \quad \xi^{ic}_{\Sigma}(k,\tau) := \sum_{j=k-\tau}^{k} \xi^{ic}\Big(\big(u_i(j),d_i(j)\big);y_i(j)\Big). \tag{6.19}$$

The matrices $Q_{ic}, Q^d_{ic}, S_{ic}, R_{ic}$ will be determined in alliance with the dissipativity matrices of the corresponding open-loop subsystem from the LMIs derived in the stabilisability conditions for network systems.

6.2.4 $\beta\tau-$Asymptotically Positive Realness Constraint

Similarly to the APRC, the $\tau-$PRC is achievable with finite power controls using the $\beta\tau-$asymptotically positive realness constraint ($\beta\tau-$APRC) defined in the sequel:

Definition 7 *The input and output pair $\big(u_i(k),y_i(k)\big)$ is said to satisfy the asymptotically positive realness constraint with respect to $\xi^{ic}_{\Sigma}(k)$ and the coupling delay τ, or simply $\beta\tau-$APRC, if there are $k_0 \in \mathbb{Z}^+$, $0 \leq \zeta^k_i < 1$ and*
$|\beta^k_i| \geq \sqrt{(1-\zeta^k_i)|\xi^{ic}_{\Sigma}(k-1)|}$ such that

$$\xi^{ic}_{\Sigma}(k,\tau) \geq \frac{1}{2}\,\sigma^k_i \left(\xi^{ic}_{\Sigma}(k-1,\tau) + (\sigma^k_i - 1)(\beta^k_i)^2\right) \quad \forall k > k_0 + \tau_{max}, \tag{6.20}$$

where $\sigma^k_i = 1 - \mathrm{sgn}(\xi^{ic}_{\Sigma}(k-1,\tau))$.

Lemma 4 *If there is $0 \leq \zeta^k_i < 1$ such that whenever $k > k_0 + \tau_{\max}$, ξ^k_{ic} satisfies the $\beta\tau-$APRC* (6.20)*, then for every real number $\delta_i \geq 0$, there exists $\bar{k}_i \in \mathbb{N}$ such that*

$$\xi^{ic}_{\Sigma}(k,\tau) \geq -\delta_i \quad \forall k \geq \bar{k}_i. \tag{6.21}$$

Proof: There are two possibilities:

1. $\xi^{ic}_{\Sigma}(k_0+\tau_{\max},\tau) \geq 0$, then (6.21) means $\xi^{ic}_{\Sigma}(k,\tau) \geq 0$ $\forall k > k_0 + \tau_{\max}$.

2. $\xi_{\Sigma}^{ic}(k_0+\tau_{\max},\tau)<0$, then (6.21) and $0\le\zeta_i(k)<1$ result in

   (a) $\zeta_i(k)\neq 0\ \forall k>k_0+\tau_{\max}$:

$$\begin{aligned}
0>\xi_{\Sigma}^{ic}(k,\tau) &\ge \xi_{\Sigma}^{ic}(k-1,\tau)+(\beta_i^k)^2\\
&\ge \zeta_i^k\,\xi_{\Sigma}^{ic}(k-1,\tau),\ |\beta_i^k|\ge\sqrt{(1-\zeta_i^k)|\xi_{\Sigma}^{ic}(k-1,\tau)|}\\
&\ge \xi_{\Sigma}^{ic}(k_0+\tau_{\max})\prod_{\ell=k_0}\zeta_i(\ell)\\
&\ge \xi_{\Sigma}^{ic}(k_0+\tau_{\max})(1-\epsilon_i)^{k-k_0+\tau_{\max}+1},\ 0<\epsilon_i<1.\\
\Rightarrow 0 &> \xi_{\Sigma}^{ic}(k,\tau)\to 0 \text{ as } k\to+\infty.
\end{aligned}$$

   This means, for every real number $\alpha_i>0$, there exists a natural number n such that for every $k>n$ we have $|\xi_{\Sigma}^{ic}(k,\tau)|<\delta_i$.

   (b) There exist $\bar{k}_i>k_0+\tau_{\max}$ such that $\xi_{\Sigma}^{ic}(k,\tau)\ge 0\ \forall k\ge\bar{k}_i$.

The proof is complete. ■

It is straight forward to see that $\beta\tau-$APRC is an asymptotic sector constraint on the window of τ time steps.

The stabilisability condition based on the accumulative dissipativity and accumulative supply rate is firstly stated in below theorem. The stabilisability condition based on the delay-robust dissipativity criterion and the one-time-step quadratic supply rate is subsequently stated following.

6.2.5 $\beta\tau-$APRC Based Stabilisability Conditions

Denote

$$\begin{aligned}
\xi_{\Sigma}^{ic}(k,\tau) &= \sum_{j=k-\tau}^{k}\xi^{ic}\Big(\big(u_i(j),d_i(j)\big),y_i(j)\Big)\\
&:= \breve{u}_i^T\breve{Q}_{ic}\breve{u}_i+2\breve{y}_i^T\breve{S}_{ic}\breve{u}_i+\breve{y}_i^T\breve{R}_{ic}\breve{y}_i+\breve{d}_i^T\breve{Q}_{ic}^d\breve{d}_i,
\end{aligned}$$

where $\breve{Q}_{ic} := \text{diag}[Q_{ic}]_1^{\tau+1}$, $\breve{S}_{ic} := \text{diag}[S_{ic}]_1^{\tau+1}$, $\breve{R}_{ic} := \text{diag}[R_{ic}]_1^{\tau+1}$, $\breve{Q}_{ic}^d := \text{diag}[Q_{ic}^d]_1^{\tau+1}$.

The LMI-based stabilisability condition is established from the accumulative dissipativity of all open-loop subsystems and their controller $\tau-$PRCs, as stated in Theorem 14 below. Theorem 14 provides a delay-dependent condition. Define a block diagonal matrix $\breve{H} := \mathsf{diag}\{H\}_{\tau_{\max}}$.

Theorem 14 *Consider h subsystems $\mathcal{S}_i$ (6.1) interconnected with each other by the coupling delay process H, and h nonnegative storage function of the Lyapunov-Krasovskii functional form (6.6). System Σ associated with h controller $\mathcal{C}_i$ is stabilised asymptotically if the controller trajectories are bounded by $\tau-$PRCs as defined in (6.19) and the following LMIs are feasible in $\breve{\boldsymbol{Q}}_i, \breve{\boldsymbol{S}}_i, \breve{\boldsymbol{R}}_i, \breve{\boldsymbol{R}}_{di}, \breve{\boldsymbol{Q}}_{ic}, \breve{\boldsymbol{Q}}_{ic}^d, \breve{\boldsymbol{S}}_{ic}, \breve{\boldsymbol{R}}_{ic}, \breve{\boldsymbol{Q}}_{\tau i}, \breve{\boldsymbol{S}}_{\tau i}, \breve{\boldsymbol{R}}_{\tau i}$:*

$$\mathsf{diag}[\breve{\boldsymbol{Q}}_{\tau i}]_1^h + \breve{H}^T \mathsf{diag}[\breve{\boldsymbol{R}}_{\tau i}]_1^h \breve{H} + \breve{H}^T \mathsf{diag}[\breve{\boldsymbol{S}}_{\tau i}^T]_1^h + \mathsf{diag}[\breve{\boldsymbol{S}}_{\tau i}]_1^h \breve{H} \prec 0, \quad \textit{and} \tag{6.22}$$

$$\begin{bmatrix} \breve{y}_i(k)^T(\breve{\boldsymbol{Q}}_i + \breve{\boldsymbol{R}}_{ic})\breve{y}_i(k) & \breve{y}_i(k)^T(\breve{\boldsymbol{S}}_i + \breve{\boldsymbol{S}}_{ic}) \\ * & \breve{\boldsymbol{R}}_i + \breve{\boldsymbol{Q}}_{ic} \end{bmatrix} \prec 0, \quad \breve{\boldsymbol{R}}_{di} + \breve{\boldsymbol{Q}}_{ic}^d \prec 0, \tag{6.23}$$

$$\textit{and} \quad (6.9), \quad \breve{\boldsymbol{Q}}_{ic} \prec 0, \quad \breve{\boldsymbol{R}}_{ic} \succ 0, \qquad \textit{for} \;\; i = 1, 2, \ldots, h.$$

Proof: This is a direct result of Theorem 1 with the state vector $\breve{x}_{si}$, control vector $\breve{u}_{si}$ and output vector $\breve{y}_{si}$. ■

6.2.6 APRC-Based Stabilisability Conditions

A stabilisability condition based on the delay-robust dissipativity criterion and the APRC, restated below, is provided in this subsection. For decentralised control problems, we consider a block diagonal controller $\mathcal{C}$, which can be implemented by h decoupled local controllers $\mathcal{C}_i$. Denote

$Q := \text{diag}[Q_i]_1^h,\ S := \text{diag}[S_i]_1^h,\ R := \text{diag}[R_i]_1^h,\ R_d := \text{diag}[R_{di}]_1^h,\ Q_\tau := \text{diag}[Q_{\tau i}]_1^h,$
$S_\tau := \text{diag}[S_{\tau i}]_1^h,\ R_\tau := \text{diag}[R_{\tau i}]_1^h, R_c := \text{diag}[R_{ic}]_1^h,\ S_c := \text{diag}[S_{ic}]_1^h, Q_c := \text{diag}[Q_{ic}]_1^h,$
$Q_c^d := \text{diag}[Q_{ic}^d]_1^h,$

The LMI-based stability condition is established from the delay-robust dissipativity of all open-loop subsystems and their corresponding APRCs, as stated in below two theorems. Theorem 15 is a delay-independent condition. Denote

$$\xi_{ic}^k := \xi^{ic}\big((u_i(k), d_i(k), y_i(k)\big).$$

Definition 8 *The controller $\mathcal{C}_i$ is said to satisfy the APRC with respect to ξ_{ic}^k* (6.18)*, if there is $k_0 > \tau_{\max}$, $0 \le \gamma_i^k < 1$ and $|\beta_i^k| \ge \sqrt{\big(1-\gamma_i^k\big)|\xi_{ic}^{k-1}|}$, such that*

$$\begin{cases} \xi_{ic}^k \ge 0, & \text{if } \xi_{ic}^{k-1} \ge 0, \\ \xi_{ic}^k \ge \xi_{ic}^{k-1} + (\beta_i^k)^2, & \text{if } \xi_{ic}^{k-1} < 0, \end{cases} \quad \forall k > k_0. \tag{6.24}$$

Theorem 15 *Consider h nonnegative storage function of the Lyapunov-Krasovskii functional form* (6.14)*. Suppose that:*

1. *The following LMIs are feasible in $\boldsymbol{Q}_i, \boldsymbol{S}_i, \boldsymbol{R}_i, \boldsymbol{R}_{di}, \boldsymbol{Q}_\tau, \boldsymbol{S}_\tau, \boldsymbol{R}_\tau, \boldsymbol{Q}_{ic}, \boldsymbol{Q}_{ic}^d, \boldsymbol{S}_{ic}, \boldsymbol{R}_{ic}$:*

$$\begin{gathered} F^T\boldsymbol{Q}_\tau F + F^T\boldsymbol{S}_\tau^T HF + F^T H^T\boldsymbol{S}_\tau F + F^T H^T\boldsymbol{R}_\tau HF \prec 0, \qquad \textit{and} \\ \begin{bmatrix} \boldsymbol{Q}_i + \boldsymbol{R}_{ic} & \boldsymbol{S}_i + \boldsymbol{S}_{ic} \\ * & \boldsymbol{R}_i + \boldsymbol{Q}_{ic} \end{bmatrix} \prec 0, \quad \boldsymbol{R}_{di} + \boldsymbol{Q}_{ic}^d \prec 0, \\ \textit{and} \quad (6.15), \qquad \boldsymbol{Q}_{ic} \prec 0, \quad \boldsymbol{R}_{ic} \succ 0, \qquad \textit{for} \quad i = 1, 2, \ldots, h; \end{gathered} \tag{6.25}$$

2. *The APRCs* (6.24)*, $i = 1 \ldots h$, are feasible in $u_i(k)$;*

Then h unconstrained control vectors $u_i(k)$, $i = 1 \ldots h$, stabilise Σ.

Proof: Denote the APRC supply rate of $\mathcal{C}$ as follows:

$$\xi^c\Big(\big(u(k), d(k)\big); y(k)\Big) = y^T(k)R_c y(k) + 2y^T(k)S_c u(k) + u^T(k)Q_c u(k) + d^T(k)Q_c^d d(k).$$

Consider the large-scale vectors x, u, d, y, v and w integrated with the delayed interconnection process H in large-scale supply rate of the form:

$$\xi\Big(\big(w(k), y(k)\big); \big(v(k), u(k), d(k)\big)\Big) := y^T(k)Qy(k) + 2y^T(k)Su(k) + u^T(k)Ru(k)$$
$$+d^T(k)R_d d(k) + w^T(-\tau)(k)Q_\tau w(-\tau)(k) + 2w^T(-\tau)(k)S_\tau Hw(-\tau)(k)$$
$$+w^T(-\tau)(k)H^T R_\tau Hw(-\tau)(k).$$

Denote $\sigma(k) := \xi\Big(\big(w(k), y(k)\big); \big(v(k), u(k), d(k)\big)\Big) + \xi^c\Big(\big(u(k), d(k)\big); y(k)\Big)$.

Omitting the time index k for conciseness, we have:

$$\begin{aligned}
\sigma = \quad & y^T R_c y + y^T S_c u + u^T Q_c u + d^T Q_c^d d + y^T Q y + 2y^T S u + u^T R u + d^T R_d d \\
& + w^T(-\tau)Q_\tau w(-\tau) + 2w^T(-\tau)S_\tau Hw(-\tau) + w^T(-\tau)H^T R_\tau Hw(-\tau), \\
= \quad & x^T C^T R_c C x + 2xTC^T S_c u + u^T Q_c u + d^T Q_c^d d + x^T C^T Q C x + 2x^T C^T S u + u^T R u \\
& + d^T R_d d \\
& + x^T(-\tau)F^T Q_\tau w(-\tau) + 2x^T(-\tau)F^T S_\tau HFx(-\tau) + x^T(-\tau)F^T H^T R_\tau HFx(-\tau),
\end{aligned}$$

$$\begin{aligned}
= \quad & x^T C^T(R_c + Q)Cx + 2x^T C^T(S_c + S)u + u^T(Q_c + R)u + d^T(Q_c^d + R_d)d \\
& + x^T(-\tau)F^T Q_\tau w(-\tau) + 2x^T(-\tau)F^T S_\tau HFx(-\tau) + x^T(-\tau)F^T H^T R_\tau HFx(-\tau),
\end{aligned}$$

$$= \quad \begin{bmatrix} Cx \\ u \end{bmatrix}^T \begin{bmatrix} Q+R & S+S \\ * & R+Q \end{bmatrix} \begin{bmatrix} Cx \\ u \end{bmatrix} + d^T(Q_c^d + R_d)d$$
$$+x^T(-\tau)\big(F^T Q_\tau F + 2F^T S_\tau HF + F^T H^T R_\tau HF\big)x(-\tau).$$

With feasible APRCs, there are $\bar{k} > k_0$ such that $\xi^c\Big(\big(u(k), d(k)\big); y(k)\Big) \geq 0\ \forall k \geq \bar{k}$. From this and (6.15), which govern the delay-robust dissipativity of $\mathcal{S}_i$, we obtain for $V_\Sigma(\breve{x}(k)) := x^T(k)Px(k) + \sum_{j=k-\tau}^{k-1} x^T(j)P_\tau x(j),\ P := \mathrm{diag}[P_i]_1^h,\ P_\tau := \mathrm{diag}[P_{\tau i}]_1^h$,

$$V_\Sigma(\breve{x}(k+1)) - \alpha V_\Sigma(\breve{x}(k)) \leq \sigma(k), \quad 0 < \alpha < 1.$$

Using (6.25) and the observability of Σ yield

$$V_{\Sigma}(\breve{x}(k+1)) - \alpha V_{\Sigma}(\breve{x}(k)) \leq -\eta_1 \|x(k)\|_2^2 - \eta_2 \|x(k-\tau)\|_2^2 \ \forall k \geq \bar{k},$$

with $\eta_1 > 0$ and $\eta_2 > 0$. Applying this inequality for the period from $k-\tau$ to k and the chain rule, we have for all $k \geq \bar{k}$:

$$V_{\Sigma}(\breve{x}(k+1)) - \alpha V_{\Sigma}(\breve{x}(k-\tau)) \leq -\eta_1 \|\breve{x}(k)\|_2^2 - \eta_2 \|\breve{x}(k-\tau)\|_2^2 \leq -\eta_2 \|\breve{x}(k-\tau)\|_2^2.$$

By virtue of $P_i \succ 0$ and $P_{\tau i} \succ 0$, there exist $0 < \alpha_1 < \alpha_2$ such that $\alpha_1 \|\breve{x}\|_2^2 \leq V_{\Sigma}(\breve{x}) \leq \alpha_2 \|\breve{x}\|_2^2$. Hence, $V_{\Sigma}(\breve{x}(k+1)) \leq \rho^2 V_{\Sigma}(\breve{x}(k-\tau))$, $\rho^2 := 1 - \frac{\eta_2}{\alpha_2}$. By choosing $\alpha_2 > \eta_2$, we have $\rho < 1$.

As a result, $\breve{x}^T(k+\tau)\breve{x}(k+\tau) \leq \alpha_2 \alpha_1^{-1} \rho^{2(k+\tau-k_0)} \breve{x}^T(k_0)\breve{x}(k_0)$.

Then, for a given $\epsilon > 0$, let $\delta := (\sqrt{\frac{\alpha_2}{\alpha_1}})^{-1}\epsilon$, it follows that,

$\|x(k_0)\|_2 < \delta \Rightarrow \|x(k+\tau)\|_2 < \epsilon$, for all $k > k_0$. ■

6.3 Decentralised Model Predictive Control

To streamline the developments, the outline of model predictive control problems is reproduced in this section. In a model predictive control problem, the two main elements of interest are the predictive control sequence and initial state vector. The predictive control sequence $\{u_i(k), u_i(k+1), \ldots, u_i(k+N)\}$ is denoted by $\hat{u}_i(k, x_i^k, N)$ or simply $\hat{u}_i$, where N is the predictive horizon, $x_i(k)$ is a known state vector, and k is the current time step in the discrete-time domain. The predictive state sequence, denoted as $\hat{x}_i(k, \hat{u}_i, N) = \big\{x_i^{\hat{u}_i}\big(k+1, x_i(k)\big), ..., x_i^{\hat{u}_i}\big(k+N, x_i(k)\big)\big\}$, is obtained as a result of applying the predictive control sequence $\hat{u}$ to the open-loop subsystem $\mathcal{S}_i^s$. The control objective is to steer the state to the origin. For an initial state vector

$x_i(k)$, the cost function is generally defined by

$$\mathcal{J}^i\big(\hat{u}_i(k, x_i^k, N)\big) = F_i\big(x_i(k+N)\big) + \sum_{j=k}^{k+N} \ell_i\big(x_i(j), u_i(j)\big).$$

The terminal time $k+N$, which could vary with time k but should not be decreasing, is often referred to as a predictive horizon. $\ell_i\big(x_i(j), u_i(j)\big)$ is the objective function. $F_i\big(x_i(k+N)\big)$ is the terminal cost function which is used to emulate the infinite-horizon optimising performance for the above finite-horizon problem. At every time step k, the open-loop optimisation problem of minimizing $\mathcal{J}^i\big(\hat{u}_i(k, x_i^k, N)\big)$ subject to $u_i(j) \in \mathbb{U}_i$ is solved to yield the minimizer of the form $\hat{u}_i^0(k, x_i^k, N) = \{u_i^0(k, x_i(k)), u_i^0(k+1, x_i(k)), \ldots, u_i^0(k+N, x_i(k))\}$. The first control move $u_i^0(k, x_i(k))$ is then applied to control the plant. The whole process is repeated in a rolling or receding manner in the next iteration. Since N is finite, a global minimum of $\mathcal{J}^i\big(\hat{u}_i(k, x_i^k, N)\big)$ exists if $\ell_i\big(x_i, u_i\big)$ are continuously differentiable, $\mathbb{U}_i$ is bounded. A comprehensive literature review for MPC methods is given in [94].

Here, the MPC problem is formulated in semi-definite programming in association with a new stability constraint to be developed. Stability constraints are artificial constraints adding to the optimisation problem of MPC solely for stability assurance purposes [121].

Optimisation Objective Function

For each stand-alone subsystem or subsystem in a decentralised control scheme, a quadratic objective function of the states and control inputs is considered in association with the adequately chosen weighting matrices $\mathcal{Q}^x, \mathcal{Q}^u$, as follows:

$$\mathcal{J}^i\Big(\hat{u}_i(k, x_i^k, N)\Big) = \sum_{j=1}^{N+1} x_i^T(k+j)\mathcal{Q}_i^x x_i(k+j) + \sum_{j=0}^{N} u_i^T(k+j)\mathcal{Q}_i^u u_i(k+j), \quad (6.26)$$

which is subject to the equality constraints of the state space model (6.1) and the inequality constraints deduced from physical limits of x_i and u_i.

The problem is formulated by predicting the state vector x_i in N time steps using the current value of state vectors $x_i(k)$. The predictive state vector is obtained from the perfect model without the input disturbance vector d_i, as follows:

$$\hat{x}_i(k, \hat{u}_i, N) = \mathit{\Theta}_i x_i(k) + \mathit{\Gamma}_i \hat{u}_i(k, x_i^k, N),$$

where $\quad \hat{u}_i(k, x_i^k, N) := [u_i(k)\ u_i(k+1) \dots u_i(k+N)]^T,$

$$\hat{x}_i(k, \hat{u}_i, N) := [x_i(k+1)\ x_i(k+2) \dots x_i(k+N+1)]^T,$$

$$\mathit{\Gamma}_i := \begin{bmatrix} B_i & 0 & \dots & 0 \\ A_i B_i & B_i & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A_i^N B_i & \dots & A_i B_i & B_i \end{bmatrix}, \quad \text{and} \quad \mathit{\Theta}_i := \begin{bmatrix} A_i \\ A_i^2 \\ \dots \\ A_i^{N+1} \end{bmatrix}.$$

By denoting $x_i(k, \hat{u}_i, N)$ as $\hat{x}_i(k)$ or just $\hat{x}_i$, and $u_i(k, x_i^k, N)$ as $\hat{u}_i(k)$ or just $\hat{u}_i$, we have:

$$\hat{x}_i = r_i^k + \mathit{\Gamma}_i \hat{u}_i, \tag{6.27}$$

where $r_i^k := \mathit{\Theta}_i x_i(k)$.

The objective function is now rewritten as,

$$\mathcal{J}^i\Big(\hat{u}_i(k, x_i^k, N)\Big) = [r_i^k + \mathit{\Gamma}_i \hat{u}_i]^T \mathcal{Q}_{iN}^x [r_i^k + \mathit{\Gamma}_i \hat{u}_i] + \hat{u}_i^T \mathcal{Q}_{iN}^u \hat{u}_i,$$

where $\mathcal{Q}_{iN}^x := \mathrm{diag}[\mathcal{Q}_i^x]_1^N$, $\mathcal{Q}_{iN}^u := \mathrm{diag}[\mathcal{Q}_i^u]_1^N$, which can be cast in the form of

$$\mathcal{J}^i\Big(\hat{u}_i(k, x_i^k, N)\Big) = \hat{u}_i^T \mathit{\Phi}_i \hat{u}_i + 2\mathit{\Upsilon}_i^k \hat{u}_i + \delta_i^k, \tag{6.28}$$

where $\mathit{\Phi}_i := \mathit{\Gamma}_i^T \mathcal{Q}_{iN}^x \mathit{\Gamma}_i + \mathcal{Q}_{iN}^u, \mathit{\Upsilon}_i^k := r_i^{kT} \mathcal{Q}_{iN}^x \mathit{\Gamma}_i, \delta_i^k := r_i^{kT} \mathcal{Q}_{iN}^x r_i^k$.

Constraint on Decision Variables

Let us consider a bounded constraint on decision variables of the following form:

$$\|u_i\|_2^2 \le \eta_i, \tag{6.29}$$

where η_i is the upper bound deduced from the physical limits of the control input vector u_i. The constraint set for the variable vector $\hat{u}_i$ will be as follows:

$$\hat{\mathbb{U}}_i \triangleq \{\hat{u}_i = [u_i^T(k)\ldots u_i^T(k+N_i)]^T : \|u_i(.)\|_2^2 \leq \eta_i\}. \tag{6.30}$$

Semi-Definite Program

The problem of minimising $\mathcal{J}^i\Big(\hat{u}_i(k, x_i^k, N)\Big)$ subject to the required constraints can be written in SDP by applying the Schur complement to (6.28) and (6.29) as:

$$\begin{gathered}\min_{\hat{u}_i,\ \gamma_i(k)} \gamma_i(k) \\ \text{subject to} \quad \gamma_i(k) > 0, \\ \begin{bmatrix} -\gamma_i(k) + \delta_i^k + 2\Upsilon_i^k \hat{\boldsymbol{u}}_i & \hat{\boldsymbol{u}}_i^T \\ \hat{\boldsymbol{u}}_i & -\Phi_i^{-1} \end{bmatrix} \preccurlyeq 0, \quad \text{and} \quad (6.30).\end{gathered} \tag{6.31}$$

The parameters Φ_i, Υ_i^0 and δ_i^0 are found from the state space realization matrices A_i, B_i, the weighting matrices $\mathcal{Q}_{iN}^x, \mathcal{Q}_{iN}^u$ of the objective function and the initial state vector $x_i(0)$. $\eta_{\Sigma i}$ is the bound of the control sequence $\hat{u}_i$ deduced from (6.29). Υ_i^k and δ_i^k are subsequently updated at every step using the updated state vector $x_i(k)$.

For each subsystem, the optimisation problem (6.31) is solved by the local controller for the minimising vector sequence $\hat{u}_i$ which comprises N elements of u_i. Only the first element $u_i(k)$ of the sequence $\hat{u}_i$ is applied to control the plant. This rolling process is repeated at the next time step, and continues thereon. Solving this problem does not, however, guarantee the stability of the closed-loop system according to the MPC literature [94]. The APRC-based stability constraints are implemented in the same manner as those in previous chapters. They are, therefore, not repeated in this chapter. In the next section, we will develop a dynamic stability constraint based on

$\beta\tau-$APRC to tackle this issue.

6.3.1 MPC Stability Constraint

Among various MPC formulations, the stability-enforced MPC achieves the closed-loop stabilization by imposing an additional constraint to the optimisation problem. In this context, a robust control Lyapunov function (CLF) with Sontag's formula has been used in two inequality constraints [121] to guarantee the stability of the closed-loop system and approximate the optimal performance of the unconstrained predictive controller, regardless of the choice of the horizon lengths.

In this chapter, we propose to add a point-wise constraint on the control vectors to the optimisation problem of MPC. This constraint, being of a quadratic type, is based on the $\beta\tau-$APRC of controller trajectories subject to the satisfaction of the interconnection stability condition.

By rewriting (6.20) as

$$\begin{aligned} & u_i^T Q_{ic}(1) u_i + 2u_i^T S_{ic}^T(1) y_i + y_i^T R_{ic}(1) y_i + d_i^T Q_{ic}^d(1) d_i + \\ & -\sigma_i^k \zeta_i^k \ \xi_{\Sigma}^{ic}(k-1,\tau) + \sum_{j=k-\tau}^{k-1} \xi^{ic}\big((u_i^j, d_i^j); y_i^j\big) \geq 0, \end{aligned} \tag{6.32}$$

where $Q_{ic}(1) \prec 0, Q_{ic}^d(1) \prec 0$, $\|d_i\|_2 \leq 1$, and $-\sigma_i^k \zeta_i^k \ \xi_{\Sigma}^{ic}(k-1,\tau) + \sum_{j=k-\tau}^{k-1} \xi^{ic}\big((u_i^j, d_i^j); y_i^j\big)$ is the historical data, y_i is the current measurement output vector, the $\beta\tau-$APRC (6.20) will be developed into a one-time-step quadratic constraint on the initial control vector $u_i^o\big(k, x_i(k)\big)$ for the optimisation problem of MPC. The convexity of the inequality constraint will be ensured if $Q_{ic}(1) \prec 0$. The stability constraint on control vectors u_i that is not depend on the input disturbances can be derived when $Q_{ic}^d(1) \prec 0$. Both $Q_{ic}(1) \prec 0$ and $Q_{ic}^d(1) \prec 0$ can be obtained from the LMIs in Theorem 15.

6.3.2 $\beta\tau-$APRC Stability Constraint

In this section, a sectoral stability constraint, that correlates the supply rate at the current time $\xi^{ic}_{\Sigma}(k,\tau)$ with that at the immediate future $\xi^{ic}_{\Sigma}(k+1,\tau)$ in its formation, is presented. It is the $\beta\tau-$APRC *sectoral stability constraint.* $\beta\tau-$APRC is a sectoral constraint on the control vectors within the window of τ time steps. To form the $\beta\tau-$APRC sectoral constraint, it is assumed that the interactive vector $v_i(k)$ is locally measured; i.e. it is known. As a result, a quadratic constraint on the control vector $u_i(k)$ is obtained. The $\beta\tau-$APRC of the form

$\xi^{ic}_{\Sigma}(k+1,\tau) \geq \sigma^k_i\, \zeta^k_i\, \xi^{ic}_{\Sigma}(k,\tau)$ is expressed in a quadratic constraint in $u_i(k)$ as follows:

$$\begin{aligned} & \boldsymbol{u}_i^{kT}(\Psi_{ic}+Q_{ic})\boldsymbol{u}_i^k + 2(\Upsilon^k_{ic}+S_{ic})\boldsymbol{u}_i^k + \delta^k_{ic} + R_{ic} + \sum_{j=k-\tau+2}^{k-1} \xi^{ic}\big((u^j_i,d^j_i);y^j_i\big) \\ \geq\ & \sigma^k_i\, \zeta^k_i \left(\boldsymbol{u}_i^{kT}Q_{ic}\boldsymbol{u}_i^k + 2y_i^{kT}S_{ic}\boldsymbol{u}_i^k + y_i^{kT}R_{ic}y^k_i + \sum_{j=k-\tau+1}^{k-1} \xi^{ic}\big((u^j_i,d^j_i);y^j_i\big)\right), \end{aligned} \tag{6.33}$$

where

$\Psi_{ic} := \Gamma_i^T R_{ic}\Gamma_i + 2\Gamma_i^T S_{ic} + Q_{ic},\ \Upsilon^k_{ic} := {r^k_{ic}}^T(R_{ic}\Gamma_i + S_{ic}),$

$\delta^k_{ic} := {r^k_{ic}}^T R_{ic} r^k_{ic} - \|Q^d_{ic}\| + \sum_{j=k-\tau+2}^{k-1} \xi^{ic}\big((u^j_i,d^j_i);y^j_i\big) - \sigma^k_i\zeta^k_i \sum_{j=k-\tau+1}^{k-1} \xi^{ic}\big((u^j_i,d^j_i);y^j_i\big),$

which is equivalent to

$$\boldsymbol{u}_i^{kT}\Psi_{\tau ic}\boldsymbol{u}_i^k + 2\Upsilon^k_{\tau ic}\boldsymbol{u}_i^k + \delta^k_{\tau ic} \geq 0, \tag{6.34}$$

where

$\Psi_{\tau ic} := \Gamma_i^T R_{ic}\Gamma_i + 2\Gamma_i^T S_{ic} + (2-\sigma^k_i\zeta^k_i)Q_{ic},\ \Upsilon^k_{ic} := {r^k_{ic}}^T(R_{ic}\Gamma_i + S_{ic}) + (1-\sigma^k_i\zeta^k_i)S_{ic},$

$\delta^k_{ic} := {r^k_{ic}}^T R_{ic} r^k_{ic} + (1-\sigma^k_i\zeta^k_i)R_{ic} - \|Q^d_{ic}\| + \varrho^k_{\tau i}$, in which

$\varrho^k_{\tau i} := \sum_{j=k-\tau+2}^{k-1} \xi^{ic}\big((u^j_i,d^j_i);y^j_i\big) - \sigma^k_i\zeta^k_i \sum_{j=k-\tau+1}^{k-1} \xi^{ic}\big((u^j_i,d^j_i);y^j_i\big).$

r^k_{ic} has been defined in Chapter 4.

6.3.3 Constraint Feasibility for $\beta\tau-$APRC Stability Constraint

To guarantee the constrained feasibility of (6.34), it is necessary that its feasible region is non-empty at every step k. The feasibility condition is stated next.

Theorem 16 *If the following LMIs are feasible:*

$$\varPsi_{\tau ic}(\boldsymbol{Q}_{ic},\ \boldsymbol{S}_{ic},\ \boldsymbol{R}_{ic}) \prec 0 \quad \textit{and} \quad \delta^k_{\tau ic}(\boldsymbol{R}_{ic}) \geq 0, \tag{6.35}$$

where

$$\varPsi_{\tau ic} = \varGamma_i^T C_i^T \boldsymbol{R}_{ic} C_i \varGamma_i + 2\varGamma_i^T C_i^T \boldsymbol{S}_{ic} + (2 - \sigma_i^k \zeta_i^k)\boldsymbol{Q}_{ic},$$

$$\delta^k_{ic} = r^{kT}_{ic} C_i^T \boldsymbol{R}_{ic} C_i r^k_{ic} + (1 - \sigma_i^k \zeta_i^k)\boldsymbol{R}_{ic} - \|Q^d_{ic}\| + \varrho^k_{\tau i};$$

Then, the feasible region of (6.34) *w.r.t* u_i^k *is nonempty recursively in* k.

Proof: With $\varPsi_{\tau i} \prec 0$, the inequality constraint (6.34) is convex. It is then sufficiently that (6.34) is feasible in $u_i(k)$ if $\delta^k_{\tau ic} \geq 0$ and $\varPsi_{\tau ic} \prec 0$ which is fulfilled by (6.35). ■

The decentralised MPC problem is formulated by applying the multiplier matrices that are found from the feasible solutions to the LMIs given in Theorem 14 and those in Theorem 16 to the $\beta\tau-$APRC based sectoral stability constraint (6.34) to be imposed on the SDP (6.31). It is as follows:

$$\min_{\hat{u}_i,\ \gamma_i(k)} \gamma_i(k)$$

$$\text{subject to} \quad \gamma_i(k) > 0,$$

$$\begin{bmatrix} -\gamma_i(k) + \delta_i^k + 2\varUpsilon_i^k \hat{\boldsymbol{u}}_i & \hat{\boldsymbol{u}}_i^T \\ \hat{\boldsymbol{u}}_i & -\varPhi_i^{-1} \end{bmatrix} \preccurlyeq 0, \quad \begin{bmatrix} -\delta^k_{\tau ic} - 2\varUpsilon^k_{\tau ic} M_i \hat{\boldsymbol{u}}_i & \hat{\boldsymbol{u}}_i^T M_i^T \\ M_i \hat{\boldsymbol{u}}_i & \varPsi^{-1}_{\tau ic} \end{bmatrix} \preccurlyeq 0, \text{ and } (6.30) \tag{6.36}$$

where $\delta_i^k, \varUpsilon_i^k, \varPsi_i$ are defined in the predictive objective function (6.28), and $M_i := [I_{m_i}\ 0_{m_i} \ldots 0_{m_i}]_{1\times h}$.

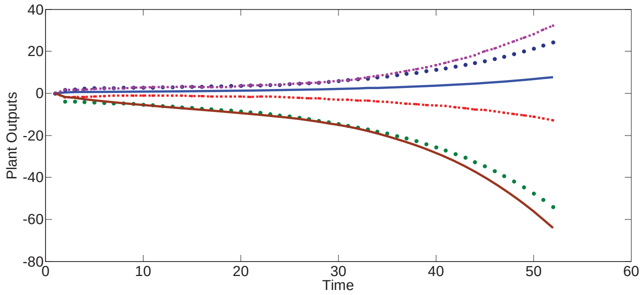


Figure 6.1 : Decentralised MPCs with Coupling Delay without Stability Constraints - Plant-Output Trends $\big(y_1(k), y_2(k), y_3(k)\big)$.

6.4 Illustrative Example

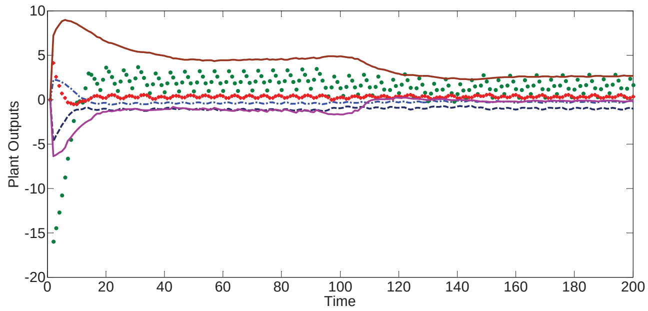
In this section, simulation results for a network system consisting of three coupled subsystems that suffer from an arbitrary coupling delay are presented to demonstrate the effectiveness of the proposed decentralised MPC strategy. Consider the following continuous-time state space models of three subsystems with the random coupling delay occurring within 7 time steps, i.e. $\tau_{max} = 7$ steps. The output constraints are ignored to emulate the system instability.

$$A_1 = \begin{bmatrix} -1 & 0.6 \\ 1 & -0.7 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0.8 \\ 0.1 \end{bmatrix} \quad E_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.2 & -0.7 \\ 1.2 & 3.1 \end{bmatrix},$$

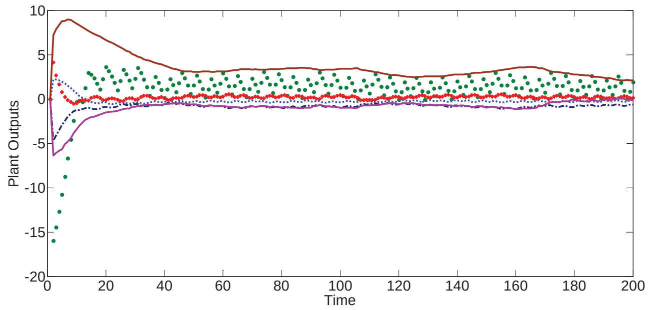
$$A_2 = \begin{bmatrix} -0.6 & 0 \\ 1 & -0.4 \end{bmatrix} \quad B_2 = \begin{bmatrix} 1.2 \\ 0.2 \end{bmatrix} \quad E_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1.9 & 0.1 \\ -0.1 & -1.5 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.8 & 0 \\ 0.4 & -0.3 \end{bmatrix} \quad B_3 = \begin{bmatrix} 1.5 \\ 0.1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} -1.5 & -0.1 \\ 0.5 & 1.3 \end{bmatrix},$$

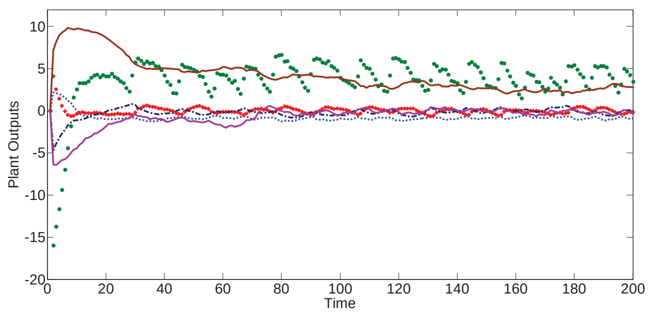
$$F_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \; F_2 = \begin{bmatrix} 1 & 2 \end{bmatrix}, \; F_3 = \begin{bmatrix} 0.5 & 4 \end{bmatrix}, H = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$



(a) Predictive Horizons of 12 Steps



(b) Predictive Horizons of 16 Steps Without Input Disturbances



(c) Predictive Horizons of 16 Steps With Input Disturbances

Figure 6.2 : Decentralised MPCs with a Coupling Delay of $\tau_{max} = 7$ steps, Delay-Dependent Dissipativity and Stability Criteria - Plant-Output Trends $\big(y_1(k), y_2(k), y_3(k)\big)$ with Local Input Disturbances $\|d_i(k)\| \leq 1$.

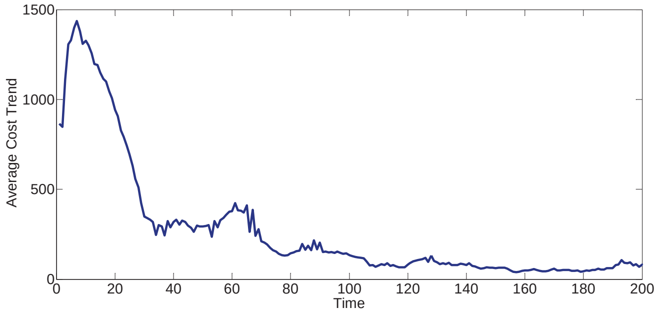


Figure 6.3 : The Average Optimising Cost Trend. The Coupling Delay time $\tau_{max} = 7$ steps.

The control and state weighting coefficients of the quadratic objective function, and constraints on the manipulation variables are as follows:

$$\mathcal{Q}_1^u{=}2, \qquad \mathcal{Q}_1^x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \eta_1{=}5,$$

$$\mathcal{Q}_2^u{=}0.5, \qquad \mathcal{Q}_2^x = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad \eta_2{=}5,$$

$$\mathcal{Q}_3^u{=}0.5, \qquad \mathcal{Q}_3^x = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad \eta_3 = 5.$$

The simulation starts with the formulations of three model predictive controllers in SDP without any stability constraints (6.36). The predictive horizon of 16 time steps is implemented for all three subsystems, using the Matlab LMI toolbox. The output time responses are depicted by Figure 6.1, which clearly imply unstable subsystems.

To demonstrate the efficacy of the APRC-based stability constraint, only offline multiplier matrices are used in this simulation study. When the predictive horizon of 12 time steps is implemented for all subsystems, the output time responses from Procedure 6.1 are depicted by Figure 6.2(a). When the predictive horizon of 16 time steps is implemented for all subsystems, the output time responses are depicted by Figure 6.2(b). The output time responses of 16 step horizon having a random input disturbance of absolute values less than 1 are given in Figure 6.2(c). The average optimising cost trend from Procedure 6.1 of the last simulation with input disturbances is provided in Figure 6.3.

The initial control and state vectors of three subsystems are as follows:

$$u_1 = \begin{bmatrix} -0.1 \end{bmatrix} \quad x_1 = \begin{bmatrix} -3 \\ -4 \end{bmatrix} u_2 = \begin{bmatrix} -0.5 \end{bmatrix} x_2 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$$
$$u_3 = \begin{bmatrix} -1 \end{bmatrix} \quad x_3 = \begin{bmatrix} 0.4 \\ 4 \end{bmatrix}.$$

Simulation results have proved that the control algorithm of Procedure 6.1 successfully stabilised the interconnected system suffering from an arbitrary coupling delay. The chosen predictive horizon play a significant role in the decentralised MPC strategy implemented with the control algorithm of Procedure 6.1.

6.5 Concluding Remarks

We have presented a decentralised model predictive control strategy for interconnected systems subject to an arbitrary coupling delay between subsystems. The dissipative systems theory is used to find solutions to the time delay problem. The delay-robust dissipativity criteria for subsystems and the interconnection stabilisability condition for interconnected systems are derived and served as the main contribution of this chapter. The condition for global state constraint satisfaction has also been developed herein. With the employments of input and output properties in the interconnection stabilisability condition, the present approach is naturally extendible to the problem of multiple state and control delays.

Chapter 7

Conclusion

The LMI-based stabilisability conditions for constrained interconnected systems in the discrete time domain have been derived in this Thesis based on the newly introduced *asymptotically positive realness constraint* (APRC). The employment of APRC in the constructive methods for distributed and decentralised MPCs constitute an essential part of the Thesis. The constructive approach of MPC *stability constraints* have been developed for recursively feasible implementations. Details of thesis contributions have been presented in Chapter 1.

The thesis has provided a decentralised perspective on the control of modular systems. We have started with a brief review on the applicability of model predictive controllers and the stabilising methods for their decentralised and distributed approaches (Chapters 1 and 2).

The APRC-based stabilisability conditions are developed into *stabilising constraints* for the local MPCs in decentralised MPC strategies for complex interconnected systems that may have parallel connections of unknown splitting ratios in Chapter 3. Alternative constructive methods of *stabilising agent* and *receding-horizon stabilising constraint* have been presented in Chapter 4 as the extensions to the development in Chapter 3. The developments for networked control systems and coupling-delayed systems are provided in Chapter 5 and Chapter 6, respectively. The multiplier matrices of the stability constraints may also be updated online at every iterative steps for the recursive feasibility. The decentralised control strategy in this Thesis is cutting-edge by employing only decoupled controls and precluding artificial constraints on the coupling variables. The former improves the system reliability. The latter mitigates any conservativeness of limiting energy and material flows between dynamically-coupled subsystems.

The work in this Thesis has laid a fundamental foundation for a new method of constrained stabilisations for interconnected large-scale systems in the time domain for the computerised control platforms. The computational costs of the presented method will be compensated for by the ever increasing computer power and the improving communication technology. Whilst being applicable to continuous nonlinear input-affine systems by virtue of using dissipative systems and quadratic constraints, the results have been emphasised on linear time invariant systems. It is, therefore, an initial step of a much broader scope which covers Lipschitz continuous and other types of nonlinear systems.

Bibliography

[1] Acar L. Boundaries for receding horizon control of interconnected systems. *Journal of Optimization Theory and Applications*, **84**[2]:251–271, 1995.

[2] Acikmese A. B. and M. J. Corless. Observers for systems with nonlinearities satisfying incremental quadratic constraints. *Automatica*, **47**[7]:1339–1348, 2011.

[3] Allgöwer F., R. Findeisen, and L. T. Biegler. *Assessment and Future Directions of Nonlinear Model Predictive Control*. LNCIS Vol. 358, Springer, 2007.

[4] Allgöwer F. and A. Zheng. *Nonlinear Model Predictive Control*. Birkhauser, 2000.

[5] Amato F., M. Ariola, and P. Dorato. Finite-time control of linear systems subject to parametric uncertainties and disturbances. *Automatica*, **37**[9]:1459–1463, 2001.

[6] Apkarian P. and D. Noll. IQC analysis and synthesis via nonsmooth optimization. *Systems and Control Letters*, **55**[12]:971–981, 2006.

[7] Apkarian P. and H. D. Tuan. Parameterized linear matrix inequalities in control theory. *SIAM Journal on Control and Optimization*, **38**[4]:1241–1264, 2003.

[8] Aske E. M. B., S. Strand, and S. Skogestad. Coordinator model predictive control for maximizing plant throughput. *Computers and Chemical Engineering*, **32**:195–204, 2008.

[9] Barrabasi A. L. and R. Albert. Emerging of scaling in random networks. *Science*, **286**[5439]:509–512, 1999.

[10] Bemporad A. and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, **35**:407–427, 1999.

[11] Bemporad A., M. Morari, V. Dua, and E. N. Pistikopoulos. The explicit linear quadratic regulator for constrained systems. *Automatica*, **38**:3–20, 2002.

[12] Blanchini F. Set invariant in control. *Automatica*, **35**:1747–1767, 1999.

[13] Blevins T., G. K. McMillan, W. K. Wojsznis, and M. W. Brown. *Advanced Control Unleashed : Plant Performance Management for Optimum Benefit*. ISA Press, 2003.

[14] Boyd S. and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

[15] Brocket R. W. Stabilization of motor networks. *Proceedings of Conference on Decision and Control CDC'95*, pages 1484–1488, 1995.

[16] Brogliato B., R. Lozano, B. Maschke, and O. Egeland. *Dissipative Systems Analysis and Control: Theory and Applications*. Springer, 2006.

[17] Byrnes C. I., A. Isidori, and J. Willems. Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE*

Transactions on Automatic Control, **36**[11]:1228–1240, 1991.

[18] Camacho E. F. and C. Bordons. *Model Predictive Control.* Springer-Verlag, Springer, 2004.

[19] Camacho E. F. and C. Bordons. Nonlinear model prediction control: an introductory survey. *Proceedings of International Workshop on Assessment and Future Directions of Nonlinear Predictive Control NMPC'05*, pages 15–30, 2005.

[20] Camponogara E., D. Jia, B. H. Krogh, and S. Talukdar. Distributed model predictive control. *IEEE Control System Magazine*, **1**:44–52, 2002.

[21] Camponogara E. and S. Talukdar. Distributed model predictive control: Synchronous and asynchronous computation. *IEEE Trans. Sys., Man, and Cybernetics*, **37**:732–745, 2007.

[22] Cannon M., B. Kouvaritakis, and V. Deshmukh. Enlargement of polytopic terminal region in NMPC by interpolation and partial invariance. *Automatica*, **40**:311–317, 2004.

[23] Cannon M., B. Kouvaritakis, S. V. Rakovic, and Q. Cheng. Stochastic tubes in model predictive control with probabilistic constraints. *IEEE Transactions on Automatic Control*, **56**[1]:194–200, 2011.

[24] Canon M., Q. Cheng, B. Kouvaritakis, and S. V. Rakovic. Stochastic tube mpc with state estimation. *Automatica*, **48**[3]:536–541, 2012.

[25] Chen H. and F. Allgöwer. A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. *Automatica*, **34**[10]:1205–1218, 1998.

[26] Cheng R., J. F. Forbes, and W. S. Yip. Dantzig-Wolfe decomposition and plant-wide MPC coordination. *Computers and Chemical Engineering*, **32**:1507–1522, 2008.

[27] Chisci L., J. A. Rossiter, and G. Zappa. Systems with persistent disturbances: predictive control with restrictive constraints. *Automatica*, **37**:1019–1028, 2001.

[28] Clarke D. W., C. Mohtadi, and P. S. Tuffs. Generalized predictive control Part I - The basic algorithm. *Automatica*, **23**[2]:137–148, 1987.

[29] Coales J. F. and A. R. M Noton. An on-off servomechanism with predicted changeover. *Proceedings of the IEEE*, **128**:227–232, 1956.

[30] Cutler C. R. and B. L. Ramaker. Dynamic matrix control - a computer control algorithm. In *Proceedings of the Joint Automatic Control Conference*, 1980.

[31] D'Andrea R. and G. E. Dullerud. Distributed control of spatially interconnected systems. *IEEE Transactions on Automatic Control*, **48**:1478–1495, 2003.

[32] De Oliveira L. B. and E. Camponogara. Multi-agent model predictive control of signaling split in urban traffic networks. *Transportation Research Part C: Emerging Technologies*, **18**[1]:120–139, 2010.

[33] Desborough L. and R. Miller. Increasing customer value of industrial control performance monitoring - Honeywell's experience. *Honeywell Hi-Spec Solution White Paper*, 2001.

[34] Desoer C. A. and M. Y. Wu. Input-output properties of discrete systems. *Journal of the Franklin Institute*, **290**:11–24, 1970.

[35] Ding B., L. Xie, and W. Cai. Distributed model predictive control for constrained linear systems. *International Journal of Robust and Nonlinear Control*, **20**[11]:1285–1298, July 2010.

[36] Du B., J. Lam, and Z. Shu. Stabilization for state/input delay systems via static and integral output feedback. *Automatica*, **46**[12]:2000–2007, 2010.

[37] Dunbar W. B. and R. M. Murray. Distributed receding horizon control with application to multi-vehicle formation stabilization. *Automatica*, **42**[4]:1549–558, 2006.

[38] El-Farra, N. H., P. Mhaskar, and P. D. Christofides. Uniting bounded control and MPC for stabilization of constrained linear systems. *Automatica*, **40**[1]:101–110, 2004.

[39] Elaiw A. M. Stabilizing output feedback receding horizon control of sampled-data nonlinear systems via discrete-time approximations. *Bulletin of Mathematical Analysis and Applications*, **2**[1]:42–55, 2010.

[40] Fax J. A. and R. M. Murray. Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control*, **49**[9]:1465–1476, 2004.

[41] Ferrari-Trecate G., L. Galbusera, M. P. E. Marciandi, and R. Scattolini. Model predictive control schemes for consensus in multi-agent systems with single- and double-integrator dynamics. *IEEE Transactions on Automatic Control*, **54**[11]:2560–2572, 2009.

[42] Findeisen R., L. Imsland, F. Allgöwer, and B. A. Foss. State and output nonlinear model predictive control: An overview. *European Journal of Control*, **9**:190–206, 2003.

[43] Findeisen R. and P. Varutti. Stabilizing nonlinear predictive control over nondeterministic communication networks. *LNCIS - Nonlinear Model Predictive Control*, **384**:167–179, 2008.

[44] Formion V., G. Scorletti, and G. Ferrere. Nonlinear performance of a PI controlled missle: An explanation. *International Journal of Robust and Nonlinear Control*, **9**:485–518, 1999.

[45] Fridman F. and U. Shaked. On reachable sets for linear systems with delay and bounded peak inputs. *Automatica*, **39**[11]:2005–2010, 2003.

[46] García C. E. and A. M. Morshedi. Quadratic programming solution of dynamic matrix control (QDMC). *Chemical Engineering Communications*, **46**:73–87, 1986.

[47] Gerry J. P. Control loop optimization. In: *Process Software and Digital Networks,* ISA Instrument Engineer's Handbook, CRC Press, 2002.

[48] Goodwin G. C. and D. E. Quevedo. A moving horizon approach to networked control system design. *IEEE Transactions on Automatic Control*, **49**[9]:1427–1445, 2004.

[49] Gossner J. R., B. Kouvaritakis, and J. A. Rossiter. Stable generalized predictive control with constraints and bounded disturbances. *Automatica*, **33**[4]:551–568, 1997.

[50] Goulart P. J., E. C. Kerrigan, and J. M. Maciejowski. Optimization over state feedback policies for robust control with constraints. *Automatica*, **42**:523–533, 2006.

[51] Gupta V., A. F. Dana, J. P. Hespanha, R. M. Murray, and B. Hassibi. Data transmission over networks for estimation and control. *IEEE Transactions on Automatic Control*, **54**[8]:1807–1819, 2009.

[52] Gupta V., B. Sinopoli, S. Adlakha, A. Goldsmith, and R. Murray. Receding horizon networked control. *44*$^{\text{th}}$ *Annual Allerton Conference on Communication, Control, and Computing*, pages 169–176, 2006.

[53] H. D. Tuan, Tri Tran, Hung T. Nguyen, and Q. P. Ha. Distributed model predictive control with stability constraints and its application in alumina refining process. *Unbuplished Report*, May 2011 / 2012.

[54] Haddad W. M. and V. Chellaboina. *Nonlinear Dynamical Systems and Control: a Lyapunov-based approach.* Priceton University Press, 2008.

[55] Halevi Y. and A. Ray. Integrated communication and control systems: Part I and II. *Journal of Dynamical Systems, Measurement and Control*, **10**:367–381, 1988.

[56] Hassibi A., S. P. Boyd, and J. P. How. Control of asynchronous dynamical systems with rate constraint on events. *Proceedings of Conference on Decision and Control CDC'99*, pages 1345–1351, 1999.

[57] Hill D. L. and P. J. Moylan. The stability of nonlinear dissipative systems. *IEEE Transactions on Automatic Control*, **21**[3]:708–711, 1976.

[58] Hines G. H., M. Arcak, and A. K. Packard. Equilibrium-independent passivity: A new definition and numerical certification. *Automatica*, **47**[9]:1949–1956, 2011.

[59] Imsland L., J. A. Rossiter, B. Pluymers, and J. Suykens. Robust triple mode MPC. *International Journal of Control*, **81**[4]:679–689, 2008.

[60] Johansson B., A. Speranzon, M. Johansson, and K. H. Johansson. On decentralized negotiation of optimal consensus. *Automatica*, **44**[0]:1175–1179, 2008.

[61] Kalman R. E. Contributions to the theory of optimal control. *Bulletin Soc. Math. Mex.*, **5**:102–119, 1960.

[62] Kano M. and M. Ogawa. The state of the art in chemical process control in Japan: Good practice and questionnaire survey. *Journal of Process Control*, **20**[9]:969–982, 2010.

[63] Katebi M. R. and M. A. Johnson. Predictive control design for large-scale systems. *Automatica*, **33**:421–425, 1997.

[64] Keerthi S. S. and E.G. Gilbert. Optimal infinite-horizon feedback laws for a general class of constrained discrete-time systems: Stability and moving horizon approximations. *Journal of Optimization Theory and Applications*, **57**[2]:265–29, 1988.

[65] Kerrigan E. C. and J. M. Maciejowski. Invariant sets for constrained nonlinear dicretetime systems with application to feasibility in model predictive control. *Proceedings of the 39*$^{\text{th}}$ *IEEE Conference on Decision and Control,* Sydney, pages 4951–4956, 2000.

[66] Kerrigan E. C. and J. M. Maciejowski. Feedback min-max model predictive control using a single linear program: robust stability and the explicit solution. *International Journal of Robust and Nonlinear Control*, **14**:395–413, 2004.

[67] Keviczky T., F. Borrelli, and G. J. Balas. Decentralized receding horizon control for large scale dynamically decoupled systems. *Automatica*, **42**[12]:2105–2115, 2006.

[68] Keviczky T., F. Borrelli, K. Fregene, D. Godbole, and G. J. Balas. Decentralized receding horizon control and coordination of autonomous vehicle formations. *IEEE Transactions on Control Systems Technology*, **16**[1]:19–33, 2008.

[69] Kim J-H. Improved ellipsoidal bound of reachable sets for time-delayed linear systems with disturbances. *Automatica*, **44**[11]:2940–2943, 2008.

[70] Kolmanovsky I. and E. Gilbert. Theory and computation of disturbance invariant sets for discrete-time linear systems. *Mathematical Problems in Engineering: Theory - Methods and Applications*, **4**:317–367, 1998.

[71] Körouğlu H. and C. W. Scherer. Robust performance analysis for structured linear time-varying perturbations with bounded rates-of-variation. *IEEE Transactions on Automatic Control*, **52**[3]:197–211, 2007.

[72] Kothare S. L. O. and M. Morari. Contractive model predictive control for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, **45**[6]:1053–1071, 2000.

[73] Kouvaritakis B. and M. Cannon. *Nonlinear Predictive Control.* IEE Control Engineering Series 61, Springer, 2001.

[74] Kouvaritakis B., M. Cannon, S. V. Rakovic, and Q. Cheng. Explicit use of probabilistic distributions in linear predictive control. *Automatica*, **46**[10]:1719–1724, 2010.

[75] Langport C., V. Gupta, and R. M. Murray. Distributed control over failing channels. *LNCIS - Networked Embedded Sensing and Control*, pages 325–342, 2006.

[76] Lavaei J., A. Momeni, and A. Aghdam. A model predictive decentralized control scheme with reduced communication requirement for spacecraft formation. *IEEE Transactionson Control System Technology*, **16**[2]:268–278, 2008.

[77] Lazar M. *Model Predictive Control of Hybrid Systems: Stability and Robustness.* PhD Thesis, Eindhoven University, 2006.

[78] Lazar M. and R. H. Gielen. On parameterized dissipation inequalities and receding horizon robust control. *Proceedings of the 18*$^{\text{th}}$ *IFAC World Congress*, pages 172–178, Milano, Italy, 2011.

[79] Leigh J. R. *Control Theory,* 2^{nd} *Edition.* IEE Control Engineering Series 64, 2004.

[80] Levines W. *The Control Handbook.* CRC Press, 1996.

[81] Lewis F. L. *Applied Optimal Control and Estimation - Digital Design and Implementation.* Prentice Hall, 1992.

[82] Li C. and G. Chen. Synchronization in general complex dynamical networks with coupling delays. *Physica-A: Stat. Mech. and its Apps.*, **343**:263–278, 2004.

[83] Li S., Y. Zhang, and Q. Zhu. Nash-optimization enhanced distributed model predictive control applied to a Shell benchmark problem. *Information Sciences*, **170**:329–34, 2005.

[84] Linh Vu and K. A. Morgansen. Stability of time-delay feedback switched linear systems. *IEEE Transactions on Automatic Control*, **55**[10]:2385–2390, 2010.

[85] Liu J., D. M. de la Pena, and P. D. Christophides. Distributed model predictive control of nonlinear processes. *AIChE Journal*, **55**[5]:1171–1184, 2009.

[86] Lotz F. I. The importance of optimal control for practical engineer. *Automatica*, **6**:749–753, 1970.

[87] Lunze J. *Feedback Control of Large Scale Systems*. Prentice Hall, 1992.

[88] Luyben W. L. and B. D. Tyreus. *Plant-Wide Process Control*. McGraw-Hill, 1999.

[89] Maciejowski J. M. *Predictive Control With Constraints*. Prentice Hall, 2002.

[90] Magni L., D. M. Raimondo, and F. Allgöwer. *Nonlinear Model Predictive Control: Towards New Challenging Applications*. LNCIS Vol. 384, Springer, 2009.

[91] Magni L. and R. Scattolini. Stabilizing decentralized model predictive control of nonlinear systems. *Automatica*, **42**[7]:1231–1236, 2006.

[92] Mahmoud M. S. *Robust Control and Filtering for Time-Delay Systems*. Marcel Dekker, 2000.

[93] Mayne D. Q. and H. Michalska. Receding horizon control of nonlinear system. *IEEE Transactions on Automatic Control*, **35**[7]:814–824, July 1990.

[94] Mayne D. Q., J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert. Constraint predictive control: Stability and optimality. *Automatica*, **36**:789–814, 2000.

[95] Meadows E. S., M. A. Henson, J. W. Eaton, and J. B. Rawlings. Receding horizon control and discontinuous state feedback stabilization. *Internationl Journal of Control*, **62**[5]:1217–1229, 1995.

[96] Megretski A. and A. Rantzer. System analysis via integral quadratic constraints. *IEEE Transactions on Automatic Control*, **42**[6]:819–830, 1997.

[97] Mercangöz M. and F. J. Doyle III. Distributed model predictive control of an experimental four-tank system. *Journal of Process Control*, **17**:297–308, 2007.

[98] Mhaskar, P., N. H. El-Farra, and P. D. Christofides. Stabilization of nonlinear systems with state and control constraints using Lyapunov-based predictive control. *Systems and Control Letters*, **55**:650–659, 2006.

[99] Michalska H. and D. Q. Mayne. Robust receding horizon control of constrained nonlinear systems. *IEEE Transactions on Automatic Control*, **38**[11]:1623–1633, 1993.

[100] MONTESTRUQUE L. A. AND P. J. ANTSAKLIS. On the model-based control of networked systems. *Automatica*, **39**:1837–1843, 2003.

[101] MOYLAN P. J. AND D. L. HILL. The stability criteria for large-scale systems. *IEEE Transactions on Automatic Control*, **23**[2]:143–149, 1978.

[102] MUELLER A. M., M. REBLE, AND F. ALLGOEWER. A general distributed MPC framework for cooperative control. *Proceedings of the 18*th *IFAC World Congress*, pages 7987–7992, Milano, Italy, 2011.

[103] MUSKE K. R. AND J. B. RAWLINGS. Model predictive control with linear models. *AIChE Journal*, **39**[2]:262–287, 1993.

[104] MUTAMBARA A. G. O AND H. F. DURRANT-WHYTE. Estimation and control for a modular wheeled mobile robot (DDEC). *IEEE Transactions on Control Systems Technology*, **8**:35–46, 2000.

[105] NEGENBORN R. R. *Multi-Agent Model Predictive Control with Applications to Power Networks*. PhD thesis, Delft University of Technology, 2007.

[106] NEGENBORN R. R., G. HUG-GLANZMANN, B. DE SCHUTTER, AND G. ANDERSSON. A novel coordination strategy for multi-agent control using overlapping subnetworks with application to power systems. In MOHAMMADPOUR J. AND GRIGORIADIS K.M., editors, *Efficient Modeling and Control of Large-Scale Systems*, pages 251–278. Springer, Norwell, Massachusetts, 2010.

[107] NEWMAN M. *Networks: An Introduction*. Oxford University Press, 2010.

[108] NICULESCU S. I. *Delay Effects on Stability - A Robust Control Approach*. Springer, 2001.

[109] NOCEDAL J. AND S. J. WRIGHT. *Numerical Optimization, 2^{nd} Edition*. Springer Verlag, 2006.

[110] OLFATI-SABER R., J. A. FAX, AND R. M. MURRAY. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, **95**[1]:215–233, 2007.

[111] ONG C. J. AND E. G. GILBERT. The minimal disturbance invariant set: Outer approximations via its partial sums. *Automatica*, **42**:1563–1568, 2006.

[112] ORTEGA R. *Passivity-Based Control of Euler-Lagrange Systems: Mechanical, Electrical, and Electromechanical Applications*. Springer, Springer, 1998.

[113] OUCHERIAH S. Robust exponential convergence of a class of linear delayed systems with bounded controllers and disturbances. *Automatica*, **42**[11]:1863–1867, 2006.

[114] PAVLOV A. AND L. MARCONI. Lyapunov-based control for switched power converters. *Proceedings of the 45^{th} IEEE Conference on Decision and Control*, pages 5263–5268, 2008.

[115] PEARSON R. K. Outliers in process modeling and identification. *IEEE Transactions on Control Systems Technologies*, **10**[1]:55–63, 2002.

[116] PENA D. M. AND P. D. CHRISTOPHIDES. Lyapnunov-based model predictive control of nonlinear systems subject to data loss. *IEEE Transactions on Automatic Control*, **53**[9]:2076–2089, 2008.

[117] PHAT V. N., Q. P. HA, AND H. TRINH. Parameter-dependent H_∞ control for time-varying delay polytopic systems. *Journal of Optimization Theory and*

Applications, **147**:58–70, 2010.

[118] Pin G. and T. Parisini. Networked predictive control of constrained nonlinear systems: recursive feasibility and input-to-state stability analysis. *IEEE Transactions on Automatic Control*, **56**[1]:72–87, 2011.

[119] Prett D. M. and C. E. Garcia. *Fundamental Process Control.* Butterworths, 1988.

[120] Primbs J. A., V. Nevistic, and J. C. Doyle. On receding horizon extensions and control Lyapunov functions. *Proceedings of IEEE American Control Conference ACC'98,* Philadelphia, pages 3276–3280, 1998.

[121] Primbs J. A., V. Nevistic, and J. C. Doyle. A receding horizon generalization of pointwise min-norm controllers. *IEEE Transactions on Automatic Control*, **45**[5]:898–909, 2000.

[122] Qin S. J. and T. A. Badgwell. An overview of industrial model predictive control technology. In Kantor J. C., Garcia C. E., and B. Carnahan, editors, *AIChE Symposium Series: Fifth International Conference on Chemical Process Control*, **316**, pages 232–256. AIChE press edition, 1997.

[123] Qin S. J. and T. A. Badgwell. An overview of nonlinear model predictive control applications. *In: Nonlinear Model Predictive Control,* Birkhauser, 2000.

[124] Qin S. J. and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, **11**:733–764, 2003.

[125] Scattolini R. Architectures for distributed and hierarchical model predictive control - A Review. *Journal of Process Control*, **19**:723–731, 2009.

[126] Rakovic S. V., B. Kouvaritakis, M. Cannon, C. Panos, and R. Findeisen. Parameterized tube model predictive control. *Internal Report No. OUEL 2315/10*, page Oxford University, 2010.

[127] Rawlings, J. and D. Mayne. *Model Predictive Control: Theory and Design.* Nob Hill Publishing, Wisconsin, 2009.

[128] Rawlings J. B. and B. T. Stewart. Coordinating multiple optimization-based controllers: New opportunities and challenges. *Journal of Process Control*, **18**:839–845, 2008.

[129] Richalet J. Industrial application of predictive functional control. *Workshop on Nonlinear Model Based Control - Software and Applications (NMPC-SOFAP'07)*, Loughborough UK, 2007.

[130] Richalet J., A. Rault, J. L. Testud, and J. Papon. Model predictive heuristic control: Application to industrial processes. *Automatica*, **14**:413–428, 1978.

[131] Richards A. and J. P. How. Robust distributed model predictive control. *International Journal of Control*, **80**[9]:1517–1531, 2007.

[132] Ricker N. L. Decentralised control of the Tennessee Eastman challenge process. *Journal of Process Control*, **6**[4]:205–221, 1996.

[133] Robert D. G. and D. J. Stilwell. Control of autonomous vehicle platoon with a switch communication network. *Proceedings of IEEE American Control Conference ACC'05*, pages 4333–4338, 2005.

[134] Rossiter J. A. *Model-Based Predictive Control-A Practical Approach.* CRC Press, 2003.

[135] Ryua J. H., J. Artigasb, and C. Preusche. A passive bilateral control scheme for a teleoperator with time-varying communication delay. *Mechatronics*, **20**[7]:812–823, 2010.

[136] Sanders S. R. and G. C. Verghese. Lyapunov-based control for switched power converters. *IEEE Transactions on Power Electronics*, **7**[1]:17–24, 1992.

[137] Schenato L., M. Franceschetti, K. Poolla, and Shankar Sastry. Foundations of control and estimation over lossy networks. *Proceedings of the IEEE*, **95**[1]:164–187, 2007.

[138] Scherer C. , P. Gahinet, and M. Chilali. Multiobjective output-feedback control via LMI optimization. *IEEE Transactions on Automatic Control*, **42**[7]:896–911, 1997.

[139] Scherer C. Dissipation-based controller synthesis: successes and challenges. *Plenary Lecture at the 18*$^{\text{th}}$ *IFAC World Congress*, Milano, Italy, 2011.

[140] Scherer C. and M. Javad (Eds.). *Control of Linear Parameter Varying Systems with Applications.* Springer-Verlag, 2012.

[141] Schooman M. L. *Reliability of Computer Systems and Networks: Fault Tolerance, Analysis and Design.* Wiley-Interscience, 2001.

[142] Scokaert P. O. M., D. Q. Mayne, and J. B. Rawlings. Suboptimal model predictive control. *IEEE Transactions on Automatic Control*, **44**[3]:648–654, 1999.

[143] Scorletti G. and G. Duc. An LMI approach to dencentralized H_∞ control. *International Journal of Control*, **74**[3]:211–224, 2001.

[144] Siljak D. D. *Decentralized Control of Complex Systems.* Academic Press, 1991.

[145] Siljak D. D. Dynamic graphs. *Proceedings of Conference on Hybrid Systems and Applications 06*, pages 544–567, 2006.

[146] Singh M. G. and A. Titli. *Systems: Decomposition, Optimisation and Control.* Franklin Book Company, 1978.

[147] Smith O. J. M. A controller to overcome dead time. *ISA Journal*, **6**:28–33, 1959.

[148] Honeywell Process Solutions. *Profit Controller Concepts Reference Guide RM09-400 R320.* Honeywell International Process Solutions, 2500 West Union Hills, Phoenix, AZ 85027, November 2008.

[149] Sun Y. and N. H. El-Farra. Quasi-decentralized model-based networked control of process systems. *Computers and Chemical Engineering*, **32**[9]:2016–2029, 2008.

[150] Takatsu H., T. Itoh, and M. Araki. Future needs for the control theory in industries - Report and topics of the control technology survey in Japanese industry. *Journal of Process Control*, **8**:369–374, 1997.

[151] Tang P. and C. de Silva. Stability and optimality of constrained model predictive control with future input buffering in networked control systems.

Proceedings of IEEE American Control Conference ACC'05, pages 1245–1250, 2005.

[152] Tri Tran. Distributed model predictive control with flexible communication network. To appear in *Proceedings of the 1st Intl. Conf. on Control, Automation and Information Science ICCAIS'12,* SaiGon, VietNam, December 2012.

[153] Tri Tran and J. Bao. A real-time trajectory-based stability constraint for model predictive control. *Proceedings of IEEE International Conference on Control and Automation ICCA'09*, pages 2094–2099, 2009.

[154] Tri Tran and Q. P. Ha. Set-point tracking semi-automatic control of interconnected systems with local input disturbances. In *Proceedings of the 1st Australian Control Conference*, pages 224–229. Melbourne, Australia, 2011.

[155] Tri Tran and Q. P. Ha. Parameterised quadratic constraints for network systems subject to multiple communication topologies. To appear in *Proceedings of the 12th Intl. Conf. on Control, Automation, Robotics and Vision ICARCV'12,* GuangZhou, China, December 2012.

[156] Tri Tran, H. D. Tuan, Q. P. Ha, and Hung T. Nguyen. Stabilising agent design for the control of interconnected systems. *International Journal of Control*, **84**[6]:1140–1156, July 2011.

[157] Tri Tran, H. D. Tuan, Q. P. Ha, and Hung T. Nguyen. Decentralized model predictive control of time-varying splitting parallel systems. In Javad Mahammadpour and Carsten Scherer, editors, *Control of Linear Parameter Varying Systems with Applications*, **ISBN 978-1-4614-1832-0**, pages 217–251. Springer, Berlin, April 2012.

[158] Trinh P. The unnatural nature of asymptotic approximations. *iSquared Magazine*, pages 18–23, September 2008.

[159] Tzafestas S., E. Kyriannakis, and G. Kapsiotis. Decentralized model based predictive control of large scale systems. *Systems Analysis Modeling and Simulation*, **29**:1–25, 1997.

[160] Van Der A. Schaft. *L_2-Gain and Passivity Techniques in Nonlinear Control*. Springer, 1996.

[161] Venkat A. N., I. A. Hisken, J. B. Rawlings, and S. J. Wright. Distributed MPC strategies with applications to power system automatic generation control. *IEEE Transactions on Control Systems Technology*, **16**[6]:1192–1206, 2008.

[162] Venkat A. N., J. B. Rawlings, and S. Wright. Distributed model predictive control of large-scale systems. *Proceedings of International Workshop on Assessment and Future Directions of NMPC*, pages 591–606, 2005.

[163] Walsh C. C. and L. G. Bushnell. Stability analysis of networked control systems I. *IEEE Transactions on Control Systems Technology*, **10**:438–446, 2002.

[164] Willems J. C. Dissipative dynamical systems , Parts I and II. *Arch. Rational Mechanics Analysis*, **45**:321–393, 1972.

[165] Willems J. C. Dissipative dynamical systems. *European Journal of Control*, **13**:134–151, 2007.

[166] Wong W. S. and R. W. Brockett. Systems with finite communication bandwidth constraints II. *IEEE Transactions on Automatic Control*, **44**:1049–1053, 1999.

[167] Wright S. J. Applying new optimization algorithms to model predictive control. *Proceedings of the 5th International Conference on Chemical Process Control: Assessment and New Directions for Research*, pages 147–155, 1997.

[168] Xu X. M., Y. G. Xi, and Z. J. Zhang. Decentralized predictive control of large scale systems. *Information and Decision Technologies*, **14**[3]:307–322, 1988.

[169] Ydstie E., E. Perea-López, and I. E. Grossmann. A model predictive control strategy for supply chain optimization. *Computers and Chemical Engineering*, **27**:1201–1218, 2003.

[170] Zheng A. and M. Morari. Robust stability of constrained model predictive control. *Proceedings of IEEE American Control Conference ACC'93*, pages 379–383, June 1993.