

PERFORMANCE ANALYSIS OF COOPERATIVE RELAY NETWORKS IN PRESENCE OF INTERFERENCE

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Abstract

In the past decade, cooperative communication has emerged as an attractive technique for overcoming the shortcomings of point-to-point wireless communications systems. Cooperative relaying improves the performance of wireless networks by forming an array of multiple independent virtual sources transmitting the same information as the source node. In addition, when relays are deployed near the edge of the network, they can provide additional coverage in network dead spots. Interference in the network can also be reduced in cooperative communications systems as the nodes can transmit at lower power levels compared to equivalent point-to-point communications systems.

Optimum design of a cooperative network requires an accurate understanding of all factors affecting performance. In order to parameterize the performance of cooperative systems, this thesis introduces mathematical models for different performance metrics, such as symbol error probability, outage probability and random coding error exponent, in order to analytically estimate network capacity.

A dual-hop network is introduced as the most basic type of relay network. Random coding error exponent results have been obtained using this simple network model are presented along with corresponding channel capacity estimates based on the assumption of Gaussian input codes. Next, a general multihop network error and outage performance model are developed.

Detailed mathematical and statistical models for interference relay networks are presented. The basic statistical parameters, cumulative distribution function and probability density function for interference cooperative dual hop relay networks are derived and explored. A partial formulation for the random coding error exponent (RCEE) result is also presented.

Simulation results over Rayleigh and Nakagami- m fading channel models are included in each chapter for all of the selected performance metrics in order to

validate the theoretical analysis, under the assumption that channels are flat over the duration of one symbol transmission. These results are in close agreement with the predictions of the analytical models.

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LIST OF ACRONYMS

SINR	Signal to interference plus noise ratio
SNR	Signal to noise ratio
SEP	Symbol error probability
CSI	Channel state information
AF	Amplify and forward
DF	Decode and forward
CF	Compress and forward
S-R	Source to relay link
R-D	Relay to destination link
PDF	Probability density function
CDF	Cumulative distribution function
MGF	Moment generating function
QOS	Quality of service
MRC	Maximal ratio combining
RCEE	Random coding error exponent
MIMO	Multiple input multiple output
i.n.i.d.	Independent and non identically distributed

LIST OF SYMBOLS

$\mathcal{C}^{m \times n}$	A $m \times n$ matrix with complex elements
$\mathbb{P}(X)$	Probability of random variable X
$f_X(x)$	Probability density function of X
$F_X(x)$	Cumulative distribution function of X
$\Phi_X(s)$	Moment generating function of X
$\mathbb{E}_X(x)$	Statistical expectation over random variable X
$I(X; Y)$	Mutual information between random variables X and Y
$K_\nu(x)$	ν th order modified Bessel's function of second kind
$H_{p,q}^{m,n} \left[x \left \begin{matrix} (a_p) \\ (b_q) \end{matrix} \right. \right]$	Fox- H function
$G_{p,q}^{m,n} \left[x \left \begin{matrix} (a_p) \\ (b_q) \end{matrix} \right. \right]$	Meijer- G function
$\Gamma(x)$	Euler's Gamma function
$\Gamma(a, x)$	The upper incomplete Gamma function
$\gamma(a, x)$	The lower incomplete Gamma function

DEDICATION

To my parents

Chapter 1

INTRODUCTION

Since the early 20th century, impressive developments in wireless communications technology have dramatically changed the way we live and communicate. This progress will continue for many more years in the future as the demand for wireless connectivity to systems and devices is increasing day by day. Current wireless technology not only provides connectivity for voice and video data but also redefines the way in which we interact with technology. Extensive progress in research has resulted in many improvements in the data rate of communication channels and mobile devices. Progress in VLSI/ULSI technology has transformed desktop computers into a mobile hand-held devices such as cellular smart phones and tablets that can run complex applications similar to a desktop PC. Such devices creates huge data traffic for each application that they run; since many phone applications store their data via Internet cloud platforms. Even though, many new efficient network protocols, modulation and coding schemes have improved network data rates and the quality of service significantly, coping with the ever increasing data rate demand is still a challenge.

1.1 BACKGROUND

During the last decade, the development of multiple antenna systems is probably the most important research achievement to meet the future data rate challenge. Multiple-input-multiple-output (MIMO) system create virtual parallel data channels between source and destination, thus increasing network capacity. Interestingly, some degrees of freedom provided by multi-antenna systems can alternatively be used to increase the channel diversity of the system as well as

capacity. MIMO systems were first implemented in wireless local area networks (WLAN) in the IEEE 802.11n standard and then in 4G cellular networks. Nevertheless, implementing MIMO system in smaller cellular mobile devices is not easy. Particularly, a minimum half wavelength of antenna separation is required (depending on the implementation scheme) to fully realize the benefits of MIMO. To solve this problem, another technique has evolved in which a virtual antenna array is created in space rather than with the device itself. Wireless channels are broadcast in nature, i.e. the wireless devices broadcast their information to neighboring users. Utilizing this broadcast phenomenon, a cooperative communication scheme is introduced that can guarantee higher quality of service (higher throughput, lower error or outage probability) in wireless networks. In cooperative networks, users can help each other by amplifying and retransmitting signals. Independent wireless networks, such as ad-hoc and wireless sensor networks can also use user cooperation strategy to increase the reliability of data transmission¹. In addition to user cooperation, dedicated relay nodes can be used to forward data as well.

1.1.1 *Wireless Channels*

The physical wireless communication channel encounters many difficulties during transmission of wireless signals. Two different types of fading exist in wireless channels. Large-scale fading is introduced due to the path loss factor and occurs mainly as a result of the distance the signal travels to reach the receiver. Large obstacles that cause a shadowing effect also contribute to large-scale fading. By contrast, small-scale fading involves rapid fluctuations in received signal over a short distance and time. Random reflectors and scatterers in the propagation path of the radio channel produce multiple copies of the original signal. These

¹we will use the term reliability to denote a state where the probability of error or outage is sufficiently low.

multiple copies arrive at the receiver at different times and with different phase shifts, causing attenuation of the signal. Small-scale fading is the key fading problem in wireless communication systems. Many channel models has been proposed over the past few decades to characterize signal propagation for indoor and outdoor mobile and other wireless systems. Statistical channel models, such as Rayleigh and Nakagami- m fading models are commonly used for urban mobile channel models, while Rician models are appropriate when a dominant direct link between the transmitter and the receiver exists [1, 2]. For mobile systems operating around the frequency bands of 800 and 900 MHz, Weibull fading models are used for indoor and outdoor environments [3].

1.1.2 Cooperative Communication

Since wireless channels often encounter deep fades and random fluctuations due to multipath fading, the reliability of such communication channel can be problematic. Diversity schemes can improve network reliability by introducing several independent versions of the same signal that are combined at the receiver using different combining techniques. Depending on the system design there are many diversity schemes, such as time diversity, frequency diversity, spatial diversity and recently introduced cooperative diversity. Cooperative diversity schemes can be seen as an extension of spatial diversity systems, where distributed antennas are placed in relay nodes distributed in space as compared to a single multi-antenna source or receiver in conventional spatial diversity systems. Such cooperative communication systems improve the diversity gain significantly [4]. This distributed antenna fashion relaxes the design requirements of many communication devices, since cellular phone devices are too small to place realistically multiple antennas in them. Other benefits of cooperative schemes are, a reduced the requirement of high transmission power in a single node, thus reducing the interference level in the network, and extended network coverage achieved when

nodes are deployed on the edge or near dead spots of cellular coverage area.

1.1.3 Wireless System Performance Measures

Due to the uncertainty involved in communication channels, system performance modeling is crucial for any wireless system in order to estimate the system performance and reliability before implementation. Depending on the system design and the environment where the system is going to be used, the performance of the system will vary. Research for wireless systems with greater reliability and quality of service (QOS) has been ongoing for many years. The channel target data rate has a nonlinear relationship with error or outage probability. The probability of error increases exponentially for any rate beyond the physical limit of the communication channel, the channel capacity. Thus, if the target data rate is below the channel capacity we can achieve it with an arbitrarily low error rate using different modulation and/or channel coding schemes. Performance measures such as the bit error probability (BEP), outage probability (OP) and the capacity of a communication system over wireless channel models are the key focus of research. Even though there are many system and channel parameters which are responsible for the performance of the system, most of performance measures can be represented simply as a function of receiver signal to noise (SNR) in noise limited system or the signal to interference plus noise ratio (SINR) in interference network. Here, we define the symbol error probability (SEP)², OP and capacity terms are defined as below:

Bit Error Probability: The probability of bit error is defined as the probability that a bit x_s is sent and the decoded receiver decision is \hat{x} , such that $\hat{x} \neq x_s$. In general, if we have $|\mathcal{X}|$ number of possible bit source,

²SEP defines the probability of error when a data symbol is sent through the channel. A data symbol usually consists of a number of bits, while practically an M -bit code symbol represents one of 2^M number of possible index codes.

$$P_e = \sum_{i \in \mathcal{X}} \mathbb{P}(e|x_i) \mathbb{P}(x_i) \quad (1.1)$$

where $\mathbb{P}(e|x_i) = \mathbb{P}(\hat{x} = x_j | x_j \neq x_i)$. \mathcal{X} is the sample space of the transmitting bits. The BEP or SEP can generally be defined as a nonlinear function of receiver SNR or SINR for any wireless network. The conditional BEP under some fading channels defined as is:

$$\mathbb{P}(e|\gamma) = \int_{\xi_1}^{\xi_2} Ch(\xi) \exp(-ag(\xi)\gamma) d\xi \quad (1.2)$$

where C and a are arbitrary constants and $h(\xi)$ and $g(\xi)$ are arbitrary functions of integrand variables [3]. Our concern is to formulate the average probability of error, which can be obtained by integrating the conditional BEP with the SNR probability function, i.e.,

$$\bar{P}_e = \int_{\gamma} \mathbb{P}(e|\gamma) f_{\gamma}(\gamma) d\gamma \quad (1.3)$$

$f_{\gamma}(\gamma)$ is the probability density function (PDF) of the instantaneous receiver SNR or SINR. Later in this thesis, the term error probability will be used to indicate the SEP. A moment generating function (MGF) based approach will be used to express the SEP results over M -ary PSK signals [3]:

$$\mathbb{P}(e|\gamma) = \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \exp\left(-\frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\theta} \gamma\right) d\theta \quad (1.4)$$

$P_{e|\gamma}$ is the conditional SEP. Now, taking the average over the channel fading PDF, the average SEP is given by,

$$\begin{aligned}
\overline{P}_e &= \frac{1}{\pi} \int_0^\infty \int_0^{\pi - \frac{\pi}{M}} \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma \right) f_\gamma(\gamma) d\theta d\gamma \\
&= \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \Phi_\gamma(s) d\theta
\end{aligned} \tag{1.5}$$

where $s = \frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta}$. $\Phi_\gamma(s)$ is the MGF of instantaneous SNR γ defined as, $\Phi_\gamma(s) \triangleq \mathbb{E}[e^{-s\gamma}]$. \mathbb{E} stands for the statistical expectation operation.

Outage Probability: An outage event generally defined as an event when a system fails to maintain reliable communication between the source and sink. Since the received SNR/SINR characterizes the channel quality, outage probability will be defined in terms of SNR and SINR. The outage probability of a system is therefore defined as the probability that the instantaneous SNR or SINR falls below a predefined threshold SNR and SINR γ_{th} [3]. However, in some literature it is defined as a function of target rate R , i.e. the probability that the data rate of a channel drops below the target rate [5]. According to this definition,

$$\begin{aligned}
P_{\text{out}} &= \mathbb{P}(\gamma \leq \gamma_{\text{th}}) = \int_0^{\gamma_{\text{th}}} f_\gamma(\gamma) d\gamma \\
&= F_\gamma(\gamma_{\text{th}})
\end{aligned} \tag{1.6}$$

where $F_\gamma(\gamma_{\text{th}})$ is the CDF of γ evaluated at γ_{th} . Thus, the outage probability of a network can be obtained from the CDF expressed in terms of receiver SNR.

Capacity: Channel capacity is mainly studied in the context information theory. If a signal x is sent over a communication channel and is the signal received y at the receiver, the capacity of a channel is defined as the supremum of all achievable rates R such that,

$$C = \sup_{f_X(x)} I(X; Y) \quad (1.7)$$

where $I(X; Y)$ is the mutual information between transmitting and receiving signal space random variables X and Y respectively [6]. In particular, the output random variable Y depends on source random variable X and the communication channel is given by the conditional probability function $f_{Y|X}(y|x)$. Shannon provided theoretical capacity results for an AWGN channel that serves a benchmark for practical system designs and provides information about the achievable spectral efficiency in a system. The capacity over fading channels is given by,

$$C = \int_{\gamma} B \log(1 + \gamma) f_{\gamma}(\gamma) d\gamma, \quad \gamma \geq 0 \quad (1.8)$$

where B is the channel bandwidth and $f_{\gamma}(\gamma)$ is the PDF of the receiver SNR [7]. An interesting procedure using random coding error exponent (RCEE) to derive channel capacity was proposed by Gallager in [8], that provides a lower bound on the Shannon error exponent or reliability function described in [9]. In this thesis, we will use RCEE procedure to analyze the capacity of network.

1.2 KEY CONTRIBUTIONS

In this thesis, several physical layer performance metrics will be derived, and simulation results will be presented to compare the analytical derivations.

- The random coding error exponent and the capacity of a dual hop cooperative relay network using CSI assisted amplify and forward relay in a single antenna source-destination network are derived [10].
- The symbol error probability and outage probability of a multihop multi

branch network using decode and forward relay network are derived [11,12] and

- The probability density function and cumulative distribution function for a dual hop CSI assisted hypothetical gain amplify and forward interference relay network are derived. A mathematical framework and procedure for RCEE of such an interference cooperative network are also presented [13].

Detailed numerical results on the analysis are presented which verify the analytical results.

1.3 THESIS OVERVIEW

In the following chapter, a detailed literature review on the cooperative communications is presented, networks including multihop and interference cooperative network. Detailed published results on this area verify the contribution presented in this thesis work.

In Chapter 3, a cooperative dual hop network, the most basic structure for cooperative networks is introduced. The RCEE of such a network using a CSI assisted AF relay in single source-destination network is derived [10]. To complete the understanding for such a network, some published results from other authors on error and outage performance are included for comparison.

In Chapter 4, the performance of a general multihop multi-branch network is analyzed. The statistical symbol error probability and outage probability result for a network with arbitrary number of hops and branches using decode and forward relaying protocol are derived [11,12].

In Chapter 5, a mathematical model for cooperative interference networks is derived. A single relay network where an arbitrary number of interferers affect both the relay and the destination node is considered. The probability density function and the cumulation distribution function for such a network are derived.

Finally, a mathematical procedure is developed which used this statistical results to prove RCEE of interference relay networks [13].

1.4 PUBLISHED WORKS

[1] Bappi Barua, Mehran Abolhasan, Daniel R. Franklin and Farzad Safaei, “Outage Probability Analysis of Dual Hop Relay Networks in Presence of Interference” Submitted to the *IEEE Transactions on Vehicular Technology*, July 2012

[2] Bappi Barua, Mehran Abolhasan, Farzad Safaei and Daniel R. Franklin, “On the Error Exponent of Amplify and Forward Relay Networks” *IEEE Communications Letters*, vol.15, no.10, pp.1047-1049, October 2011

[3] Bappi Barua, Mehran Abolhasana and Farzad Safaei, “On the Symbol Error Probability of Multihop Parallel Relay Networks” *IEEE Communications Letters*, vol.15, no.7, pp.719-721, July 2011

[4] Bappi Barua, Farzad Safaei and Mehran Abolhasan, “On the Outage of Multihop Parallel Relay Networks” in Proc. IEEE 72nd Vehicular Technology Conference Fall (VTC 2010-Fall), Ottawa, CA 6-9 Sept. 2010, pp.1-5

Chapter 2

LITERATURE REVIEW

Relaying techniques have been used in microwave wireless systems to carry radio signals over long distances since the middle of the 20th century. However, recent research shows that relaying strategies, when used in a quite different way than in traditional microwave relaying, can improve the reliability of wireless systems that are perturbed by drastic multipath fading and interference over the communication channel. It has been shown that a cooperative relay communication technique can mitigate the fading effects significantly in communication channels. In cooperative communications, certain users or dedicated relay nodes overhear other users signals and then try to retransmit the signals to the desired receivers using a forwarding strategy. Fig. 2.1 shows such a cooperation strategy, where two users U_1 and U_2 are trying to communicate to their designated receivers D_1 and D_1 respectively. Due to the broadcast nature of the wireless medium, user 2, U_2 overhears the transmission of U_1 , then using a predefined relaying protocol U_2 will retransmit U_1 's signal. This cooperation between users (or

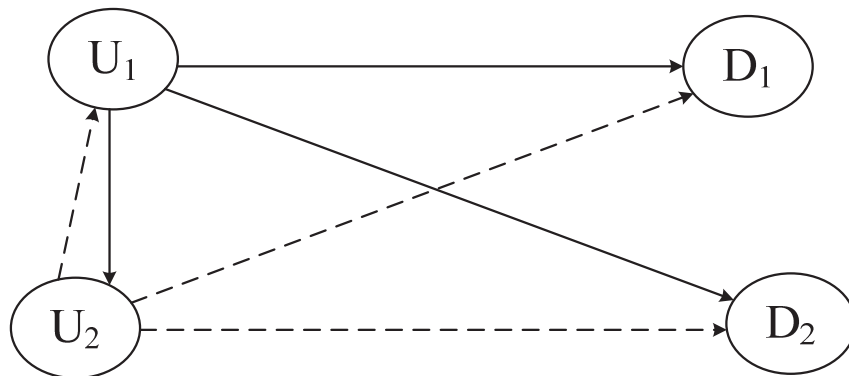


Figure 2.1: User cooperation cooperative network.

by relays) creates multiple independent virtual sources transmitting the same information as the source node, thus reducing the probability of error when making decisions about the information bits at the receiver. In this thesis, it is assumed that all the cooperating nodes are dedicated relaying nodes by default.

Depending on the network configuration and available resources, three main relaying schemes have been proposed in cooperative networks: amplify-and-forward (AF), decode-and-forward (DF), compress and forward (CF) [14, 15]. However, amplify-and-forward and decode-and-forward are the most widely used protocols in practice. In AF relaying, relays simply scale the received signal by a predefined gain factor and retransmit it. Thus AF relays have relatively less complex circuitry and cost. However, as simple repeaters, AF relays are well known for transmitting an amplified version of noise signals over the communication channel since it amplifies the noise parts as well as the signal during the scaling process. In contrast to amplify-and-forward relaying, decode-and-forward scheme limits the propagation of noise at the expense of circuit complexity. However, if the signal is commuted over multihop paths, DF relays propagate the errors over the network.

In the following sections, the literature related to cooperative relay networks will be starting with basic dual hop cooperative relay networks followed by cooperative multihop networks and interference networks.

2.1 COOPERATIVE COMMUNICATIONS

The very first classical relay channel was introduced by van der Meulen in his PhD thesis in 1971 [16,17]. Cover and Gamal extended the three terminal model of van der Meulen for discrete memoryless and Gaussian degraded relay channels assumptions [18]. Past research efforts focusing on dual hop cooperative relay network are extensively discussed in the literature such as [4, 14, 18–53] and references therein. Dual hop relaying is the basic structure in relaying technique,

can be seen as a bent satellite communication between two nodes.

Characterization of relay network channel capacity has been an open problem in information theory for almost half a century. References [17–23] mainly focus on relay channel capacity bounds of such a dual hop network. Gaussian relay network has been analyzed in [17–19]. The capacity and achievable rate of Gaussian degraded general relay channel with one relay node with and without feedback has been studied in [18]. Later the work has extended the network for two relay nodes and shows the upper and lower bound on channel capacity on such a relay network [19]. Authors in [20–22] have studied the relay channel in a fading environment, introducing the capacity and achievable rate bounds analysis for dual to multihop relay networks in [20], while [21] and [22] developed capacity bounds on dual hop parallel relay network models. Moreover, [22] studies the resource allocation problem maximizing the data rate in relay network. It is shown that resource allocation plays a very important role in capacity optimization when S-R and R-D channels are quite asymmetric.

Other than information theoretic analysis the statistical performance of dual hop networks have been studied [14, 24–28]. The authors in [14] have developed cooperative communication strategies and performed diversity analysis of a basic two source and two corresponding receiver network, where the sources help each other by relaying the information. The diversity of a user cooperative system using AF and DF relaying strategies has been studied and shows that with the exception of the fixed DF protocol (where DF relays decode, re-encode and forward the signals without deciding on feedback) all relaying protocols i.e. fixed AF, selection and incremental DF and AF protocols achieve full diversity. An AF scaling gain G has been applied at the relay such that [14],

$$G^2 = \frac{P_R}{P_S |h_1|^2 + \sigma_1^2} \quad (2.1)$$

where P_S and P_R are the source and the relay power respectively, h_1 is the channel gain of source-relay (S-R) hop and σ_1^2 is the noise spectral density at the relay node. This relay gain is also called CSI assisted AF relay gain since the relay node requires to estimate the instantaneous channel information of the S-R channel. Gain G provides proper amplification to the relay to counter the effect of the channel fading and prevents the relay gain from saturating when the backward relay channel undergoes deep fade. This relay gain has become the basis for most AF relaying network models. However, due to the noise variance term in the AF gain expression, the mathematical complexity is greatly increased, making it difficult to provide closed form PDF and CDF expressions for receiver SNR for many AF cooperative relay networks. In [24] the authors have modified the CSI assisted AF relay gain of [14] removing the denominator noise parameter and denoting it hypothetical/ideal gain, although it still requires the instantaneous CSI of backward relay channel S-R. Avoiding the noise density in the gain parameter allows the authors to express the PDF and CDF of the end-to-end SNR function in a more mathematically tractable form for many cooperative networks. Interestingly, the end-to-end SNR expression takes the form of a harmonic mean of two random variables when using hypothetical relay gain in dual hop networks. The authors derive the PDF, CDF and the MGF of the receiver SNR for the dual hop network. Furthermore, the closed mathematical expression of average symbol error probability (SEP) and outage probability based on the proposed hypothetical gain have also been derived. In subsequent work, the authors have extended the idea of AF relaying to fixed AF and semi-blind AF relaying¹ [25]. In fixed AF, the scaling to the received signal is performed using a fixed constant, thus the relay does not need to measure the instantaneous CSI, whereas, in semi-blind it is assumed that the AF relay has

¹A semi-blind relay refers to AF relays that have average knowledge of the fading channels instead of instantaneous channel knowledge as in CSI assisted AF relays.

some average knowledge of the S-R channel. These two strategies make amplify-and-forward relays very simple and easy implementable without sacrificing much in performance. In [26], the authors have extended the idea of [24] in Nakagami- m fading channels.

2.1.1 Resource Allocation and Relay Selection

Resource allocation protocols have been studied in [40–44] and shown to be a very effective tool to optimize network performance and quality of service (QoS) in cooperative networks. It can ensure longer network lifetime and keeps the interference level lower in the network. However, pursuing an optimization protocol is not the only way to optimize the network performance; other strategies such as selecting the best relay node and the use of beamforming techniques can help the network operate with optimum performance. Depending on the available CSI at the relay node, or using a centralized cooperating node with total network CSI, relay selection schemes can perform better over networks employing a resource optimization scheme. Nevertheless, these two strategies are not mutually exclusive and can be used together in a network. Many interesting relay selection schemes are studied in [4, 28–33, 44]. The most interesting one is discussed in [4], where Bletsas proposed the idea of totally distributed relay selection, i.e. the network does not require centralized CSI to select the best relay. Relays only need to estimate their forward and backward channels. This idea is more practical than resource allocation and beamforming schemes where a full CSI of the total network is essential to implement the protocols. In later work, Bletsas shows that such an opportunistic relaying (best relay selection) scheme can provide the optimum outage performance over multiple relay operation scheme [29]. In a comparison study of relay selection with resource allocation strategies the authors have shown that relay selection schemes outperform the optimal power allocation strategy in terms of system throughput and outage probability [44].

Assuming that the forward relay channel state information is not available at the relay nodes, a partial relay selection scheme is proposed in [30–32] where best relay is selected depending on the backward relay channels of the network. CSI assisted hypothetical relays are used in [30] with Rayleigh fading channels, while [31] and [32] use semi-blind AF relays assuming Rayleigh and Nakagami- m fading channels respectively. In [54], authors derive the high SNR approximations for the error probability of a dual hop cooperative AF relay network for best relay selection and partial relay selection schemes in presence of noise and interference. All the channels in the network (main and the interfering channels) are assumed to be independent and non-identically distributed Rayleigh fading channels. An optimal power allocation algorithm also proposed minimizing the asymptotic error probability under total power constraint. A relay subset selection strategy is also discussed showing that relay subset selection enables significant reduction in feedback signaling overhead at an expense of little loss in performance. Derived results show that the diversity gains of best relay selection and partial relay selection are independent of the type of noise but their SNR gains do depend on the type of noise.

Compared to relay selection and resource allocation, beamforming strategy studied in [35–39, 55] have quite different characteristics and capabilities. Beamforming allows simultaneous transmission in multiple relay networks without introducing interference and hence can reduce total power use in the network. In practice, if the channel state information of the forward channels are already known, beamforming schemes can be implemented together with resource allocation or relay selection strategy to further optimize the network [36]. Beamforming schemes usually require instantaneous forward channel knowledge in order to apply the beamforming weights at the transmitting nodes, and in fact, networks with multiple antenna nodes can be very efficient if they implement a beamforming strategy [38, 55]. In [35] a beamforming vector is applied at the source node

only in a multiple antenna source-destination system with a single antenna relay. The receiver combines the signals using MRC weights at the receiver. Closed-form expressions for the outage probability, MGF and generalized moments of the end-to-end SNR is derived. The derived expressions is also applicable to general operating scenarios with distinct Nakagami- m fading parameters and average SNRs between the hops. In [38], the authors have studied a multiple antenna source, receiver and AF relay system using a Grassmannian beamforming vector. Optimal source and relay beamforming vectors were derived maximizing the receiver SNR. In [55] the authors have used a multiple antenna source with multiple multi-antenna AF relays and a single antenna receiver. An N_T (number of transmit antennas) dimensional source beamforming vector and optimal beamforming matrices for amplify-and-forward relays has been derived for individual and joint relay power constraints.

Regarding the problem of how to maximize diversity and throughput, several researchers have applied the idea of space-time block codes (STBC) in distributed systems. STBC codes were first introduced by Alamouti in [56] in a single hop multi-antenna transmitter and single antenna receiver system. The idea quite elegant and achieve transmit diversity in a system without requiring the knowledge of CSI at the transmitter side. The transmitter, in practice, transmits a specially designed code that is orthogonally separated in space and time. A receiver with full CSI knowledge can combine this signal and achieve diversity equal to the of number of transmit antennas, although the system loses spectral efficiency due to the use of multiple time slots for transmitting the code matrix. In a cooperative network, where the relays implement multiple antenna systems in a distributed fashion, there are some significant challenges in implementation. Firstly, a STBC requires a set of transmit antennas to transmit its code column at different time indices. Thus the distributed system will need to ensure that a fixed number of transmitting nodes will participate in relaying. Secondly, the

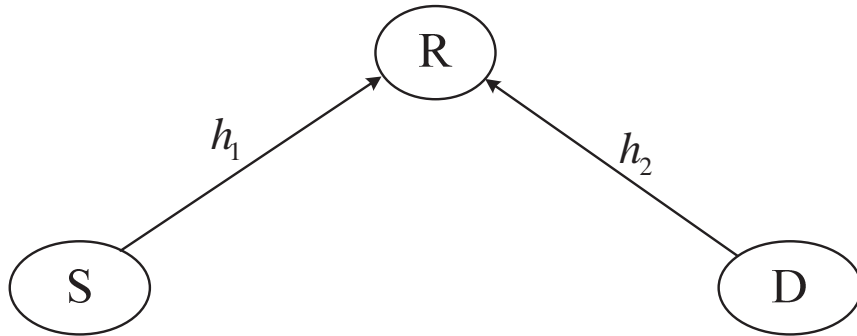


Figure 2.2: Two-way relaying communication phase 1.

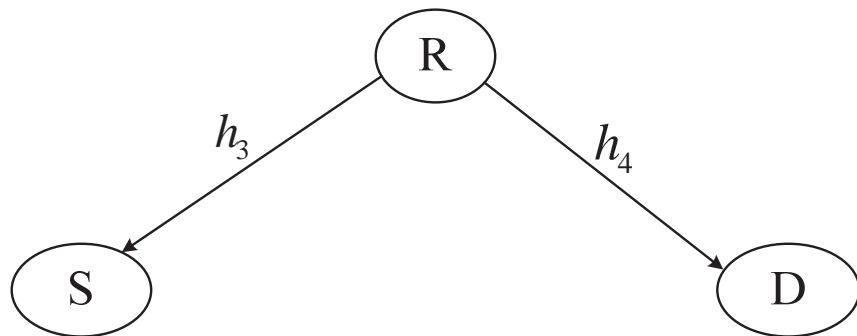


Figure 2.3: Two-way relaying communication phase 2.

problem of synchronization at the receiver; as the relay nodes are not located in single area or may be mobile, synchronizing the received signal that from same code block may prove a challenging issue [48]. In some literatures authors used linear dispersion space-time codes at the relay nodes to achieve the full diversity in Rayleigh fading channels [49, 50].

A very attractive relaying protocol ‘two-way relaying’ using half duplex communication has been studied in [53, 57–59] and enjoys significant support in the research community. The idea of bidirectional communication was first introduced by Shannon in [60]. By exploiting the broadcast nature of the wireless medium and the knowledge of terminal’s own message, the information exchanging phases between the relay and the source nodes can be reduced by half. Fig-

ures 2.2 and 2.3 show the two way relaying technique. In transmission phase 1, both the source and destination transmit their information simultaneously to the relay, and in phase 2 the relay amplify the received signal and broadcast the information.

2.1.2 Random Coding Error Exponent

At the beginning of this section several papers from information theoretic context that studied capacity bounds for cooperative networks were cited. However, a capacity result for cooperative relay networks is still an open problem in information theory. In this thesis, capacity analysis in a different way will be approached. In the analysis of system capacity, Shannon defines a reliability function or error exponent to describe the probability of the error as a function of code rate R and code length W as,

$$E(R) \triangleq \lim_{W \rightarrow \infty} \sup \frac{-\ln P_e^{\text{opt}}(R, W)}{W} \quad (2.2)$$

where $P_e^{\text{opt}}(R, W)$ is the average block error probability for the optimal block code of length W and rate R [9]. In practice, derivation of exact error exponent (1.3) involves quite complex mathematical procedures, however, a lower bound on the error exponent known as random coding error exponent (RCEE), (defined in [8,61]) exists. This RCEE measurement provides important information about the design requirements of a codeword to achieve a given target rate R below the capacity C of the channel at an affordable probability of error. In particular, important capacity terms such as the ergodic capacity, cut-off rate and the critical rate of a network can derived utilizing the RCEE expression [8]. Recently, [52] and [53] studied the RCEE for cooperative systems with CSI assisted ideal gain AF relays and derive ergodic capacity and cutoff rate for the network.

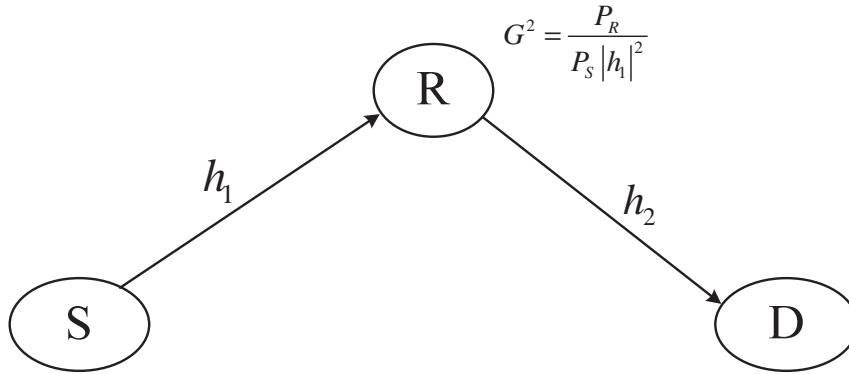


Figure 2.4: Dual-hop cooperative relay network.

Furthermore, in [53] the authors use two way relay network and obtain optimal power allocation that maximizes the error exponent function. A relay gain $G = \sqrt{\frac{P_R}{P_s |h_1|^2 + P_d |h_2|^2 + N_0}}$ is considered, where p_s and p_d are the transmission power of the source and destination nodes respectively. P_R is the relay's transmission power. h_1 and h_2 are fading channels from source to relay (S-R) and destination to relay (D-R) respectively as in figure 2.3, and N_0 is the AWGN noise at the relay node. The AF relay used here can be considered as an half duplex instantaneous CSI assisted relay since it uses the instantaneous fading channel gain of both S-R and D-R channels. In addition the fading channels are assumed reciprocal in this thesis, thus $h_1 = h_3$ and $h_2 = h_4$ in figure 2.3. A dual-hop communication system with hypothetical AF gain relay as figure 2.4 is studied in [52]. Direct link between source and destination is not considered. Authors derive the random coding error exponent of the network assuming Nakagami- m fading channels. The critical and cut-off rate expressions are also derived from the basic random coding error exponent expression. We will use the knowledge of these works to study the RCEE of an one way relay network in Chapter 3 of this thesis.

Literature discussed above illustrates the most significant published works on dual hop cooperative relay networks in different areas. Our interest is to analyze

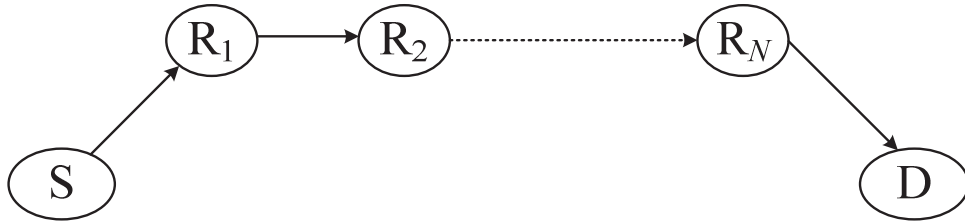


Figure 2.5: Multihop series relay network.

the random coding error exponent of the network. In Chapter 3, expressions for the RCEE for dual hop cooperative network using CSI assisted relay gain are derived. Compared to work [52], where the authors use a CSI assisted hypothetical AF relay to avoid mathematical complexity in RCEE derivation, a CSI assisted AF relay gain (2.1) will be used in this thesis.

2.2 COMMUNICATION OVER MULTIPLE HOPS

Communication over multiple hops frequently occurs in ad-hoc and sensor network, where signals need to be relayed via many intermediate nodes. In cellular networks, this multihop communication may happen when a temporary extension of a network is required without substantial infrastructure development or to bring network dead-spots (such as underground shopping malls or tunnels) into the network coverage zone. Considerable research has been undertaken regarding physical layer problems in multihop cooperative networks [11, 12, 62–76].

2.2.1 Multihop Series Relay Communication

Statistical outage analysis on a very basic multihop network, where transmission occurs via N serial nonregenerative relay nodes as figure 2.5 has been performed in [62]. A limited closed form expression of the MGF of the end-to-end SNR with relays that use the inverted channel gain (CSI assisted hypothetical gain) of the preceding channel for scaling is presented. In particular, the inverted channel gain

is practically convenient for large networks of serial relays or as an approximation for high SNR conditions. Using a different approach, Karagiannidis studied the series network considering the well known inequality of harmonic and geometric mean to simplify the end SNR [63, 64, 77]. In fact, this upper bound on SNR transforms the statistical sum of the random variables into a problem of the product of random variables. Average error probability and outage probability has been derived. However, the numerical results show that, the bound becomes loose as the SNR increases. More recently, BER (bit error rate) of K parallel path (KPP) networks with diversity branch have been investigated using an MGF based approach with AF relaying in generalized fading channels [68, 72]. However, the closed form expressions for the CDF or PDF of the end-to-end SNR are not presented. In particular, exact expressions of the PDF and CDF of the end-to-end SNR involve inverse Laplace transform in a mathematical intractable form, which usually confines authors to evaluating the performance numerically rather than expressing in closed form [62, 68].

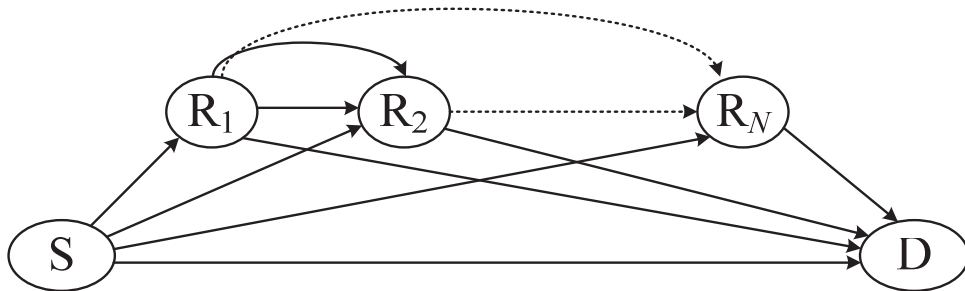


Figure 2.6: Multihop series relay network with diversity links.

A multihop network with diversity links has been studied in [67]. The outage and error probability upper bounds for multihop networks using DF and AF relays is derived. Four basic network models has been considered: AF multihop networks, DF multihop networks, AF relaying in multihop diversity channels and DF relaying in multihop diversity channels (Fig.2.6). It shows multihop

diversity networks outperform multihop series type networks. Also, AF networks perform better than DF networks. A feedback and feed-forward interference in the multihop network has also been discussed. The end-to-end SINR expressions of AF and DF multihop diversity networks provided here will help us to derive some basic equations of interference relay networks discussed in Chapter 5 in this thesis. However, statistical results such as, the PDF and the CDF of the receiver SNR and SINR (signal to interference plus noise ratio) are not provided, which are very interesting for analysis of the network performances with different fading channels. Statistical properties, the PDF and the CDF of a dual hop network in interference will be derived in Chapter 5 of this thesis.

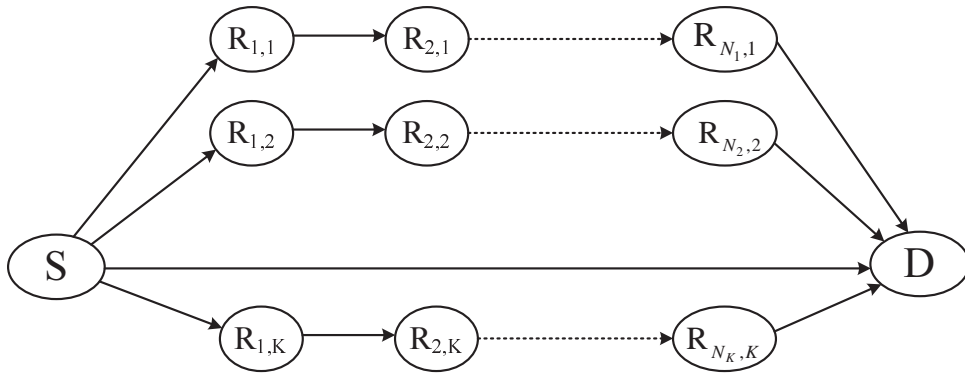


Figure 2.7: Multihop multi-branch relay network with diversity link.

2.2.2 Multihop Multi-branch Relay Communication

Second category of multihop relay networks consists of multiple branches to carry the information message in a parallel fashion and each branch has a series of relaying nodes (Fig.2.7). Such communication networks with multiple parallel channels in multihop have been studied in [11, 12, 68–72]. Relaying layers with multiple relays using both single and multiple antennas in a flat Rayleigh fading channel have been discussed in [70] and [71] respectively, for both DF and AF relaying protocols. Authors in [70] analyze the diversity of a multihop series

relay network of $N - 1$ layers with fixed network size at high SNR. The upper bound on diversity multiplexing tradeoff has been derived for clustered DF and non-clustered AF relaying. Results show, in non-clustered network AF relays can achieve the maximum multiplexing gain. Furthermore, a flip-and-forward (FF) relaying strategy has been proposed that achieves both the maximum diversity and multiplexing gains in a distributed multihop networks of arbitrary network size. Application of distributed STBC is a very interesting problem in cooperative wireless networks, however, the lack of node cooperation realizability and channels subject to fast fading process may raise challenges in implementation. Using STBCs, the authors in [71] have studied a multihop network for both flat and frequency-selective fading channels. The mathematical analysis has been provided for dual hop scenarios only. Since multihop parallel networks can offer multiplexing gains as well as diversity gain, high SNR diversity multiplexing tradeoff (DMT) behavior in such networks has been investigated in [69, 70, 72] and in multihop MIMO network in [70]. In [69], authors show that a multiple antenna source-destination network with k layers of relaying hops can achieve a degree of freedom equal to the number of source antenna at asymptotic SNR when the number of layers is fixed and each relaying layer has a number of relays which is similar to the number of source node antennas. DMT results using many different parallel path and layered networks has been presented in [72] showing that a regular (K, N) network (K parallel path and N hops) can achieve a diversity of $K(1 - r)$, where r is the multiplexing gain defined as for any target rate R , $R = r \log(\text{SNR})$.

Considering diversity link in a general multihop multi-branch network, a general MGF based performance analysis has been performed in [68]. CSI assisted hypothetical AF relays has been used which is practical for large networks reducing the complexity in end-to-end SNR expression. Authors provide results on outage probability and average error probability in integral form using MGF-

based approach. However, the MGF integral has not been solved for end-to-end receiver SNR, which is mathematically quite complex; instead simulation results for SEP and outage are presented. The authors develop the MGF model for many different fading channels, such as Rayleigh, Nakagami- m , n , q , LogNormal, Weibull and composite of Gamma and LogNormal fading models. The analysis provides a mathematical framework to study a similar network in this thesis. In Chapter 4, we develop mathematical performance model for multihop-multi-branch network Fig.2.6 without the diversity link. We use selective DF relays instead of hypothetical AF relays in [68].

2.2.3 Opportunistic Relaying in Multihop Relay Communication

In this subsection we will discuss different relay and optimum route selection results in short. In multihop networks relay selection problem can be seen as best route selection in large networks. Different selection protocols for multihop networks have been widely discussed in the context of network layer routing problems in ad-hoc [78] or sensor networks [79]. Some literature [28, 73, 80, 81] has investigated this as an optimal relay selection problem. Optimal selection of relays with the objective of maximizing the SNR [28], minimizing SEP [80] or maximizing bandwidth efficiency with optimal diversity [81] has been considered by many authors in dual hop networks. In [28], authors derive the closed form SEP of a dual hop multi-branch relay network. The best relaying path is being selected that provides maximum receiver SNR at the destination. The authors in [73] study an optimal path selection optimizing outage metric in selective DF relay networks. It shows that the optimal route will achieve full diversity in a centralized network. However, to implement such a network optimization protocol in a large cooperative network will require huge CSI feedback overhead. Taking that feedback problem into account, it also proposes an ad-hoc routing protocol where relays selected at every hop using local channel knowledge. Result

shows that the proposed ad-hoc routing protocol can achieve full diversity as well as optimal routing, however, compared with optimal routing outage performance degrades linearly with the number of hops. In this thesis, we will not develop any best relay selection protocols, however the results of Chapter 4 can be used to design a best relay/route selection strategy as future work.

2.3 COMMUNICATION IN INTERFERENCE

Wireless networks by their nature generate interference due to multipath propagation. Such interference produces multiple replicas of the same signal at the receiver, and a well-designed receiver can add up all the multipath signals. However, the term interference is usually used for the disturbing signals generated by other users operating in the same communication channel. In practice, users in multiuser network quite often share the same channel with others and the receiver intelligently separates the signals. Also, unlicensed networks, such as, WiFi and bluetooth networks can introduce interference to each other as well as in wideband communication systems operating in the same area. Considering the practical implications, interference modeling in wireless networks is a very interesting problem. Interference induced in networks, termed the cochannel interference, depends mainly on physical factors, such as the spatial distribution of interferers, interfering channel fading and the power of the interferers.

A general mathematical framework has been developed in [82–84] which considers a network in which interfering nodes are scattered according to a Poisson process and operate asynchronously in the wireless environment (Fig.2.8). A probabilistic characterization of the interference in a network generated by the other nodes has been developed in [82]. It also studies the interference model for cognitive radio networks, characterizing the statistical distribution of the network interference generated by the secondary users and analyzes the effect of the wireless propagation characteristics on such distribution. Finally statistical

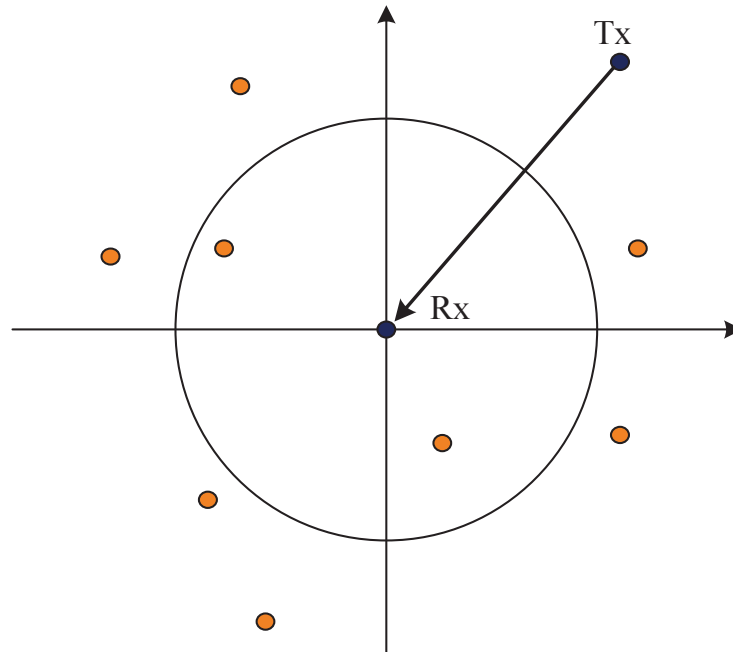


Figure 2.8: Interference in wireless network. Tx and Rx are the transmitter and the receiver respectively, and all other nodes on the plane are interfering nodes. The interferers inside the circle are the only active interferers that has sufficient transmission power to contribute in interference to the receiver Rx.

distribution of the aggregate ultra-wide band interferers at any location in the two-dimensional plane is investigated to analyze the probability of error on a narrow band system subject to the aggregate UWB interference. The statistical models are developed for single source-destination network. A performance model for a similar single hop network in Fig.2.8 has been developed in [83] in presence of interference. Authors present a mathematical framework where the interferers are scattered randomly according to a spatial Poisson process. Error and outage probabilities have been presented for aggregated interference as a function of receiver SNR and interference-to-noise ratio (INR), path loss exponent, and spatial density of the interferers. In [84], the authors present the capacity analysis of the network subject to both network interference and noise. The power spectral density of aggregate interference of the network is also derived.

Statistical modeling for interference network has been performed for SIMO channel models [85–97]. The statistical models for receiver end SNR or SINR have been developed for systems using a maximal ratio combining (MRC) receiver [85–93]. These mathematical models for multiantenna receiver systems can provide a basic analytical procedures applicable to cooperative networks. However, unlike SIMO network, most cooperative networks try to avoid the interference problem by assuming orthogonal channel allocations to cooperating nodes. Cooperative networks subject to interference have been studied extensively in [98–106].

Maximal ratio combining receiver usually performs best in AWGN or fading channels, however, in interference networks a suboptimal performance is obtained compared to optimum combining (OP) receivers. While implementing optimum combining requires the CSI knowledge of the interfering channels, MRC can be used by simply estimating the signal channels only. In [85], the authors derive the PDF and the CDF of receiver signal to interference ratio (SIR) of an interference limited system for MRC over independent and identically distributed (i.i.d.) Rayleigh and Rician fading channels. The results are then compared with OP combining case. In [88] and [89] authors derive the outage probability when the interference system is under correlated Rayleigh fading channels assuming equal and unequal power interferers respectively in interference limited system. Fading behaviors for each user are assumed to be correlated with identical correlation coefficient matrices. The error performance and outage probability in equal power interference networks over Nakagami- m fading channels are studied in [92]. Systems using an optimum combining receiver have been studied in [94–97, 107]. Winters et. al. studies the error probability performance in Rayleigh fading channels using optimum combining receiver [94]. In [95], authors assume an interference limited system where the number of equal power interferers is larger than the number of receiver antennas, and derives the bit

error probability (BEP) of the networks. An exact SEP result is derived for an arbitrary number of receiver antennas and interferers in Rayleigh fading channels in [97], assuming interferers have equal power.

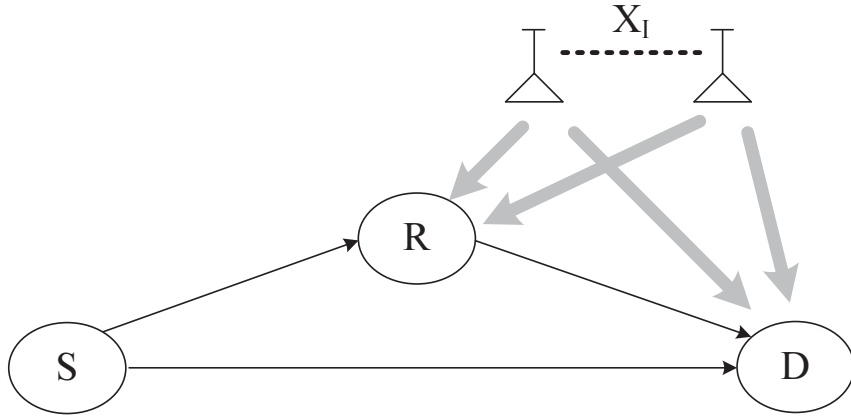


Figure 2.9: Dual hop relay network in presence of interference with diversity link.

In [98], authors has analyzed a cooperative network in interference for a single antenna source-destination network communicating via K DF relays. All the fading channels (S-R and R-D and the interfering channels) are independent and identically distributed (i.i.d.). The CDF of the received signal to interference ratio (SIR) is derived assuming an interference limited network. In [100], the authors have investigated a cooperative relay network subject to interference with one source and arbitrary number of relays and interfering nodes. The authors have concluded their study with numerical evaluations of the effect of interference on the relay network. A similar experimental performance analysis has been performed in [101]. Bit error rate (BER) performance has been studied in [102] for a multiple access relay network where all nodes are equipped with multiple antenna. A DSTBC has been used at the relay node to forward the signal. Moreover, a different transmission strategy named "IC-Relay-TDMA" has been proposed to cancel interference at the relay node. In [103], the authors have investigated the effect of cooperation in an interference-limited system utilizing

DF relaying protocol over Rayleigh fading channels. It divides the cooperating nodes into different cooperating regions and quantifies the relation between cooperative region radius and the interference level. It also analyzes the network sum rate optimization problem as a function of cooperative region radius. Relay selection strategies have been discussed in [104–106] for cooperative interference networks. When a cooperative network has CSI available at the source and at the relay nodes, a beamforming strategy can be performed. Beamforming essentially reduce the necessity of orthogonalizing the relaying channels and thus improving the spectral efficiency of the network. Furthermore, a co-transmission may be permitted in such kind of network still avoiding harmful interference.

Physical layer models for cooperative relay networks in interference has been extensively studied in previous publications [108–118]. Authors of [108] and [109] consider a scenario where only the relay node experiences interference in interference limited networks and the receiver node remains interference free. The relays here can estimate the instantaneous CSI of interfering channels to scale the gain. A fixed gain AF relay with interference limited destination is considered in [110] for Nakagami- m fading channels. The outage probability and the average BER for both interference-free and interference-limited receptions at the destination are studied. Finally an asymptotic result on outage probability also is presented. However, the assumption that the AF relay gain parameter includes the instantaneous or average channel information of interfering channels (as assumed in [108–111, 113, 115, 116]) requires additional computational capability at the relaying node, and in certain cases where the interfering signals are not known to the relay a priori, the technique can not be applied. Furthermore, in many previous results authors consider interference limited cooperative networks [113–118]. Outage performance of a dual hop network has been studied using a fixed gain relay in [113] and hypothetical gain AF relay in [114] with an arbitrary number of interferers. The system is assumed to be an interference dominated network

where noise power is negligible compared to interfering signal power. Closed form PDF and CDF of signal to interference ratio (SIR) are derived. Avoiding the noise in receiver systems usually provides an opportunity to express some of the integrals used to obtain the CDF and the PDF in very standard formats. Outage probability using DF relays in Nakagami- m fading channels is studied in [118]. The authors consider that the destination faces a negligible amount of interference, but non-negligible noise. The statistical expressions, the PDF and the CDF of the corresponding receiver SNR are derived.

Research results discussed above show that cooperative networks in presence of interference to date consider mostly dual hop systems in interference limited cases. In many results fixed gain relaying network or a CSI assisted relay with knowledge of interference channels as well as the source-relay channels has been considered. In [114], authors use hypothetical gain AF relay and derive closed form outage probability for DF and AF relaying in Rayleigh fading channels. Interference has been considered on both the relay and destination nodes with arbitrary transmit power. In this thesis, outage probability of a similar dual hop relay network using hypothetical AF relay will be presented in Chapter 5. We will consider a noise plus interference network both in relay and destination as compared to interference network in [114].

2.4 SUMMARY

In this thesis, we will present mathematical performance models of cooperative relay networks. Error exponent of a dual hop network will be studied followed by multihop parallel relay networks. Finally, performance of dual hop cooperative relay networks in presence of interference will be analyzed. In summary:

In Chapter 3 of this thesis we will present a detail analysis of random coding error exponent and capacity of a dual hop CSI assisted AF relay networks. Relevant work [52] has presented a closed form result of RCEE using hypothetical

AF relay. A CSI assisted AF relay where the relay gain parameter includes a noise variance factor that normalizes the relay transmission power preventing it to saturate when source-relay channel is in deep fade will be used in this thesis.

In Chapter 4, a general multihop multibranch relay network will be analyzed. Symbol error probability and outage probability will be derived using selective DF relays. A similar network was considered in [68] with AF relays, however the authors did not provide the closed form analytical results. Selective DF relays will be used in multihop networks. In [14] authors show that compared to fixed DF relay selective DF relay offers full diversity.

In Chapter 5, mathematical model for a cooperative relay network in presence of arbitrary number of interferers will be derived. Our interest in this thesis is to provide mathematical formulation of relay networks in interference and analyze the performance utilizing the derived analytical model. A similar network model using hypothetical AF relay gain is analyzed in [114]. However, the authors investigated an interference limited network that helped authors to use standard form of integrations deriving the PDF and the CDF of the network. In this thesis, a cooperative network with interference plus noise will be considered.

Chapter 3

COOPERATIVE RELAY NETWORK: DUAL HOP TRANSMISSION

Cooperative relay communication has been proven to provide better reliability against the multipath fading process. Depending on available CSI and allowable complexity relays retransmit the received signals utilizing different relaying schemes. amplify-and-forward is the commonly used relaying strategy due to its simplicity and ease of deployment. In this chapter, we will mainly focus our study on deriving the random coding error exponent (RCEE) of AF relay network using CSI assisted relay gain [10]. Basic results on error and outage performance for similar dual hop cooperative network from references will also be provided. Performance analysis utilizing random coding error exponent has received considerable attention in recent years. The random coding error exponent defined in [8, 61] is an exponent of a function of codeword length and data rate that imposes a tight upper bound on the probability of error. Error exponent measurement provides information about the design requirement of a codeword to achieve any target rate R below the capacity C of the channel at a specific error probability. Eventually, we can derive basic capacity results such as, the ergodic capacity, cut-off rate and the critical rate of a network utilizing the error exponent expression [8].

The most relevant result on RCEE with our network model is published in [52], where the authors have derived the RCEE using the hypothetical relay gain which is, in fact, a simplified assumption of CSI assisted power constraint relay gain factor $G^2 = \frac{P_R}{P_S|h_1|^2 + \sigma_1^2}$ proposed by [14] when σ_1^2 is set to zero. P_S and P_R are the source and the relay power respectively, and h_1 is the channel gain of

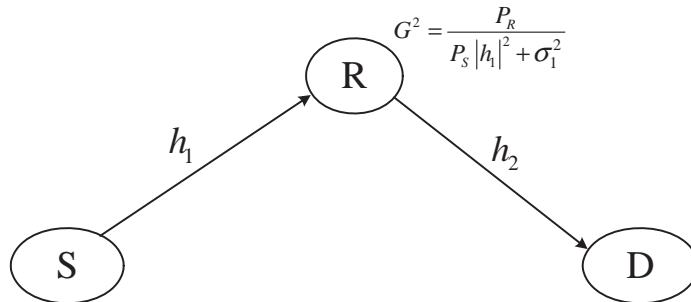


Figure 3.1: Cooperative relay network without diversity link.

Source-Relay hop. σ_1^2 is the one-sided noise spectral density at the relay node. Avoiding the noise figure in the denominator of the relaying gain factor allows authors to get the probability density function (PDF) of the receiver SNR in more mathematically tractable form that can later be used to derive the RCEE. However this assumption may not be practical in a power constraint AF relay network when the instantaneous channel gain of the source-relay hop drops very low.

3.1 SYSTEM MODEL

Consider a single source-destination pair communicating via a single antenna relay without any direct link (Fig. 3.1). We denote source-relay and relay-destination links as S-R and R-D respectively. A half duplex AF protocol is considered over independent and non identically distributed (i.n.i.d.) Rayleigh fading channels. The destination is assumed to have full channel state information (CSI) of the two main channels, S-R and R-D, while the relay has full CSI of the backward S-R channel only. The source and relay have no CSI of forwarding transmitting channels.

The instantaneous and average signal to noise ratio (SNR) of 1st and 2nd hops are denoted as $\gamma_i \triangleq \frac{P|h_i|^2}{\sigma_i^2}$ and $\lambda_i \triangleq \frac{P\Omega_i}{\sigma_i^2}$ respectively, where $i \in \{1, 2\}$, P is the corresponding source and relay power; h_i and Ω_i are the instantaneous

and average channel gain of the i th hop respectively¹ and σ_i^2 is the one sided additive white Gaussian noise (AWGN) power at relay or destination node, i.e. $i \in \{R, D\}$. We assume the total power of the network (source and relay) is constrained to P_{tot} . This total power is split between the source and the relay by a power sharing coefficient $\zeta \in (0, 1]$ such that the source and the relay powers are given by, $P_S = (1 - \zeta)P_{\text{tot}}$ and $P_R = \zeta P_{\text{tot}}$ respectively. Thus if equal power sharing strategy is adopted ζ will be 0.5.

For the desired network model as shown in Fig. 3.1 the received signal at the relay node R is

$$y_R = h_1 x_s + n_1 \quad (3.1)$$

The signal at the destination node is

$$y_D = Gh_2 h_1 x_s + Gh_2 n_1 + n_2 \quad (3.2)$$

where G is the AF relay gain. $n_1 \sim \mathcal{CN}(0, \sigma_1^2)$ and $n_2 \sim \mathcal{CN}(0, \sigma_2^2)$ are additive white Gaussian noise at the relay and destination respectively.² At the receiver node, the signal to noise ratio (SNR) will be

$$\gamma_d = \frac{G^2 |h_1|^2 |h_2|^2 P_S}{G^2 |h_2|^2 \sigma_1^2 + \sigma_2^2} \quad (3.3)$$

Using the CSI assisted relay gain $G^2 = \frac{P_R}{P_S |h_1|^2 + \sigma_1^2}$ [14], the end-to-end signal-to-noise ratio (SNR) at the receiver is given by,

¹The average channel gain, Ω_i is in fact the statistical average of the squared instantaneous channel gain h_i , i.e. $\Omega_i \triangleq \mathbb{E} [|h_i|^2]$.

² $\mathcal{CN}(\lambda, \sigma^2)$ denotes a circularly symmetric complex Gaussian random variable with mean λ and variance σ^2 .

$$\gamma_{d_1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \quad (3.4)$$

Due to the Rayleigh fading assumption, the first and second hop SNR γ_1 and γ_2 are exponentially distributed with parameter λ_1 and λ_2 respectively³. However in many literature an ideal relay gain proposed by Hasna [24] has been used AF relaying where $G^2 = \frac{P_R}{P_S |h_1|^2}$ which is quite easier to implement in many practical scenario. By simply ignoring the noise figure in the gain parameter the following statistical analysis using this gain become easier and in this case the end-to-end SNR expression is given by

$$\gamma_{d_2} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \quad (3.5)$$

Equation (3.5) provides an asymptotically tight upper bound on end-to-end SNR (3.4) and the statistical expressions the PDF, CDF and the MGF can be expressed in more mathematical tractable format to proof the error and outage probability. There are some other forms of AF relay gains also proposed in different literatures such as fixed and semi-blind relay gain as discussed in the introduction section will not be used here. In this thesis all of our discussions will be concentrating on CSI assisted relay gain only.

In the following sections we will consider the end-to-end SNR (3.4) and (3.5) to provide the performance and error exponent results.

³where, $\lambda_i = \frac{1}{\gamma_i}$; $i \in \{1, 2\}$ is the inverse of the average SNR of the corresponding (1st or 2nd) hop.

3.2 PERFORMANCE ANALYSIS: ERROR AND OUTAGE PROBABILITY

In this section we will recall some published results on the performance of a basic dual hop relay networks. For the CSI assisted receiver SNR in (3.4) the PDF and the CDF of the end-to-end SNR γ_{d_1} can be invoked from [28] respectively as,

$$f_{\gamma_{d_1}}(\gamma) = 2e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} \left[\frac{1}{\lambda_1\lambda_2} (2\gamma + 1) K_0 \left(2\sqrt{\frac{\gamma(\gamma + 1)}{\lambda_1\lambda_2}} \right) + \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \sqrt{\frac{\gamma(\gamma + 1)}{\lambda_1\lambda_2}} K_1 \left(2\sqrt{\frac{\gamma(\gamma + 1)}{\lambda_1\lambda_2}} \right) \right] \quad (3.6)$$

$$F_{\gamma_{d_1}}(\gamma) = 1 - 2e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} \sqrt{\frac{\gamma(\gamma + 1)}{\lambda_1\lambda_2}} K_1 \left(2\sqrt{\frac{\gamma(\gamma + 1)}{\lambda_1\lambda_2}} \right) \quad (3.7)$$

where, $K_\nu(z)$ is the ν th order modified Bessel's function of second kind [119, eq. 8.432.6].

When the CSI assisted ideal/hypothetical relay gain is used at the relay node the PDF and the CDF of the end-to-end SNR γ_{d_2} can be written respectively as [24],

$$f_{\gamma_{d_2}}(\gamma) = 2\gamma e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} \left[\frac{2}{\lambda_1\lambda_2} K_0 \left(\frac{2\gamma}{\sqrt{\lambda_1\lambda_2}} \right) + \frac{1}{\sqrt{\lambda_1\lambda_2}} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) K_1 \left(\frac{2\gamma}{\sqrt{\lambda_1\lambda_2}} \right) \right] \quad (3.8)$$

$$F_{\gamma_{d_2}}(\gamma) = 1 - \frac{2\gamma}{\sqrt{\lambda_1\lambda_2}} e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} K_1 \left(\frac{2\gamma}{\sqrt{\lambda_1\lambda_2}} \right) \quad (3.9)$$

The MGF of the end-to-end SNR γ_{d_2} for i.i.d. fading channels with equal power allocation between the source and the relay is given by [24].

$$\Phi_{\gamma_{d_2}}(s) = \frac{\sqrt{\frac{\lambda}{4}s(\frac{\lambda}{4}s+1)} + \operatorname{arcsinh}\left(\sqrt{\frac{\lambda}{4}s}\right)}{2\sqrt{\frac{\lambda}{4}s(\frac{\lambda}{4}s+1)}^{3/2}} \quad (3.10)$$

Now, using MGF based approach we can derive the SEP for M -ary PSK signals as described in Chapter 1. The MGF expression for end-to-end SNR γ_{d_1} in (3.4) can be obtained from [28]. We avoid to recite the equation here due to its size. The final SEP expression of a CSI assisted AF relay network can also be found there. The outage probability of end-to-end SNR γ_d or γ_{d_2} can be derived from there CDF expressions.

3.3 ERROR EXPONENT

The random coding error exponent is defined as a function of input distribution function $Q(x)$, a factor $\rho \in (0, 1]$ and rate $R \leq C$ (for details please read ch. 5 of [8]), which is jointly optimized over $Q(x)$ and ρ at a desired rate R . The Gaussian input distribution has often been used in many publications such as in [53, 120] to avoid the mathematical complexity involved in the joint optimization of the reliability function. This assumption provides near optimal result for the error exponent at a rate near the channel capacity. The error exponent of the dual hop AF network with Gaussian input distribution can be written as [8],

$$E_r(R) = \max_{0 \leq \rho \leq 1} \{E_0(\rho) - 2\rho R\} \quad (3.11)$$

with

$$E_0(\rho) = -\ln \mathbb{E}_{\gamma_{d_1}} \left\{ \left(1 + \frac{\gamma}{1+\rho} \right)^{-\rho} \right\} \quad (3.12)$$

$\mathbb{E}_X \{x\}$ denotes the statistical expectation operation over the random variable X . We will show here the error exponent for γ_{d_1} only. The error exponent results using end-to-end SNR γ_{d_2} can be found in [52] for Nakagami- m fading model. Now using (3.6) we modify eq.(3.12) as,

$$E_0(\rho) = -\ln [\mathcal{J}_1 + \mathcal{J}_2] \quad (3.13)$$

where,

$$\mathcal{J}_1 \triangleq \int_0^\infty \frac{2}{\lambda_1 \lambda_2} \left(1 + \frac{\gamma}{1+\rho} \right)^{-\rho} e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} (2\gamma + 1) K_0 \left(2\sqrt{\frac{(\gamma+1)\gamma}{\lambda_1 \lambda_2}} \right) d\gamma \quad (3.14)$$

and,

$$\begin{aligned} \mathcal{J}_2 \triangleq & 2 \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \int_0^\infty \left(1 + \frac{\gamma}{1+\rho} \right)^{-\rho} \sqrt{\frac{(\gamma+1)\gamma}{\lambda_1 \lambda_2}} \\ & \times e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} K_1 \left(2\sqrt{\frac{(\gamma+1)\gamma}{\lambda_1 \lambda_2}} \right) d\gamma \end{aligned} \quad (3.15)$$

From [119, eq. (8.447.3)] we have,

$$\begin{aligned} K_0 \left(2\sqrt{\lambda_1 \lambda_2 \gamma (\gamma+1)} \right) &= \sum_{k=0}^{\infty} \frac{\left(2\sqrt{\lambda_1 \lambda_2 \gamma (\gamma+1)} \right)^{2k}}{2^{2k} (k!)^2} \\ &\times \left[\psi(k+1) - \ln \left(\sqrt{\lambda_1 \lambda_2 \gamma (\gamma+1)} \right) \right] \end{aligned} \quad (3.16)$$

Using binomial expansion we can modify (3.16) as,

$$\begin{aligned}
K_0 \left(2\sqrt{\lambda_1 \lambda_2 \gamma (\gamma + 1)} \right) &= \sum_{k=0}^{\infty} \sum_{l=0}^k \binom{k}{l} \frac{(\lambda_1 \lambda_2)^k \gamma^{2k-l}}{2(k!)^2} \left[2 \left\{ \psi(k+1) - \frac{1}{2} \ln(\lambda_1 \lambda_2) \right\} \right. \\
&\quad \left. - (\ln \gamma + \ln(1 + \gamma)) \right] \tag{3.17}
\end{aligned}$$

Similarly, using [119, eq. (8.446)] we have,

$$\begin{aligned}
K_1 \left(2\sqrt{\lambda_1 \lambda_2 (\gamma^2 + \gamma)} \right) &= \frac{1}{2\sqrt{\lambda_1 \lambda_2 (\gamma^2 + \gamma)}} + \sum_{v=0}^{\infty} \frac{\left(2\sqrt{\lambda_1 \lambda_2} \sqrt{\gamma^2 + \gamma} \right)^{2v+1}}{v!(v+1)!2^{2v+2}} \\
&\quad \times \left[\ln \gamma + \ln(1 + \gamma) + 2 \ln \frac{2\sqrt{\lambda_1 \lambda_2}}{2} - \psi(v+1) - \psi(v+2) \right] \tag{3.18}
\end{aligned}$$

Now, using (3.17) and (3.18) and then transforming binomial power function to H -function using [121, eq. (8.4.2.5)] and [121, eq. (8.3.2.21)], we can modify (3.14) and (3.15) as,

$$\begin{aligned}
\mathcal{J}_1 &= \sum_{k=0}^{\infty} \sum_{l=0}^k \binom{k}{l} \frac{2C(k, \lambda)}{(\lambda_1 \lambda_2)^{k+1} (k!)^2 \Gamma(\rho)} \left[\int_0^{\infty} \gamma^{2k-l} e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} H_{1,1}^{1,1} \left[\frac{\gamma}{1+\rho} \middle| \begin{matrix} (l-\rho, 1) \\ (0, 1) \end{matrix} \right] \right. \\
&\quad \times (2\gamma + 1) \left\{ 1 - \frac{(\gamma - 1)}{2C(k, \lambda)} H_{2,2}^{2,2} \left[\gamma \middle| \begin{matrix} (0, 1), (0, 1) \\ (0, 1), (0, 1) \end{matrix} \right] \right. \\
&\quad \left. \left. - \frac{1}{2C(k, \lambda)} H_{2,2}^{1,2} \left[\gamma \middle| \begin{matrix} (1, 1), (1, 1) \\ (1, 1), (0, 1) \end{matrix} \right] \right\} d\gamma \right] \tag{3.19}
\end{aligned}$$

$$\begin{aligned}
\mathcal{J}_2 &= \frac{(\lambda_1 + \lambda_2)}{\lambda_1 \lambda_2 \Gamma(\rho)} \int_0^\infty e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} H_{1,1}^{1,1} \left[\frac{\gamma}{1 + \rho} \left| \begin{matrix} (l - \rho, 1) \\ (0, 1) \end{matrix} \right. \right] d\gamma + \sum_{v=0}^\infty \sum_{u=0}^{v+1} \frac{(\lambda_1 + \lambda_2)}{v! (v+1)!} \\
&\times \frac{C(v, \lambda)}{(\lambda_1 \lambda_2)^{v+2} \Gamma(\rho)} \binom{v+1}{u} \left[\int_0^\infty \gamma^{2v+2-u} e^{-\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\gamma} H_{1,1}^{1,1} \left[\frac{\gamma}{1 + \rho} \left| \begin{matrix} (l - \rho, 1) \\ (0, 1) \end{matrix} \right. \right] \right. \\
&\times \left. \left\{ 1 + \frac{(\gamma - 1)}{C(v, \lambda)} H_{2,2}^{2,2} \left[\gamma \left| \begin{matrix} (0, 1), (0, 1) \\ (0, 1), (0, 1) \end{matrix} \right. \right] + \frac{1}{C(v, \lambda)} H_{2,2}^{1,2} \left[\gamma \left| \begin{matrix} (1, 1), (1, 1) \\ (1, 1), (0, 1) \end{matrix} \right. \right] \right\} d\gamma \right]
\end{aligned} \tag{3.20}$$

where, $C(k, \lambda) = \psi(k+1) + \frac{1}{2} \ln(\lambda_1 \lambda_2)$ and, $C(v, \lambda) = -\ln(\lambda_1 \lambda_2) - \psi(v+1) - \psi(v+2)$, $H_{p,q}^{m,n} \left[\gamma \left| \begin{matrix} (a_p) \\ (b_q) \end{matrix} \right. \right]$ is the Fox- H function defined as [121, eq. 8.3.3.1].

To arrive at (3.19) and (3.20) we represent the natural log functions in terms of H -functions by using [121, eq. 8.4.6.11] and [121, eq. 8.4.6.5]. Now using [122, eq. 2.19] and [123, eq. 2.6.2] and after some modifications we can represent the solution of eq. (3.19) and (3.20), as eq. (3.21) and (3.22), shown at the bottom of the next page, where, $s = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$, $a = 2k - l$, $b = 1 - \rho$ and $d = 2v - u$. Finally replacing \mathcal{J}_1 and \mathcal{J}_2 from (3.21) and (3.22) in (3.13) we will have the desired error exponent expression.

3.4 ERGODIC CAPACITY

The ergodic capacity of this dual hop network is given by,

$$\begin{aligned}
C &= \frac{1}{2} \left[\frac{\partial E_0(\rho)}{\partial \rho} \right] \Big|_{\rho=0} \\
&= \frac{1}{2} \int_0^\infty \ln(1 + \gamma) f_{\gamma_{d_1}}(\gamma) d\gamma
\end{aligned} \tag{3.23}$$

Equation (3.23) can easily be solved first by replacing $H_{1,1}^{1,1} \left[\begin{matrix} \frac{\gamma}{1+\rho} \\ (0,1) \end{matrix} \middle| \begin{matrix} (l-\rho,1) \\ (0,1) \end{matrix} \right]$ with $H_{2,2}^{1,2} \left[\begin{matrix} \gamma \\ (1,1), (0,1) \end{matrix} \middle| \begin{matrix} (1,1), (1,1) \\ (1,1), (0,1) \end{matrix} \right]$ and some related scalar modifications in eq. (3.19) and (3.20) and then utilizing [122, eq. 2.19] and [123, eq. 2.6.2] we will arrive to a similar form of equations as eq. (3.21) and (3.22). Alternatively, we can apply

$$\begin{aligned}
\mathcal{J}_1 = & \sum_{k=0}^{\infty} \sum_{l=0}^k \frac{2(\lambda_1 \lambda_2)^{-k-1}}{(k!)^2 \Gamma(\rho)} \binom{k}{l} s^{-a-2} \left[2C(k, \lambda) H_{2,1}^{1,2} \left[\begin{matrix} 1 \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (-a-1,1), (b,1) \\ (0,1) \end{matrix} \right] \right. \\
& + sC(k, \lambda) H_{2,1}^{1,2} \left[\begin{matrix} 1 \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (-a,1), (b,1) \\ (0,1) \end{matrix} \right] - \frac{1}{s} H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+3,1) \\ (0,1), (0,1); (b,1) \end{matrix} \right] \\
& + \frac{1}{2} H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+2,1) \\ (0,1), (0,1); (b,1) \end{matrix} \right] - H_{1,[2:1],0,[2:1]}^{1,2,1,1,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+2,1) \\ (1,1), (1,1); (b,1) \end{matrix} \right] \\
& \left. + \frac{s}{2} \left\{ H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+1,1) \\ (0,1), (0,1); (b,1) \end{matrix} \right] - H_{1,[2:1],0,[2:1]}^{1,2,1,1,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+1,1) \\ (1,1), (1,1); (b,1) \end{matrix} \right] \right\} \right] \\
& \left. \left[H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+1,1) \\ (0,1), (0,1); (0,1) \end{matrix} \right] - H_{1,[2:1],0,[2:1]}^{1,2,1,1,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (a+1,1) \\ (1,1), (0,1); (0,1) \end{matrix} \right] \right] \right\} \right] \\
& \qquad \qquad \qquad (3.21)
\end{aligned}$$

$$\begin{aligned}
\mathcal{J}_2 = & \frac{1}{\Gamma(\rho)} H_{2,1}^{1,2} \left[\begin{matrix} 1 \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (0,1), (b,1) \\ (0,1) \end{matrix} \right] + \sum_{v=0}^{\infty} \sum_{u=0}^{v+1} \frac{(\lambda_1 \lambda_2)^{-v-1}}{v!(v+1)! \Gamma(\rho)} \binom{v+1}{u} s^{-d-2} \\
& \times \left[C(v, \lambda) H_{2,1}^{1,2} \left[\begin{matrix} 1 \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (-d-2,1), (b,1) \\ (0,1) \end{matrix} \right] + \frac{1}{s} H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (d+4,1) \\ (0,1), (0,1); (b,1) \end{matrix} \right] \right. \\
& - H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (d+3,1) \\ (0,1), (0,1); (b,1) \end{matrix} \right] + H_{1,[2:1],0,[2:1]}^{1,2,1,1,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (d+3,1) \\ (1,1), (1,1); (b,1) \end{matrix} \right] \\
& \left. \left[H_{1,[2:1],0,[2:1]}^{1,2,1,2,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (d+3,1) \\ (0,1), (0,1); (0,1) \end{matrix} \right] - H_{1,[2:1],0,[2:1]}^{1,2,1,1,1} \left[\begin{matrix} \frac{1}{s} \\ s(1+\rho) \end{matrix} \middle| \begin{matrix} (d+3,1) \\ (1,1), (0,1); (0,1) \end{matrix} \right] \right] \right] \\
& \qquad \qquad \qquad (3.22)
\end{aligned}$$

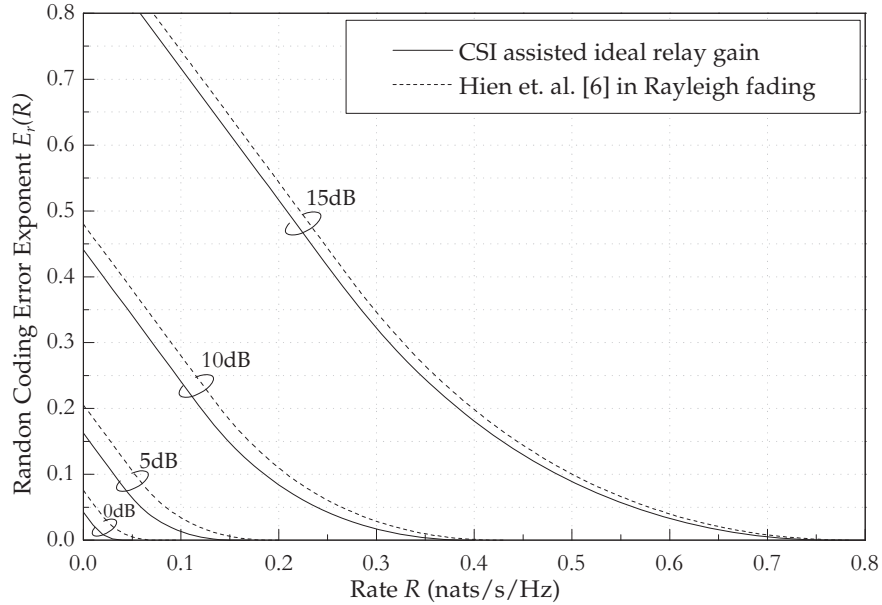


Figure 3.2: Random coding error exponent vs rate R in nats/s/Hz with various total signal to noise power allocation.

the partial differentiation over the error exponent with respect to ρ using [121, eq. 8.3.2.15] and we will get the expression of ergodic capacity.

$$\frac{d}{dx} \left[x^\sigma H_{p,q}^{m,n} \left(x \left| \begin{matrix} a_p \\ b_q \end{matrix} \right. \right) \right] = x^{\sigma-1} H_{p+1,q+1}^{m,n+1} \left(x \left| \begin{matrix} (-\sigma, 1), a_p \\ b_q, (1-\sigma, 1) \end{matrix} \right. \right) \quad (3.24)$$

3.5 NUMERICAL RESULTS

For numerical evaluation we assume that the Gaussian noise at the relay node and at the receiver has the same variance. Since the transmitter does not have any knowledge of CSI the total system power has been allocated equally between the source and the relay node. To assist with comparison the numerical results have been plotted with the result of [52], which has studied the error exponent with AF relays avoiding the noise figure at the denominator of the relay gain factor.

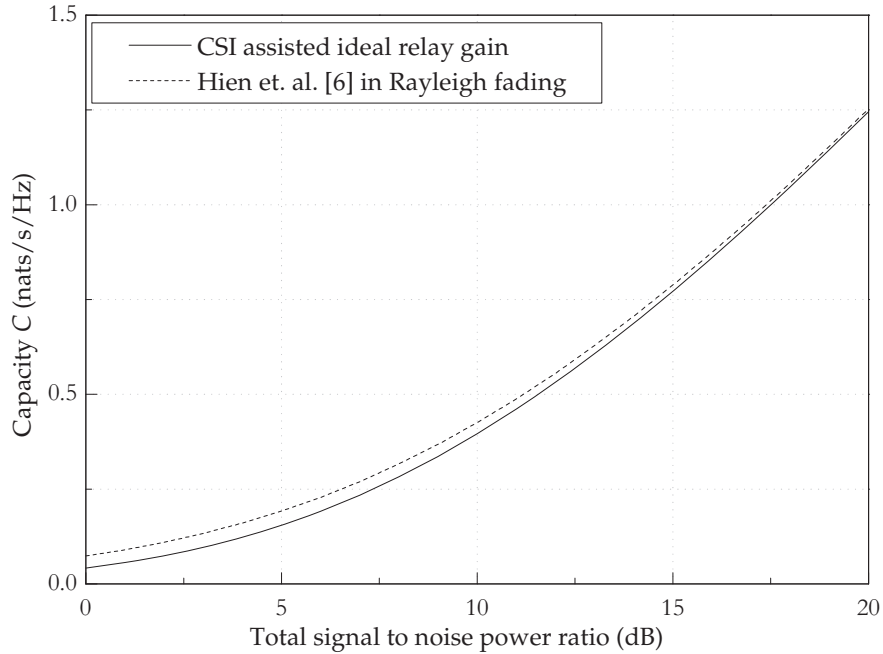


Figure 3.3: Capacity (nats/s/Hz) vs total signal power to noise ratio in dB.

Fig. 3.2 plots the error exponent as a function of rate R for total average SNR 0, 5, 10 and 15dB. The result of [52] sets a clear upper bound at these low SNR regions on our results. For example, at a fixed rate 0.1 nats/s/Hz and at total SNR of 10dB, the error exponent differs by 0.04 due to neglecting the noise figure in the denominator of the amplification factor. However, at SNR values e.g. SNR above 25dB the difference between two models becomes negligible. It also reveals that, an increase in transmit SNR from 10dB to 15dB offers a reduction of the codeword length almost 3 times while achieving the same error performance as 10dB at a fixed rate 0.1 nats/s/Hz. Fig. 3.3 shows the capacity of the channel behavior with total SNR in Rayleigh fading channels. We can see that the channel capacity increases with the total SNR in the system and about 10dB increase in total SNR can double the channel capacity. In both figures the result of [52] is an upper bound on our result, which becomes progressively loose at the low SNR region.

3.6 DISCUSSION

In this chapter we derive closed form RCEE of a dual hop cooperative network using a CSI assisted AF relay over Rayleigh fading channels. Presented numerical results reveal a quantitative measurement on RCEE showing how much a source encoder codeword construction can be relaxed or increased in the system to achieve a specific performance. We observe that at specific error probability, rate and SNR, the required codeword length is higher for CSI assisted relay than a cooperative network with hypothetical AF relay when compared with the result of [52].

Chapter 4

MULTIHOP RELAYING

4.1 INTRODUCTION

In this chapter a mathematical performance model on a multihop relay network is presented. Closed form expressions of SEP and outage probability for a general multihop relay network without centralized CSI are derived for Rayleigh and Nakagami- m fading channels. The multihop network is divided into arbitrary K parallel relaying paths, each path with an arbitrary number of relays. We use selective decode-and-forward (DF) relays [14] and M -PSK modulated signals at all transmitting nodes.

At first, mathematical error probability model for Rayleigh fading channels is developed. In subsequent sections the performance modeling is extended for Nakagami- m fading channels. Nakagami- m fading model gives the best approximation for multipath land mobile as well as indoor mobile environments. It also encompass other various wireless fading models such as Hoyt or Rician fading channels including the Rayleigh fading channels. Numerical results have been presented for non-identical and non-symmetric network assuming a network consists of any arbitrary number of paths or hops. Finally, numerical results are presented to analyze the performance behavior with different network parameters such as, power sharing factor among the nodes and network size. We observe that for a specific total power constraint we can optimize the performance by choosing an appropriate network size in terms of the number of parallel paths K or number of hops N . In the analysis of this chapter, interference signals from other adjacent nodes are neglected, assuming that the distance between two adjacent relaying paths are far away from each other or they are transmit-

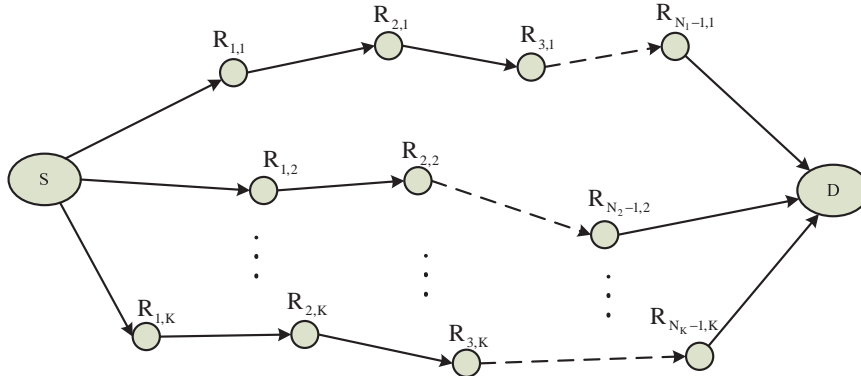


Figure 4.1: Multihop parallel relaying network.

ting in different channels. The results of this chapter has been published in IEEE Communications Letter and Conference on Vehicular technology [11, 12].

4.2 SYSTEM MODEL

Consider a single source-destination pair communicating via a total of R single antenna relays in a multihop parallel network shown in Fig. 4.1. There are K possible parallel relay paths with arbitrary number of relays in each path. We let path $k \in \mathcal{K}$, $\mathcal{K} = \{1, 2, \dots, K\}$, has a total $N_k - 1$ relays in series (i.e. total N_k hops in path k). A selective decode-and-forward (DAF) protocol has been considered over independent and non-identically distributed (i.n.i.d.) fading channels. We assume the relays on a certain path transmit over different time slots. However the same time slots can be used by the relays on different parallel relay paths because the paths are far away from each other [67]. The details of resource and channel allocation is not in the scope of this study here.

We consider the total power of the system is constrained to P_{tot} . This total power is split into the whole network by a power sharing coefficient $\zeta \in (0, 1]$ so that the source and the total relay power is given by $P_S = (1 - \zeta)P_{\text{tot}}$ and $P_R = \zeta P_{\text{tot}}$ respectively. We consider the network has no centralized controller and each relaying path will equally share the power P_R . For example, the relay path

k has total power constraint $P_{R,k}$ such that, $P_{R,k} = \frac{P_R}{K}$, and the individual relays of path k share this total power $P_{R,k}$ equally, i.e. the power of the relay node $R_{n,k}$, $n = 1, 2, \dots, N_k - 1$, is $P_{R_{n,k}} = \frac{P_{R,k}}{N_k - 1}$. The instantaneous and the average SNR at relay $R_{n,k}$ are denoted as $\gamma_{n,k} \triangleq \frac{P|h_{n,k}|^2}{\sigma_{n,k}^2}$ and $\lambda_{n,k} \triangleq \frac{P\Omega_{n,k}}{\sigma_{n,k}^2}$ respectively, where, P is the corresponding transmit power from the source or the relay. $h_{n,k}$ and $\Omega_{n,k}$ are the instantaneous and average channel gain of n th hop on k th path respectively and $\sigma_{n,k}^2$ is the variance of a zero mean circularly symmetric complex Gaussian noise (AWGN) at relay node $R_{n,k}$. To illustrate a wireless network we will use the term $\text{Net} = [N_1, N_2, N_3, \dots, N_K]$ such that 1st path has N_1 number of hops, 2nd path has N_2 number of hops, 3rd path has N_3 number of hops and so on. Thus the total number of paths in the network is $|\text{Net}| = K$. To avoid the synchronization problem due to transmission delays between the relay paths at the destination, we assume $N_i \approx N_j$, for $i \neq j$ [74].

4.3 SEP AND DIVERSITY ANALYSIS: RAYLEIGH FADING CHANNELS

In decode-and-forward relaying error propagates through a multihop series network. Thus error in any arbitrary relaying links¹ will result an error at the receiver. We assume that the relays can sense the decoding error and therefore transmit only if there is no error in the decoded message. Consequently if any error happens at any arbitrary hop $n \in \mathcal{N}_k$ on path k , the relays from the next hops $n + 1$ onwards will remain idle. Due to non centralized CSI network in consideration, the error message on a path will be unknown to the network and the system will be penalized by the power lost in that path. At the end, the signals from all decodable paths will be combined using maximal ratio combining (MRC) at the receiver. To begin our analysis we define a decodable path set \mathcal{D} as,

¹Any link $A \rightarrow B$ refers to the communication channel from node A to node B

$$\mathcal{D} \triangleq \{k \in \mathcal{K} : P_{e,k} = 0\} \quad (4.1)$$

where, $P_{e,k}$ is the probability of error of path k . If all the relays on path k can decode a message without error then path k will be in the decodable path set \mathcal{D} . Let $\mathcal{D}_p \subseteq \mathcal{K}$ such that, the cardinality of \mathcal{D}_p is p , i.e. $|\mathcal{D}_p| = p$. Now utilizing the law of total probability we can express the average probability of error at the destination of the multihop parallel relay network as [124],

$$\bar{P}_e = \sum_{p=0}^K \sum_{\mathcal{D}_p} \bar{P}(e|\mathcal{D}_p) \mathbb{P}(\mathcal{D}_p) \quad (4.2)$$

where, $\bar{P}(e|\mathcal{D}_p)$ is the probability of error at the receiver with p branch MRC. The second *sum* in the above expression is over all possible paths in \mathcal{K} with exact p decodable paths. The probability of \mathcal{D} with p decodable paths can be written as,

$$\mathbb{P}(\mathcal{D}_p) = \prod_{k \in \mathcal{D}_p} (1 - \bar{P}_{e,k}) \prod_{k' \in \mathcal{K} \setminus \mathcal{D}_p} \bar{P}_{e,k'} \quad (4.3)$$

$\bar{P}_{e,k}$ and $\bar{P}_{e,k'}$ are the average probability of error of path k and k' respectively. To proceed to the solution of (4.3) we first determine the error of any hop n on path k of the network that has $N_k - 1$ number of relays. The instantaneous conditional probability of error with M -PSK modulation with i.n.i.d. SNR γ_n at the corresponding node [3, eq. 8.22] is,

$$P(e|\gamma_n) = \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma_n \right) d\theta \quad (4.4)$$

Now, taking average over the fading channel the average probability of error at relay node n is,

$$\bar{P}(e) = \frac{1}{\pi} \int_0^\infty \int_0^{\pi - \frac{\pi}{M}} \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma_n \right) f_{\gamma_n}(\gamma_n) d\theta d\gamma_n \quad (4.5)$$

where $f_{\gamma_n}(\gamma_n)$ is the pdf of the SNR of hop n given as in [3, eq. (2.7)]. With the assumption that the fading channels in all hops are independent to each other, the end-to-end average probability of error at the last relay node on path k is,

$$\bar{P}_{e,k} = 1 - \prod_{n=1}^{N_k-1} \left[1 - \frac{1}{\pi} \int_0^\infty \int_0^{\pi - \frac{\pi}{M}} \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma_n \right) f_{\gamma_n}(\gamma_n) d\theta d\gamma_n \right] \quad (4.6)$$

Using MGF form representation in (4.6) for our model we can modify eq. (4.3) as,

$$\begin{aligned} \mathbb{P}(\mathcal{D}_p) &= \prod_{k \in \mathcal{D}_p} \left[\prod_{n=1}^{N_k-1} \left[1 - \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \Phi_{\gamma_{n,k}}(s) d\theta \right] \right] \\ &\times \prod_{k' \in \mathcal{K} \setminus \mathcal{D}_p} \left[1 - \prod_{n=1}^{N_{k'}-1} \left[1 - \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \Phi_{\gamma_{n,k'}}(s) d\theta \right] \right] \end{aligned} \quad (4.7)$$

where $s = \frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta}$. $\Phi_{\gamma_{n,k}}(s)$ is the MGF of SNR $\gamma_{n,k}$ defined as, $\Phi_{\gamma_{n,k}}(s) \triangleq \mathbb{E} [e^{-s\gamma_{n,k}}]$. \mathbb{E} stands for statistical expectation operation. Note that, the second product in (4.7) over n to $N_k - 1$ is because we calculate the decodable path before the last hop and then use MRC at the receiver for the signals from the

decodable paths. Now, using [3, eq. (5.79)] for any SNR $\gamma_{n,k}$ we can solve the integration of MGF in (4.7) as,

$$\begin{aligned} \mathcal{I}(M, n, k) &\triangleq \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \Phi_{\gamma_{n,k}} \left(\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \right) d\theta \\ &= \left(\frac{M-1}{M} \right) \left\{ 1 - \sqrt{\frac{\lambda_{n,k} \sin^2 \left(\frac{\pi}{M} \right)}{1 + \lambda_{n,k} \sin^2 \left(\frac{\pi}{M} \right)}} \left(\frac{M}{\pi(M-1)} \right) \right. \\ &\quad \left. \times \left[\frac{\pi}{2} + \tan^{-1} \left(\sqrt{\frac{\lambda_{n,k} \sin^2 \left(\frac{\pi}{M} \right)}{1 + \lambda_{n,k} \sin^2 \left(\frac{\pi}{M} \right)}} \cot \frac{\pi}{M} \right) \right] \right\} \end{aligned} \quad (4.8)$$

The conditional average probability of error at the receiver end with exactly p decodable paths,

$$\bar{P}(e|\mathcal{D}_p) = \frac{1}{\pi} \int_0^\infty \int_0^{\pi - \frac{\pi}{M}} \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma_r \right) f_{\gamma_r}(\gamma_r) d\theta d\gamma_r \quad (4.9)$$

where γ_r is the instantaneous receiver SNR at the destination with expectation λ_r , which is in fact the sum of SNR's of decodable paths, i.e. $\gamma_r = \sum_{p \in \mathcal{D}_p} \gamma_p$. Invoking the results for the pdf of the sum of exponential random variables from [29, eq. 29] and then putting in (4.9) we have,

$$\begin{aligned} \bar{P}(e|\mathcal{D}_p) &= \frac{1}{\pi} \sum_{i=1}^{\alpha(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} \int_0^{\pi - \frac{\pi}{M}} \int_0^\infty \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma_r \right) \\ &\quad \times X_{i,j}(\Lambda) \frac{\lambda_{<i>}^{-j}}{(j-1)!} \gamma_r^{j-1} e^{-\gamma_r/\lambda_{<i>}} d\gamma_r d\theta \\ &= \frac{1}{\pi} \sum_{i=1}^{\alpha(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} X_{i,j}(\Lambda) \int_0^{\pi - \frac{\pi}{M}} \left[1 + \frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \lambda_{<i>} \right]^{-j} d\theta \end{aligned} \quad (4.10)$$

where, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_p)$. $\alpha(\Lambda)$ is the number of distinct diagonal elements of Λ , $\lambda_{(1)} > \lambda_{(2)} > \dots > \lambda_{(\alpha(\Lambda))}$ are the distinct diagonal elements in decreasing order. $\tau_i(\Lambda)$ is the multiplicity of $\lambda_{(i)}$ and $X_{i,j}(\Lambda)$ is the (i, j) th characteristic

coefficient of Λ [125].

The second equality in (4.10) can be arrived at by using [119, eq. 3.351.3]. Then using [3, eq. (5.80)] we can write equation (4.10) as (4.16) shown at the bottom of the next page, where, $\varphi_i \triangleq \sqrt{\frac{\lambda_{<i>\sin^2(\frac{\pi}{M})}}{1+\lambda_{<i>\sin^2(\frac{\pi}{M})}} \cot \frac{\pi}{M}$ and, $T_{ql} \triangleq \frac{\binom{2l}{l} 4^{-q}}{\binom{2(l-q)}{l-q} [1+2(l-q)]}$. Finally utilizing the results of (4.8), (4.7) and (4.16) and substituting in (4.2) we will have the desired SEP expression of the network.

Diversity: At high SNR the average SEP can be approximated as $\bar{P}_e \approx [C_g(K, M, N) \lambda]^{-K}$, where $C_g(K, M, N)$ is the coding gain as $\lambda \rightarrow \infty$. Thus the achievable diversity of this network is K which also agrees with [126]. We can easily attain to the asymptotic SEP expression by considering i.i.d case and replacing all $\left(1 + \frac{\sin^2(\frac{\pi}{M})}{\sin^2\theta} \lambda\right)$ with $\frac{\sin^2(\frac{\pi}{M})}{\sin^2\theta} \lambda$ in the integral form SEP expression provided by eq. (4.2), (4.7) and (4.15) and then taking limit for $\lambda \rightarrow \infty$.

4.4 SEP ANALYSIS: NAKAGAMI- m FADING CHANNELS

In this section we consider the multihop network Fig. 4.1 operating over fading Nakagami- m fading channels. The PDF of each hop channel distribution is given by [3, eq. (2.20)].

$$f_h(h) = \frac{2m^m h^{2m-1}}{\omega^m \Gamma(m)} e^{-mh^2/\Omega}; \quad h \geq 0 \quad (4.12)$$

$$\begin{aligned} \bar{P}(e|\mathcal{D}_p) &= \sum_{i=1}^{\alpha(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} X_{i,j}(\Lambda) \left[\frac{M-1}{M} - \frac{1}{\pi} \sqrt{\frac{\lambda_{<i>\sin^2(\frac{\pi}{M})}}{1+\lambda_{<i>\sin^2(\frac{\pi}{M})}} \left\{ \left(\frac{\pi}{2} + \tan^{-1} \varphi_i \right) \right. \right. \\ &\times \sum_{l=0}^{j-1} \binom{2l}{l} \frac{4^{-l}}{(1+\lambda_{<i>\sin^2(\frac{\pi}{M})})^l} + \sin(\tan^{-1} \varphi_i) \\ &\times \left. \left. \sum_{l=1}^{j-1} \sum_{q=1}^l \frac{T_{ql}}{(1+\lambda_{<i>\sin^2(\frac{\pi}{M})})^l} [\cos(\tan^{-1} \varphi_i)]^{2(l-q)+1} \right\} \right] \quad (4.11) \end{aligned}$$

where, m is Nakagami- m parameter and $\Omega \triangleq \mathbb{E}[h^2]$. From last section of error analysis on Rayleigh fading channels we can say finding the solution of general eq. (4.2) entirely dependent on two probability functions $\bar{P}(e|\mathcal{D}_p)$ and $\mathbb{P}(\mathcal{D}_p)$. Therefore, solving equations (4.7) and (4.9) for Nakagami- m channels will complete the error analysis of Nakagami- m fading channels. The PDF of i.n.i.d. SNR of any arbitrary hop n , $f_{\gamma_n}(\gamma_n)$ in (4.6) is Gamma distributed given by [3, eq. (2.21)], and, thus the MGF of SNR $\gamma_{n,k}$ have to be modified for Nakagami- m fading channels given as [3, eq. 2.22],

$$\begin{aligned}\Phi_{\gamma_{n,k}}(s) &\triangleq \mathbb{E}[e^{-s\gamma_{n,k}}] \\ &= \left(1 + \frac{s\gamma_{n,k}}{m}\right)^{-m}\end{aligned}\quad (4.13)$$

Inserting (4.13) in (4.7) the integrations in (4.7) can directly be solved by using [3, eq. (5.80)]. At this point to solve the conditional probability of error for a given decodable set \mathcal{D}_p , we recall eq. (4.9),

$$\bar{P}(e|\mathcal{D}_p) = \frac{1}{\pi} \int_0^\infty \int_0^{\pi - \frac{\pi}{M}} \text{Exp}\left(-\frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\theta}\gamma\right) f_{\gamma_r}(\gamma) d\theta d\gamma$$

The pdf of the sum of the instantaneous receiver SNR's, γ_r for arbitrary Nakagami parameter m can be written as (4.14) by using [127, eq. 2.9],

$$f_{\gamma_r}(\gamma) = \prod_{p \in \mathcal{D}_p} \left(\frac{\alpha_1}{\alpha_p}\right)^{m_p} \sum_{h=0}^{\infty} \delta_h \frac{\gamma^{\beta+h-1} e^{-\gamma/\alpha_1}}{\alpha_1^{\beta+h} \Gamma(\beta+h)} \quad \gamma \geq 0; \quad (4.14)$$

where, $\alpha_1 = \min_{p \in \mathcal{D}_p} \{\alpha_p\}$, $\beta = \sum_{p \in \mathcal{D}_p} m_p$, $\alpha_p = \frac{\lambda_r}{m_p}$ and,

$$\begin{cases} \delta_0 &= 1 \\ \delta_{h+1} &= \frac{1}{h+1} \sum_{i=1}^{h+1} \left[\sum_{j=1}^{|\mathcal{D}_p|} m_j \left(1 - \frac{\alpha_1}{\alpha_j}\right)^i \right] \delta_{h+1-i} \end{cases}$$

Substituting the results of equation (4.14) in (4.9) we have,

$$\begin{aligned}
\bar{P}(e|\mathcal{D}_p) &= \frac{1}{\pi} \prod_{p \in \mathcal{D}_p} \left(\frac{\alpha_1}{\alpha_p} \right)^{m_p} \sum_{h=0}^{\infty} \frac{\delta_h}{\alpha_1^{\beta+h} \Gamma(\beta+h)} \\
&\times \int_0^{\pi - \frac{\pi}{M}} \int_0^{\infty} \text{Exp} \left(-\frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \gamma \right) \gamma^{\beta+h-1} e^{-\gamma/\alpha_1} d\gamma d\theta \\
&= \frac{1}{\pi} \prod_{p \in \mathcal{D}_p} \left(\frac{\alpha_1}{\alpha_p} \right)^{m_p} \sum_{h=0}^{\infty} \delta_h \int_0^{\pi - \frac{\pi}{M}} \left[1 + \frac{\sin^2 \left(\frac{\pi}{M} \right)}{\sin^2 \theta} \alpha_1 \right]^{-(\beta+h)} d\theta \quad (4.15)
\end{aligned}$$

The second equality in (4.15) can arrive at by using [119, eq. 3.381.4]. Then using [3, eq. (5.80)] we can write equation (4.15) as (4.16), which is shown at the bottom of the page, where, $a = \sin \left(\frac{\pi}{M} \right)$, $\varphi_1 \triangleq \sqrt{\frac{\lambda_1 a^2 / m_1}{1 + \lambda_1 a^2 / m_1}} \cot \frac{\pi}{M}$ and, $T_{ql} \triangleq \frac{\binom{2l}{l} 4^{-q}}{\binom{2(l-q)}{l-q} [1+2(l-q)]}$. Finally, using (4.16) and (4.7) we can solve (4.2). Note that, even though, eq. (4.14) provides the sum of Nakagami- m variates for arbitrary non-integer m , since the solution of the integral (4.15) provided by [3, eq. (5.80)] is valid only for integer values of $\beta + h$, the final SEP expression will not hold for non-integer Nakagami parameter m .

4.5 OUTAGE ANALYSIS

In this section we analyze the outage performance for the network Fig. 4.1 using similar selective decode-and-forward relays over Nakagami- m fading channels.

$$\begin{aligned}
\bar{P}(e|\mathcal{D}_p) &= \prod_{p \in \mathcal{D}_p} \left(\frac{\alpha_1}{\alpha_p} \right)^{m_p} \sum_{h=0}^{\infty} \delta_h \left[\frac{M-1}{M} - \frac{1}{\pi} \sqrt{\frac{\lambda_1 a^2 / m_1}{1 + \lambda_1 a^2 / m_1}} \left\{ \left(\frac{\pi}{2} + \tan^{-1} \varphi_1 \right) \right. \right. \\
&\times \sum_{l=0}^{\beta+h-1} \binom{2l}{l} \frac{4^{-l}}{(1 + \lambda_1 a^2 / m_1)^l} + \sin(\tan^{-1} \varphi_1) \\
&\times \left. \left. \sum_{l=1}^{\beta+h-1} \sum_{q=1}^l \frac{T_{ql}}{(1 + \lambda_1 a^2 / m_1)^l} [\cos(\tan^{-1} \varphi_1)]^{2(l-q)+1} \right\} \right] \quad (4.16)
\end{aligned}$$

We define a relaying path k to be in outage if any link SNR of path k falls below the predefined threshold SNR value γ_{th} . Thus an event where all paths are in outage, will cause the outage of the whole network. We assume all possible relays will try to participate in cooperation if their links are not in outage, and at the last hop the signals from decodable paths are coherently combined by MRC at the destination. To cope with this context here we define decodable path set D_p as,

$$D_p \triangleq \{k \in \mathcal{K} : \gamma_{n,k} \geq \gamma_{\text{th}}\}; \quad n \in \{1, 2, \dots, N_k - 1\} \quad (4.17)$$

where, \mathcal{K} is the parallel path set such that, $\mathcal{K} = \{1, 2, \dots, K\}$. Thus we can determine the probability of decodable path D_p as,

$$\begin{aligned} \mathbb{P}(D_p) &= \prod_{k \in D_p} \mathbb{P} \left\{ \min_{n \in \{1, 2, \dots, N_k - 1\}} \{\gamma_{n,k}\} > \gamma_{\text{th}} \right\} \\ &\times \prod_{k' \in \mathcal{K} \setminus D_p} \mathbb{P} \left\{ \min_{n \in \{1, 2, \dots, N_{k'} - 1\}} \{\gamma_{n,k'}\} < \gamma_{\text{th}} \right\} \end{aligned} \quad (4.18)$$

where, $\gamma_{n,k}$ is the Gamma distributed SNR expressed as [3, eq. (2.21)] of the link of relay hop n of path k . To calculate the outage, we simplify the model of a multihop series path to a single relay link, denoting the link is active if the path is decodable. The total probability of the outage at the destination will be the sum of the signals from all decodable relay paths. Thus, the total outage will be conditional on the set of the decodable relay paths. Taking the cardinality of D_p as p , we can write the outage probability of the system using the law of total probability similar as eq. (4.2),

$$P_{\text{out}} = \sum_{p=0}^K \sum_{D_p} \mathbb{P}(\text{outage} | D_p) \mathbb{P}(D_p) \quad (4.19)$$

where

$$\mathbb{P}(\text{outage}|D_p) = \mathbb{P}\left\{\sum \gamma_{N_k,k} < \gamma_{\text{th}} | k \in D_p\right\} \quad (4.20)$$

using (4.18) and (4.20) in (4.19),

$$\begin{aligned} P_{\text{out}} &= \sum_{p=0}^K \sum_{D_p} \mathbb{P}\left\{\sum_{i \in D_p} \gamma_{N_k,k} < \gamma_{\text{th}}\right\} \mathbb{P}\{D_p\} \\ &= \sum_{p=0}^K \sum_{D_p} \mathbb{P}\left\{\sum_{k \in D_p} \gamma_{N_k,k} < \gamma_{\text{th}}\right\} \prod_{k \in D_p} \mathbb{P}\left\{\min_{n \in \{1,2,\dots,N_k-1\}} \{\gamma_{n,k}\} > \gamma_{\text{th}}\right\} \\ &\quad \times \prod_{k' \in \mathcal{K} \setminus \mathcal{D}_p} \mathbb{P}\left\{\min_{n \in \{1,2,\dots,N_{k'}-1\}} \{\gamma_{n,k'}\} < \gamma_{\text{th}}\right\} \end{aligned} \quad (4.21)$$

Now invoking the result from order statistics and using the CDF of Nakagami- m channels equation (4.21) can be written as,

$$\begin{aligned} P_{\text{out}} &= \sum_{p=0}^K \sum_{D_p} \left[\mathbb{P}\left\{\sum_{k \in D_p} \gamma_{N_k,k} < \gamma_{\text{th}}\right\} \prod_{k \in D_p} \prod_{n=1}^{N_k-1} \left[\frac{\Gamma(m_{n,k}, m_{n,k} \gamma_{\text{th}} / \lambda_{n,k})}{\Gamma(m_{n,k})} \right] \right. \\ &\quad \left. \times \prod_{k' \in \mathcal{K} \setminus \mathcal{D}_p} \left\{ 1 - \prod_{n=1}^{N_{k'}-1} \frac{\Gamma(m_{n,k'}, m_{n,k'} \gamma_{\text{th}} / \lambda_{n,k'})}{\Gamma(m_{n,k'})} \right\} \right] \end{aligned} \quad (4.22)$$

Proof. See Appendix A.1. □

The CDF sum of eq.(4.22) can be calculated by using the random sum of random variables. If the gamma variates with parameter m and α are restricted to only integer m 's, then for distinct α 's we have the CDF of the random sum

Y ,

$$F_Y(y) = \left(\prod_{i=1}^p \frac{1}{\alpha_i^{m_i}} \right) \sum_{i=1}^p \sum_{l=1}^{m_i} \frac{d^{l-1}}{ds^{l-1}} \left\{ \prod_{\substack{q=1 \\ q \neq i}}^p \left(s + \frac{1}{\alpha_q} \right)^{-m_q} \right\} \Big|_{s=-\alpha_i^{-1}} \\ \times \frac{\alpha_i^{a_i+1}}{(a_i)!(l-1)!} \left\{ \Gamma(a_i+1) - \Gamma\left(a_i+1, \frac{y}{\alpha_i}\right) \right\} \quad (4.23)$$

where, $a_i = m_i - l$, Gamma distribution parameter $\alpha_i = \frac{\Omega_i}{m_i}$ and $\Gamma(a, z)$ is upper incomplete Gamma function defined as, $\Gamma(a, z) \triangleq \int_z^\infty t^{a-1} e^{-t} dt$.

Proof. See Appendix A.2. □

When the value of m is not restricted to integer values the CDF can be written as Moschopoulos et. al. [127]

$$F_Y(y) = \prod_{r=1}^p \left(\frac{\alpha_1}{\alpha_r} \right)^{m_r} \sum_{u=0}^{\infty} \theta_u \left[1 - \frac{\Gamma(\beta + u, y/\alpha_1)}{\Gamma(\beta + u)} \right], \\ y \geq 0; \quad (4.24)$$

where, $\alpha_1 = \min_r \{\alpha_r\}$, $\beta = \sum_{r=1}^p m_r$ and,

$$\begin{cases} \theta_0 = 1 \\ \theta_{u+1} = \frac{1}{u+1} \sum_{i=1}^{u+1} \left[\sum_{j=1}^N m_j \left(1 - \frac{\alpha_1}{\alpha_j} \right)^i \right] \theta_{u+1-i}; \quad u = 0, 1, 2, \dots \end{cases}$$

Substituting the above result in eq. (4.23) and putting $\alpha_i = \frac{\lambda_i}{m_i}$ we will have the total outage expression as eq. (4.25), shown at the bottom of the next page. Again using (4.24) we can express the outage probability of the system as (4.26) shown at the bottom of the next page, where θ_u is given by eq. (4.24).

4.6 NUMERICAL RESULTS

In numerical evaluation over Rayleigh fading channels case a multihop wireless network with equal number of relays in different paths is considered, i.e. $N_k = N_{k'} = N$. For fair comparison with similar studies in all simulation results we consider the average channel gain of all hops in the network are unity and the noise variance at all nodes equal to σ^2 . Furthermore we assume the total power, P_{tot} is equally divided into every hops i.e. $\zeta = \frac{N-1}{N}$ in our numerical results unless otherwise specified.

$$\begin{aligned}
P_{\text{out}} &= \sum_{p=0}^K \sum_{D_p} \left[\prod_{k \in D_p} \prod_{n=1}^{N_k-1} \left(\frac{\Gamma(m_{n,k}, m_{n,k} \gamma_{\text{th}} / \lambda_{m_{n,k}})}{\Gamma(m_{n,k})} \right) \right. & (4.25) \\
&\times \prod_{k' \in \mathcal{K} \setminus D_p} \left(1 - \prod_{n=1}^{N_{k'}-1} \frac{\Gamma(m_{n,k'}, m_{n,k'} \gamma_{\text{th}} / \lambda_{m_{n,k'}})}{\Gamma(m_{n,k'})} \right) \\
&\times \left\{ \left(\prod_{k \in D_p} \frac{m_k^{m_k}}{\lambda_k^{m_k}} \right) \sum_{k \in D_p} \sum_{l=1}^{m_k} \frac{d^{l-1}}{d s^{l-1}} \left\{ \prod_{\substack{q=1 \\ q \neq k}}^p \left(s + \frac{m_q}{\lambda_q} \right)^{-m_q} \right\} \right\} \\
&\times \frac{\lambda_k^{m_k-l+1}}{m_k^{m_k-l+1} (m_k - l)! (l - 1)!} \left. \left\{ \Gamma(m_k - l + 1) - \Gamma(m_k - l + 1, m_k \gamma_{\text{th}} / \lambda_k) \right\} \right\} \Bigg|_{s = -\frac{m_k}{\lambda_k}}
\end{aligned}$$

$$\begin{aligned}
P_{\text{out}} &= \sum_{p=0}^K \sum_{D_p} \left[\prod_{r=1}^{|D_p|} \left(\frac{m_r \lambda_1}{m_1 \lambda_r} \right)^{m_r} \sum_{u=0}^{\infty} \theta_u \left[1 - \frac{\Gamma\left(\sum_{r \in D_p} m_r + u, \gamma_{\text{th}} m_1 / \lambda_1\right)}{\left(\sum_{r \in D_p} m_r + u\right)} \right] \right] \\
&\times \prod_{k \in D_p} \prod_{n=1}^{N_k-1} \left(\frac{\Gamma(m_{n,k}, m_{n,k} \gamma_{\text{th}} / \lambda_{m_{n,k}})}{\Gamma(m_{n,k})} \right) \prod_{k' \in \mathcal{K} \setminus D_p} \left(1 - \prod_{n=1}^{N_{k'}-1} \frac{\Gamma(m_{n,k'}, m_{n,k'} \gamma_{\text{th}} / \lambda_{m_{n,k'}})}{\Gamma(m_{n,k'})} \right) & (4.26)
\end{aligned}$$

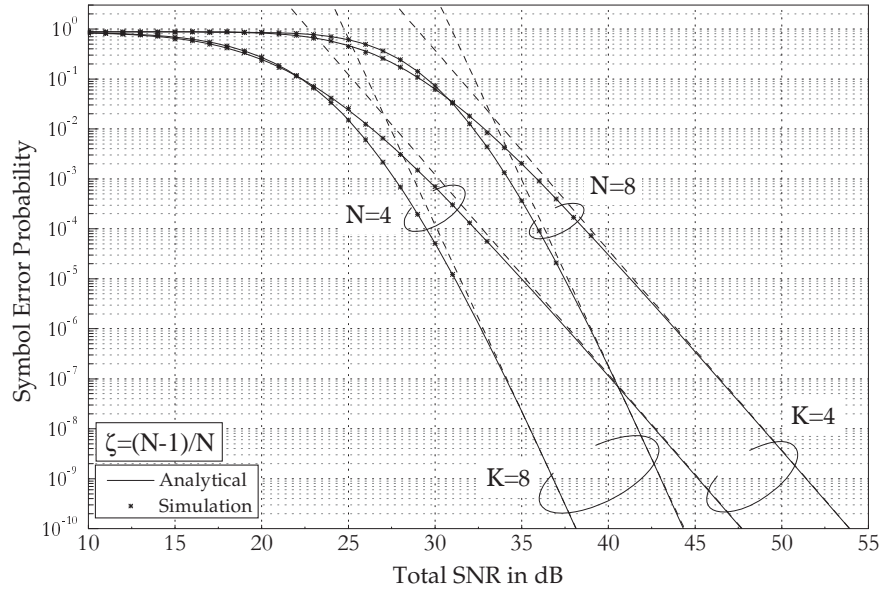


Figure 4.2: Symbol error probability of multihop relay network as a function of total signal to noise power ratio in Rayleigh fading channels.

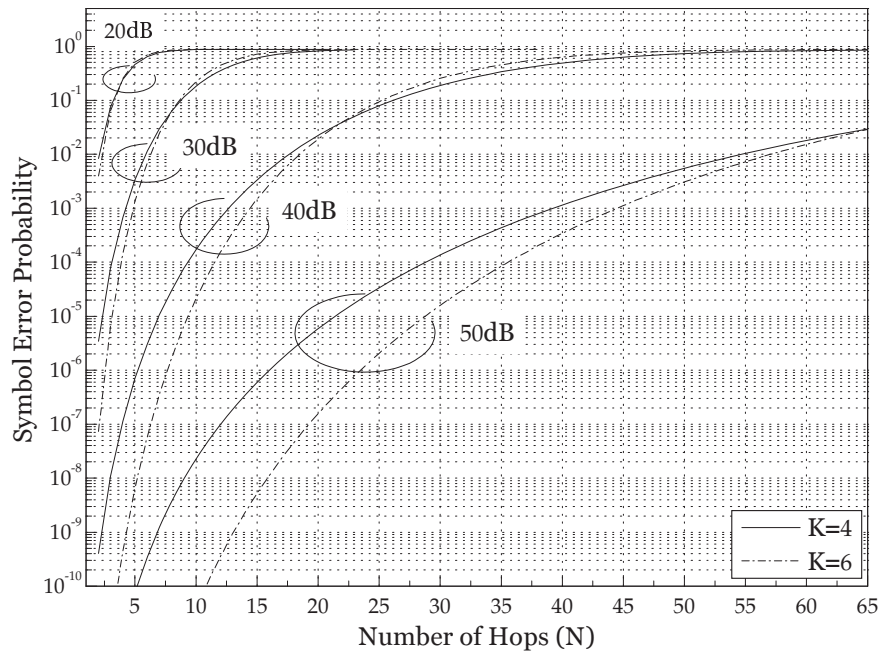


Figure 4.3: Symbol error probability of multihop relay network with the number of hops N in Rayleigh fading channels.

Fig. 4.2 plots the symbol error probability in Rayleigh fading channels as a function of total SNR (P_{tot}/σ^2). The figure shows that regardless of the number

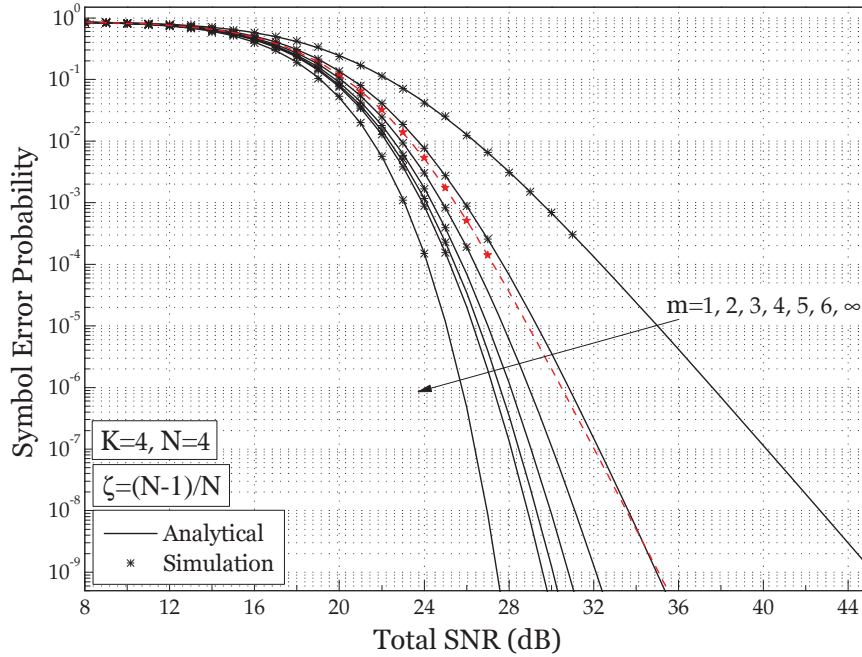


Figure 4.4: Symbol error probability as a function of total signal to noise power ratio in Nakagami- m fading channels with $m = 1, 2, 3, 4, 5, 6$, and ∞ in a $[4, 4, 4, 4]$ network.

of hops N the high SNR slopes are parallel to each other for any specific number of parallel paths K . However, increasing the number of hops affects the coding gain of the system. Fig. 4.3 plots the SEP behavior with respect to the number of hops N . The figure divides the SEP into four different total SNR groups. It reveals that with higher total SNR values the reliability (lower SEP) can significantly increase by using a larger number of possible paths. For example, using 6 paths as opposed to 4, at total SNR of 50dB, the SEP is improved by an order of magnitude over a wide range. In addition, the figure shows that by increasing the total SNR, the SEP is improved. For instance, given a required SEP of 10^{-3} , an increase of total SNR from 30 to 40dB can provide the opportunity to use 15 hops as opposed to 5 in a 6 path network.

For Nakagami- m channel model evaluation we use two specific networks defined as, $\text{Net}_1 \triangleq [4, 4, 4, 4]$ and $\text{Net}_2 \triangleq [4, 5, 5, 4]$. Thus, unlike 8 path in Rayleigh

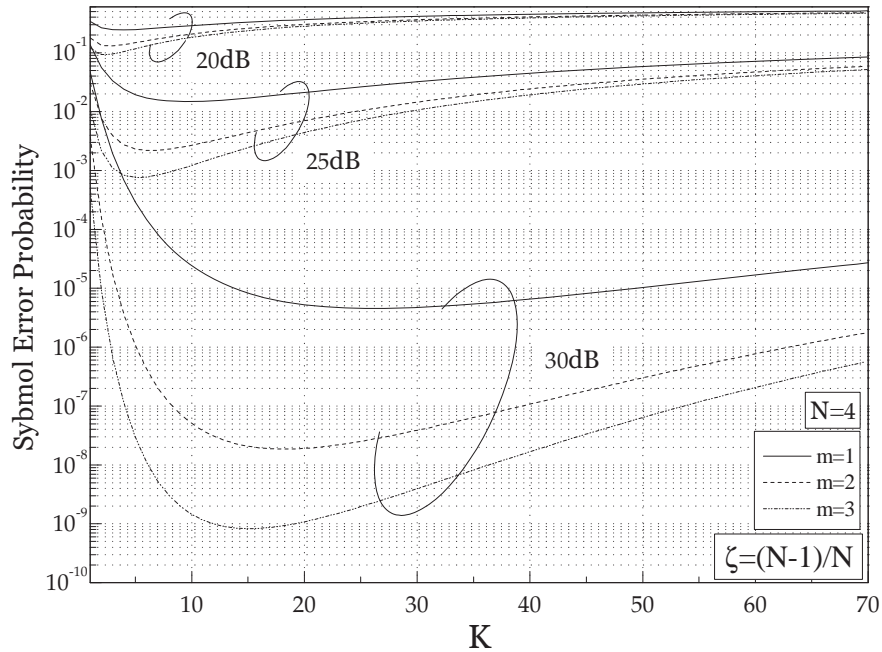


Figure 4.5: Symbol error probability of multihop parallel relay network with the number of parallel paths K in a 4 hop network over Nakagami- m fading channels with $m = 1, 2$ and 3 .

fading channels network simulation here we use only 4 path networks. Note that, Nakagami parameter $m = 1$ represents the Rayleigh fading environment in the network.

Fig. 4.4 plots the symbol error probability in Nakagami- m fading channels for Nakagami parameter $m = 1, 2, 3, 4, 5, 6$, and ∞ as a function of total SNR (P_{tot}/σ^2). We observe the high SNR slopes in Fig. 4.4 increase with the increase of Nakagami parameter m , this is due to the fact that the fading in the channel decreases as the value of m goes high, and eventually the plot $m = \infty$ refers to the AWGN channels. For comparison a *red* dashed line in the figure plots the SEP in Net_2 model with random Nakagami parameter m values as given in (4.27) \mathbf{m} below. For example, the 1st path has 4 hops and the value of m from 1st hop to 4th hop are 2, 5, 7 and 9 respectively. We can see, despite having higher m values in some random hops in Net_2 , when compare to i.i.d. network $[4, 4, 4, 4]$,

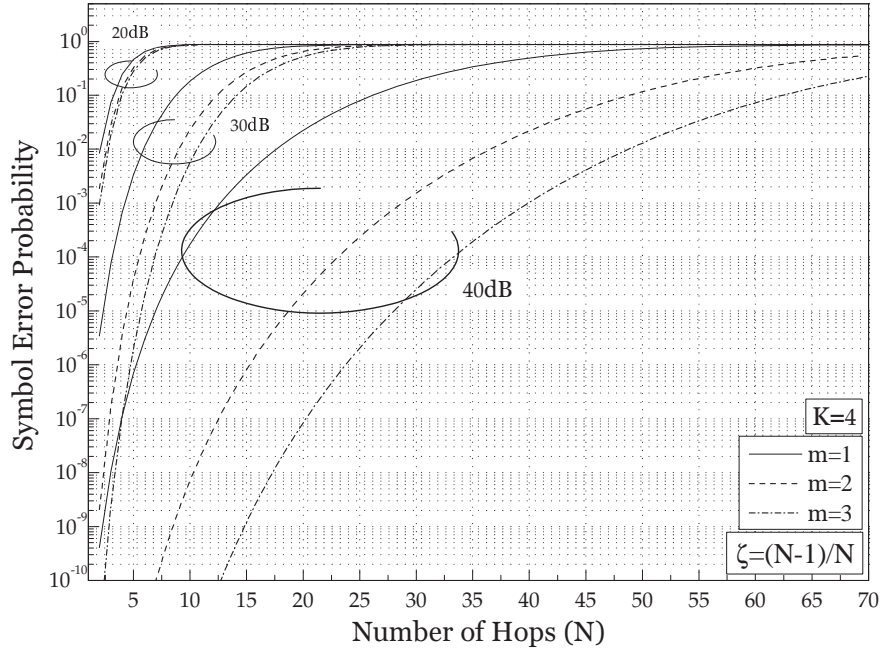


Figure 4.6: Symbol error probability of a 4 path multihop parallel relay network as a function of the number of hops N over Nakagami- m fading channels with $m = 1, 2$ and 3 .

the network $[4, 5, 5, 4]$ can provide only a reliability on average close to $m = 3$.

$$\mathbf{m} = \begin{bmatrix} 2 & 5 & 7 & 9 & . \\ 6 & 4 & 2 & 1 & 7 \\ 5 & 7 & 1 & 4 & 6 \\ 2 & 4 & 5 & 3 & . \end{bmatrix} \quad (4.27)$$

Fig. 4.5 plots the SEP with respect to the number of parallel paths K . We divide the figure in three different total SNR groups 20dB, 25dB and 30dB for three different Nakagami- m values $m = 1, 2$ and 3 . The SEP vs K curves take an upward bell shaped form which implies that there is a minimum point that optimizes the error performance. As K increases the allocation of power to each relay node is reduced and thus after a certain number of parallel paths the error

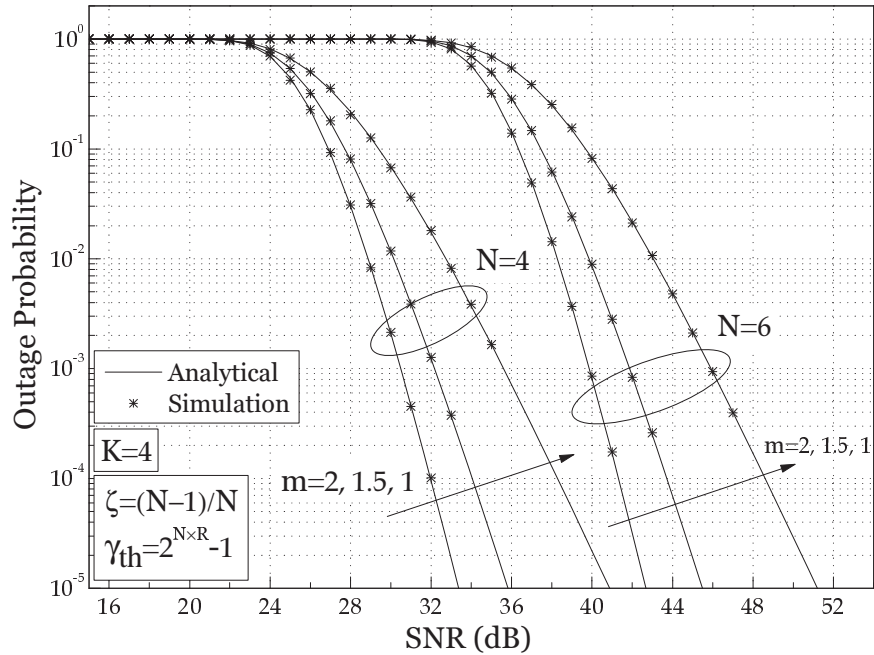


Figure 4.7: Outage probability of 4 path multihop relay network as a function of total power in Nakagami- m fading channels.

probability increases with K at fixed total SNR. However, before reaching the optimal number of path point the SEP reduces as well due to low diversity gain. As example, with a total SNR constraint to 30dB, in a 4 hop network we have to use 14 paths to achieve optimum SEP performance.

Fig. 4.6 plots the SEP behavior with respect to the number of hops N for same three different Nakagami- m values $m = 1, 2$ and 3 in a 4 path network. The figure divides the SEP into 3 different total SNR groups. It reveals that at an specific fading (constant m) with higher total SNR values the reliability (lower SEP) can significantly increase by using lower number of hops. The figure shows the the SEP increases exponentially with N . For instance, at fixed fading $m = 3$ and 40dB total SNR in a 4 parallel path network we can ensure a SEP of 10^{-6} by using 24 hops as opposed to a SEP of 10^{-3} in 40 hops network.

Since the analytical expression on outage probability supports non-integer Nakagami- m values compare to the SEP expression of Nakagami model, figures

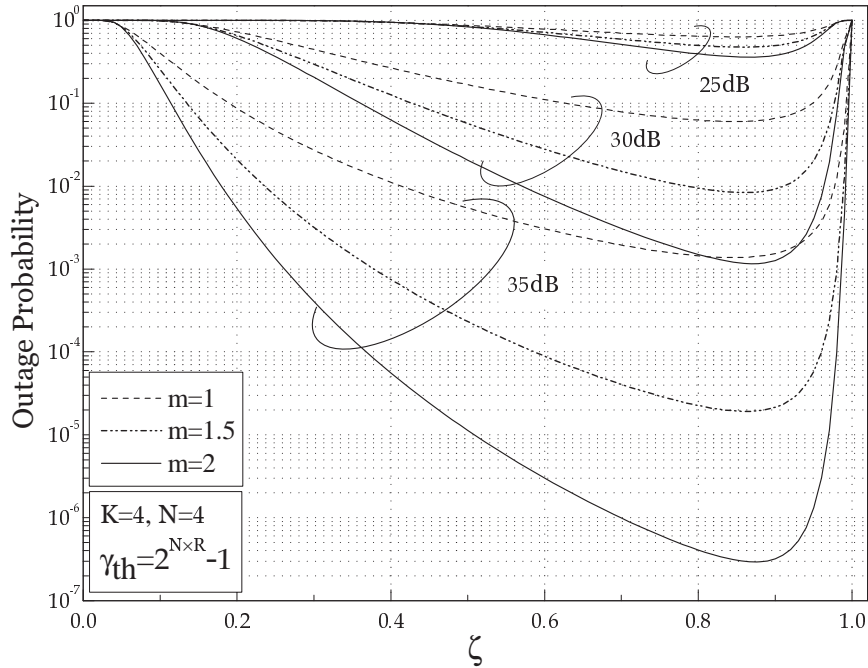


Figure 4.8: Outage probability of 4 path and 4 hop relay network as a function of power sharing coefficient ξ with different SNR groups in Nakagami- m fading channels.

on outage performance are plotted for three distinct Nakagami parameter $m = [1, 1.5, 2]$ considering a symmetrical multihop network $N_k = N_{k'} = N$. Similar as other results above, we assume the SNR threshold $\gamma_{\text{th}} = 2^{NR} - 1$, with a data rate $R = 1$ bps.

Fig. 4.7 plots the outage probability in Nakagami- m fading channels as a function of total power measured in SNR (dB) with 4 relays per layer scenario in 4 and 6 hop networks. In general outage probability is inversely related to the SNR. The network size in terms of the number of hops exposes a similar behavior on the outage. The figure indicates that an increase of 2 hops from 4, requires an SNR penalty of around 9dB to ensure an outage about 10^{-3} . Moreover the high SNR slope suggests that the diversity of the network is not effected by the number of hops rather than by the number of relaying paths. Fig. 4.8 and Fig. 4.9 show the outage probability behavior as a function of power sharing factor, ξ and

4.7 DISCUSSION

In this chapter closed form expression of symbol error probability and outage for a multihop K parallel path relay network over i.n.i.d. Rayleigh and Nakagami- m fading channels without centralized CSI using selective DF relays is presented. A general multihop multibranch network is considered for investigation. The derived expression can be used to assess the SEP performance of the network with arbitrary different network parameters such as, network size (K or N) and power sharing factor ζ . The SEP and outage possess a similar behavior when plotted as a function of total transmit SNR. Numerical results show that the outage and error probability can be reduced by increasing the number of relay nodes in each stage with sufficiently high SNR. Observation shows that the total power constraint plays the dominant role in case of power sharing between the relays and the source or in selecting the size of the network in terms of parallel paths.

Chapter 5

RELAY NETWORK IN INTERFERENCE

5.1 INTRODUCTION

Wireless channels often suffer severe performance degradation due to multipath fading and interference. The increase of spectral reuse in wireless systems exposes the network to ever-large number of interfering nodes. Ad-hoc and multiuser networks often face interference from other user nodes operating in multiple parallel channels. Even though, a substantial amount of research effort has been devoted to cooperative relay networks and interference networks, statistical analysis of cooperative network under interference and noise has not been performed to date. In this chapter, our goal is to derive the basic statistical expression for a cooperative relay networks in presence of interference. These expressions are fundamental to develop the systems mathematical performance model. We will derive the statistical properties, the CDF and the PDF of signal to interference plus noise ratio (SINR) of a dual hop interference relay network operating under arbitrary number of interferes. A CSI assisted hypothetical gain AF relay will be considered for cooperation in the network. Benefits of such hypothetical gain AF relays has been discussed in Chapter 2 in details. Using the statistical properties later, we will describe the procedures to derive the random coding error exponent of the interference relay network [13].

5.2 SYSTEM AND CHANNEL MODEL

Consider a single source-destination pair communicating via a single antenna relay without any direct link. We will denote source-relay and relay-destination links as S-R and R-D respectively. A half duplex AF protocol has been consid-

ered over independent and non-identically distributed (i.n.i.d.) Rayleigh fading channels. In this thesis, two different system models are investigated: system model 1 (SM 1), in which the interferers are only at the relay node; and system model 2 (SM 2), where interferers affect both the relay and destination nodes. In both models, all the interfering channels are i.n.i.d. Rayleigh faded. The destination is assumed to have full channel state information (CSI) of the two main channels, S-R and R-D, while the relay has full CSI of the S-R channel only. The source and relay have no CSI of forwarding transmitting channels. None of the nodes, source (S), relay (R) and destination (D) possess information about the interfering channels.

The instantaneous and average signal to noise ratios (SNR) of 1st and 2nd hops are denoted as $\gamma_i \triangleq \frac{P|h_i|^2}{\sigma_i^2}$ and $\lambda_i \triangleq \frac{P\Omega_i}{\sigma_i^2}$ respectively, where $i \in \{1, 2\}$, P is the corresponding source and relay power; h_i and Ω_i are the instantaneous and average channel gain of the i th hop respectively¹ and σ_i^2 is the one sided additive white Gaussian noise (AWGN) power at relay or destination node, i.e. $i \in \{R, D\}$. We assume the total power of the network (source and relay) is constrained to P_{tot} . This total power is split between the source and the relay by a power sharing coefficient $\zeta \in (0, 1]$ such that the source and the relay powers are given by, $P_S = (1 - \zeta)P_{\text{tot}}$ and $P_R = \zeta P_{\text{tot}}$ respectively. Thus if equal power sharing protocol is adopted, ζ will be 0.5.

Let there be a total of L interferers in the system, and define an interferer set \mathfrak{I} , the set of all interfering source nodes. For example, any interferer $I_l \in \mathfrak{I}$, where $l \in \mathcal{L}$, $\mathcal{L} = \{1, 2, 3, \dots, L\}$. The elements of interfering channel row vectors $\mathbf{h}_{I,i} \in C^L$ represent the corresponding interference channels from the source nodes of the interfering signal source vectors $\mathbf{x}_{I,i} \in C^L$, $i \in \{R, D\}$.²

¹The average channel gain, Ω_i is in fact the statistical average of the squared instantaneous channel gain h_i , i.e. $\Omega_i \triangleq \mathbb{E} [|h_i|^2]$.

² C^L denotes a L -dimensional complex vector.

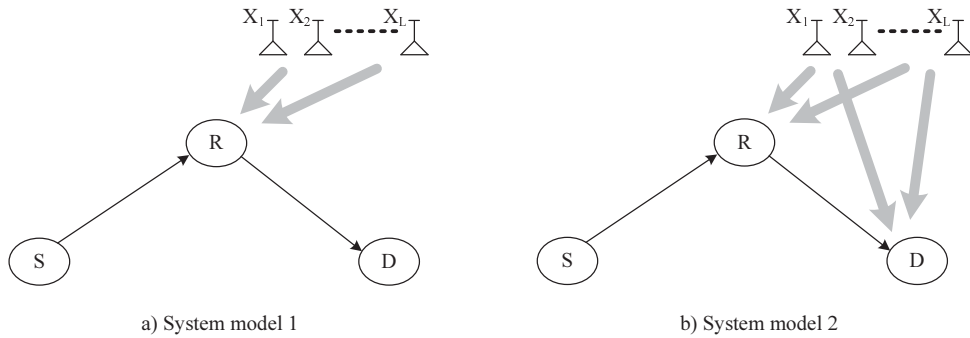


Figure 5.1: Interference relay network.

Furthermore, we assume the interferer I_l has individual transmit power $P_{I,l}$. Thus the instantaneous and the average interference power to noise ratio (INR) for any interferer I_l is $\gamma_{I,l} \triangleq \frac{P_{I,l}|h_{I,l}|^2}{\sigma_i^2}$ and $\lambda_{I,l} \triangleq \frac{P_{I,l}\Omega_{I,l}}{\sigma_i^2}$ respectively. $h_{I,l}$ and σ_i^2 are the fading channel gain from the interfering source I_l to the node i and the noise power at node i respectively, $l \in \mathcal{L}$ and $i \in \{R, D\}$. $\Omega_{I,l}$ is the average interfering channel gain, $\Omega_{I,l} \triangleq \mathbb{E}[|h_{I,l}|^2]$. Throughout this chapter the term INR will be used to indicate individual interferer signal power to noise ratio unless otherwise specified.

5.2.1 System Model 1: Interference at Relay

In the network model as shown in Fig. 5.1, interference occurs only at the relay node. The received signal at the relay node R is

$$y_R = h_1 x_s + \mathbf{h}_{I,1} \mathbf{x}_{I,1}^T + n_1 \quad (5.1)$$

where, $\mathbf{h}_{I,1} \in C^L$ and $\mathbf{x}_{I,1} \in C^L$ are the row vectors for the interference channels and the corresponding interfering signal sources respectively. The notation $(\cdot)^T$ represents the transpose operation on a vector. The signal at the destination

node is

$$y_D = Gh_2h_1x_s + Gh_2\mathbf{h}_{I,1}\mathbf{x}_{I,1}^T + Gh_2n_1 + n_2 \quad (5.2)$$

where G is the AF relay gain. $n_1 \sim \mathcal{CN}(0, \sigma_1^2)$ and $n_2 \sim \mathcal{CN}(0, \sigma_2^2)$ are additive white Gaussian noise at the relay and destination respectively.³ At the receiver node, the signal to interference plus noise ratio (SINR) will be

$$\gamma_{\text{SINR}_1} = \frac{G^2|h_1|^2|h_2|^2P_S}{G^2|h_2|^2\mathbf{h}_{I,1}\mathbf{\Sigma}_{I,1}\mathbf{h}_{I,1}^\dagger + G^2|h_2|^2\sigma_1^2 + \sigma_2^2} \quad (5.3)$$

where the diagonal matrix $\mathbf{\Sigma}_I = E\{\mathbf{x}_{I,1}^\dagger\mathbf{x}_{I,1}\}$ is composed of interference signal powers. With hypothetical AF relay gain $G^2 = \frac{P_R}{P_S|h_1|^2}$, the end-to-end SINR is [24]⁴,

$$\gamma_{\text{SINR}_1} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + \gamma_2\gamma_I} \quad (5.4)$$

where γ_1 and γ_2 are the instantaneous SNRs for S-R and R-D links respectively. γ_I is the *total* interference to noise ratio at the relay node, which is literally the sum of all individual INRs.

Note that due to the Rayleigh fading assumption, the first and second hop SNRs γ_1 and γ_2 are exponentially distributed with mean parameter λ_1 and λ_2 respectively. Later, it will be shown that under an equal power sharing protocol at the source and relay nodes, the system performance metric of system model 1 does not vary if the interferers are switched from the relay to the destination node.

³ $\mathcal{CN}(\lambda, \sigma^2)$ denotes a circularly symmetric complex Gaussian random variable with mean λ and variance σ^2 .

⁴A CSI assisted relay gain $G^2 = \frac{P_R}{|h_1|^2P_S + \sigma_1^2}$ proposed in [14] render the end-to-end SINR as $\gamma_{\text{SINR}} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + \gamma_2\gamma_I + 1}$. Eq. (5.4) proposes a tight upper bound on the CSI assisted SINR γ_{SINR} .

5.2.2 System Model 2: Interference at Relay and Destination

In this network model, interference occurs both at the relay and at the destination nodes. We let there be L_1 interferers at the relay node and L_2 interferers at the destination. All the interfering channels are independent and non-identically distributed. In this case, the received signal at the destination is

$$y_D = Gh_2h_1x_s + Gh_2\mathbf{h}_{I,1}\mathbf{x}_{I,1}^T + \mathbf{h}_{I,2}\mathbf{x}_{I,2}^T + Gh_2n_1 + n_2 \quad (5.5)$$

where $\mathbf{x}_{I,1} \in C^{L_1}$ and $\mathbf{x}_{I,2} \in C^{L_2}$ are the vectors with interference sources for the relay and receiver nodes respectively, and $\mathbf{h}_{I,1} \in C^{L_1}$ and $\mathbf{h}_{I,2} \in C^{L_2}$ are the corresponding fading channels from interferers to the relay and receiver nodes respectively. Again, as for SM 1, G is the AF relay gain, $n_1 \sim \mathcal{CN}(0, \sigma_1^2)$ and $n_2 \sim \mathcal{CN}(0, \sigma_2^2)$ are AWGN at the relay and the destination respectively. Thus the SINR with arbitrary relay gain is given by

$$\gamma_{\text{SINR}_2} = \frac{G^2|h_1|^2|h_2|^2P_S}{G^2|h_2|^2\mathbf{h}_{I,1}\boldsymbol{\Sigma}_{I,1}\mathbf{h}_{I,1}^\dagger + \mathbf{h}_{I,2}\boldsymbol{\Sigma}_{I,2}\mathbf{h}_{I,2}^\dagger + G^2|h_2|^2\sigma_1^2 + \sigma_2^2} \quad (5.6)$$

$\boldsymbol{\Sigma}_{I,1} = E\{\mathbf{x}_{I,1}^\dagger\mathbf{x}_{I,1}\}$ and $\boldsymbol{\Sigma}_{I,2} = E\{\mathbf{x}_{I,2}^\dagger\mathbf{x}_{I,2}\}$ are diagonal matrices of the transmission powers of interfering signals at the relay node and the destination respectively. Applying the similar hypothetical relay gain as used in SM 1 at the AF relay, the receiver SINR is given by

$$\gamma_{\text{SINR}_2} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + \gamma_1\gamma_{I,2} + \gamma_2\gamma_{I,1}} \quad (5.7)$$

where $\gamma_{I,1}$ and $\gamma_{I,2}$ are the total interference to noise ratio at the relay node and the destination respectively.

5.3 STATISTICAL ANALYSIS

In the following sections statistical theorems related to the network of interest are developed.

Definition 5.1: Let random variables X , Y and U be statistically independent, where $X \sim \mathcal{E}\left(\frac{1}{\lambda_x}\right)$ and $Y \sim \mathcal{E}\left(\frac{1}{\lambda_y}\right)$.⁵ Random variable $U = \sum_{l=1}^L U_l + 1$, where $U_l \sim \mathcal{E}\left(\frac{1}{\lambda_{u_l}}\right)$ are i.n.i.d. exponentially distributed random variables and U has the PDF, $f_U(u) = \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} \frac{X_{i,j}(\Lambda)(u-1)^{j-1} e^{-\frac{(u-1)}{\lambda_{u(i)}}}}{\Gamma(j)\lambda_{u(i)}^j}$,⁶ $u \geq 1$ [29]. A r.v. W is defined as

$$W = \frac{XY}{X + YU} \quad (5.8)$$

Theorem 5.1 (CDF and PDF): The cumulative distribution function (CDF) and probability density function (PDF) of random variable W are given in (5.9) and (5.10) respectively shown at the top of page 72, where $\psi_k \triangleq \frac{1}{2} \psi(k+1) + \frac{1}{2} \psi(k+2)$, and $\mathcal{I}_1(w, \lambda) \triangleq G_{3,2}^{1,3} \left(\begin{matrix} \lambda_x \lambda_{u(i)} \\ \lambda_x + w \lambda_{u(i)} \end{matrix} \middle| \begin{matrix} 1-j-n, 1, 1 \\ 1, 0 \end{matrix} \right) + 2\Gamma(j+n) \left\{ \ln \left(\frac{w}{\sqrt{\lambda_x \lambda_y}} \right) - \psi_k \right\}$. $G_{p,q}^{m,n} \left(x \middle| \begin{matrix} a_p \\ b_q \end{matrix} \right)$ and $\psi(x)$ are the Meijer-G function and the Euler psi function defined as [121, eq. 8.2.1.1] and [119, eq. 8.360.1] respectively.

Note that, exchanging X and Y in *Definition 1*, represents the switching of the interference from relay to destination node in (5.4). Hence, the CDF and PDF equations of SM1 are also applicable to the general case when either the relay or destination is subject to interference. In addition, the performance metric will be the same if an equal power sharing is adopted between the source and relay nodes, that is, when $\zeta = 0.5$.

⁵ $\mathcal{E}\left(\frac{1}{\lambda_x}\right)$ denotes an exponential distribution with a hazard rate $\frac{1}{\lambda_x}$.

⁶where $\Lambda = \text{diag}(\lambda_{u_1}, \lambda_{u_2}, \dots, \lambda_{u_L})$. $\zeta(\Lambda)$ is the number of distinct diagonal elements of Λ and $\lambda_{u(1)} > \lambda_{u(2)} > \dots > \lambda_{u(\zeta(\Lambda))}$ are the distinct diagonal elements in decreasing order. $\tau_i(\Lambda)$ is the multiplicity of $\lambda_{u(i)}$ and $X_{i,j}(\Lambda)$ is the (i, j) th characteristic coefficient of Λ [125].

$$\begin{aligned}
F_W(w) &= 1 - \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} X_{i,j}(\Lambda) e^{-w\left(\frac{1}{\lambda_x} + \frac{1}{\lambda_y}\right)} \left(\frac{\lambda_x}{\lambda_x + w\lambda_{u(i)}}\right)^j \left[1 + \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \binom{k+1}{n}\right] \\
&\times \frac{w^{2k+2}}{\Gamma(j)(\lambda_x\lambda_y)^{k+1}k!(k+1)!} \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}}\right)^n \left[2\Gamma(j+n) \left\{\ln\left(\frac{w}{\sqrt{\lambda_x\lambda_y}}\right) - \psi_k\right\}\right. \\
&\left. + G_{3,2}^{1,3} \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} \middle| \begin{matrix} 1-j-n, 1, 1 \\ 1, 0 \end{matrix} \right) \right] \quad (5.9)
\end{aligned}$$

$$\begin{aligned}
f_W(w) &= \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} X_{i,j}(\Lambda) e^{-w\left(\frac{1}{\lambda_x} + \frac{1}{\lambda_y}\right)} \left(\frac{\lambda_x}{\lambda_x + w\lambda_{u(i)}}\right)^j \left[\frac{1}{\lambda_x} + \frac{1}{\lambda_y} + \frac{j\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}}\right. \\
&- \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \binom{k+1}{n} \frac{w^{2k+1}}{\Gamma(j)(\lambda_x\lambda_y)^{k+1}k!(k+1)!} \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}}\right)^n \\
&\times \left[2\Gamma(j+n) + \mathcal{I}_1(w, \lambda) \left\{2k+2 - \frac{w(\lambda_x + \lambda_y)}{\lambda_x\lambda_y} - \frac{(j+n)w\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}}\right\}\right. \\
&\left. + \frac{w\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} G_{4,3}^{2,3} \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} \middle| \begin{matrix} 1-j-n, 1, 1, 0 \\ 1, 1, 0 \end{matrix} \right) \right] \quad (5.10)
\end{aligned}$$

Proof. See Appendix B.1. □

Definition 5.2: Let random variables X, Y, U and V are statistically independent where $X \sim \mathcal{E}\left(\frac{1}{\lambda_x}\right)$ and $Y \sim \mathcal{E}\left(\frac{1}{\lambda_y}\right)$. Random variables $U = \sum_{l=1}^{L_1} U_l + 1$ and $V = \sum_{l=1}^{L_1} V_l + 1$, U_l and V_l are i.n.i.d. exponentially distributed random variables with PDFs [29],

$$f_U(u) = \sum_{i=1}^{\zeta(\Lambda_1)} \sum_{j=1}^{\tau_i(\Lambda_1)} \frac{X_{i,j}(\Lambda_1)}{\Gamma(j)\lambda_{u(i)}^j} (u-1)^{j-1} e^{-\frac{(u-1)}{\lambda_{u(i)}}}, \quad u \geq 1 \quad (5.11)$$

$$f_V(v) = \sum_{p=1}^{\zeta(\Lambda_2)} \sum_{q=1}^{\tau_p(\Lambda_2)} \frac{X_{p,q}(\Lambda_2)}{\Gamma(q)\lambda_{v(p)}^q} (v-1)^{q-1} e^{-\frac{(v-1)}{\lambda_{v(p)}}}, \quad v \geq 1 \quad (5.12)$$

respectively. We define a r.v. Z such that

$$Z = \frac{XY}{XU + YV} \quad (5.13)$$

Theorem 5.2 (CDF and PDF): The cumulative distribution function (CDF) and probability density function (PDF) of random variable Z are given by eq. (5.14) and (5.15) respectively shown at the top of page 74.

Proof. See Appendix B.2. □

Proposition 5.1: Consider a system with L interferers in both relay and destination nodes where the interfering channels are i.i.d. Rayleigh faded. Adopting an equal power allocation protocol for source and relay node the cumulative distribution function of γ_{SINR_2} in (5.7) can be written as (5.16) shown at the top of the page 75.

Proof. See Appendix B.3. □

5.4 INTERFERENCE AT RELAY: I.I.D. NAKAGAMI- m INTERFERERS

Proposition 5.2: Suppose the interfering channels are i.i.d. Nakagami- m distributed while the main channels (S-R and R-D) are Rayleigh faded. In this case the CDF of γ_{SINR_2} can be written as (5.17) shown at page 75, where $\alpha_{I,i} = \lambda_{I,i}/m_i$, $i \in \{1, 2\}$. $\lambda_{I,1}$ and $\lambda_{I,2}$ are the average INR at the relay and the destination respectively; similarly, m_1 and m_2 are i.i.d. Nakagami- m parameter at the relay and destination respectively. The average SNRs of the 1st and 2nd hop are λ_1 and λ_2 respectively.

Proof. See Appendix B.4. □

5.5 OUTAGE PROBABILITY

In this section the derived analytical results are used to the investigate wireless network outage probability performance. We define outage probability as the

$$\begin{aligned}
F_Z(z) &= 1 - \sum_{i=1}^{\zeta(\Lambda_1)} \sum_{j=1}^{\tau_i(\Lambda_1)} \sum_{p=1}^{\zeta(\Lambda_2)} \sum_{q=1}^{\tau_p(\Lambda_2)} X_{i,j}(\Lambda_1) X_{p,q}(\Lambda_2) e^{-z\left(\frac{1}{\lambda_x} + \frac{1}{\lambda_y}\right)} \left(\frac{\lambda_y}{\lambda_y + z\lambda_{u\langle i \rangle}}\right)^j \\
&\times \left(\frac{\lambda_x}{\lambda_x + z\lambda_{v\langle p \rangle}}\right)^q \left[1 + \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \sum_{r=0}^{k+1} \binom{k+1}{n} \binom{k+1}{r} \frac{z^{2k+2}}{\Gamma(j)\Gamma(q)(\lambda_x\lambda_y)^{k+1} k!(k+1)!} \right. \\
&\times \left.\left(\frac{\lambda_y\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}}\right)^r \left(\frac{\lambda_x\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}}\right)^n \left[2\Gamma(j+r)\Gamma(q+n) \left\{ \ln\left(\frac{z}{\sqrt{\lambda_x\lambda_y}}\right) - \psi_k \right\} \right. \right. \\
&+ \Gamma(j+r) G_{3,2}^{1,3} \left. \left. \left(\frac{\lambda_x\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}} \middle| \begin{matrix} 1-q-n, 1, 1 \\ 1, 0 \end{matrix} \right) \right. \right. \\
&+ \left. \left. \Gamma(q+n) G_{3,2}^{1,3} \left(\frac{\lambda_y\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}} \middle| \begin{matrix} 1-j-r, 1, 1 \\ 1, 0 \end{matrix} \right) \right] \right] \quad (5.14)
\end{aligned}$$

$$\begin{aligned}
f_Z(z) &= \sum_{i=1}^{\zeta(\Lambda_1)} \sum_{j=1}^{\tau_i(\Lambda_1)} \sum_{p=1}^{\zeta(\Lambda_2)} \sum_{q=1}^{\tau_p(\Lambda_2)} X_{i,j}(\Lambda_1) X_{p,q}(\Lambda_2) e^{-z\left(\frac{1}{\lambda_x} + \frac{1}{\lambda_y}\right)} \left(\frac{\lambda_y}{\lambda_y + z\lambda_{u\langle i \rangle}}\right)^j \\
&\times \left(\frac{\lambda_x}{\lambda_x + z\lambda_{v\langle p \rangle}}\right)^q \left[\left\{ \frac{1}{\lambda_x} + \frac{1}{\lambda_y} + \frac{q\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}} + \frac{j\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}} \right\} - \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \sum_{r=0}^{k+1} \right. \\
&\times \frac{\binom{k+1}{n} \binom{k+1}{r} z^{2k+1}}{\Gamma(j)\Gamma(q)k!(k+1)!(\lambda_x\lambda_y)^{k+1}} \left(\frac{\lambda_y\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}}\right)^r \left(\frac{\lambda_x\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}}\right)^n \left[2\Gamma(q+n) \right. \\
&\times \left. \Gamma(j+r) + \mathcal{I}_2(z, \lambda) \left\{ 2k+2 - \frac{z(\lambda_x + \lambda_y)}{\lambda_x\lambda_y} - \frac{(j+r)z\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}} - \frac{(q+n)z\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}} \right\} \right. \\
&+ \frac{\Gamma(j+r)z\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}} G_{4,3}^{2,3} \left. \left. \left(\frac{\lambda_x\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}} \middle| \begin{matrix} 1-q-n, 1, 1, 0 \\ 1, 1, 0 \end{matrix} \right) \right. \right. \\
&+ \left. \left. \frac{\Gamma(q+n)z\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}} G_{4,3}^{2,3} \left(\frac{\lambda_y\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}} \middle| \begin{matrix} 1-j-r, 1, 1, 0 \\ 1, 1, 0 \end{matrix} \right) \right] \right] \quad (5.15)
\end{aligned}$$

where, $\mathcal{I}_2(z, \lambda) \triangleq 2\Gamma(q+n)\Gamma(j+r) \left\{ \ln \frac{z}{\sqrt{\lambda_x\lambda_y}} - \psi_k \right\} +$

$$\begin{aligned}
&\Gamma(j+r) G_{3,2}^{1,3} \left(\frac{\lambda_x\lambda_{v\langle p \rangle}}{\lambda_x + z\lambda_{v\langle p \rangle}} \middle| \begin{matrix} 1-q-n, 1, 1 \\ 1, 0 \end{matrix} \right) \\
&+ \Gamma(q+n) G_{3,2}^{1,3} \left(\frac{\lambda_y\lambda_{u\langle i \rangle}}{\lambda_y + z\lambda_{u\langle i \rangle}} \middle| \begin{matrix} 1-j-r, 1, 1 \\ 1, 0 \end{matrix} \right)
\end{aligned}$$

probability that the instantaneous receiver SINR falls below a predefined threshold value of SINR γ_{th} . We consider $\gamma_{\text{th}} = \rho(2^{NR} - 1)$, where ρ varies from 1 to 6.4 depending on the degree of coding, N be the number of hops and R the data

$$\begin{aligned}
F_{\gamma_{\text{SINR}_2}}(\gamma) &= 1 - e^{-2\gamma/\lambda} \left(\frac{\lambda}{\lambda + \gamma\lambda_I} \right)^{2L} - \frac{1}{\Gamma(L)^2} e^{-2\gamma/\lambda} \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \sum_{r=0}^{k+1} \binom{k+1}{n} \binom{k+1}{r} \\
&\quad \times \frac{\lambda_I^{n+r} \gamma^{2k+2}}{\lambda^{2k+2} k! (k+1)!} \left(\frac{\lambda}{\lambda + \gamma\lambda_I} \right)^{2L+n+r} \left[2\Gamma(L+n)\Gamma(L+r) \left\{ \ln \frac{\gamma}{\lambda} - \psi_k \right\} \right. \\
&\quad + \Gamma(L+r) G_{3,2}^{1,3} \left(\frac{\lambda\lambda_I}{\lambda + \gamma\lambda_I} \middle| \begin{matrix} 1-L-n, 1, 1 \\ 1, 0 \end{matrix} \right) \\
&\quad \left. + \Gamma(L+n) G_{3,2}^{1,3} \left(\frac{\lambda\lambda_I}{\lambda + \gamma\lambda_I} \middle| \begin{matrix} 1-L-r, 1, 1 \\ 1, 0 \end{matrix} \right) \right] \tag{5.16}
\end{aligned}$$

$$\begin{aligned}
F_{\gamma_{\text{SINR}_2}}(\gamma) &= 1 - e^{-\gamma(\frac{1}{\lambda_1} + \frac{1}{\lambda_2})} \left(\frac{\lambda_2}{\lambda_2 + \gamma\alpha_{I,1}} \right)^{m_1 L_1} \left(\frac{\lambda_1}{\lambda_1 + \gamma\alpha_{I,2}} \right)^{m_2 L_2} \\
&\quad \times \left[1 + \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \sum_{r=0}^{k+1} \binom{k+1}{n} \binom{k+1}{r} \frac{\gamma^{2k+2} (\lambda_1 \lambda_2)^{-k-1}}{\Gamma(m_1 L_1) \Gamma(m_2 L_2) k! (k+1)!} \left(\frac{\lambda_2 \alpha_{I,1}}{\lambda_2 + \gamma\alpha_{I,1}} \right)^r \right. \\
&\quad \times \left(\frac{\lambda_1 \alpha_{I,2}}{\lambda_1 + \gamma\alpha_{I,2}} \right)^n \left[2\Gamma(m_1 L_1 + r) \Gamma(m_2 L_2 + n) \left\{ \ln \left(\frac{\gamma}{\sqrt{\lambda_1 \lambda_2}} \right) - \psi_k \right\} \right. \\
&\quad + \Gamma(m_1 L_1 + r) G_{3,2}^{1,3} \left(\frac{\lambda_1 \alpha_{I,2}}{\lambda_1 + \gamma\alpha_{I,2}} \middle| \begin{matrix} 1 - m_2 L_2 - n, 1, 1 \\ 1, 0 \end{matrix} \right) \\
&\quad \left. \left. + \Gamma(m_2 L_2 + n) G_{3,2}^{1,3} \left(\frac{\lambda_2 \alpha_{I,1}}{\lambda_2 + \gamma\alpha_{I,1}} \middle| \begin{matrix} 1 - m_1 L_1 - r, 1, 1 \\ 1, 0 \end{matrix} \right) \right] \right] \tag{5.17}
\end{aligned}$$

rate in bits/s/Hz [73]. The CDF equations (5.9), (5.14) and (5.17) may be used to evaluate the outage probabilities in SM 1 and SM 2 with an arbitrary number of interferers and interfering powers.

5.6 ERROR EXPONENT AND ERGODIC CAPACITY

The analysis of fundamental capacity performance of interference network can be performed by formulating the random coding error exponent of the network. If we assume Gaussian input distribution as considered in Chapter 3 in a noise limited dual hop network, we can express the random coding error exponent

as [8],

$$E_r(R) = \max_{0 \leq \rho \leq 1} \{E_0(\rho) - 2\rho R\} \quad (5.18)$$

with

$$E_0(\rho) = -\ln \mathbb{E}_{\gamma_{d_1}} \left\{ \left(1 + \frac{\gamma}{1 + \rho} \right)^{-\rho} \right\} \quad (5.19)$$

$\mathbb{E}_X \{x\}$ denotes the statistical expectation operation over the random variable X . For the sake of simplicity we will study here in this section RCEE when interference occurs only at the relay node. However, the result can easily be extended to SM 2. Revoking the PDF of SINR_1 from (5.10) we modify (5.19)

$$\begin{aligned} E_0(\rho) = & -\ln \left[\int_0^\infty \left(1 + \frac{\gamma}{1 + \rho} \right)^{-\rho} \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} X_{i,j}(\Lambda) e^{-w(\frac{1}{\lambda_x} + \frac{1}{\lambda_y})} \left(\frac{\lambda_x}{\lambda_x + w\lambda_{u(i)}} \right)^j \right. \\ & \times \left[\frac{1}{\lambda_x} + \frac{1}{\lambda_y} + \frac{j\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} - \sum_{k=0}^\infty \sum_{n=0}^{k+1} \binom{k+1}{n} \frac{w^{2k+1}}{\Gamma(j)(\lambda_x\lambda_y)^{k+1} k!(k+1)!} \right. \\ & \times \left. \left. \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} \right)^n \left[2\Gamma(j+n) + \mathcal{I}_1(w, \lambda) \left\{ 2k+2 - \frac{w(\lambda_x + \lambda_y)}{\lambda_x\lambda_y} - \frac{(j+n)w\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} \right\} \right. \right. \right. \\ & \left. \left. \left. + \frac{w\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} G_{4,3}^{2,3} \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} \middle| \begin{matrix} 1-j-n, 1, 1, 0 \\ 1, 1, 0 \end{matrix} \right) \right] \right] \right] \right] d\gamma \quad (5.20) \end{aligned}$$

where $\psi_k \triangleq \frac{1}{2} \psi(k+1) + \frac{1}{2} \psi(k+2)$, and $\mathcal{I}_1(w, \lambda) \triangleq G_{3,2}^{1,3} \left(\frac{\lambda_x\lambda_{u(i)}}{\lambda_x + w\lambda_{u(i)}} \middle| \begin{matrix} 1-j-n, 1, 1 \\ 1, 0 \end{matrix} \right) + 2\Gamma(j+n) \left\{ \ln \left(\frac{w}{\sqrt{\lambda_x\lambda_y}} \right) - \psi_k \right\}$.

From (5.20) we can see, solving the part of integral involve Meijer-G function will solve the total error exponent problem. Thus, we will focus on the solution of expression below in general,

$$\begin{aligned}
\mathcal{J}_1 &= \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \binom{k+1}{n} \frac{X_{i,j}(\Lambda)}{\Gamma(j) (\lambda_1 \lambda_2)^{k+1} \lambda_1 \lambda_{I(i)}^j k! (k+1)!} \\
&\times \int_0^{\infty} \left(1 + \frac{\gamma}{1+\rho}\right)^{-\rho} \gamma^{2k+2} e^{-\gamma\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)} \left(\frac{\lambda_1 \lambda_I}{\lambda_1 + \gamma \lambda_{I(i)}}\right)^{n+j+1} \\
&\times G_{4,3}^{2,3} \left(\frac{\lambda_1 \lambda_{I(i)}}{\lambda_1 + \gamma \lambda_{I(i)}} \middle| \begin{matrix} 1-j-n, 1, 1, 0 \\ 1, 1, 0 \end{matrix} \right) d\gamma \tag{5.21}
\end{aligned}$$

In equation (5.21), the argument of Meijer- G function is a scalar shifted function of γ , we don't have any solution exists integral of a nonlinear argument of Meijer- G function. To avoid this problem we can transform the independent variable of the integral to a linear argument of Meijer- G function. Since only one interfering parameter involve in equation (5.21) we can drop the subscript i for $\lambda_{I(i)}$ if we want, let $\frac{\lambda_1 \lambda_I}{\lambda_1 + \gamma \lambda_I} = \frac{1}{\vartheta}$ we have,

$$\begin{aligned}
\mathcal{J}_{11} &= \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} \sum_{k=0}^{\infty} \sum_{n=0}^{k+1} \sum_{p=0}^{2k+2} \binom{k+1}{n} \binom{2k+2}{p} \frac{X_{i,j}(\Lambda)}{\Gamma(j) (\lambda_1 \lambda_2)^{k+1} \lambda_{I(i)}^{j+p} k! (k+1)!} \\
&\times a^{-\rho} \lambda_1^{2k+2} e^{\frac{\lambda_1}{\lambda_{I(i)}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)}} \int_{1/\lambda_{I(i)}}^{\infty} \left(1 + \frac{\lambda_1 \vartheta}{a(1+\rho)}\right)^{-\rho} \vartheta^{2k+2-p} e^{-\vartheta \lambda_{I(i)}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)} \\
&\times G_{4,3}^{2,3} \left(\frac{1}{\vartheta} \middle| \begin{matrix} 1-j-n, 1, 1, 0 \\ 1, 1, 0 \end{matrix} \right) d\vartheta \tag{5.22}
\end{aligned}$$

where, $a = 1 - \frac{\lambda_1}{\lambda_{I(i)}(1+\rho)}$. The solution of the integral is not possible in this format due to the integral range and is left for future work related to capacity analysis of cooperative relay network in interference.

5.7 NUMERICAL ANALYSIS

In this section, numerical results on the derived equations are presented and compared with Monte-Carlo simulations. Due to lack of transmitter CSI we assume

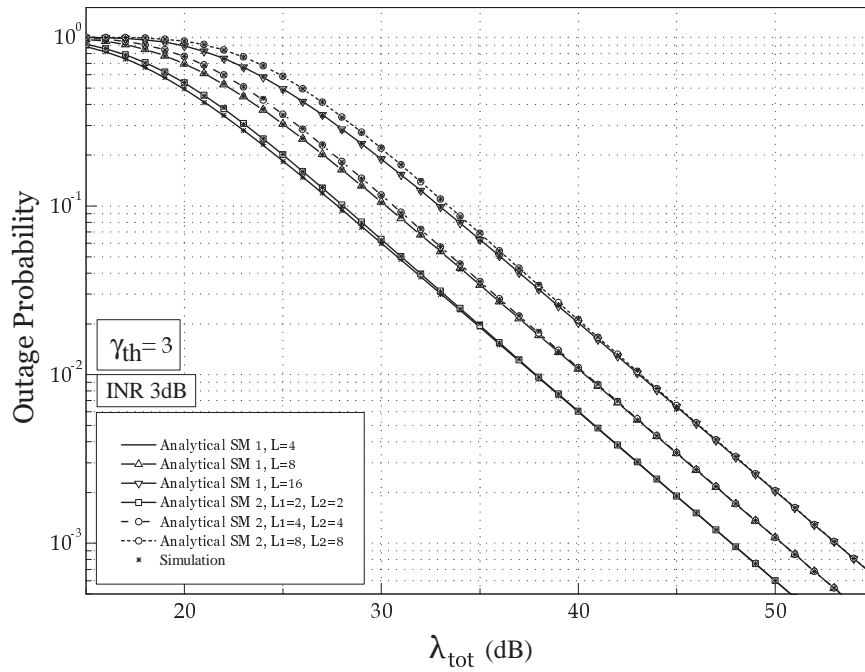


Figure 5.2: Outage probability vs total SNR in system model 1 and 2.

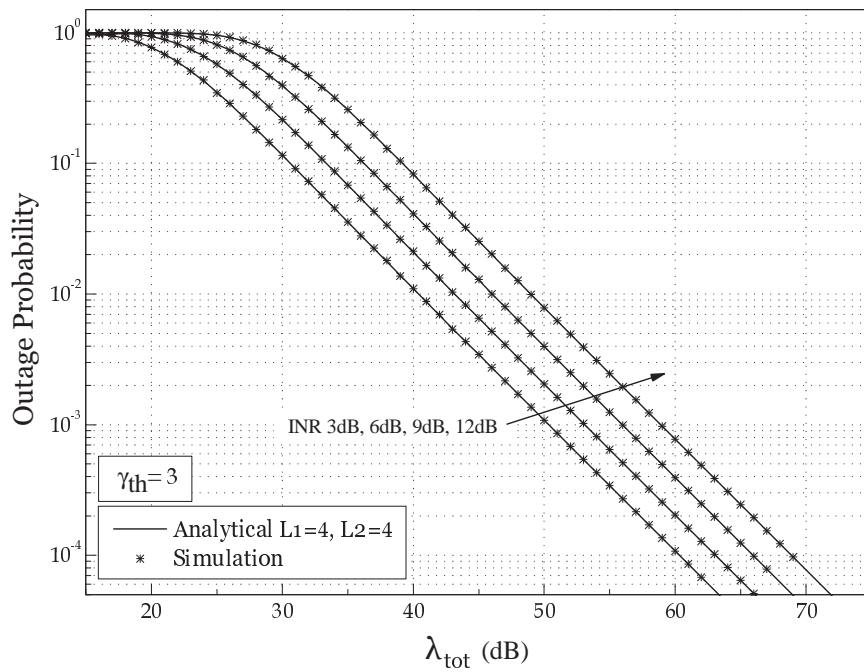


Figure 5.3: Outage probability vs total SNR in system model 2 with different level of INR's.

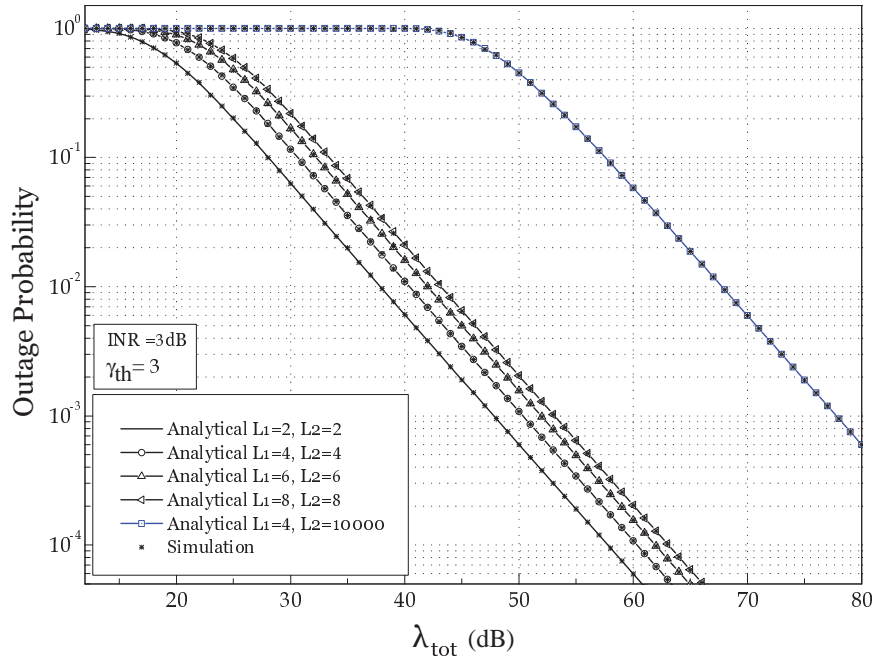


Figure 5.4: Outage probability vs total SNR in system model 2 with different number of interferers when INR=3 dB.

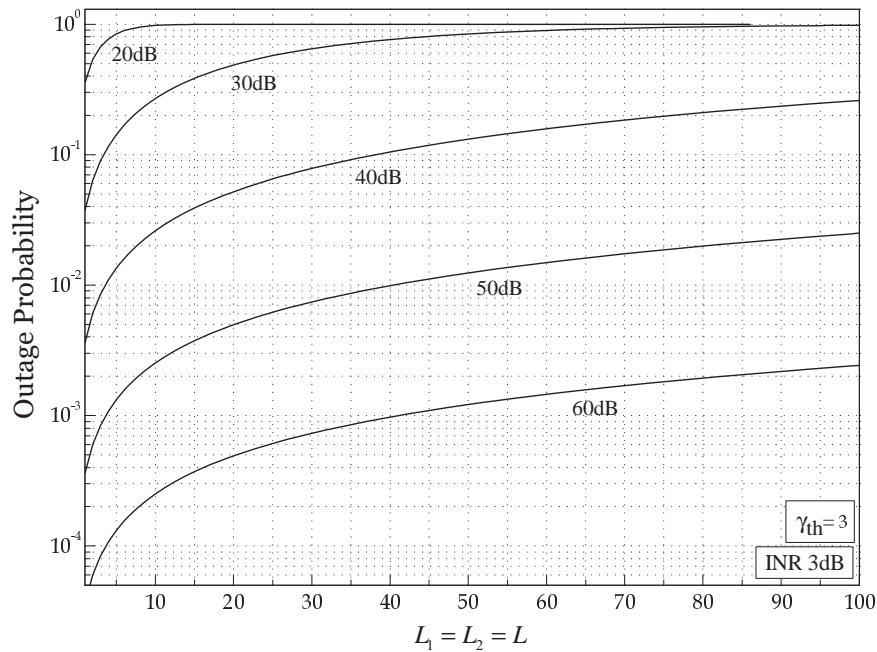


Figure 5.5: Outage probability as a function of total interferers at the relay and destination when the INR of each interferer is 3 dB.

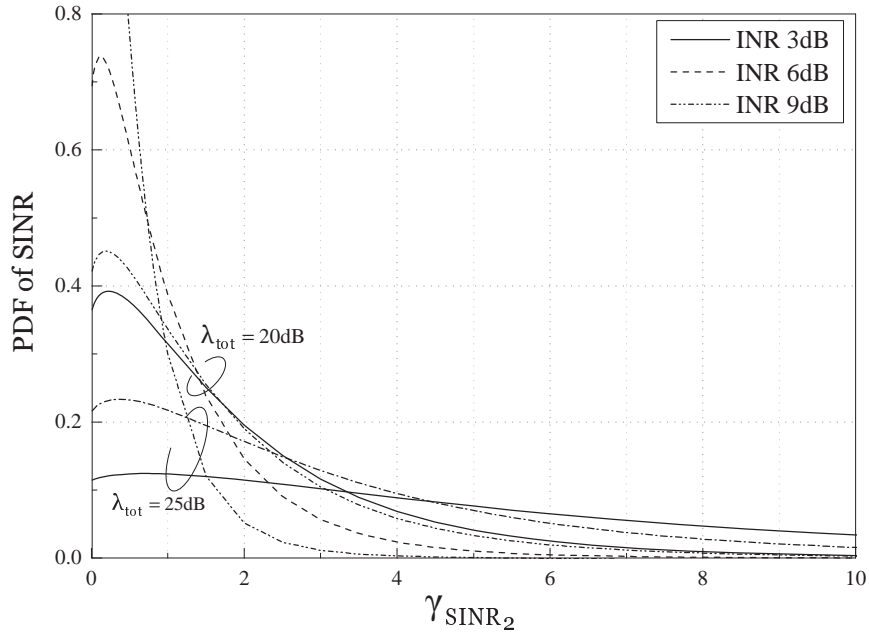


Figure 5.6: Probability density function of SINR of system model 2.

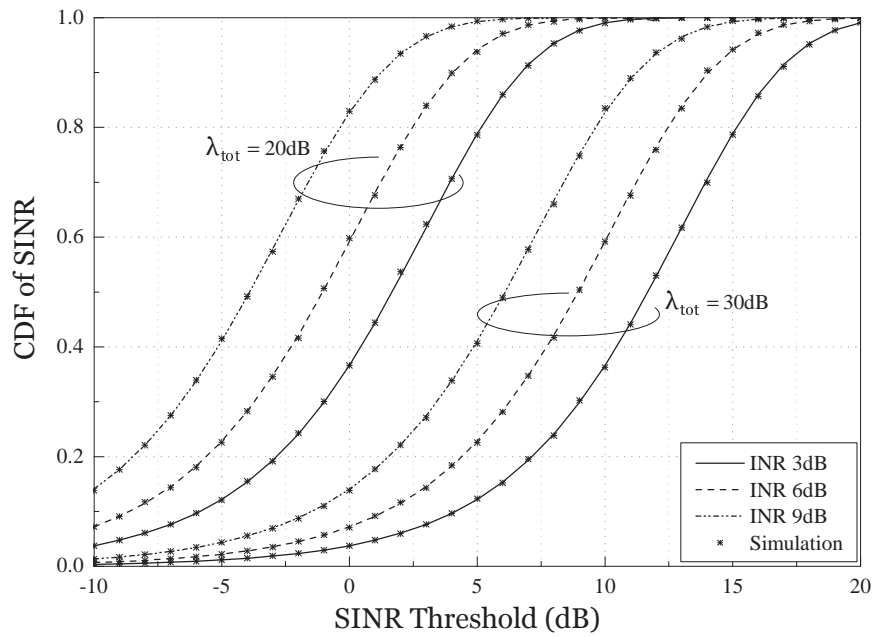


Figure 5.7: Cumulative distribution function of SINR of system model 2.

the source and the relay evenly share the total system power P_{tot} . Furthermore, for fair comparison with relevant studies the average channel gain of all hops in

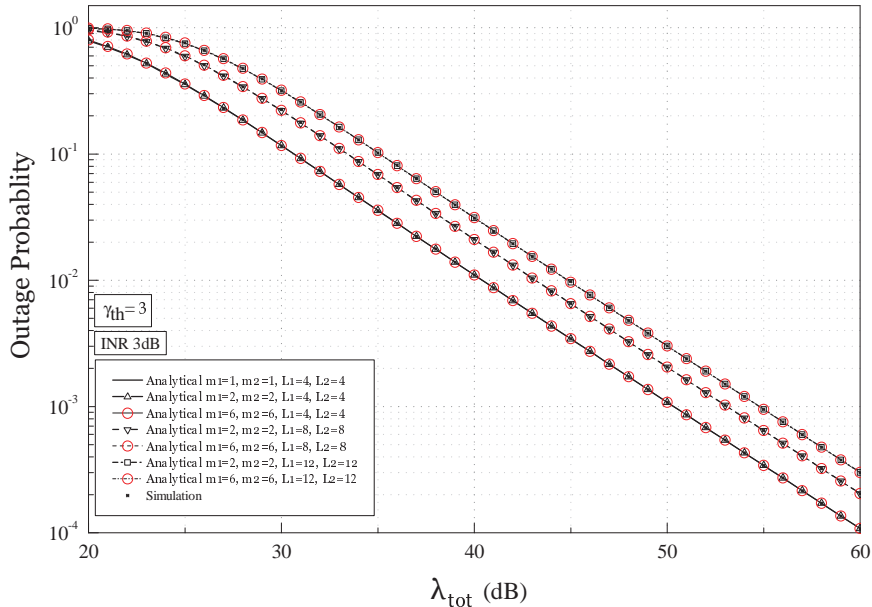


Figure 5.8: Outage probability vs total SNR over Nakagami- m faded interfering channels in system model 2 with different number of interferers.

this network are set to unity and the noise variance at the relay and destination nodes are set to σ^2 . We consider $\gamma_{\text{th}} = 3$ which achieves data rate $R = 1$ in a dual hop network unless otherwise specified. The maximum value of $k = 100$ for sum terms in (5.9) and (5.14) is used. The match with simulation results verify that $k = 100$ is sufficient to compute the analytical data and it also indicates the fast convergence of the relevant equations (5.9) and (5.14). The number of interferers at the relay and destination are assumed to be same ($L_1 = L_2$) in SM 2 networks, and all the interferers have equal transmit power. Throughout this section we will use total average transmit SNR, $\lambda_{\text{tot}} \triangleq P_{\text{tot}}/\sigma^2$ as a function in performance of SINR plots.

Fig. 5.2 shows the outage probability as a function of total transmit SNR for a total of $\{4, 8, 16\}$ interferers in SM 1 and SM 2 with INR fixed at 3 dB. All wireless channels (main channels and the interfering channels) are assumed to be Rayleigh faded. As expected, outage probability increases with an increase in the number of interferers. However, when SM 1 has the same number of

interferers as the total number of interferers for SM 2, SM1 shows slightly better performance compared to SM2 at a moderately low SINR region only. To study the OP performance with different interfering powers Fig. 5.3 shows outage probability as a function of λ_{tot} for a system with 4 interferers at the relay and 4 interferers at the destination. The INR values $\{3, 6, 9, 12\}$ dB are considered. Interestingly, the figure suggests that every 3 dB increase in average INR values requires a subsequent increase of 3 dB total SNR on average to ensure an OP of around 10^{-3} . In contrast, Fig. 5.4 shows OP in a fixed 3 dB INR system in SM 2 with a total of $\{4, 8, 12, 16\}$ number of interferers at the relay and destination respectively. To see how the interference system behaves if one of the nodes experiences most of the interference, Fig. 5.4 shows OP with a large L_2 and $L_1 = 4$. For very high interference levels e.g. $L_2 = 10000$ interferers at the destination node, a $\lambda_{\text{tot}} = 77$ dB is required to achieve an outage probability of 10^{-3} compared to 50dB when $L_2 = 4$.

Fig. 5.5 shows outage probability with number of interferers L in relay and destination where $L_1 = L_2 = L$ and each interferer has an INR of 3dB. The figure shows how the outage probability increases exponentially as the number of interferers increases. The plot includes graphs for transmit SNRs of 20, 30, 40, 50 and 60 dB. we observe that at 60 dB of total transmit SNR the system can support up to 40 interferers at the relay and destination, ensuring an outage probability of 10^{-3} , however, with a reduction of 10 dB in total transmit SNR the performance reduces drastically and it can support only 4 interferers at the relay and destination for the same outage probability.

The PDF of γ_{SINR_2} is presented in Fig. 5.6 for two different λ_{tot} , 20 dB and 25 dB, when the individual interferer INRs are $\{3, 6, 9\}$ dB. The figure implies that a lower INR per interferer increases the probability of higher output SINR at the receiver and similarly when the INR is constant, an increase in average total signal power results in a higher output SINR. In addition, Fig. 5.7 shows

the CDF of γ_{SINR_2} as a function of γ_{th} in dB for two λ_{tot} groups, 20 dB and 30 dB. Each group contains plots for 3 INRs 3, 6 and 9 dB. It reveals that higher average total SNR and lower INR per interferer decreases the probability of outage.

Finally, Fig. 5.8 presents a plot over Nakagami- m faded interfering channels. The outage probability is plotted as a function of λ_{tot} for different number of interferers and different Nakagami channel parameter m in SM 2. Surprisingly, varying the Nakagami m parameter does not result in any significant effects on outage performance if the number of interferers and INR remain constant in the network.

5.8 CONCLUSION

In this chapter, we derive the statistical functions such as the cumulative distribution function and the probability density function of a dual hop interference relay network with arbitrary number of interferers, where the main channels and the interfering channels are i.n.i.d. Rayleigh faded. Later the analysis is extended for i.i.d. Nakagami- m faded interfering channels. The analysis has been performed for two system models; in SM 1 interference is only at the relay node while in SM 2, interference is at both the relay and destination nodes. The derived CDF and PDF expressions for SM 1 can also be used for analyzing an interference network where interference occurs only at the destination node. Numerical results for outage probability performance for different network parameter configurations are presented. The Monte-Carlo simulations show an exact match with the analytical expressions. Results indicate that in a total INR constrained interference network, the number of interferers do not affect the performance of the system in a large scale. Similarly, if the interfering channels are Nakagami- m distributed, changing in Nakagami-parameter m does not affect the performance of the system if the number of interferers and the INR per interferer remain constant. Finally, we discuss about the random coding error exponent of interference relay

network that leads us to the capacity analysis of the network.

Chapter 6

NUMERICAL RESULTS AND DISCUSSION

In this chapter a comparison of numerical results on all the proposed analytical models is presented. The average channel gain of all channel hops is assumed to be unity and the noise variance at all relay nodes and at the receiver node is σ^2 . For related symbols, refer to the relevant chapters.

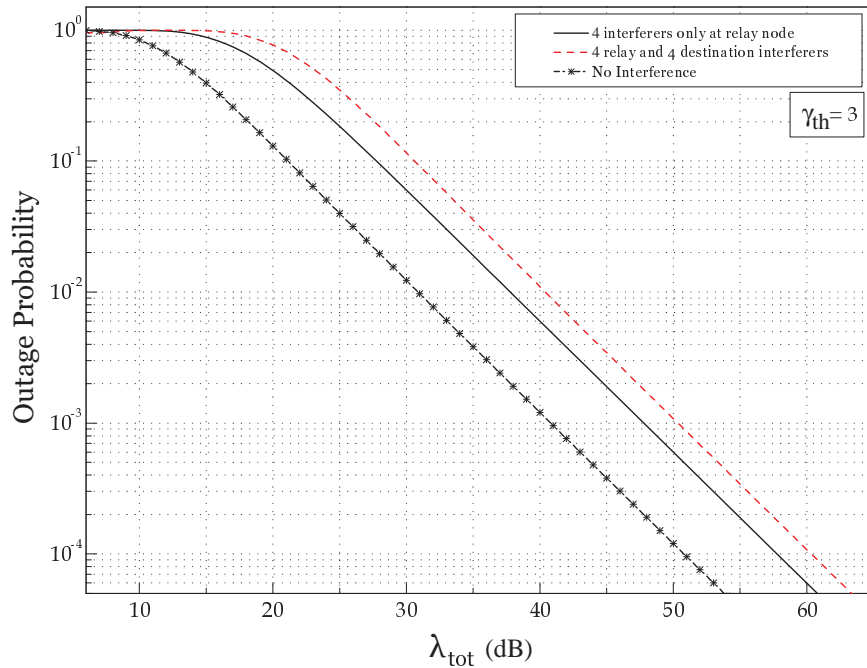


Figure 6.1: Comparison of outage probability as a function of total transmit SNR of cooperative relay network with and without interference.

In Fig. 6.1, the outage probability as a function of transmit SNR is compared for networks with and without interference. Furthermore, it is assumed that interference occurs only at the relay or both at the relay and destination nodes. Fig. 6.1 shows that network performance degrades due to the presence of inter-

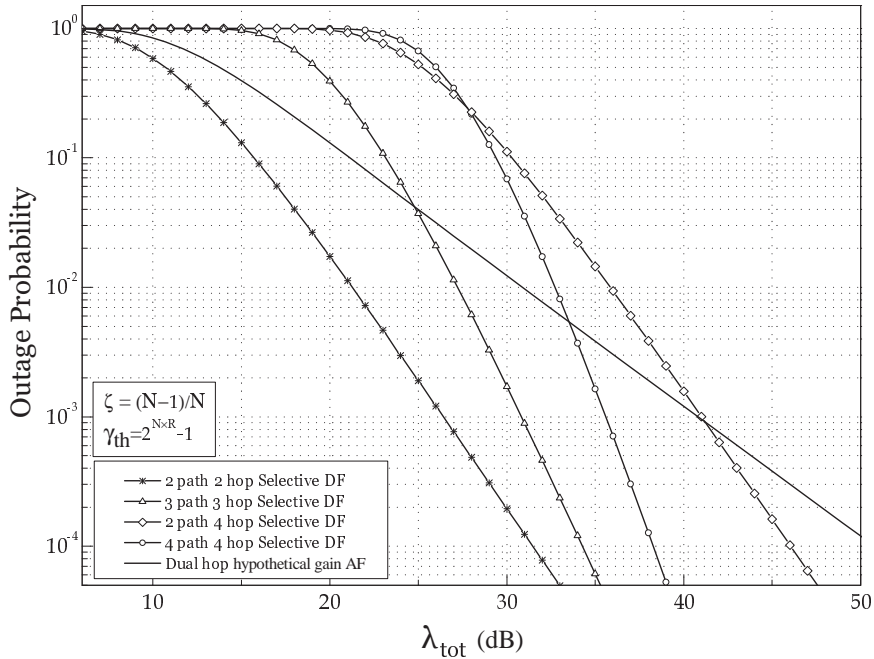


Figure 6.2: Outage probability as a function of total transmit SNR comparing multihop selective DF relay networks with dual hop hypothetical AF relay networks.

ference. Quantitatively, for a probability of outage 10^{-3} and total transmit SNR 47 dB the network can support 4 interferers at the relay node, while it requires 3 dB more power when 4 independent interferers are added at the destination as well as the 4 interferers at the relay node. An interference free network can provide the same outage probability only with 40 dB transmit SNR. It is also observed that diversity of the compared networks remains the same, even though interference in the network shifts the outage curves horizontally, penalizing networks by requiring more with total transmit power to achieve the target outage performance.

Fig. 6.2 shows the outage probability of multihop parallel relay networks and dual hop interference free networks as a function of total transmit SNR. Multihop network relays use selective DF protocol while dual hop single relay uses a hypothetical AF gain relay. Different combination of 2, 3, and 4 paths

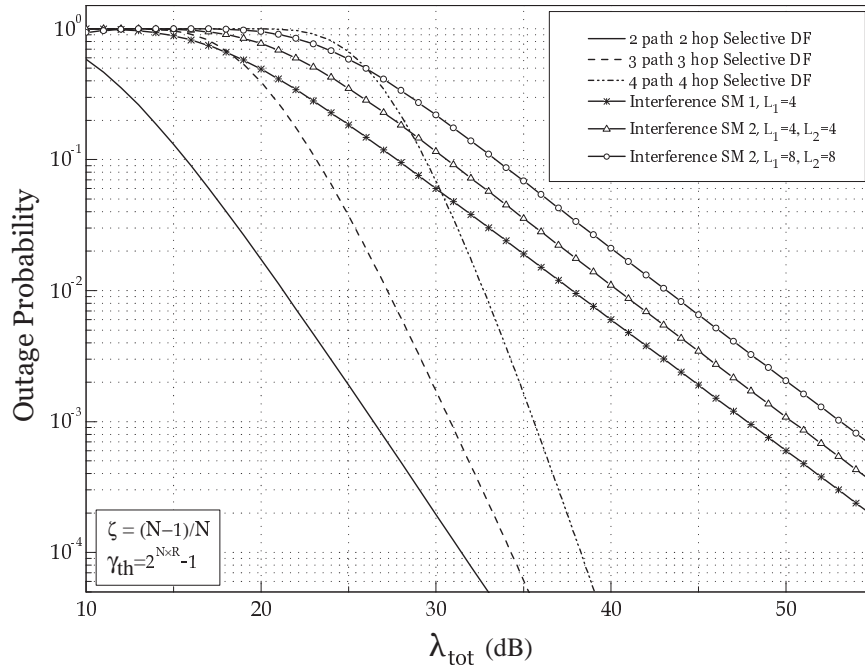


Figure 6.3: Outage probability as a function of total transmit SNR comparing multihop selective DF relay networks with dual hop interference network using hypothetical AF relay.

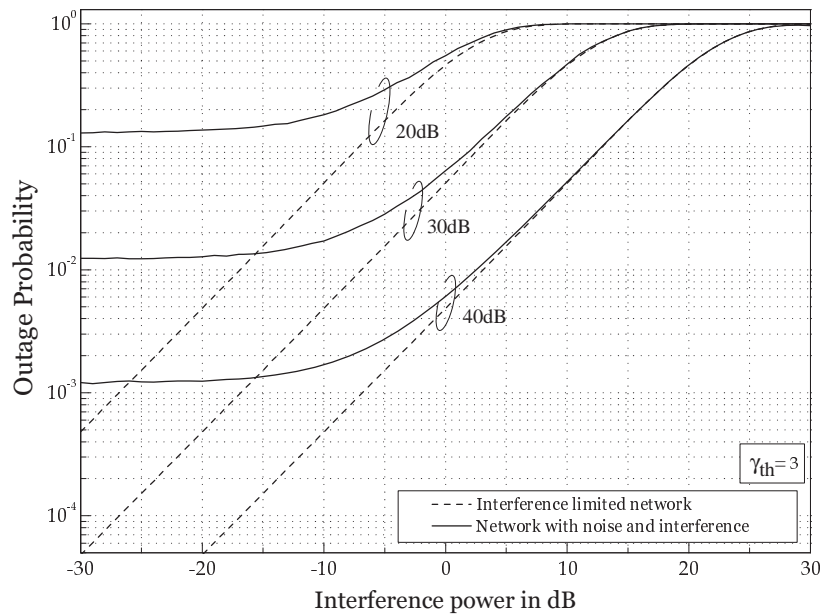


Figure 6.4: Outage probability as a function of interference power when the relay and the destination are subject to 4 interferers.

with 2, 3, and 4 hops are considered for the multihop network configuration. Fig. 6.3 plots outage probability behavior as a function of total transmit SNR for multihop parallel network and dual hop interference networks. Both figures show that above some value of total transmit SNR, multibranch networks outperform over both interference free and interference networks. Diversity gain provides the essential benefit for multihop parallel networks. From Fig. 6.2, it may be observed that as the number of parallel paths increases, the high-SNR diversity slopes increase while increasing the number of hops in the network shifts the outage curves horizontally only. Fig. 6.3 shows that interference networks offer the worst performance compared to all other networks (interference free dual hop and multihop parallel network with selective DF relays).

Fig. 6.4 shows the outage probability as a function of interference power when the relay and destination are both subject to 4 interferers. The figure compares the outage probability of an interference limited system (dashed line) studied in [114] to a system where both noise and interference are present. To express total transmit power and interference power in dB we normalize the terms considering noise power equal to 1 unit. Total transmission power of 20, 30 and 40 dB are considered in the figure and the power of each interferer is rated from -30 dB to +30 dB. Figure 6.4 shows that when the transmission power of each interferer is more than 10 dB, the outage probability of the considered network (noise plus interference) is similar to an interference limited network. However, performance will vary if the number of interferers in the network changes. The figure also illustrates how noise dominates the outage probability metric when interference power is low.

In summary, Fig. 6.1, Fig. 6.2, Fig. 6.3 and Fig. 6.4 provide an overview of the performance of different networks when compared in terms of outage metric. Even though, in dual hop networks hypothetical AF gain relays are used while selective DF relays are in multihop parallel networks, the performance measures

do not differ significantly if the relaying protocol is changed. The analytical approach to computing the outage probability in multihop networks is quite different to those using CSI assisted AF relays, and is not considered in this context. BER and outage performance using AF and DF relays have been analyzed in [24] showing that the performance metrics are very similar higher values of transmit SNR in dual hop relay networks.

Chapter 7

CONCLUSION

In this thesis, mathematical frameworks for performance metrics, such as the symbol error probability, outage probability and capacity were developed for cooperative relay networks. Cooperative communication increases the reliability of signal transmission and extends the coverage of networks while reducing interference. The main idea of cooperative communication is to utilize the broadcast nature of the wireless medium and send replicas of information messages using different channels. However, the key drawback of this technique is reduction in data rate. When large numbers of relaying nodes use multihop paths to reach the destination, the data rate reduced in the network by a factor of N , where N is the number of hops. The rate does not improve even if a number of parallel paths are created as compared to MIMO systems, where the data rate increases with the number of antenna systems in the transmitter and receiver. In Chapters 3, 4 and 5 mathematical performance models of cooperative networks were developed to assess cooperative system performance using different physical network parameters.

In Chapter 3, the random coding error exponent was discussed. The topic of RCEE is usually studied in information theory context to achieve a network capacity bound under some coding assumption. In this work, the RCEE result for dual hop amplify-and-forward relay networks has been derived assuming the relay can estimate the backward relay channel instantaneous CSI at the relay node. It was observed that the derived formula is mathematically intractable when CSI assisted relay gain is used compared to hypothetical relay gain. Some interesting numerical results provided an insight into the system, for example,

in a certain system setting increasing input SNR only 5 dB can provide a 3-fold reduction in the codeword length, which is highly beneficial when higher reliability at a reduced rate is desired.

In Chapter 4, multihop networks with decode-and-forward relays were investigated. The symbol error probability, outage probability and diversity of arbitrary multihop network were derived using different parameters. In this thesis, the resource allocation optimization problem regarding the network setup was not discussed. This may be an interesting problem for future study. Previous research results show that optimum relay selection schemes outperform optimum power allocation and equal power strategies. Thus, an opportunistic relay selection strategy can be formulated in future work. Despite not having any mathematical model, by using the presented numerical results it is possible to set the network parameters to operate the network at close to optimum performance. The numerical results show that the outage and error probability can be reduced by increasing the number of relay nodes in each stage beyond some high SNR levels. In addition, during error analysis it has been shown that a K -parallel path network can achieve its full diversity order of K under Rayleigh fading channels.

In Chapter 5, the dual hop network model was investigated for error exponent and statistical network parameters in presence of interference. To encompass different network scenarios, models where interference can occur only at the relay node or both at relay and receiver nodes were considered. The source was assumed to have no knowledge of interference or was assumed to not be affected by interference. When interference is known to the transmitter a priori, a code can be constructed utilizing the interference signal as information and thus can reduce the effect of interference. In this chapter, mathematical theories were developed which allow to observe the statistical nature of the performance variation of relay networks in interference to be predicted. Numerical results for

the outage probability function as a function of input signal to interference plus noise ratio were presented and the probability distribution of output signal to interference plus noise ratio were shown. Interesting results in numerical analysis show that when total power is equally divided between the transmitters (the source and the relay), interchanging the interferers from relay node to the receiver does not change outage performance. It also revealed that total interference power is the main factor that dominates the network performance rather than the number of interferers in the network. Considering all these analytical results, it is possible to design a cooperative network that is exposed to an arbitrary number of interferers.

In conclusion, utilizing the mathematical models developed using a wide range of network parameters in this thesis, it is possible to analyze relevant cooperative system performance and also to design efficient cooperative networks for optimum performance.

Appendix A

A.1 Proof of ordered X_n

Let the random variables X_n 's are i.n.i.d. Gamma distributed such that, $X_n \sim \Gamma(m_n, \lambda_n)$. We want to find,

$$\begin{aligned} \mathbb{P} \left\{ \min_{n \in \{1, 2, 3, \dots, N\}} X_n > \delta \right\} &= \prod_{n=1}^N \mathbb{P}(X_n > \delta) \\ &= \prod_{n=1}^N \int_{\delta}^{\infty} f_{X_n}(x) dx \end{aligned} \quad (\text{A.1})$$

The PDF of X_n is given as [3, eq. (2.21)],

$$f_{X_n}(x) = \frac{m_n^{m_n} x^{m_n-1}}{\lambda_n^{m_n} \Gamma(m_n)} e^{-xm_n/\lambda_n}$$

Thus, using the PDF in (A.3)

$$\mathbb{P} \left\{ \min_{n \in \{1, 2, 3, \dots, N\}} X_n > \delta \right\} = \prod_{n=1}^N \frac{\Gamma\left(m_n, \frac{m_n \delta}{\lambda_n}\right)}{\Gamma(m_n)} \quad (\text{A.2})$$

Similarly for,

$$\begin{aligned} \mathbb{P} \left\{ \min_{n \in \{1, 2, 3, \dots, N\}} X_n < \delta \right\} &= 1 - \prod_{n=1}^N \mathbb{P}(X_n > \delta) \\ &= 1 - \prod_{n=1}^N \int_{\delta}^{\infty} f_{X_n}(x) dx \\ &= 1 - \prod_{n=1}^N \frac{\Gamma\left(m_n, \frac{m_n \delta}{\lambda_n}\right)}{\Gamma(m_n)} \end{aligned} \quad (\text{A.3})$$

A.2 Proof of the CDF of random sum

Let the random variable $Y = \sum_{n=1}^N X_n$, X_n 's are gamma distributed i.n.i.d. random variables with parameters (m_n, λ_n) such that,

$$f_{X_n}(x) = \frac{x^{m_n-1} m_n^{m_n}}{\lambda_n^{m_n} \Gamma(m_n)} e^{-\frac{x m_n}{\lambda_n}}, \quad x \geq 0. \quad (\text{A.4})$$

Defining $\alpha_n = \frac{\lambda_n}{m_n}$, we have the MGF of Y as,

$$\begin{aligned} \Phi_Y(s) &= \mathbb{E} [e^{-sY}] \\ &= \prod_{n=1}^N (1 + s\alpha_n)^{-m_n} \end{aligned} \quad (\text{A.5})$$

For distinct values of m_n , the PDF of Y is the inverse Laplace transform of eq. (A.5), using [128, eq. (2.1.4.8)],

$$f_Y(y) = \left(\prod_{n=1}^N \frac{1}{\alpha_n^{m_n}} \right) \sum_{n=1}^N \sum_{l=1}^{m_n} \frac{\mathcal{A}_{n_l} \left(-\frac{1}{\alpha_n} \right) y^{m_n-l}}{(m_n-l)! (l-1)!} e^{-\frac{y}{\alpha_n}}, \quad y \geq 0. \quad (\text{A.6})$$

where $\mathcal{A}_{n_l}(s) = \frac{d^{l-1}}{ds^{l-1}} \left\{ \prod_{\substack{q=1 \\ q \neq n}}^N \left(s + \frac{1}{\alpha_q} \right)^{-m_q} \right\}$

Appendix B

B.1 Proof of Theorem 5.1

According to the PDF of X , Y and U as defined in *Definition 1*, the random variables X , Y and U are nonnegative, thus $F_W(w) = 0$ for $w < 0$. For $w \geq 0$ we have

$$\begin{aligned}
 F_W(w) &= \mathbb{P} \left\{ \frac{XY}{X + YU} \leq w \right\} \\
 &= \mathbb{P} \{X(Y - w) \leq wYU\} \\
 &= 1 - e^{-w/\lambda_y} + \int_1^\infty \int_w^\infty \int_0^{\frac{wyu}{y-w}} f_X(x) f_Y(y) f_U(u) dx dy du \quad (\text{B.1})
 \end{aligned}$$

The two inner integrals in (B.1) can easily be solved by using [119, eq. 3.351.3] and [119, eq. 3.471.9] and after some manipulation the CDF of W is obtained,

$$\begin{aligned}
 F_W(w) &= 1 - \sum_{i=1}^{\zeta(\Lambda)} \sum_{j=1}^{\tau_i(\Lambda)} \frac{2w X_{i,j}(\Lambda) e^{-w/\lambda_y}}{\Gamma(j) \lambda_{u^{(i)}}^j \sqrt{\lambda_x \lambda_y}} \int_1^\infty (u-1)^{j-1} \\
 &\quad \times \sqrt{u} e^{-(u-1)/\lambda_{u^{(i)}}} e^{-wu/\lambda_x} K_1 \left(2w \sqrt{\frac{u}{\lambda_x \lambda_y}} \right) du \quad (\text{B.2})
 \end{aligned}$$

where $K_\nu(x)$ is the ν th order modified Bessel function of the second kind. Using [119, eq. 8.446] we expand the first order modified Bessel function of second kind, then [121, eq. 8.4.6.5], [129, eq. 3.40.1.1] and [119, eq. 3.351.3] are applied to solve the related integrals. After some manipulation we arrive at the desired result (5.9). The PDF of W follows directly from the differentiation of the CDF

of W in eq. (5.9) w.r.t. w by using [121, eq. 8.2.2.30],

$$\frac{d}{dx} \left[x^\sigma G_{p,q}^{m,n} \left(x \left| \begin{matrix} a_p \\ b_q \end{matrix} \right. \right) \right] = -x^{\sigma-1} G_{p+1,q+1}^{m+1,n} \left(x \left| \begin{matrix} a_p, -\sigma \\ 1-\sigma, b_q \end{matrix} \right. \right) \quad (\text{B.3})$$

B.2 Proof of Theorem 5.2

Following a similar argument as used in the proof of *Theorem 5.1*, the random variables X , Y , U and V are nonnegative, and thus $F_Z(z) = 0$ for $z < 0$. For $z \geq 0$,

$$\begin{aligned} F_Z(z) &= \mathbb{P} \left\{ \frac{XY}{XU + YV} \leq z \right\} \\ &= \mathbb{P} \{ X(Y - zU) \leq zYV \} \\ &= \underbrace{\int_1^\infty \int_0^{uz} f_Y(y) f_U(u) dy du}_{\triangleq I_{uy}} \\ &\quad + \underbrace{\int_1^\infty \int_1^\infty \int_{uz}^\infty \int_0^{\frac{zyv}{y-zu}} f_X(x) f_Y(y) f_V(v) f_U(u) dx dy dv du}_{\triangleq I_{uvyx}} \end{aligned} \quad (\text{B.4})$$

The first part of this expression, integral I_{uy} can be solved by using [119, eq. 3.351.3], resulting in

$$I_{uy} = 1 - \sum_{i=1}^{\zeta(\Lambda_1)} \sum_{j=1}^{\tau_i(\Lambda_1)} X_{i,j}(\Lambda_1) e^{-z/\lambda_y} \left(\frac{\lambda_y}{\lambda_y + z\lambda_{u\langle i \rangle}} \right)^j \quad (\text{B.5})$$

In I_{uvyx} , the first two integrals for variables x and y can be solved using a similar approach, via [119, eq. 3.351.3], [119, eq. 3.471.9], [119, eq. 8.446], [121, eq. 8.4.6.5] and [129, eq. 3.40.1.1]. Substituting (B.5) in (B.4) and after some manipulation the CDF of Z can be written as (B.6) at the top of the next page, where $\psi_k \triangleq \frac{1}{2} \psi(k+1) + \frac{1}{2} \psi(k+2)$. Now using [119, eq. 3.351.3], [121, eq. 8.4.6.5]

$$\begin{aligned}
F_Z(z) &= 1 - \sum_{i=1}^{\zeta(\Lambda_1)} \sum_{j=1}^{\tau_i(\Lambda_1)} \sum_{p=1}^{\zeta(\Lambda_2)} \sum_{q=1}^{\tau_p(\Lambda_2)} \frac{X_{i,j}(\Lambda_1) X_{p,q}(\Lambda_2)}{\Gamma(j) \Gamma(q) \lambda_{u\langle i \rangle}^j \lambda_{v\langle p \rangle}^q} \left[e^{-z/\lambda_x} \Gamma(q) \left(\frac{\lambda_x \lambda_{v\langle p \rangle}}{\lambda_x + z \lambda_{v\langle p \rangle}} \right)^q \right. \\
&\times \int_1^\infty (u-1)^{j-1} e^{-\frac{u-1}{\lambda_{u\langle i \rangle}} - \frac{uz}{\lambda_y}} du + \sum_{k=0}^\infty \sum_{n=0}^{k+1} \binom{k+1}{n} \frac{z^{2k+2} e^{-z/\lambda_x}}{k! (k+1)! (\lambda_x \lambda_y)^{k+1}} \left(\frac{\lambda_x \lambda_{v\langle p \rangle}}{\lambda_x + z \lambda_{v\langle p \rangle}} \right)^{q+n} \\
&\times G_{3,2}^{1,3} \left(\frac{\lambda_x \lambda_{v\langle p \rangle}}{\lambda_x + z \lambda_{v\langle p \rangle}} \middle| \begin{matrix} 1-q-n, 1, 1 \\ 1, 0 \end{matrix} \right) \int_1^\infty u^{k+1} (u-1)^{j-1} e^{-\frac{u-1}{\lambda_{u\langle i \rangle}} - \frac{uz}{\lambda_y}} du \\
&+ \sum_{k=0}^\infty \sum_{n=0}^{k+1} \binom{k+1}{n} \frac{2z^{2k+2} e^{-z/\lambda_x} \Gamma(q+n)}{k! (k+1)! (\lambda_x \lambda_y)^{k+1}} \left(\frac{\lambda_x \lambda_{v\langle p \rangle}}{\lambda_x + z \lambda_{v\langle p \rangle}} \right)^{q+n} \\
&\times \int_1^\infty u^{k+1} (u-1)^{j-1} e^{-\frac{u-1}{\lambda_{u\langle i \rangle}} - \frac{uz}{\lambda_y}} \left\{ \ln \sqrt{\frac{u}{\lambda_x \lambda_y}} z - \psi_k \right\} du \Big] \tag{B.6}
\end{aligned}$$

and [129, eq. 3.40.1.1] it is possible to solve (B.6), which then results in the desired eq. (5.14). Eq (5.15) directly follows the differentiation of eq.(5.14) utilizing the property [121, eq. 8.2.2.30].

B.3 Proof of Proposition 5.1

When the interfering signals are from L equal power sources with i.i.d. interfering channels, $\lambda_{u\langle i \rangle} = \lambda_u = \lambda_I$, $\lambda_{v\langle p \rangle} = \lambda_v = \lambda_I$ and the characteristic coefficient $X_{i,j}(\Lambda)$ of Λ becomes [29],

$$X_{i,j}(\Lambda) = \begin{cases} 0 & j = 1, 2, 3.. \\ 1 & j = L \end{cases} \tag{B.7}$$

Using the above facts the proof of the *Proposition 5.1* immediately follows from *Theorem 5.2*. Due to equal power allocation, average 1st and 2nd hop SNR $\lambda_1 = \lambda_2 = \lambda$.

B.4 Proof of Proposition 5.2

If the interfering channels are i.i.d. Nakagami- m faded, in *Definition 5.2*, the PDF of r.v. U and V can be modified to,

$$f_U(u) = \frac{(u-1)^{L_1 m_1 - 1}}{\Gamma(L_1 m_1) \alpha_u^{L_1 m_1}} e^{-\frac{(u-1)}{\alpha_u}}, \quad u \geq 1 \quad (\text{B.8})$$

and

$$f_V(v) = \frac{(v-1)^{L_2 m_2 - 1}}{\Gamma(L_2 m_2) \alpha_v^{L_2 m_2}} e^{-\frac{(v-1)}{\alpha_v}}, \quad v \geq 1 \quad (\text{B.9})$$

respectively, where $\alpha_u = \lambda_u/m_u$ and $\alpha_v = \lambda_v/m_v$ respectively, and m is the corresponding Nakagami- m channel parameter. The r.v.s X and Y are exponentially distributed with hazard rate $1/\lambda_x$ and $1/\lambda_y$ respectively. The CDF of $F_{\gamma_{\text{SINR}_2}}(\gamma)$ for i.i.d. Nakagami- m interfering channels can be obtained by replacing $\lambda_u = \alpha_u$, $\lambda_v = \alpha_v$, $j = L_1 m_1$ and $q = L_2 m_2$ in (5.14).

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