Qualitative Constraint Satisfaction Problems: Algorithms, Computational Complexity, and Extended Framework

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Student

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Abstract

Qualitative Spatial and Temporal Reasoning (QSTR) is a subfield of artificial intelligence that represents and reasons with spatial/temporal knowledge in a qualitative way. In the past three decades, researchers have proposed dozens of relational models (known as qualitative calculi), including, among others, Point Algebra (PA) and Interval Algebra (IA) for temporal knowledge, Cardinal Relation Algebra (CRA) and Cardinal Direction Calculus (CDC) for directional spatial knowledge, and the Region Connection Calculus RCC-5/RCC-8 for topological spatial knowledge. *Relations* are used in qualitative calculi for representing spatial/temporal information (e.g. Germany is to the east of France) and constraints (e.g. the to-be-established landfill should be disjoint from any lake).

The reasoning tasks in QSTR are formalised via the *qualitative constraint satisfaction problem* (QCSP). As the central reasoning problem in QCSP, the *consistency problem* (which decides the consistency of a number of constraints in certain qualitative calculi) has been extensively investigated in the literature. For PA, IA, CRA, and RCC-5/RCC-8, the consistency problem can be solved by composition-based reasoning. For CDC, however, composition-based reasoning is incomplete, and the consistency problem in CDC remains challenging.

Previous works in QCSP assume that qualitative constraints only concern completely unknown entities. Therefore, constraints about *landmarks* (i.e., fixed entities) cannot be properly expressed. This has significantly restricted the usefulness of QSTR in real-world applications.

The main contributions of this thesis are as follows.

(i) The composition-based method is one of the most important reasoning methods in QSTR. This thesis designs a semi-automatic algo-

rithm for generating composition tables for general qualitative calculi. This provides a partial answer to the challenge proposed by Cohn in 1995.

(ii) Schockaert et al. (2008) extend the RCC models interpreted in Euclidean topologies to the fuzzy context and show that composition-based reasoning is sufficient to solve fuzzy QCSP, where 31 composition rules are used. This thesis first shows that only six of the 31 composition rules are necessary, and then introduces a method which consistently fuzzifies any classical RCC models. This thesis also proposes a polynomial algorithm for realizing solutions of consistent fuzzy RCC constraints.

(iii) Composition-based reasoning is incomplete for solving QCSP over the CDC. This thesis provides a cubic algorithm which for the first time solves the consistency problem of complete basic CDC networks, and further shows that the problem becomes NP-complete if the networks are allowed to be incomplete. This draws a sharp boundary between the tractable and intractable subclasses of the CDC.

(iv) This thesis proposes a more general and more expressive QCSP framework, in which a variable is allowed to be a landmark (i.e., a fixed object), or to be chosen among several landmarks. The computational complexity of the consistency problems in the new framework is then investigated, covering all qualitative calculi mentioned above. For basic networks, the consistency problem remains tractable for Point Algebra, but becomes NP-complete for all the remaining qualitative calculi. A special case in which a variable is either a landmark or is totally unknown has also been studied.

(v) A qualitative network is *minimal* if it cannot be refined without changing its solution set. Unlike the assumptions in the literature, this thesis shows that computing a solution of minimal networks is NP-complete for (partially ordered) PA, CRA, IA, and RCC-5/RCC-8. As a by-product, it has also been proved that determining the minimality of networks in these qualitative calculi is NP-complete.