

New optimization techniques for point feature and general curve feature based SLAM

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Certificate

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Abstract

This doctoral thesis deals with the feature based Simultaneous Localization and Mapping (SLAM) problem. SLAM as defined in this thesis is the process of concurrently building up a map of the environment and using this map to obtain improved estimates of the location of the robot. In feature based SLAM, the robot relies on its ability to extract useful navigation information from the data returned by its sensors. The robot typically starts at an unknown location without priori knowledge of feature locations. From relative observations of features and relative pose measurements, estimates of entire robot trajectory and feature locations can be derived. Thus, the solution to SLAM problem enables an autonomous vehicle navigates in a unknown environment autonomously. The advantage of eliminating the need for artificial infrastructures or a priori topological knowledge of the environment makes SLAM problem one of the hot research topics in the robotics literature. Solution to the SLAM problem would be of inestimable value in a range of applications such as exploration, surveillance, transportation, mining etc.

The critical problems for feature based SLAM implementations are as follows: 1) Because SLAM problems are high dimensional, nonlinear and non-convex, when solving SLAM problems, robust optimization techniques are required. 2) When the environment is complex and unstructured, appropriate parametrization method is required to represent environments with minimum information loss. 3) As robot navigates in the environment, the information acquired by the onboard sensor increases. It is essential to develop computationally tractable SLAM algorithms especially for general curve features.

This thesis presents the following contributions to feature based SLAM. First, a convex optimization based approach for point feature SLAM problems is developed. Using the proposed method, a unique solution can be obtained without any initial state estimates. It will be shown that, the unique SDP solution obtained from the proposed method is very close to the true solution to the SLAM problem. Second, a general curve feature based SLAM formulation is presented. Instead of scattered points, in this formulation, the envi-

ronment is represented by a number of continuous curves. Using the new formulation, all the available information from the sensor is utilized in the optimization process. Third, method for converting curve feature to point feature is presented. Using the conversion method, the curve feature SLAM problem can be transferred to point feature SLAM problem and can be solved by the convex optimization based approach.