

# Concurrent Pattern Unification

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## Certificate of Authorship/Originality

I certify that the work in this dissertation has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that this dissertation has been written by me. Any help that I have received in my research work and the preparation of this dissertation has been acknowledged. In addition, I certify that all information sources and literature used are indicated within this dissertation.

Signature of Student

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**Abstract**

Ever since Milner showed that Church's  $\lambda$ -calculus can be subsumed by  $\pi$ -calculus, process calculi have been expected to subsume sequential computation. However, Jay & Given-Wilson show that extensional sequential computation as represented by  $\lambda$ -calculus is subsumed by intensional sequential computation characterised by pattern-matching as in  $SF$ -calculus. Given-Wilson, Gorla & Jay present a concurrent pattern calculus (CPC) that adapts sequential pattern-matching to symmetric pattern-unification in a process calculus. This dissertation proves that CPC subsumes both intensionality sequential computation and extensional concurrent computation, respectively  $SF$ -calculus and  $\pi$ -calculus, to complete a computation square. A behavioural theory is developed for CPC that is then exploited to prove that CPC is more expressive than several representative sequential and concurrent calculi. As part of its greater expressive power, CPC provides a natural language to describe interactions involving information exchange. Augmenting the pattern-matching language **bondi** to implement CPC yields a Concurrent **bondi** that is able to support web services that exploit both sequential and concurrent intensionality.

