Credit Risk Modelling in Markovian HJM Term Structure Class of Models with Stochastic Volatility

A Thesis Submitted for the Degree of
Doctor of Philosophy

by

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Certificate

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Glossary of Notations

- ATM = At-the-money.
- CIR = Cox-Ingersoll-Ross.
- HJM = Heath-Jarrow-Morton model.
- CDS = Credit Default Swaps.
- HW = Hull-White model.
- ODE = Ordinary differential equation.
- OTM = Out-the-money.
- PDE = Partial differential equation.
- SDE = Stochastic differential equation.
- SIE = Stochastic integral equation.
- $B(t), B_t$: Money market account at time t, bank account at time t.
- $P(t,T,\cdot)$: Price at time t of a default-free zero coupon bond with maturity T.
- $\bar{P}^d(t,T,\cdot)$: Pre-default price at time t of a defaultable zero coupon bond with maturity T.
- $P^d(t,T,\cdot)$: Price at time t of a defaultable zero coupon bond with maturity T.
- $f(t,T,\cdot)$: Instantaneous default-free forward rate of interest prevailing at time t for instantaneous borrowing at T.
- $f^d(t,T,\cdot)$: instantaneous defaultable forward rate of interest prevailing at time t for instantaneous borrowing at T.
- $r(t,\cdot)$: Instantaneous default free short rate of interest at time t.
- $r^d(t,\cdot)$: Instantaneous defaultable short rate of interest at time t.
- $\lambda(t,T,\cdot)$: Instantaneous forward credit spread.
- $c(t,\cdot)$: Instantaneous short term credit spread at time t.

- $N(t)$: Marked point Process at time t.
- $h(t)$: Intensity of a Marked point process at time t under the real world probability measure; $\tilde{h}(t)$: Intensity of a Marked point process at time t under the risk-neutral probability measure.
- $\mathcal{R}(t)$: Fractional recovery process at time t.
- τ_i: Random default time.
- $q(\tau_i)$: Loss rate on the bond's face value at each default time τ_i.
- $V(t)$: Stochastic volatility process at time t.
- $W(t) = \{W^f(t), W^\lambda(t), W^V(t)\}$: Wiener Process at time t under the real world measure;
  $\tilde{W}(t)$: Wiener Process at time t under the risk-neutral measure;
- $X(t)$: Markov chain Process at time t.
- τ_i^x: Jump times of the Markov chain.
- $\mu(\omega; dt, dq)$: Random measure associated with the Marked point process, N.
- $h_{i,j}^X(t)$, $\tilde{h}_{i,j}^X(t)$: Transition intensity at time t of a Markov chain from state i to j.
- $\mathcal{F}^W(t)$, $\mathcal{F}^N(t)$, $\mathcal{F}^X(t)$ Filtrations generated at time t by the Wiener process, Marked point process and Markov chain respectively.
- $\rho_{12} \equiv \rho^{V\lambda}$: Correlation between stochastic volatility and short term credit spread processes; $\rho_{13} \equiv \rho^{Vf}$: Correlation between stochastic volatility and default-free short rate processes; $\rho_{23} \equiv \rho^{\lambda f}$: Correlation between short term credit spread and default-free short rate processes.
- $\phi(t)$: Market price of diffusion risk; $\psi(t)$: Market price of jump risk.
- κ^f, κ^λ, κ^V: Speeds of mean reversion for the risk-free short rate, the short term credit spread and the stochastic volatility processes, respectively.
- $\mathcal{P}(t, r^d, T; T_0, K)$: Price at time t for the put option with maturity T_0, strike K that is knocked out on default of an underlying defaultable bond with a maturity T, under stochastic volatility.
- $\mathcal{C}(t, \bar{P}^d, X(t))$: Price at time t of a call option under regime-switching stochastic volatility.

- $\pi_f(t)$, $\tilde{\pi}_f(t)$, $\bar{\pi}_f(t)$, $\pi_{cpr}(t)$: Price at time t of a credit default swap.

- $\mathcal{C}_{swpt}(t)$, $\tilde{\mathcal{C}}_{swpt}(t)$: Price at time t of a credit default swaption.

- $[X]_t$: Quadratic variation at time t of the process X; $[X,Y]_t$: Quadratic covariation of two processes, X and Y.

- $< \mathbf{a}, \mathbf{b} >=< (a_1, \ldots, a_n), (b_1, \ldots, b_n) >= \sum_{i=1}^{n} a_i b_i$: Inner product.

Abstract

Empirical evidence strongly suggests that interest rate volatility is stochastic and correlated to changes in interest rates. In addition, the intensity process has been shown to generate heavy-tailed behavior and this has been attributed to stochastic volatility. A good credit risk model should incorporate the correlation between the short rate and credit spread processes as changes in interest rates can directly affect and change the credit spread or indirectly influence the market's perception of default risk which has an impact on credit spreads.

The objective of this thesis is to model credit risk within a Markovian Heath, Jarrow, and Morton [1992] (hereafter HJM) term structure model with stochastic volatility by extending the defaultable framework developed in Schönbucher [1998]. Adapting the HJM framework to include default risk results in a generalised framework that incorporates all the information on the current risk free term structure as well as the credit spread curve. Under some conditions on the specification of the volatility functions, the model admits finite dimensional Markovian realisations and as a result, the default-free yield curve as well as the credit spread curves can be calculated with low computational cost at any given time.

The main contributions of this thesis are:

◇ *Markovian Defaultable HJM Term Structure Models with Unspanned Stochastic Volatility - Chapter 2.* Stochastic volatility is introduced into the Schönbucher [1998] model and we generalise it to allow for a correlation structure between the default-free forward rate, the forward credit spread and stochastic volatility. Under certain level dependent volatility specifications, we derive a Markovian representation of the defaultable short rate in terms of a finite number of state variables which we then express in terms of economic quantities observed in the market, specifically in terms of discrete tenor forward rates. A numerical experiment is then conducted to investigate the distributional properties of the defaultable bond price and bond returns which reveals the existence of a left tail.

◇ *Credit Derivative Pricing under a Markovian HJM Term Structure Model with (Diffusion Driven) Humped Volatility - Chapter 3.* We verify that under the assumption of a humped volatility specification, the defaultable forward rates admits finite dimensional affine realisations. The default of the underlying reference entity is modelled as a Cox process and we derive exponential affine bond price formulas in the presence of stochastic volatility. We then investigate the pricing of single-name credit default swaps both in the presence and absence of counterparty risk and derive formulas for the valuation of credit default swaptions within the framework. On relaxing the level dependency assumption within the humped volatility specification, we price knocked-out put options on defaultable bonds using the Fourier transform approach.

◇ *Valuation of Bond Options under a Defaultable HJM Class of Models with Regime-Switching Volatility - Chapter 4.* We allow the defaultable forward rate volatility to depend on the current forward rate curve as well as on a modulating continuous time Markov chain making use of the results in Valchev [2004] and Elhouar [2008]. Stochasticity is then introduced to the volatility function by a separable volatility specification which guarantees finite-dimensional Markovian realisations under regime switching. A special case of the short rate class of models, the Hull-White-Extended-Vasicek type of model is obtained in the defaultable setting from which an explicit bond pricing formula is derived. We then apply finite difference methods to price European options under two-state regimes.

We give a summary of all the thesis findings in Chapter 5 where we also present the concluding remarks and directions for future research work.