Exact modelling of the long wavelength properties of the fundamental mode in microstructured optical fibres

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Abstract
We report a novel, exact technique for modelling defect modes in photonic crystal structures with infinite cladding. Applying this, we establish conclusively that the fundamental mode in an index-guided MOF has no cut off.

Introduction. The structure of the bound mode spectrum of conventional optical fibres is well understood, based on calculations which assume the cladding is infinite. At short wavelengths λ, many bound modes are supported, with these being progressively cut off with increasing λ until only a single propagating mode, the *fundamental mode*, remains. Since most fibres operate in the single mode regime, the fundamental is the most important of the modes and in most conventional fibres, this mode is not cut off, remaining bound as λ increases to infinity [1].

In the case of index guiding MOFs, which have a cladding comprised of low index holes, the situation is less clear, largely because of the absence of closed form solutions due to the complexity of the geometry. To date, there have been no theoretical/computational methods that can model a defect mode in an infinite PC cladding. Until now, such modelling has been restricted to essentially finite structures—either explicitly, where a only finite number of rings of holes is considered, or implicitly, by periodically replicating a supercell [2] that contains a finite structure and thereafter exploiting the theoretical methods associated with Bloch's theorem. Such methods work well for strongly confined modes but can be quite problematic when the mode becomes arbitrarily extended (e.g., near cut off), necessitating vast computation in order to deal with sufficiently large structures, and possibly generating inaccurate results if the mode is inadequately confined.

Recently, our group investigated the long-wavelength behaviour of the second MOF mode and concluded that this mode does cut off, using numerical techniques to extrapolate the properties of a finite cladding to an infinite structure [3]. Similar techniques were used to consider the fundamental mode [4] where it was demonstrated that the mode is limited to the core at short wavelengths, while it extends over the cladding at long wavelengths, leading to increasing confinement losses. The extrapolation showed that the width of this transition region remained finite as the cladding was extrapolated to infinite size (in contrast to the second mode), leading to the conjecture that the fundamental had a cut off—at variance with the notion of "endlessly single mode" behaviour [5].

To resolve this dilemma we set about developing a new computational method that models exactly defect modes in an infinite two-dimensional lattice. The following section outlines the principles on which the method is based, after which we apply it to settle definitively the issue of whether of not the fundamental mode in an index guided MOF has a cutoff.

Outline of the theoretical method. Here, we outline our new technique known as the *fictitious source superposition method* (FSS), referring the reader to Ref. [6] for a comprehensive description. The method is based on *three key ideas*, the *first* of which is the use of *fictitious sources*, which, when placed within any particular scatterer, can be chosen to compensate exactly the reflected field generated by the incident field. In this way, it is possible to generate an exterior field that is identical to that which would be observed if the scatterer was absent, i.e., as in a defect. While such a calculation is simple for a single cylinder, the complexity of the interrelationships between the infinity of scatterers makes the direct solution of the problem for a non-periodic structure exceedingly difficult, if not impossible. Instead, we choose to devise the defect mode from a *superposition of solutions of qua-*

siperiodic field problems—the *second of the three main ideas*. Each problem corresponds to fictitious quasiperiodic multipole sources $\mathbf{q}_p = \mathbf{q}\exp(i\,\mathbf{k}_0 \cdot \mathbf{r}_p)$ embedded in each of the cylinders of the lattice at $\mathbf{r}=\mathbf{r}_p$. The superposition is then formed by integrating over the Brillouin zone (BZ) of the reciprocal lattice, thus generating a solution that satisfies the wave equations and boundary conditions, and which is associated with the fictitious source distribution $\mathbf{q}\int_{BZ}\exp(\mathrm{i}\,\mathbf{k}_0 \cdot \mathbf{r}_p)\,d\mathbf{k}_0$ at each cylinder ($\mathbf{r}=\mathbf{r}_p$). The BZ integration eliminates all of the sources except that which lies within the primary cylinder at $\mathbf{r}=\mathbf{r}_0=\mathbf{0}$, and which remains available to modify the response field, thereby generating the defect mode. In its present form, the necessary BZ integration is two-dimensional and is thus very time consuming. This difficulty can be alleviated, however, by the *third key idea* which reformulates the problem to involve only a *single integration*. Here, we model the structure as a diffraction grating (with cylinders containing a phased line of sources) sandwiched between two semi-infinite photonic crystals, the action of which is modelled by the Fresnel reflection matrix $\mathbf{R}_\infty$ [6]. This new field problem is quasiperiodic in only one dimension, satisfying the Bloch condition $F(\mathbf{r}+p\,d\,\hat{x}) = F(\mathbf{r})\exp(\mathrm{i}\,k_{0x}d)$. Quasiperiodicity in the second dimension is captured through the mode structure of the lattice which is embedded in the calculation of $\mathbf{R}_\infty$. The diffractive action of the grating is characterized by plane wave scattering matrices which we calculate using a multipole method that encapsulates the quasiperiodic contributions to the scattered field due to the periodic array of cylinders in terms of lattice sums. Finally, by integrating over the one-dimensional BZ [6], we deduce a resonance condition which appears as a homogeneous system of equations $\mathbf{Zq} = \mathbf{0}$. The matrix $\mathbf{Z}$ is a function of λ and thus we form the mode by searching for zeros of $\det\mathbf{Z}=0$, and reconstructing it from the corresponding null vector(s) $\mathbf{q}$.

Results: fundamental mode cutoff in a MOF. Figure 1 shows the axial Poynting vector for a MOF comprising a single defect (core) surrounded by a hexagonal lattice cladding of period Λ, hole diameter $d = 0.24\Lambda$, and with the background index $n_b = 1.45$. In the bottom row of Fig. 1, this lattice is infinite, whereas in the top row the core is surrounded by only three rings of holes.

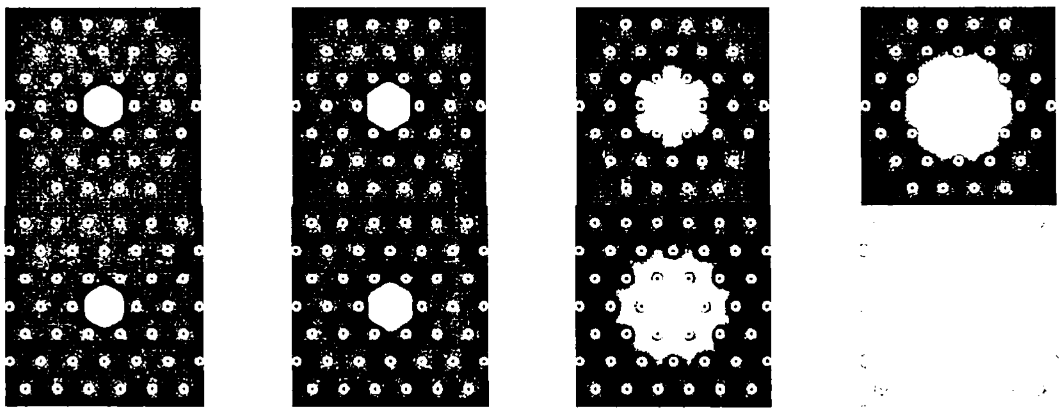
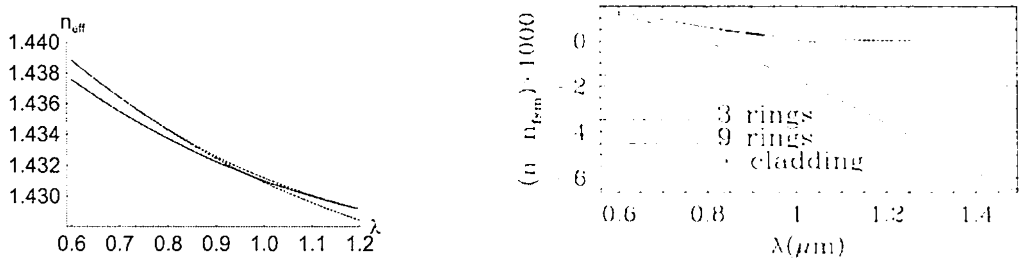


Fig. 1 Axial Poynting vector for the fundamental mode in a finite structure with 3 rings of holes (top row) and an infinite cross section (bottom row) with λ/Λ =0.133, 0.50, 1.1 and 1.6 respectively in columns 1 through 4. The small circles show the air holes

At the shortest wavelength $\lambda/\Lambda = 0.133$, the mode is well confined, while for the longest wavelength $\lambda/\Lambda = 1.6$, well beyond the previously estimated cut off wavelength [4], the mode, though poorly confined, does exist. For the finite MOF, the mode largely fills the cladding, but is confined by antiguiding at the outer cladding boundary. For the infinite MOF, without this outer boundary, the mode extends even further into the cladding.

To understand the properties of the fundamental mode we need to consider the relationship between the effective refractive index n_{eff}, and the effective index of the fundamental space filling mode n_{fsm}, which plays the same role as the cladding index in a conventional fibre. The field in the cladding region is evanescent when $n_{eff} > n_{fsm}$, and is propagating otherwise. Now, at short wavelengths, we see from Fig. 2(a) that for a finite MOF, n_{eff} is well above n_{fsm}. From Fig. 2(b), it is evident that at short wavelengths n_{eff} has approximately the same value (well above n_{fsm}) irrespective of the size of the cladding—a consequence of the mode being confined to the core and decaying strongly towards the cladding boundary. With increasing λ, however, the n_{eff} curves for finite structures deviate from one another and ultimately drop below the n_{fsm} curve—at which point the cladding fields cease to be evanescent, losses increase rapidly and confinement is effectively lost [4]. In contrast, for the infinite cladding, $n_{eff} \rightarrow n_{fsm}$ from above, never crossing it. The cladding field is thus always evanescent and the fundamental mode never cuts off—consistent with the heuristic arguments in Ref. [5]. The transition region [4], which led to the incorrect conjecture that the fundamental mode was cut off, is nevertheless genuine and is the wavelength range over which the modal area increases rapidly.



(a) (b)

Fig. 2 (a) Effective index n_{eff} of the fundamental mode of an infinite structure (short dash), n_{eff} for a finite 7 ring MOF (long dash) and n_{fsm} (solid). (b) the difference of n_{eff} and n_{fsm} for 3 and 9 ring MOFs.

Discussion and conclusions

We have outlined a unique theoretical approach that allows the exact calculation of defect modes in PC structures with an infinite cladding. While the discussion has been restricted to a single defect, the method extends readily to handle general defects and may also be extended to structures with 3D geometry and periodicity. It handles those extreme situations in which the mode becomes arbitrarily extended, for which other methods fail. Indeed for $\lambda / \Lambda \approx 1.6$, it is possible, by analogy with conventional fibres [7], to estimate that the modal field extends over hundreds of periods. To the best of our knowledge, the modelling of such extreme cases is beyond the capability of all previously existing methods, and can be tackled only by the technique described here. This novel approach allows us to elucidate the long-wavelength behaviour of the fundamental mode in an index guided MOF. Its effective index approaches the effective index of the fundamental space filling mode from above, ensuring endlessly single mode behaviour. For longer wavelengths we experience some numerical difficulties in our present implementation due to the very small value of the transverse wave-number $k_\perp = k\sqrt{n_b^2 - n_{eff}^2}$, and problems which arise in evaluating lattice sums (that behave asymptotically as powers of $1/k_\perp$). Nevertheless, since this occurs well beyond the predicted cut off wavelength, we conclude that the fundamental mode for a MOF with infinite cross section does not cut off.

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