

Commodity Derivative Pricing under the Benchmark Approach

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**UNIVERSITY OF TECHNOLOGY, SYDNEY
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Signature of Author

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Abstract

This thesis models commodity prices and derivatives, written on commodity prices, under the benchmark approach. Under this approach, the commodity prices are modeled under the real world probability measure while the corresponding numéraire is the numéraire portfolio (NP), which is the growth optimal portfolio that maximizes expected logarithmic utility. The existence of an equivalent risk neutral probability measure is not required. Under the proposed new concept of benchmarked risk minimization, the minimal price for a nonhedgeable contingent claim is identified, and the fluctuations of the benchmarked profit and loss, when denominated in units of the NP, are minimized. The resulting real world pricing formula generalizes the classical risk neutral pricing formula. The NP will be approximated by a well-diversified stock index. New forward and futures price formulas will be derived, which generalize their classical counterparts. Stylized empirical facts for the dynamics of the NP in a selected commodity denomination will be identified. These lead to a model which falls outside classical no-arbitrage assumptions, but is covered by the benchmark approach. Under this model, some long dated derivatives will be shown to be less expensive than under the classical paradigm.

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