Commodity Derivative Pricing
under the Benchmark Approach

Ke Du

A thesis submitted for the degree of
Doctor of Philosophy
at the University of Technology, Sydney.
27 February 2013.
I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Author
Acknowledgements

First and foremost, I would like to thank Professor Eckhard Platen, my supervisor, for providing the opportunity to undertake this PhD and for his consistent and enthusiastic support. I would also like to thank Professor Erik Schlögl and Professor Tony He for their help throughout my research. This thesis would not have been possible without their support and encouragement. Further, I wish to express gratitude to my best friends, Rei Chen, Jan Baldeaux, Hardy Hulley, Boda Kang, Renata Rendek and Nicholas Yap. They have given me lots of good advice on various issues from my research to career development. I am also grateful to the staff and students of the Finance Discipline Group for their support and for providing a friendly atmosphere. Last but not least, I would like to thank my parents and my wife, Li Zeng, for their moral support and undying love. Without them, I would not have been able to accomplish anything.
Abstract

This thesis models commodity prices and derivatives, written on commodity prices, under the benchmark approach. Under this approach, the commodity prices are modeled under the real world probability measure while the corresponding numéraire is the numéraire portfolio (NP), which is the growth optimal portfolio that maximizes expected logarithmic utility. The existence of an equivalent risk neutral probability measure is not required. Under the proposed new concept of benchmarked risk minimization, the minimal price for a nonhedgeable contingent claim is identified, and the fluctuations of the benchmarked profit and loss, when denominated in units of the NP, are minimized. The resulting real world pricing formula generalizes the classical risk neutral pricing formula. The NP will be approximated by a well-diversified stock index. New forward and futures price formulas will be derived, which generalize their classical counterparts. Stylized empirical facts for the dynamics of the NP in a selected commodity denomination will be identified. These lead to a model which falls outside classical no-arbitrage assumptions, but is covered by the benchmark approach. Under this model, some long dated derivatives will be shown to be less expensive than under the classical paradigm.
Contents

Contents iv

List of Figures vii

1 Introduction 1
   1.1 Brief Survey of Results  1
   1.2 Commodity Price Modeling  2
   1.3 A Benchmark Approach to Commodities  6
   1.4 Forward-Futures Spread  9
   1.5 Forward and Futures Prices  10
   1.6 Diversified Indices as Proxy for the NP  12
   1.7 Main Contributions of the Thesis  14

Nomenclature 1

2 Benchmarked Risk Minimization 16
   2.1 Financial Market  18
   2.2 Real World Pricing  23
   2.3 Benchmarked Profit and Loss  24
   2.4 Benchmarked Risk Minimization  26
   2.5 Regular Benchmarked Contingent Claims  29
   2.6 A Jump Diffusion Market  32
   2.7 Illustrative Comments and Examples  36
      2.7.1 Regular Claims in Markovian Markets  36
      2.7.2 BRM Strategies and a Quadratic Criterion  37
## CONTENTS

2.7.3 An Example with Nonhedgeable Contingent Claim .......................... 39
2.7.4 Classical and BRM Prices .............................................. 42
2.7.5 Evolving Information on Nonhedgeable Claims ............................ 43
2.7.6 Minimal Market Model with Gamma Type Jumps .......................... 44
2.7.7 Pricing of Real World Zero Coupon Bond .................................. 46
2.7.8 Hedging of Real World Zero Coupon Bond .................................. 47

2.8 Conclusions on Benchmarked Risk
Minimization ................................................................. 51

3 Commodity Forward and Futures Contracts ................................. 53
3.1 Real World Forward and Futures Contracts ................................. 54
3.1.1 Spot Price ............................................................... 55
3.1.2 Forward Price ........................................................... 56
3.1.3 Futures Price ............................................................ 60
3.1.3.1 Discrete Time Futures Price and Margin Account ................. 61
3.1.3.2 Continuous Time Approximation .................................. 65
3.1.4 Options on Futures ..................................................... 68

3.2 An Alternative Model for Commodity Prices ................................. 69
3.2.1 Minimal Market Model .................................................. 70
3.2.2 Forward and Futures Prices under the MMM ............................. 72
3.2.3 Options on Commodities ............................................... 76
3.2.3.1 Option on Zero Coupon Bond .................................... 77
3.2.3.2 Option on Forward Price ......................................... 79
3.2.3.3 Option on Futures Prices .......................................... 82
3.2.4 Non-existence of a Risk Neutral Probability Measure ....................... 84
3.2.5 Empirical Observations and Model Fitting ................................. 87

3.3 Conclusions on Commodity Forward and Futures Contracts ...................... 97

4 Modeling of Oil Prices ...................................................... 99
4.1 Empirical Observations .................................................... 100
4.2 Modeling of Oil Prices ..................................................... 105
List of Figures

3.1 $R(Y^i_t, c, \xi^i_t, \tau)$ as a function of $Y^i$ and $\tau$, with $c = 0.05$, $\xi^i = 0.02$ 73
3.2 $G(Y^i_t, c, \xi^i, \tau)$ as a function of $Y^i$ and $\tau$, with $c = 0.05$, $\xi^i = 0.12$ 74
3.3 Relative pricing error of “risk neutral” zero coupon bond 86
3.4 The EWI in US dollar denomination (upper panel) and benchmarked US savings account (lower panel) 88
3.5 Spot prices of three commodities: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 89
3.6 EWI in three commodity denominations: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 90
3.7 Estimated convenience yields of three commodities: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 92
3.8 Benchmarked savings accounts of three commodities: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 92
3.9 Logarithm of discounted EWI in commodities denominations of three commodities: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 93
3.10 Normalized NP of three commodities: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 94
3.11 Quadratic covariation between convenience yield and respective normalized NP of three commodities: crude oil (upper panel), copper (middle panel), aluminium (lower panel) 95
3.12 Calculated term structure of the real world futures price of crude oil 97
4.1 Autocorrelation function for log-returns of the oil discounted EWI. 101
LIST OF FIGURES

4.2 Autocorrelation function for the absolute log-returns of the oil discounted EWI. .......................... 101
4.3 Logarithms of empirical density of normalized log-returns of the oil discounted EWI and Student-t density with 3.13 degrees of freedom. 102
4.4 Estimated volatility from log-returns of the oil discounted EWI. . 103
4.5 Logarithms of normalized EWI for oil (upper graph) and its volatility (lower graph). .......................... 104
4.6 Quadratic covariation between the logarithms of normalized EWI for oil and its volatility. .......................... 104
4.7 Logarithm of the oil discounted EWI and linear fit. ............... 110
4.8 Normalized EWI for oil. ........................................... 110
4.9 Market activity. ...................................................... 111
4.10 Quadratic variation of the square root of $\frac{1}{M}$. .................. 112
4.11 Histogram of market activity with inverse gamma fit. .............. 113
4.12 Simulated path of $M$. ............................................ 114
4.13 Simulated $\tau$-time, the market activity time. ................. 115
4.14 Simulated trajectory of the normalized index $Y_\tau$. ............ 116
4.15 Simulated volatility of the index. ............................... 117
4.16 Estimated market activity of the simulated index. ............. 117
4.17 Quadratic variation of the square root of the inverse of estimated market activity. ......................... 118
4.18 Autocorrelation function for log-returns of the simulated index. . 119
4.19 Autocorrelation function for absolute log-returns of the simulated index. .................................. 119
4.20 Logarithms of the empirical distribution of the normalized log-returns of the simulated index and Student-t density with four degrees of freedom. .......................... 120
4.21 Estimated volatility of the simulated index. ...................... 120
4.22 Logarithm of simulated index with linear fit. .................. 121
4.23 Logarithms of simulated normalized index (upper graph) and its estimated volatility (lower graph). .......... 122
4.24 Quadratic covariation between the logarithms of simulated normalized index and its estimated volatility. .......................... 123