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Design of a Common Disturbance Decoupled Observer for Two Linear Systems

H. Trinh and P. S. Teh
School of Engineering and Information Technology
Deakin University
Geelong, 3217, Australia
Email: hmt@deakin.edu.au, pste@deakin.edu.au

Q. P. Ha
Faculty of Engineering
University of Technology, Sydney
NSW 2007, Australia
Email: quangha@eng.uts.edu.au

Abstract— **This paper considers the problem of designing a common observer that can observe a partial set of the state vector of two linear systems with known/unknown inputs. A novel structure for a common disturbance decoupled functional observer which only uses the available output information is proposed. Existence conditions and a design procedure for constructing a common observer are given. A numerical example is given to illustrate the attractiveness of the design procedure.**

I. INTRODUCTION

It is always intriguing to know whether or not there exists a common observer that can observe the states for each of a given set of plants. The problem of simultaneous observation ([1]-[3]) of two or several linear plants is of interest when, for example, a robust observer is designed that converges despite of failures in some components in a system. In the observer design of a nonlinear system at various operating conditions, the nonlinear system is often linearized at various operating points and some of linearized models can be obtained. If the states of each of these linearized models can be observed using a common observer, then a fixed observer can be used for the nonlinear system.

The simultaneous observation problem of linear systems with known inputs was first studied in [2] using the coprime factorization technique. Here, the authors presented necessary and sufficient conditions for the existence of a simultaneous observation. In [3], based on the stable inverse approach, a general result for parameterization of all simultaneous observers was given. More recently, Moreno [1] provided a state-space characterization for simultaneous observation of several linear plants. For the case of two linear systems with known inputs, the simultaneous observation problem was completely solved [1] and the author also presented an algorithm and a structure for simultaneous observers. So far, we observe that the authors of references [1]-[3] only treated the case where all plants are subjected to known inputs and that the common observer required a complete knowledge ([2]-[3]) or partial knowledge ([1]) of the input signals. This requirement may not always be met in practice since input signals may be completely unknown or measurement of the system inputs is either too expensive or perhaps physically not possible [4]-[5]. In such a situation, one has to use only the available output information to design a common observer.

In this paper, motivated by the work of [1]-[3], we consider the problem of designing a common linear functional observer for two linear systems with unknown inputs. A new structure for a common linear functional observer which only uses the available output information is proposed. Here, for the proposed structure, we show that the simultaneous observation problem of two plants is reduced to a problem of designing two observers: The first is an unknown input observer for one of the two systems and the second is a reduced-order unknown input linear functional observer of a system comprises two-connected systems. We show, via a numerical example, that for the case of two linear systems, our existence conditions are less conservative than those presented by Moreno [1]. This is a nice finding given that our common observer only uses the output information instead of both output information and a partial input information as was the case of Moreno [1].

The organization of this paper is as follows: Section II presents the problem statement. Section III proposes a new structure for a common disturbance decoupled functional observer. We present existence conditions and a design procedure for constructing a common observer. Section IV presents a numerical example. We show that for the given numerical example, the proposed structure and design method of Moreno [1] can not produce a common linear functional observer whereas with our results, we can design a common observer. Section V provides a conclusion.

Throughout this paper, $\hat{x}(t) \in \mathbb{R}^n$ represents n-dimensional estimated state vector of $x(t)$ and $\hat{x}(t) \rightarrow x(t)$ means that $\hat{x}(t)$ converges asymptotically to $x(t)$.

II. PROBLEM STATEMENT

Let us consider two linear time-invariant systems, Φ_i (i=1,2), as described by [1]:

$$\Phi_i : \begin{cases} \dot{x}_i(t) = A_i x_i(t) + D_i d_i(t) \\ y_i(t) = C_i x_i(t) \\ z_i(t) = L_i x_i(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector, $y_i(t) \in \mathbb{R}^p$ is the measured output vector, $d_i(t) \in \mathbb{R}^{m_i}$ is treated in this paper as *unknown* input vector and $z_i(t) \in \mathbb{R}^r$ is a vector of signals to be estimated. Matrices A_i, D_i, C_i and L_i are known real constant and of appropriate dimensions.

The aim of this paper is to use only the available output information, $y_i(t)$, to construct a single disturbance decoupled linear functional observer, called a Common Functional Observer (CFO), such that it is a linear functional observer for every element of the family of LTI systems described by (1). Figure 1 shows the block diagram of a CFO. In Figure 1, Φ_i (i=1,2), represents the actual plant, Ω_c is the CFO and $\hat{z}_i(t)$ denotes the estimate of $z_i(t)$ such that $\hat{z}_i(t) \to z_i(t)$.

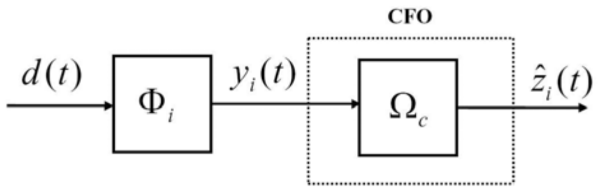

Fig. 1. Block diagram of a Common Functional Observer.

III. Main Results

In this section, we first propose a novel structure for the CFO. We then derive existence conditions and a design procedure. Let us define the following error function:

$$e(t) = \sum_{j=1}^{p} |\hat{y}_1^j(t) - y_i^j(t)|, \tag{2}$$

where $\hat{y}_1^j(t)$ $(j = 1, 2, ..., p)$ denotes the estimated j-th output of the system Φ_1 and $y_i^j(t)$ $(i = 1, 2)$ denotes the j-th output of the i-th system, Φ_i. Thus for the case where $\hat{y}_1(t)$ is an asymptotic estimate of $y_1(t)$ and that the measurement is coming from $y_1(t)$ then we expect that the error function (2) would be exponentially diminished to zeros (i.e. $e(t) \to 0$ as $t \to \infty$). Base on the properties of $e(t)$, let us propose the following decision logic as shown in Figure 2.

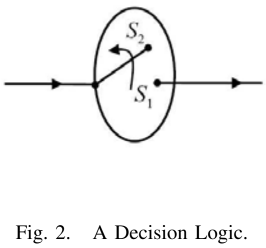

Fig. 2. A Decision Logic.

The logic is modelled as a switch, where the switch being open or closed is determined from the asymptotic convergence of $e(t)$. Let us define the logic decision as follows:

$$J(e) = \begin{cases} 1, & e \geq \delta \\ 0, & e < \delta, \end{cases} \tag{3}$$

where $\delta > 0$ is a threshold value of the error function $e(t)$. Thus, generally speaking, for the above logic, the switch is open when it is determined that the error, $e(t)$, is small (i.e., $\hat{y}_1(t)$ is actually estimating the measurement signal $y_i(t)$). Otherwise the switch is closed.

We are now in a position to propose a novel structure for a CFO as depicted in Figure 3. As can be seen from Figure 3, the only information that feeds into the CFO is the output information, $y_i(t)$. The proposed CFO observer for Φ_1 and Φ_2 consists of the interconnection of two unknown input functional observers (UIFOs), namely Ω_1 and Ω_u. In Figure 3, $\tilde{y}_1(t)$ and $v_1(t)$ are defined as:

$$\tilde{y}_1(t) = \hat{y}_1(t) - y_i(t) \tag{4}$$

and

$$v_1(t) = \hat{z}_1(t) - z_i(t). \tag{5}$$

Now, let Ω_1 be a full-order unknown input observer of the form:

$$\Omega_1 : \begin{cases} \dot{\omega}_1(t) = N_1\omega_1(t) + F_1 y_i(t) \\ \hat{y}_1(t) = M_{11}\omega_1(t) + G_{11} y_i(t) \\ \hat{z}_1(t) = M_{12}\omega_1(t) + G_{12} y_i(t), \end{cases} \tag{6}$$

where $\omega_1(t) \in \mathbb{R}^{n_1}$, N_1 (Hurwitz), F_1, M_{11}, G_{11}, M_{12} and G_{12} are matrices of appropriate dimensions to be determined such that for $i = 1$, $\hat{z}_1(t) \to z_1(t)$ and $\hat{y}_1(t) \to y_1(t)$ for Φ_1.

Remark 1: Necessary and sufficient conditions for the existence of Ω_1 for system Φ_1 are well-known and existing observer design methods reported in the literature (see, for example [5]-[7]) can be used to obtain matrices N_1, F_1, M_{11}, G_{11}, M_{12} and G_{12} such that $\hat{z}_1(t) \to z_1(t)$ and $\hat{y}_1(t) \to y_1(t)$.

Assuming that the switch is closed, then the interconnection of the observer Ω_1 and the system Φ_2 has the form:

$$\begin{bmatrix} \dot{\omega}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} N_1 & F_1 C_2 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ D_2 \end{bmatrix} d_2(t) \tag{7}$$

$$\begin{aligned} \bar{y}(t) &= \begin{bmatrix} \tilde{y}_1(t) \\ \hat{z}_1(t) \end{bmatrix} = \begin{bmatrix} \hat{y}_1(t) - y_2(t) \\ M_{12}\omega_1(t) + G_{12} y_2(t) \end{bmatrix} \\ &= \begin{bmatrix} M_{11} & (G_{11}C_2 - C_2) \\ M_{12} & G_{12}C_2 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{8}$$

$$\begin{aligned} v_1(t) &= \hat{z}_1(t) - z_2(t) \\ &= \begin{bmatrix} M_{12} & (G_{12}C_2 - L_2) \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ x_2(t) \end{bmatrix}. \end{aligned} \tag{9}$$

For brevity, let the interconnected system defined by equations (7)-(9) be called Φ_u. Let Ω_u be a reduced-order unknown input linear functional observer of Φ_u, where

$$\Omega_u : \begin{cases} \dot{\omega}_2(t) = N_2\omega_2(t) + F_2\bar{y}(t) \\ \hat{v}_1(t) = M_2\omega_2(t) + G_2\bar{y}(t), \end{cases} \tag{10}$$

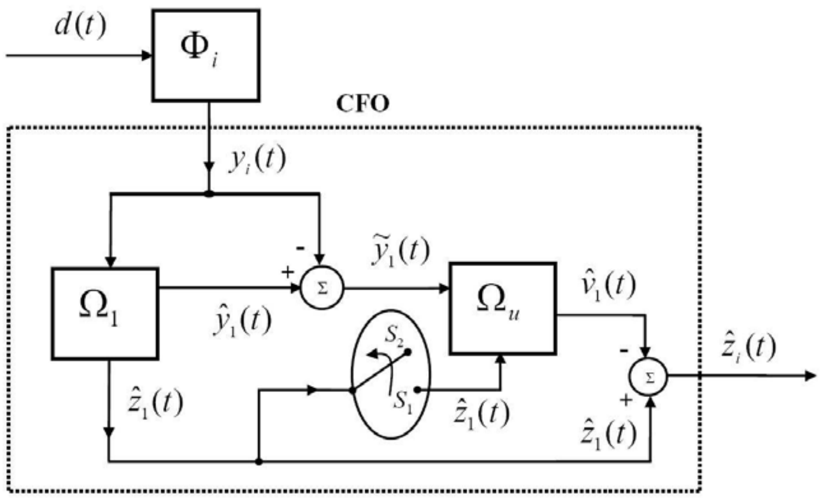

Fig. 3. Block diagram of a CFO for two systems.

$\omega_2(t) \in \mathbb{R}^q$, matrices N_2 (Hurwitz), F_2, M_2 and G_2 are matrices of appropriate dimensions to be determined such that $\hat{v}_1(t) \to v_1(t)$.

The following theorem establishes an existence condition for the proposed CFO of Figure 3.

Theorem 1: There exists a CFO of the structure depicted in Figure 3 such that $\hat{z}_i(t) \to z_i(t)$ $(i = 1, 2)$ provided that there exist two UIFOs Ω_1 and Ω_u for Φ_1 and Φ_u, respectively.

Proof: Firstly, let us suppose that the information that feeds into Ω_1 is from the first plant, (i.e., $i = 1$, $\Phi_i = \Phi_1$ and $y_i(t) = y_1(t)$). Accordingly, if Ω_1 exists, then Ω_1 provides both estimates $\hat{z}_1(t)$ and $\hat{y}_1(t)$ such that $\hat{z}_1(t) \to z_i(t)$ and $\hat{y}_1(t) \to y_i(t)$. Since $\hat{y}_1(t) \to y_i(t)$, it follows that $\tilde{y}_1(t) \to 0$. From (2), it is also clear that the error function $e(t)$ is exponentially diminishing to zeros. This in turn would activate the logic gate and the switch is open. Hence the signal $\bar{y}(t)$ that feeds into Ω_u is now $\bar{y}(t) = \tilde{y}_1(t)$ (i.e., $\bar{y}(t) \to 0$). Accordingly, provided that Ω_u is internally stable (i.e., matrix N_2 is Hurwitz) then $\hat{v}_1(t) \to 0$. From Figure 3, it is clear that $\hat{z}_i(t) \to z_1(t)$. In this case, the CFO provides the estimate $\hat{z}_1(t)$ of system Φ_1. Here, the CFO has achieved what it is supposed to do, i.e., when the information is from system Φ_1, the CFO produces the estimate $\hat{z}_1(t)$ for Φ_1.

Secondly, let us consider the case where the information is now coming from the second plant (i.e., $i = 2$, $\Phi_i = \Phi_2$ and $y_i(t) = y_2(t)$). In this case, Ω_1 is not an actual observer for Φ_2. Therefore $\tilde{y}_1(t) = (\hat{y}_1(t) - y_2(t)) \nrightarrow 0$ and $e(t) \nrightarrow 0$. Here, the logic switch would be closed and the interconnection of systems Ω_1 and Φ_2 is now described by Φ_u of equations (7)-(9). Accordingly, if Ω_u for Φ_u exists, then Ω_u provides the estimate $\hat{v}_1(t)$ such that $\hat{v}_1(t) \to v_1(t)$. From Figure 3, it is clear that $\hat{z}_i(t) = (\hat{z}_1(t) - \hat{v}_1(t)) \to z_2(t)$. In this case, the CFO provides the estimate $\hat{z}_2(t)$ of system Φ_2. Again, the CFO has achieved its objective, i.e., when the information is from Φ_2, the CFO produces the estimate $\hat{z}_2(t)$. This completes the proof of Theorem 1.

Remark 2: The CFO proposed in this paper has an advantage over those common observers proposed in [1]-[3]. Here, the proposed CFO caters for both known/unknown inputs and it only uses the available output information $y_i(t)$. In [1], the input/output pair $(d_1(t), y_i(t))$ is required. Note that even when $d_1(t)$ is a known input vector, sometimes it is not possible to have assess to $d_1(t)$ when $i \neq 1$. The performance of the CFO of this paper, however, depending on the timely detection, dictated by the logic decision (3). To improve on this, we can either refine the logic decision (3) or to incorporate a microprocessor based detection system that can take more measurements of the error function (2) at more frequent intervals in order to provide a more timely detection. In the numerical example, we show how early detection can improve the convergence rate of the estimates.

Remark 3: The existence conditions of Ω_u for Φ_u in this paper are less conservative than those of Moreno [1]. Note that in [1], the interconnection of systems Ω_1 and Σ_2 (i.e., $\Pi^0_{1,2}$) has more unknown inputs and less outputs than Φ_u in this paper. Since it is well-known in the design of unknown-input observers ([5]-[7]) that the more unknown inputs and the less available outputs, the harder it is to satisfy existence conditions, particularly the so-called observer matching condition [5]-[7] (i.e., $\mathrm{rank}(CD) = \mathrm{rank}(D)$). Indeed, this observation is true when we show via a numerical example that the design method of Moreno [1] can not produce a common linear functional observer whereas we can design a common observer.

Remark 4: The design procedure of a CFO can now be proceeded as follows. In the first step, we design a full-order unknown input observer, Ω_1 for system Φ_1. For this, we can adopt some existing design methods ([5]-[7]) to obtain matrices N_1, F_1, M_{11}, G_{11}, M_{12} and G_{12} such that $\hat{z}_1(t) \rightarrow z_1(t)$ and $\hat{y}_1(t) \rightarrow y_1(t)$. In the second step, we obtain the interconnected system Φ_u and design a reduced-order observer Ω_u for Φ_u. Again, for this step, we can adopt design methods (for example, [5]-[8]) to obtain matrices N_2 (Hurwitz), F_2, M_2 and G_2 such that $\hat{v}_1(t) \rightarrow v_1(t)$.

IV. A Numerical Example

We consider the numerical example reported in [1], where:

System 1, Φ_1: $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -5 \end{bmatrix}$, $D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} 2 & 7 \end{bmatrix}$ and $L_1 = \begin{bmatrix} 1 & -1 \end{bmatrix}$.

System 2, Φ_2: $A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -10 \end{bmatrix}$, $D_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 2 & 7 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}$.

Note that according to Moreno [1], there does not exist an unknown input functional observer (UIFO), Ω_{ui}, for the interconnected system $\Pi^0_{1,2}$ because there is an *unstable fixed mode* (eigenvalue) at $s = 0$. Hence a common functional observer [1] cannot be designed.

In the following, let us design a CFO for the above systems based on the new structure and design procedure of this paper. First, we design an observer Ω_1 of the form (6) for system 1, Φ_1, such that $\hat{z}_1(t) \rightarrow z_1(t)$ and $\hat{y}_1(t) \rightarrow y_1(t)$. By using the design method of [7], the following observer parameters for Ω_1 are obtained:

$N_1 = \begin{bmatrix} -7.9 & -22.4 \\ 1.4 & 3.4 \end{bmatrix}$, $\text{eig}(N_1) = \{-1.5, -3\}$, $F_1 = \begin{bmatrix} 1.75 \\ -0.5 \end{bmatrix}$, $M_{11} = \begin{bmatrix} 2 & 7 \end{bmatrix}$, $G_{11} = 1$, $M_{12} = \begin{bmatrix} 1 & -1 \end{bmatrix}$ and $G_{12} = 0.5$.

The next step in the design of a CFO is to design an observer Ω_u for Φ_u of (7)-(9). By substituting the above observer parameters into (7)-(9), the system Φ_u is obtained, where:

$$\begin{bmatrix} \dot{\omega}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -7.9 & -22.4 & 3.5 & 12.25 \\ 1.4 & 3.4 & -1 & -3.5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & -10 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} d_2(t),$$

$$\bar{y}(t) = \begin{bmatrix} 2 & 7 & 0 & 0 \\ 1 & -1 & 1 & 3.5 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ x_2(t) \end{bmatrix},$$

$$v_1(t) = \begin{bmatrix} 1 & -1 & 0 & 4.5 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ x_2(t) \end{bmatrix}.$$

We can now easily design an observer, Ω_u, for Φ_u such that $\hat{v}_1(t) \rightarrow v_1(t)$. Here, by adopting the design method proposed in [8], a reduced-order observer of second-order is obtained for Ω_u, where:

$N_2 = \begin{bmatrix} -15.2344 & -15.7299 \\ 7.1823 & 6.2344 \end{bmatrix}$, $\text{eig}(N_2) = \{-3, -6\}$, $F_2 = \begin{bmatrix} -4.4075 & -4.5 \\ 4.8189 & 1.7835 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 0.016 & 0 \end{bmatrix}$.

Simulation Results: In the simulation studies, we first consider the case where the output signal that feeds into the CFO is from system 1 (i.e., $y_i(t) = y_1(t)$). Figure 4 shows the unknown input signal, $d_1(t)$, arbitrarily generated. For the case when the switch is activated (open) within one second delay, Figure 5 shows the simulated responses $z_1(t)$ and $\hat{z}_1(t)$, with initial conditions $x_1(0) = 0$, $\omega_1(0) = \begin{bmatrix} -20 \\ 20 \end{bmatrix}$ and $\omega_2(0) = 0$. Figure 6 shows the simulated responses $z_1(t)$ and $\hat{z}_1(t)$ for the case where the switch is activated (open) within five seconds delay. Observe that we still have $\hat{z}_1(t) \rightarrow z_1(t)$. It is also clear that the earlier the detection time is, the better the tracking becomes.

Next in the simulation studies, we consider the case where after thirty seconds, the output signal that feeds into the CFO is from system 2 (i.e., $y_i(t) = y_2(t)$). Figure 7 shows the unknown input signals, $d_1(t)$ for the first thirty seconds and $d_2(t)$ for the last thirty seconds. Figure 8 shows the simulated responses $z_i(t)$ and $\hat{z}_i(t)$ for the case where the switch is activated (open) within five seconds delay and the switch is closed after thirty four seconds. It is clear from Figure 8 that $\hat{z}_i(t) \rightarrow z_i(t)$ $(i = 1, 2)$.

V. Conclusion

In this paper, we have proposed a novel common disturbance decoupled linear functional observer for two linear systems with unknown inputs. Existence conditions and a design procedure for constructing a common linear functional observer have been given. We showed, via a numerical example, that for the case of two linear systems, our existence conditions are less conservative than those existence conditions derived by Moreno [1]. Further research work is underway to extend current work to more than two plants and how to (best) design a decision logic to provide early detection which in turn improves the convergence rate of the estimates.

References

[1] J. A. Moreno, "Simultaneous observation of linear systems: A state-space interpretation," *IEEE Trans. Autom. Contr.*, vol. 50, no. 7, pp. 1021-1025, 2005.

[2] Y. X. Yao, M. Darouach and J. Schaefers, "Simultaneous observation of linear systems," *IEEE Trans. Autom. Contr.*, vol. 40, no. 4, pp. 696-699, 1995.

[3] R. Kovacevic, Y. X. Yao and Y. M. Zhang, "Observer parameterization for simultaneous observation," *IEEE Trans. Autom. Contr.*, vol. 41, no. 2, pp. 255-259, 1996.

[4] M. Corless and J. Tu, "State and input estimation for a class of uncertain systems," *Automatica*, vol. 34, pp. 757-764, 1998.

[5] M. Hou and P. C. Muller, "Design of observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 871-875, 1992.
[6] Y. Guan and M. Saif, "A novel approach to the design of unknown-input observers," *IEEE Trans. Automat. Contr.,* vol. 36, no. 5, pp. 632–635, 1991.
[7] M. Darouach, M. Zasadzinski and S. J. Xu, "Full-order observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.,* vol. 39, no. 3, pp. 606–609, 1994.
[8] H. Trinh, T. Fernando and S. Nahavandi, "Design of reduced-order functional observers for linear systems with unknown inputs," *Asian Journal of Control*, vol. 6, no. 4, pp. 514-520, 2004.

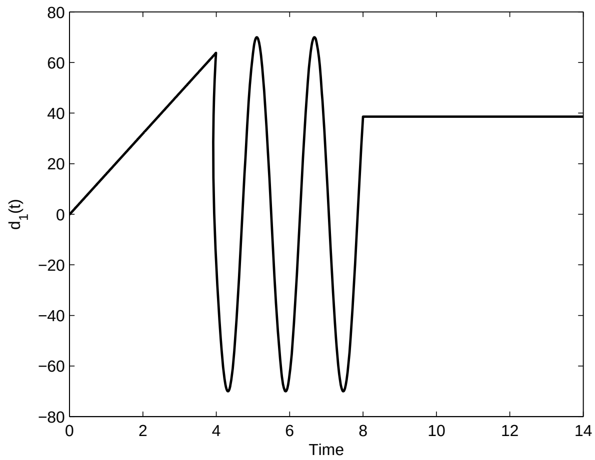

Fig. 4. Unknown input $d_1(t)$.

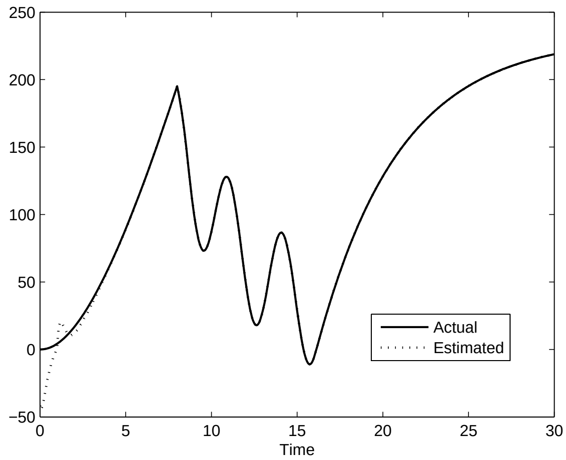

Fig. 5. Responses of $z_1(t)$ and $\hat{z}_i(t)$ (one second delay).

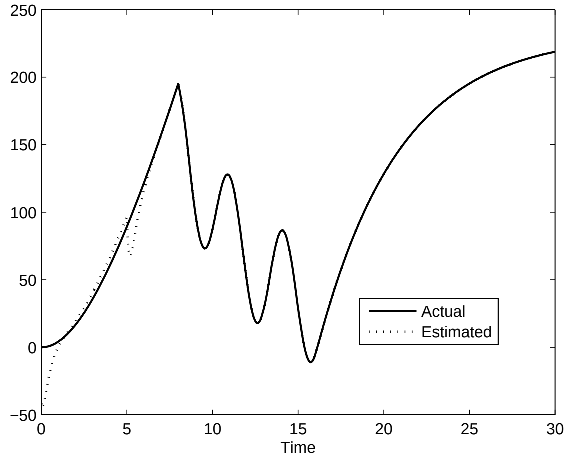

Fig. 6. Responses of $z_1(t)$ and $\hat{z}_i(t)$ (five seconds delay).

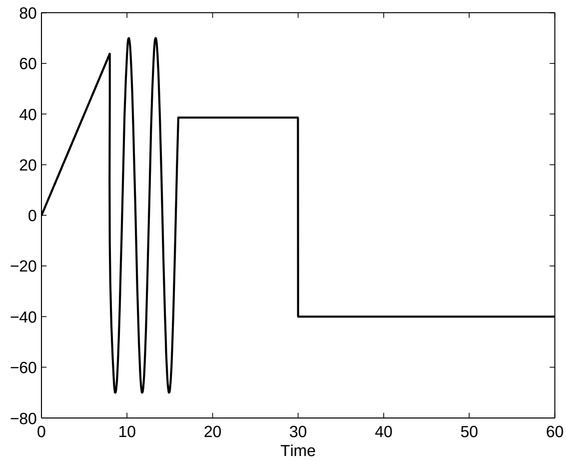

Fig. 7. Unknown input $d_i(t)$.

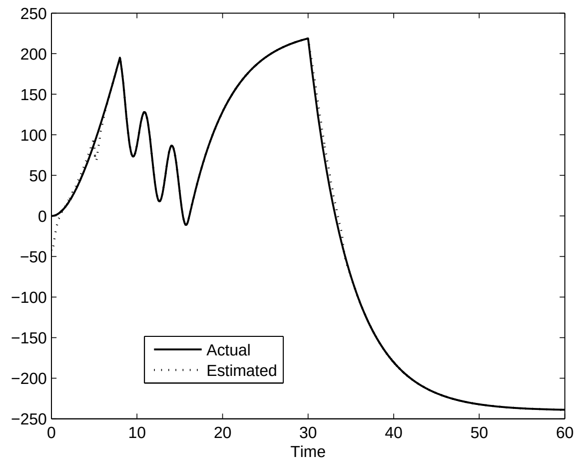

Fig. 8. Responses of $z_i(t)$ and $\hat{z}_i(t)$.

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