

UNIVERSITY OF TECHNOLOGY, SYDNEY

**Robotic Manipulation by Pushing at a  
Single Point with Constant Velocity:  
Modeling and Techniques**

by

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degree of Doctor of Philosophy

in the

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## Declaration of Authorship

I, Michael James Behrens, declare that this thesis titled, Robotic Manipulation by Pushing at a Single Point with Constant Velocity: Modeling and Techniques, and the work presented in it are my own. I confirm that:

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## *Abstract*

*In many mobile robotic manipulation tasks it is desirable to interact with the robot's surroundings without grasping the object being manipulated. Non-prehensile manipulation allows a robot to interact in situations which would otherwise be impossible due to object size or mass. This thesis investigates the most general pushing mode, that of a single contact point, formed either as a fixed point at a vertex or as a single rolling contact between two curved surfaces with a view to enable the manipulation of common household objects such as bins or coffee tables by a simple mobile robot. The investigation is limited to objects which possess a flat base and are able to slide on a flat, horizontal support surface. The derivation of a mathematical model is presented for an object pushed under these conditions, where the system accelerations influence the object motion through the dynamic effects of inertia and friction rendering the quasi-static assumption invalid. Numerical simulations explore the system behavior under a variety of configurations revealing the effect of the dynamic forces on the object motion and the existence of stable configurations, under certain conditions, where an object is pushed by a curved fence. The stable pushing behavior is confirmed experimentally. The mathematical model is utilized to generate near time-optimal pushing trajectories to manipulate an object to a desired goal location and control strategies to compensate for uncertainty in the physical parameters.*

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