The Design of Efficient Stated Choice Experiments

by

Stephen A. BUSH

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate

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α	the primitive root in $GF[\ell]$ (Chapter 6 only)
$\alpha_{\boldsymbol{e},a,b}$	the number of times $\boldsymbol{e} = (e_1, e_2, \ldots, e_k)$ appears as a difference between the items in positions a and b of the choice set
$\boldsymbol{\beta}$	a vector containing the effects of each level of each attribute, and the effects of each combination of levels
$\delta_{T_i \text{ in pos } a}$	an indicator variable which equals 1 if item T_i appears in position a of the choice set
η	the utility threshold for equality of preferences
ϵ_i	the stochastic component of the utility of the item T_i
$\boldsymbol{\gamma}$	a vector containing the log merits for each item
$\lambda_{\{i,j\}}$	the proportion of choice sets that are $\{T_i, T_j\}$
$\lambda_{(i,j)}$	the proportion of choice sets that are (T_i, T_j)
λ_C	the proportion of the choice sets that are the choice set C
$\lambda_{T_i \text{ in pos } a}$	the proportion of choice sets with item T_i in position a of the choice set
$\lambda_{T_i \text{ in pos } a, T_j \text{ in pos } b}$	the proportion of choice sets with item T_i in position a of the choice set and T_j in position b of the choice set
$\lambda_{\text{att } q=x \text{ in pos } a}$	the proportion of choice sets where the item in the item in position a of the choice set has the q^{th} attribute at the x^{th} level
$\lambda_{\text{att } q_1=x_1, q_2=x_2 \text{ in pos } a}$	the proportion of choice sets where the item in the item in position a of the choice set has the q_1^{th} attribute at the x_1^{th} level and the q_2^{th} attribute at the x_2^{th} level
$\Lambda(\boldsymbol{\pi})$	the information matrix for the estimation of the entries in $\boldsymbol{\gamma}$
$\Lambda(\boldsymbol{\pi}_0)_{\text{B–T}}$	the information matrix for the estimation of contrasts in $\boldsymbol{\gamma}$ when the Bradley–Terry model is used
$\Lambda(\boldsymbol{\pi}_0)_{\text{MNL}}$	the information matrix for the estimation of contrasts in $\boldsymbol{\gamma}$ when the MNL model is used

$\Lambda(\boldsymbol{\pi}_0, \nu)_{\text{DAV}}$	the information matrix for the estimation of the entries in $\boldsymbol{\gamma}$ and ν when the (generalised) Davidson ties model is used
$\Lambda(\boldsymbol{\pi}_0, \boldsymbol{\psi})_{\text{D–B}}$	the information matrix for the estimation of contrasts in $\boldsymbol{\gamma}$ and $\boldsymbol{\psi}$ when the (generalised) Davidson–Beaver position effects model is used
ν	the ties parameter for the Davidson ties model and the generalised Davidson ties model
π_i	the merit of item T_i
π_{i_b}	the merit of the item in position b of the choice set
$\boldsymbol{\pi}$	a vector containing the merits of each item
$\boldsymbol{\pi}_0$	the vector of π_is under the null hypothesis of equal selection probabilities
ψ_a	the position effect parameter for position a of the choice set
ψ_L	the linear component of the position effect
ψ_Q	the quadratic component of the position effect
$\boldsymbol{\psi}$	the vector containing $\psi_1, \ldots, \psi_m$
$\Sigma(\boldsymbol{\pi})_a$	the variance–covariance matrix for $\sqrt{sN}\boldsymbol{\gamma}$
$\Sigma(\boldsymbol{\pi})$	the variance–covariance matrix for the parameters of interest
θ	the ties parameter in the Rao–Kupper ties model
τ_i	the main effect of the i^{th} attribute, which takes 2 levels
$\tau_{i,\text{LIN}}$	the linear component of the main effect of the i^{th} attribute
ξ	a design
$\mathfrak{X}$	the class of competing designs
$\otimes$	Kronecker product
BlkDiag[]	a block diagonal matrix
$\binom{m}{x}$	the number of ways of choosing x objects from a set of m distinct objects without repetitions
$\bar{A}$	the set of items that do not appear in the choice experiment
$A_{\text{eff}}(\xi)$	the A–efficiency of a design $\xi \in \mathfrak{X}$ over the class of competing designs
$a_{k,i}$	the frequency of choice sets that differ in i attributes
B	a contrast matrix
B_a	a contrast matrix containing those effects that we assume are negligible
$B_{\bar{A}}$	the contrast coefficients that correspond to those items that do not appear in the choice experiment

$B_{F(q)}$	the rows of a contrast matrix corresponding to the main effects of attribute q
$B_{F,1}$	the rows of the contrast matrix for the items in F that correspond to the main effects of the first $k-p$ attributes
$B_{F,2}$	the rows of the contrast matrix for the items in F that correspond to the main effects of the last p attributes
B_h	a contrast matrix containing those effects that we are interested in estimating
B_ℓ	a contrast matrix containing the orthogonal polynomial contrast coefficients for an ℓ level attribute
B_M	a contrast matrix containing the contrast coefficients for the attribute main effects
B_T	a contrast matrix containing the contrast coefficients for the two–factor interactions between attributes
$\boldsymbol{b}_q$	the column of an orthogonal array corresponding to the q^{th} attribute
C	the information matrix for the estimation of the effects of interest
$C = \{T_{i_1}, \ldots, T_{i_m}\}$	the unordered choice set containing the items $T_{i_1}, \ldots, T_{i_m}$
$C = (T_{i_1}, \ldots, T_{i_m})$	the ordered choice set containing the items $T_{i_1}, \ldots, T_{i_m}$
$C(\boldsymbol{\pi}_0)_{\text{B–T}}$	the information matrix for the estimation of contrasts in $B\boldsymbol{\beta}$ when the Bradley–Terry model is used
$C(\boldsymbol{\pi}_0)_{\text{MNL}}$	the information matrix for the estimation of contrasts in $B\boldsymbol{\beta}$ when the MNL model is used
$C(\boldsymbol{\pi}_0, \nu)_{\text{DAV}}$	the information matrix for the estimation of contrasts in $B\boldsymbol{\beta}$ and ν when the (generalised) Davidson ties model is used
$C(\boldsymbol{\pi}_0, \boldsymbol{\psi})_{\text{D–B}}$	the information matrix for the estimation of contrasts in $B\boldsymbol{\beta}$ and $B_\psi\boldsymbol{\psi}$ when the (generalised) Davidson–Beaver position effects model is used
$C(\boldsymbol{\pi}_0, \boldsymbol{\psi})_{MP}$	the information matrix for the estimation of main effects and position effects
$(C(\boldsymbol{\pi}_0, \boldsymbol{\psi})_{MP})_{CLS}$	the information matrix for the estimation of main effects and position effects when a complete Latin square based design is used
$(C(\boldsymbol{\pi}_0, \boldsymbol{\psi})_{MP})_{S-B}$	the information matrix for the estimation of main effects and position effects when a design from Burgess and Street [2005] is used
$C(\boldsymbol{\pi}_0, \boldsymbol{\psi})_{MTP}$	the information matrix for the estimation of main effects plus two–factor interactions and position effects

$c_{\boldsymbol{v}_j}$	the number of choice sets containing the item $00\ldots0$ with difference vector $\boldsymbol{v}_j$
$c_{\boldsymbol{v}_j,a}$	the number of ordered choice sets with ordered difference vector $\boldsymbol{v}_j$ that contain the item $00\ldots0$ in position a of the choice set
$D_{\boldsymbol{d}}$	a $t \times t$ matrix with entries either 0 or 1 such that there is a 1 is position (i, j) if the items T_i and T_j have difference $\boldsymbol{d}$
$D_{\text{eff}}(\xi)$	the D–efficiency of a design $\xi \in \mathfrak{X}$ over the class of competing designs
d_{ij}	the number of attributes that differ between items T_i and T_j
$\boldsymbol{d}$	a difference
F	a starting design
$f(\boldsymbol{x}, \boldsymbol{\theta})$	the joint probability distribution function of the independent sample $\boldsymbol{X} = (x_1, \ldots, X_n)$
G	a set of m generators
$G()$	the Lagrangian function for obtaining the maximum likelihood estimators
$\boldsymbol{g}$	a generator
I_ℓ	an $\ell \times \ell$ identity matrix
$I(\boldsymbol{\pi})$	the information matrix for the estimation of the entries in $\boldsymbol{\pi}$
i_q	an indicator that equals 1 if the q^{th} position of the difference $\boldsymbol{d}$ equals 1
$i_{\boldsymbol{v}_j}$	an indicator of whether all choice sets with difference vector $\boldsymbol{v}_j$ appear in the experiment
$I_\xi(\boldsymbol{\theta})$	the Fisher information matrix for the estimation of the entries in $\boldsymbol{\theta}$
J_ℓ	an $\ell \times \ell$ matrix of 1s
$\boldsymbol{j}_\ell$	a vector containing ℓ 1s
k	the number of attributes describing an item
ℓ_i	the number of levels that the i^{th} attribute may take
$L_{i,j}$	the $(i, j)^{\text{th}}$ entry in a Latin square
$L(\boldsymbol{x}, \boldsymbol{\theta})$	the likelihood function
m	the number of options in each choice set
m_{ijk}	the expected number of times the k^{th} outcome will occur when the choice set $\{T_i, T_j\}$ or (T_i, T_j) are presented
N	the number of choice sets in a choice experiment

$n_{\{i,j\}}$	the number of times that the choice set $\{T_i, T_j\}$ appears in the experiment
$n_{(i,j)}$	the number of times that the choice set (T_i, T_j) appears in the experiment
n_C	the number of times that the choice set C appears in the experiment
p	the number of attribute contrasts being estimated
P_i	an $\ell \times \ell$ permutation matrix where $(P_i)_{x,y} = 1$ if and only if $x + i = y \mod \ell$
P_{ℓ_q,e_q}	a $t \times t$ matrix with entries either 0 or 1 such that there is a 1 is position (t_1, t_2) if $t_2 - t_1 = e_q$
Q_i	an $\ell \times \ell$ permutation matrix where $(Q_i)_{x,y} = 1$ if and only if $x + i = y$ in $GF[\ell]$
S_q	the optimal number of non–zero entries in a difference vector corresponding to a particular q level attribute
T_i	an item
T_{i_b}	the item in position b of the choice set C
U_i	the utility of the item T_i
V_i	the deterministic component of utility of the item T_i
$\boldsymbol{v}$	a difference vector
$w_{i\|C,\alpha}$	an indicator variable that equals 1 if respondent α selects item T_i when presented with the choice set C
$w_{\{i,j\}\|C,\alpha}$	an indicator variable that equals 1 if respondent α selects finds items T_i and T_j equally attractive when presented with the choice set C
$\boldsymbol{w}$	a vector containing all of the selection indicators for an experiment
$x_{\boldsymbol{v}_j;\boldsymbol{d}}$	the number of times the difference $\boldsymbol{d}$ appears in the difference vector $\boldsymbol{v}_j$
$x_{\boldsymbol{v}_j;\boldsymbol{d},a,b}$	the number of times that the difference $\boldsymbol{d}$ appears as a difference between positions a and b of the ordered difference vector $\boldsymbol{v}_j$
y_i	the proportion of pairs in a choice experiment that differ in the levels of i attributes
$y_{\boldsymbol{d}}$	the proportion of pairs of items in the choice experiment with difference $\boldsymbol{d}$
$y_{\boldsymbol{d},a,b}$	the proportion of pairs in the choice experiment that are in positions a and b of the choice set and have difference vector $\boldsymbol{d}$

Abstract

Making choices is a fundamental part of life. Whether it be the food that we eat, how we get from A to B, or the things that we do or do not purchase, choices are made all of the time. The ability to understand and influence these choices is valuable in many areas such as marketing, health economics, tourism, transportation research, and public policy. Choice experiments allow researchers in these areas to show respondents sets of options, described by attributes, and use the attributes of the chosen options to determine how important each of the attributes are to the 'attractiveness' of any option. From this information market share or policy acceptability can be predicted.

In this thesis we look at optimal designs for the multinomial logit (MNL) model, and for two extensions of this model. The first extension incorporates tied preferences, and is based on the extension of the Bradley–Terry model introduced by Davidson [1970]. The second extension allows the researcher to estimate the effect that the position of an item in the set of alternatives has on the perceived merit of the item. This extension is based on the extension of the Bradley–Terry model introduced by Davidson and Beaver [1977]. We prove results that give optimal designs, both for the extensions of the Bradley–Terry model and the extensions of the MNL model, and conduct simulations of these models. Finally, we prove results that give optimal designs for the MNL model when the starting design is an orthogonal array constructed using the Rao–Hamming construction, rather than a complete factorial design.