

Faculty of Engineering and Information Technology
University of Technology, Sydney

Volatility Modeling and Analysis via Coupled Wishart Process

A thesis submitted in partial fulfillment of
the requirements for the degree of
Master of Science in Computing Sciences

by

Zhong She

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

Volatility refers to the measure for price fluctuation of specific financial instrument over time. It is a very important factor that can greatly influence investor's decisions and concerns every other participant in the stock market. High volatility implies great insatiability and will definitely increase liquidity whereas low volatility indicates poor activeness. Hence the research on volatility draws great attention and interest of researchers from different backgrounds. Including the methods from data mining and machine learning is essential to improve the quality of volatility analysis.

There are two main types of models on volatility analysis: the deterministic models and stochastic models. The deterministic models assume the volatility at particular time is a deterministic function of the past. The generalized autoregressive conditional heteroskedasticity (GARCH) model and its variations are in such category. The stochastic volatility (SV) models take the assumption that the volatility follows certain random process. Recent literature has shows that the stochastic models outperform the deterministic models to some extent. Among them, the Wishart process is a hot tool for modeling multivariate volatility.

However, the stock market is closely connected with the society and human behavior, which makes it difficult to model. Almost all the existing models assume independence between our target objects: prices or the hidden covariance matrices behind them. These assumption works well for rough research or when the relationship between objects is weak. For a more solid research, the coupling relationship must be taken into account.

In this thesis, we present two kinds of coupled Wishart process to model volatility: the homogenous coupled Wishart process and heterogenous coupled Wishart process. And corresponding algorithms are developed based on the models. The homogenous coupled Wishart process refers to model that our target objects belong to the same category. A two-chain coupled Wishart process is introduced in this thesis. Within such a model, the matrix in one chain is not only related with the past one from its own chain but also from its neighbors. After the derivation of its learning procedures, synthetic data are tested. Then, experiments are implemented with real data from two markets: U.S. and Hong Kong. In the two-chain coupled Wishart process, one chain indicates the volatility from U.S. stock market and the other the volatility indicates Hong Kong stock market.

The latter one is the heterogenous coupled Wishart process. Unlike the homogenous one, in such a model, the covariance matrices are coupled with vectors, scalars or even a system. We aim to model how the outside influence from other kinds of data affect the evolving of covariance matrices. For time limitation, we make a simplified setup to illustrate how the heterogeneous coupling works. Then we construct the learning algorithm based on the setups and test it on synthetic data.

To conclude, we include the thought of coupling into the analysis of volatility via Wishart process, with machine learning techniques. Sufficient experiments have proved the effectiveness of coupling in volatility analysis.