STUDY OF SOME FUNCTIONALS OF STANDARD AND FRACTIONAL BROWNIAN MOTIONS WITH APPLICATIONS IN QUANTITATIVE FINANCE AND STATISTICS

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Abstract

This thesis contains results on two important problems arising in quantitative finance and statistics.

The first problem is about option pricing with a volume weighted average price (VWAP) as an underlying process. The VWAP is a very important quantity in finance; it appears in Australian taxation law to specify the price of share-buybacks for publicly-listed companies and it is a standard benchmark price for market participants. Pricing options on VWAP is challenging problem from a mathematical point of view because it involves two sources of randomness: the price of the asset and its traded volume. To solve the problem we have applied the moment-matching approach to a range of "stock and volume" models and as a result obtained an accurate approximation for prices of "call" options. All results have been verified by intensive Monte Carlo simulation.

The second problem is concerned with study of analytical properties and simulation algorithms for a fractional Brownian motion (fBM) which is considered a good alternative to modelling stochastic processes with long range dependence in modern Mathematical finance. In particular, we have reviewed the known simulation algorithms and have implemented the fastest of them ("circulant embedding") on a modern multicore computer. We applied the algorithm to two longstanding open problems in statistics, namely, to study distributions of exponential functionals of fBm and the maximum of fBm. The results of our simulations exhibit new and striking properties of these distributions.

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Student:

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