

CAPACITIVE OBJECT RANGING AND MATERIAL TYPE CLASSIFYING SENSOR

N.Kirchner, D.K. Liu, T.Taha and G.Paul

Centre for Autonomous Systems
University of Technology, Sydney
Sydney, Australia

{n.kirchner, d.liu, t.taha and g.paul} @cas.edu.au

Abstract—This paper presents modifications made to the Adaptive Capacitive Sensor for Obstacle Ranging (ACSOR) [1] that have enabled the sensor to provide material type data for an object in the sensing field. The ACSOR, previously capable of obstacle ranging (up to 500mm) in various densities of particle laden air, has been modified and fitted with a noise and stimuli-response analyzing algorithm allowing the sensor to determine the material type of a sensed object. Experimental results have demonstrated the modified sensor's ability to successfully classify the material type of several different objects whilst fitted to an in-motion 6-DOF anthropomorphic robotic arm in typical indoors conditions; 3.56% incorrect classifications (from a set of known material types) with an unintelligent 'hard logic' classifier.

Keywords—component; Capacitive, range sensor, material type, classification

I. INTRODUCTION

Bridges are a vital component of transport system infrastructure worldwide and with their exceedingly high construction costs there is significant motivation to extend their life spans. Research into premature bridge failure has identified rust as a primary cause and stripping the structure back to clean, untainted metal and then applying a paint coating as an effective means of protection [1]-[3]. The most effective method of large-scale metal stripping, such as that necessary for a bridge, is sandblasting and herein lies the problem. Sandblasting is a labor intensive and hazardous [4] operation. Not only are workers required to spend long periods of time handling forces in excess of 100N [5]-[6], but a large portion of the bridges in Australia are painted with lead and/or asbestos based paints. These types of paint pose a serious health risk to the workers tasked with their removal. With the long-term health damage of lead and asbestos now common knowledge [7], the appeal of replacing manual labor with robotic labor is high. This, along with changing workplace laws, which are slowly evolving to prohibit humans from working in such environments, leaves little alternative other than robotics in order to complete this necessary task.

In order to facilitate the use of autonomous robotics in construction/maintenance operations such as sandblasting, there is a need for the robots to be capable of self deriving an

understanding of the environment that they are to interact with; the environments are too complex and unstructured for the environmental knowledge to be derived manually [8]-[9]. Furthermore, the knowledge of occupied space alone is not adequate for true robot-environment interaction [10]. The robot must also be able to determine the material type of the objects occupying the space in order to adjust the method of interaction accordingly. Consider a robot designed to autonomously sandblast, the robot would cause considerable damage if it were to blast a section of plasterboard rather than metal. Clearly there is a need for the robot to be capable of determining the material type of the surface with which it intends to interact.

There are several mature sensors available for material type classification: with the sensors falling into the broad categories of contact/near-contact or non-contact. Due to the specific application targeted in this paper, contact or near-contact sensors, such as tactile sensors [11] and laser spectroscopy [12], are not desirable. This is largely due to the small sensing areas ($<25\text{mm}^2$) typical with this category of sensor. Physical contact with the surface is also an issue in some cases, for instance, it is not acceptable for the robot to make contact with a human.

Non-contact sensors, such as Impact Acoustic Sensors [13], Spectrometers (light based) and light diffusion measuring techniques [14], typically have larger sensing areas ($<1000\text{mm}^2$) and do not require physical contact with the surface. However, these sensors are again not suitable for the intended application due to the underlying technology on which they are based. The ambient noise typical in construction environments will saturate the receiver of the acoustic based method rendering it unusable. The light based methods are not appropriate for this application as they do not penetrate the surface of the object, thus it is possible for two materials of different types with the same coating (e.g. paint) to be indistinguishable from each other.

A capacitive-based approach offers many advantages. The broad distribution of the electric field allows large areas of coverage with a relatively small sensor size. Additionally, capacitive sensors are insensitive to lighting, noise, or the color, shape, surface or texture of the obstacle [15]. Although a significant amount of work has been done by manufactures and

researchers [16]-[20] developing capacitive sensors designed for ranging obstacles there is currently no capacitive based technology for determining material type available. Even though, the fundamental principles of operation of such sensors suggest that they will respond differently to sensed objects of different material types; which implies that material type classification may be practically possible.

This paper presents the Modified Adaptive Capacitive Sensor for Object Ranging (MACSOR). The MACSOR is capable of material type classification and object ranging in air heavily laden with particles. The MACSOR is a non-contact surface penetrating sensor whose base technology has proven to be immune to the conditions present in typical sandblasting environments [6]. The MACSOR is an extension of the sensor developed by Kirchner, et al. in 2006 [1], which is based on the work of Novak, et al. in 1992 [15]. The MACSOR distinguishes itself from these two sensors with its ability to classify the material type of the sensed object. Neither Novak's sensor, nor the ACSOR are capable of determining material type of the sensed object. they provide range information only. This paper details the adaptations made to the sensor that allow for the material type of the sensed object to be determined. Experimental results have demonstrated the MACSOR's ability to successfully classify the material type, from a set of known material types, of several different sensed objects whilst fitted to an in-motion 6-DOF anthropomorphic robotic arm in conditions typical to common indoors environments.

The breakdown of this paper is as follows; firstly the fundamentals of the capacitive technology that allows for material type classification will be discussed followed by a discussion of the modifications to the ACSOR that allow it to classify. Section 4 will detail the results from MACSOR testing, these results will then be discussed with conclusions drawn and future work proposed.

II. FUNDAMENTALS

Fig. 1 shows a schematic of the active sensing component of the MACSOR. The MACSOR is built on the same fundamental technology as Novak's sensor and the reader is referred to [15] for an in-depth explanation of the technology. In the following example a conductive obstacles will be considered as the intended application is for sensing bridge structures. The obstacle is assumed to be a flat plate orientated parallel to the sensor. This assumption is reasonable considering the sensor size (7cm x 7cm) compared to the size of the intended object being sensed; a part of a bridge structure.

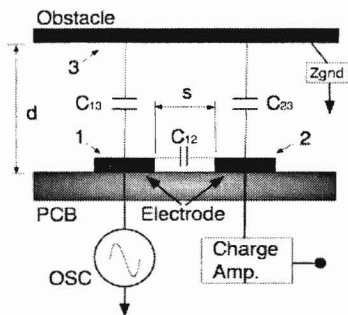


Figure 1. Schematic of Active Sensing Component

Referring to Fig. 1, electrode 1 is connected to a drive oscillator and electrode 2 is connected to a charge amplifier – this is, a charge to voltage converter. As the magnitude of the electric field generated by the drive oscillator on electrode 1 is fixed and as electric fields will follow the path of least resistance, then in the case where no obstacles are present the electric field tends to electrode 2 and gives rise to a maximum charge, Q , between electrodes 1 and 2, shown in (1).

$$Q = \oint_S \epsilon E dS \quad (1)$$

where E is the electric field vector and S is a surface completely enclosing the conductors. In the model shown in Fig.1 the surface, S reduces to a continuous path around the conductor; the obstacle.

The magnitude of the charge, Q , is determined by the drive oscillator voltage and frequency and remains fixed whilst the drive oscillator voltage and frequency is fixed. In the case shown in Fig. 1 where an obstacle with high impedance to ground is present Q becomes the sum of capacitances C_{12} , C_{23} and C_{13} and as Q is fixed and C_{23} and C_{13} are greater than zero, C_{12} must be of reduced magnitude (conservation of energy). Further to this, as capacitance is proportional to the distance between electrodes, the distance d is directly related to C_{23} and C_{13} and thus C_{12} . Due to the electrical configuration of the charge amplifier circuit the charge amplifier measures the sensor capacitance, C_{12} , only. It is this property of the sensor that is exploited to facilitate range measurements.

Fig. 2 shows the electrostatic field created by the sensor. The electric field lines are spaced by a repulsive force between adjacent field lines. The field lines actively follow the path of least resistance, in this case, the length of the obstacle. In the case of the field following through the path of least resistance a larger number of field lines will occupy the same physical space as a smaller number of field lines through a path of higher resistance. This is, the repulsive force will be partially negated by the attractive force of traveling through the path of lesser resistance and adjacent field lines will become closer.

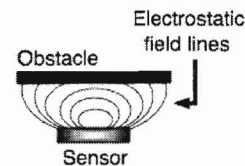


Figure 2. Electrostatic Field

III. MACSOR

To understand how the MACSOR is responsive to the material being sensed we again refer to the interaction of the electric field created by the sensor and the obstacle. Fig. 3 shows the electric fields interaction with a conductive and with a non-conductive obstacle present. As can be seen in the figure, the attraction to travel along a conductive path (low resistance to field flow) is sufficiently high to cause the majority of field lines to travel along the length of the obstacle. Conversely, the

attraction to travel along a non conductive path is not sufficient to cause the field lines to travel the length of the obstacle.

The sensor's maximum response is limited by the magnitude of the electrostatic field flowing along the obstacle and between the obstacle and sensor. At the range where the electric field is of sufficient intensity so that the repulsive force between adjacent field lines will cause the field lines to penetrate the lesser conductive obstacle rather than travel along it, the sensor's reading will be lower than with a conductive obstacle in the same position [21]. Thus, the MACSOR will respond to a physical property (resistivity) of a sensed object. Assuming that the materials of interest have sufficiently different resistivity to field line flow then this property can be used for material type classification.

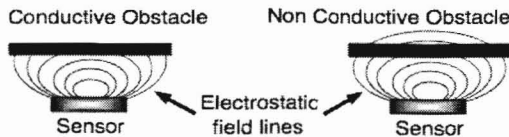


Figure 3. Electrostatic Field Interaction with Conductive and Non-Conductive Obstacles

Further to this, as the electric field is not truly static and has some variation in intensity (due to fluctuations/noise in the circuit and the environment) the instantaneous magnitude of the electric field flowing through the obstacle and between the obstacle and sensor will vary depending on the resistivity of the material. For instance, in the case of an obstacle with high resistivity, a small increase in the field intensity may cause the outer most field line to penetrate through the obstacle thus reducing the magnitude of the field acting on the sensor. Conversely, in the case of an obstacle with low resistivity a large change in the field intensity would be required to cause a similar effect. This theory suggests that the variance in a set of readings of the same material will be larger if the material has a higher resistivity. Assuming that the materials of interest have sufficiently different resistivity, then the variance in consecutive readings of the same material is potentially suitable for use as the basis on which to classify the material type.

The deviation of a new reading, at a known sensor-object distance, from a reference reading and the variance of one-thousand consecutive readings (20ms to acquire) are compared to the corresponding values for known materials. The result of this comparison enables the MACSOR to classify a sensed object's material type; from a set of known materials.

IV. RESULTS

A. Evaluating the Sensors Response

The first test was designed to evaluate the sensor's response with objects of various material types in the sensor field. The MACSOR was fitted to the end-effector of an anthropomorphic robotic arm (6-DOF); Fig. 4 shows the MACSOR and Fig. 5 shows the MACSOR fitted to the robot. The robot was programmed to place the sensor parallel to the object's surface at 10mm from the object. The robot then moved directly away from the object (with no lateral or vertical displacement - as indicated by the darker arrow in the Fig. 5) whilst taking ten

sets of one thousand readings at each ten millimeter interval, up to 300mm. The test was repeated with the objects of the material types; Metal, Wood, Concrete and Human and with no object present (a control). A total of one million, five hundred thousand readings were taken, approximately $20\mu s$ per reading.

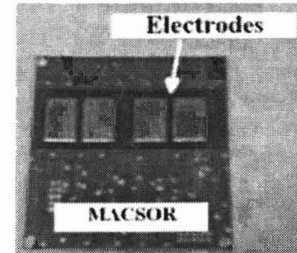


Figure 4. MASOR Sensor

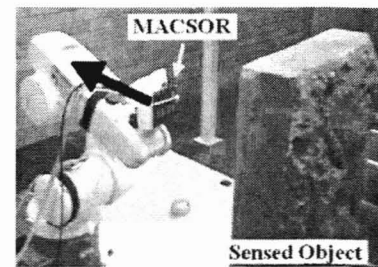


Figure 5. MASOR Sensor on Robot

Fig. 6 shows one set, randomly selected from the ten, of readings at 100mm from the object for each material tested. As can be seen from the figure there is considerable variation between consecutive readings for all of the materials tests, however, it is observable that the magnitude of the variation is different for the various materials. For instance, when comparing the variance of the readings for Wood and Metal, as the theory in Section 3 suggested, the less resistive material has a smaller variance. The darker, seemingly straight lines on the figure are in fact three hundred reading moving averages: three hundred readings takes $6ms$ to complete. As can be seen the moving average produces a stable output; which is used for object ranging as described in [1].

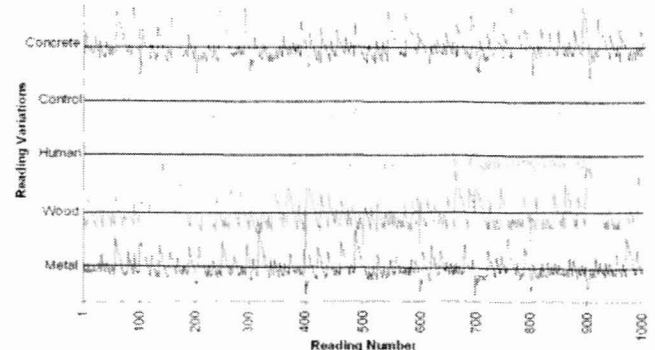


Figure 6. 1000 Readings for Materials at 100mm

Fig. 7 shows the standard deviation of the ten thousand readings taken for each of the materials at each 10mm increment over the range of 10mm-300mm from the object. As can be seen from the figure at an object-sensor distance of less

than 150mm the difference between the standard deviation of the readings enables the partial classification (Metal, Human/Concrete or Wood) of the sensed material.

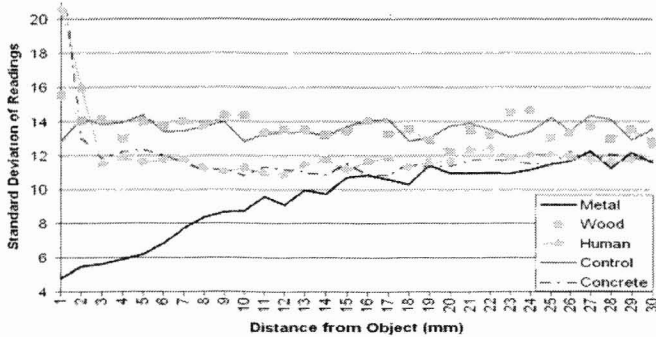


Figure 7. Readings' Standard Dev. for Various Materials (10mm-300mm)

Fig. 8 shows the difference between the sensors output-reading (from hereto to be considered the stable value produced from the aforementioned moving average) and the value for the control reading (this will be referred to hereafter as the Δ FAR) for each of the materials at each 10mm increment over the range of 10mm-300mm from each of the objects. As can be seen from the figure; at an object-sensor distance of less than 100mm, the Δ FARs for each of the materials are sufficiently differentiable to enable the partial classification (Metal, Human, Concrete or Wood/Control) of the sensed material. Further to this, Wood and Control is distinguishable if the sensor is moved to 60mm or if an object is known to be present. Based on this and the previous result the readings taken at object-sensor distance of approximately 100mm will yield the most information in a single output-reading.

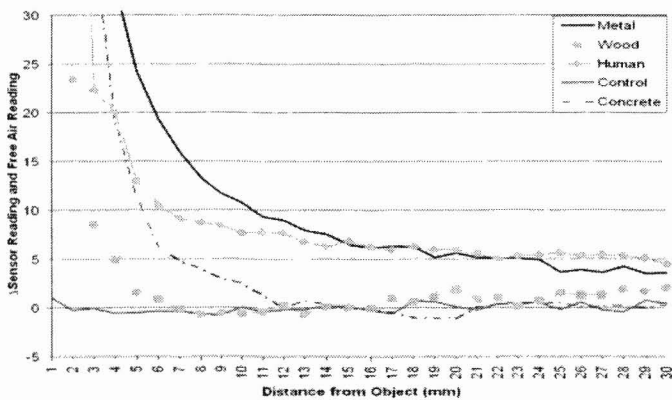


Figure 8. Readings for Various Materials (10mm-300mm)

At this stage an unintelligent hard-logic classifier was designed. The classifier takes a reading from the MACSOR (at a known object-sensor distance) and determines the Δ FAR and the intra output-reading standard deviation. The Δ FAR is compared to the Δ FARs obtained from Fig. 8 (at the same object-sensor distance) and an intermediate classification is made by finding the Δ FAR from the graph closest to the Δ FAR of the reading at the known range. A similar process is then repeated for the intra output-reading standard deviation and if both intermediate classifications agree upon the same material the classifier outputs a material type. If the intermediate

classifications disagree, the classifier output indicates an unknown material is present. At this point it must be emphasized that the primary research goal of this work was to develop a sensor that provides information useful for classification and not to develop a classifier. The simple classifier mentioned above was implemented as a means to evaluate the sensor's potential.

B. The Sensor's Ability to Classify

The next test was designed to determine the MACSOR's ability to classify. The MACSOR was again fitted to the end-effector of the 6-DOF anthropomorphic robotic arm as described in the previous test. The MACSOR was then placed at a distance of 100mm from various objects (as can be seen in Fig. 9) and three hundred output-readings for each material were taken; the readings were taken at spaced intervals over a total of one hundred and twenty minutes.

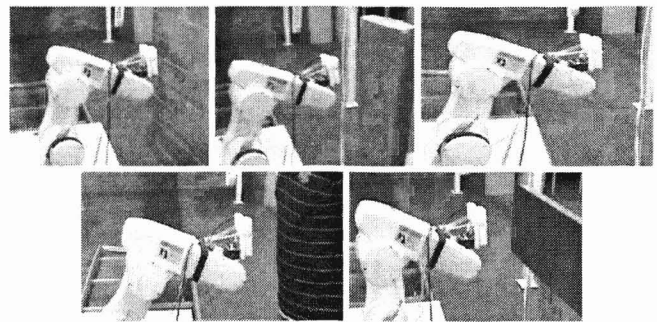


Figure 9. MACSOR Testing Various Materials (Wood, Concrete, Control, Human, Metal)

As this test was performed on a different day than the previous test and using a different objects for the Wood, Metal and Human tests, a new set of Δ FARs and intra output-reading standard deviations were required to classify with. This new set of classifier reference values was obtained by randomly selecting nine output-readings from the three hundred taken, for each of the materials. Table I shows the Δ FARs and intra output-reading standard deviations derived from these readings.

TABLE I. CLASSIFIER REFERENCE VALUES

Material	Δ FARs	σ
Metal	19.397	7.153
Human	18.934	8.717
Concrete	10.690	11.023
Wood	3.641	12.729
Control	0.843	16.107

With the classifier reference values now determined the remaining output-readings where inputted into the previously described classifier. Table II presents the results from the classifier. As can be seen from the table using a single output-reading from the MACSOR the classifier was able to correctly classify the material of the sensed object with a 0.8231 probability. The remaining probability of 0.1769 is spread over the classifier being unable to classify the sensed objects material (0.1324) and incorrect classifications (0.0445). On further analysis of the results it can be seen that all errors

consisted of classifying either Metal or Concrete as Human. This is significant as, for the intended application (sandblasting) this type of incorrect classification will result in non-blasting of the area – this is, the classifier has failed in a safe manner. These results clearly demonstrate the MACSOR's can be used to successfully classify material type.

TABLE II. CLASSIFIER RESULTS

Object Material	Correct (%)	Incorrect (%)	Unknown n (%)	Most Common Error (%)
Metal	84.04	16.60	0	Human (16.60)
Human	77.25	0	22.25	-
Concrete	73.34	1.11	25.55	Human (1.11)
Wood	94.60	0	5.40	-
Overall	82.31	4.45	13.24	Human (4.45)

The next test was designed to evaluate the MACSORs ability to classify whilst in motions. This test saw the MACSOR fitted to the end-effector of the 6-DOF anthropomorphic robotic arm as described in the previous tests, however, in this test the robot was programmed to continuously move between the different materials collecting data and attempting to classify the material when located 100mm from the surface of the objects. A total of fifteen hundred output-readings (spread evenly between the materials) were taken. An image sequence of this test is shown in Fig. 10.

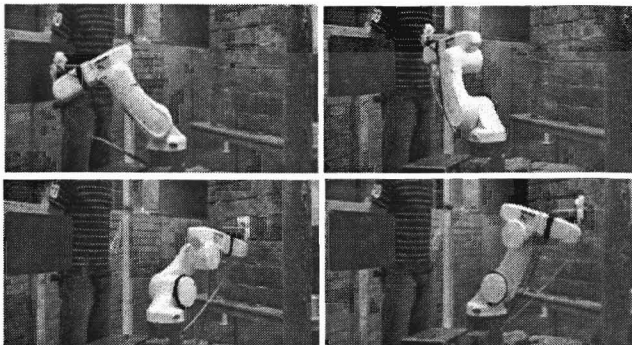


Figure 10. In-Motion Classification

As with the previous tests, a new set of Δ FARs and intra output-reading standard deviations were required to in order to classify. This new set of classifier reference values was obtained by randomly selecting nine output-readings of each of the materials from the fifteen hundred readings taken. Table III shows the Δ FARs and intra-reading standard deviations derived from these readings. As can be seen, although the magnitudes vary the trends are again similar.

TABLE III. CLASSIFIER REFERENCE VALUES

Material	Δ FARs	σ
Metal	12.646	12.426
Human	18.060	9.9045
Concrete	7.835	14.067
Wood	2.695	14.673
Control	1.469	17.528

With the classifier reference values now determined the remaining output-readings were inputted into the classifier. Table 4 presents the results from the classifier. As can be seen from the table, using a single output-reading from the MACSOR the classifier was able to correctly classify the material of the sensed object with a 0.8867 probability. An incorrect classification accounts for the remaining probability (0.1163). On further analysis of the results it can be seen that the major source of errors consist of classifying Metal as Human. Again the classifier has failed in a safe manner. The next largest source of errors arises from classifying Concrete as Metal. This is not a critical error, however, it is not desirable as it would result in concrete being sandblasted. These results clearly demonstrate the MACSORs ability to successfully classify material type whilst fitted to an in-motion robotic arm. The results also suggest a more sophisticated classifier is required for in application use; in order to ensure safety.

TABLE IV. CLASSIFIER RESULTS

Object Material	Correct (%)	Incorrect (%)	Unknown (%)	Most Common Error (%)
Metal	72.95	27.05	0	Human (27.05)
Human	100	0	0	-
Concrete	89.95	10.05	0	Metal (7.61)
Wood	100	0	0	-
Overall	88.67	11.33	0	Human (8.57)

The final result presented here details the effect of a statistical-correlation based modification to the classifier criteria. This modification is based on the assumption that; as three consecutive readings are taken in 60ms the readings are statistically likely to be from the same obstacle. This change has been included in order to illustrate the error reduction possible by introducing a more intelligent classifier (set down as future work). To implement the change the final output stage was modified so that in order for a classification to be outputted two of three consecutive classifications must report the same material and the third must be either the same material again or an unknown material but, cannot be a different material. As an output-reading are obtained from the sensor in 20ms, requiring three for a single classification does not create a significant time lag for obtaining a classification.

Table 5 presents the result of the aforementioned criteria modification made to the classifier. As can be seen from the table the introduction of this criteria modification before a final classification may be outputted has resulted in a significant decrease in the probably of the classifier outputting an incorrect classification or reporting the material as unknown. The probability of the classifier outputting an error while static has gone from 4.45%→0.58%, the probability of a classifier error whilst in-motion has gone from 11.33%→3.56%. As a final note, this result suggests that the accuracy of the system is higher when the sensor is in-motion, this is not the case however. as can be seen from comparing Table I to Table III, the Δ FARs and the intra-reading standard deviations for the most commonly misclassified materials are considerably more differentiable for the in-motion test and it is this that has enabled the classifier to operate with a higher accuracy.

TABLE V. MODIFIED CRITERIA CLASSIFIER

Test	Correct (%)	Incorrect (%)	Unknown (%)
Pre Mod.			
Static	82.31	4.45	13.24
In-motion	88.67	11.33	0
Post Mod.			
Static	91.72	0.58	7.70
In-motion	96.44	3.56	0

V. CONCLUSIONS AND FUTURE WORK

This paper has presented the MACSOR which has been designed to provide information on the material type of an object in the sensing field.

Experiments have demonstrated the MACSORs ability to classify material type from a set of known materials common to sandblasting environments with a 0.58% error when the sensor is static and with a 3.56% error when fitted to an in-motion 6-DOF anthropomorphic robotic arm. The results have clearly demonstrated that the output from the MACSOR can be analyzed to yield material type information during use in our stated application and thus the technology will be pursued.

Future work will primarily focus on developing a more intelligent classifier in order to reduce classification errors and to allow for the classification between a larger set of material types and evaluating the MACSOR under different environmental conditions.

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