The Eighth International Conference on Intelligent Technologies (InTech'07)
Sydney, Australia, 12-14 December 2007

Takagi-Sugeno Fuzzy Modelling of Multivariable Nonlinear System via Genetic Algorithms

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***Abstract*—In this paper, the Takagi-Sugeno (T-S) fuzzy model of a multivariable nonlinear system in state space form is obtained using the developed fuzzy modelling algorithm. In this fuzzy model, the system sate space equation is expressed as the fuzzy summation of the state variables, disturbance and control input. To obtain this model with high accuracy, the genetic algorithm (GA) with a new encoding method is applied to search for the optimal model parameters. The proposed hybrid intelligence technique can evolve the fuzzy rule structure (number of rules and selection of rules, number of premise inputs and selection of premise inputs) so that the obtained fuzzy model has the simplest structures without decreasing the modelling accuracy. To validate the proposed approach, the algorithm is applied to model a building structure with a magneto-rheological (MR) damper, which is a multivariable nonlinear system. The modelling errors between the system outputs and the corresponding fuzzy model outputs are compared with the automatically selected rules. It is confirmed by the validation results that the proposed hybrid intelligence technique can find the optimal T-S fuzzy model for the nonlinear system.**

Keywords- Takagi-Sugeno fuzzy modelling ; multivariable nonlinear system ; genetic algoritms

I. INTRODUCTION

Nowadays, the T-S fuzzy modeling technique is becoming powerful engineering tools for modelling and control of complex nonlinear dynamic systems. The T-S fuzzy model is a system described by fuzzy if-then rules which can give local linear representation of the nonlinear system. For the reason that it employs linear model in the consequent part, conventional linear system theory can be applied for the system analysis and synthesis easily. The methods for learning T-S fuzzy models from data are based on the idea of consecutive structure and parameter identification [1]. To accommodate new input data, adaptive online learning of T-S fuzzy model has been developed [2]. On the other hand, design of a fuzzy model can be formulated as a search problem in multidimensional space where each point represents a possible fuzzy model with different rule structure, membership functions (MFs), and related parameters. Due to the search capability, evolutionary algorithms (EAs), such as genetic algorithms (GAs) and evolution strategies (ESs), have been utilised greatly in evolutionary fuzzy modelling. In some of EA-based fuzzy models, only parameters of fuzzy models are optimised using EAs while the structure itself is fixed [3]. Since parameters and rule structure of fuzzy models are codependent, they should be designed or evolved simultaneously. Thus, methodologies that try to change the rule structure by encoding all the information into the chromosome have been developed [4]. In this paper, the GA-based fuzzy modelling algorithm is developed. Especially, an encoding scheme that consists of three kinds of genes in one chromosome, which allows simultaneous optimisation of parameters of antecedent MFs, rule structure (number of rules and selection of rules), and input structure (number of premise inputs and selection of premise inputs) is proposed. For simplicity in the specified application, the fitness function only considers one evaluation criterion (accuracy) in terms of the mean square error (MSE), and the other aspect, compactness (number of rules) is constrained with the maximal number.

To demonstrate the effectiveness of the obtained T-S fuzzy model, the presented approach is applied to approximate the dynamic behaviour of a building structure with a magneto-rheological (MR) damper, which is a nonlinear structural system, in the form of the T-S fuzzy model. The use of the T-S model to emulate the dynamic behaviour of the nonlinear building-MR damper system is validated by numerical values. The obtained T-S fuzzy model could be used for designing a nonlinear fuzzy controller for the nonlinear system in a systematic way.

II. TAKAGI-SUGENO FUZZY MODELLING OF NONLINEAR SYSTEM

The T-S fuzzy model is a system described by fuzzy IF-THEN rules which can give local linear representation of the nonlinear system by decomposing the whole input space into several partial fuzzy spaces and representing each output space with a linear equation. Such a model is capable of approximating a wide class of nonlinear systems. For the reason that it employs linear model in the consequent part, conventional linear system theory can be applied for the system analysis and synthesis accordingly. And hence, the T-S fuzzy models are becoming powerful engineering tools for modelling

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and control of complex dynamic systems. To obtain the T-S fuzzy model of a multivariable nonlinear system, we need using the fuzzy modelling technique as following:

IF $\theta_1(t)$ is M_1^1 and $\theta_2(t)$ is M_2^1 and ... and $\theta_p(t)$ is M_p^1,
THEN $\dot{x}(t) = A_i x(t) + B_{1i} w(t) + B_{2i} u(t)$,

.
.
.

IF $\theta_1(t)$ is M_1^r and $\theta_2(t)$ is M_2^r and ... and $\theta_p(t)$ is M_p^r,
THEN $\dot{x}(t) = A_r x(t) + B_{1r} w(t) + B_{2r} u(t)$,

where M_j^i is a fuzzy set on the jth premise variable defined by the MF, $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]^T$ are the premise variables, p is the number of premise variables, $x(t) = [x_1(t), \ \ldots \ , x_n(t)]^T$ are the state variables, $w(t)$ is the disturbance input, $u(t)$ is the control input. Scalar r is the number of IF-THEN rules, and A_i, B_{1i}, and B_{2i} are constant matrices.

Then, the overall fuzzy model is inferred as follows:

$$\dot{x}(t) = \sum_{i}^{r} h_i(\theta(t))[A_i x(t) + B_{1i} w(t) + B_{2i} u(t)], \qquad (1)$$

where

$$h_i(\theta(t)) = \frac{\mu^i(\theta(t))}{\sum_{i=1}^{r} \mu^i(\theta(t))}, \qquad (2)$$

$$\mu^i(\theta(t)) = \prod_{j=1}^{p} M_j^i(\theta(t)), \qquad (3)$$

$M_j^i(\theta(t))$ is the grade of the MF of $\theta_j(t)$ in $M_j^i(\theta(t))$. When a Gaussian function is used, then

$$\mu^i(\theta(t)) = \prod_{j=1}^{p} M_j^i(\theta(t)) = \prod_{j=1}^{p} e^{-\frac{(\theta_j - c_j^i)^2}{(b_j^i)^2}} \qquad (4)$$

where c_j^i and b_j^i represent the centres and widths of the MFs, respectively. It is obvious that

$$h_i(\theta(t)) \geq 0, \qquad \sum_{i=1}^{r} h_i(\theta(t)) = 1 \ . \qquad (5)$$

For system (1), it is easy to derive the stability conditions of system (1) by using Lyapunov function and condition (5) as:

$$A_i^T P + P A_i < 0, \qquad i = 1,2,\ldots,r \ , \qquad (6)$$

$$\left(\frac{A_i^T + A_j^T}{2}\right) P + P \left(\frac{A_i + A_j}{2}\right) < 0, \qquad 1 \leq i \leq j \leq r \ . \qquad (7)$$

Most of the studies on T-S fuzzy models consider that all inputs used in the premise variables are used in the consequents. However, in general, the premises of the rules describe different operating regions which depend on some antecedent inputs, while the consequents are linear (or affine) descriptions of the behaviour of the system in each of the operating regions that do not necessarily depend on the same inputs. So, in many applications, the approximation of a nonlinear system by local linear models requires many antecedent inputs to characterise the regions where the dynamics of the system can be considered as linear. Hence, we consider that the antecedent vector $\theta(t)$ is not necessary the same as the vector [$x(t)$, $w(t)$, $u(t)$] which was used in the consequent affine models.

Therefore, to represent a nonlinear system with the T-S fuzzy model accurately, the premise variable $\theta(t)$ and the number of premise variable p, the membership function parameters c_j^i, b_j^i, and the number of fuzzy rules r, the constant matrices A_i, B_{1i}, and B_{2i} are needed to be determined. Generally, the selections of fuzzy rules and premise inputs are made by trial and error method or using some clustering methods. In this paper, the selections will be made by GAs with a new encoding scheme so that they can be automatically selected in the modelling process.

III. ENCODING SCHEME

Using GAs to design the T-S fuzzy model, one of the first important things is to encode the T-S fuzzy model parameters into the chromosome with an efficient method. When the rule structure (number of rules and selection of rules), the input structure (number of premise inputs and selection of premise inputs), and the parameters of MFs associated are specified, the T-S fuzzy model will be specified. In order to realise the automatic selection of rules and inputs, a new encoding scheme is presented. The proposed encoding scheme uses a chromosome that consists three parts as shown in Fig. 1. The first part deals with the rule selection and the optimisation of number of rules, the second part deals with the input selection and the optimisation of number of inputs, and the third part deals with the optimisation of parameters of MFs. Here, we adopt the binary-coded GAs and every gene in the chromosome is represented by a binary value `1' or `0'.

In the first and second parts, each gene represents one rule or one input. The position of one gene in the first part will denote the corresponding sequence of one rule in all the rule sets, and the position of one gene in the second part denotes the corresponding sequence of one input in all the input sets. The selection of rules or inputs is made by checking the binary value of the gene. If a specified gene in the first part is zero, then the corresponding rule is not valid and vice versa. If a

1st part			2nd part			3rd part					
rule 1	… …	rule N_R	input 1	… …	input N_I	centre 1	… …	centre $N_I x N_R$	width 1	… …	width $N_I x N_R$
1	… …	0	1	… …	1	01...11	… …	10...10	01...10	… …	01...01

Fig. 1. Encoding scheme

specified gene in the second part is one, then the corresponding input is valid and vice versa. So, the information of genes in the first and second parts represents whether a certain rule or input is used or not for the current rule structure or input structure of an individual.

IV. Modelling Algorithm

Using the standard GAs together with the presented encoding scheme, the T-S fuzzy model can be obtained by the following steps:

Step 1: Generate training and validation data from a nonlinear system to be modelled.

Step 2: Set parameters for the T-S fuzzy models. The maximum numbers of fuzzy rules and premise inputs, the premise variable candidates, the consequent part variables, etc., are needed to be given.

Step 3: Encode some of the T-S model parameters, such as $r_{\max}$, $p_{\max}$, c_j^i , and b_j^i , into chromosome using the presented encoding scheme.

Step 4: Generate initial population for chromosomes.

Step 5: Calculate objective functions.

5.1, after the centres (c_j^i), widths (b_j^i), and the numbers of rules (r) and inputs (p) are generated, the weights for every state variable are calculated using the pseudo-inverse algorithm. The weights form the row of matrices A_i , B_{1i} , and B_{2i} corresponding to every state variable, i.e., $\dot{x}_k(t) = a_{ik}x(t) + b_{1ik}w(t) + b_{2ik}u(t)$, where a_{ik} , b_{1ik} , and b_{2ik} are weights, $k = 1,2,\ldots,n$, n is the number of state variable.

5.2, calculate the objective function for every state variable $\dot{x}_k(t)$. The mean square error (MSE) for the training data or the testing data is regarded as the objective function of each chromosome. If necessary, the evolved number of rules and number of inputs can be added into the objective function to obtain the reasonable sizes of the rules and inputs. The summation of MSE for every state variable will be the overall objection function, i.e., $MSE(\dot{x}(t)) = \sum_{k=1}^{n} MSE(\dot{x}_k(t))$. It is noted that the system stability is important for designing a controller for the obtained T-S fuzzy system. Therefore, the check of the stability of the obtained system will be made using the stability conditions (6) and (7). If the obtained T-S fuzzy system is not stable, then its objective function will be assigned a large value instead of using the MSE value in order to reduce its opportunity to survive in the next generation.

5.3, record every objective function that corresponds to every set of parameters to a suitable fitness value according to the rank-based fitness assignment approach.

Step 6: Apply evolutionary operators: selection, crossover, and mutation.

Step 7: Use the elitist reinsertion approach.

Step 8: Evaluate the fitness of each individual.

Steps 5 to 8 correspond to one generation. The evolution process will repeat for a fixed number of generations or will end when the search process converges with a given accuracy. The best chromosome will be used to determine the optimal numbers of rules and inputs, centres and widths. At last, the T-S fuzzy model (1) will be constructed using the obtained fuzzy rules, premise variables, centres, widths, and matrices A_i , B_{1i} , and B_{2i} .

V. Application Example

In this section, a three-storey building model with a single magneto-rheological (MR) damper as shown in Fig. 2 is considered. This model has been used by many researchers, e.g. [6], to study the control problem of MR damper. The MR damper is a semi-active control device that employs a special type of controllable fluids, the magneto-rheological fluids, which typically consist of micron-sized, magnetically polarisable particles dispersed in a carrier medium such as mineral or silicone oil. When a magnetic field is applied to the fluid, the particles are lined up in chains so that the fluid becomes semisolid within a few milliseconds, exhibiting a plastic behaviour. However, the practical use of MR dampers for control is still hindered by their inherently hysteretic and highly nonlinear dynamics. This makes the modelling of MR dampers more important for their applications. The integrated structure and nonlinear damping device behave nonlinearly although the structure itself is usually assumed to remain linear. Because the building-MR damper system is intrinsically nonlinear, development of an appropriate model of the system that includes interaction between the structural system and nonlinear device plays a key role in control system design. In this paper, the nonlinear system will be modelled as T-S fuzzy model so that nonlinear fuzzy controller could be designed to reduce the vibration of the system.

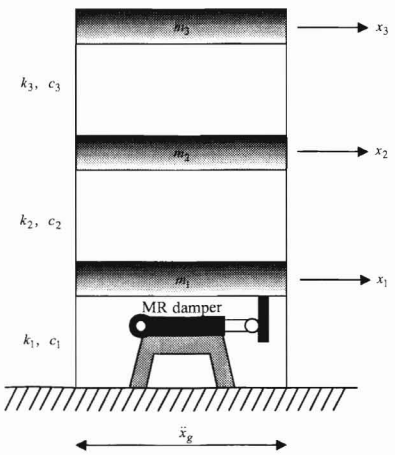

Fig. 2. Three-storey building structure with a single MR damper

The equation of motion for the linear structure model with external disturbance and control input can be written as

$$\overline{M}\ddot{x}(t)+\overline{C}\dot{x}(t)+\overline{K}x(t)=Hf(t)+Ew(t)\,, \tag{8}$$

where $x(t)=[x_1(t)\ \ x_2(t)\ \ x_2(t)]$ is the floor displacement vector; $f(t)$ is the control force generated by MR damper; H gives the location of the control forces; $w(t)$ is the ground excitation disturbance; E is an vector denoting the influence of disturbance; $\overline{M}$, $\overline{C}$, and $\overline{K}$ are the mass, damping, and stiffness matrices of the structure, respectively. The MR damper is installed between the ground and the first floor. The system matrices are

$$\overline{M}=\begin{bmatrix}98.3 & 0 & 0\\ 0 & 98.3 & 0\\ 0 & 0 & 98.3\end{bmatrix}kg\,,$$

$$\overline{C}=\begin{bmatrix}175 & -50 & 0\\ -50 & 100 & -50\\ 0 & -50 & 50\end{bmatrix}Ns/m\,,$$

$$\overline{K}=\begin{bmatrix}12.0 & -6.84 & 0\\ 6.84 & 13.7 & -6.84\\ 0 & -6.84 & 6.84\end{bmatrix}N/m\,,$$

$$H=\begin{bmatrix}-1\\ 0\\ 0\end{bmatrix},\quad E=\begin{bmatrix}-1\\ -1\\ -1\end{bmatrix}.$$

A phenomenological model has been proposed by Spencer et al [5] to portray the behaviour of a prototype MR damper. This model is governed by the following seven simultaneous equations:

$$\begin{aligned}
&f=c_1\dot{y}+k_1(x-x_0)\\
&\dot{y}=\frac{1}{c_0+c_1}\left[\alpha z+c_0\dot{x}+k_0(x-y)\right]\\
&\dot{z}=-\gamma|\dot{x}-\dot{y}|z|z|^{n-1}-\beta(\dot{x}-\dot{y})|z|^n+A(\dot{x}-\dot{y})\\
&\alpha=\alpha_a+\alpha_b u\\
&c_1=c_{1a}+c_{1b}u\\
&c_0=c_{0a}+c_{0b}u\\
&\dot{u}=-\eta(u-v)
\end{aligned}\,, \tag{9}$$

where f is the force generated by the MR damper; x is the displacement of the damper; y is an internal pseudo-displacement of the MR damper; u is the output of a first-order filter; v is the command voltage sent to the current driver. In this model, k_1 is the accumulator stiffness; c_0 and c_1 are the viscous damping coefficients observed at large and low velocities, respectively; k_0 is the gain to control the stiffness at large velocities, and x_0 is the initial displacement of spring k_1 associated with the nominal damper force due to the accumulator; γ, β, A are hysteresis parameters for the yield element, and α is the evolutionary coefficient. In this model, there are a total of 14 model parameters to characterize the MR damper. The obtained values for the 14 parameters can be determined by fitting the model to the experimental data obtained in the experiments. As an example, a set of parameters which was obtained by Spencer at al [5] was listed in that paper.

A block diagram of the integrated nonlinear structural system to be modelled is shown in Fig. 3. There are two external inputs to the nonlinear system, including the ground excitation $\ddot{x}_g(t)$ acted at the ground floor of the structure and the applied command voltage $v(t)$ sent to the MR damper. The measured outputs are the system state variables, i.e., $x(t)=[x_1(t),x_2(t),x_3(t),\dot{x}_1(t),\dot{x}_2(t),\dot{x}_3(t)]^T$. The aim of the modelling process is to build a T-S fuzzy model as shown in Eq. (1) according to the measured state variables, ground excitation, and command voltage.

In this paper, the phenomenological model of the MR damper is used to simulate the MR damper. The parameters used for MR damper are same to those given in reference [6]. To get the T-S fuzzy model, data for training and testing of the T-S fuzzy model are generated firstly. In order to obtain a high quality trained model, a high quality training and testing data must be obtained. To make the identified model fully represent the underlying system, the training samples should cover all possible combinations and ranges of input variation in which the system will operate. This is to ensure that the T-S fuzzy models trained using these samples can accurately represent the dynamic behaviour of the integrated nonlinear system. Normally, the limits of these input signals are dependent upon the characteristics and specific applications of the system. Advanced knowledge of the input signals enables the creation of more useful training data. Given this idea, note that the

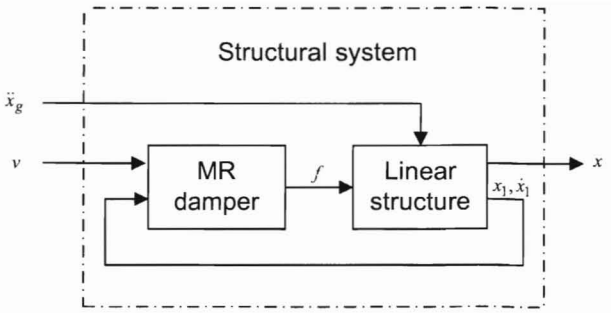

Fig. 3. System identification block diagram

maximum operational voltage of the MR damper is 2.25 V, which is defined as the saturation voltage of the damper and is obtained experimentally, and the situation of zero voltage will also be common during operation of the MR damper. Therefore, ranges of the voltage signal and its frequency are set as 0-2.25 V and 0-6 Hz, respectively, in this study. The ground excitation is used as a random signal (0-50 Hz) with peak amplitude around 0.6 g. Signals of displacement and voltage used for training are produced using band-limited Gaussian white noise and some specified filters are used to obtain such random signals in indicated frequency ranges. Fig. 4 shows the histories of ground acceleration and command voltage signals. Once the ground excitation and command voltage signals are sent to the integrated nonlinear system, the responded state variables are measured. Then, using these obtained signals and the training algorithm presented in Section IV, the T-S fuzzy model of the nonlinear system can be obtained.

For brevity, in this study, we assume all the signals used can be obtained, and we define the fixed premise variables as the previous floor displacements, floor absolute accelerations, and command voltage. We define the consequent variables as the current state variables, ground excitation, and command voltage. The objective is to predict the next state variables. The maximum fuzzy rule number is set as 50. After finishing the training process, the automatically selected fuzzy rule number

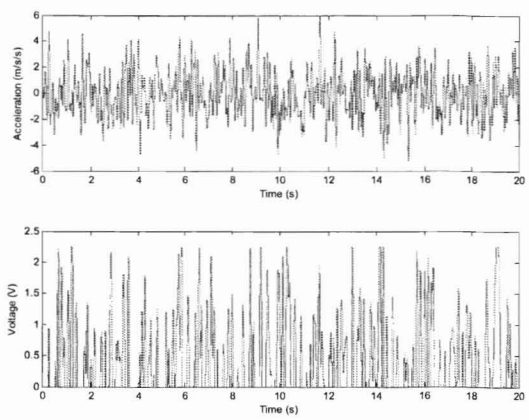

Fig. 4. Training data

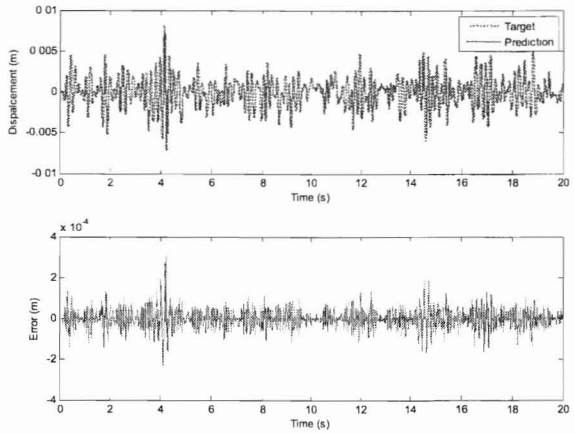

Fig. 5. Modelling results for the first floor displacement

by the presented algorithm is given as 31. The modeling result is shown in Fig. 5. For brevity, only the displacement of the first floor is plotted in this figure. The error between the prediction (output of the T-S fuzzy model) and the target (output of the integrated building-MR damper nonlinear system) is shown in the figure as well. It can be seen from this figure that the modeling error is very small. It validates that the T-S fuzzy model can represent the nonlinear system well. The velocity of the first floor is plotted in Fig. 6, where the prediction and target velocities and the error between the prediction and the target are plotted, respectively. It is also shown that the prediction error is very small. The root mean square error (RMSE) values for the all the state variables are listed in Table I.

To further validate the obtained T-S fuzzy model, the scaled El Centro 1940 earthquake data is applied to both the integrated structure and the obtained T-S fuzzy model. The command voltage signal is also a random signal with amplitude 0-2 V and frequency 0-3 Hz. The earthquake data and the command voltage signal are shown in Fig. 7. The modeling results for the first floor displacement and first floor velocity are shown

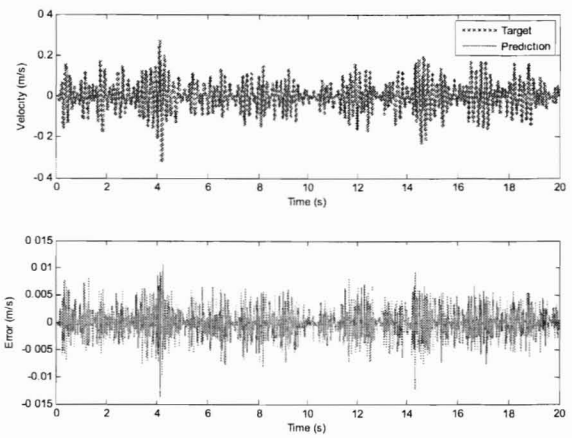

Fig. 6. Modelling result for the first floor velocity

in Figs. 8-9, respectively. The RMSE values for all the state variables are listed in Table II. It can be seen from Figs. 8-9 and Table I that the prediction errors are very small even when the ground excitation and command voltage signals are different from those given in the training process, the obtained T-S fuzzy model can still approximate the nonlinear system dynamic outputs very well. It validates that the obtained T-S fuzzy model can represent the nonlinear system reasonably.

VI. CONCLUSIONS

In this paper, a T-S fuzzy model is developed to approximate the dynamic behaviour of a nonlinear system. The rule structure, input structure, and the MF parameters are simultaneously obtained by GA with the objective to reduce the MSE between the predicted output and the true output. Numerical simulation on a three-storey building structure with a single MR damper is used to testify the presented training algorithm. It is certified by the training and validation data that the presented T-S fuzzy model can emulate the dynamic behaviour of the nonlinear system.

TABLE I. RMSE VALUE FOR TRAINING AND VALIDATION DATA

RMSE	Training data	Validation data
$x_1(t)$	5.0767×10^{-5}	1.9539×10^{-5}
$x_2(t)$	7.7847×10^{-5}	3.6912×10^{-5}
$x_3(t)$	9.2097×10^{-5}	5.1182×10^{-5}
$\dot{x}_1(t)$	1.7658×10^{-3}	1.5040×10^{-3}
$\dot{x}_2(t)$	2.4340×10^{-3}	2.0126×10^{-3}
$\dot{x}_3(t)$	2.9467×10^{-3}	2.5005×10^{-3}

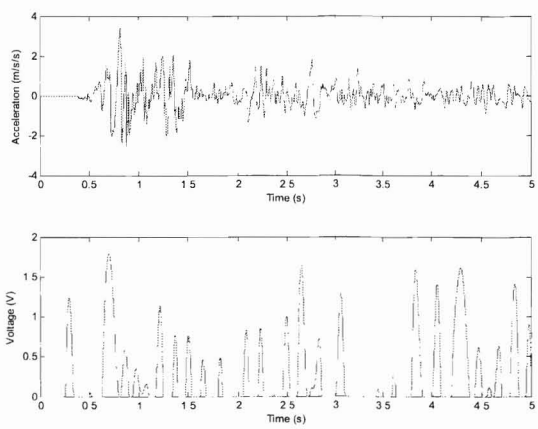

Fig. 7. Validation data

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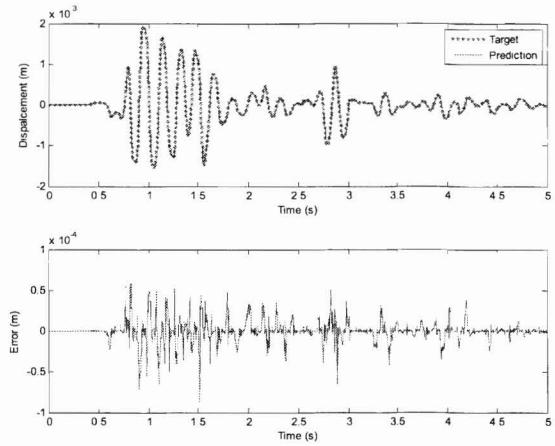

Fig. 8. Validation result for the first floor displacement

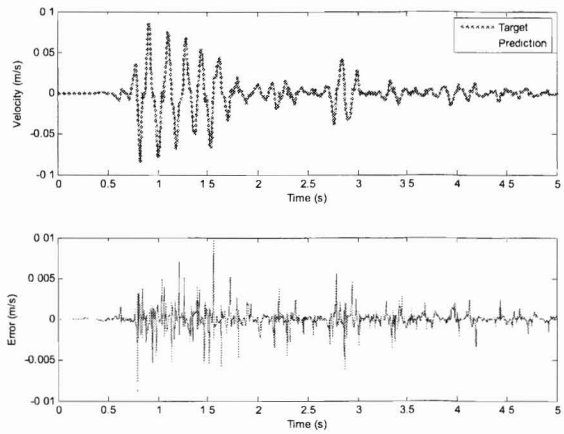

Fig. 9. Validation result for the first floor velocity