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A Solution for Estimating the Tensile Yield Strength From Small Specimens

ABSTRACT: The small punch test is an innovative test that utilises small disc-shaped specimens to assess the mechanical behaviour of materials. The main advantage is the relatively small specimen size. In this article, a modified analytical solution for the small punch maximum bend strength is proposed that is based on classical plate theory. A clear linear relationship is observed between the tensile yield strength σ_{YS} and the small punch maximum bend strength σ_y for both alloys and metal matrix composites.

KEYWORDS: small punch test, metal matrix composite, yield strength, equivalent contact radius

Introduction

Studies have been conducted regarding prediction of the tensile yield strength σ_{YS} from small punch load–displacement curves. Mao and Takahashi proposed an empirical relation of the small punch elastic-plastic load P_y and the tensile yield strength σ_{YS} for ferritic steels as shown in Eq 1 [1], where P_y is the small punch elastic-plastic load and t_0 is the original thickness. The theory is based on the assumption that the strength of a material is proportional to the load via an empirical relationship. Furthermore, the small punch elastic-plastic load P_y is described by many authors as representing the end of elastic bending and the beginning of localised plastic deformation [2].

$$\sigma_y = 0.360 \times \frac{P_y}{t_0^2} \quad (1)$$

This assumption was further analysed by Xu and Zhao, who developed an analytical solution based on analysis of the elastic-plastic bulge deformation behaviour of a circular plate and for very small deflections as described in Eq 2 [3].

$$\sigma_y = \frac{3}{2\pi} \times \frac{P_y}{t_0^2} \quad (2)$$

Cheon and Kim described this relationship between the tensile yield strength σ_{YS} and the small punch elastic-plastic load σ_y

using a normalisation parameter, α [4]. Furthermore, Finarelli et al. claimed that α is influenced by material properties and test geometry [5] as expressed in Eq 3.

$$\sigma_y = \alpha \times \frac{P_y}{t_0^2} \quad (3)$$

Eskner and Sandström analysed the initial elastic deformation using classical plate bend theory as shown in Eq 4 [6], where again P_y is the small punch elastic-plastic load, t_0 is the original thickness, R_1 is the radius of the lower die bore, v is Poisson's ratio, and r' is the small punch equivalent contact radius. In their work, uniaxial and biaxial flow properties are found to be in good agreement for 1Cr-0.5Mo low alloy and 18Cr-9Ni steels. The solution takes into account the geometry of the spherical punch contact radius with respect to the small punch configuration for the clamped condition. Note that this analysis assumes primarily elastic bending, for which the effects imparted by punch indentation are assumed to be negligible [2].

$$\sigma_y = \frac{3P_y(1+\nu)}{2\pi t_0^2} \ln \frac{R_1}{r'} \quad (4)$$

The main problem in this interpretation is the characterisation of the equivalent contact radius r', which has been proposed by a number of authors as shown in Table 1.

The current view on the equivalent contact radius r' is based on Westergaard's solution [10]; however, Fleury and Ha provide a

TABLE 1—*Proposed small punch equivalent contact radius r'.*

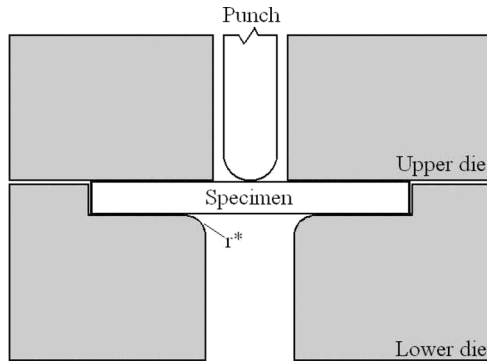| Author	r'	Constraint
Westergard [7]	$\sqrt{1.6r_o^2 + t_0^2} - 0.675t_0$	$0 \leq r_o \leq \frac{t_0}{2}$
Fleury and Ha [8]	$0.721(Pd\gamma)^{1/3}$	$r_o \geq 0$
Shetty [9]	$\frac{t_0}{3}$	$r_o \geq 0$

FIG. 1—*Schematic representation of the small punch test.*

better solution, as it acknowledges the influence of Young's modulus E and Poisson's ratio v [8] for both the spherical punch and the specimen. Nevertheless, the solution requires prior knowledge of the material's mechanical properties, which are often not readily available.

Another approach for determining the yield strength from a small punch test (SPT) is based on considering the plasticity of the material. This approach attempts to overcome the potential problems of determining yield strength simply from the linearity deviation. A number of works involving interpretations that include full consideration of plasticity from initial loading through to failure are found in the literature. This includes the work of Manahan et al. [11,12] and Foulds et al. [13] and the relatively recent work of Catherine et al. [14] and Isselin et al. [15], all of which employs some form of stress analysis using an inverse predictive method for stress–strain behaviour that includes full consideration of plasticity. However, the work reported here focuses on the need for a more straightforward approach that can yield results at a lower cost.

Method

The experimental procedures are performed on aluminum [16] and titanium [17] metal matrix composites (MMCs) with increasing amounts of ceramic reinforcement content. The SPT specimens are initially machined into 6 mm diameter disc-shaped specimens with a thickness of 1 mm. Fine grinding with Struers 4000 grit SiC paper and final polishing with 50 nm colloidal silica

(OP-S) on MD CHEM cloth further reduces the thickness to 0.5 ± 0.005 mm.

The SPT is conducted with a 5 kN load cell Instron 8562 universal mechanical testing machine. The SPT is performed at ambient temperature with a speed of 0.2 mm/min. The SPT specimen is firmly clamped in place. As can be seen in Fig. 1, the SPT jig incorporates a spherical punch diameter of 2 mm with a lower die bore diameter of 4 mm and a lower die chamber edge r^* of 0.22 mm. The spherical ball punch is made of cast iron with a hardness of 55 HRC.

Results

In this article, a new small punch equivalent contact radius r' is proposed based on geometric analysis of a sphere and experimental small punch load displacement curve as expressed in Eq 5. The equation below takes into account the small punch elastic-plastic displacement δ_y, which describes the evolution of the contact radius as the punch is displaced. Furthermore, r' represents the theoretical maximum contact radius that can be achieved by the SPT using Pythagoras on the triangle in Fig. 2(*b*).

$$r' = \sqrt{2r_o\delta_y - \delta_y^2} \tag{5}$$

Combining Eqs 4 and 5 leads to Eq 6, which describes the small punch maximum bend strength σ_y, described further in Fig. 2.

$$\sigma_y = \frac{3P_y(1+\nu)}{2\pi t_0^2} \ln \frac{R_1}{\sqrt{2r_o\delta_y - \delta_y^2}} \tag{6}$$

Discussion

The SPT can be thought of as a multi-mechanical test with applications in creep, stress, and fracture toughness analysis. An advantage is that samples are small enough that sampling can potentially occur in a manner that preserves the structure as a whole. In this research the SPT is utilised for assessing yield strength from small disc-shaped specimens. A correlation of the tensile yield strength σ_{YS} and the small punch maximum bend strength σ_y is shown in Table 2 and Fig. 3. STE690 steel [19], 12Cr-1Mo steel [20], 6061 aluminum [21], and Zircaloy-4 [22] were analysed based on previously published work; all the other
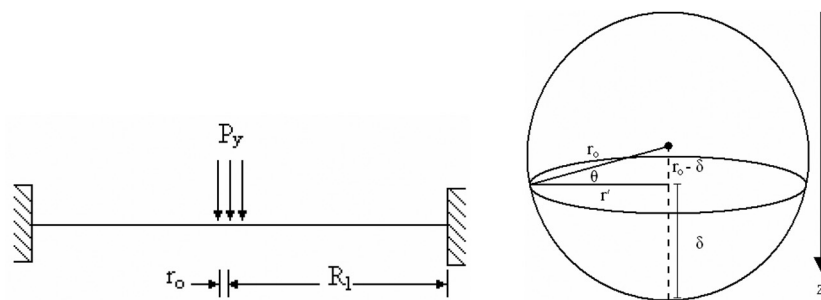

FIG. 2—*(a) A flat circular plate with constant thickness [18]. (b) A proposed geometric method for determining the small punch equivalent contact radius r′, where P_y is the small punch elastic-plastic load, R_l is the radius of the lower die bore, r_o is the punch radius, r′ is the equivalent contact radius, and δ is the small punch displacement.*

TABLE 2—*Tabulated 0.2% offset proof stress and small punch maximum bend strength σ_y. At least three tests were conducted for each material.*

Materials	0.2% Offset Proof Stress, (MPa)	Small Punch Maximum Bend Strength σ_y, (MPa)	Percent Difference, (%)
7A04-T6 base alloy	462	442	5
7A04/SiC/7.5p-T6	555	502	1
7A04/SiC/10p-T6	557	576	−3
TC4 base alloy	900	976	−8
TC4/TiB, TiC/2.5p, 2.5w	909	1041	−12
TC4/TiB, TiC/5p, 5w	1023	1088	−6

materials were tested in this study. The small punch maximum bend strength σ_y was determined using Eq 6. A clear linear relationship is observed between the tensile yield strength σ_{YS} and the small punch maximum bend strength σ_y over a large stress range from around 300 MPa to 1000 MPa.

The point of non-linearity is applied in order to determine the small punch elastic-plastic load P_y. It identifies the yield load at the point of non-linearity as shown in Fig. 4. It is determined by drawing a straight line through zero and rotating the line until it is tangent to, and does not cross, the curve.

It is determined that yielding occurs early on in the deformation process of the SPT. Therefore, the small punch elastic-plastic load P_y must be identified at the point of non-linearity. The associated small punch elastic-plastic displacements δ_y are identified at values less than 0.05 mm. Therefore, careful analysis of the small punch elastic-plastic load P_y and small punch elastic-plastic displacements δ_y is needed. Isselin and Shoji disputed the use of the small punch elastic-plastic load P_y, arguing that the complex biaxial stresses cannot be interpreted by analytical techniques alone [23]; however, in their research they conceded that the offset method provides a straightforward approach for identifying the small punch elastic-plastic load P_y with reasonably good results. In addition, similar to this research, small punch displacements δ
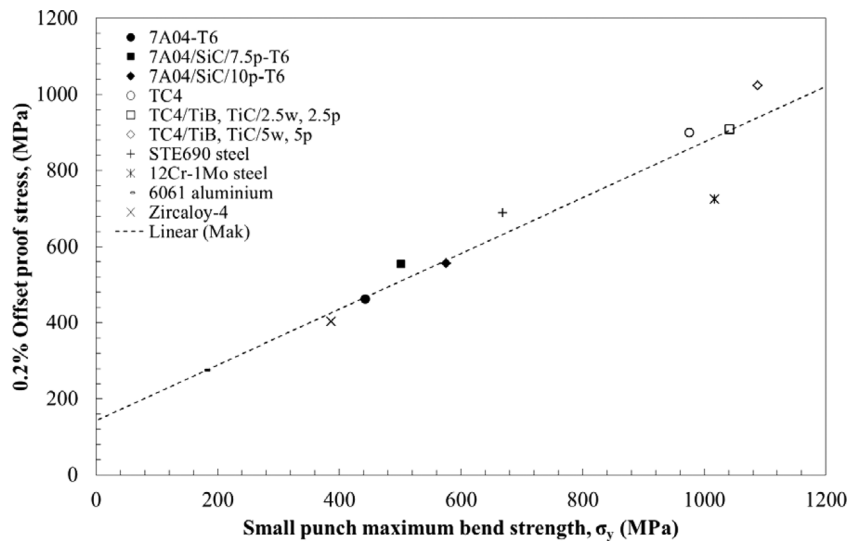

FIG. 3—*A graph showing the tensile yield strength σ_{YS} plotted against the small punch maximum bend strength σ_y.*
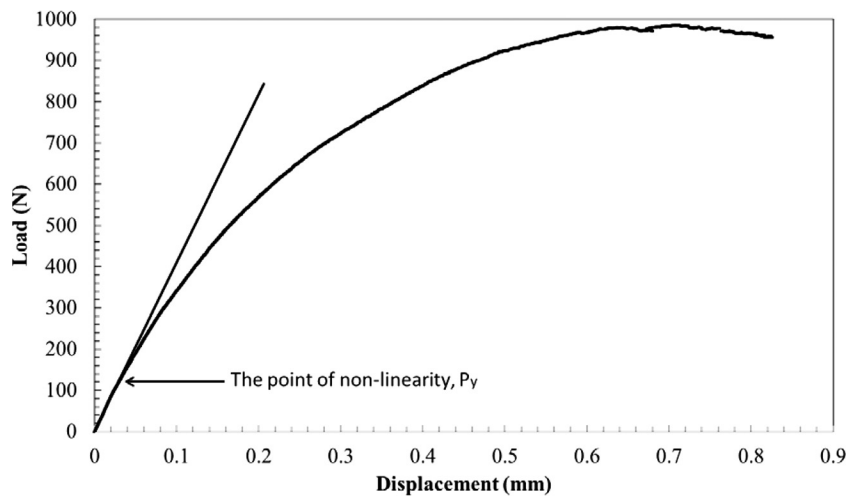

FIG. 4—*Representative plot of the small punch test curve for Ti-6Al-4V showing the "point of non-linearity" method employed for identifying the small punch elastic-plastic load P_y.*

are as well identified at small displacements, which for them are below 0.02 mm. It is true that plastic deformation occurs early on with respect to the punch and specimen; however, it is these authors' opinion that the deviation from linearity is a consequence of yielding at the free surface and not through the thickness of the specimen.

Conclusions

A new analytical solution for determining the yield strength σ_{YS} from the SPT is proposed that provides a convenient and simple method for determining tensile strength properties from alloys and MMCs. The method is based on classical plate theory of circular plates and incorporates a new analytical solution for the small punch equivalent contact radius r'. The small punch equivalent contact radius r' is based on geometric analysis of a sphere and an experimental small punch load displacement curve. In particular, the small punch elastic-plastic load P_y and corresponding small punch elastic-plastic displacement δ_y values are utilised. This study furthers the development of the SPT as a multi-mechanical test to be employed on all kinds of alloys and advanced MMCs. To this end, more research is required in order to standardise the test method and procedures, which will further improve the mechanical evaluation of material properties.

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