Model Predictive Control of Permanent Magnet Synchronous Machine with Reduced Torque Ripple

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Abstract —The conventional direct torque control (DTC) method features fast dynamic response, but it also has notable drawbacks such as high torque and flux ripples, variable switching frequency and acoustic noise. The new DTC scheme based on model predictive control (MPC) is studied in the paper for permanent magnet synchronous machine drive. The paper aims to reduce the torque and flux ripple. Firstly one- and two-step delay MPC methods are employed with or without three torque/flux and switching frequency conditions. Furthermore linear extrapolations N-step delay MPC method is taken into account. Compared to simulation results, the two-step delay DTC scheme is better than the one-step scheme in the steady response and dynamic performance, at the same time the one the torque and frequency compensations can reduce switching frequency from high value to a low constant one. Simulation results are presented to validate the effectiveness of the proposed schemes in this paper.

I. INTRODUCTION

Permanent magnet synchronous machine (PMSM) drives are nowadays widely used due to its low inertia, high efficiency, high power/torque density and high reliability, and are considered superior to induction motor and DC motor. For the applications requiring high dynamic performance, field oriented control (FOC) is usually employed, which requires relatively complex rotary transformation and accurate rotor position. The dynamic response is limited by the bandwidth of inner current loop.

Conventional DTC directly manipulates the final output voltage, and hence eliminates the delay and features a high dynamic response [1-2]. For DTC, all calculations are implemented in stationary coordinate, so the structure is simple. Despite the merits above, conventional switching-table-based DTC presents some notable drawbacks, such as high torque and flux ripples, variable switching frequency and acoustic noise [3-5]. To address these problems, a number of methods have been proposed in the literature, of which the space vector modulation (SVM) is usually employed [6]. The SVM based DTC provides low torque ripple and fixed switching frequency. However, it requires rotary coordinate transformation, knowledge of many motor parameters and high computational ability; these negate the simplicity and robustness of the conventional DTC.

Apart from SVM-DTC, the predictive control method can combine more control variables in one cost function, such as torque and flux, and control them at the same time. The cost function is employed to weight the effects of different voltage vectors and select the one which minimizes the cost function. The cost function employed MPC features its flexibility in incorporating any nonlinear constraints that can be modeled mathematically, which can decrease torque and flux ripples [7-17].

Another kind of approach is to introduce the duty cycle control in conventional DTC. By adjusting the duty ratio of the active vector from the switching table, the torque ripple can be reduced significantly.

In this paper, the model based predictive torque control (MPTC) is introduced to overcome this problem .The principle of model predictive control is to predict the future behavior of the variables over a time frame based on the model of the system . All predictions are evaluated based on a cost function to find the voltage vector which results in the minimum cost.

II. MACHINE EQUATIONS OF PMSM

For a PMSM, the model in the rotor synchronous coordinate is the most popular, because all the parameters become constant. The machine equations of PMSM in the synchronous frame are expressed as follows.

$$u_d = R_s i_d + L_d \frac{di_d}{dt} - \omega L_q i_q \tag{1}$$

$$u_q = R_s i_q + L_q \frac{di_q}{dt} + \omega L_d i_d + \omega \psi_f \tag{2}$$

$$\psi_d = L_d i_d + \psi_f \tag{3}$$

$$\psi_q = L_q i_q \tag{4}$$

The electromagnetic torque is expressed as below.

$$\begin{aligned}
T_e &= \frac{3}{2} p(\psi_d i_q - \psi_q i_d) \\
&= \frac{3}{2} p(\psi_f i_q + (L_d - L_q) i_d i_q) \\
&= \frac{p_n |\psi_s|}{2 L_d i_q} [2(\psi_f L_q \sin\delta + (L_d - L_q)|\psi_s| \sin 2\delta)]
\end{aligned} \tag{5}$$

where

u_s stator voltage vector
R_s stator resistance
i_s, i_d, i_q stator current vector, d-axis and q-axis component
ω rotor speed
ψ_s stator flux vector

L_d, L_q d-axis and q-axis inductance

ψ_f permanent magnet flux

T_e electromagnetic torque

p number of pole pairs

III. MODEL PERDICTIVE CONTROL ON PMSM

A. Basic principle

A DC/AC inverter is employed in the main circuit, which outputs six effective voltage vectors and two naught voltage vector as shown in Fig. 1. In conventional DTC [2], hysteresis comparators and switching table are used to select the appropriate voltage vector. The principle of model predictive control is to predict the future behavior of the variables over a time frame based on the above model of the system. All predictions are evaluated based on a cost function to find the voltage vector which results in the minimum cost. In the improved DTC control diagram [1] as shown in Fig. 3 the cost function is selected in the case which both torque and flux is as close as possible to the commanding value at end of the cycle. In the paper the cost function is defined as

$$g = |T_e^* - T_e| + k\left(|\psi_s^*| - |\psi_s|\right)$$ (6)

$$u_s \in \{V_0, V_1, \ldots V_6, V_7\}$$ (7)

where k is the weighting factor. In this paper, torque and flux are given the same weight, so k is selected as T_n/φ_n , where T_n and φ_n are the rated value for the torque and stator flux. The MPTA is the maximum value of torque/current, expressed as below

$$\varphi_s = \frac{T_e}{p i_s}$$ (8)

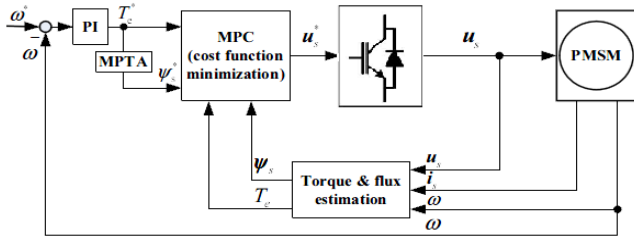Torque and flux are calculated according to formulae (3), (4) and (5).

Fig. 1. An improved DTC control diagram

B. Principle of Model Predictive Control

The 2L-VSI converter can be defined as the determination of an appropriate control action $S(i)(i=1-8)$ that will drive a generic system variable $x(t)$ as close as possible to a desired reference value $x^*(t)$. Consider the qualitative behaviour [3] of $x(t)$ and its regularly sampled value $x(t_k)$ over a sample period

T_s for a system with a finite number of control actions n, as shown in Fig. 4. They can be evaluated together with the measured value $x(t_k)$, based on a prediction function f, to predict all the possible system transitions $x_i(t_{k+1}) = f\{x(t_k), S_i\}$, for i = 1, . . . , 8. To determine which of the control actions is to be selected, a cost function f is used, which is usually dependent on absolute error between the predictions and the reference $g_i = |x^*(t_{k+1}) - x_i(t_{k+1})|$. Here variable x represents torque or flux.
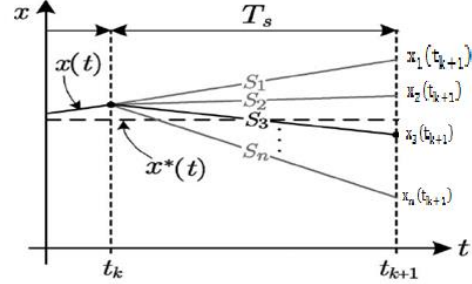

Fig. 2. One-step delay MPC operating principle

Combining (1) and (2), the PMSM machine model can be rewritten in terms of state pace equations[4] as

$$\frac{di_d}{dt} = \frac{-R_s i_d + \omega L_q i_q + u_d}{L_d}$$ (9)

$$\frac{di_q}{dt} = \frac{-R_s i_q - \omega L_d i_d + u_{q-\omega\varphi_f}}{L_q}$$ (10)

The equations of PMSM in (11) and (12) are used to obtain a prediction of the stator current at the next sampling instant, i_s^{k+1}, based on a given stator voltage u_s^k and measured current i_s^k at current sampling instant. The prediction is expressed as

$$i_d^{k+1} = i_d^k + \frac{1}{L_d}(-R_s i_d^k + \omega^k L_q i_q^k + u_d^k)T_s$$ (11)

$$i_q^{k+1} = i_q^k + \frac{1}{L_q}(-R_s i_q^k - \omega^k L_d i_d^k + u_q^k - \omega^k \varphi_f)T_s$$ (12)

After obtaining i_s^{k+1}, both the torque and flux at the (k +1)th instant can be obtained from (3) (4) and （5）.

As it is well known that torque and flux are important variable,a cost functioncan be defined as below.

$$g = |T_e^* - T_e^{k+1}| + k\left(|\psi_s^*| - |\psi_s^{k+1}|\right)$$

$$u_s^k \in \{V_0, V_1, \ldots V_6, V_7\}$$ (13)

The coefficient k can be decided theoretically by the weight between torque and flux,which can be modified flexibly by MATLAB simulation [5][6].

B. Consideration of Two Step Delay

In （13） it is the one-step delay to define the cost function.From (11) and (12) the variable i_d^{k+1} and i_q^{k+1} at the (k+1) th instant is derived from the variable $i_d^k, \omega^k, i_q^k, u_d^k$ together with the constant variable R_s, L_q, L_d, φ_f and T_s .From Fig.2 i_d and i_q can be obtained by abc/dq transformation.
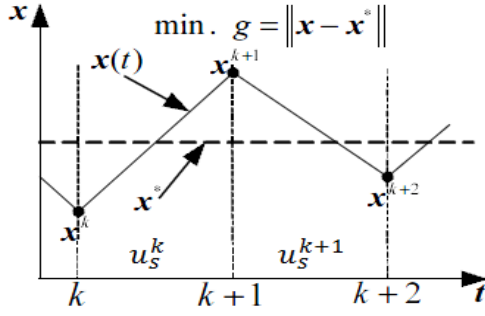
Fig. 3. Two-step delay in digital control system

Considering the calculation of one-step delay, the cost function in (6) should be changed to (14) as shown below very necessary for predictive control in digital implementation.That is to say Two-step delay in digital control system:

$$min \quad g = |T_e^* - T_e^{k+2}| + k(|\psi_s^*| - |\psi_s^{k+2}|)$$
$$s.t \quad u_s^{k+1} \in \{V_0, V_1, \ldots V_6, V_7\} \tag{14}$$

To obtain T_e^{k+2} and ψ_s^{k+2} in (14), a two-step prediction is required. $i_s^{'k+1}$ is firstly obtained from (11) and (12) and then in a similar way, $i_s^{'k+2}$ can be predicted with estimated $i_s^{'k+1}$ as new initial state and u_s^{k+1} as input using (11) and (12). Knowing i_s^{k+2}, T_e^{k+2} and ψ_s^{k+2} can be obtained from (3) ,(4) and (5). All the possible vector from V_0 to V_7 for u_s^{k+1} will be evaluated to obtain the one minimizing (14).

C. Consideration of N Step Delay

The expression of the proposed cost function is

$$min. \quad g = |T_e^* - T_e^{k+2}| + k(|\psi_s^*| - |\psi_s^{k+2}|)$$
$$+ A(|T_e^* - T_e^{k+N}| + k(|\psi_s^*| - |\psi_s^{k+N}|))$$
$$s.t. \quad u_s^{k+1} \in \{V_0, V_1, \ldots V_6, V_7\} \tag{15}$$

the stator flux T_e^{k+N} and torque ψ_s^{k+N} at $(k+N)th$ instant are predicted from the value at $(k+1)th$ and $(k+2)th$ instants using linear extrapolations, which are expressed as

$$T_e^{k+N} = T_e^{k+1} + (N-1)(T_e^{k+2} - T_e^{k+1})$$
$$|\psi_s^{k+N}| = |\psi_s^{k+1}| + (N-1)(|\psi_s^{k+2}| - |\psi_s^{k+1}|) \tag{16}$$

Where N larger than 2.

D. Consideration of reduced switching frequency

One of the well-known disadvantages of the hysteresis based DTC scheme are variable high switching frequency. One of the merits of the cost-function based predictive control is its flexibility in incorporating nonlinear constraints. In this section, the switching frequency reduction will be modelled and combined into the cost function based on (9).
The expression of the item is as follows.

$$g_f = \sum_{i=a,b,c} |u_s^{k+1}(S_i) - u_s^k(S_i)|$$
$$s.t. \quad u_s^{k+1} \in \{V_0, V_1, \ldots V_6, V_7\} \tag{17}$$

the item is related to commutation frequency reduction, where $u_s^k(S_i)$ and $u_s^{k+1}(S_i)$ represent the switching state of phase i(i = a, b, c) at the present cycle (during kth and (k+1)th instant) and the next cycle (during (k+1)th and (k+2)th instant), espectively.

IV. SIMULATION RESULT

To validate the proposed strategy, simulations using Matlab/Simulink are carried out. The system and machine parameters are listed in Tab. I, II, which are referenced to [12].

TABLE I
MOTOR AND SYSTEM PARAMETERS

Number of pole pairs	p	3
Permanent magnet flux	ψ_f	0.1057 Wb
Stator resistance	R_s	1.8Ω
d-axis and q-axis inductance	L_d, L_q	15 mH
Rated speed	n_N	2000 rpm
Rated torque	T_N	4.5 Nm
Rated line-line voltage	U_N	128 V
Sampling period	T_s	100 μs
DC bus voltage	U_{dc}	200 V
control parameters	k	25.6

The performances of model predictive control are comparatively investigated in this section by using simulations in the environment of Simulink.

A. Steady Responses and dynamic performance

Firstly the steady responses and dynamic performance is tested in Fig4-6. The paper presents the chatacteristics for One-step delay DTC and two-step delay DTC. From top to bottom, the curves shown in Fig4-6 are the current of phase a,speed,torque and stator flux respectively. In the section

(a)

(b)
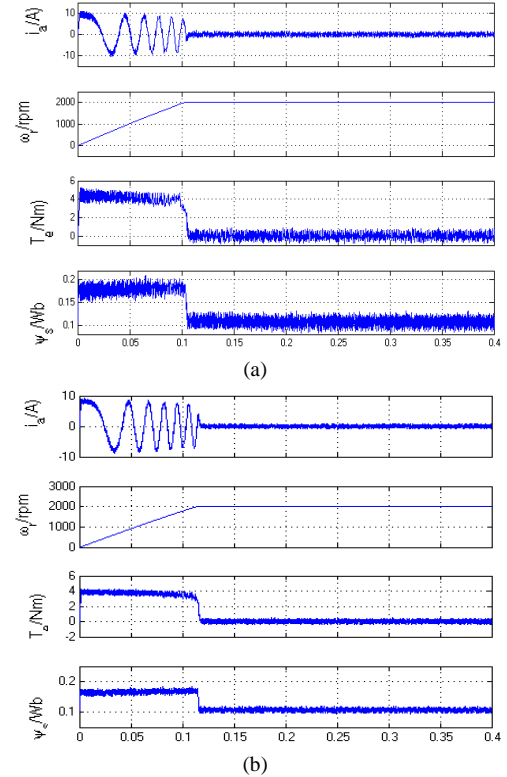
Fig. 4. Steady responses at rated speed without load:(a) One-step delay DTC, (b) Two-step delay DTC

The motor starts from standstill to 100% rated speed without load, There are lower torque and flux ripples for two-step delay DTC without compensation shown in Figs. 4 compared to One-step delay DTC.
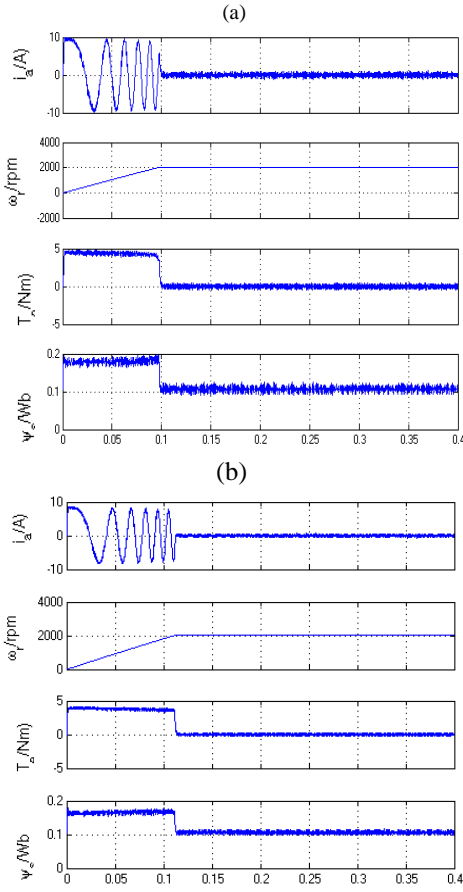
(a)

(b)

Fig. 5. Steady responses at rated speed without load for MPTC with torque compensation
(a) One-step delay DTC   (b) two-step delay DTC

Under the above same condition, there are lower stator current,torque and flux ripples for two-step delay DTC without compensation shown in Figs. 5 compared to One-step delay DTC.
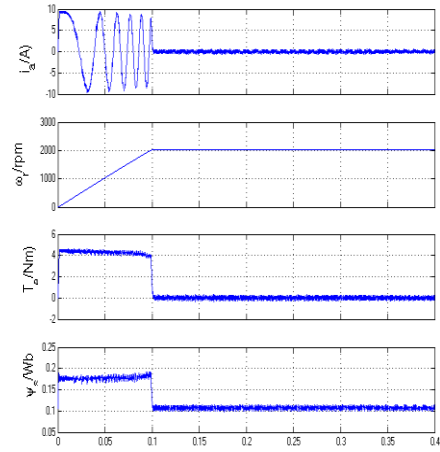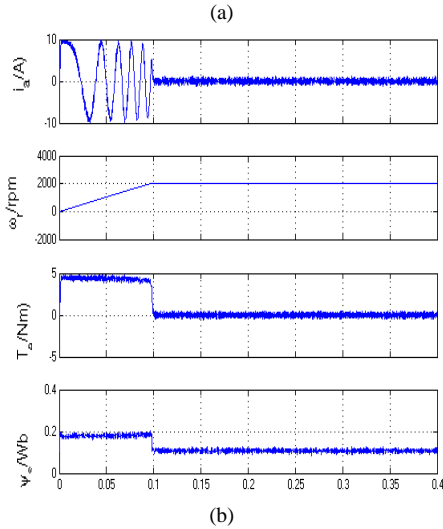
(a)

(b)

Fig. 6. Steady responses at rated speed without load for MPTC with torque and frequency compensation
(a) One-step delay DTC   (b) two-step delay DTC

Under the above same condition, there are lower stator current, torque and flux ripples for two-step delay DTC without compensation shown in Figs. 6 compared to One-step delay DTC further.

B. Consideration of switching frequency

Figs. 7 to 8 illustrate the switching frequency characteristics without compensation , with torque compensation and with torque compensation and frequency compensation, espectively. Fig. 7 presents Switching characteristics of One-step delay DTC in the above three conditions.
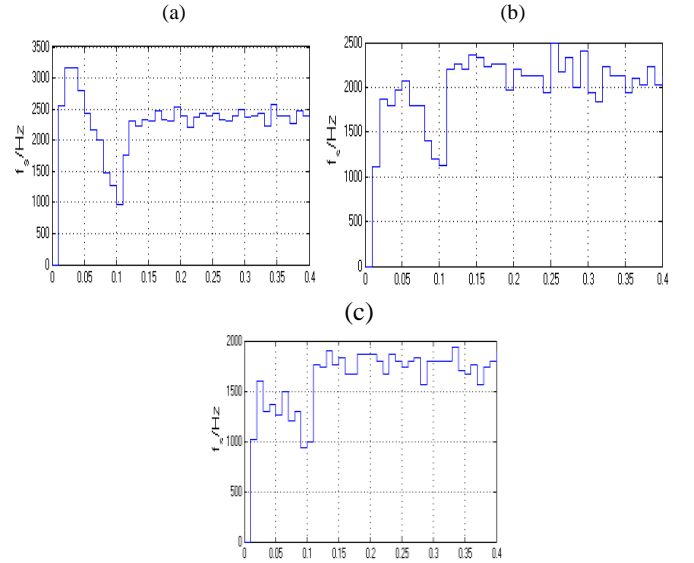
(a)   (b)

(c)

Fig. 7. Switching characteristics of One-step delay DTC method using cost function in (13);
(a)without compensation (b) with torque compensation;
(c) with torque compensation and frequency compensation

There are lowest switching frequency with torque compensation and frequency compensation,highest without compensation under the condition of One-step delay DTC method.
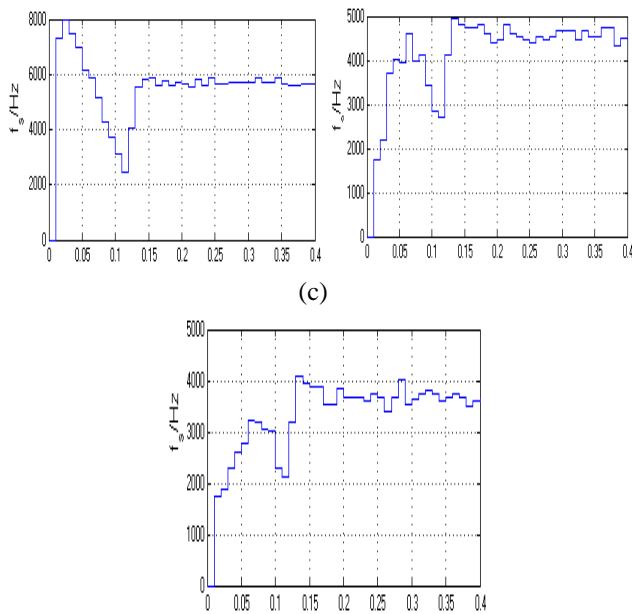
(a)   (b)

Fig. 8. Switching characteristics of two-step delay DTC method using cost
function in (13);
(a)without compensation (b) with torque compensation;
(c) with torque compensation and frequency compensation

There are lowest switching frequency with torque compensation and frequency compensation,highest without compensation under the condition of two-step delay DTC method.The results are the same as of one-step delay DTC method.

V. CONCLUSION

A novel model based predictive torque control method is studied in this paper. Some cost functions are studied to get better dynamic performance and reduce the torque ripple. Simulation results can be summarized as follows:

1) The two-step delay DTC scheme is better than one-step delay DTC in the steady response and dynamic performance.
2) The two-step delay DTC scheme features lower torque and flux ripples than the one-step delay.
3) In the two-step delay DTC scheme the torque and frequency compensations are employed can furtherly optimize,at the same time reduce switching frequency from high value to a low constant one.

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