Modeling and Performance Analysis of Energy Regeneration System in Electric Vehicle with Permanent Magnet DC Motor Driving System

Jiaxin Chen[1,2], Jianguo Zhu[2], Youguang Guo[2],
[1]College of Electromechanical Engineering, Donghua University, Shanghai 200051, China
[2]Faculty of Engineering, University of Technology, Sydney, NSW 2007, Australia

***Abstract*-This paper presents the modeling and performance analysis of energy regeneration system (ERS) of electric vehicle with permanent magnet DC motor driving system. Based on the boost equivalent circuit with average current mode controlled ERS, a detailed switching model (DSM) is built to predict the dynamical performance. The DSM includes four blocks: system dynamics block, permanent magnet DC motor block, boost converter block and system performance calculation block. An automatic mechanism of linearizing the nonlinear system at the local operating point is enrolled to obtain the small signal average model (SSAM) of ERS, by which the frequency domain performance can be acquired. The proposed models are implemented in MATLAB/Simulink. By running the simulation model, several important performances of ERS are obtained.**

I. INTRODUCTION

Since the 1980's, the technique of electric vehicle (EV) has been greatly developed. However, as being limited by the development of current storage battery technique, the EV has not shown an overall advantage over the car with fuel engine yet. One of serious problems is its short rated distance. It is well known that the technique of energy regeneration system (ERS) can save much energy which can prolong the rated distance by over 20% in the surrounding of city road, and has become one of important research points at present.

To advance ERS to the practical level, an accurate model is always necessary. Paper [1] has introduced the principles and analytical models for four kinds of ERS; and paper [2] has presented the dynamic performance analysis by using a large signal average model. As their complement, this paper presents the frequency domain performance analysis. For this, a detailed switching model is given first, including four blocks: system dynamics block, permanent magnet (PM) DC motor block, boost converter block and system performance calculation block. Then, the small signal average model around a desired operating point is deduced by the method of averaging the switching model. In order to increase the efficiency for obtaining the small signal model from the detailed switching model, an automatic mechanism of exchanging data between them is employed. As an example, an EV with permanent magnet DC motor driving system is studied in this paper. All the proposed models are implemented in MATLAB/Simulink. By running the simulation models, the comprehensive performances of ERS can be obtained efficiently.

II. EQUIVALENT CIRCUIT OF ENERGY REGENERATION SYSTEM

Both the circuit of ERS based on the PMDC motor driving system and the boost equivalent circuit with average current mode controlled ERS and the actual equivalent components are shown in Fig. 1.

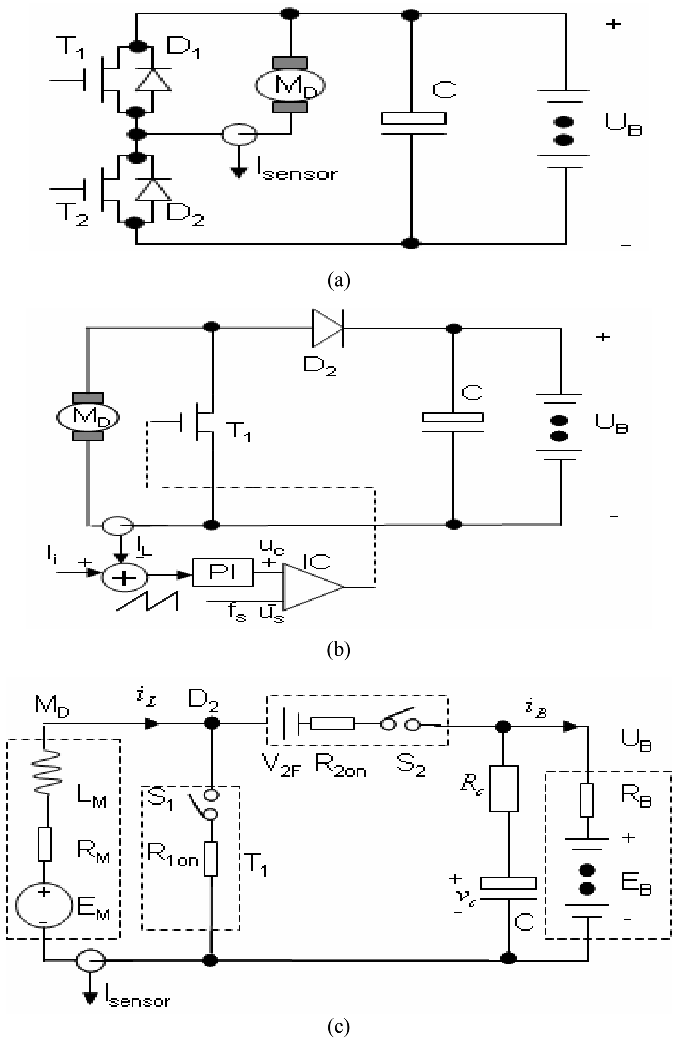


Fig. 1. Circuit of ERS based on the permanent magnet DC motor driving system: (a) ERS circuit, (b) ERS with average current model control, and (c) equivalent circuit with actual equivalent components.

In the figure, E_m, R_m, and L_m are the back *emf*, the armature resistance, and the armature inductance of the DC motor, respectively. The diode in the ON state is modeled by a constant voltage battery, V_{2F} and a constant forward resistance R_{2ON}, and in the OFF state by an infinite resistance. All its junction capacitances and lead inductances are omitted. The power MOSFET in the ON state is modeled by a constant resistance R_{1ON} and in the OFF state by an infinite resistance, and its output capacitance and lead inductance are omitted. The rechargeable battery is modeled by a constant *emf* battery, E_B and a constant resistance, R_B. R_c is the equivalent resistance of the capacitance.

III. Dynamics Model of Electric Vehicle

According to the theory of mechanical kinematics, the dynamics model of EV with two wheels running in a straight course is shown as the follows:

$$\begin{cases} F_w = \frac{1}{2} C_d A \rho (v + v_w)^2 \\ F_f = mgf \cos\alpha \\ F_i = mg \sin\alpha \\ F_a = \delta m \frac{dv}{dt} \end{cases} \tag{1}$$

$$\delta = 1 + \frac{J}{mr^2} \tag{2}$$

$$F_q = F_w + F_f + F_i + F_a \tag{3}$$

where v is the EV speed, v_w the windward speed, J the rotational inertia of two wheels, C_d the wind baffle coefficient, A the windward area, ρ the air mass density, r the radius of the wheel, m the sum of EV mass and load, g the gravitation coefficient, f the roll hinder coefficient, δ the equivalent inertia coefficient, α the gradient angle of road, and F_q the propulsive force of motor.

The model of PMDC motor driving system is shown as

$$\begin{cases} E_m = K_e \omega \\ T_e = K_T i_L \\ F_q = (T_e - \sigma_0 \omega)/r \\ v = \omega r \end{cases} \tag{4}$$

where K_e is the coefficient of back *emf*, K_T the coefficient of electromagnetic output torque, ω the angular speed, and σ_0 the damp coefficient of the motor. To the motor with hubcap structures, both the transfer ratio and transfer efficiency are 1.

IV. Detailed Switching Model of Boost Converter with Average Current Model Control

A. Detailed Switching Model (DSM)

According to Fig. 1, the current flowing through the inductance, L_m, may be in continuous conduction mode (CCM) or in discontinuous conduction mode (DCM). The typical current waveform flowing through the inductance in DCM is shown in Fig. 2 [3], where, the switching interval T_s is divided into two subintervals (k=1, 2, 3) for (t_0, t_1), (t_1, t_2), and (t_2, t_3), and the corresponding duty ratios are d_1, d_2 and d_3, respectively.

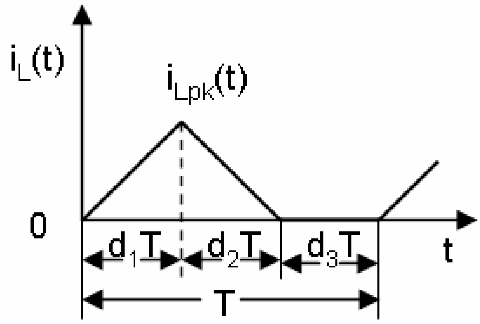


Fig. 2. Typical inductor current waveform in DCM

When d_3=0, CCM can be seen as a special condition of DCM. According to the KVL and KCL circuit laws, the detailed switching dynamical model of boost converter can be obtained as a set of state space differential equations:

When $0 \le t < d_1 T$:

$$\begin{cases} \frac{di_L}{dt} = \frac{1}{L}[-(R_M + R_{1on})i_L + E_M] \\ \frac{dV_c}{dt} = -\frac{1}{(R_B + R_c)C} V_c + \frac{1}{(R_B + R_c)C} E_B \end{cases} \tag{5}$$

When $d_1 T \le t < (d_1 + d_2)T$:

$$\begin{cases} \frac{di_L}{dt} = -\frac{1}{L_m}[(R_M + R_{2on} + \frac{R_B R_c}{R_B + R_c})i_L + \frac{R_B}{R_B + R_c} V_c + -\frac{R_C}{R_B + R_c} E_B + V_{2F} - E_m] \\ \frac{dV_c}{dt} = \frac{1}{C}[\frac{R_B}{R_B + R_c} i_L - \frac{1}{R_B + R_c} V_c + \frac{1}{R_B + R_c} E_B] \end{cases} \tag{6}$$

When $(d_1 + d_2)T \le t < T$:

$$\begin{cases} i_L = 0 \\ \frac{dV_c}{dt} = -\frac{1}{(R_B + R_c)C} V_c + \frac{1}{(R_B + R_c)C} E_B \end{cases} \tag{7}$$

Assuming that the operation states of the converter, s_1, s_2 and s_3, corresponds to the duty ratios d_1, d_2 and d_3, respectively, then according to Fig. 1, the operation states can be decided by the following equations:

$$\begin{cases} s_1^k = \bar{s}_1^{k-1} * (u_c > u_s) * cp + s_1^{k-1} * (u_c \ge u_s) \\ s_2^k = \bar{s}_1^k * (i_L > 0) \\ s_3^k = \bar{s}_1^k * \bar{s}_2^k * (i_L \le 0) \end{cases} \tag{8}$$

and

$$\begin{cases} u_s = \dfrac{f_s}{m_c} \\ e(t) = i_i - i_L \\ u_c = K_p e(t) + K_i \int e(t)dt \end{cases} \tag{9}$$

where K_p and K_i are the proportional and integral coefficients, f_s and m_c are the frequency and slop of sawtooth wave, and i_i and i_L are the given current and actual current of the motor, respectively. The physical definitions of u_c and u_s are the input voltages at two input ports of the comparator as shown in Fig. 1. The upward scripts (k) and (k-1) refer to the kth and *(k-1)*th time steps, respectively, and cp is the rising edge of square pulse wave which has the same frequency as that of sawtooth wave.

B. Model Implementation

To simplify the realization of the DSM in Matlab/Simulink, (5)-(7) can be written in a uniform form as shown in the following equations:

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(s_1 * V_1 + s_2 * V_2 + s_3 * V_3) \\ \dfrac{dv_c}{dt} = \dfrac{1}{C}[s_2 * \dfrac{R_B}{R_B+R_c} i_L - \dfrac{1}{R_B+R_c} V_c + \dfrac{1}{R_B+R_c} E_B] \end{cases} \tag{10}$$

and

$$\begin{cases} V_1 = -(R_M + R_{1on}) i_L + E_M \\ V_2 = (R_M + R_{2on} + \dfrac{R_B R_c}{R_B+R_c}) i_L + \dfrac{R_B}{R_B+R_c} V_c + -\dfrac{R_C}{R_B+R_c} E_B + V_{2F} - E_m \\ V_3 = 0 \end{cases} \tag{11}$$

During a period of the vehicle running from the start speed V_{start} to the stop speed V_{stop}, the power of energy regeneration P_r can be calculated by

$$P_r = \frac{1}{T_s} \int_0^{T_s} E_B I_B dt \tag{12}$$

The braking distance, S, and the efficiency of energy regeneration, η, are calculated by

$$S = \int_0^{t_1} v dt, \qquad v(0) = v_0, v(t_1) = v_1 \tag{13}$$

$$\eta = \frac{W_E}{W_Z} = \frac{\int_0^{t_1} p_r dt}{0.5 \sigma m (v_0^2 - v_1^2)} \tag{14}$$

All the above equations, i.e. (1)-(14), constitute the detailed switching simulation model of ERS, which is shown in Fig. 3. The parameters for simulation are: K_e=K_T=1.31, r=0.28 m, v_{start}=20 km/h, v_{stop}=15 km/h, f_s=10 KHz, m_c=10000, E_B=36 V, L_m=1.26 mH, v_f=0, a=0, m=110 kg, f=0.007, A=0.6 m^2, C_d=0.9, ρ=1.225 kg/m, δ=1.05, σ_0=0.015, K_p=100, and K_i=1.

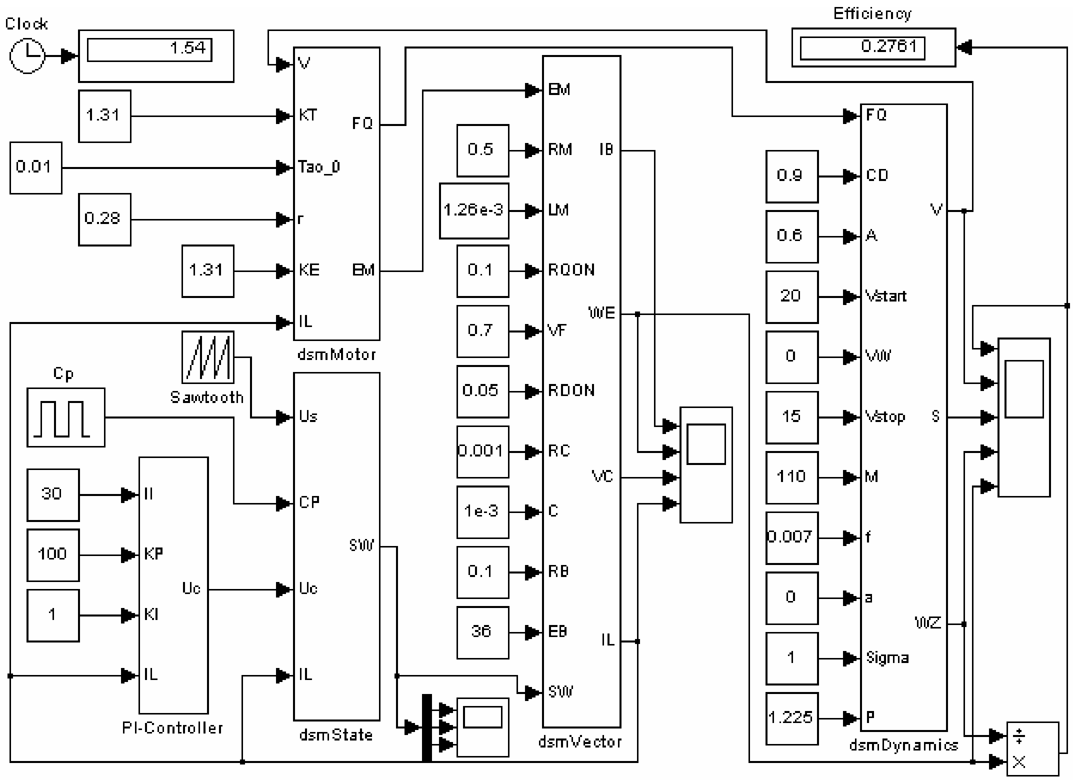
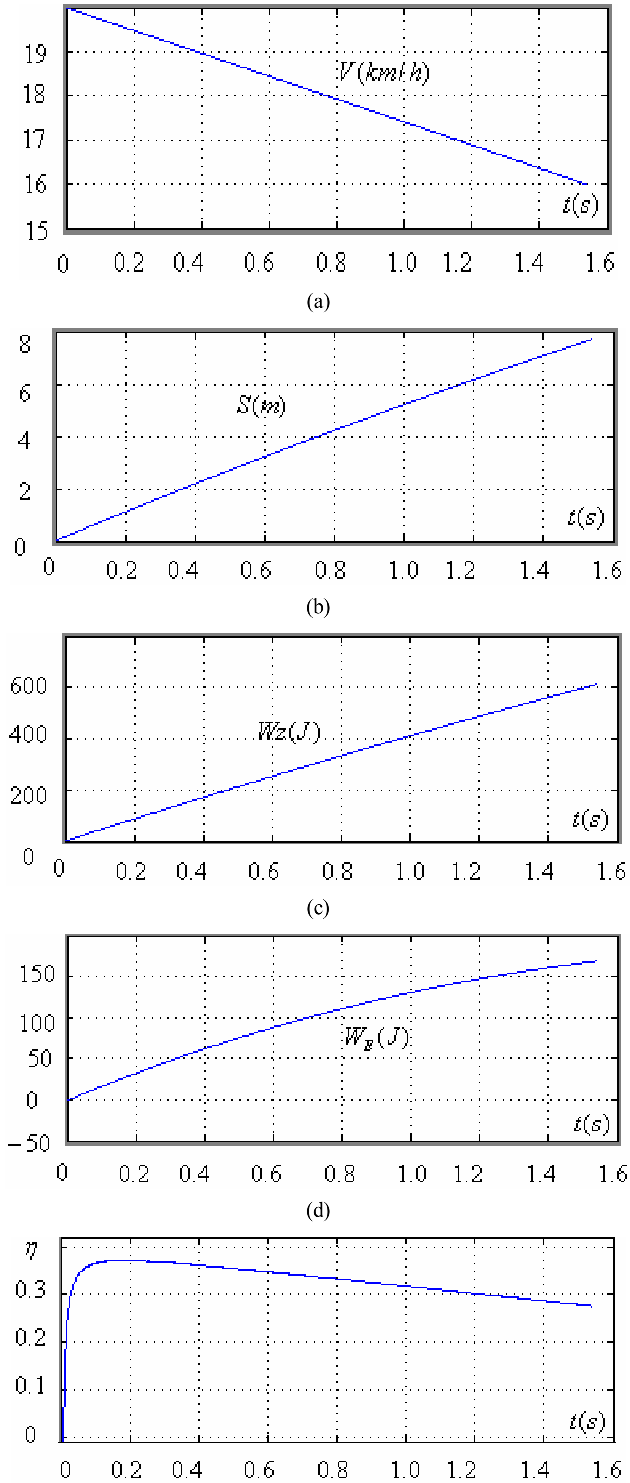


Fig. 3. Detailed switching simulation model of the ERS

By inputting the above data into the simulation model, under the given control average current, I_i=30 A, several simulation results such as the curves of velocity, $v(t)$, braking distance, $S(t)$, kinetic energy consumed, $W_Z(t)$, energy regenerated, $W_E(t)$, efficiency of energy regeneration, η, voltage across the capacitor, V_c, current charging the battery, I_B and current flowing through the motor can be obtained and shown in Fig. 4.



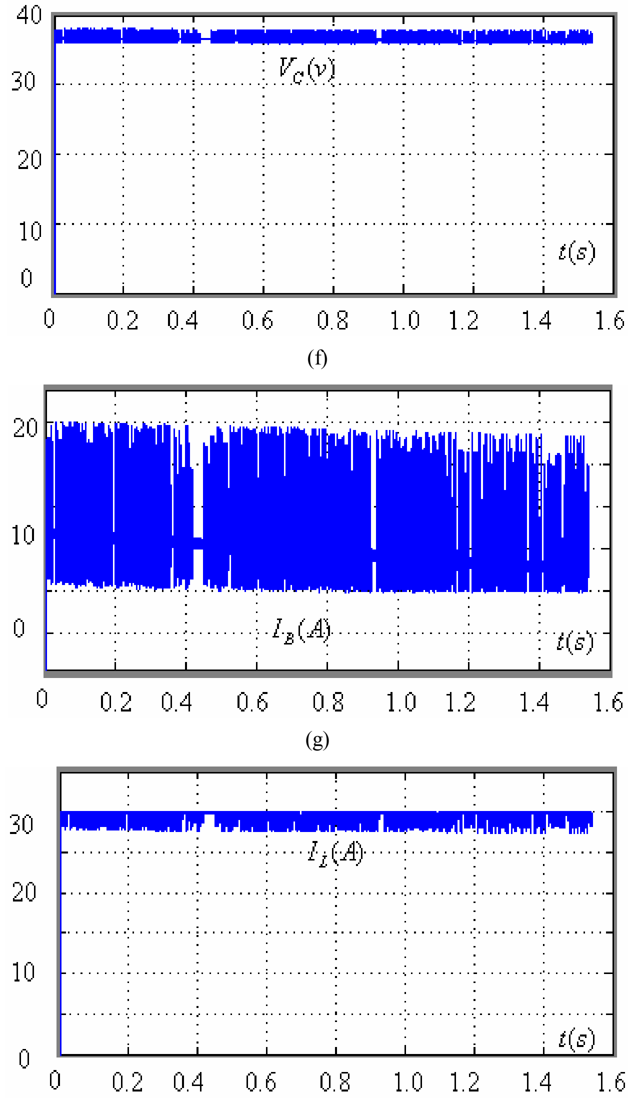


Fig. 4. Simulated performances of the ERS which runs from 20 km/h to 16 km/h under a given control current of 30 A: (a) velocity, (b) braking distance, (c) kinetic energy consumed, (d) sum of energy regenerated, (e) energy regeneration efficiency, (f) voltage across the capacitor, (g) current charging the battery, and (h) current flowing through the motor, all against braking time.

V. AVERAGE MODEL OF ENERGY REGENERATION SYSTEM WITH CONSTANT BRAKING TORQUE (CBT)

A. Large Signal Average Model

According to (4), the electromagnetic torque of the PMDC motor is proportional to the armature current, so the constant braking torque model corresponds to the average current mode controlled converter. In ERS, comparing with the 10 KHz high frequency of the PWM, the system performance varies much more slowly, and hence the back *emf*, E_m of the PMDC motor can be seen as a constant during one PWM cycle. This reduces the order of the system from three to two. By using the method of averaging the switching model, the average model is produced as the follows:

$$\begin{cases} \dot{\bar{\mathbf{x}}} = \sum_{k=1}^{3} \dot{\bar{\mathbf{x}}}_{\mathbf{k}} = \sum_{k=1}^{3} (d_k \mathbf{A}_{\mathbf{k}} \bar{\mathbf{x}} + d_k \mathbf{B}_{\mathbf{k}} \mathbf{u}) \\ \bar{\mathbf{y}} = \sum_{k=1}^{3} d_k \mathbf{C}_{\mathbf{k}} \bar{\mathbf{x}} \end{cases} \tag{15}$$

In the above equation, the state vector, $\mathbf{x}$, is defined as $[i_L\ V_c]^T$, where i_L is the armature current and V_c is the voltage across the output filter capacitor. (k=1, 2, 3) corresponds to three subintervals (t_0, t_1), (t_1, t_2) and (t_2, t_3), and $\mathbf{A}_\mathbf{k}$, $\mathbf{B}_\mathbf{k}$ and $\mathbf{C}_\mathbf{k}$ corresponds to the system matrices of three subintervals, respectively.

B. Small Signal Average Model

By using the method of linearizing the large signal average model around a desired local operating point, the small signal average model is deduced as the follows:

$$\begin{cases} \dot{\hat{\mathbf{x}}} = \sum_{k=1}^{3} [(D_k \mathbf{A}_{\mathbf{k}} \hat{\mathbf{x}} + D_k \mathbf{B}_{\mathbf{k}} \hat{\mathbf{u}}) + (\mathbf{A}_{\mathbf{k}} \bar{\mathbf{X}} + \mathbf{B}_{\mathbf{k}} \mathbf{U}) \hat{d}_k] \\ \hat{\mathbf{y}} = \sum_{k=1}^{3} (D_k \mathbf{C}_{\mathbf{k}} \hat{\mathbf{x}} + \mathbf{C}_{\mathbf{k}} \bar{\mathbf{X}} \hat{d}_k) \end{cases} \tag{16}$$

$$\begin{cases} \sum_{k=1}^{3} D_k = 1 \\ \sum_{k=1}^{3} \hat{d}_k = 0 \end{cases} \tag{17}$$

where, d_1, d_2, d_3, $\mathbf{x}(t)$, $\mathbf{y}(t)$ and $\mathbf{u}(t)$ are transient variables, and D_1, D_2, D_3, $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{U}$ represent their stable ones, and $\hat{d}_1, \hat{d}_2, \hat{d}_3, \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{u}}$ represent their disturbance ones, respectively.

By using the Laplace transform, one has

$$\begin{cases} \hat{\mathbf{x}}(s) = (s\mathbf{I} - \sum_{k=1}^{3} D_k \mathbf{A}_{\mathbf{k}})^{-1} \sum_{k=1}^{3} [D_k \mathbf{B}_{\mathbf{k}} \hat{\mathbf{u}} + (\mathbf{A}_{\mathbf{k}} \bar{\mathbf{X}} + \mathbf{B}_{\mathbf{k}} \mathbf{U}) \hat{d}_k] \\ \hat{\mathbf{y}}(s) = \sum_{k=1}^{3} [D_k \mathbf{C}_{\mathbf{k}} \hat{\mathbf{x}}(s) + \mathbf{C}_{\mathbf{k}} \bar{\mathbf{X}} \hat{d}_k(s)] \end{cases} \tag{18}$$

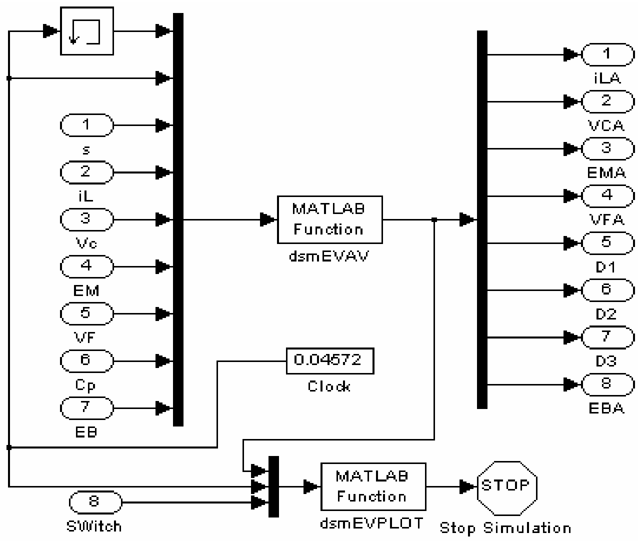
By using (18), some small signal transfer functions can be obtained. For example, $\mathbf{G}_{\mathbf{vdk}}$ is derived as

$$\mathbf{G}_{\mathbf{vd}_k} = \frac{\hat{\mathbf{y}}(s)}{\hat{d}_k(s)} = \sum_{k=1}^{3} (\mathbf{C}_{\mathbf{k}} \bar{\mathbf{X}})(s\mathbf{I} - \sum_{k=1}^{3} \mathbf{D}_{\mathbf{k}} \mathbf{A}_{\mathbf{k}})^{-1} \sum_{k=1}^{3} (\mathbf{A}_{\mathbf{k}} \bar{\mathbf{X}} + \mathbf{B}_{\mathbf{k}} \mathbf{U}) \tag{19}$$

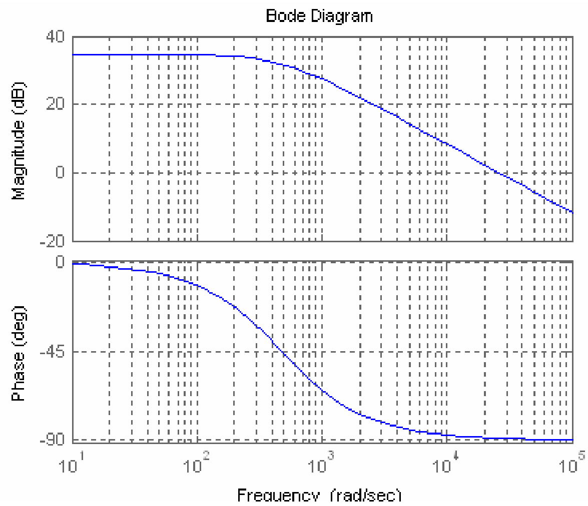
Through the transfer functions, some frequency domain behaviors of the converter can be obtained.

C. Model implementation

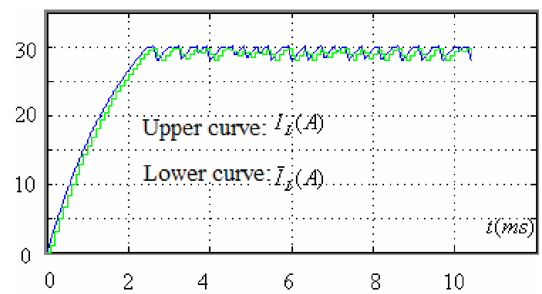
To efficiently obtain the small signal average model from large signal average model, the system matrices $\mathbf{A}_\mathbf{k}$, $\mathbf{B}_\mathbf{k}$ and $\mathbf{C}_\mathbf{k}$, and D_k, $\bar{\mathbf{X}}$, $\mathbf{U}$ in (16) are transferred from large signal average model in each PWM cycle automatically. Based on (15)-(19), the simulation model of small signal average model is obtained and shown in Fig. 5(a). The average values of D_k, $\bar{\mathbf{X}}$ and $\mathbf{U}$ in each PWM cycle are calculated by using the function of dsmEVAV and then transferred to the function of dsmEVPLOT for the frequency domain performance analysis. By inputting the parameters into the proposed model and running the model, the Bode plot of G_{vd1} when the converter operates with the rated condition is obtained, as shown in Fig. 5(b). Fig. 5(c) plots both the transient current and the average current flowing through the motor in one PWM cycle, and Fig. 5(d) shows both the transient voltage and the average voltage across the power filter capacitor in one PWM cycle. Fig. 5(e) gives the PWM duty ratio, *D1*.



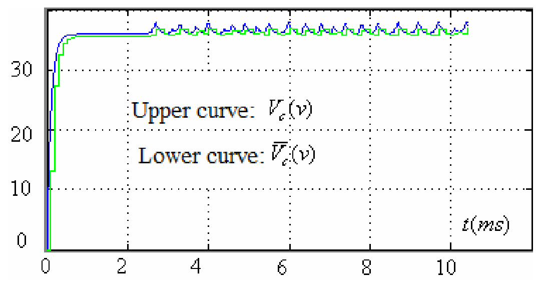
(a)



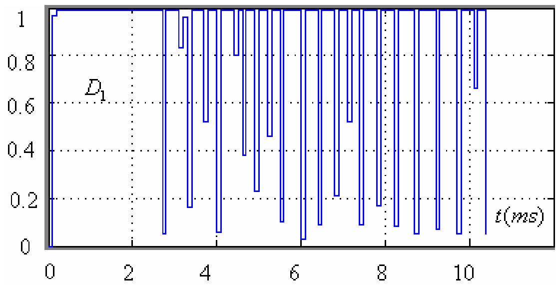
(b)



(c)



(d)



(e)

Fig. 5. Implementation of small signal average model of ERS with constant braking torque: (a) simulation model, (b) Bode plot of G_{vd1}, (c) armature current, (d) voltage across the capacitance, and (e) PWM duty ratio, D_1.

VI. CONCLUSION

In this paper, a detailed switching model for accurately analyzing the performance of energy regeneration system of a permanent magnet DC motor drive is proposed, and then the small signal average model around a desired operating point is deduced by using the method of averaging the switching model. The automatic mechanism of exchanging data between them is introduced. The proposed models have been implemented in MATLAB/Simulink to efficiently predict the comprehensive performances of the energy regeneration system.

REFERENCES

[1] J. X. Chen, J. G. Zhu, Y. G. Guo, and J. X. Jin, "Performance analysis of energy regeneration system of electric vehicle with two wheels under the mode of constant braking torque," in *Proc. 2nd Int. Conf. Industrial Electronics and Applications*, Harbin, China, May 2007, pp. 891-895.

[2] J. X. Chen, H. Cai, and M. S. Sun, "The research of energy regeneration system and its reliability in electric vehicle," in *Proc. IEEE 5th Int. Conf. Power Electronics and Drive System*, Singapore, Nov. 2003, pp. 864-867.

[3] J. X. Chen, J. G. Zhu, and Y. G. Guo, "A unified hybrid model with two level networks for peak current mode controlled buck-boost DC to DC converters operating in DCM and CCM," in *Proc. Int. Conference on Electrical Machines and Systems*, Seoul, Korea, Oct. 8-11, 2007.