Adaptive Fuzzy Sliding Mode Control for Uncertain Nonlinear Underactuated Mechanical Systems

R. M. T. Raja Ismail, Nguyen D. That and Q. P. Ha

Abstract— **Sliding mode control has been shown to be a robust and effective control approach for stabilization of nonlinear systems. However the dynamic performance of the controller is a complex function of the system parameters, which is often uncertain or partially known. This paper presents an adaptive fuzzy sliding mode control for a class of underactuated nonlinear mechanical systems. An adaptive fuzzy system is used to approximate the uncertain parts of the underactuated system. The adaptive law is designed based on the Lyapunov method. The proof for the stability and the convergence of the system is presented. The robust performance of the adaptive fuzzy sliding mode control is illustrated using a gantry crane system. The performance of the adaptive fuzzy sliding mode controller is illustrated using a gantry crane system. The simulation results demonstrate that the system output can track the reference signal in the presence of modelling uncertainties, external disturbances and parameter variation.**

I. Introduction

Sliding mode control (SMC) is a robust control law that computes quickly, making it suitable for system with fast dynamics. The approach is based on defining exponentially stable sliding surface as a function of the system states and using the Lyapunov theory to ensure that all closed-loop system trajectories reach this surface in finite time. Due to the closed-loop system dynamics on the surface are exponentially stable, the system trajectories slide along the surface until they reach the origin [1]. The dynamic performance of a sliding mode controller, however, is a complex function of the effort, surface, and system parameters [2].

SMC has been widely used for underactuated mechanical systems. A SMC law for a class of underactuated mechanical systems that can be represented in normal form and satisfy invertability conditions has been proposed in [3]. Sliding mode controllers for two degrees-of-freedom (DOF) underactuated mechanical systems using dry friction to essentially lock a joint when positioning the other has been developed in [4]. Futhermore, the author in [5] present a SMC algorithm to stabilize a class of underactuated mechanical systems that are not linearly controllable. In [6], a constructive method to determine exponential stability of the closed-loop system while on the sliding surface is developed, which further allows to determine a subset of the domain of attraction for the closed-loop system during the sliding phase. In addition, [7] presented a sliding mode control for two different classes of mechanical systems based on the existence of isolated equilibrium. However, some bounds on system uncertainties must be estimated in order to guarantee the stability of the closed-loop system, and its implementation in practice will cause a chattering problem, which is undesirable in application.

Fuzzy logic control has been an active research topic in automation and control theory. The basic concept of fuzzy logic control is to utilize the qualitative knowledge of a system for designing a practical controller. Generally, fuzzy logic control is applicable to plants that are ill-modeled, but qualitative knowledge of an experienced operator is available. The principle of SMC has been introduced in designing fuzzy logic controllers to guarantee the stability. This combination which is known as adaptive fuzzy sliding mode control (AFSMC) provides the mechanism to design robust controllers for nonlinear systems with uncertainty. Many works have been reported on the application of AFSMC for specific mechanical systems which include robot manipulator and translational oscillations by a rotational actuator. Adaptive fuzzy has been either used to adjust the control gain of the sliding mode [8], [9] or approximate the sliding function of the SMC [10]–[12]. Furthermore, the applications of AFSMC on the crane system, which is a typical underactuated system have been reported in [13], [14]. In recent years, the developments of AFSMC for uncertain mechanical systems to tackle more generic problems have been reported, which include the adjustment of control gain [15] and approximation of sliding function [16]–[22].

In this paper, an adaptive fuzzy logic sliding mode control is proposed for a class of underactuated mechanical systems. The considered system is uncertain and subject to nonlinear frictions and disturbances. The uncertainties of system dynamics are approximated with fuzzy logic system. Based on the approximated functions of the system dynamics, a sliding function is composed and a fuzzy adaptive law for sliding mode controller is proposed. The adaptive law is designed based on the Lyapunov method. Besides, the stability of the closed-loop system is presented in the Lyapunov sense. The robust performance of the adaptive fuzzy sliding mode control is illustrated using a gantry crane system. The simulation results demonstrate that the chattering and the steady state errors, which usually occur in the classical sliding mode control, are eliminated and satisfactory trajectory tracking is achieved. The robust performance of the AFSMC is illustrated using a gantry crane system. The simulation results indicate that a satisfactory trajectory tracking is achieved in the presence of uncertainties and external disturbances.

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II. Problem Formulation

By using the Euler-Lagrange's formulation, taking into account the viscous and Coulomb friction terms and uncertainties, the dynamic model of a mechanical system can be cast in a Lagrangian system of the form:

$$M(q)\ddot{q} + h(q,\dot{q}) + \eta(q,\dot{q}) = \tau, \tag{1}$$

where $M(q) \in \Re^{n\times n}$ is the inertia matrix, $h(q,\dot{q}) \in \Re^n$ is the vector of centrifugal-Coriolis and gravity, and $\eta(q,\dot{q}) \in \Re^n$ is the vector of viscous and Coulomb frictions, bounded disturbances and uncertainties in the system. The inertia matrix is symmetry and positive definite, that is, $M(q) = M^T(q)$ and $M(q) > 0$. The vector of generalized coordinates $q \in \Re^n$ can be partitioned as $q = [q_a^T, q_u^T]^T$, where $q_a \in \Re^m$ is the vector of actuated coordinates and $q_u \in \Re^{n-m}$ is the vector of unactuated coordinates. Similarly, the input forces vector $\tau \in \Re^n$ can be partitioned as $\tau = [u^T, 0]^T$, where $u \in \Re^m$ is the control input. Then (1) can be rewritten as

$$\begin{bmatrix} M_{aa}(q) & M_{au}(q) \\ M_{ua}(q) & M_{uu}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_a \\ \ddot{q}_u \end{bmatrix} + \begin{bmatrix} h_a(\dot{q},q) \\ h_u(\dot{q},q) \end{bmatrix} + \begin{bmatrix} \eta_a(\dot{q},q) \\ \eta_u(\dot{q},q) \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}. \tag{2}$$

By substituting $\ddot{q}_u = -M_{uu}^{-1}(M_{ua}\ddot{q}_a + h_u + \eta_u)$ obtained from the second row of (2) into the first row, we get

$$\ddot{q}_a = \tilde{h}_a(q,\dot{q}) + \tilde{\eta}_a(q,\dot{q}) + D_a(q)u \tag{3}$$

where $D_a = (M_{aa} - M_{au}M_{uu}^{-1}M_{ua})^{-1}$, $\tilde{h}_a = -D_a^{-1}(h_a - M_{au}M_{uu}^{-1}h_u)$ and $\tilde{\eta}_a = -D_a^{-1}(\eta_a - M_{au}M_{uu}^{-1}\eta_u)$. Consequently, it follows that

$$\ddot{q}_u = \tilde{h}_u(q,\dot{q}) + \tilde{\eta}_u(q,\dot{q}) + D_u(q)u \tag{4}$$

where $D_u = -M_{uu}^{-1}M_{ua}D_a$, $\tilde{h}_u = -M_{uu}^{-1}(M_{ua}\tilde{h}_a + h_u)$ and $\tilde{\eta}_u = -M_{uu}^{-1}(M_{ua}\tilde{\eta}_a + \eta_u)$.

Let the vector of tracking error is $e = q - q^d = [e_a^T, e_u^T]^T$, where $q^d \in \Re^n$ is the desired trajectory, in which $e_a = q_a - q_a^d \in \Re^m$ and $e_u = q_u - q_u^d \in \Re^{n-m}$. The problem is to find a suitable control strategy to drive the tracking error to zero subject to nonlinearities and uncertainties of the system.

III. Adaptive Fuzzy Sliding Mode Control

This section presents the design of the adaptive fuzzy sliding mode control for trajectory tracking problem. At first, a basic fuzzy logic control is presented, followed by the introduction of sliding function and sliding mode controller for underactuated system. Then the adaptive fuzzy sliding mode controller for the system is proposed.

A. Fuzzy Logic Control

For a fuzzy logic controller with p inputs, $v_1, \ldots, v_j, \ldots, v_p$, and using the center of average defuzzification, it can be represented as

$$u = \frac{\sum_{k=1}^{l} \prod_{j=1}^{p} \mu_{F_j^k}(v_j)\theta_k}{\sum_{k=1}^{l} \prod_{j=1}^{p} \mu_{F_j^k}(v_j)}, \tag{5}$$

where l is the number of fuzzy rules, F_j^k is the jth fuzzy set corresponding to the kth fuzzy rule, and θ_k is the centroid of the kth fuzzy set corresponding to the controller's output, u. By introducing $\theta = [\theta_1, \ldots, \theta_k, \ldots, \theta_l]^T$ as the vector of the centroid of fuzzy sets, and $\xi = [\xi_1, \ldots, \xi_k, \ldots, \xi_l]^T$ as the regressor vector, in which $\xi_k = \prod_{j=1}^{p} \mu_{F_j^k}(v_j) / \sum_{k=1}^{l} \prod_{j=1}^{p} \mu_{F_j^k}(v_j)$, then (5) can be rewritten as

$$u = \theta^T \xi. \tag{6}$$

Now we consider the vector of the centroid of fuzzy sets as a function of time, that is, $\theta(t) = [\theta_1(t), \ldots, \theta_k(t), \ldots, \theta_l(t)]^T$, where $\dot{\theta}_k(t) = \alpha_k(t)$, $k = 1, \ldots, l$. It can be ensured to lie between its lower bound $\underline{\theta}_k$ and its upper bound $\overline{\theta}_k$, i.e., $\underline{\theta}_k \le \theta_k \le \overline{\theta}_k$ by defining the following projection function:

$$\dot{\theta}_k(t) = \underset{\theta_k}{\mathrm{proj}}(\alpha_k) = \begin{cases} 0 & \text{if } \theta_k = \underline{\theta}_k \text{ and } \alpha_k(t) < 0 \\ 0 & \text{if } \theta_k = \overline{\theta}_k \text{ and } \alpha_k(t) > 0 \\ \alpha_k(t) & \text{otherwise} \end{cases} \tag{7}$$

B. Sliding Mode Control

For the underactuated mechanical system (2), we consider the vector of sliding functions $\sigma \in \Re^m$ as

$$\sigma = \alpha_a \dot{e}_a + \alpha_u \dot{e}_u + \lambda_a e_a + \lambda_u e_u \tag{8}$$

where $\alpha_a, \lambda_a \in \Re^{m\times m}$ and $\alpha_u, \lambda_u \in \Re^{m\times(n-m)}$ are the matrices of sliding surface parameters. Since $e_a = q_a - q_a^d$ and $e_u = q_u - q_u^d$, (8) can be rewritten as

$$\sigma = \alpha_a \dot{q}_a + \alpha_u \dot{q}_u - \dot{q}^r, \tag{9}$$

where $\dot{q}^r = \alpha_a \dot{q}_a^d + \alpha_u \dot{q}_u^d - \lambda_a(q_a - q_a^d) - \lambda_u(q_u - q_u^d)$. Then the derivative of the sliding function is

$$\dot{\sigma} = \alpha_a \ddot{q}_a + \alpha_u \ddot{q}_u - \ddot{q}^r. \tag{10}$$

Substituting (3),(4) into (10) yields

$$\begin{aligned} \dot{\sigma} &= \alpha_a(\tilde{h}_a + \tilde{\eta}_a + D_a u) + \alpha_u(\tilde{h}_u + \tilde{\eta}_u + D_u u) - \ddot{q}^r \\ &= F(q,\dot{q}) + G(q)u - \ddot{q}^r + \tilde{\eta}(q,\dot{q}), \end{aligned} \tag{11}$$

where $F(q,\dot{q}) = \alpha_a \tilde{h}_a + \alpha_u \tilde{h}_u$, $G(q) = \alpha_a D_a + \alpha_u D_u$ and $\tilde{\eta}(q,\dot{q}) = \alpha_a \tilde{\eta}_a + \alpha_u \tilde{\eta}_u$. Thus, from (11), the sliding mode controller $u \in \Re^m$ is proposed as

$$u = G^{-1}(q)[-F(q,\dot{q}) - \mu\sigma + \ddot{q}^r] - K \,\mathrm{sign}\,\sigma, \tag{12}$$

where μ and K are diagonal matrices of designed parameters, i.e., $\mu = \mathrm{diag}(\mu_1, \ldots, \mu_m)$ and $K = \mathrm{diag}(K_1, \ldots, K_m)$.

C. Adaptive Fuzzy Sliding Mode Controller

Suppose that for $i = 1, \ldots, m$, the sliding function (11) can be expressed as

$$\dot{\sigma}_i = f_i(q,\dot{q}) + g_i(q,\dot{q})u_i - \ddot{q}_i^r + \tilde{\eta}_i(q,\dot{q}). \tag{13}$$

Here, we assume that $|\tilde{\eta}_i(q,\dot{q})| \le d_i$, $\forall i = 1, \ldots, m$, where $d_i > 0$ is known. It follows that, the sliding mode controller (12) can be written as

$$u_i = \frac{1}{g_i(\boldsymbol{x})}[-f_i(\boldsymbol{x}) - \mu_i\sigma_i + \ddot{q}_i^r] - K_i \,\mathrm{sign}\,\sigma_i, \tag{14}$$

where $\boldsymbol{x} = [q^T, \dot{q}^T]^T$. We now consider that the functions $f_i(\boldsymbol{x})$ and $g_i(\boldsymbol{x})$ are uncertain. Hence, we approximate $f_i(\boldsymbol{x})$ and $g_i(\boldsymbol{x})$ with fuzzy logic systems $\theta_{f_i}^T \xi_{f_i}$ and $\theta_{g_i}^T \xi_{g_i}$, respectively.

Let $\theta_{f_i}^*$ and $\theta_{g_i}^*$ are optimal vectors such that

$$\begin{cases} \theta_{f_i}^* = \underset{\theta_{f_i}}{\operatorname{argmin}} \sup_{\boldsymbol{x} \in \Omega} |f_i(\boldsymbol{x}) - \theta_{f_i}^T \xi_{f_i}(\boldsymbol{x})|, \\ \theta_{g_i}^* = \underset{\theta_{g_i}}{\operatorname{argmin}} \sup_{\boldsymbol{x} \in \Omega} |g_i(\boldsymbol{x}) - \theta_{g_i}^T \xi_{g_i}(\boldsymbol{x})|, \end{cases} \tag{15}$$

where $\Omega \subseteq \Re^{2n}$ is a region to which the state $\boldsymbol{x}$ is constrained to reside. We assume that

$$\begin{cases} |f_i(\boldsymbol{x}) - \theta_{f_i}^{*\,T} \xi_{f_i}(\boldsymbol{x})| \leq d_{f_i}, \\ |g_i(\boldsymbol{x}) - \theta_{g_i}^{*\,T} \xi_{g_i}(\boldsymbol{x})| \leq d_{g_i}, \end{cases} \quad \forall \boldsymbol{x} \in \Omega, \tag{16}$$

where $d_{f_i} > 0$ and $d_{g_i} > 0$, and each kth element of $\theta_{f_i}^*$ and $\theta_{g_i}^*$ is constant and bounded as follows:

$$\begin{cases} \underline{\theta}_{f_{i_k}} \leq \theta_{f_{i_k}}^* \leq \overline{\theta}_{f_{i_k}}, \\ \underline{\theta}_{g_{i_k}} \leq \theta_{g_{i_k}}^* \leq \overline{\theta}_{g_{i_k}}, \end{cases} \quad \forall k = 1, \ldots, l. \tag{17}$$

For the purpose of designing the sliding mode control, we choose the following adaptation laws:

$$\begin{cases} \dot{\theta}_{f_i} = \underset{\theta_{f_i}}{\operatorname{proj}}(\gamma_{f_i} \sigma_i \xi_{f_i}), \\ \dot{\theta}_{g_i} = \underset{\theta_{g_i}}{\operatorname{proj}}(\gamma_{g_i} \sigma_i \xi_{g_i} w_i), \end{cases} \tag{18}$$

where $\gamma_{f_i} > 0$ and $\gamma_{g_i} > 0$ are design parameters and

$$w_i = \frac{1}{\theta_{g_i}^T \xi_{g_i}} (-\theta_{f_i}^T \xi_{f_i} + \ddot{q}_i^r). \tag{19}$$

Then, we define the adaptation parameter errors δ_{f_i} and δ_{g_i} respectively as

$$\begin{cases} \delta_{f_i} = \theta_{f_i} - \theta_{f_i}^*, \\ \delta_{g_i} = \theta_{g_i} - \theta_{g_i}^*. \end{cases} \tag{20}$$

Since $\theta_{f_i}^*$ and $\theta_{g_i}^*$ are constants and from (18), the time derivatives of the adaptation parameter errors are obtained as

$$\begin{cases} \dot{\delta}_{f_i} = \dot{\theta}_{f_i} = \underset{\theta_{f_i}}{\operatorname{proj}}(\gamma_{f_i} \sigma_i \xi_{f_i}), \\ \dot{\delta}_{g_i} = \dot{\theta}_{g_i} = \underset{\theta_{g_i}}{\operatorname{proj}}(\gamma_{g_i} \sigma_i \xi_{g_i} w_i). \end{cases} \tag{21}$$

Finally, by assuming that $g_i(q, \dot{q})$ has a positive lower bound, i.e., there exist a constant $\underline{g}_i$ such that $g_i(q, \dot{q}) \geq \underline{g}_i > 0$, $\forall i = 1, \ldots, m$, we propose the adaptive fuzzy sliding mode controller as follows:

$$\begin{aligned} u_i &= w_i - \frac{1}{\underline{g}_i} \mu_i \sigma_i - K_i \operatorname{sign} \sigma_i \qquad (22) \\ &= \frac{1}{\theta_{g_i}^T \xi_{g_i}} (-\theta_{f_i}^T \xi_{f_i} + \ddot{q}_i^r) - \frac{1}{\underline{g}_i} \mu_i \sigma_i - K_i \operatorname{sign} \sigma_i. \end{aligned}$$

D. Stability Analysis

To prove the stability by means of control algorithm (22), we choose the following Lyapunov function candidate:

$$V = \frac{1}{2} \sum_{i=1}^{m} \left(\sigma_i^2 + \frac{1}{\gamma_{f_i}} \delta_{f_i}^T \delta_{f_i} + \frac{1}{\gamma_{g_i}} \delta_{g_i}^T \delta_{g_i}. \right)$$

Then, its time derivative is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{m} \left(\sigma_i \dot{\sigma}_i + \frac{1}{\gamma_{f_i}} \delta_{f_i}^T \dot{\delta}_{f_i} + \frac{1}{\gamma_{g_i}} \delta_{g_i}^T \dot{\delta}_{g_i} \right) \\ &= \sum_{i=1}^{m} \left(\sigma_i (f_i + g_i u_i - \ddot{q}_i^r) + \frac{1}{\gamma_{f_i}} \delta_{f_i}^T \dot{\delta}_{f_i} + \frac{1}{\gamma_{g_i}} \delta_{g_i}^T \dot{\delta}_{g_i} \right). \end{aligned} \tag{23}$$

From (19), we have $\ddot{q}_i^r = \theta_{f_i}^T \xi_{f_i} + \theta_{g_i}^T \xi_{g_i} w_i$. Substituting this equation together with (21) and (22) into (23) yields

$$\begin{aligned} \dot{V} = \sum_{i=1}^{m} \Bigg[& \sigma_i \bigg(-\frac{g_i}{\underline{g}_i} \mu_i \sigma_i + (f_i - \theta_{f_i}^T \xi_{f_i}) \\ & + (g_i - \theta_{g_i}^T \xi_{g_i}) w_i - g_i K_i \operatorname{sign} \sigma_i \bigg) \\ & + \frac{1}{\gamma_{f_i}} \delta_{f_i}^T \underset{\theta_{f_i}}{\operatorname{proj}}(\gamma_{f_i} \sigma_i \xi_{f_i}) + \frac{1}{\gamma_{g_i}} \delta_{g_i}^T \underset{\theta_{g_i}}{\operatorname{proj}}(\gamma_{g_i} \sigma_i \xi_{g_i} w_i) \Bigg]. \end{aligned}$$

From (16) and (20), the last equation becomes

$$\begin{aligned} \dot{V} \leq \sum_{i=1}^{m} \Bigg[& \sigma_i \bigg(-\frac{g_i}{\underline{g}_i} \mu_i \sigma_i - \delta_{f_i}^T \xi_{f_i} \\ & - \delta_{g_i}^T \xi_{g_i} w_i - g_i K_i \operatorname{sign} \sigma_i \bigg) \\ & + \frac{1}{\gamma_{f_i}} \delta_{f_i}^T \underset{\theta_{f_i}}{\operatorname{proj}}(\gamma_{f_i} \sigma_i \xi_{f_i}) + \frac{1}{\gamma_{g_i}} \delta_{g_i}^T \underset{\theta_{g_i}}{\operatorname{proj}}(\gamma_{g_i} \sigma_i \xi_{g_i} w_i) \Bigg] \\ = \sum_{i=1}^{m} \Bigg[& -\frac{g_i}{\underline{g}_i} \mu_i \sigma_i^2 - g_i K_i |\sigma_i| \\ & + \delta_{f_i}^T \left(\frac{1}{\gamma_{f_i}} \underset{\theta_{f_i}}{\operatorname{proj}}(\gamma_{f_i} \sigma_i \xi_{f_i}) - \sigma_i \xi_{f_i} \right) \\ & + \delta_{g_i}^T \left(\frac{1}{\gamma_{g_i}} \underset{\theta_{g_i}}{\operatorname{proj}}(\gamma_{g_i} \sigma_i \xi_{g_i} w_i) - \sigma_i \xi_{g_i} w_i \right) \Bigg]. \end{aligned} \tag{24}$$

From the definition of projection function (7), one can verify that

$$\begin{cases} \delta_{f_i}^T \left(\frac{1}{\gamma_{f_i}} \underset{\theta_{f_i}}{\operatorname{proj}}(\gamma_{f_i} \sigma_i \xi_{f_i}) - \sigma_i \xi_{f_i} \right) \leq 0, \\ \delta_{g_i}^T \left(\frac{1}{\gamma_{g_i}} \underset{\theta_{g_i}}{\operatorname{proj}}(\gamma_{g_i} \sigma_i \xi_{g_i} w_i) - \sigma_i \xi_{g_i} w_i \right) \leq 0, \end{cases}$$

By applying the above inequalities to (24), it gives

$$
\begin{aligned}
\dot{V} &\le \sum_{i=1}^{m}\left(-\frac{g_i}{\underline{g}_i}\mu_i\sigma_i^2 - g_iK_i|\sigma_i|\right) \\
&\le \sum_{i=1}^{m}\left(-\frac{g_i}{\underline{g}_i}\mu_i\sigma_i^2\right) \\
&\le \sum_{i=1}^{m}(-\mu_i\sigma_i^2),
\end{aligned}
$$

since $g_i \ge \underline{g}_i$. Thus, it implies that the surface $\sigma = 0$ is globally reached in a finite time.

IV. Results and Discussion

In this example, the adaptive fuzzy sliding mode control is applied to the gantry crane system of Figure 1 where x is the trolley position, ϕ is the swing angle of the hoisting rope, l is the rope length, M and m are the masses of the trolley and payload, respectively, and F_x is the trolley driving force. The crane dynamics is described by the following equation:

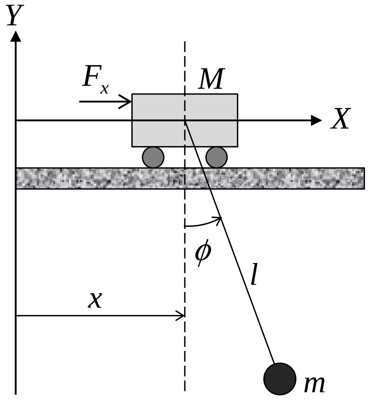


Fig. 1: Motion of the gantry crane system.

$$
\begin{bmatrix} M+m & ml\cos\phi \\ ml\cos\phi & ml^2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\phi} \end{bmatrix}
+ \begin{bmatrix} -ml\sin\phi\dot{x}\dot{\phi} + B_x\dot{x} + P_x \operatorname{sign}\dot{x} \\ mg_0 l\sin\phi + B_\phi\dot{\phi} + P_\phi \operatorname{sign}\dot{\phi} \end{bmatrix} + \begin{bmatrix} \eta_x(t) \\ \eta_\phi(t) \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}, \tag{25}
$$

in which the vector of generalized coordinates is defined as $q = [q_a, q_u]^T = [x, \phi]^T$. In (25), g_0 is the gravitational acceleration, B_x and B_ϕ denote the viscous friction coefficients, and P_x and P_ϕ denote the Coulomb friction coefficients associated with the trolley and rope sway motions, respectively.

The trajectory tracking problem consists of the positional control of the trolley while the sway angle must be suppressed to zero. Thus, the desired trajectory vector is defined as $q^d(t) = [q_a^d, q_u^d]^T = [x^d(t), 0]^T$. Therefore, from (9), the sliding function is determined as

$$
\sigma = \alpha_a\dot{x} + \alpha_u\dot{\phi} - \dot{q}^r(x,\phi)
$$

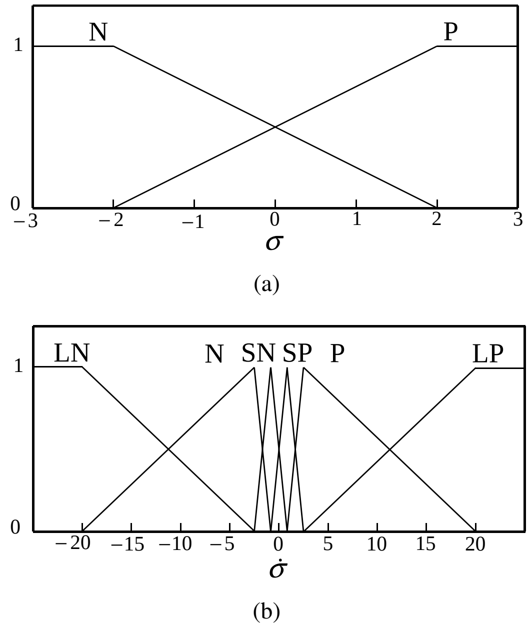


Fig. 2: Fuzzy sets for (a) σ; and (b) $\dot{\sigma}$.

where $\dot{q}^r(x,\phi) = (\alpha_a\dot{x}^d(t) - \lambda_a(x - x^d(t)) - \lambda_u\phi)$. Then time derivative of the sliding function is

$$
\dot{\sigma} = \alpha_a\ddot{x} + \alpha_u\ddot{\phi} - \ddot{q}^r(x,\phi)
$$

which can be expressed in the form of

$$
\dot{\sigma} = f(x,\phi,\dot{x},\dot{\phi}) + g(x,\phi)u - \ddot{q}^r(x,\phi) + \eta(x,\phi,\dot{x},\dot{\phi}).
$$

By assuming that the functions $f(x,\phi,\dot{x},\dot{\phi})$ and $g(x,\phi)$ are unknown apart from their bounds, we can propose the adaptive fuzzy sliding mode control, in which the control signal $u = F_x$ is defined as follows:

$$
u = \frac{1}{\theta_g^T\xi_g}(-\theta_f^T\xi_f + \ddot{q}^r) - \frac{1}{\underline{g}}\mu\sigma - K\operatorname{sign}\sigma. \tag{26}
$$

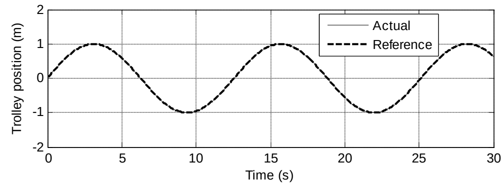
where θ_f and θ_g are as the vectors of the centroid of the membership functions and ξ_f and ξ_g are the corresponding regressor vectors, $\underline{g}$ is the known positive lower bound of $g(x,\phi)$, μ is the positive design parameter, and K is the control gain.

In this work, we use fuzzy sets for σ and $\dot{\sigma}$ as shown in Figure 2. Since there are two fuzzy sets for σ and six fuzzy sets for $\dot{\sigma}$, we have 12 fuzzy rules possible in the following form:

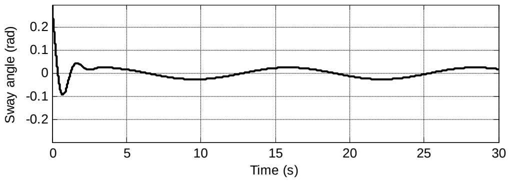
Rule k: IF σ is A^k AND $\dot{\sigma}$ is B^k, THEN $y = \theta_k$

for $k = 1, \ldots, 12$, where A^k and B^k are fuzzy sets as described in Figure 2(a) and (b) respectively.

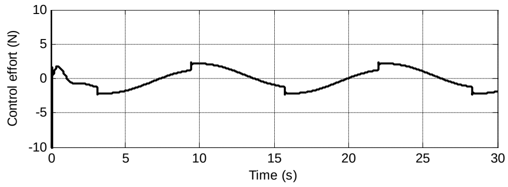
For the simulation, the values of the crane parameters are listed as $M = 2.70$ kg, $m = 2.24$ kg, $l = 0.795$ m, $g_0 = 9.8065$ m-s^{-2}, $B_x = 0.17$ N/m-s^{-1}, $B_\phi = 0.04$ N-m/rad-s^{-1}, $P_x = 0.90$ N, $P_\phi = 0.45$ N-m and the disturbances, are assumed bounded such that $|\eta_x(t)| \le 10$ N and $|\eta_\phi(t)| \le 20$ N-m. The controller parameters used in the simulations are $\alpha_a = 1$, $\alpha_u = 0.5$, $\lambda_a = 40$, $\lambda_u = -10$, $\gamma_f = 5000$,



(a)

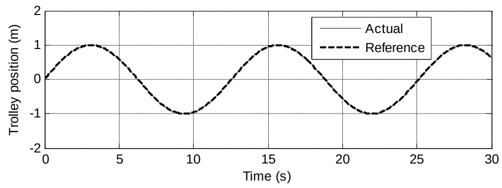


(b)

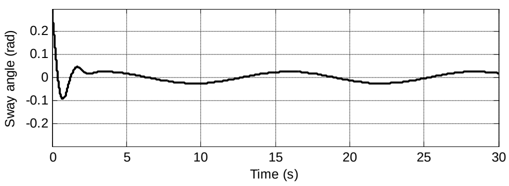


(c)

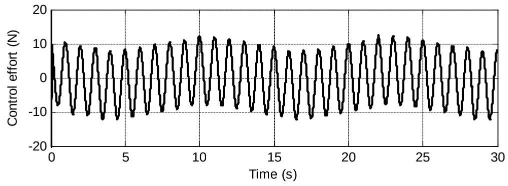
Fig. 3: (a) Trolley position; (b) Sway angle; and (c) Control effort; when $\eta_x = \eta_\phi = 0$.



(a)

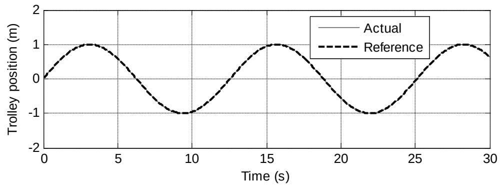


(b)

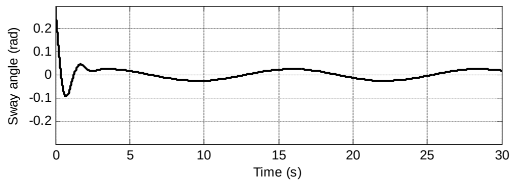


(c)

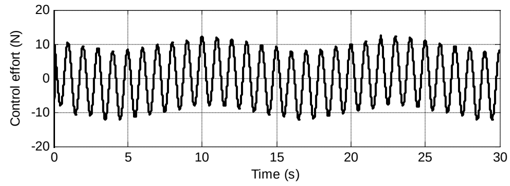
Fig. 4: (a) Trolley position; (b) Sway angle; and (c) Control effort; when $\eta_x \neq 0$ and $\eta_\phi \neq 0$.



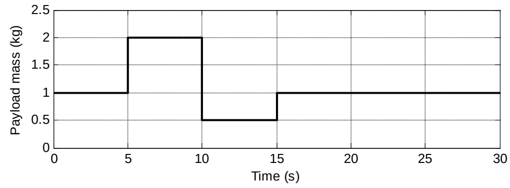
(a)



(b)



(c)



(d)

Fig. 5: (a) Trolley position; (b) Sway angle; and (c) Control effort (d) Payload mass; when $\eta_x \neq 0$, $\eta_\phi \neq 0$, and the payload mass is varied.

$\gamma_g = 1000$, $\underline{g} = 1/(M+m)$, $\mu = 250$, and $K = 100$. The bounds on the adaptation parameters are chosen as $\underline{\theta}_f = -200$, $\overline{\theta}_f = 200$, $\underline{\theta}_g = 50$, and $\overline{\theta}_g = 150$. The initial cart position is $(x_0, \phi_0) = (0 \text{ m}, 0.2 \text{ rad})$.

Figure 3(a) and (b) show the trolley position and sway angle responses without the presence of external disturbance. Figure 3(a) shows a good trajectory tracking whereas Figure 3(b) shows the sway angle of the hoisting rope is suppressed from an initial value. The control effort for this corresponding case is shown in Figure 3(c).

Figure 4 show the system responses and control effort with the presence of external disturbances. As can be seen in Figure 4(a) and (b), the proposed control system appears to be insensitive to the presence of the disturbances. However, higher control effort is required as shown in Figure 4(c).

To demonstrate the robustness of the controller, the payload is varied between 0.5 to 2.0 kg, which reflects the process of loading/unloading of the gantry crane. From Figure 5(a) and (b), it is shown that the trajectory tracking of the trolley position and the sway angle of the hoisting rope are unperturbed by the presence of payload variation.

V. Conclusion

In this paper we have proposed an adaptive fuzzy logic sliding mode control for trajectory tracking for a class of underactuated Lagrangian systems. By using fuzzy system, the uncertainties of system dynamics are approximated. Then, a sliding function is composed based on the approximated functions of the system dynamics. The adaptive law for the sliding mode control is designed based on the Lyapunov method. The performance of the AFSMC is evaluated by applying the controller to a gantry crane system with the presence of nonlinear frictions. High robust control performance is obtained when the system is subject to parametric uncertainties, external disturbances, and parameter variations.

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