Power Electronic-Controlled High Q Resonator Theory with HTS Technology

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Abstract—**Power electronic-controlled switch has been used with a high-temperature superconductor (HTS) to develop an advanced high Q resonant circuit. With a HTS, a very high Q circuit can be achieved; consequently special aspects such as high voltage generation can be theoretically and practically realized. Theoretical study has been carried out for this concept and method, and this paper describes the theory of this high Q resonant circuit and the operational principle.**

I.   INTRODUCTION

An electronic resonant circuit with a power electronic-controlled switch has been explored with regard to its special aspects of using a high T_c superconductor (HTS) [1,2]. The circuit special aspects have been studied, and especially to form a method of very high voltage generation from a low voltage source. This high voltage generation method using the high Q resonant circuit mainly consists of a HTS inductor, a DC battery source, a high voltage capacitor, and an electronic switch. As a basic principle of operational approach, a low voltage DC power source can be used and its polarity is reversed at a certain frequency and this control is achieved with an electronic switch. As a fundamental principle, the resonant circuit generates the voltage which is proportional to the circuit Q value. Resistance in the circuit will limit the Q value and therefore voltages that can be achieved in practice; however HTS technology can dramatically reduce the resistance and present a very high Q value in its circuit.

With regard to the power electronic-controlled HTS high Q resonator, both theoretical analysis and practical operation principle will be presented in this paper with the advantages of potentially using a high Q electronic circuit which can be made with HTS Ag-clad $(Bi,Pb)_2Sr_2Ca_2Cu_3O_{10+x}$ multi-filament wires.

II.   CIRCUIT PRINCIPLE

A.   Q Value and Voltage Feature of a Resonant Circuit

In a resistive RCL series resonant circuit, the quality factor of a resonant circuit is $Q = \omega_o L/R$ (= X/R), and $\omega_o = [(1/LC)-(R^2/4L^2)]^{1/2}$ is the resonant frequency in rad s^{-1}. The potential difference across the capacitor V_c at resonance is Q times as great as the applied emf V_p (rms) to the circuit. For a sinusoidal power supply, the voltage across the capacitor V_{Cmax} at resonance is given by: $V_{Cmax} = Q\ V_p$. For using a DC power supply with an electronic switch to reverse the polarity, the maximum voltage V_{Cmax} can be expressed as: $V_{Cmax} = Q\ V_B$', where V_B' is the effective voltage of the low voltage power source. If the switching controller used has rectangular switching wave form with low voltage source polarity switching frequency f, the build-up voltage wave form F(t) can be expressed by Fourier series as

$$F(t) = (4V_B/\pi)[\sin\omega t + (1/3)\sin3\omega t + (1/5)\sin5\omega t + ...]\ \ (1)$$

The generator only resonates on the first harmonic with amplitude of 4 V_B/π. Therefore, V_B' = 4 V_B/π, and the maximum build-up voltage is related with the RCL resonant circuit Q value by: $V_{Cmax} = Q\ (4V_B/\pi)$. Reducing the RCL resonant circuit resistance is achieved by introducing the superconducting inductor; the circuit Q value then will be dramatically increased, therefore leading to a very high voltage across the capacitor.

B.   Resistance-Less Circuit

For the RCL resonant circuit, when a DC source is switched to the circuit, the instantaneous current i(t) and capacitor voltage V_C(t) solutions respectively are

$$i(t) = e^{\frac{-Rt}{2L}} \left[\frac{(V_{CO} + V_B)\sin\omega t}{\omega L} \right] \qquad (2)$$

$$V_C(t) = (V_{CO} + V_B)\left[1 - e^{\frac{-Rt}{2L}}\left(\cos\omega t + \frac{R}{2L\omega}\sin\omega t\right)\right] - V_{CO} \qquad (3)$$

where $R < 2(L/C)^{1/2}$, V_{CO} is the initial capacitor voltage. Both equations describe decaying sinusoids, with $V_C(t)$ approaching a steady state value of V_B, and $i(t)$ approaching a steady state value of zero.

Now in a circuit using a superconducting inductor and no separate resistor, then R will become very small. If $R = 0\ \Omega$, then (2) and (3) can be simplified to

$$i(t) = \frac{(V_{CO} + V_B)\sin\omega t}{\omega L} \qquad (4)$$

$$V_C(t) = -(V_{CO} + V_B)\cos\omega t + V_B \qquad (5)$$

where $\omega = (LC)^{-1/2}$ rad s^{-1}. These two equations describe constant magnitude sinusoids, with the average values of $i(t)$ and $V_C(t)$ being zero and V_B respectively.

When the DC source is switched in, $V_C(t) = V_C(0) = -V_{CO}$. One half a resonant cycle later, this voltage will have increased to

$$V_C(t) = V_C(\pi/\omega) = -(V_{CO} + V_B)(-1) + V_B = V_{CO} + 2V_B \quad (6)$$

If at this point of time the DC source is disconnected, and then reconnected in the opposite polarity for the next half cycle, then the initial capacitor voltage V_{CO} is changed to $V_{CO\text{-new}}$, and is given by

$$V_{CO\text{-new}} = -V_C(t) = -(V_{CO} + 2V_B) \qquad (7)$$

Half a cycle later, $V_C(t) = V_C(2\pi/\omega)$ becomes

$$V_C(t) = -[-(V_{CO} + 2V_B) + (-V_B)](-1) + (-V_B) = -(V_{CO} + 4V_B) \qquad (8)$$

If the DC source polarity is reversed every half cycle thereafter then

$$V_C(t) = (V_{CO} + 6V_B); -(V_{CO} + 8V_B); (V_{CO} + 10V_B); \ldots \text{etc} \qquad (9)$$

This is the build-up voltage for an ideal non-resistive circuit. Therefore the positive and negative peak voltages can be described by (9). Consequently in a resistance-less circuit, the voltage across the capacitor C after n cycles will be:

$$V_C(n) = (-1)^{n+1}(V_{CO} + 2nV_B) \qquad (10)$$

where n is the iteration number, and V_{CO} is the initial capacitor voltage.

C. Practical Resistive Circuit

From (3), when $t = \pi/\omega$, $i = 0$, if the DC source changes polarity, the capacitor voltage is given by

$$V_{C1} = (V_{CO} + V_B)(1 + e^{-R\pi/2L\omega}) - V_{CO} \qquad (11)$$

After the polarity is changed n times, the capacitor voltage becomes

$$V_{Cn} = (V_{Cn-1} + V_B)(1 + e^{-R\pi/2L\omega}) - V_{Cn-1} \qquad (12)$$

If $V_{CO} = 0$, then

$$V_{C1} = V_B(1 + e^{-R\pi/2L\omega}) \qquad (13)$$

$$V_{C2} = (V_{C1} + V_B)(1 + e^{-R\pi/2L\omega}) - V_{C1}$$
$$= V_B(1 + 2e^{-R\pi/2L\omega} + e^{-2R\pi/2L\omega})$$

$$\ldots$$

$$V_{Cn} = (V_{Cn-1} + V_B)(1 + e^{-R\pi/2L\omega}) - V_{Cn-1}$$
$$= V_B(1 + 2e^{-R\pi/2L\omega} + 2e^{-2R\pi/2L\omega} \ldots + 2e^{-(n-1)R\pi/2L\omega} + e^{-nR\pi/2L\omega})$$

$$= (1 + e^{-nR\pi/2L\omega}) + 2V_B \sum_{i=1}^{n-1} e^{-iR\pi/2L\omega} \qquad (14)$$

Assuming that the power supply polarity changes at $t = n\pi/\omega$, after the polarity is changed n times, the build-up V_{Cn} is given by

$$V_{Cn} = V_B(1 + e^{-nR\pi/2L\omega}) + 2V_B \frac{e^{-R\pi/2L\omega} - e^{-(n-1)R\pi/2L\omega}}{1 - e^{-R\pi/2L\omega}} \qquad (15)$$

When $n \to \infty$, $V_{Cn} \to V_{Cmax}$, therefore

$$V_{Cmax} = \lim_{n\to\infty} V_{Cn} = V_B\left(1 + \frac{2}{e^{R\pi/2L\omega} - 1}\right) \qquad (16)$$

From (16), when $R \to 0$, $V_{Cmax} \to \infty$. When $R \to \infty$, $V_{Cmax} \to V_B$. Since $Q = \omega L/R$, therefore (16) can be expressed as

$$V_{Cmax} = V_B + \frac{2V_B}{e^{\pi/2Q} - 1} \qquad (17)$$

D. Step Voltage Source

The low voltage sources can be an ac power supply, a dc battery or a rectangular pulse source. When a rectangular pulse source u_s is used, consequently the RCL circuit has its differential equation $LC\dfrac{d^2 v_c}{dt^2} + RC\dfrac{dv_c}{dt} + v_c = u_s$ where its characteristic roots are $S_{1,2} = -\dfrac{R}{2L} \pm \sqrt{\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}}$,

if $R^2 < \dfrac{4L}{C}$, then its natural frequencies are conjugate complex numbers,

$$S_{1,2} = -\dfrac{R}{2L} \pm j\sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2} = -\alpha \pm j\omega_d \ , \quad \text{where} \quad \alpha = \dfrac{R}{2L} \quad \text{is}$$

attenuation coefficient, $\omega_d = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}$ is damping

angular frequency. $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ is series resonant angular frequency.

If the exciting source is a step voltage source, $u_s = V_s\varepsilon(t)$, then $u_s = v_c(t) = e^{-\alpha t}(K_1\cos\omega_d t + K_2\sin\omega_d t) + V_s$. If the circuit has zero initial state, then $v_c(0) = K_1 + V_s = 0$, $\dfrac{dv_c(t)}{dt}\Big|_{t=0} = -\alpha K_1 + \omega_d K_2 = 0$; when $K_1 = -V_s$, $K_2 = -V_s\alpha/\omega_d$, then

$$v_c(t) = -V_S e^{-\alpha t}\left(\cos\omega_d t + \dfrac{\alpha}{\omega_d}\sin\omega_d t\right) + V_S = V_S\left[1 - e^{-\alpha t}\sqrt{1 + \dfrac{\alpha^2}{\omega_d^2}}\left(\dfrac{1}{\sqrt{1 + \dfrac{\alpha^2}{\omega_d^2}}}\cos\omega_d t + \dfrac{\alpha}{\omega_d\sqrt{1 + \dfrac{\alpha^2}{\omega_d^2}}}\sin\omega_d t\right)\right]$$

$$= V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha t}\cos(\omega_d t - \theta)\right], \quad t \geq 0, \quad (\theta = \operatorname{arctg}\dfrac{\alpha}{\omega_d}) \tag{18}$$

If exciting source is a causality pulse voltage source

$$u_s = \varepsilon(t) - 2\varepsilon(t - \dfrac{T}{2}) + 2\varepsilon(t - T) - 2\varepsilon(t - T - \dfrac{T}{2}) + 2\varepsilon(t - 2T) - 2\varepsilon(t - 2T - \dfrac{T}{2}) + 2\varepsilon(t - 3T)....... , \tag{19}$$

for the first cycle, $u_s(t) = \begin{cases} \varepsilon(t), & 0 < t < \dfrac{T}{2} \\ \varepsilon(t) - 2\varepsilon(t - \dfrac{T}{2}), & \dfrac{T}{2} < t < T \end{cases}$

$$\tag{20}$$

after the first cycle,

$$u_s(t) = \begin{cases} \varepsilon(t) - 2\sum\limits_{k=1}^{k}\varepsilon\left[t - (k-1)T - \dfrac{T}{2}\right] + 2\sum\limits_{k=1}^{k}\varepsilon(t - kT), & kT < t < kT + \dfrac{T}{2} \\ \varepsilon(t) - 2\sum\limits_{k=1}^{k}\varepsilon\left[t - (k-1)T - \dfrac{T}{2}\right] + 2\sum\limits_{k=1}^{k}\varepsilon(t - kT) - 2\varepsilon(t - kT - \dfrac{T}{2}), & kT + \dfrac{T}{2} < t < (k+1)T \end{cases}$$

$$\tag{21}$$

V_c in the first cycle,

$$v_c(t) = \begin{cases} V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha t}\cos(\omega_d t - \theta)\right], & 0 < t < \dfrac{T}{2} \ ; \\ V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha t}\cos(\omega_d t - \theta)\right] - 2V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha(t - \frac{T}{2})}\cos\left(\omega_d(t - \dfrac{T}{2}) - \theta\right)\right], & \dfrac{T}{2} < t < T \end{cases}$$

$$\tag{22}$$

After the first cycle,

$$
v_c(t) = \begin{cases}
V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha t}\cos(\omega_d t - \theta)\right] - 2\sum_{k=1}^{k}V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha[t-(k-1)T-\frac{T}{2}]}\cos\left(\omega_d[t-(k-1)T-\dfrac{T}{2}]-\theta\right)\right] \\
\quad + 2\sum_{k=1}^{k}V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha(t-kT)}\cos(\omega_d(t-kT)-\theta)\right], \quad kT < t < kT + \dfrac{T}{2} \quad ; \\[2ex]
V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha t}\cos(\omega_d t - \theta)\right] - 2\sum_{k=1}^{k}V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha[t-(k-1)T-\frac{T}{2}]}\cos\left(\omega_d[t-(k-1)T-\dfrac{T}{2}]-\theta\right)\right] \\
\quad + 2\sum_{k=1}^{k}V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha(t-kT)}\cos(\omega_d(t-kT)-\theta)\right] \\
\quad - 2V_s\left[1 - \dfrac{\omega_0}{\omega_d}e^{-\alpha(t-kT-\frac{T}{2})}\cos\left(\omega_d(t-kT-\dfrac{T}{2})-\theta\right)\right], \quad kT + \dfrac{T}{2} < t < (k+1)T
\end{cases}
$$

(23)

For the steady state after a number of cycles, (23) can be simplified as

$$
v_c(t) \approx \begin{cases}
V_s - 2V_s\dfrac{\omega_0}{\omega_d}e^{-\alpha(t-kT)}\left[e^{-\alpha\frac{T}{2}}\cos\left(\omega_d(t-kT+\dfrac{T}{2})-\theta\right)+\cos(\omega_d(t-kT)-\theta)\right] \\
\quad , \quad kT < t < kT + \dfrac{T}{2} \quad ; \\[2ex]
-V_s + 2V_s\dfrac{\omega_0}{\omega_d}e^{-\alpha(t-kT)}\left[\begin{array}{l} e^{-\alpha\frac{T}{2}}\cos\left(\omega_d(t-kT+\dfrac{T}{2})-\theta\right)-\cos(\omega_d(t-kT)-\theta) \\ + e^{\alpha\frac{T}{2}}\cos\left(\omega_d(t-kT-\dfrac{T}{2})-\theta\right) \end{array}\right] \\
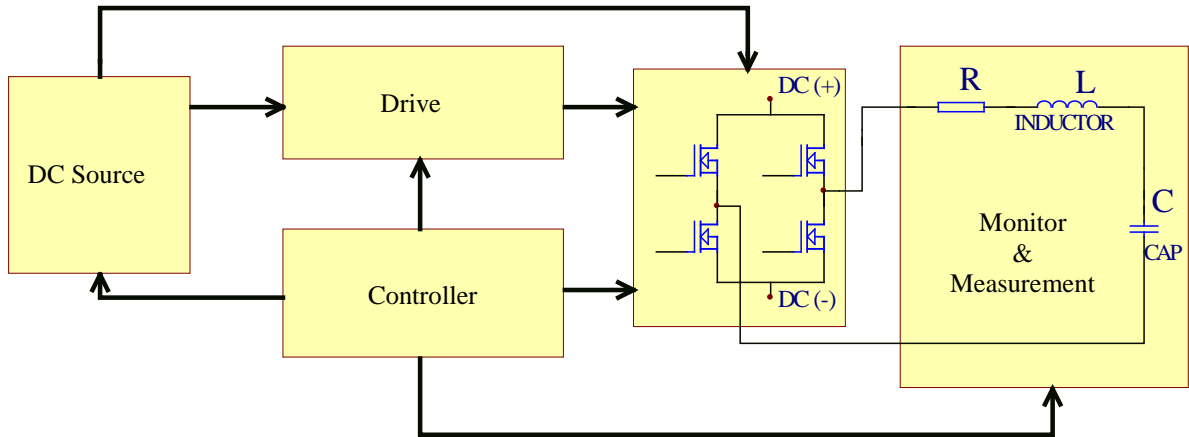\quad , \quad kT + \dfrac{T}{2} < t < (k+1)T
\end{cases}
$$

(24)

Figure 1. Circuit frame configuration of a practical experimental device.

III. DISCUSSION

Practical solution and device can be made as the configuration shown in Fig. 1. With a HTS, the resonant circuit can be a very high Q circuit, special aspects such as a very high voltage can be built up or a high current can be controlled. With regard to the voltage built up, practically any circuit resistance causes energy dissipation, and for each reversal of the DC source polarity for example, the corresponding increase in voltage is less than $2V_B$, and the magnitude of the voltage increase gets smaller with each iteration. Any resistance in the circuit will limit the final achievable voltage. To reduce the circuit resistance to near zero leads to an infinite circuit quality factor Q at resonance. In a series resonant circuit with a power supply $V_p = V_m\sin\omega t$, the rms current I_{max} in the circuit at resonance is

$$I_{max} = V_m/(R\sqrt{2}) \qquad (25)$$

and the voltage across the capacitor V_{Cmax} (rms) is

$$V_{Cmax} = X_C I_{max} = (\omega_o C)^{-1} [V_m/(R\sqrt{2})] \qquad (26)$$

By assuming that the circuit resistance R is zero, the circuit then has infinite Q at resonant frequency $\omega_o = 1/\sqrt{(LC)}$ rad s^{-1}. This leads to an infinite value of V_C generated and a very large potential current. This device therefore is able to provide controls of both high voltages and high currents.

HTS technology is critical for this method to be practical. HTS wires can be used to achieve the very high Q inductor to make this method viable; on the other hand the conventional inductor technique can not make this method applicable. The high Q inductor is able to be realized by using HTS Ag-clad $(Bi,Pb)_2Sr_2Ca_2Cu_3O_{10+x}$ wires, which have potential capability to make the inductor winding [3-5]. The HTS inductor virtually has no resistance for a DC current operation; however loss will be generated even at a relative low value and low frequency AC application, which however does not affect this application significantly [6].

IV. CONCLUSION

A high Q resonant circuit with a power electronic controller has been verified with theoretical and also practical operation analysis, which forms a base to develop this high Q resonant method with HTS technology. The application of this method generates special aspects, such as high voltage built up and high current control, which can be readily achieved by using the newly developed HTS technology.

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