Design Optimization of an Interior-type Permanent Magnet BLDC Motor using PSO and Improved MEC

Zhe Ren[1], Xu Fang[1], Shuhong Wang[1], Jie Qiu[1], Jian Guo Zhu[2], *Senior Member, IEEE*, Youguang Guo[2], *Senior Member, IEEE*, Xiaofeng Yang[3], J.H. Ha[3], Zhenyu Wang[3], Yingke Sun[3], and Jiunan Zhang[3]
[1]Department of Electrical Engineering, Xi'an Jiaotong University, Xi'an, 710049, China
[2]Faculty of Engineering, University of Technology, Sydney, NSW 2007, Australia
[3]LG Electronics (Tianjin) Appliance Co. Ltd., Tianjin, China

***Abstract*— In this paper, an improved magnetic equivalent circuit (MEC) is applied to calculate the nonlinear magnetic field in an interior-type permanent-magnet (IPM) brushless DC (BLDC) motor. Compared with the finite element method, the MEC method is much more time efficient, whereas compared with the conventional MEC method, the improved MEC is more accurate since it takes the complicate topological structure of the motor into account. A rough design of the IPM BLDC motor was firstly conducted by the improved MEC method. The particle swarm optimization (PSO) algorithm is then employed to refine the design for optimal structural parameters that result in the lowest cost and highest performance.**

***Index Terms*—Interior-type permanent-magnet brushless DC motor (BLDC), magnetic equivalent circuit (MEC) method, particle swarm optimization (PSO) algorithm.**

I. INTRODUCTION

The interior-type permanent-magnet (IPM) brushless DC (BLDC) motors can achieve high efficiency in a wide speed range, and hence are very suitable for household appliances requiring frequent start/stop and speed adjustment, such as air conditioners, refrigerators, and washing machines. In an IPM, the permanent magnets are inserted into the pre-punched slots and need not be bound like those in surface mounted PM motors [1], and the configurations of PMs in the rotor of an IPM motor are multiform.

An improved magnetic equivalent circuit (MEC), which combines the speed of the conventional MEC method and the flexibility of finite element method (FEM), is sought in this paper. The MEC is a very widely used technique for modeling electromagnetic devices, by which useful information such as torque, flux, magnetic motive force (MMF), electromotive force (EMF), and current can be estimated. The improved MEC method is different from the FEM in two aspects. Firstly, the number of elements deployed for the MEC method is much less than that required by the FEM. This reduces accuracy, but allows rapid iterative computations. Better accuracy may be achieved by concentrating the elements in critical and saturated parts of the machine. Secondly, in the MEC, the flux can pass through an element only in the specified direction, whereas in the FEM there is no restriction on the direction of flux through any element. The direction of flux in each element of the MEC model must be decided before the method is applied, which requires the user to have a good knowledge of the possible field distribution. The better knowledge the user has, the more accurate the MEC model can be. This is in contrast to the finite element analysis, where this is a result [2], and the user's knowledge contributes very little to the analysis. The MEC method has already been used for a great number of motors with different types, such as the switched reluctance motor [3,4], the permanent magnet linear synchronous motor [5], the permanent magnet hysteresis synchronous motor [6], and the brushless surface mounted permanent magnet motor [7].

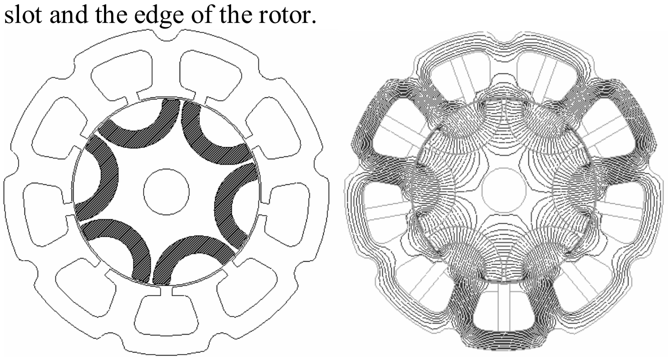
In the last decade, taking advantage of the increased availability of powerful computing platforms, optimization techniques are more and more used in electrical motor design, stimulated by the pressing demands of the highly competitive motor market and applications. The task is to achieve a design with an optimized objective function for certain desired features, e.g. minimum material cost, minimum weight, highest efficiency, maximum torque-to-current ratio, or a combination of them. The optimization procedure has to consider different, often conflicting, design objectives at the same time. In order to obtain a true optimum design, an optimization technique has to be used together with a reliable and accurate model of the electrical motor for predicting the motor performance. Because of this, numerical models based on the FEM are often chosen for the most exigent cases since it is more rigorous than any sophisticated nonlinear analytical models. However, the cost of the FEM in terms of computation time is significantly higher than that of the MEC method, and this may obstruct dramatically the optimization process.

Diverse optimization algorithms have been developed from a mathematical point of view. They are based on classical techniques such as the direct search proposed by Hooke and Jeeves, the simplex method, Rosenbrock algorithm; or based on stochastic (probabilistic) techniques such as the genetic and evolutionistic algorithms, and the simulated annealing technique. The first group of algorithms is generally faster but less safe in determining the optimum of the objective function; the second group is slower but usually securer [8].

This paper presents an evolutionistic algorithm known as the particle swarm optimization (PSO) algorithm to optimize the size parameters of an interior-type permanent-magnet brushless DC motor. For magnetic field analysis, the improved MEC model is employed. The characteristics predicted by the improved MEC method with optimized parameters are promising.

II. The IPM Brushless Motor

Fig. 1 (a) shows the cross section of the IPM BLDC motor to be optimized, and Fig. 1(b) shows the flux distribution obtained by using the FEM. When the MEC method is used for field analysis, due to the structural symmetry, only one-third of the motor is modeled. Because the fluxes of two adjacent teeth and yokes are not quite the same, the flux is not constrained to flow in one direction, and hence the conventional MEC method cannot reflect the flux distribution correctly. Moreover, there is some flux leakage between the slot and the edge of the rotor.



(a) Configuration (b) Flux distribution

Fig. 1. IPM brushless DC motor and flux distribution

III. Model of Improved MEC Method

The MEC method uses a lumped parameter network to represent a distributed magnetic circuit. The source of the network is known as the MMF and the resistive components are known as the reluctances, whose values depend on the geometry, and for ferromagnetic materials, the flux density in the region as well. The MEC model, combined with electric equivalent circuits, can give insight into the phenomena in a real, saturated machine. Effects due to spatial values like the number and shape of stator and rotor slots, saturation, type and connection of the windings, may be included [9, 10].

The MEC modeling is selected for further investigation as it seems a good technique providing high speed and acceptable accuracy compared to the FEM and the empirical methods. The conventional MEC method, which uses flux tubes to constrain magnetic flux to flow in one direction, is not accurate for predicting the field in the IPM BLDC motor. This paper presents an improved MEC model for predicting the PM motor performance with higher accuracy while maintaining the computing speed. The results are compared with those obtained from the measurements on the motor.

A. The Improved MEC Model

The model consists of flux tubes, each described by a reluctance value and optional MMF or flux sources. The distribution of these elements is crucial, as reduction of the number of elements increases the simulation speed.

Fig. 2 shows the proposed MEC model for the IPM BLDC motor. The stator yokes are modeled by unidirectional elements. These elements allow only radial or tangential flux to flow, neglecting the leakage flux in slots and outside the periphery of the motor. As the predetermination of the flux direction in other parts is impossible, bidirectional elements may be employed, allowing the flux to flow both radially and tangentially.

The MEC model follows the general principle for mesh generation in the FEM. A dense mesh is used at the air-gap to enhance accuracy. The tooth and the pole-shoe are equaled with dense meshes for non-homogeneous distribution of flux. Elsewhere, the mesh is less critical, and is generally much coarser. Flux is constrained to flow in iron, air-gap, slot opening sections, magnet and magnet retainer.

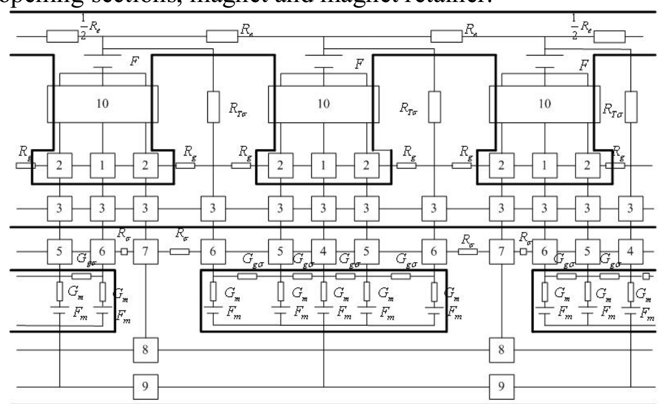


Fig. 2. MEC model for IPM brushless DC motor

In the MEC model illustrated in Fig. 2, the equivalent reluctance includes linear reluctance, and parameter nonlinear reluctance. R_e stands for a reluctance of yoke, $R_{T\sigma}$ a leakage reluctance of slot, R_g a leakage reluctance between the pole-shoes, R_σ a leakage reluctance between the adjacent magnets, G_m an inner magnetic conductance of one magnet element, $G_{g\sigma}$ a leakage conductance in magnet elements, F_m an MMF of magnet element, and F an MMF of armature tooth. Sub-networks 1, 2, 4, 5, 6, 7 and 8 are all made of bidirectional elements. Sub-network 3 uses dense mesh and bidirectional elements. The nodes connecting the air-gap to the rotor will change accordingly when the rotor rotates.

B. Equations

The reluctance can be represented by

$$R = \frac{l}{\mu S} \tag{1}$$

where l and S are the axial length and cross-sectional area, and μ is the permeability. For a nonlinear reluctance of a ferromagnetic material, μ is not a constant, and should be determined by the B-H characteristic. The MMF generated by the armature winding is

$$F = NI_a \tag{2}$$

where N and I_a are the number of turns in one slot and the current flowing in one conducting wire, respectively.

For the magnet, F_m is the MMF of the magnet and can be expressed as

$$F_m = H_c \cdot h_m \tag{3}$$

where H_c and h_m are the coercive force and the thickness of the magnet, respectively. G_m is obtained by

$$G_m = \frac{1}{R_m} = \mu \cdot l \cdot \frac{W_2 - W_1}{h} \cdot \frac{1}{\ln \frac{W_2}{W_1}} \tag{4}$$

where W_1 and W_2 are shown in Fig. 3. The leakage conductance $G_{g\sigma}$ is deduced as follows [11].

$$G_{g\sigma} = \frac{\mu}{W_1 + W_2} \cdot \left[\frac{h}{2} + \frac{W_1}{W_2 - W_1} \cdot h \cdot \left(\frac{W_1}{W_2 - W_1}\right)^2 \cdot \ln \frac{W_2}{W_1} \right] \tag{5}$$

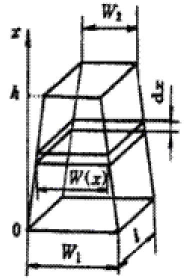


Fig. 3. Zone of solution for magnet

The flux density distribution of IPM motor may be obtained from the improved MEC, and then the EMF and torque could be acquired. Besides the loss in windings, hysteresis and eddy current loss are applied by using loss data curves. Subsequently, the unknown hysteresis constant is determined. Rotational losses are estimated using an empirical approach, suggested by Bergcluist (check the name, not the same as that in ref) [12].

C. Model Verification

The prototype motor is experimented, and the comparison of characteristics between the experiment and simulation by using the MEC model is listed in Table I.

TABLE I
COMPARISON BETWEEN EXPERIMENT AND SIMULATION

Motor	Experiment	Simulation
Speed (rpm)	3600	3600
Terminal voltage (V)	158	158
Back EMF (V)	144	145.86
Current (A)	5.2	5.03
Torque (Nm)	2	1.95
Output power (W)	736.56	697.23
Efficiency (%)	89.65	87.73

The result of comparison indicates that the improved MEC model of IPM BLDC motor is correct, and can predict the motor characteristics accurately. This comparison gives us the confidence to apply the method for design purpose.

D. Comparison of MEC, FEA and IMEC

The conventional MEC model can only reflect one position of rotor. It's not convenient to determine the parameters for diverse IPM brushless DC motor. The FEA is more accuracy than MEC. However, it sacrifices more time, and is not suitable for optimization.

The IMEC model needs one network and determines parameters one time. Computing time with one condition is about a few seconds by IMEC method, and a few minutes by FEA. The average relative error of flux density by IMEC is about 10%. Less computing time and acceptable accuracy result, which makes it have large dominance in optimization.

A motor of larger power rating with the same configuration is firstly designed by using the proposed MEC method, and then optimized by using the PSO algorithm.

IV. MODEL FOR OPTIMIZATION

A. Objective Function

For motor optimization, it is very important to choose a suitable number of optimization variables. Selection of more optimization variables can give more freedom, but it would be difficult to balance the relationship among these variables and it would take more computing time. Fewer variables can result in fast computation, but the freedom is small. Meanwhile, the influence of different variables on the objective function is also important. Following the above principle, for the motor design optimization, the design variables are chosen and listed in Table II.

TABLE II
DESIGN VARIABLES

Variables	Symbol
Inner diameter of stator	D_a
Axial length of stator	L
Stator slot height	H
Pole-shoe height of stator	B
Length of pole-shoe	t_0
Tooth width	b_t
Number of turns of winding	W
Winding conductor diameter	d
Average arc length of magnet	b_m
Thickness of magnet	h_m

The objective function for optimization of the IPM BLDC motor is derived to find the solution for the lowest total cost in manufacture as

$$\min f(\boldsymbol{x}) = P_1 M_1(W, d) + P_2 M_2(D_a, L, h, b, t_0, b_t) + P_3 M_3(b_m, h_m, L) \tag{6}$$

where P_1, P_2 and P_3 are prices for unit of copper, silicon sheet and magnet, M_1, M_2 and M_3 which are functions of design variables are the weight of winding, stator and rotor, and magnet, respectively.

B. Constraints

1) Constraints of axial length

The axial length of the IPM BLDC motor is required less than 80 mm so that it can be installed in the container and be fixed facilely. The constraint on the axial length can be expressed as

$$80 - L_{rotor} \geq 0 \tag{7}$$

where L_{rotor}, the rotor length, is 4 mm longer than L defined in Table II.

2) Constraints of winding

The winding and the insulation area in a slot must be smaller than the area of slot. The constraints of this can be expressed as

$$S_{slot} - (S_1 + S_2) \geq 0 \tag{8}$$

where the fill factor is 70%, and S_{slot} is the area of slot, S_1 and S_2 are the areas of winding and insulation in one slot. The thickness of insulation is 1 mm.

3) Constraints of efficiency

The motor efficiency η should be over 91%. Its constraint can be expressed as

$$\eta - 0.91 \geq 0 \tag{9}$$

V. THE PARTICLE SWARM OPTIMIZATION METHOD

The PSO method is a population based stochastic optimization technique developed in 1995 by Kennedy and Eberhart, inspired by the social behavior of birds flocking and fish schooling [13]. In the PSO, each potential solution, known as a 'particle', flies in the problem search hyperspace to look for the optimal position. As the time passes, a particle adjusts its position according to its own 'experience', and that of the neighboring particles.

Suppose that the search space has D-dimensions. The position of the *i*-th particle in the swarm can then be determined by its coordinates in the D-dimensional search space, and expressed as a vector $X_i(t)=(X_{i,1}(t),X_{i,2}(t),\ldots,X_{i,D}(t))$. The velocity (position change) of this particle can be represented by another vector $V_i(t)=(V_{i,1}(t),V_{i,2}(t),\ldots,V_{i,D}(t))$. The *i*-th particle also maintains a memory of its previous best position in vector $pbest_i=(pbest_{i,1},\ pbest_{i,2},\ \ldots,\ pbest_{i,D})$. In each iteration step, *gbest* is designated as the index of the best particle in the swarm. Subsequently, the swarm is manipulated according to the following two equations [14]:

$$V_{i,d}(t) = wV_{i,d}(t-1) + c_1 r_1 \times (pbest_{i,d} - X_{i,d}(t-1)) / \Delta\tau + c_2 r_2 \times (gbest_d - X_{i,d}(t-1)) / \Delta\tau \tag{10}$$

and

$$X_{i,d}(t) = X_{i,d}(t-1) + V_{i,d}(t) \times \Delta\tau \tag{11}$$

where $d=1,2,\ldots,D$, and $i=1,2,\ldots,N$, N is the size of the swarm, c_1 and c_2 are two positive constants, namely social and cognitive parameters, r_1 and r_2 two random numbers distributed within the range [0,1], t is the iteration number, Δt=1, and w is inertia weight.

Using (10), the particle updates its velocity according to its previous velocity and the distances to its current position from both its own best historical position and the best positions of the neighbors in every iteration step, and then it flies towards a new position given by (11).

VI. STRUCTURAL PARAMETER OPTIMIZATION USING PSO

The improved MEC model is used for the initial motor design, and the PSO method is used to optimize the motor design. Because of the use of time efficient MEC model, the computing time is very short.

Table III tabulates the predicted motor characteristics before and after the optimization. As shown, the optimized motor characteristics are significantly better than the initial design and the resultant motor can well meet the requirement. The cost is smaller than that of the initial design.

VII. CONCLUSION

In this paper, an improved EMC model is applied to predict the characteristics of IPM BLDC motor and conduct the initial rough design. The MEC model is proved to be accurate for the prototype motor proved by the experiment. A new heuristic algorithm known as PSO is introduced to handle the structural parameter optimization of the IPM BLDC motor for low cost and high performances. The optimized results demonstrate that the cost is reduced without sacrificing the performances, and it has short computation time. It is proved that the PSO algorithm is an attractive alternative for solving the structural parameter optimization problems of IPM BLDC motor.

TABLE III
OPTIMIZED IPM BLDC MOTOR CHARACTERISTICS

Motor	Before	After
Speed (rpm)	3600	3600
Terminal voltage (V)	158	158
Back EMF (V)	129.43	146.64
Current (A)	15.03	7.73
Torque (Nm)	0.83	2.99
Efficiency	80.6%	92.2%
Cost (￥)	206	171

VIII. REFERENCES

[1] J. F. Gieras and M. Wing, *Permanent Magnet Motor Technology: Design and Applications*, 2nd ed. New York: Marcel Dekker, 2002.

[2] C. B. Rasmussen and E. Ritchie, "A magnetic equivalent circuit approach for predicting PM motor performance," in *Proc. IEEE Industry Applications Society Annual Meeting*, Oct. 5-8, 1997, pp. 10-17.

[3] M. Modem, H. Nikkhajoei, and M. Falahi, "Predicting the performance of a switched reluctance machine using improved magnetic equivallent circuit method," in *Proc. Int. Conf. Power Electronics and Drive Systems*, Feb. 1995, pp. 198-201.

[4] M. Moallem and G. E. Dawson, "An improved magnetic equivalent circuit method for predicting the characteristics of highly saturated electromagnetic devices," *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 198-201, Sept. 1998.

[5] J. Hur, S. B. Yoon, D. Y. Hwang, and D. S. Hyun, "Analysis of PMLSM using three dimensional equivalent magnetic circuit network method," *IEEE Trans. Magn.*, vol. 33, no. 5, pp. 4143-4145,Sept. 1997.

[6] R. Qin and M. A. Rahman, "Magnetic equivalent circuit of PM hysteresis synchronous motor," *IEEE Trans. Magn.*, vol. 39, no. 5, pp. 2998-3000, Sept. 2003.

[7] K. F. Rasmussen, J. H. Davies, T. J. E. Miller, M. I. McGilp, and M. Olaru, "Analytical and numerical computation of air-gap magnetic fields in brushless motors with surface permanent magnets," *IEEE Trans. Industry Applications*, vol. 36, no. 6, pp. 1547-1554, Nov./Dec. 2000.

[8] N. Bianchi and S. Bolognani, "Design optimisation of electrical motors by an adaptive model based optimisation technique," in *Proc. 9th Int. Conf. on Electrical Machines and Drives*, Sept. 1999, pp. 291-296.

[9] O. Vlado, *Dynamics of Saturated Electrical Machines*, Berlin, Springer Verlag, 1987, ISBN 3-540-97079-7.

[10] O. Vlado, *Computer Aided Analysis of Electrical Machines*, Prentice Hall, 1994, ISBN 0-1 3-068859-2.

[11] Weixiong Zhang, Chanjun Zheng, and Jinhua Zhang, "Air-gap field analysis of PM motor based on magnetic equivlent circuit," *Small & Medium Electric Machines,* vol. 27, no. 4, pp. 12-18, 2000.

[12] A. Bergquist, *On Magnetic Hysteresis Modelling*, PhDThesis, Kungliga Tekniska Hogskolan, Stockholm, Sweden, 1994.

[13] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," in *Proc. IEEE Int. Conf. on Evolutionary Computation*, USA, 1998, pp. 69-73.

[14] R. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proc. 6th Int. Symp. on Micro Machine and Human Science*, Japan, 1995, pp. 39-43.