In memory of my father Daqun Chang

A Consistent Approach to Modelling the Interest Rate Market Anomalies Post the Global Financial Crisis

A Thesis Submitted for the Degree of
Doctor of Philosophy

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Certificate

I certify that this thesis has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signed ....

Date ....

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Glossary of Abbreviations

ATM = At the money;

ATMVOL = At the money volatility;

BF = Butterfly;

BP = Basis Pay;

bps = Basis Point (1bp = 0.0001);

BR = Basis Receive;

BS = Liquidity Basis;

CCS = Cross–Currency Swap;

CCBS = Cross–Currency Basis Swap;

CDS = Credit Default Swap;

CIR = Cox–Ingersoll–Ross;

FF = Fed Funds;

FOR–DOM = Foreign–Domestic;

FRA = Forward Rate Agreement;

FX = Foreign Exchange;

GFC = Global Financial Crisis;

HJM = Heath–Jarrow–Morton;

IRDIFF = Interest Rate Difference;

IRS = Interest Rate Swap;

ITM = In the money;

LIBID = London Interbank Bid Rate;

LIBOR = London Interbank Offered Rate;

LMM = LIBOR Market Model;

MM = Market Maker;

OIS = Overnight Indexed Swap;

OTM = Out of money;

PCA = Principal Component Analysis;

PV = Present Value;

RR = Risk Reversal;

TS = Tenor Swap;

UIP = Uncovered Interest Rate Parity.

Abstract

The thesis is focused on the phenomenon of the cross–currency swap and tenor swap basis spread in foreign exchange (FX) and interest rate markets, which contradicts textbook no arbitrage conditions and has become an important feature of these markets since the beginning of the Global Financial Crisis (GFC) in 2007.

The results demonstrate empirically that the basis spread can not be explained by transaction costs alone and is therefore due to a new perception by the market of risks involved in the execution of textbook "arbitrage" strategies. We show that using the basis spread as a proxy for the market valuation of these risks, a better empirical explanation than hitherto found in the literature can be obtained for the "uncovered interest rate parity (UIP) puzzle," i.e. the phenomenon that carry trades taking advantage of interest rate differentials between different currencies have positive excess returns on average. Furthermore, considering the single–currency basis spread (the "tenor basis"), the empirical analysis of market data since the GFC has led us to a model which reduces the dimensionality of the tenor basis from observed term structures for every tenor pair down to term structures of two factors characterising the driving liquidity risk, and demonstrates that the tenor basis swap market is in the process of maturing since the turmoil of the

GFC.

There are three main contributions in this thesis. In Chapter 3 we examine the role of transaction costs in explaining the basis spread in cross–currency basis swaps. Based upon transaction costs, we derive bounds which should eliminate arbitrage in practice. The empirical results are consistent with the conventional market wisdom that to a large extent, transaction costs alone precluded arbitrage opportunities before the GFC. However, the no–arbitrage bounds have been persistently violated since the GFC. We propose that the market is prevented from exploiting such violations and making arbitrage profit by increased market imperfections, in particular the currency liquidity risk. These imperfections have resulted in forward and currency swap prices being determined by supply and demand pressures, rather than by arbitrage considerations.

In Chapter 4 we aim to explain the UIP puzzle by a model with liquidity risk. We empirically examine the effect of FX market liquidity risk on the excess returns of currency carry trades. Based upon Chapter 3 results, we use the violations of no–arbitrage bounds as the proxy for the market expectation of liquidity risk. The liquidity proxy, along with FX market volatility factors, is significant in explaining the abnormal returns of carry trades, particularly after the GFC. Our liquidity proxy is also statistically more significant than alternative proxies for liquidity risk in related studies. Our findings provide evidence that the UIP puzzle can potentially be resolved after controlling for liquidity risk.

In Chapter 5 and 6, we focus on the high–dimensional modelling problem existing in

the single–currency tenor swap market. Based on empirical results of recent studies, we propose an intensity–based model to describe the arrival time of liquidity shocks in the interbank market. With the no–arbitrage argument and non–linear constrained optimisations, we calibrate the model parameters to quoted basis spreads in tenor swaps. Our model reduces the dimensionality of the problem down to two factors: the intensity and the loss rate characterising the driving liquidity risk. In contrast to the credit risk literature, the intensities and loss rates are calibrated simultaneously and results show that loss rates display more variations than intensities. Another advantage of our modelling approach, compared to the ad–hoc modelling approach adopted by practitioners, is that our model is motivated by the driving risk of market anomalies. It is hence more explanatory and consistent with market fundamentals. In order to account for potential randomness, we also set up stochastic models for the intensity and the loss rate. We show that under certain conditions closed form solutions exist, which can be used to tractably calibrate or estimate the model parameters.

Chapter 1

Introduction

1.1 Background and Motivation

The Global Financial Crisis (GFC), which started from August 2007 and reached its peak around the collapse of Lehman Brothers in September 2008, has caused major changes in the interest rates quoted in the market. Before the GFC, the market quotes had been generally consistent with the textbook no–arbitrage principles, which require that two floating rates should trade flat in a swap contract because floating–rate bonds are always worth the par value at initiation, regardless of the tenor length of the underlying rate (see, for example, Hull 2008). Therefore, theoretically the spread in floating–for–floating swaps should be zero to avoid arbitrage profit. Before the crisis, a non-zero but negligible spread (see, for example, Mercurio 2010) was usually added to the shorter tenor rate in a single–currency interest rate swap (IRS) or to the floating rate of one currency in a cross–currency swap (CCS). After controlling for transaction costs, such as the bid–ask spread, such a small spread generally did not constitute arbitrage profit.

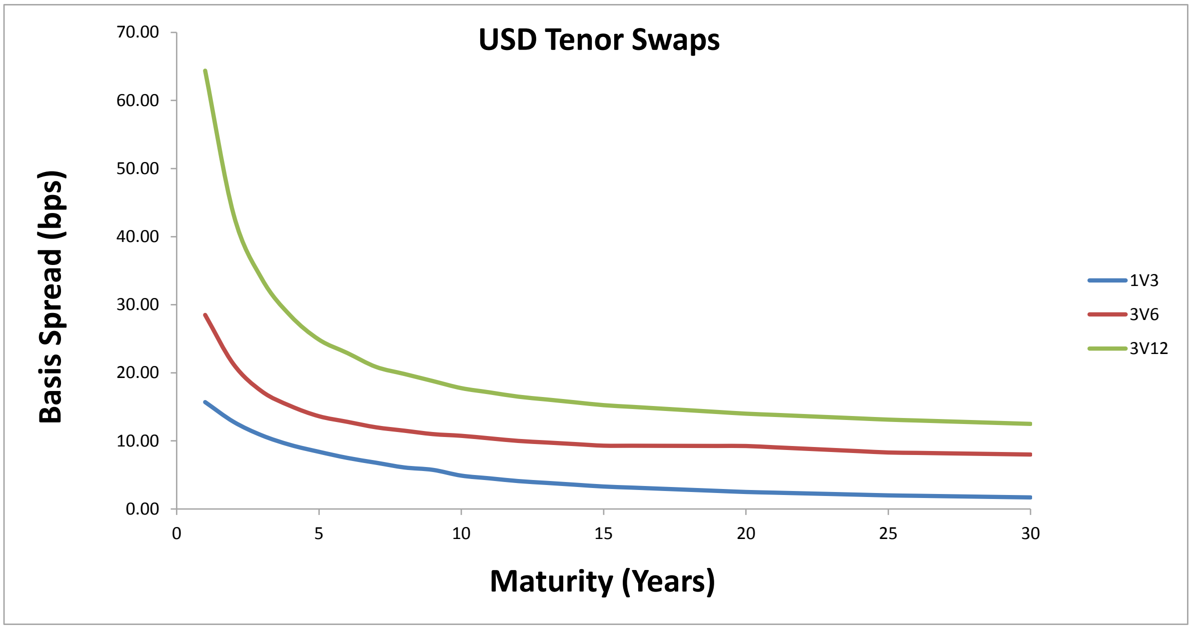


Figure 1.1: **USD tenor swap basis spread curves on 16/02/2009. (Data source: Bloomberg)**

Since the GFC, IRS and CCS have been quoted with substantially higher basis spreads than before (see Figures 1.1 and 1.2).

In the IRS market, a tenor swap (TS) exchanges payments of the same currency based on a notional amount and tenor indexes of two underlying floating rates. For example, party A pays 3–month (3M) USD LIBOR[1] quarterly to party B and in exchange receives 6–month (6M) USD LIBOR semi–annually. Only interest payments are exchanged and no notional is exchanged. The interest payment is settled on a net basis. A TS can be used

[1]London Interbank Offered Rate is a daily reference rate published by the British Banker Association (BBA) based on the interest rates at which panel banks borrow unsecured funds from each other in the London interbank market.

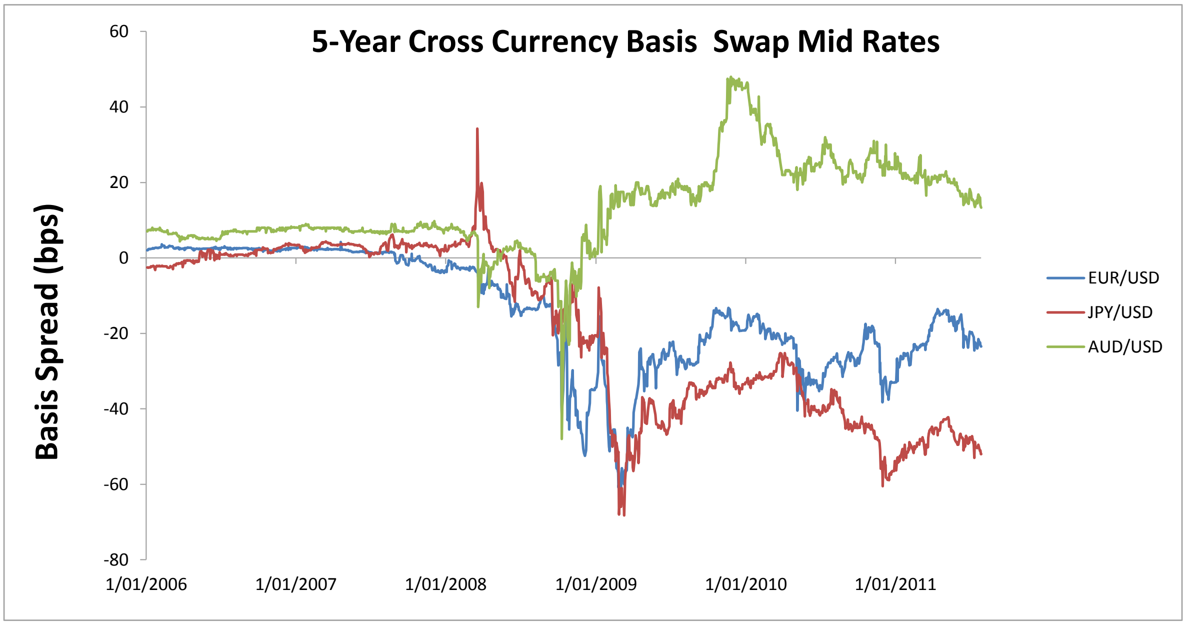


Figure 1.2: **Cross–currency swap basis (Data source: Bloomberg)**

to hedge basis risk, due to the widening or narrowing spread between the two indexes. Since the crisis, tenor swaps have displayed a persistent and unambiguous pattern. In general, the shorter tenor rate is quoted with a large and positive spread in exchange for the longer tenor rate. The magnitude of the spread tends to increase as the tenor difference increases. Figure 1.1 shows the USD TS spread curves as at 16th of February, 2009, corresponding to 1M, 3M, 6M and 12M USD LIBOR. Swap maturity ranges from 1 year to 30 years. We see at the 1–year maturity end, the spread increased from 16 basis points (bps) for the 1M vs. 3M TS to 65 bps for the 3M vs. 12M TS.

In the CCS market, a basis swap (CCBS) exchanges floating interest rates and principal payments of two different currencies. The principal amounts are exchanged both at the inception and at the end of swap period, based upon the spot exchange rate prevailing at the initiation date. The interest payment is exchanged on a net basis at each tenor of the swap term. In the CCBS market similar anomalies developed since the GFC. According to data obtained from Bloomberg, from the late 1990s until August 2007, the spread of one–year EUR/USD swaps had ranged from 0 to 2.5 bps. This swap exchanges 3M EURIBOR[2] plus the spread for 3M USD LIBOR, as well as the notional amounts. However, since the end of August 2007, the spread added to 3M EURIBOR turned significantly negative. From the collapse of Lehman Brothers in September 2008 to February 2010, the average basis was -34 bps. This means that the USD borrower pays 3M USD LIBOR flat but receives 3M EURIBOR minus 34 bps. The movements of the spread for the JPY/USD and AUD/USD basis swaps, however, demonstrated different dynamics from their European counterpart. The spreads of 5–year EUR/USD, JPY/USD and AUD/USD currency swaps from 2006 to 2011 are shown in Figure 1.2. From late 1997 to mid 1999, the spread of JPY/USD swaps dropped significantly to below -30 bps. The spread picked up from 2000 and traded between -8 to 5 bps before the crisis. During the crisis, the basis declined, once again, into substantially negative territory. The basis for AUD/USD currency swap had been historically in the positive territory (5 to 15 bps). It also turned negative, though to a lesser extent in 2008. However, from early 2009 the positive, sometimes large spread returned.

As noted in Bianchetti (2010), other market anomalies have also emerged, including large and positive spreads between LIBOR and OIS[3] rate of the same maturity. Forward Rate

[2]EURIBOR is the reference rate of unsecured borrowing of EUR between European prime banks within the euro zone.

[3]An overnight indexed swap is an interest rate swap where the floating leg of the swap is equal to the geometric average of the overnight cash rate over the swap period. Overnight lending involves little default or liquidity risk, hence the LIBOR–OIS spread is an important measure of risk factors in the

Agreement (FRA) rates[4] observed in the market also significantly diverge from the rates implied by the replication of two deposits at spot LIBOR of different maturities.

As discussed in Chapter 3 of this thesis, the aforementioned market changes would seem to present textbook arbitrage opportunities. However, since the crisis they have persisted, implying that such opportunities have not been fully exploited. Furthermore, these anomalies have caused implications for the pricing methodology for interest rate derivative products, such as the ad–hoc modelling approach of “one discount curve, multiple forward curves” adopted by practitioners. We are therefore motivated to study three issues arising from swap markets since the financial crisis. Firstly, by proposing no–arbitrage bounds in the CCBS market we investigate whether transaction costs, by themselves, are able to explain the unusually large movements of spreads. This is motivated by recent empirical studies (discussed in the literature review in Chapter 2), which propose that fundamental risks have caused such market anomalies. However, they do not explicitly control for the effect of transaction costs. Secondly, we examine whether fundamental risks are predictive for currency movements and helpful in resolving the uncovered interest rate parity (UIP) puzzle. Thirdly, we propose a consistent framework for modelling the tenor swap market, taking into account the observed anomalies.

interbank market. See, for example, Mercurio (2010).

[4] A FRA is a contract which is initiated at current time t and allows the holder to exchange, at maturity S, a fixed payment (based on the fixed rate K) for a floating payment based on the spot rate $L(T, S)$ resetting at T with maturity S, with $t \leq T \leq S$. The FRA rate is the value of K which renders the contract value 0 (i.e. fair) at t. E.g. Brigo and Mercurio (2006).

1.2 Thesis Contents

We begin in Chapter 2 with a review of the related literature. We present recent studies aiming to explain and/or model the observed large spreads in both single and cross–currency products. Broadly speaking, these studies take two distinct approaches. The ad–hoc modelling approach, mainly adopted by quantitative finance practitioners, extends the interest rate derivative pricing models. On the other hand, the fundamental approach identifies risk factors which may cause market anomalies. We also introduce literature related to the UIP puzzle to motivate a liquidity based econometric model and study the performance of currency carry trades.

Chapter 3 investigates the behaviour of the basis of cross–currency basis swaps by explicitly considering transaction costs and proposing no–arbitrage bounds. We assume a risk free market and derive bounds for the forward exchange rates and CCBS basis rates, which should eliminate arbitrage opportunities in practice. The bounds are examined with market data and results demonstrate that the no–arbitrage bounds held strongly before the GFC. Since the second half of 2007 the bounds have been persistently violated. We propose that currency liquidity risk explains these violations and has limited the effectiveness of arbitrage transactions which exploit these violations.

In Chapter 4 we empirically examine the effect of foreign exchange (FX) market volatility and liquidity risk on the excess returns of currency carry trades. We use the violations of no–arbitrage bounds in Chapter 3 as the proxy of liquidity risk, and use volatility smile data to capture FX market specific volatility. The sample data cover periods both before and after the GFC. Both proxies are significant in explaining the abnormal returns of carry trades, particularly after the GFC. Our findings provide evidence that the UIP puzzle can potentially be resolved after controlling for liquidity risk and market volatility.

Chapter 5 and 6 cover the third component of the thesis. We focus on the tenor swap market and propose an intensity–based model to describe the arrival time of liquidity shocks. With the no–arbitrage argument and non–linear constrained optimisations we calibrate the model parameters to quoted tenor swap basis spreads. In contrast to the credit risk literature, the intensities and loss rates are calibrated simultaneously and results show that loss rates display more variations than intensities. To allow also for potential stochasticity of model parameters, we set up Cox–Ingersoll–Ross (CIR) type (Cox et al. 1985) models for the intensity and the loss rate.

Chapter 7 concludes the thesis, discusses our main contributions and proposes future research avenues based upon the thesis results.

Chapter 2

Literature Review

2.1 Introduction

In this chapter we review studies which aim to explain and/or model the observed large spreads in both single–currency and cross–currency swaps. We separate these studies into two broad categories: the ad–hoc modelling approach and the fundamental approach, depending upon whether fundamental factors which drive market anomalies are explicitly examined. Since, in subsequent chapters, we will investigate the predictive power of liquidity risk in the foreign exchange (FX) market for exchange–rate movements, we also review literature related to the uncovered interest rate parity (UIP) puzzle.

2.2 Ad–hoc Approach

The first approach is mainly adopted by quantitative finance practitioners to extend the existing interest rate derivative pricing models, such as the LIBOR market model (e.g.

Brace et al. 1997). The price of interest–rate derivative products depends on the present value of future cash flows linked to interest rates. For the pricing purpose, we need a forward curve to generate future cash flows and a yield curve to discount them.

2.2.1 No Arbitrage Relationship Before and After the Crisis

Before the crisis, the standard market practice was to build a single curve to both generate and discount cash flows. A set of the most liquid interest–rate instruments based upon underlying rates of different tenors (e.g., deposits on 1M LIBOR, FRA or interest futures on 3M LIBOR and IRSs on 6M LIBOR) are selected to construct the yield curve. Discount factors off the yield curve are used to calculate the forward rates (e.g. Brigo and Mercurio 2006),

$$F(t;T_1,T_2) = \frac{1}{\tau(T_1,T_2)}\left(\frac{P(t,T_1)}{P(t,T_2)} - 1\right), \qquad t \le T_1 \le T_2, \tag{2.1}$$

where $F(t;T_1,T_2)$ is the simple compounded forward rate contracted at t and applicable between the year fraction of the time interval $\tau(T_1,T_2)$. $P(t,T)$, also known as the discount factor, is the price at time t of a zero–coupon bond maturing at T with face value of unity. The pre–crisis single curve approach ensures the no-arbitrage relationship

$$P(t,T_2) = P(t,T_1)P(t;T_1,T_2), \qquad t \le T_1 \le T_2, \tag{2.2}$$

where $P(t;T_1,T_2)$ is the forward discount factor defined by $F(t,T_1,T_2)$ and $\tau(T_1,T_2)$ via,

$$P(t;T_1,T_2) = \frac{1}{1+F(t;T_1,T_2)\tau(T_1,T_2)}. \tag{2.3}$$

Table 2.1: Arbitrage Strategy

Strategy	t	T_1	T_2
Buy 1 bond maturing T_2	-$P(t,T_2)$		1
Short $\frac{P(t,T_2)}{P(t,T_1)}$ bonds maturing T_1	$P(t,T_2)$	- $\frac{P(t,T_2)}{P(t,T_1)}$	
Borrow $\frac{P(t,T_2)}{P(t,T_1)}$ cash at $F(t,T_1,T_2)$		$\frac{P(t,T_2)}{P(t,T_1)}$	- $\frac{P(t,T_2)}{P(t,T_1)}(1+F(t;T_1,T_2)\tau(T_1,T_2))$
Net Cash Flow	0	0	0

We employ an arbitrage strategy in Table 2.1 to prove Eqn. (2.2). Because the net cash–flow is zero both at t and T_1, to eliminate arbitrage opportunity we have to ensure

$$\frac{P(t,T_2)}{P(t,T_1)}(1+F(t;T_1,T_2)\tau(T_1,T_2)) = 1. \tag{2.4}$$

Eqn. (2.2) then is proved by putting together Eqns. (2.3) and (2.4). Eqn. (2.1) can also be proved from Eqn. (2.4). Eqn. (2.2) basically states that for a cash flow at T_2, its present value at t must be unique. We can either discount the cash flow by $P(t,T_2)$ in one step, or we can first discount from T_2 to T_1 by the forward discount factor $P(t;T_1,T_2)$, then discount from T_1 to t by the discount factor $P(t,T_1)$. From the way that the single yield curve is constructed before the crisis, we see that all discount factors and forward rates are calculated from a unique curve, hence the no–arbitrage relation is guaranteed.

Now if we consider a generic LIBOR $L(T_1,T_2)$ which is simply compounded between

T_1 and T_2. $L(T_1, T_2)$ and the forward rate $F(t; T_1, T_2)$ is related by,

$$\lim_{T_1 \to t} F(t; T_1, T_2) = L(T_1, T_2). \tag{2.5}$$

It then follows from Eqn. (2.1) that

$$L(T_1, T_2) = \frac{1}{\tau(T_1, T_2)} \left(\frac{1}{P(T_1, T_2)} - 1 \right). \tag{2.6}$$

From the interest rate derivative pricing perspective, forward rate $F(t; T_1, T_2)$ is the expectation of $L(T_1, T_2)$ at t under the T_2 forward measure,[1]

$$E^{T_2}[L(T_1, T_2) \mid \mathcal{F}_t] = F(t; T_1, T_2), \qquad t \leq T_1 \leq T_2. \tag{2.7}$$

Eqn. (2.7) is an important tool to price LIBOR–linked derivatives, such as interest caps, floors and swaptions. It provides a link between LIBORs and forward rates, hence we can express the expected LIBOR under the associated forward measure by discount factors via Eqn. (2.1). Again, the internal consistency of the single curve framework is crucial in no–arbitrage pricing of such derivatives.

A yield curve is supposed to produce interest rates as a smooth function of any arbitrary time to maturity, hence a continuous function. However, in real markets we only have a

[1] See the proof of Eqn. (2.7) in Appendix A.

set of instruments of discrete maturities quoted, including zero–coupon products such as deposits at LIBORs, and coupon–bearing products such as interest rate swaps. For the short–end of this discrete set of points on the yield curve, we compute the corresponding interest rates from the zero–coupon instruments. Given these yields, the longer–maturity zero–coupon yields can be recovered from the coupon–bond products by solving for them iteratively by forward substitution. This process is the so called bootstrap method in constructing yield curves[2]. This discrete set of yields is calculated to eliminate arbitrage opportunities. For time points that fall between any two maturities in the discrete set, some interpolation scheme has to be employed because no instrument is quoted in the market corresponding to that maturity. Many arbitrary and different interpolation algorithms are used in practice (see Hagan and West 2006). Therefore together with bootstrapping, interpolation completes, in a non–unique way, the construction of yield curves. As noted by Bianchetti (2010), such a yield curve is not strictly guaranteed to be free of arbitrage because discount factors through interpolation are not always consistent with those obtained by a stochastic interest rate model which belongs to the no–arbitrage framework developed by Heath et al. (1992). Researchers have extended arbitrage–free interpolation schemes from discrete to continuous settings (e.g. Schlögl 2002). In practice the transaction costs in general cancel such arbitrage opportunities. Therefore, this drawback of the single–currency–single–curve approach, as far as practitioners are concerned, was of second-order importance.

After the crisis, the single–curve approach described above is not valid. The reason is that the interest rate market is segmented and rates of different tenors display distinct dy-

[2]See Section 3 of Chapter 3 for a detailed description of the bootstrap method.

namics, reflected in the large spreads in tenor swaps, as well as in LIBOR vs. OIS spreads of a given currency. Such "segmentation" reflects varying levels of risk premia driving rates of different tenors. The pre–crisis single curve approach which mixes instruments of different tenors of underlying rates characterized by significantly different risk premia would result in inconsistencies across market segments. To consistently account for the market segmentation, as well as explain the reason that textbook arbitrage opportunities are not exploited, approaches based on explanatory factors are required. Recent studies generally attribute such market anomalies to default and liquidity risk, but acknowledge that a consistent framework incorporating these risks is not easy to construct (see, for example, Bianchetti 2010 and Mercurio 2010) .

Bypassing a consistent framework, practitioners have tackled this issue by constructing multiple forward curves based on the length of the tenor to forecast future cash flows (i.e. 1M, 3M, 6M, 12M forward curves). Each forward curve is built with vanilla instruments homogeneous in the underlying rate tenor. For example, the 1M USD forward curve is bootstrapped with instruments on 1M USD (spot and forward) LIBOR only. On the other hand, the curve for discounting has to be unique to preclude arbitrages. By the "Law of One Price", two identical future cash flows must have same present value. The unique discount curve is constructed with the pre–crisis approach, which mixes instruments on rates of different tenors.

The current practice of "one discount curve, multiple forward curves" contradicts the single curve approach which precludes arbitrage. Forward rates of a particular tenor are calculated from the corresponding forward curve, whereas the discount factors are from the discount curve. A natural consequence of this approach is that if we calculate the forward discount factor $P(t;T_1,T_2)$ from Eqn. (2.3), each curve would give us a different result. The present value of a particular cash flow is no longer unique. If we only use $P(t;T_1,T_2)$ off the discount curve, then the relationship defined by Eqn. (2.3) is immediately invalidated. Consequently, this created a clear need for a unified, consistent framework to reconcile inconsistencies and simplify the pricing methodologies of interest rate derivatives.

2.2.2 Mercurio (2010)

The classic LIBOR Market Model (LMM) models the joint evolution of a set of consecutive forward LIBORs. Mercurio (2010) points out that two complications arise when we move to a multi-curve setting. The first is the co–existence of several yield curves. The second is that forward LIBORs are no longer equal to the corresponding ones defined by the discount curve. Mercurio addresses the first issue by adding extra dimensions to the vector of modelled rates and suitably modelling their instantaneous covariance structure. For the second issue, Mercurio models the joint evolution of forward rates calculated from the OIS discount curve[3] and the spread between OIS forward rates and forward LIBORs. For a given tenor, forward OIS rates are defined as

$$F_k(t) = F_D(t;T_{k-1},T_k) = \frac{1}{\tau_k}\left(\frac{P_D(t,T_{k-1})}{P_D(t,T_k)} - 1\right), \qquad t \leq T_{k-1} \leq T_k, \tag{2.8}$$

[3]Because OIS swap rates are perceived as entailing little default or liquidity risk, since the crisis market participants increasingly construct OIS–based discount curve to discount collateralized contracts. Section 3 in Chapter 5 provides a detailed procedure of OIS curve construction.

where the subscript D refers to the discount curve built with the OIS rates, which are considered effective risk–free rates since the GFC. There are two reasons for directly modelling OIS forward rates. First, as in Kijima et al. (2009), which proposes a three yield–curve model (discount curve, LIBOR curve and government bond curve), the pricing measures in Mercurio (2010) (including the spot LIBOR measure Q_D^{τ} and the forward measure $Q_D^{T_k}$) are associated with the OIS discount curve. Secondly, forward swap rate depends on the OIS discount factors. The spread between forward LIBOR and forward OIS rate is defined as

$$S_k(t) = L_k(t) - F_k(t), \tag{2.9}$$

where $L_k(t)$ is the forward LIBOR for the given tenor. By construction, both $L_k(t)$ and $F_k(t)$ are martingales under the forward measure $Q_D^{T_k}$ with the zero–coupon bond $P_D^{T_k}$ as the numéraire. Therefore $S_k(t)$ is also a martingale under $Q_D^{T_k}$. $S_k(t)$ is modelled with a continuous and positive martingale which is independent of the OIS forward rate. The model is calibrated to market caplet smile and model volatilities fitted the market almost perfectly, though the sample size is small.

2.2.3 Bianchetti (2010)

Bianchetti (2010) incorporated the forward basis to recover the no–arbitrage relationship between forward curves and the discount curve. The no–arbitrage relationship between two curves is expressed as

$$F_f(t;T_1,T_2)\tau_f(T_1,T_2) = F_d(t;T_1,T_2)\tau_d(T_1,T_2)BA_{fd}(t;T_1,T_2), \tag{2.10}$$

where the subscripts f and d denote forward curves and the discount curve from which forward rates (or discount factors) are extracted and obviously $\tau_f(T_1,T_2) = \tau_d(T_1,T_2)$. The multiplicative forward basis $BA_{fd}(t;T_1,T_2)$ is the ratio between forward rates (or equivalently in terms of discount factors) from forward curves and from the discount curve

$$BA_{fd}(t;T_1,T_2) = \frac{F_f(t;T_1,T_2)\tau_f(T_1,T_2)}{F_d(t;T_1,T_2)\tau_d(T_1,T_2)} = \frac{P_d(t,T_2)}{P_f(t,T_2)}\frac{P_f(t,T_1)-P_f(t,T_2)}{P_d(t,T_1)-P_d(t,T_2)}. \tag{2.11}$$

Eqn. (2.11) can be easily derived from Eqn. (2.1). Hence the forward basis is a measure of the difference between the forward rates from the forward curve and forward rates from the discount curve.

Alternatively, the additive forward basis $BA'_{fd}(t;T_1,T_2)$ is defined as

$$BA'_{fd}(t;T_1,T_2) = F_d(t;T_1,T_2)[BA_{fd}(t;T_1,T_2) - 1]. \tag{2.12}$$

In the single curve setting, the basis should be zero because there is only one curve, hence we expect $BA_{fd}(t;T_1,T_2) = 1$ and $BA'_{fd}(t;T_1,T_2) = 0$. Bianchetti (2010) then constructed the forward basis curve through bootstrapping. The finding is that the short–term forward basis is wide ranging, with the multiplicative forward basis ranging from 0.7 (12M tenor forward curve versus the discount curve) to 1.3 (1M tenor forward curve versus the discount curve). However, the longer term (up to 30 years maturity) forward basis tends to 1 (resp. 0) for the multiplicative case (resp. additive case). It is important to note that the term structure of the forward basis curve as constructed by Bianchetti (2010) oscillates. The oscillations are demonstrated especially in the longer term forward basis curve. This suggests that there may be some over–fitting in the bootstrap curve construction.

The discount curve was built with the traditional "pre–crisis" approach. The instruments included liquid deposits, FRAs on 3M EURIBOR and swaps on 6M EURIBOR. On the other hand, forward curves were constructed from instruments with homogeneous underlying tenor. For instance, 3M forward curve was based upon instruments linked to 3M EURIBOR. Hence the discount curve mixed rates of different underlying tenors with distinct dynamics, whereas a forward curve corresponded to one particular underlying tenor. Bianchetti (2010) therefore attributed oscillations in the forward basis curve to the amplification of small local differences between the two curves. The author also suggested to use the forward basis term structure as a tool to assess the distinct risk dynamics in the interest rate market because it provides a sensitive indicator of the tiny, but observable statical differences between different interest rate market sub-areas in the post GFC world.

2.2.4 Henrard (2007, 2010)

As a sequel of Henrard (2007), Henrard (2010) proposed a framework to price interest rate derivatives based on different LIBOR tenors by introducing a deterministic, and maturity dependent, spread between the forward curve and the discount curve. In Henrard (2007) the spread was assumed to be constant across maturities. Hence this extension is a natural adaptation to the post-crisis market reality. Henrard (2010) assumed that the discount curve is given and proceeded to construct the forward curves based on the spreads. Simple vanilla instruments were selected to achieve this purpose, including FRA, futures and IRS. Henrard then proposed to extend this framework to cross currency products and the object to be modelled is the cross-currency basis, which had also become substantially higher since the crisis.

2.2.5 Fujii et al. (2009)

Fujii et al. (2009) proposes a Heath–Jarrow–Morton (HJM, see Heath et al. 1992) model framework to adapt to new developments in the interest rate markets: large spreads in LIBOR vs. OIS and widespread use of collateral. The underlying quantities in the model are the instantaneous forward OIS rate and the spread, which measures the difference between the forward LIBOR under the collateralized forward measure and the OIS forward rate. The model is set up as follows,

$$dc(t,s) = \sigma_c(t,s) \cdot \left(\int_t^s \sigma_c(t,u)\, du \right) dt + \sigma_c(t,s) \cdot dW^Q(t), \tag{2.13}$$

$$\frac{dB(t,T;\tau)}{B(t,T;\tau)} = \sigma_B(t,T;\tau) \cdot \left(\int_t^s \sigma_c(t,s)\, ds \right) dt + \sigma_B(t,T;\tau) \cdot dW^Q(t), \tag{2.14}$$

where $c(t,T)$ is the instantaneous forward collateral rate and in Eqn. (2.13) the standard arbitrage–free HJM dynamics applies under the risk–neutral measure Q. $B(t,T;\tau)$ is the spread and by construction a martingale under the collateralized forward measure τ^c. τ stands for a particular LIBOR tenor. The stochastic differential equation is written as

$$\frac{dB(t,T;\tau)}{B(t,T;\tau)} = \sigma_B(t,T;\tau) \cdot dW^{\tau^c}(t). \tag{2.15}$$

The Brownian motion $W^{\tau^c}(t)$ under the measure τ^c is related to $W^Q(t)$ by the the Girsanov theorem (Girsanov 1960),

$$dW^{\tau^c}(t) = \left(\int_t^s \sigma_c(t,s)\, ds \right) dt + dW^Q(t). \tag{2.16}$$

The details of the volatility processes $\sigma_c(t,s)$ and $\sigma_B(t,T;\tau)$ are not specified in Fujii et al. (2009). It is also clear from Eqn. (2.14) that $\sigma_B(t,T;\tau)$ needs to be specified for all relevant LIBOR tenors (i.e. 1M, 3M, 6M and 12M), hence this is a high–dimensional approach.

These papers endeavor to reconcile inconsistencies caused by the multi–curve framework used by practitioners. They appear promising in fitting model prices to market prices by incorporating the spreads of LIBORs of different tenors. The drawback of this approach is that it does not relate the spreads to more fundamental risks, which preclude the exploitation of textbook arbitrage opportunities seemingly created by the presence of these spreads. Furthermore, one quickly ends with a multitude of basis spread dynamics, which should be related at a fundamental level. However, these relationships are not addressed by this ad-hoc approach.

2.3 Fundamental Approach

Different from the ad-hoc approach, the fundamental approach aims to identify the risk factors causing market anomalies. Although market anomalies are commonly considered entailing default and liquidity risk premiums, empirical evidence shows that liquidity risk plays a more significant role.

2.3.1 Default Risk

Morini (2009) examined two particular instruments in interest rate markets: FRA and tenor swaps. Before the crisis, the market FRA rate was well approximated by the LIBOR–based replication. After the crisis, the LIBOR–based replication of the FRA rate has been persistently higher than the market quotes of FRA rates. Morini used two different discount curves, the LIBOR–based curve and the OIS curve to bridge the gap between the market FRA and the replicated FRA by incorporating the basis spreads of LIBORs of different tenors. Therefore, two issues are reduced to one: why has the basis attached to the leg of the shorter–tenor LIBOR been persistently large and positive? Morini explicitly assumed an unexplained axiom proposed by Tuckman and Porfirio (2003) that lending at longer–tenor LIBOR involves higher counterparty default risk and liquidity risk than rolling lending at shorter–tenor LIBORs. Morini then argued that it is difficult to separate default risk and liquidity risk because the two risks are highly correlated. Hence Morini used default risk only to approach the question. Morini conjectured that a LIBOR panel bank today may not be a LIBOR bank in the future, due to its worsening credit rating. For example, the roll–over lender at 6M LIBOR can reassess the credit quality of the borrowing bank and may choose to replace with a counterparty that remains to be a LIBOR bank. There is a cap to how much the credit standing of a current LIBOR bank can worsen before it is excluded from the LIBOR Panel. This conjecture motivated Morini to model the spread of a generic LIBOR L^{X^0} over the market OIS rate E_M between time α and 2α as

$$S^{X^0}(\alpha, 2\alpha) = L^{X^0}(\alpha, 2\alpha) - E_M(\alpha, 2\alpha), \tag{2.17}$$

where $S^{X^0}(\alpha, 2\alpha)$ is the spread, X^0 denotes a generic LIBOR panel bank and the subscript M refers to market rate. The forward spread at time $t \leq \alpha$ is then the spread between the forward rate F_{Std} replicated by LIBORs and the forward rate E_{Std} replicated by OIS rates,

$$S^{X^0}(t; \alpha, 2\alpha) = F_{Std}(t; \alpha, 2\alpha) - E_{Std}(t; \alpha, 2\alpha). \tag{2.18}$$

A particular LIBOR counterparty is excluded from the LIBOR panel if

$$S^{X^0}(\alpha, 2\alpha) > S^{X^0}(t; \alpha, 2\alpha). \tag{2.19}$$

The interpretation of the inequality in (2.19) is that a current counterparty defaults if its LIBOR–OIS spread at α exceeds a prespecified level. The spread thus is reduced to a call option with the strike $S^{X^0}(t; \alpha, 2\alpha)$. Morini further assumed that the spread evolves as a driftless geometric Brownian motion and priced the option with the standard Black–Scholes formula (Black and Scholes 1973). The formula was calibrated to market quotes of basis of EURIBORs and closely tracked the shape of traded 6M/12M basis from July 2008 to May 2009, though there are discrepancies in levels. Morini attributed level discrepancies to a lack of more appropriate volatility inputs during the sample period.

Taylor and Williams (2009) use a no–arbitrage model of term structure to examine the effect of default risk and liquidity risk on 3M LIBOR-OIS spread. They consider a range of possible measures of default risk, such as the credit default swap (CDS) premium, TIBOR–LIBOR spread[4] and asset–backed commercial paper spread. The effect of liquidity risk is measured by a dummy variable, Term Auction Facility (TAF). TAF was provided by the US Federal Reserve to inject liquidity into financial institutions. Results find that default risk measures explain most of the variations of LIBOR–OIS spread. The TAF dummy variable is either statistically insignificant or of the wrong sign[5].

[4]TIBOR is the reference rate of unsecured lending of JPY to Japanese prime banks in the Tokyo interbank market. Taylor and Williams argue that because Japanese banks were less affected by the financial crisis than US banks, TIBOR-LIBOR spread reflected default risk differential between two markets.

[5]TAF announcements are supposed to decrease the level of LIBOR-OIS spread, hence the sign is expected to be negative.

In the cross–currency market, Baba and Packer (2009) used the difference in CDS spreads between European and US financial institutions as a measure of the default risk premium differential. They found that this measure has a significant negative correlation with the basis of EUR/USD currency swaps. In other words, the riskier the European banks than US counterparts, the more negative the basis. The effect of counterparty default risk on the movement of the basis in currency swaps is probably best illustrated in the JPY/USD pair. Kanaya and Woo (2000) provided a comprehensive account of the credit deterioration of Japanese banking industry in the 1990s. When the asset and property bubbles burst in 1990s, almost all Japanese banks were downgraded by credit rating agencies. The cost of funding of Japanese banks in the overseas interbank market dramatically increased, hence the "Japan premium". Covrig et al. (2004) examined the determinants of the "Japan premium", which is measured by the JPY TIBOR–LIBOR spread. JPY TIBOR is the reference rate of unsecured lending of JPY to Japanese banks in the Tokyo market and JPY LIBOR is the reference rate of unsecured lending of JPY to panel banks in the London market, with a majority of the panel members being non–Japanese banks. Hence the spread is a measure of the default–risk premium of Japanese banks over non–Japanese banks. Naoki (2005), Amatatsu and Baba (2008) examined the effect of "Japan premium" on the basis in JPY/USD currency swaps. They found significant negative impact of "Japan premium" on the basis. The large premium resulted in large and negative basis for JPY/USD currency swaps in late 1990s.

2.3.2 Liquidity Risk

Brunnermeier and Pedersen (2009) develop a theoretical liquidity risk model in which market liquidity and funding liquidity reinforce each other. Market liquidity is defined as the ease of trading securities, including low bid–ask spread, market depth and market resilience. On the other hand, funding liquidity is the ease of raising funds, with own capital or loans. During the financial crisis, initial losses in the sub–prime mortgage market forced financial institutions to exit positions in other asset classes (e.g. stocks) to meet margin calls and other funding needs. Funding constraints prompted traders to sell securities at "fire sale" prices, which resulted in even larger losses. In such volatile market conditions, market liquidity also deteriorated and positions in illiquid assets (e.g. structured products due to highly customized nature and held–to–maturity investment strategy) were particularly difficult to unwind. Selling such assets meant even greater losses than selling in a liquid market. Both market liquidity and funding liquidity disappeared and banks faced a double jeopardy: they found it difficult to sell assets to raise funds exactly at a time it was difficult to borrow. The double "liquidity shock" forced them to hoard cash and other liquid instruments which they might otherwise have lent to others. They were reluctant to make lending to inter–bank counterparties for longer than three months (see Mollenkamp and Whitehouse 2008). Brunnermeier (2009) identifies liquidity risk, lending channel, bank run and network effects as main amplification mechanisms through which a relatively small shock in the mortgage market transmitted to other asset classes and resulted in a full-blown financial crisis.

Ivashina and Scharfstein (2010) and Cornett et al. (2011) identify three factors which led banks to manage liquidity and reduce lending during the crisis. Firstly, the extent to which a bank is financed by short–term debt, as opposed to insured deposits. Short–term debts are subject to rollover risks[6]. On the other hand, insured deposits are a more stable source of capital. Before the crisis, financial institutions relied heavily on short–term funding, such as Asset–Backed Commercial Papers and Repurchase (Repo) Agreements, to finance their long-term assets. The average maturity of such instruments ranges from overnight to 90 days. After initial losses in mortgage securities, investors refused to roll over and banks had to refinance from other sources. The second factor is banks' exposure to credit-line draw downs. Ivashina and Scharfstein (2010) show that during the crisis firms drew on their credit lines primarily because of concerns about the ability of banks to fund these commitments, as well as due to firms' desire to enhance their own liquidity[7]. Lastly, on the asset side, banks holding illiquid loans and securities tended to increase holdings of liquid assets and decreased new lending.

In contrast to Taylor and Williams (2009), McAndrews et al. (2008) find that TAF announcements and operations significantly reduced the 3M LIBOR–OIS spread, which points to the importance of the liquidity risk premium. The authors argue that in order to test the effect of the TAF dummy variable, the dependent variable should be the change, not the level of the LIBOR–OIS spread. The use of the spread level as the dependent

[6]Rollover risk is associated with debt refinancing. It arises when existing debt is about to mature and needs to be rolled over into new debt and interest rates increase. The debt issuer hence needs to refinance at a higher interest rate and incur more interest payments in the future. Recent studies on rollover risk during the GFC include Acharya et al. (2011) and He and Wei (2012), etc.

[7]For example, FairPoint Communications drew down 200 million from the committed credit line supplied by Lehman Brothers as the lead bank on September 15th, 2008. In the SEC filing, the company "believes that these actions were necessary to preserve its access to capital due to Lehman Brothers' level of participation in the company's debt facilities and the uncertainties surrounding both that firm and the financial markets in general".

variable, as in Taylor and Williams (2009), is only valid under the assumption that the effect of TAF auction disappears immediately after the auction. If the liquidity risk premium stays low over days after the auction, the coefficient of the TAF dummy cannot be interpreted as the TAF effect.

Michaud and Upper (2008) aim to identify the drivers of the increase of the 3M LIBOR–OIS spread. Acknowledging that it is difficult to disentangle default risk and funding liquidity risk, as well as the measurement problem of bank–specific funding liquidity, Michaud and Upper examine only the effect of default risk and market liquidity risk. Funding liquidity is treated as an unobserved variable whose effects will appear as a residual once the impact of all other variables has been taken into account. The default risk is measured by the spread between the unsecured and secured interbank rate, as well as the CDS premiums. The measures of market liquidity are number of trades, volume, bid–ask spreads and price impact of trades. The finding is that while default risk plays a role, the significance is stronger in market liquidity measures. Furthermore, due to potential positive correlation between default risk and funding liquidity risk, the effect of default risk may have been overestimated.

Acharya and Merrouche (2013) examine the UK interbank market during the crisis and empirical results are in favor of precautionary liquidity hoarding over default risk in explaining the increase of the 3M LIBOR–OIS spread. They find that liquidity hoarding substantially increased after structural breaks (e.g. BNP Paribas froze withdrawals on 08/09/2007, Bear Stearns in March 2008). Secondly, the hoarding of liquidity by banks was precautionary in nature, especially for banks with large losses in sub-prime mortgage securities. Thirdly, liquidity hoarding drove up interbank lending rates, both secured and unsecured. The effect of liquidity hoarding is to raise overnight inter–bank rates after the crisis. In contrast, before the crisis an increase in the overnight liquidity buffer was associated with a decline in overnight spreads. This confirms the authors' hypothesis that in stressed conditions banks only release liquidity at a premium that exceeds the direct cost of using the emergency lending facility offered by the central bank and the indirect stigma cost (e.g. bank run, credit line draw downs). The fact that the effects on rates are similar for secured and unsecured inter-bank rates implies that the market stresses were not *per se* due to default risk concerns. Instead, the stresses were most likely due to each bank engaging in liquidity hoarding as the precautionary response to its own heightened funding risk.

Schwarz (2010) is the first paper, to our best knowledge, to deliberately separate the effect of default risk and liquidity risk on the LIBOR–OIS spread. Researchers commonly agree that it is difficult to disentangle default risk and funding liquidity risk, e.g, Michaud and Upper (2008). A bank with a funding shortage is more likely to default than a bank with ample funding. On the other hand, if a bank's credit rating worsens, it becomes more difficult to secure external funding. In fact, initial losses in the sub–prime mortgage market may have increased both default risk and funding liquidity risk. Hence, these two risk factors are highly interrelated. Schwarz measures market liquidity with the yield spread between German government bonds and KfW agency bonds. KfW bonds are fully guaranteed by the German government hence entail no default risk, but are less liquid in the bond market than the government bonds. The measure of default risk is the dispersion of borrowing rates of banks with different credit standings. Schwarz argues that a market–wide liquidity shock should have similar effect on banks' borrowing rates, hence the dispersion is relatively unchanged. On the other hand, a market–wide credit shock affects banks with bad credit rating more than banks with good credit, hence the dispersion increases. The correlation between the two risk measures is 0.07 and Schwarz claims that the regression results show the "clean" (i.e. independent) effect of each risk. The finding is that, though both risks are significant, nearly 70% of the increase of the 3M LIBOR–OIS spread and nearly 90% of the sovereign bond spread (Italy–Germany ten–year spread) increase can be explained by the market liquidity measure.

In the cross–currency market, it has been observed in the literature that there is a shortage of US Dollars in the global banking industry and the USD funding gap is especially large for European banks. Fender and McGuire (2010) and McGuire and Peter (2009) noted that European banks' total USD–denominated assets were more than 800 billion by mid-2007, immediately before the crisis. European banks have traditionally used short–term USD borrowing to fund these long–term assets. The funding channels are mainly the USD interbank market and swap market. Due to the mismatch of maturities of assets and liabilities, these banks are exposed to funding or rollover risk. As the financial crisis unfolded from 2007, it became increasingly difficult, or almost impossible, to borrow USD in the unsecured interbank market. To fund USD assets, European banks could only rely on the swap market. They raised funds domestically and then swapped them into USD. Hence there was a huge demand for USD in the swap markets. However, the demand of USD from European banks is not matched by the demand of EUR from US banks. Due to the heightened liquidity risk of USD, European banks borrowing USD in currency swaps had to pay a price for the demand and the price was reflected in the large and negative basis added to EURIBOR. Although Japanese banks had a smaller USD funding gap than European banks, the basis of JPY/USD currency swap also declined. There is anecdotal evidence that some non–Japanese banks raised funds in JPY and swapped JPY into USD. Hence the relative demand of USD over JPY may also have dislocated during the crisis.

To provide more liquidity of USD to non–US financial institutions, the US Federal Reserve established swap lines with major central banks, including ECB, SNB, BoE and BoJ[8] (see Baba and Packer 2009). Baba and Packer tested the effect of the government swap lines on the basis and found significant positive impact on the basis of EUR/USD, CHF/USD and GBP/USD currency swaps.

In contrast to European and Japanese counterparts, Australian banks are mainly funded with longer term bond issuances and not exposed to maturity mismatch and rollover risk (see Ossolinski and Zurawski 2010). The basis of AUD/USD currency swap had been generally positive. From a currency demand/supply point of view, before the crisis, there had been a persistent premium to receive AUD in currency swaps. In other words, the demand to receive AUD exceeded the supply of it in the currency swap market. Ryan (2007) shows that the offshore bond issuance denominated in foreign currencies by Australian

[8]European Central Bank, Swiss National Bank, Banks of England and Bank of Japan.

institutions far exceeds the Kangaroo bond[9] issuance by foreign investors in Australia. Australian issuers typically swap their offshore issuance proceeds back into Australian dollar, with Kangaroo bond issuers as the counterparty. There is a high demand for AUD but insufficient supply of it. The Australian banks therefore are willing to pay a premium to receive Australian dollars in a currency swap, hence the positive basis.

The basis also became negative from mid 2008. However, Blees (2010) notes that this should not be viewed as a sudden stronger demand for USD from Australian banks, nor does this reflect higher a default risk premium of Australian banks. The unusual move of the basis was a result of international banks using AUD as a source of funds to exchange for USD funding, coupled with major rehedging of Japanese power reverse dual currency (PRDC) notes[10]. The negative basis was also relatively short–lived. From early 2009, the basis returned to positive levels, which again reflected the strong demand for AUD in the swap market. The offshore issuance of Australian bank paper had a strong increase during 2009 due to high credit rating and government guarantees, while there was no Kangaroo bond issuance in the first quarter at all.

[9] A kangaroo bond is issued in the Australian market by non–Australian firms and is denominated in AUD.

[10] A PRDC note is an exotic structured product where domestic investors seek higher yields by taking advantage of the interest rate differentials between two currencies, which is a form of leveraged carry trade. PRDC note holders initially receive fixed coupon rate, then the coupon rate rises (decreases) as the domestic/foreign exchange rate depreciates (appreciates).

2.4 Uncovered Interest Parity Puzzle

In a carry trade investors borrow funds in a low–yield currency (funding currency) and lend in a high–yield currency (investment currency) to gain the interest rate differential. UIP predicts that, on average, the carry should be exactly offset by the depreciation of the investment currency. If UIP holds in practice, carry trades should have zero excess return. In practice, UIP often fails and investment currencies have been found to actually appreciate on average against funding currencies, this has been termed the UIP puzzle. The UIP puzzle is the reason that carry trades are historically profitable with high Sharpe ratios (Burnside et al. 2011).

The UIP puzzle is among the most prominent puzzles in international economics and finance, see Engel (1996) for a comprehensive survey. Original works on the failure of UIP date back to Hansen and Hodrick (1980), who reject the Foreign Exchange (FX) market efficiency hypothesis that speculations in the FX forward market should have zero return. Meese and Rogoff (1983) find that exchange rates can be modelled by a “random walk” and investors are able to exploit interest rate differentials between currencies. Fama (1984) labels the failure of UIP as the “forward discount puzzle”.

Among more recent works, Chinn and Meredith (2004) find that UIP fails at short–run horizons but recovers at long–run horizons and they attribute the failure of UIP to the interaction of random FX market shocks with endogenous monetary policy reactions. Lusting and Verdelhan (2007) assert that, conditional upon the interest–rate differential, aggregate consumption growth risk is useful in explaining the UIP puzzle. Burnside

(2008) challenges the results in Lusting and Verdelhan (2007) by arguing that the covariance between the excess portfolio return and risk factors is not significant.

Brunnermeier and Pedersen (2009) use a liquidity–based model to explain some stylized facts, including the correlation between market liquidity and volatility. They also predict that speculative investments have negative skewness, arising from the asymmetric response to liquidity shocks: shocks leading to losses are amplified through liquidity spirals and shocks leading to gains are not amplified. Built upon the theoretical liquidity framework, Brunnermeier et al. (2008) approach the UIP puzzle by examining the performance of carry trades. They propose that carry trades tend to be unwound when speculators near their funding constraints. The unwinding of carry trades results in large losses and currency crashes. Key findings in Brunnermeier et al. (2008) include: 1) In the short–run, carry trades are profitable due to under–reaction to shocks to interest rate differentials. The initial under–reaction arises from liquidity frictions in the market and speculative capital arrives slower than predicted by UIP (Mitchell et al. 2007). However, in the long–run, speculators tend to over–react and bubbles are built in exchange rates. Abreu and Brunnermeier (2003) argue that bubbles build up due to dispersion of opinions and the need for coordination among arbitrageurs, hence a "synchronization" problem. 2) Carry trade crashes are positively correlated with funding liquidity measures: VIX[11] and TED spread[12]. 3) Controlling for liquidity effects, the interest rate differential is not significant in forecasting the excess returns of carry trades, which helps resolve the UIP

[11]VIX is a trademarked ticker symbol for the Chicago Board Options Exchange Market Volatility Index, a popular measure of the implied volatility of S&P 500 index options.

[12]TED spread is the difference between 3M Eurodollar LIBOR and 3M Treasury-bill rate. Eurodollars are deposits denominated in USD outside the United States.

puzzle.

As commented in Burnside (2008), a model with liquidity frictions is a plausible candidate to explain the UIP puzzle, particularly when empirical evidence fails to support other leading explanations such as risk premium, skewness of carry trade payoffs and "peso problems"[13]. In this respect, Brunnermeier et al. (2008) provides support for the liquidity model proposed by Brunnermeier and Pedersen (2009). On the other hand, the lack of statistical significance of liquidity measures, especially TED spread, implies that these measures may be improved. For example, in the panel regression with weekly data, although the sign of the coefficient is correct, the change of TED spread is not contemporaneously significant in explaining excess return of carry trades. The significance is also only marginal with one–week delay. In the panel regression with interest rate difference, the liquidity measures are not significant in predicting excess returns for the immediate following quarters.

Among related studies, Ranaldo and Soderlind (2010) study safe–haven properties of high–frequency exchange rates with market volatility and liquidity. The proxies for volatility are realized exchange rate volatility and VIX. The TED spread is used to measure liquidity. The FX realized volatility is found to be significant in affecting the excess return of all exchange rates in the sample, while VIX is only significant for JPY/USD. The TED spread is not significant for any of the exchange rates. Christiansen et al. (2011) employ a similar factor model to study the risk exposure of carry trade returns. The risk expo-

[13]Peso problems refer to the situation that in an investment there is a high probability of small gains, and a small probability of large losses.

sures are allowed to be regime–dependent to account for FX time–varying risk premia. As regime variables, FX market volatility and TED spread are found to be more significant than VIX and bid–ask spreads.

Chapter 3

On the Behavior of the Cross–Currency Swap Basis: Empirical Observations Before and During the Recent Financial Crises

3.1 Introduction

A cross–currency basis swap (CCBS) exchanges floating interest rates of two different currencies at each tenor of the swap term. Different from basis swaps within a single currency, the notional amount of a CCBS is also exchanged both at the beginning and the end of swap term, based upon the spot exchange rate at initiation.

Currency swaps are a popular tool to swap the currency exposure and manage adverse movements of foreign exchange rates. For instance, Ossolinski and Zurawski (2010) note that Australian banks have long issued bonds overseas (e.g. Yankee bond in US, Samurai bond in Japan) to reduce funding costs. To hedge the currency risk, Australian issuers often choose to swap the foreign currency proceeds into AUD, with Kangaroo bond issuers as the counterparty. Kangaroo bonds are issued by foreign investors in Australia and denominated in AUD. As another example, European banks which need USD funding may choose to raise EUR domestically and then swap EUR proceeds into USD with US counterparties (see Fender and McGuire 2010, McGuire and Peter 2009).

By the classical no–arbitrage pricing principle, two floating rates should trade at par to ensure that the initial value of the swap is zero (see Hull 2008). In practice, a basis is often added to the floating rate of the left–hand side (LHS) currency by market convention. Chapter 2 shows that the basis spreads of major currency swaps against USD all experienced large movements during the crisis, especially after the collapse of Lehman Brothers in September 2008.

In Chapter 2 we identified potential risk factors which may have caused unusual changes in the basis spreads of currency swaps. It is important to realize that default risk and liquidity risk also existed before the crisis, but at significantly lower levels (see Tuckman and Porfirio 2003). Boenkost and Schmidt (2005) attributed the basis spreads to liquidity premium and proposed valuation methodologies taking into account the basis spreads. However, the spreads prior to the crisis were small and it is commonly agreed that arbitrage opportunities exploiting the spreads could be easily canceled by transaction costs (see, for example, Bianchetti 2010). We are hence motivated to examine whether after taking into account transaction costs, can arbitrage profits presented by much larger spreads in currency swaps still be cancelled in the post–GFC world? If transaction costs *per se* are insufficient in explaining observed spreads, what has prevented market participants from executing textbook arbitrage?

Suppose an arbitrageur is willing to exploit the large spreads in a CCBS and execute an associated arbitrage strategy, there are two main transaction costs: spread of LIBOR–LIBID[1] and bid-offer spread of FX forward rate[2]. An arbitrageur in the interbank market has to pay LIBOR when borrowing funds but only get LIBID when lending funds. He/she also must pay the offer price of the FX forward when buying foreign currency but can only sell at the bid price, which is lower than the offer price. If spreads in interest rates and forward exchange rates are sufficient to cancel the arbitrage profit, the basis spread in CCBSs must be bounded by some function of interest rates and forward exchange rates. Similarly, to preclude arbitrage, the FX forward rates should be bounded by a function of FX spot rate and interest rates of two currencies.

In this chapter we aim to examine the role of transaction costs in explaining the basis spreads of CCBSs[3]. We propose no–arbitrage bounds for FX forward rates and basis spreads of currency swaps. We then examine whether these bounds hold in practice with

[1]London Interbank Bid Rate is a bid rate at which a bank is willing to borrow from other banks, while LIBOR is the ask rate.

[2]The FX forward rate is the exchange rate at which two parties in the forward contract agree to exchange one currency for another at a future date. Maturities of major FX forward rates are normally quoted up to 12 months.

[3]This chapter extends the work done in Chang (2009).

market data. The remainder of Chapter 3 is organized as follows. Section 2 derives no–arbitrage bound formulae. Section 3 describes the data and methodologies used for empirical tests. Section 4 summarizes and analyzes the findings and Section 5 concludes.

3.2 No-Arbitrage Bounds

3.2.1 Bounds for FX Forward Rates

In order to make riskless profit from FX forward contracts, an arbitrageur must engage in transactions simultaneously in both the domestic market and the foreign market. The arbitrageur borrows funds at LIBOR, but can only lend at LIBID. In the forward exchange market, the arbitrageur can only sell a currency at the bid price but have to pay the offer price when buying. The arbitrage strategy is proposed in Table 3.1 and notations are explained as follows,

X_0: spot mid price of domestic currency/unit foreign currency at time 0;

$F_{(0,t)}$: forward price of domestic currency/unit foreign currency at time 0 with maturity $t, \quad t > 0$;

X_{bid}: spot bid price of domestic currency/unit foreign currency at time 0;

X_{offer}: spot offer price of domestic currency/unit foreign currency at time 0;

$F_{bid(0,t)}$: forward bid price of domestic currency/unit foreign currency at time 0 with maturity $t, \quad t > 0$;

$F_{offer(0,t)}$: forward offer price of domestic currency/unit foreign currency at time 0 with maturity t, $t > 0$;

$r_{LIBOR(0,t)}$: annual domestic currency LIBOR rate applicable between time 0 and t, $t > 0$;

$r_{LIBID(0,t)}$: annual domestic currency LIBID rate applicable between time 0 and t, $t > 0$;

$\overline{r}_{LIBOR(0,t)}$: annual foreign currency LIBOR rate applicable between time 0 and t, $t > 0$;

$\overline{r}_{LIBID(0,t)}$: annual foreign currency LIBID rate applicable between time 0 and t, $t > 0$;

$\tau_{(0,t)}$: year fraction of time to maturity, $t > 0$;

Table 3.1: Arbitrage Strategy 1 for FX Forward

Initiation	Maturity t
borrow unit domestic currency	$-(1 + r_{LIBOR(0,t)}\tau)$
sell at $\frac{1}{X_{offer}}$ and invest at $\overline{r}_{LIBID(0,t)}$	
buy forward contract at $F_{bid(0,t)}$	$\frac{1}{X_{offer}}(1 + \overline{r}_{LIBID(0,t)}\tau)F_{bid(0,t)}$

At maturity the arbitrageur repays his liability from domestic currency proceeds. To ensure the no–arbitrage condition, we require the net cash flow to be non–positive, i.e.

$$\frac{1}{X_{offer}}(1 + \overline{r}_{LIBID(0,t)}\tau)F_{bid(0,t)} - (1 + r_{LIBOR(0,t)}\tau) \leq 0. \tag{3.1}$$

Hence we must have

$$F_{bid(0,t)} \leq \frac{X_{offer}(1 + r_{LIBOR(0,t)}\tau)}{1 + \overline{r}_{LIBID(0,t)}\tau}. \tag{3.2}$$

(3.2) is the upper bound for the bid price of the forward exchange rate. Alternatively, the arbitrageur can start from borrowing the foreign currency and the arbitrage strategy is as in Table 3.2,

Table 3.2: Arbitrage Strategy 2 for FX Forward

Initiation	Maturity t
borrow unit foreign currency	$-(1+\overline{r}_{LIBOR(0,t)}\tau)$
sell at X_{bid} and invest at $r_{LIBID(0,t)}$	
buy forward contract at $\frac{1}{F_{offer(0,t)}}$	$X_{bid}(1+r_{LIBID(0,t)}\tau)/F_{offer(0,t)}$

No arbitrage condition requires that

$$X_{bid}\frac{(1+r_{LIBID(0,t)}\tau)}{F_{offer(0,t)}}-(1+\overline{r}_{LIBOR(0,t)}\tau)\leq 0. \tag{3.3}$$

Hence, we must have

$$F_{offer(0,t)}\geq\frac{X_{bid}(1+r_{LIBID(0,t)}\tau)}{1+\overline{r}_{LIBOR(0,t)}\tau}. \tag{3.4}$$

(3.4) is the lower bound for the offer price of the forward exchange rate. We propose that bounds (3.2) and (3.4) must hold in order to eliminate arbitrage opportunities in the forward currency market.

3.2.2 Bounds for Basis of Currency Swaps

By market convention, a generic CCBS is quoted as: left–hand side (LHS) currency LIBOR + B / right–hand side (RHS) currency LIBOR. B is the mid rate of the basis spread. The pay rate BP added to the LHS LIBOR is the price that a market maker (MM) is willing to pay for receiving the RHS currency LIBOR. The receive rate BR is the price that a MM receives when paying the RHS currency LIBOR. $BP \leq B \leq BR$ and $BR - BP$ is the profit for the MM.

Let $[0, T]$ be the term of the swap, i.e. time 0 is initiation and T is maturity. Let t_i (i =1, 2 ,..., n, where $t_n = T$) denote a pre-specified set of payment exchange dates, i.e. tenors. Let Y denote the RHS LIBOR-LIBID spread and Z denote the LHS LIBOR-LIBID spread. We examine both sides of the CCBS.

1) Pay RHS LIBOR and Receive LHS LIBOR + BP

If one counterparty receives LHS LIBOR + BP and pays RHS LIBOR, then at initiation it receives the RHS principal and pays the LHS principal. The LHS principal is borrowed at LHS LIBOR and the RHS principal can be invested at RHS LIBID. Its cash flow position is in Table 3.3.

We see that the principal amounts cancel and the net cash flow position at every tenor is LHS principal $\times$ BP $-$ RHS principal $\times$ Y. BP is a fixed quantity and Y is the LIBOR–LIBID spread of the RHS currency. Conventionally Y is fixed at 12.5 bps for all currencies

Table 3.3: Cash Flow 1 in CCBS

Cash flow	Initiation	Each tenor t_i	Maturity T
positive	RHS principal	LHS LIBOR + BP (swap), RHS LIBID (deposit)	LHS principal
negative	LHS principal	RHS LIBOR (swap), LHS LIBOR (loan)	RHS principal

quoted by British Banker Association (see, for example, Coyle 2001). If the spread is indeed fixed, cash flows are fixed at $\underline{\text{LHS principal} \times BP - \text{RHS principal} \times 12.5 \text{ bps}}$ at each tenor. The no arbitrage condition requires that the total present value (PV) of these cash flows must not be greater than zero. To properly discount the negative cash flows, we need to convert all cash flows to a common currency, say RHS currency and use RHS LIBIDs as discount rates. We use LIBIDs because the arbitrageur should discount these cash flows at his investment rate, hence LIBIDs. The total present value is calculated as

$$\sum_{i=1}^{n} \left\{ \left(\frac{\text{LHS principal} \cdot BP}{F_{offert_i}} - \text{RHS principal} \cdot 12.5 \text{ bps} \right) \cdot d_{t_i} \right\}. \tag{3.5}$$

In (3.5) d_{t_i} is the discount factor applicable between time 0 and t_i. F_{offert_i} is the forward offer price of LHS/unit RHS that can be locked in at time 0 and exercised at time t_i. We must use the offer price because $\frac{1}{F_{offert_i}}$ is the price the arbitrageur has to take when selling LHS for RHS in the forward market. We firstly solve BP for

$$\sum_{i=1}^{n} \left\{ \left(\frac{\text{LHS principal} \cdot BP}{F_{offert_i}} - \text{RHS principal} \cdot 12.5 \text{ bps} \right) \cdot d_{t_i} \right\} = 0. \tag{3.6}$$

Suppose BP_0 is the solution. By market convention, the principal amounts in a CCBS are based upon the spot mid exchange rate at initiation. Therefore X_0 is the ratio of LHS principal over RHS principal. We then simplify Eqn. (3.6) as

$$\sum_{i=1}^{n}\left\{(\frac{X_0 \cdot BP_0}{F_{offert_i}} - 12.5 \text{ bps}) \cdot d_{t_i}\right\} = 0. \tag{3.7}$$

Solving Eqn. (3.7) for BP_0 yields

$$BP_0 = \frac{\sum_{i=1}^{n}(12.5 \text{ bps} \cdot d_{t_i})}{X_0 \cdot \sum_{i=1}^{n}\frac{d_{t_i}}{F_{offert_i}}}. \tag{3.8}$$

For any $BP \leq BP_0$ the no–arbitrage inequality holds, BP_0 is hence the upper bound for the basis pay rate.

The upper bound BP_0 has been derived by assuming Y is constant. However, in the case of unfixed spread, the net position is exposed to uncertain cash flows. To fully eliminate this risk, we propose the alternative strategy. The counterparty can enter an IRS contract in the RHS currency. In the IRS, the counterparty pays fixed RHS interest rate and receives RHS LIBORs. Let H denote the fixed rate of the IRS, the cash flows are summarized in Table 3.4.

Only one component in the net cash flow of Table 3.4 is uncertain, the RHS LIBID. However, we can view this series of cash flows as a floating rate bond without notional payment at maturity. As a result, we know for certain that the PV of this bond is RHS principal $\times (1 - d_{t_n})$. Hence we have completely eliminated the risk of uncertain

Table 3.4: Cash Flow 2 in CCBS

Cash flow	Initiation	Each tenor t_i	Maturity T
positive	RHS principal	LHS LIBOR+BP, RHS LIBOR (IRS), RHS LIBID	LHS principal
negative	LHS principal	RHS LIBOR, LHS LIBOR, RHS fixed H (IRS)	RHS principal
net cash flow		LHS principal $\cdot BP -$ RHS principal $\cdot H +$ RHS LIBID	

cash flows. The PV of total net cash flows is

$$\text{RHS principal} \cdot (1 - d_{t_n}) + \sum_{i=1}^{n} \left\{ \left(\frac{\text{LHS principal} \cdot BP}{F_{offert_i}} - \text{RHS principal} \cdot H \right) \cdot d_{t_i} \right\}. \quad (3.9)$$

Solving (3.9) for BP_0 gives

$$BP_0 = \frac{\sum_{i=1}^{n} (H \cdot d_{t_i}) + d_{t_n} - 1}{X_0 \cdot \sum_{i=1}^{n} \frac{d_{t_i}}{F_{offert_i}}}. \quad (3.10)$$

2) Receive RHS LIBOR and Pay LHS LIBOR + BR

The other counterparty of the swap pays LHS LIBOR + BR and receives RHS LIBOR. At initiation he pays the RHS principal and receives the LHS principal. The RHS principal is borrowed at RHS LIBOR and the LHS principal is invested at LHS LIBID. The cash flow position is in Table 3.5.

The net cash flow position for this counterparty is different from the other counterparty

Table 3.5: Cash Flow 3 in CCBS

Cash flow	Initiation	Each tenor t_i	Maturity T
positive	LHS principal	RHS LIBOR (swap), LHS LIBID (deposit)	RHS principal
negative	RHS principal	LHS LIBOR + BR (swap), RHS LIBOR (loan)	LHS principal
net cash flow		$-$LHS principal $\cdot$ $(BR + Z)$	

because there is only LHS currency is involved. Therefore there is no need to involve FX forward rates for this side of the swap. The total PV of the cash flow positions should be non-positive to ensure the no arbitrage condition

$$\sum_{i=1}^{n} -(\text{LHS principal} \cdot (BR + Z) \cdot d_{t_i}) \leq 0. \tag{3.11}$$

Because LHS principal and the sum of discount factors must be both positive, if Z is a fixed quantity, we must have

$$BR \geq -Z. \tag{3.12}$$

Hence $-Z$ is the lower bound for the basis receive rate. If we assume that Z is fixed at 12.5 bps, then the lower bound is simply

$$BR \geq \text{-12.5 bps.} \tag{3.13}$$

Alternatively, to eliminate the risk of uncertain cash flows, we assume the IRS swap fixed rate is G for the LHS currency. The lower bound then becomes

$$BR \geq \frac{1 - d_{t_n}}{\sum_{i=1}^{n} d_{t_i}} - G. \tag{3.14}$$

3.3 Data and Methodology

3.3.1 Description of Data

We collect daily data from January 3, 2006 to August 12, 2011. This sample period is chosen to span pre–GFC, GFC and the ongoing Eurozone crisis that started since late 2009.

1) Spot and Forward Exchange Rates

We select four major currencies in the FX and swap markets: USD, JPY, EUR and AUD. We use USD as the foreign currency and the other three currencies respectively used as the domestic currency. All exchange rates, including spot and forward, represent the price of one unit foreign currency in domestic currency. For example, the EUR/USD spot rate is the spot price of unit USD in EUR. Hence we have three currency pairs. The data source is Bloomberg. Excluding missing data, there are 1419 trading days during this period. One each trading day, we collected the spot rate, 1-day (1D), 1M, 3M, 6M, 12M, 2-year (2Y), 3Y, 4Y and 5Y forward rates. Because we explicitly examine the effect of transaction costs, we also obtain the mid, bid and offer rate for each spot and forward rate.

2) LIBOR / LIBID and IRS Rates

We collect USD LIBOR/ LIBID, JPY LIBOR/ LIBID, EURIBOR/EURIBID[4] (source: Bloomberg) and AUD Bank Bill Bid and Offer Rates[5] (source: Reserve Bank of Australia). These rates are selected because they represent the borrowing and lending costs for an arbitrageur in each currency. They are also the references rates used in currency swaps. We use Bank Bill rates to proxy AUD Bank Bill Swap rate (BBSW)[6], which is the reference rate used in AUD currency swaps.

We need IRS bid and offer rates for two purposes. Firstly, LIBOR/LIBID rates have maturities up to 12 months. Because LIBOR is often used as the reference rate in an IRS, IRS rates can be used as proxies to extend the LIBOR zero curve beyond 12M maturity. Secondly, in deriving the bounds for the basis spreads of CCBSs, we eliminate the uncertain cash flow risk by entering an IRS. We use the IRS bid rate to approximate LIBID and the ask rate to approximate LIBOR. All IRS data are sourced from Bloomberg.

We obtain daily data of overnight (O/N), 1M, 3M, 6M and 12M USD LIBOR/ LIBID,

[4]EURIBID is the bid rate at which European prime banks are willing to borrow unsecured EUR from each other within the Euro zone, while EURIBOR is the ask rate.

[5]The bank bill interest rate is the wholesale interbank rate within Australia published by the Australian Financial Markets Association (AFMA). It is the borrowing rate among the country's top market makers, and is widely used as the benchmark interest rate for financial instruments

[6]The BBSW rates are independent and transparent rates for the pricing and revaluation of privately negotiated bilateral Australian dollar interest swap transactions, published by AFMA. BBSW data are not available for all maturities and on all sample period trading days. Based upon available BBSW data, we computed the error of using Bank Bill rates to approximate. The average error is 0.3 basis points (bps) for 1M maturity and 0.1 bps for 3M rates. The errors are sufficiently small.

JPY LIBOR/ LIBID and EURIBOR/EURIBID. AUD Bank Bill bid and offer rates are only available for O/N, 1M, 3M and 6M. Hence we use 1Y AUD IRS rates to proxy. The maturities of IRS data are as follows, USD and JPY: 18M, 2Y, 3Y, 4Y and 5Y; EUR: 2Y, 3Y, 4Y and 5Y; AUD: 1Y, 2Y, 3Y, 4Y and 5Y. Maturities vary for different currencies due to different payment frequencies of IRS rates and data availability. USD, JPY and AUD swap rates are paid semi-annually and EUR swap rates are paid annually.

3) CCBS Basis Spread

Daily data of CCBS basis rates, including mid rate, pay rate and receive rate, are collected for the sample period. Maturities include 1Y, 2Y, 3Y, 4Y and 5Y. We have three sets of CCBSs. For each set, the LHS currency and the RHS currency are respectively EUR/USD, JPY/USD and AUD/USD.

3.3.2 Methodology

To test no-arbitrage bounds proposed in section 3.2, we build two discount curves, one based upon LIBOR with IRS ask rates and the other upon the LIBID with IRS bid rates. Because it is easier to work with discount factors due to different compounding frequencies of interest rates across currencies, we rewrite the FX forward rate bounds (3.2) and (3.4) respectively as

$$F_{bid(0,t)} \leq \frac{X_{offer} \cdot \bar{d}_{LIBID(0,t)}}{d_{LIBOR(0,t)}}, \tag{3.15}$$

and

$$F_{offer(0,t)} \geq \frac{X_{bid} \cdot \overline{d}_{LIBOR(0,t)}}{d_{LIBID(0,t)}}. \tag{3.16}$$

In (3.15) and (3.16), $\overline{d}_{LIBOR(0,t)}$ ($\overline{d}_{LIBID(0,t)}$) represents the foreign currency discount curve while $d_{LIBOR(0,t)}$ ($d_{LIBID(0,t)}$) represents the domestic currency discount curve.

1) Discount Curves with LIBORs (LIBIDs)

We firstly construct the discount curve out to 12M maturity with LIBORs (LIBIDs). They are inherently zero coupon rates and we can directly convert them to discount factors. For the sake of consistency, all LIBORs (LIBIDs), which are simply compounded by market conventions, are converted to continuously compounded rates $LIBOR_c$ ($LIBID_c$):

$$LIBOR_c = \frac{1}{\tau} \cdot \ln(1 + LIBOR \cdot \tau). \tag{3.17}$$

where τ is the year fraction of the LIBORs of a particular maturity. We assume the Act/365 day count convention. For instance, the O/N LIBOR has year fraction of 1/365. The continuously compounded discount factor then is

$$d_{LIBOR(0,t)} = \exp(-LIBOR_c \cdot \tau(0,t)). \tag{3.18}$$

2) Discount Curves with IRS Rates

Discount curves beyond one year and out to five-year maturity are extracted from IRS rates. Different from LIBORs, par swap rates are not zero coupon rates. An IRS can be considered as a contract in which a coupon bearing bond is exchanged for a floating-rate bond (see Brigo and Mercurio 2006). Because the initial value of an IRS must be zero to preclude arbitrage and floating-rate bonds always trade at par, the swap fixed rate is simply the coupon rate for a par-value coupon bond. Assuming unit notional amount, swap fixed rate S_{t_N} must satisfy

$$(S_{t_N} \cdot F) \cdot \sum_{i=1}^{N} d_{t_i} + d_{t_N} = 1. \tag{3.19}$$

where F is the payment frequency of the IRS rate and N is the total number of coupon payments. The payment frequency of IRS rates varies across different currencies. For USD, JPY and AUD the frequency is semi-annual, hence F is $\frac{1}{2}$. F is 1 for EUR IRS rates. From Eqn. (3.19) we obtain the final discount factor

$$d_{t_N} = \frac{1 - (S_{t_N} \cdot F) \cdot \sum_{i=1}^{N-1} d_{t_i}}{1 + (S_{t_N} \cdot F)}. \tag{3.20}$$

To apply Eqn. (3.20), we need all discount factors and swap rates before the final maturity, i.e. from t_1 to t_{N-1}. We start from available discount factors and work iteratively to obtain d_{t_N}, which is the bootstrap method for discount curve construction (see, for example, Ron

2000). Suppose we want to construct the 5Y USD discount curve based on LIBORs and IRS ask rates. Firstly, we use the 18M USD IRS ask rate and Eqn. (3.20) to obtain 18M discount factor, based upon the 6M and 12M discount factors calculated from Eqn. (3.18). We then use the 2Y USD IRS ask rate to calculate the 2Y discount factor with Eqn. (3.20), based upon the 6M, 12M and 18M discount factors. Because the USD IRS rate is paid-semiannually, if we have 2.5Y 3Y, 3.5Y, 4Y, 4.5Y and 5Y IRS ask rates, repeating this process we eventually get the 5Y discount factor. However, the 2.5Y , 3.5Y and 4.5Y IRS rates are not available from Bloomberg. We hence linearly interpolate these rates from available swap rates:

$$S_t = S_{t_i} + (\frac{t - t_i}{t_{i+1} - t_i}) \cdot (S_{t_{i+1}} - S_{t_i}), \quad t_i < t < t_{i+1}. \tag{3.21}$$

3) Formulae Specification

We use (3.15) and (3.16) to test the bounds of forward exchange rates. We test the upper bound of the CCBS basis pay rate with (3.8) and the lower bound with (3.13).

We decide not the use expression (3.10) and (3.14) after analysing the data. For the sample period, the LIBOR-LIBID spread for USD and EUR stays constant at 12.5 bps and the AUD Bank Bill bid-offer spread is constantly 10 bps. The only exception is JPY LIBOR-LIBID spread. However, the variation of the spreads is sufficiently small. For example, the 3M JPY LIBOR-LIBID spread is only different from 12.5 bps on 36 trading days and the average spread of the sample period is 12.49 bps. For such a small uncertainty, it is not worthwhile to engage in the strategy in Table 3.4, because the arbitrageur also bears transaction costs in the IRS contract. The transaction costs would clearly more than offset the benefit. Hence we use (3.8) and (3.13) to test the bounds. We also change 12.5 bps to 10 bps when testing AUD/USD swap basis bounds.

Another modification we make is the IRS bid-ask spread. Our data show that the IRS bid-ask spread is much smaller than the LIBOR-LIBID spread. For instance, the average 2Y USD IRS bid-ask spread during the sample period is merely 0.74 bps, while the LIBOR-LIBID spread over the same period is 12.5 bps. Because we proxy LIBID rates by IRS bid rates for maturities beyond one year, without modifications we would have to assume that the arbitrageur's borrowing-lending spread drops by more than 10 bps beyond 1Y maturity. This is clearly unrealistic. To better reflect reality, we assume that the borrowing-lending spread is constant for all maturities. The IRS ask rates are from the market data, but bid rates are obtained by subtracting the constant spread from the ask rate. As a result, the USD, JPY and EUR IRS bid-ask spread is fixed at 12.5 bps and AUD spread is fixed at 10 bps for all maturities[7].

[7]Note that this is a conservative approach in our context, as we seek to identify violations of no-arbitrage bounds, which are wider if the bid-ask spreads are wider.

3.4 Results

3.4.1 Forward Exchange Rate

We tested bounds of 1D, 1M, 3M and 6M forward rates because FX forwards are mostly liquid for maturities less than one year[8]. The results are summarized by two quantities,

1) Upper bound minus forward bid rate.

2) Forward offer rate minus the lower bound.

If bounds strictly hold, these two quantities should be non-negative. The results are plotted for each currency pair in Figures 3.1, 3.2 and 3.3. These figures clearly demonstrate a change of pattern, with the GFC as the turning point. In addition, tests of 4 different maturities demonstrate very similar results. Hence, in the following discussions we focus on the 6M maturity only.

We find that for the EUR/USD currency pair, the upper bound of the 6M forward bid rate holds strongly. In Figure 3.1 the test result stayed positive and stable on all trading days before the GFC. From August 2007, it showed more fluctuations and peaked on September 30th 2008, two weeks after the fall of Lehman Brothers [9]. It then fluctuated but remained significantly positive.

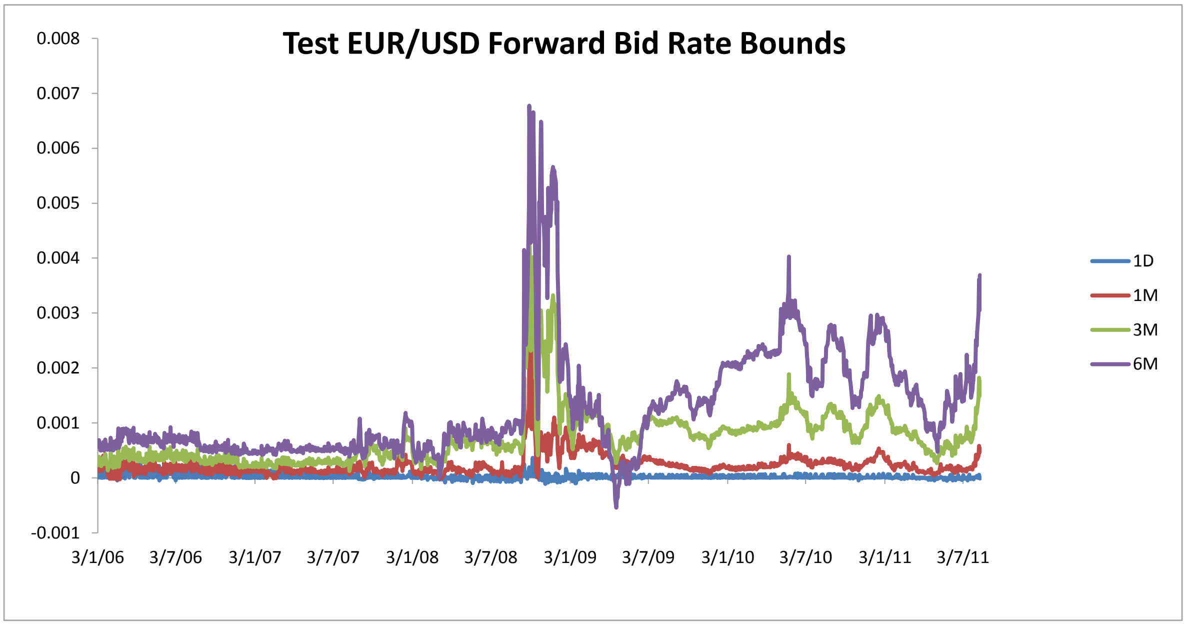
[8]Investors tend to hedge currency risks with forward contracts for maturities less than one year. For terms greater than or equal to one year, currency swaps provide greater liquidity. See Baba et al. 2008.

[9]Lehman Brothers filed for bankruptcy protection on September 15, 2008.

On the other hand, the lower bound test result of the 6M forward offer rate shows almost an mirror image. Before the crisis, it held tightly. Starting from the crisis, the bound has been persistently violated. The difference between the offer rate and its lower bound dropped to the lowest level on September 30th 2008. From there it went back to the positive region on only 27 trading days but remained negative on others.

The JPY/USD findings are similar, with the upper bound strongly holds for the whole sample period but the lower bound violated since the GFC. There are two notable differences between JPY/USD and EUR/USD 6M forward rates. Firstly, the lower bound of the JPY/USD 6M forward offer rate did not hold as strongly as its EUR/USD counterpart before the crisis, though only a small portion (12.47%) in Figure 3.2 went below zero. Secondly, unlike the lower bound of EUR/USD 6M offer rate, the JPY/USD lower bound held for a majority of the period from January to May of 2009.

Lastly, the AUD/USD 6M forward rate bounds behave significantly different from JPY/USD and EUR/USD. In Figure 3.3 we see both upper and lower bounds held strongly before the crisis. From January to May of 2009, the upper bound of the forward bid rate was violated on 50% of the trading days. It then remained stable and held until present. The lower bound test shows even greater differences from JPY/USD and EUR/USD forward rates. After a relatively short period of violations (roughly from August to November 2008), the lower bound of the offer rate held on a majority (77.15%) of total number of trading days since the beginning of 2009.



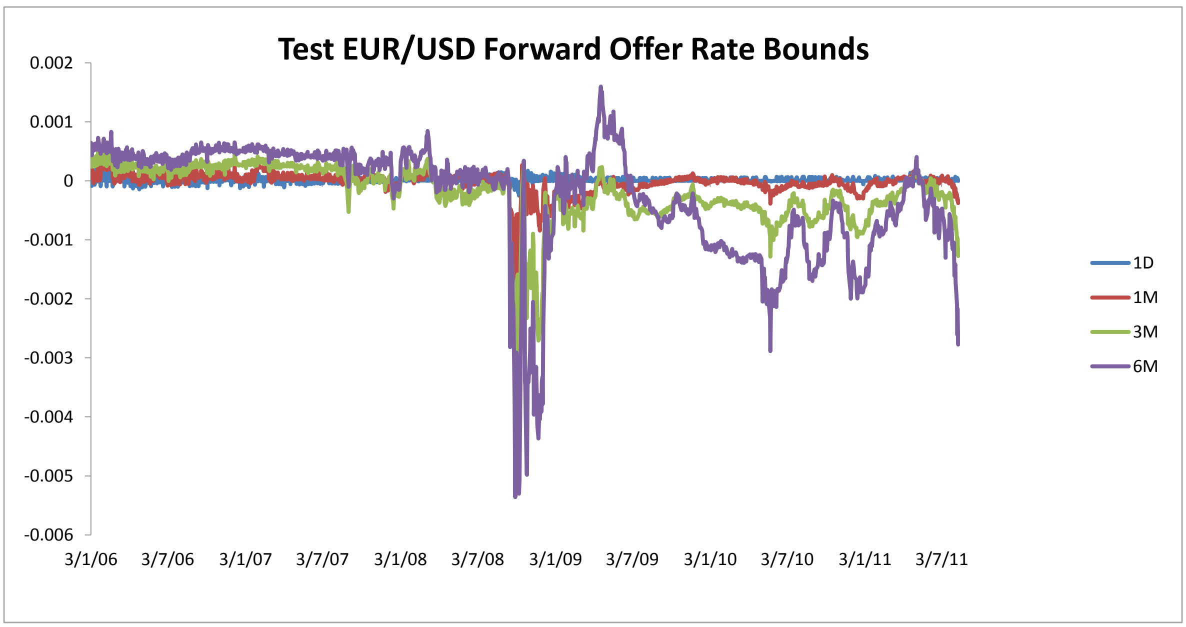
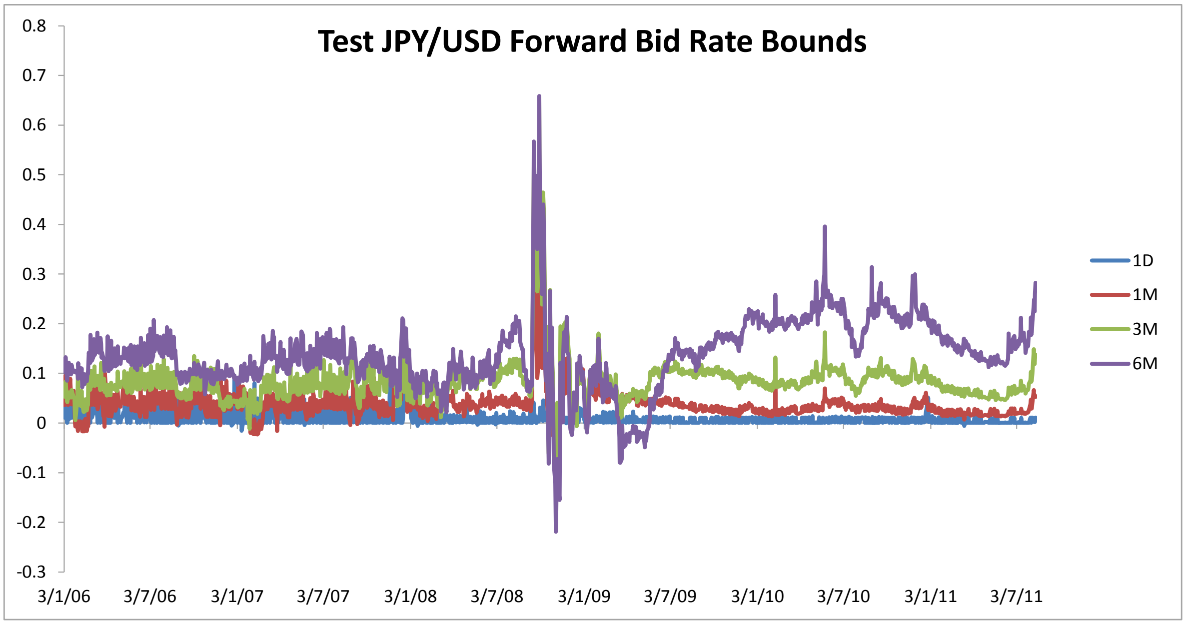


Figure 3.1: **Violations of no-arbitrage bounds of EUR/USD forward rates. Positive parts in the figure imply no arbitrage opportunities considering transaction costs. Negative parts of the figure indicate that arbitrage opportunities are present in the forward market.**



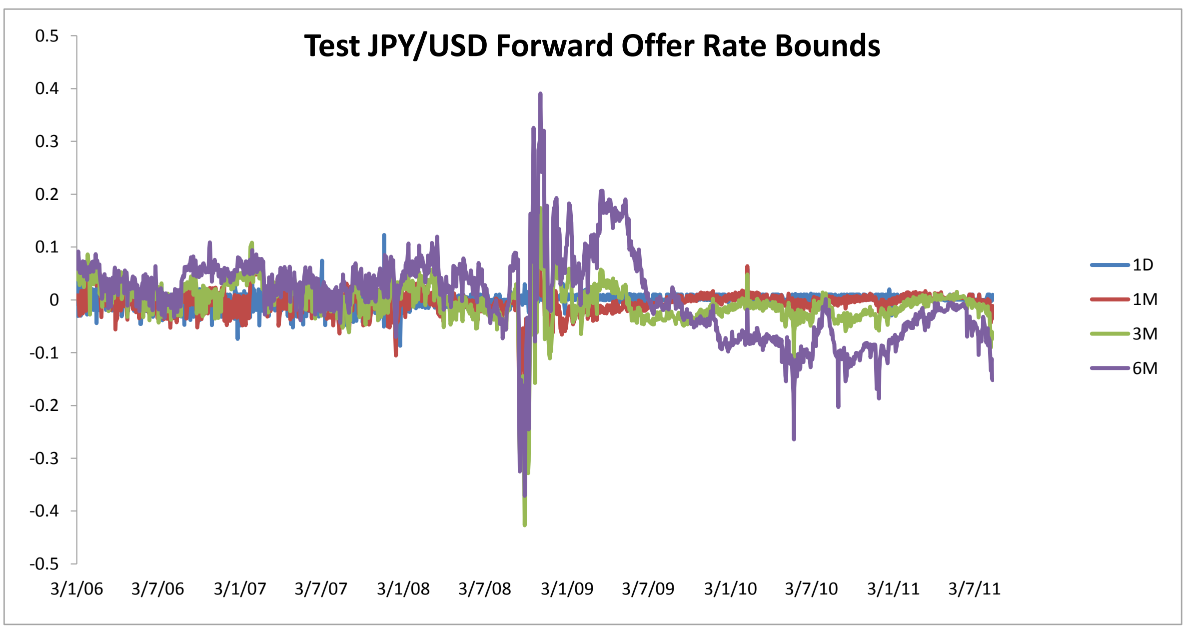


Figure 3.2: **Violations of no-arbitrage bounds of JPY/USD forward rates. Positive parts in the figure imply no arbitrage opportunities considering transaction costs. Negative parts of the figure indicate that arbitrage opportunities are present in the forward market.**

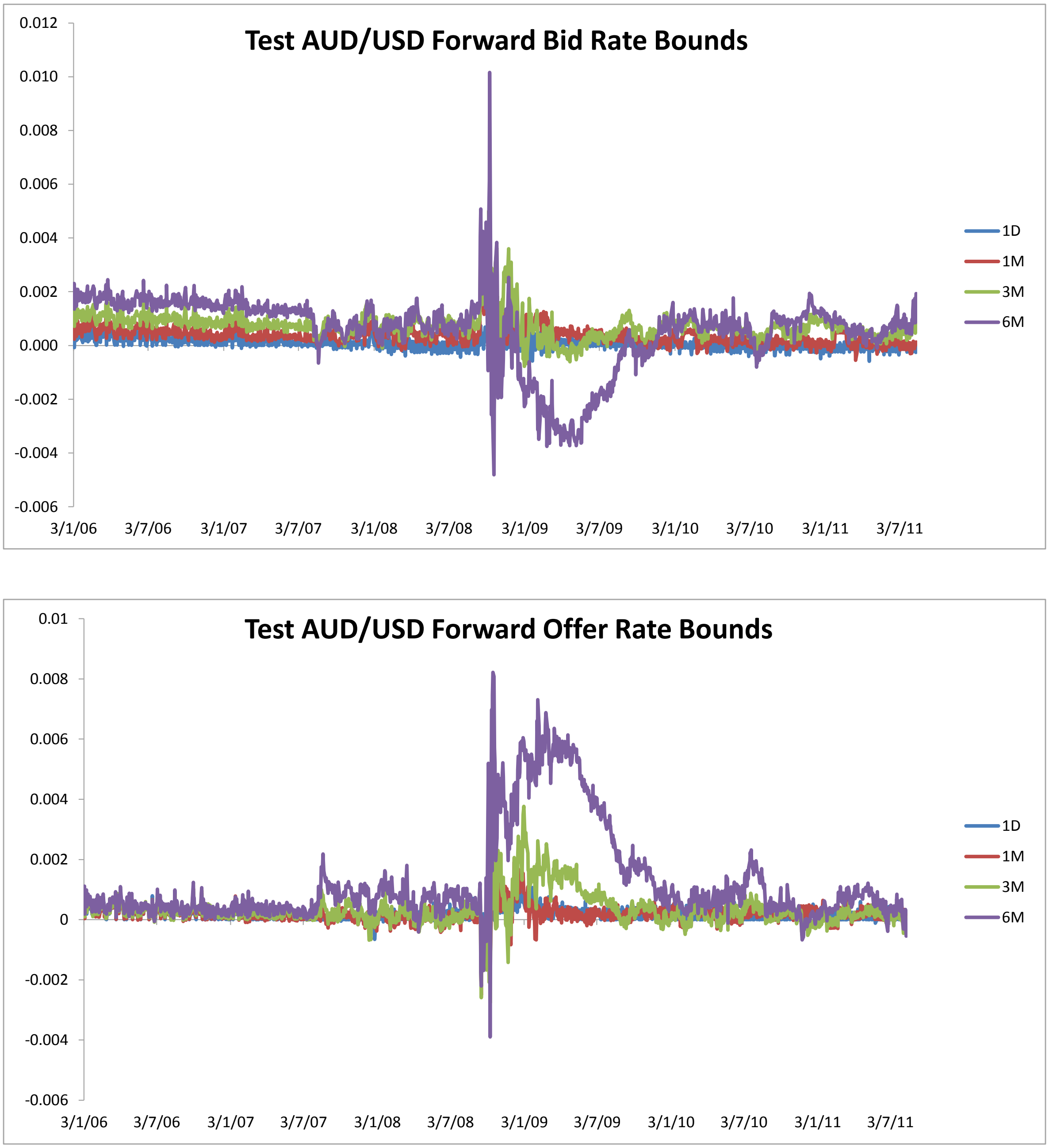


Figure 3.3: **Violations of no-arbitrage bounds of AUD/USD forward rates. Positive parts in the figure imply no arbitrage opportunities considering transaction costs. Negative parts of the figure indicate that arbitrage opportunities are present in the forward market.**

3.4.2 Currency Swap Basis Rates

To test the upper bound and lower bound of the CCBS basis rate, we calculate two quantities,

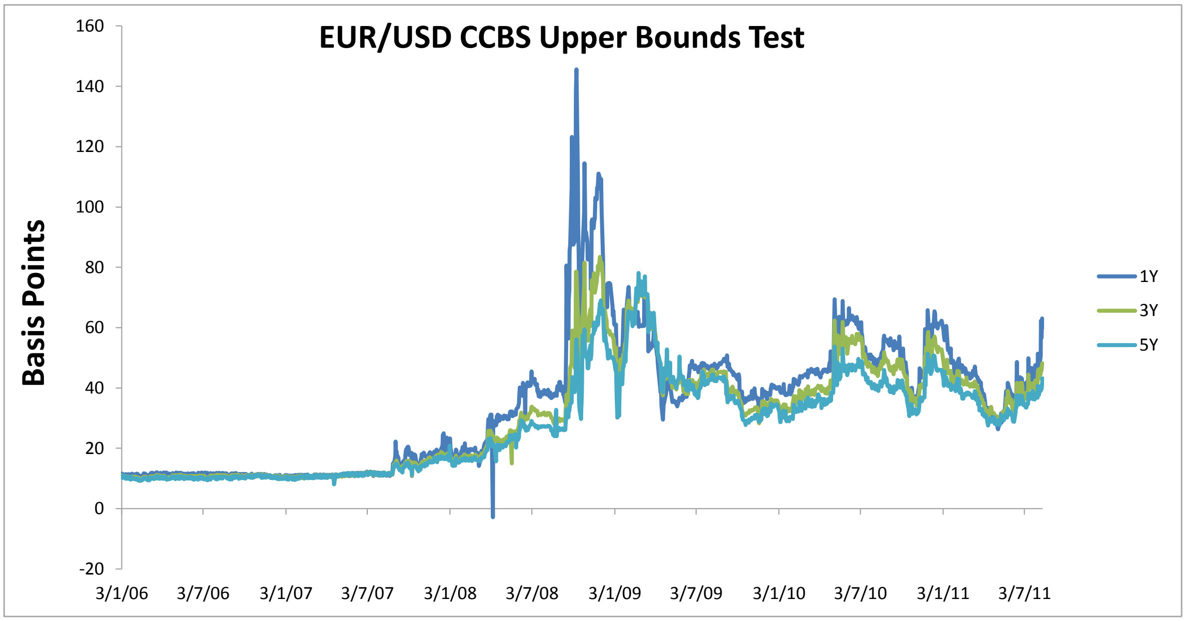
1) Upper bound minus the pay rate.

2) Receive rate minus the lower bound.

If bounds hold, these two quantities should not be negative. Results are presented in Figures 3.4, 3.5 and 3.6 for 1Y, 3Y and 5Y swaps of each currency pair[10]. Similar to what we find in the forward rate tests, the GFC is a destabilizing event for the bounds.

For the EUR/USD CCBS, we see in Figure 3.4 that both upper and lower bounds held before the crisis. Starting from October, 2007, two bounds went in the opposite directions. Both results showed significantly greater volatility. However, the upper bound result became even more positive for the whole post-crisis period whereas the lower bound result stayed mostly negative since the second half of 2008. As a result, the no-arbitrage lower bound for the receive rate has been consistently violated. The JPY/USD results in Figure 3.5 show some resemblances to the EUR/USD counterpart. Both upper and lower bounds held tightly before the GFC. For a majority of the post-crisis sample period, the lower bound is violated but the upper bound holds.

[10] For clarity of presentation, 2Y and 4Y results are omitted. The results are very similar to 1Y, 3Y and 5Y results.



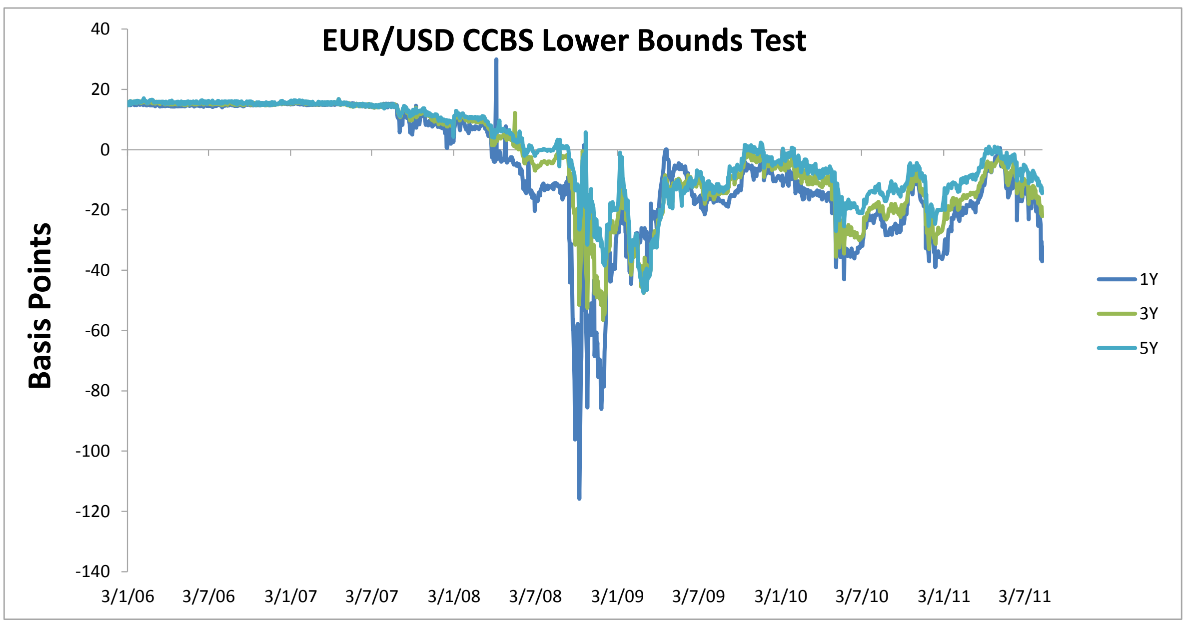
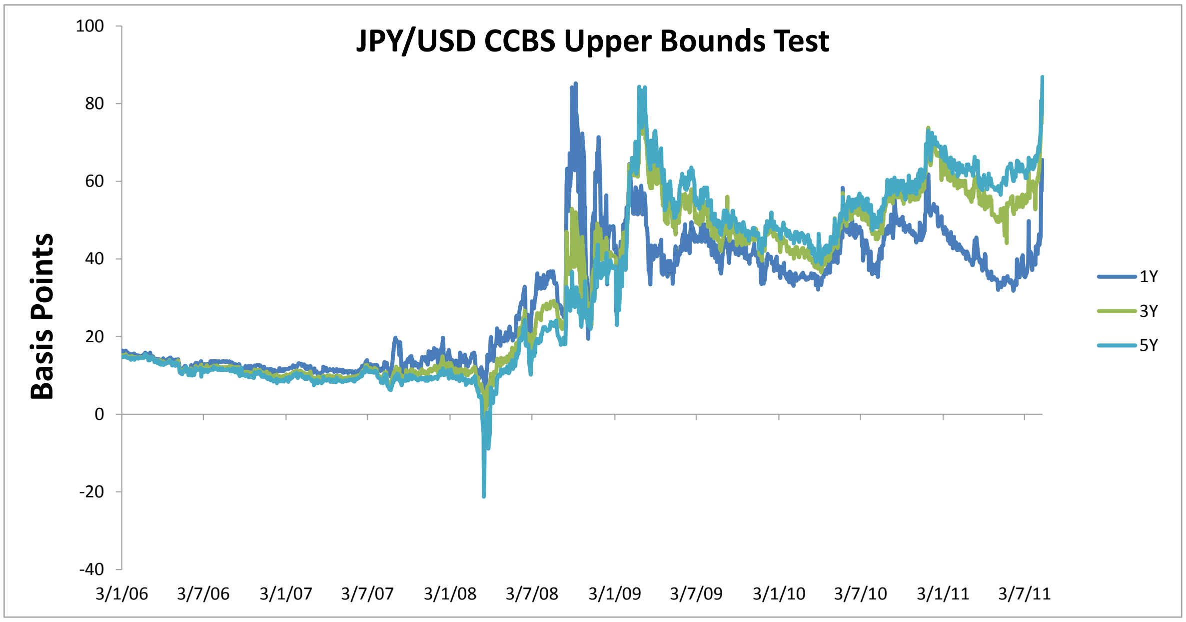


Figure 3.4: **Violations of no-arbitrage bounds of EUR/USD CCBS basis rates. Positive parts in the figure imply no arbitrage opportunities considering transaction costs. Negative parts of the figure indicate that arbitrage opportunities are present in the currency swap market.**



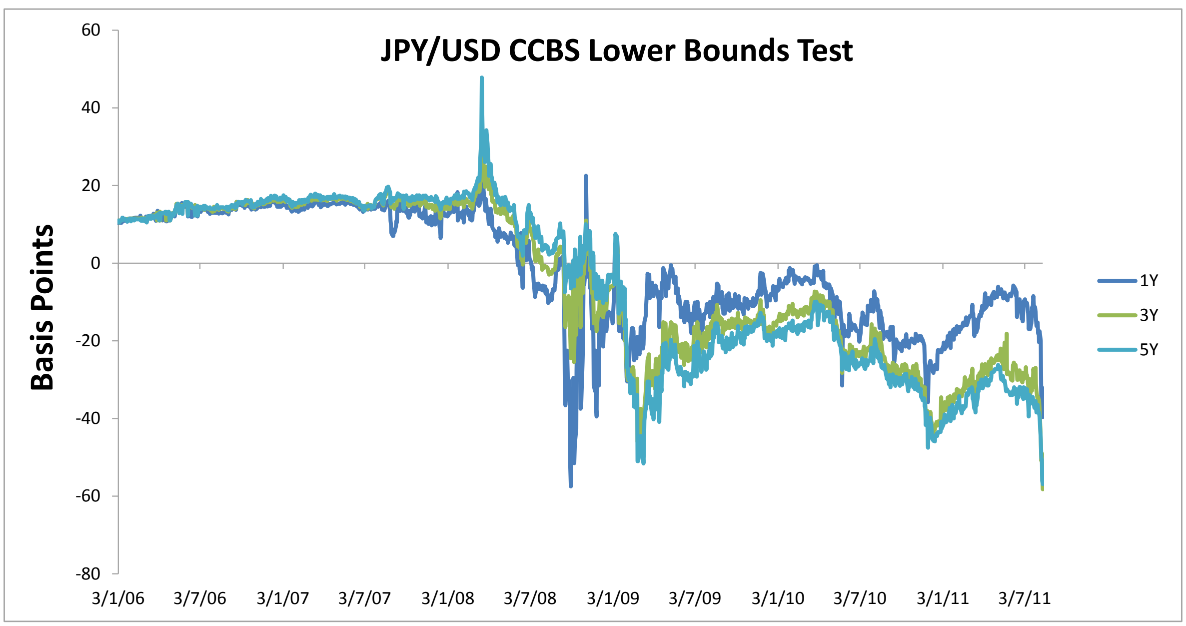
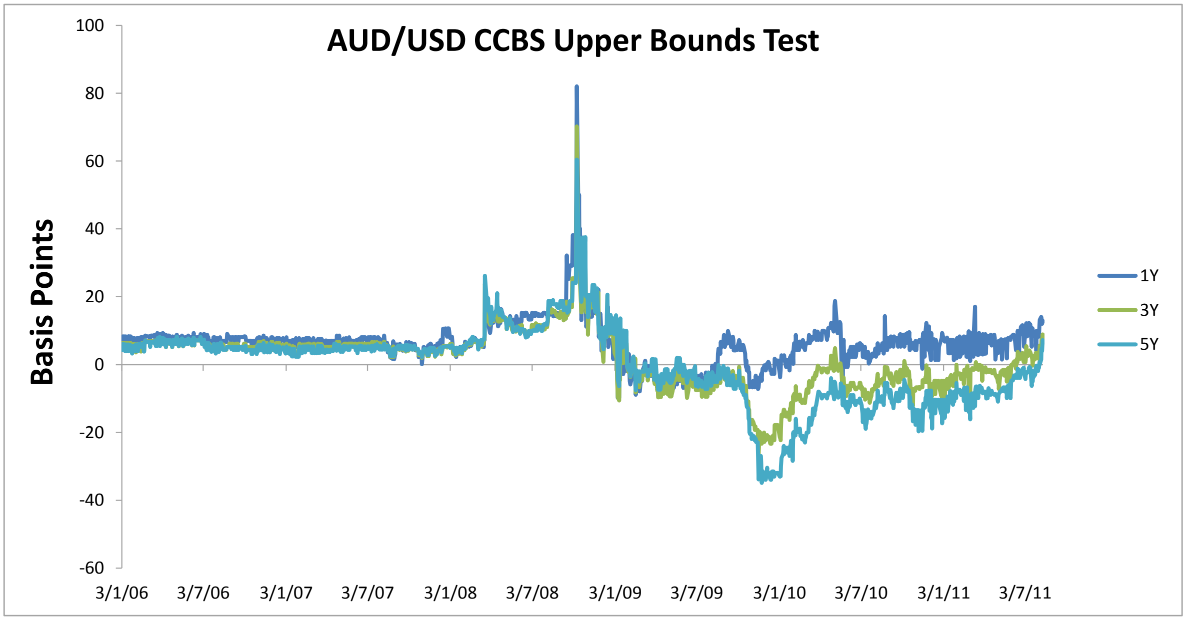


Figure 3.5: **Violations of no-arbitrage bounds of JPY/USD CCBS basis rates. Positive parts in the figure imply no arbitrage opportunities considering transaction costs. Negative parts of the figure indicate that arbitrage opportunities are present in the currency swap market.**



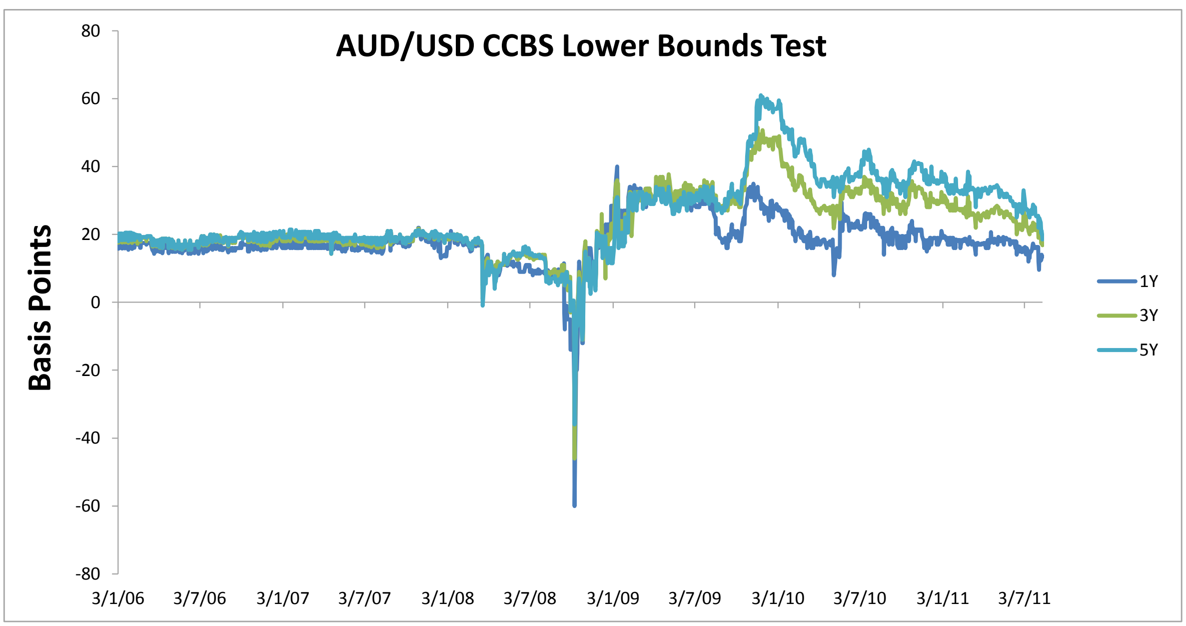


Figure 3.6: **Violations of no-arbitrage bounds of AUD/USD CCBS basis rates. Positive parts in the figure imply no arbitrage opportunities considering transaction costs. Negative parts of the figure indicate that arbitrage opportunities are present in the currency swap market.**

The results in Figure 3.6 show that the AUD/USD bounds responded to the GFC from March 2008. The immediate effect of the shock is that the lower bound was violatedbut the upper bound became stronger. However, since November 2008 the pattern has been completely different from EUR/USD and JPY/USD: the lower bound result remains positive and with the exception of 1Y maturity, the upper bound no longer holds.

3.4.3 Exploit Violations

If violations of no-arbitrage bounds do admit arbitrage opportunities, an arbitrageur can exploit and make riskless profits. We present two examples.

1) Forward Offer Rate

The lower bound of the 3M EUR/USD forward offer rate has been persistently violated since the crisis. For example, on September 30th 2008, the lower bound calculated by (3.16) should be 0.6996, but the market offer rate is 0.6958. The 3M USD LIBOR discount factor is 0.9900 and EURIBID discount factor is 0.9873.

The arbitrageur can borrow unit USD at USD LIBOR and sell for EUR in the spot market. The proceeds is 0.6977 EUR. EUR is then lent at EURIBID for 3 months. Simultaneously the arbitrageur enters a 3M forward contract to sell the EUR investment for USD and the rate $1/0.6958 = 1.4372$ is secured.

At maturity, the investment grows to 0.6977/0.9873 = 0.7067 EUR. It can be sold for USD at the forward rate and the proceeds is 1.0157 USD. The USD loan now grows to 1/0.9900 = 1.0101. The arbitrageur repays the loan and makes a profit of 0.0056 USD.

2) Currency Swap Basis Receive Rate

Violations of the lower bound of the receive rate of 5Y EUR/USD swap can be exploited. On March, 2nd of 2009 the lower bound by (3.13) should be -12.5 bps, but the observed receive rate is -60 bps. The arbitrageur should enter the 5Y EUR/USD CCBS in which he/she pays 3M EURIBOR minus 60 bps (receive rate for the market maker) and receives 3M USD LIBOR. At initiation the arbitrageur also pays the USD principal and receives the EUR principal. The principal amounts are based on the spot mid rate, say USD 1 million for EUR 0.7942 million. The USD principal is borrowed at USD LIBOR and EUR principal is invested at EURIBID. According to Table 3.5, the arbitrageur's net cash flow at each tenor is 47.5 bps of the EUR principal, which becomes the profit.

3.4.4 Making Sense of Violations

The bound violations contradict the no-arbitrage methodology in pricing forward rates and CCBSs. Clearly the persistence of the observed bound violations demonstrate that the market has not taken advantage of the apparent opportunities. We propose that the market is prevented from doing so by increased market imperfections, in particular the currency liquidity risk. These imperfections have developed in the forward and spot currency market since the GFC and result in forward and currency swap prices being determined by supply and demand pressures, rather than by arbitrage considerations.

In this study all forward rates are quoted as the price of unit USD in the domestic currency. If supply exceeds demand, the MM can buy the forward contract at a rate lower than the no-arbitrage level and the upper bound of the bid rate holds even tighter. On the other hand, the MM can afford to sell at a level lower than the no-arbitrage lower bound. We therefore propose that the lower bound of the offer rate can be violated if supply exceeds demand. This implies that the relative demand for USD is lower in the forward market than in the spot market, which is consistent with the USD liquidity risk. The USD funding shortage during the crisis has driven the demand to secure USD in the spot market, hence the demand for USD is relatively lower in the forward market. On the other hand, the upper bound violations of the AUD forward bid rate indicate that the relative demand for USD is lower in the spot market than in the forward market. This is also consistent with the observation in Ossolinski and Zurawski (2010) that Australian banks do not have USD funding shortage. Instead they have high demand of AUD funding in international markets.

The supply/demand imbalance also occurred in the CCBS market. Since the crisis, in the currency swap market the demand for USD borrowing far exceeds demand for EUR and JPY. The USD borrowers are willing to pay a liquidity premium to secure USD. Hence the lower bounds of EUR/USD and JPY/USD basis receive rates are substantially violated. On the other hand, the violation of the upper bound of AUD/USD basis pay rates implies that demand for AUD exceeds the demand for USD.

3.4.5 Limits to Arbitrage

If the no-arbitrage bound violations did represent practical opportunities, we would expect that arbitrageurs take large positions in forward contracts and currency swaps to make profit. Standard theories in finance, such as the Arbitrage Pricing Theory (Ross 1976), assume that such violations should be exploited and no-arbitrage equilibria should be quickly restored. However, since the crisis the violations have persisted. We propose that USD liquidity risk is the plausible factor which may have rendered textbook arbitrage strategies ineffective. For instance, our arbitrage example in CCBS assumes that the arbitrageur can borrow USD principal at USD LIBOR for 5 years. However, during the crisis, banks faced liquidity squeeze and were reluctant to make lending to LIBOR counterparties for longer than three months (see, for example, Mollenkamp and Whitehouse 2008). Consequently, although the market may appear rife with arbitrage opportunities, the strategy may break down.

Under extreme market circumstances, arbitrage is risky and ineffective. The theory of "limits of arbitrage" was first proposed in the seminal work of Shleifer and Vishny (1997). Arbitrageurs invest clients' money. In the short run, if mispricing persists due to uncertain market conditions, arbitrageurs may face margin calls or withdrawal of funds. To meet funding needs, arbitrageurs have to unwind positions at losses. A high profile example of 'limits to arbitrage' is the failure of Long Term Capital Management (LTCM) in 1998. The Russian government's default on its debt caused investors to panick and trade against LTCM's positions. Although the arbitrage strategy of LTCM was sound in principle, the bond prices which were supposed to converge in the long run were driven further apart. LTCM was forced to unwind postilions and eventually failed.

3.5 Conclusion

In this study we formulate no-arbitrage bounds for forward exchange rates and currency swap basis rates with transaction costs. We examine these bounds with market data and find that some bounds have been persistently violated since the GFC. In theory, violations admit arbitrage opportunities. However, in real markets such transactions may have been limited, due to heightened currency liquidity risk.

These arbitrage violations may carry information about supply and demand pressures in currency markets. In the next chapter, we explore whether this information may be helpful in resolving the uncovered interest parity (UIP) puzzle.

Chapter 4

Carry Trade and Liquidity Risk: Evidence from Forward and Currency Swap Markets

4.1 Introduction

This chapter empirically tests the effect of liquidity risk and volatility in the FX market on the performance of currency carry trades. In a carry trade investors borrow funds in a low-yield currency (funding currency) and lend in a high-yield currency (investment currency) to gain the interest rate differential. The uncovered interest parity (UIP) predicts that on average the carry should be exactly offset by the depreciation of the investment currency. If UIP holds in practice, carry trades should have zero expected excess return. In practice, UIP often fails and investment currencies have been found often to actually appreciate on average against funding currencies, this has been termed the UIP puzzle.

Researchers have tackled the UIP puzzle from various approaches, see Section 2.4 for a detailed review of the related literature. Among them, Brunnermeier, Nagel and Pedersen (2008) employ a model with funding liquidity risk to approach the UIP puzzle by studying carry trade performance. In this model carry trades tend to be unwound and incur losses when traders' funding constraints become binding. Therefore proxies for liquidity risk should be significant in explaining carry trade excess returns. These findings provide support for the theoretical liquidity model developed in Brunnermeier and Pedersen (2009). A key result is that after controlling for liquidity proxies, the interest rate differential is not significant in predicting carry returns, which points to a potential resolution of the UIP puzzle. However, the statistical significance of Brunnermeier et al. (2008) is not sufficiently strong. In particular, the liquidity proxy TED spread, is in general insignificant or only marginally significant. Nevertheless Brunnermeier et al. (2008) offers an alternative approach to explaining the UIP puzzle.

We propose that the liquidity measures employed by Brunnermeier et al. (2008), VIX and the TED spread, may not proxy FX market liquidity risk well. Firstly, VIX, which is the implied volatility index for stock options, is not directly related to the FX market. Burnside (2008) also points out that the link between VIX and the FX market is less clear if changes in VIX are interpreted as changes in implied or actual volatility of the underlying asset. Hence VIX is not an ideal measure of FX market liquidity risk. Secondly, the TED spread is also not specific to the FX market. We conjecture that these issues may have caused the lack of statistical significance. We therefore are motivated to identify alternative proxies of FX market liquidity to tackle the UIP puzzle. In addition, we build upon the theoretical liquidity model of Brunnermeier and Pedersen (2009) and allow for the effect of market volatility.

We propose a FX market specific proxy for liquidity risk - violations of no arbitrage bounds in the forward and currency swap markets - which measures market expectation of future liquidity risk. The bounds are examined with market data in Chapter 3 and results demonstrate that the no-arbitrage bounds held strongly before the GFC, but were substantially violated after the crisis. By construction, we propose that this proxy incorporates both market liquidity risk and funding liquidity risk. Market liquidity risk is proxied by transaction costs, i.e. bid-ask spreads quoted in the forward and currency swap contracts, whereas funding liquidity risk is proxied by the magnitude of violations of the no-arbitrage bounds. In this study we specifically examine if this alternative liquidity proxy can better explain the UIP puzzle by testing its effect on carry trade excess returns.

To allow for the effect of volatility specific to the FX market, we use volatility smile data from the FX option market. Volatility smile information represents market participants' views of future volatility of the underlying exchange rate until the option maturity. We propose that this proxy should have a significant effect on explaining exchange rate movement and hence carry trade return.

Our sample data cover periods both before and after the GFC because both liquidity risk and market volatility have substantially heightened since the GFC. It is thus interesting to investigate if there are structural breaks in the effects of our proposed proxies.

Econometric results demonstrate that both proxies have significant effects on carry trade performance, particularly after the GFC. Furthermore, the interest-rate differential is not significant in predicting carry trade returns after controlling for volatility and liquidity, hence providing a potential resolution of the UIP puzzle.

The remainder of this chapter is organized as follows. Section 2 describes the data. Section 3 develops methodologies used for our econometric tests. Section 4 presents and analyzes results and Section 5 concludes.

4.2 Data

The data on our proxy of FX liquidity risk - violations of no arbitrage bounds in the forward and currency swap markets - are based upon the results of Chapater 3.

For FX market volatility, we consider two measures: realized volatility of exchange rates and FX option market volatility smile. Due to lack of higher frequency data, we can only calculate realized volatilities with daily data. The common practice in calculating daily volatility is to square that day's return (see, for example, Brooks 2008). This way of computing volatility with daily data depends on the assumption that the expected daily return is zero. However, the squared daily return is a potentially poor proxy for the true volatility and ideally we should compute the volatility by using higher frequency, intra-day data (see Andersen and Bollerslev 1998). We therefore decide not to use this measure and instead collect volatility proxies from FX option markets. To better understand the mechanics of FX volatility smiles, we describe basic terminologies and conventions.

4.2.1 FX Option Pricing Formula

Based upon the seminal work of Black and Scholes (1973), Garman and Kohlhagen (1983) model the underlying spot exchange rate as a geometric Brownian Motion

$$dS_t = (r_d - r_f)S_t dt + \sigma S_t dW_t, \tag{4.1}$$

where S_t is quoted as FOR-DOM (foreign-domestic) (see Wystup 2006) and represents the price of one unit of foreign currency in terms of domestic currency. r_d (r_f) denotes the continuously compounded domestic (foreign) risk free rate. σ is the volatility of underlying asset and W_t is a standard Brownian Motion. An application of Iτ̂o's formula (Iτ̂o 1944) to the natural logarithm of S_t produces the solution for the geometric Brownian Motion under the domestic risk-neutral measure P^d:

$$S_t = S_0 \cdot e^{((r_d - r_f - \frac{1}{2}\sigma^2)t + \sigma W_t)}. \tag{4.2}$$

Under the domestic risk-neutral measure P^d, the current value of an European call option is the discounted expected value of the final payoff:

$$
\begin{aligned}
V_t &= e^{-r_d \tau} E^d[(S_T - K)^+] \\
&= e^{-r_d \tau} E^d[(S_T - K) I_{(S_T \geq K)}] \\
&= e^{-r_d \tau} E^d[S_T I_{(S_T \geq K)}] - K e^{-r_d \tau} E^d[I_{(S_T \geq K)}] \\
&= e^{-r_d \tau} E^d[S_T I_{(S_T \geq K)}] - K e^{-r_d \tau} P^d[S_T \geq K], \qquad (4.3)
\end{aligned}
$$

where τ is the time to maturity and I is the indicator function. The change of measure from domestic risk-neutral to foreign risk-neutral measure P^f via Radon-Nikodym derivative yields

$$
V_t = S_t e^{-r_f \tau} P^f[S_T \geq K] - K e^{-r_d \tau} P^d[S_T \geq K]. \qquad (4.4)
$$

We compute probabilities in Eqn. (4.4) and obtain the Garman-Kolhagen formula for a vanilla European option:

$$
V_t = \omega S_t e^{-r_f \tau} N(\omega d_1) - \omega K e^{-r_d \tau} N(\omega d_2). \qquad (4.5)
$$

In Eqn. (4.5), ω is 1 for a call and -1 for a put. $N(x) = \int_{-\infty}^{x} n(u)\, \mathrm{d}u$ and $n(x) = \frac{1}{\sqrt{2\pi}} e^{(\frac{-x^2}{2})}$. $N(x)$ is the cumulative distribution function (CDF) and $n(x)$ is the probability density function (PDF) for the standard normal distribution $N \sim (0,1)$. Lastly,

$$d_{1,2} = \frac{ln(\frac{S_t}{K}) + (r_d - r_f \pm \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. \tag{4.6}$$

The Garman-Kolhagen formula is often expressed as

$$V_t = \omega e^{-r_d\tau}[F(t,T)N(\omega d_1) - KN(\omega d_2)], \tag{4.7}$$

where $F(t,T) = S_t e^{(r_d - r_f)\tau}$ is the forward price of the underlying exchange rate at time t, with maturity T and (4.6) is rewritten as

$$d_{1,2} = \frac{ln(\frac{F(t,T)}{K}) \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}. \tag{4.8}$$

Formula (4.7) recovers the Black (1976) model, in which the forward process $F_{(t,T)}$ is modeled as a driftless geometric Brownian motion.

4.2.2 FX Volatility Smile

Although the Black-Scholes model (or Garman-Kolhagen formula for FX options) is a benchmark for the industry, the assumption of constant volatility throughout all maturities and all moneyness levels (deltas) is clearly a limitation. A constant volatility σ implies that the volatility surface, when plotted as a function of time to maturity and delta, should be flat. However, the implied volatility levels for out-of-money (OTM) options, generated by market data, are usually higher than for the at-the-money (ATM) options based upon the same underlying asset. Hence, the two-dimensional plot of implied volatility as a function of option deltas, takes a “smile” shape.

The volatility smile arises from a key limitation of Black-Scholes model: underlying assets follow a lognormal distribution. In FX markets, the natural logarithm of exchange rate returns is in general not normally distributed (see Hull 2008). Instead, the distribution is leptokurtic in the sense that it exhibits excess kurtosis, hence excess probability mass in tails and more peakedness around the mean of the distribution. Leptokurtic distribution of exchange rates is consistent with empirical observations in the FX market, which often experiences extreme moves. For instance, Clark (2010) notes that the likelihoods for rare events (beyond ± 3 standard deviations) under leptokurtic distribution are considerably greater than under normal distribution.

A natural consequence of the excess kurtosis of exchange rates is that FX option traders place higher level of probabilities for “tail events” than assumed by Black-Scholes model. Therefore the implied volatility are higher for OTM options than for ATM options. If the market takes no view on the movement of underlying exchange rate, the volatility smile is symmetric around the ATM option volatility. Otherwise, the smile is skewed either to the left or right, depending on the market expectations. Below is an illustration of volatility smile from 1M EUR/USD options as at May 14th, 2007.

Figure 4.1: **Volatility Smile of 1-month EUR/USD options. Source: Bloomberg Finance L.P.**

We see from Figure 4.1 that both OTM call and OTM put have higher implied volatilities than ATM option. However, the smile is not symmetric and the volatility of an OTM call is greater than the corresponding OTM put, such as the 25-delta (25-d) options. Options' time value is an increasing function of volatility, hence in this scenario the 25-d call is more valuable than the 25-d put. The call (put)option is quoted as EUR call(put)/USD put(call), therefore the market expects that EUR will appreciate against USD. We also find the implied volatilities are plotted against delta (moneyness) levels rather than against strikes. Strike level is not chosen to parameterize volatility smiles because a particular strike may correspond to options with different deltas. For example, depending on time to maturity, an OTM option strike for a small τ may be close to an ATM strike for a larger τ.

The strike K of an ATM FX option is equal to the forward rate $F(t,T)$. Differentiating (4.7) w.r.t the forward rate we get the delta[1]:

$$\frac{\partial V}{\partial F} = \omega e^{-r_d \tau} N(\omega d_1) = \omega e^{-r_d \tau} N(\omega \frac{1}{2} \sigma \sqrt{\tau}). \tag{4.9}$$

From (4.9) we see that for a relatively small volatility σ and time to maturity τ, the ATM call (put) delta is approximately 50% (-50%). This is consistent with Figure 4.1 where ATM options corresponds to approximately 50% absolute delta. OTM options therefore have less than 50% absolute delta. The smile usually quotes the volatility levels for 25-d options. In order to capture the skewness and kurtosis of the smile, market also quotes risk reversals (RR) and butterflies (BF). RR is defined as the implied volatility of OTM call options minus the implied volatility of OTM put options with the same absolute delta, which is a measure of the skewness of the smile and can be interpreted as the market view of the direction of the spot exchange rate movement until option maturity. This can be seen from Figure 4.1. A positive (negative) RR indicates that the option market participants place a higher value on the call (put) option and expects the currency on the call (put) side of the quote to appreciate against the currency on the put (call) side. On the other hand, BF measures the kurtosis of the smile and represents the market view

[1]See the Appendix B for the proof.

on the likelihood of large moves in the spot price towards either direction. It is defined as the difference between the average volatility of the OTM call and the OTM put and the corresponding ATM volatility. A high (low) BF indicates the market expectation of higher (lower) volatility in the exchange rate movements until option maturity. By using 25-d OTM options, the relationship between ATM volatility ($ATMVOL$), RR and BF is as follows:

$$RR = \sigma_{(25-d-call)} - \sigma_{(25-d-put)}, \tag{4.10}$$

$$BF = \frac{\sigma_{(25-d-call)} + \sigma_{(25-d-put)}}{2} - ATMVOL, \tag{4.11}$$

$$\sigma_{(25-d-call)} = ATMVOL + BF + \frac{1}{2}RR, \tag{4.12}$$

$$\sigma_{(25-d-put)} = ATMVOL + BF - \frac{1}{2}RR. \tag{4.13}$$

Graphically, the relationship between $ATMVOL$, RR and BF can be illustrated in Figure 4.2 (source: Wystup 2006). From the above description of the FX option volatility smile conventions we see that $ATMVOL$, RR and BF provide us with three measures of FX market volatility. $ATMVOL$ measures the market expectation of future volatility of underlying exchange rate until the option's maturity, while RR and BF respectively measure the skewness and kurtosis of the volatility smile. It is intuitively appealing to investigate whether these proxies have predictive power on spot exchange rate movements,

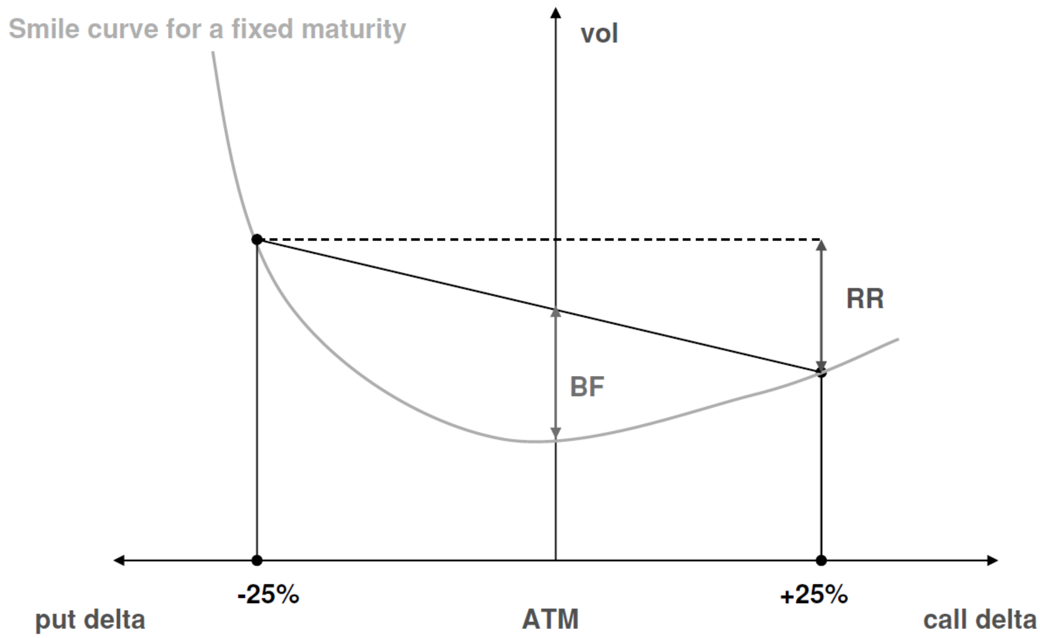


Figure 4.2: **ATM VOL, RR and BF for a given FX vanilla option**

hence affecting carry trade excess returns. To this end, we collect the most liquid 1M $ATMVOL$, 25-d RR and 25-d BF data on EUR/USD, JPY/USD and AUD/USD options from Bloomberg. Following Christiansen et al. (2011), we collect daily closing data. The sample period is from January 3, 2006 to August 12, 2011.

4.3 Empirical Methodology

4.3.1 Specifications of Model Variables

We follow Brunnermeier et al. (2008) and calculate the carry trade return in excess of the return predicted by UIP, hence the abnormal return. We follow the FOR-DOM quotation style and treat USD as the foreign currency in all three pairs. In each currency carry trade, USD is the funding currency. Hence funds are borrowed at the USD money market rate and invested at the EUR, JPY and AUD interest rates respectively. The daily excess return Z_t^k is thus

$$Z_t^k = (r_{t-1}^k - r_{t-1}^{USD}) - (X_t^k - X_{t-1}^k). \tag{4.14}$$

In Eqn. (4.14) X_t^k is the logarithm spot exchange rate at the end of day t for currency k and r_{t-1}^k is the continuously compounded overnight money market rate at the end of day $t-1$ for currency k. Hence the excess return is equal to the interest rate differential between currency k and USD locked in at the end of day $t-1$, minus the return of the spot rate from day $t-1$ to t. The term $X_t^k - X_{t-1}^k$ stands for the depreciation of currency k against USD. The UIP predicts that the interest rate differential $r_{t-1}^k - r_{t-1}^{USD}$ should be exactly offset by the depreciation of currency k, hence $E_{t-1}(Z_t) = 0$. The UIP puzzle arises from empirical observations that the abnormal return Z_t^k is often positive, i.e. investment currencies do not depreciate as much as predicted by UIP, instead in many cases they appreciate against the funding currency.

We define the violations of no-arbitrage bounds for forward rates and currency swaps as liquidity basis BS, which is the proxy for USD liquidity risk. In Subsection 4.4 of Chapter 3 we proposed that violations of the no-arbitrage lower bound of the forward offer rate and of the cross-currency basis swap receive rate represent the higher demand of USD borrowing in the spot market, hence the increased USD liquidity risk. Therefore, we calculate BS for each currency pair as follows. We use the observed rate to subtract the lower bound. Negative results imply that the lower bounds are violated. We then change the sign of all results. For example, if the quoted basis receive rate for a 5Y EUR/USD currency swap is -30 bps and the no-arbitrage lower bound is -12.5 bps, then BS is 17.5 bps after the sign change. The reason we change the sign is for the ease of the interpretation of the estimated coefficient of BS in the test of the econometric model (proposed in Subsection 4.3.2). The more positive the BS, the higher the USD liquidity risk. Therefore the sign of the estimated coefficient of BS can be directly interpreted as the effect of USD liquidity risk on the excess return of the carry trade Z_t.

The maturities of BS include 1M, 3M and 6M for forward rates and 1Y, 2Y, 3Y, 4Y and 5Y for currency swaps for each currency pair. We conjecture that BS of different maturities are closely related to each other because they are driven by some common liquidity events. They are not sufficiently independent of each other and from a statistical point of view, it is sensible to include all maturities in our empirical tests. Since there are 8 maturities, in order to reduce dimensionality we employ the principal component analysis (PCA) to transform these closely related variables into uncorrelated new variables [2].

4.3.2 Econometric Model

To test the effects of FX market volatility and liquidity basis on the excess return of carry trades, we propose a linear factor distributed lag model for each currency pair (i.e. USD/JPY, USD/EUR and USD/AUD):

[2] See Appendix C for a brief introduction of PCA.

$$Z_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 IRDIFF_{t-1} + \beta_3 ATMVOL_t + \beta_4 ATMVOL_{t-1} + \beta_5 RR_t + \beta_6 RR_{t-1} + \beta_7 BF_t + \beta_8 BF_{t-1} + \beta_9 BS_t + \beta_{10} BS_{t-1} + \epsilon_t, \quad (4.15)$$

where Z_t is the abnormal return of carry trade between day $t-1$ and t, $IRDIFF_{t-1}$ is the overnight interest rate difference at the end of day $t-1$ between the investment currency and USD, $ATMVOL_t$, RR_t and BF_t are respectively the ATM volatility, 25-delta risk reversal and 25-delta butterfly at the end of day t for the 1M FX option, quoted as USD/investment currency. Lastly, BS_t is the liquidity basis based on PCA at the end of day t. We include one-day lags to account for potential autocorrelations in the exchange rates. The inclusion of lags in the regression model also enables us to capture both inertia of the dependent variable to explanatory factors and contemporaneous effects.

Compared to related literature (Brunnermeier et al. 2008, Ranaldo and Söderlind 2010), variables in our regression model may better proxy risk factors. Firstly, $ATMVOL_t$, RR_t and BF_t, which represent FX option market practitioners' expectations. Therefore they should better measure FX market volatility. Secondly, TED spread is also not specific to the FX market. This may potentially explain the lack of significance of TED in related works. In our study TED is replaced with a new measure, liquidity basis of forward exchange rates and currency swaps, which captures both market liquidity and funding liquidity risks specific to the FX market.

We make several hypotheses with respect to the effects of explanatory variables. Firstly, the effect of $ATMVOL_t$ is negative if the investment currency on average depreciates against USD when FX market volatility increases. Secondly, RR is negatively correlated with Z_t because FX options in our study are quoted as USD/investment currency. If RR increases, then the USD call is more favored than the USD put, hence the market expects that the investment currency depreciates against the USD. Thirdly, we take an agnostic view on butterfly because higher BF indicates larger movements of exchange rates towards either direction. Fourthly, we expect that BS negatively impacts Z_t if carry traders unwind positions when liquidity risk increases, hence suffering losses. Finally, after controlling for these factors, if $IRDIFF$ is not significant in affecting carry returns, we have contributed to the resolution of the UIP puzzle.

4.3.3 Panel Regression

The model in Eqn. (4.15) is used to test effect of explanatory variables for each individual currency pair. In order to control for individual currency heterogeneity we also construct panel data, which combine both time series of all variables in the distributed lag model and cross-sectional elements, namely three currency pairs. Compared to the distributed lag model, which is based on pure time-series data for each currency pair, panel data offer several benefits. Baltagi (2008) summarizes the advantages of panel data over time-series data as,

(1) Panel data control for individual heterogeneity and account for any common struc-

tures present, hence avoid the risk of obtaining biased results.

(2) Panel data present more information, more variability, more degrees of freedom and less collinearity. The latter is especially important because time-series data are often characterized by multicollinearity. Hence the problem of multicollinearity can be mitigated with panel data.

(3) The appropriate structure of panel data can remove certain forms of omitted variables bias.

We construct a balanced panel dataset with same number of time-series observations for each currency pair. The panel technique we employ is the currency-fixed effects model. In this panel regression, we allow the intercept to differ cross-sectionally but not over time, and the slope coefficients are fixed both cross-sectionally and over time. We follow Brooks (2008) to describe the properties of panel regression with cross-sectionally fixed effects. The model is set up as

$$y_{it} = \alpha + \beta x_{it} + u_{it}, \tag{4.16}$$

where y_{it} is the dependent variable, α is the constant term, β is a $k \times 1$ vector of parameters to be estimated on the independent variables and x_{it} is a $1 \times k$ vector of independent variable observations, $t = 1, ..., T$ is the index of the time series and $i = 1, ..., N$ is the index of cross sections. The total number of observations in the panel regression is thus

$N \times T$. In our study $N = 3$ and $T = 1419$.

To estimate the fixed effects cross-sectionally, we decompose the error term u_{it} into an individual effect μ_i, which encapsulates all information that affects the dependent variable cross-sectionally but does not vary over time; and the "remainder disturbance" term ν_{it}. ν_{it} is allowed to vary both over time and cross sectionally. The regression model (4.16) is rewritten as

$$y_{it} = \alpha + \beta x_{it} + \mu_i + \nu_{it}. \tag{4.17}$$

This model is estimated by the least squares dummy variable (LSDV) approach:

$$y_{it} = \beta x_{it} + \mu_1 D1 + \mu_2 D2 + \mu_3 D3 + \cdots + \mu_N DN + \nu_{it}. \tag{4.18}$$

where $D1$ is the dummy variable that takes the value of 1 for all observations on the first cross-sectional entity and 0 otherwise. $D2$ is the dummy variable that takes the value of 1 for all observations on the second cross-sectional entity and 0 otherwise, and so on. The constant term α does not appear in the equation to avoid the problem of perfect multicollinearity between the dummy variables and the intercept. In Eqn. (4.18), there are $N + K$ parameters to estimate.

4.4 Results

4.4.1 Principal Component Analysis

Table 4.1 shows the results of PCA based upon the correlation matrix for the liquidity basis. The first PC explains 84.42% of total variance of liquidity basis for EUR. For JPY and AUD, two PCs are needed to explain at least 80% of total variance. In order to reduce dimensionality and ease the interpretation, we take the first PC for EUR, but first and second PC for JPY and AUD respectively. The factor loadings on the first PC all take positive values and are approximately equally weighted for maturities beyond one year. We interpret that the first PC captures the common level change of the bases and the second PC represents the slope change of the term structure of the bases.

Table 4.1: Principal Component Analysis

	JPY	EUR	AUD
Eigenvalue number 1	0.6717	0.8442	0.5926
Eigenvalue number 2	0.1984	0.0978	0.2596
cumulative variance explained	0.8702	0.9420	0.8520

4.4.2 Summary Statistics

Table 4.2, 4.3 and 4.4 show the summary statistics of each variable for each currency pair, both before and after the crisis. We split the full sample period at August 10th of 2007 and treat the period before and after as two sub-samples, in order to capture potential structural breaks in the relationship between carry trade return and explanatory variables.

From mid-August 2007 the financial markets started to experience turmoils (see Baba and Packer 2009). Our calculations of liquidity bases also show that in the forward and currency swap markets, the no-arbitrage bounds of JPY and EUR started to be violated around this time[3].

In Table 4.2 we see that for the USD/JPY pair, all variables show large movements in level and standard deviation after the crisis. For example, the mean of the first PC of the liquidity basis increased by 55 bps. The standard deviation of the basis surged to more than 10 times its level before the crisis. Similarly, FX option market volatility variables: $ATMVOL$, RR and BF, also significantly changed in both mean and standard deviation. The interest rate difference substantially decreased after the crisis, due to the stimulatory policies taken by the US Federal Reserve. The carry trade excess return on average has been profitable after the crisis, due to the much smaller interest rate difference and the substantial appreciation of JPY against USD. The results are similar in the EUR and AUD summary statistics. The standard deviation of the first PC of EUR liquidity basis increased about 25 times after the crisis, and about 10 times for AUD basis. FX option volatility measures in both currencies demonstrate much greater variations. In all three currency pairs, most of the variables experienced greater kurtosis, reflecting large market movements and heightened uncertainties since the crisis. Finally, for all the variables (except EUR RR and first PC of AUD liquidity basis before the crisis), normal distribution is rejected at 5% significance level. The summary statistics clearly show different dynamics of the variables before and after the crisis, which provide us with further motivation to investigate if there exist structural changes.

[3]The AUD bounds violations started from March of 2008.

Table 4.2: JPY Summary Statistics

Variables	Z	IRDIFF	ATMVOL	BF	RR	BS1	BS2
Before Crisis							
Mean	-0.0001	-0.0479	8.0525	0.1766	-0.9864	-31.23	8.89
Median	-0.0007	-0.0482	8.0500	0.1750	-0.8250	-31.84	9.23
Maximum	0.0250	-0.0424	11.4750	0.2500	-0.1485	-23.37	15.84
Minimum	-0.0160	-0.0526	5.7750	0.1250	-2.5740	-37.24	-0.82
Std. Dev.	0.0055	0.0019	1.2122	0.0355	0.4714	3.50	3.22
Skewness	0.7506	0.7178	0.2854	-0.1158	-0.8677	0.39	-0.61
Kurtosis	4.3516	3.4778	2.4602	1.7887	3.5067	1.94	3.08
Jarque-Bera	69.03	38.72	10.44	25.73	55.29	29.22	25.28
Probability	0.0000	0.0000	0.0054	0.0000	0.0000	0.0000	0.0000
After Crisis							
Mean	0.0004	-0.0091	13.2211	0.2892	-2.3635	24.24	-7.95
Median	0.0003	-0.0013	12.1500	0.2990	-1.9650	32.40	-9.82
Maximum	0.0538	0.0036	43.0000	0.8787	0.9000	116.86	40.95
Minimum	-0.0348	-0.0584	6.3500	0.1260	-10.4000	-76.42	-40.65
Std. Dev.	0.0081	0.0143	3.9948	0.0771	2.0261	35.72	13.88
Skewness	0.3828	-1.5297	2.3474	2.0217	-1.7958	-0.47	0.33
Kurtosis	6.2908	4.0158	12.0836	15.3900	6.6356	2.18	2.32
Jarque-Bera	481.35	438.19	4408.64	7162.52	1101.28	65.66	38.51
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4.3: EUR Summary Statistics

Variables	Z	IRDIFF	ATMVOL	BF	RR	BS1
Before Crisis						
Mean	0.0003	-0.0193	6.9864	0.1298	-0.1880	-31.69
Median	0.0004	-0.0197	6.9375	0.1250	-0.2000	-31.65
Maximum	0.0177	-0.0110	9.6750	0.1800	0.2500	-28.55
Minimum	-0.0106	-0.0262	4.6500	0.0957	-0.7500	-36.58
Std. Dev.	0.0043	0.0039	1.3141	0.0160	0.1831	1.2618
Skewness	0.4067	0.2218	0.1295	0.1896	0.0658	-0.3058
Kurtosis	4.0785	2.0101	1.8180	2.1343	3.3733	3.1645
Jarque-Bera	30.87	19.91	24.77	15.11	2.65	6.78
Probability	0.0000	0.0000	0.0000	0.0005	0.2657	0.0336
After Crisis						
Mean	0.0001	0.0058	12.4031	0.3126	0.7642	25.39
Median	0.0004	0.0040	11.2898	0.2625	0.6840	24.27
Maximum	0.0351	0.0329	29.5000	0.9555	3.3500	156.57
Minimum	-0.0430	-0.0270	6.1500	0.1125	-1.4000	-31.31
Std. Dev.	0.0079	0.0088	3.8731	0.1577	0.8771	31.6158
Skewness	-0.1798	0.2971	1.5265	2.1142	0.5010	0.6381
Kurtosis	5.3384	3.4659	5.5565	6.9245	3.0745	3.9736
Jarque-Bera	236.03	24.04	668.63	1403.36	42.57	108.65
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4.4: AUD Summary Statistics

Variables	Z	IRDIFF	ATMVOL	BF	RR	BS1	BS2
Before Crisis							
Mean	0.0004	0.0089	8.0011	0.1539	0.3913	-39.52	8.51
Median	0.0007	0.0097	7.9500	0.1500	0.3750	-39.61	8.87
Maximum	0.0189	0.0126	11.5500	0.2004	1.5000	-32.78	12.97
Minimum	-0.0246	0.0043	5.9750	0.1250	-0.1750	-45.94	0.35
Std. Dev.	0.0054	0.0017	1.1114	0.0145	0.1901	2.3680	2.34
Skewness	-0.6296	-0.5062	0.7352	0.9210	0.8852	0.1550	-0.73
Kurtosis	4.9953	3.1192	3.3047	4.4039	6.4794	2.8169	3.45
Jarque-Bera	94.17	17.58	38.14	90.74	257.82	2.19	39.54
Probability	0.0000	0.0002	0.0000	0.0000	0.0000	0.3342	0.000
After Crisis							
Mean	0.0003	0.0379	15.7495	0.3296	2.0029	-54.67	9.43
Median	0.0012	0.0418	13.7000	0.3000	1.6813	-56.81	10.40
Maximum	0.0724	0.0567	48.5000	0.8250	8.0000	99.11	36.52
Minimum	-0.0824	0.0018	8.4936	0.1340	-0.6000	-107.89	-48.18
Std. Dev.	0.0120	0.0102	6.2230	0.1001	1.4798	22.2220	12.38
Skewness	-0.5053	-0.8182	2.2310	1.4285	1.8265	0.6974	-0.73
Kurtosis	9.5888	2.9997	8.6008	5.5514	6.6963	5.4547	4.38
Jarque-Bera	1873.62	112.93	2162.26	618.66	1138.78	336.13	171.81
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

4.4.3 Unit Root Tests

Given likely serial correlations, we perform the Phillips–Perron test of unit roots with Newey–West automatic bandwidth using Bartlett kernel. The tests are taken on both level and first difference of the variables, before and after the crisis. Table 4.5 presents the probabilities of t statistics of the tests, with the null hypothesis that unit root is present. We see that the first differences of all variables are stationary, which is expected for financial time–series data. However, the levels of all independent variables except BF are shown to be integrated of order one ($I(1)$) at 5% significance level[4], at least for a particular currency during a particular sub-period. To avoid non-stationarity we rewrite the regression model in Eqn. (4.15) by taking the first differences of independent variables:

$$\begin{aligned} Z_t = \beta_0 &+ \beta_1 \triangle IRDIFF_{t-1} + \beta_2 \triangle ATMVOL_t + \beta_3 \triangle ATMVOL_{t-1} \\ &+ \beta_4 \triangle RR_t + \beta_5 \triangle RR_{t-1} + \beta_6 \triangle BF_t + \beta_7 \triangle BF_{t-1} + \beta_8 \triangle BS_t + \beta_9 \triangle BS_{t-1} + \epsilon_t. \end{aligned} \tag{4.19}$$

4.4.4 Factor Model Regression Results

Regression results of the model in (4.19) for each currency pair are presented in Table 4.6, 4.7 and 4.8. To find if there is a structural break before and after the crisis, we use the

[4] In Table 4.5, if the reported probabilities exceed 5%, the null hypothesis of non–stationarity cannot be rejected at 5% significance level. If a non-stationary series must be differenced d times before it becomes stationary, then it is said to be integrated of order d.

Table 4.5: Unit Root Test

		JPY		EUR		AUD
Before Crisis	Level	1st Diff	Level	1st Diff	Level	1st Diff
Z	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$IRDIFF$	0.1034	0.0000	0.4937	0.0000	0.1063	0.0000
$ATMVOL$	0.0537	0.0000	0.2767	0.0000	0.4629	0.0000
BF	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RR	0.4071	0.0000	0.0178	0.0000	0.0002	0.0000
$BS1$	0.0320	0.0000	0.0547	0.0000	0.0113	0.0000
$BS2$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
After Crisis						
Z	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$IRDIFF$	0.0240	0.0000	0.0007	0.0000	0.0012	0.0000
$ATMVOL$	0.0003	0.0000	0.1095	0.0000	0.1105	0.0000
BF	0.0000	0.0000	0.0098	0.0000	0.0000	0.0000
RR	0.1367	0.0000	0.0719	0.0000	0.0362	0.0000
$BS1$	0.5112	0.0000	0.0379	0.0000	0.0423	0.0000
$BS2$	0.1178	0.0000	0.0291	0.0000	0.0012	0.0000

Chow Breakpoint test to identify if parameters are stable over the whole period, with August 10th, 2007 as the break date. Test results[5] show that the break date is supported for JPY and EUR, but not for AUD. Figure 4.2 shows a plot of the first principal component of AUD liquidity bases. The jump is around observation 560, corresponding to March 18th, 2008. Chow test result supports March 18th, 2008 as the break point for AUD. We hence use August 10th, 2007 for JPY and EUR and March 18th, 2008 for AUD to split the full sample into two sub-periods. The model is then tested for both sub-periods as well as the full period.

The factor model (4.19) is estimated with ordinary least squares for each currency pair. To account for heteroskedasticity and autocorrelation [6], the test statistics are based upon Newey-West estimator with two lags.

Table 4.6 results show that in the USD/JPY carry trade, $ATMVOL$ has significant positive contemporaneous effect on excess returns in all periods. RR is significantly negative for all three periods. BF and BS principal components are not significant. Controlling for other variables, $IRDIFF$ is insignificant in explaining carry trade excess returns. The one-day lag terms have no predictive power for excess returns.

The EUR regression results in Table 4.7 show that $ATMVOL$ is only significant before the crisis. The first PC of BS is insignificant before the crisis, but significantly negative after the crisis. Findings of other variables are similar to those of the JPY regression.

[5] See the Chow Breakpoint test results in the Appendix D.

[6] Diagnostic tests are performed for each currency pair. Heteroskedasticity (White's test) and autocorrelation (Breusch-Godfrey test) are both present.

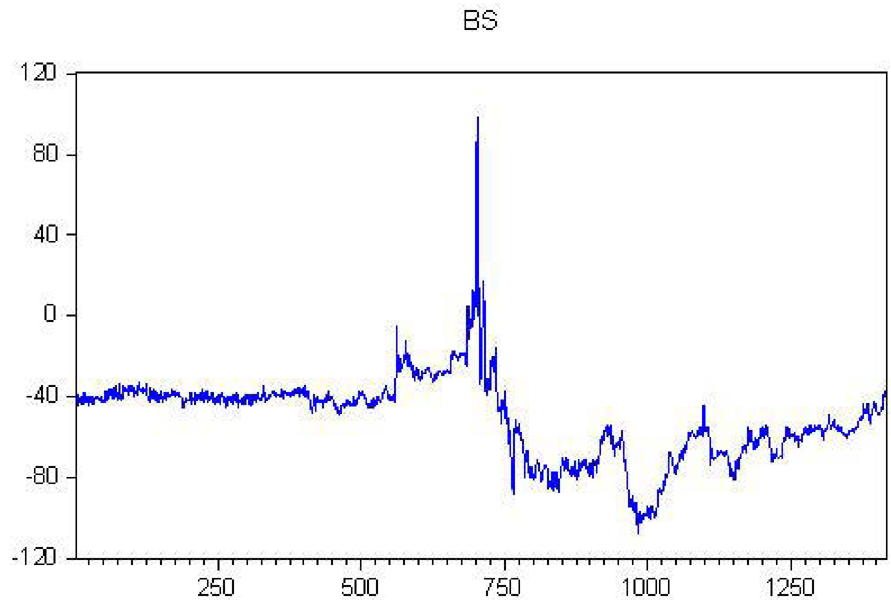


Figure 4.3: **First Principal Component of AUD Liquidity Basis.**

Table 4.6: JPY Regression Results

	Full Sample		**Before**		**After**	
Variables	Coeffi.	Prob.	Coeffi.	Prob.	Coeffi.	Prob.
C	0.0002	0.08	-0.0002	0.12	**0.0004**	**0.01***
$\triangle IRDIFF(-1)$	0.0046	0.94	0.2272	0.72	-0.0048	0.95
$\triangle ATMVOL$	**0.0026**	**0.00****	**0.0058**	**0.00****	**0.0025**	**0.00****
$\triangle ATMVOL(-1)$	-0.0004	0.13	-0.0015	0.06	-0.0003	0.16
$\triangle RR$	**-0.0094**	**0.00****	**-0.0130**	**0.00****	**-0.0090**	**0.00****
$\triangle RR(-1)$	-0.0007	0.36	-0.0027	0.13	-0.0006	0.39
$\triangle BF$	-0.0028	0.48	-0.0064	0.34	-0.0027	0.53
$\triangle BF(-1)$	-0.0008	0.72	0.0035	0.70	-0.0016	0.47
$\triangle BS1$	0.0000	0.80	0.0003	0.24	0.0000	0.80
$\triangle BS1(-1)$	-0.0001	0.08	0.0000	0.93	-0.0001	0.07
$\triangle BS2$	0.0000	0.71	0.0000	0.67	-0.0001	0.38
$\triangle BS2(-1)$	0.0000	0.76	0.0001	0.67	0.0000	0.83
R^2	0.4475		0.4635		0.4649	

*Regression results of model (4.19) for JPY. The dependent variable Z is the carry trade excess return by borrowing USD and investing in JPY. The table presents the regression coefficients and probabilities of test statistics. To account for heteroskedasticity and autocorrelation, the test statistics are based upon Newey-West estimator with two lags. Individual coefficients are statistically significant at the *5% or **1% significance level. Results are based on daily data. The full sample period is from 01/04/2006 to 08/12/2011. The before crisis period is 01/04/2006 to 08/10/2007 and the after crisis period is 08/12/2007 to 08/12/2011.

Table 4.7: EUR Regression Results

	Full Sample		**Before**		**After**	
Variables	Coeffi.	Prob.	Coeffi.	Prob.	Coeffi.	Prob.
C	0.0002	0.19	**0.0003**	**0.03***	0.0001	0.43
$\triangle IRDIFF(-1)$	-0.0652	0.41	0.0785	0.72	-0.0581	0.48
$\triangle ATMVOL$	-0.0006	0.29	**0.0049**	**0.00****	-0.0010	0.12
$\triangle ATMVOL(-1)$	0.0007	0.06	-0.0015	0.13	0.0008	0.05
$\triangle RR$	**-0.0239**	**0.00****	**-0.0272**	**0.00****	**-0.0232**	**0.00****
$\triangle RR(-1)$	-0.0004	0.78	-0.0035	0.20	-0.0006	0.70
$\triangle BF$	0.0076	0.17	0.0020	0.92	0.0073	0.19
$\triangle BF(-1)$	0.0055	0.18	0.0150	0.49	0.0047	0.25
$\triangle BS1$	**-0.0002**	**0.00****	-0.0002	0.22	**-0.0002**	**0.00****
$\triangle BS1(-1)$	0.0000	0.29	-0.0002	0.46	0.0000	0.27
R^2	0.3218		0.2874		0.3392	

*Regression results of model (4.19) for EUR. The dependent variable Z is the carry trade excess return by borrowing USD and investing in EUR. The table presents the regression coefficients and probabilities of test statistics. To account for heteroskedasticity and autocorrelation, the test statistics are based upon Newey-West estimator with two lags. Individual coefficients are statistically significant at the *5% or **1% significance level. Results are based on daily data. The full sample period is from 01/04/2006 to 08/12/2011. The before crisis period is 01/04/2006 to 08/10/2007 and the after crisis period is 08/12/2007 to 08/12/2011.

Table 4.8: AUD Regression Results

	Full Sample		**Before**		**After**	
Variables	Coeffi.	Prob.	Coeffi.	Prob.	Coeffi.	Prob.
C	**0.0004**	**0.04***	**0.0005**	**0.02***	0.0003	0.23
$\triangle IRDIFF(-1)$	0.0090	0.96	-0.4312	0.09	-0.0018	0.99
$\triangle ATMVOL$	**-0.0046**	**0.00****	**-0.0042**	**0.00****	**-0.0044**	**0.00****
$\triangle ATMVOL(-1)$	0.0006	0.14	-0.0004	0.55	0.0006	0.17
$\triangle RR$	**-0.0121**	**0.00****	**-0.0079**	**0.00****	**-0.0146**	**0.00****
$\triangle RR(-1)$	-0.0015	0.19	-0.0003	0.89	-0.0018	0.12
$\triangle BF$	-0.0029	0.53	**-0.0165**	**0.02***	0.0023	0.67
$\triangle BF(-1)$	0.0007	0.90	0.0058	0.22	0.0021	0.76
$\triangle BS1$	**-0.0001**	**0.04***	0.0001	0.59	**-0.0002**	**0.01***
$\triangle BS1(-1)$	0.0000	0.40	0.0000	0.86	0.0001	0.29
$\triangle BS2$	-0.0001	0.09	0.0000	0.68	-0.0002	0.10
$\triangle BS2(-1)$	**-0.0003**	**0.00****	-0.0002	0.06	**-0.0002**	**0.02***
R^2	0.4569		0.2803		0.5080	

*Regression results of model (4.19) for AUD. The dependent variable Z is the carry trade excess return by borrowing USD and investing in AUD. The table presents the regression coefficients and probabilities of test statistics. To account for heteroskedasticity and autocorrelation, the test statistics are based upon Newey-West estimator with two lags. Individual coefficients are statistically significant at the *5% or **1% significance level. Results are based on daily data. The full sample period is from 01/04/2006 to 08/12/2011. The before crisis period is 01/04/2006 to 03/18/2008 and the after crisis period is 03/19/2008 to 08/12/2011.

In Table 4.8, the AUD results show that $ATMVOL$ effect is significant for all periods. However, different from JPY and EUR, the contemporaneous effect is negative. BF has significant negative effect on excess returns before the crisis. The first PC of BS is significantly negative after the crisis. The second PC has no significant contemporaneous effect on excess returns, but the one-day lag term has significant negative effect. This implies that if the slope of the term structure of the liquidity bases increases, the excess return of the next day is expected to decrease, holding other principal components fixed. Other findings are similar to those of the JPY and EUR regressions .

We discuss several important findings in the regression results. Firstly, for all currencies and in all test periods, $IRDIFF$ is not significant in explaining the carry trade excess return. This points to a potential resolution of the UIP puzzle. Secondly, among our proposed FX market volatility proxies, $ATMVOL$ is in general significant, but the effect is different for different currencies. RR is significantly negative for all currencies and BF is only significant for AUD during the pre-crisis period. Thirdly, the newly proposed liquidity proxy, BS, is highly significant for EUR and AUD after the crisis. However, this effect does not appear in the USD/JPY trade.

4.4.5 Panel Regression Results

The individual currency regression results show that the liquidity basis is insignificant in the USD/JPY carry trade. This is possibly driven by fundamental differences across currencies. For example, Ranaldo and Soderlind (2010) find that JPY possesses “safe haven" properties. Safe-haven currencies provide hedging benefits during market stress periods. The sharp appreciation of JPY since the GFC offers further support for the haven status of JPY. On the other hand, Ranaldo and Soderlind (2010) find no significant evidence that EUR is a safe–haven currency. AUD generally appreciates and depreciates with investors' risk appetite (Melvin and Taylor 2009). In order to control for currency individual heterogeneity and identify common structures, we test the model in (4.19) with the balanced panel data with currency-fixed effects. The results are summarized in Table 4.9.

We see in Table 4.9 that after controlling for currency specific effects, $IRDIFF$ is insignificant, both before and after the crisis. This suggests that the UIP puzzle can potentially be explained after accounting for the effects of FX market volatility and liquidity. $ATMVOL$ is insignificant before the crisis, but significantly negative after the crisis. RR is significantly negative both before and after the crisis. BF is not significant. Since the crisis the effect of the level (first PC) of BS is significantly negative. For all variables the one-day lags are not predictive of the excess returns[7]. Lastly, the R^2 increases from 24.70% before the crisis to 34.31% after the crisis, which supports the assertion that the break date is reasonably chosen.

[7]Thus one should interpret the regression model as explanatory rather than predictive, i.e. the explanatory variables which are significant point to risks in carry trades for which investors are compensated by expected return - and not to variables which would allow investors to anticipate positive returns on carry trades.

Table 4.9: Panel Regression Results

	Full Sample		**Before**		**After**	
Variables	Coeffi.	Prob.	Coeffi.	Prob.	Coeffi.	Prob.
C	**0.0003**	**0.04***	0.0001	0.49	0.0003	0.05
$\triangle IRDIFF(-1)$	-0.0703	0.50	-0.2497	0.36	-0.0710	0.52
$\triangle ATM$	**-0.0011**	**0.00****	0.0012	0.14	**-0.0011**	**0.00****
$\triangle ATM(-1)$	0.0000	0.92	-0.0009	0.08	0.0000	0.98
$\triangle RR$	**-0.0184**	**0.00****	**-0.0213**	**0.00****	**-0.0182**	**0.00****
$\triangle RR(-1)$	0.0003	0.72	-0.0012	0.39	0.0003	0.74
$\triangle BF$	-0.0013	0.66	-0.0127	0.09	-0.0011	0.73
$\triangle BF(-1)$	0.0015	0.63	0.0096	0.24	0.0010	0.76
$\triangle BS1$	**-0.0001**	**0.00****	-0.0001	0.37	**-0.0001**	**0.00****
$\triangle BS1(-1)$	0.0000	0.81	-0.0001	0.36	0.0000	0.76
$\triangle BS2$	-0.0001	0.41	0.0001	0.23	-0.0001	0.32
$\triangle BS2(-1)$	-0.0001	0.22	0.0000	0.87	-0.0001	0.27
R^2	0.3281		0.2470		0.3431	

*This table reports panel regression results with currency-fixed effects. Regression coefficients and probabilities of test statistics are presented. Standard errors are robust to cross-section heteroscedasticity and contemporaneous correlation among cross-sections, with White cross-section method. Individual coefficients are statistically significant at the *5% or **1% significance level. Results are based on daily data. The full sample period is from 01/04/2006 to 08/12/2011. The before crisis period is 01/04/2006 to 08/10/2007 and the after crisis period is 08/12/2007 to 08/12/2011.

4.4.6 Robustness Check

The settlement convention in the FX market is $t+2$ days. Strictly speaking, the overnight carry trade return based on X_t^k and X_{t-1}^k should be calculated with r_{t+1}^k and r_{t+1}^{USD}. We hence modify the dependent variable and re-estimate all regressions:

$$Z_t^k = (r_{t+1}^k - r_{t+1}^{USD}) - (X_t^k - X_{t-1}^k). \tag{4.20}$$

Results based upon the adjusted excess return are almost unchanged. We therefore propose that the effect of settlement lag is negligible.

4.5 Conclusion

This study provides empirical support for a liquidity based model to explain the UIP puzzle. We study the effects of liquidity and volatility on carry trade excess returns. We develop an alternative proxy for FX market liquidity - violations of no-arbitrage bounds for forward exchange rates and currency basis swaps. We also propose FX market specific volatility proxies. Our hypothesis is that both proxies should be significant in explaining carry trade performance and hence useful for a resolution of the UIP puzzle.

The sample is chosen to cover periods both before and after the GFC in order to capture structural breaks. A linear factor model is proposed and tested for both individual currencies and the panel data. Both proxies are significant. The results are robust to the settlement lag in the FX market.

We contribute to the extant literature from three perspectives. Firstly, our liquidity proxy captures both market liquidity risk and funding liquidity risk. Secondly, we demonstrate that liquidity and volatility factors change effects since the GFC. Lastly, we provide evidence that the UIP puzzle may be resolved after controlling for liquidity risk and market volatility.

Chapter 5

A Consistent Framework for Modelling Spreads in Tenor Basis Swaps

5.1 Introduction

In this chapter we focus on the single currency tenor swaps (TS) and propose a consistent framework to reconcile the differences between the classic "single curve" approach and practitioners' "multiple curve" approach. We conjecture that liquidity risk is the fundamental factor that has caused large spreads in tenor swaps since the GFC. We then construct an intensity model to describe the arrival time of liquidity shocks with a time-inhomogeneous Poisson process. With the no-arbitrage argument and non-linear constrained optimisations, we calibrate the model parameters to quoted basis spreads in tenor swaps .

We present an arbitrage strategy to exploit the large basis in a TS. Assume that for a given currency, the current market quote is 3M LIBOR + 50 bps exchanging 6M LIBOR flat for 6 months. The notional amount is 1 unit and there are no transaction costs. An arbitrageur, which we assume is a LIBOR counterparty, such as an AA-rated bank, can then make arbitrage profit by,

(1) Enter the TS in which the arbitrageur pays 6M LIBOR and receives 3M LIBOR + 50 bps.

(2) Roll over 3M borrowing at 3M LIBOR for 6 months, with unit notional.

(3) Deposit the notional at 6M LIBOR.

The net cash flows are summarized in Table 5.1. In Table 5.1, $L^{3m}(0)$ refers to the 3M LIBOR fixed at time 0. $L^{3m}(0.25)$ is the 3M LIBOR fixed at the end of 3 months and 0.25 is the year fraction. $L^{6m}(0)$ refers to the 6M LIBOR fixed at time 0.

The notional is canceled by the loan and deposit at time 0 and at the end of 6 months. The floating payment of the loan is canceled by the receipt from the TS. The payment of 6M LIBOR in the TS is financed by the interest income of the deposit. All cash flows are netted out except the spread of the TS, which becomes the profit every 3 months. Because the arbitrageur has zero initial cost, this is clearly an arbitrage.

Table 5.1: Arbitrage Strategy for Tenor Swap

Strategy	$t = 0$	$t = 0.25$	$t = 0.5$
Loan	1	-$L^{3m}(0) \cdot 0.25$	-$L^{3m}(0.25) \cdot 0.25 - 1$
Deposit	-1	0	$L^{6m}(0) \cdot 0.5 + 1$
Swap	0	$(L^{3m}(0) + 50bps) \cdot 0.25$	$(L^{3m}(0.25)) + 50bps) \cdot 0.25 - L^{6m}(0) \cdot 0.5$
Net Cash Flow	0	12.5 bps	12.5 bps

If the arbitrage strategy in Table 5.1 is practical, we would expect that arbitrageurs take large positions in TSs to make riskless profit. However, during the crisis such large spreads persisted and exceeded transaction costs from bid/ask spreads, and apparent arbitrage opportunities do not seem to be taken.

The remainder of this chapter is organized as follows. Section 2 sets up the model framework. Section 3 presents the empirical results and Section 4 concludes.

5.2 Model Set-up and Implementation

5.2.1 Liquidity risk, Basis Spreads and Limits to Arbitrage

We propose that liquidity risk is the fundamental factor that led to anomalies in the TS market, as well as prevented arbitrage opportunities from being fully exploited. We choose a liquidity based model for TS spreads for two reasons. Firstly, liquidity is the factor which empirically seems to be driving the spreads in the TS market [1]. Secondly, there is a need to reduce dimensionality from having a separate basis spread term structure for each tenor pair, which is the case in the "multiple curve" modelling approach. To illustrate the effect of liquidity risk, we revisit the arbitrage strategy in Table 5.1.

During the crisis, suppose a lender in the interbank (i.e. LIBOR) market rolls over two consecutive 3-M lending. At the end of 3 months if there is funding shortage, the lender can choose not to make the second lending. In contrast, if the lender makes a 6M lending, there is no such flexibility. Ceteris paribus, because the 6M lending involves higher liquidity risk than the 3M roll-over lending, 6M LIBOR should entail a liquidity premium over the 3M LIBOR. As the crisis developed and intensified, liquidity risk was amplified, which led to large liquidity risk premium for longer term loans over the short term ones.

However, since the GFC tenor swaps are "almost" free of counterparty credit risk due to the widespread use of collateral. Johannes and Sundaresan (2007) find that due to collateral, market participants commonly view swaps as risk free and swap cash flows should be discounted at the risk-free rate. Bianchetti (2010), Mercurio (2010) and Piterbarg (2010) also note that it makes sense to discount collateralized cash flows by the OIS rate, which is regarded as the best proxy for the risk free rate. It is hence incorrect to compensate the party receiving 6M LIBOR in a swap (rather than a loan) with the liquidity premium. To make the contract fair, a positive spread equal to the liquidity premium should be added to the 3M LIBOR. We propose that this is the reason the

[1]See Subsection 2.3.2 for for a review of recent studies.

spread is always added to the shorter tenor rate. Large liquidity risk premium during the crisis hence also explains the large spreads quoted in tenor swaps .

In the arbitrage strategy proposed in Table 5.1, if a liquidity shock arrives between initiation and the end of 3 months, the lender may refuse to roll over the loan to the arbitrageur. Because the arbitrageur is committed to the 6M lending, he/she has to refinance in a stressed market. The potential loss due to refinancing (i.e. at a higher rate than $L^{3m}(0.25)$) could offset or even exceed the gain from the spreads in the swap. Therefore, although the market may appear rife with arbitrage opportunities, the strategy may break down.

5.2.2 Model and Implementation

We use an intensity model to describe the arrival time of liquidity shocks with a time-inhomogeneous Poisson process $N(t)$, with deterministic intensity $\lambda(t)$. The basic idea of intensity models is to describe the shock time τ as the first jump time of a Poisson process. Although shocks are not induced by observed market information or economic fundamentals, by formulation intensity models are suited to model credit spreads and calibrate to credit default swap (CDS) data (see Brigo and Mercurio 2006). In this study we adopt this technique and propose an intensity model for basis spreads in TSs and calibrate to market data. See Appendix E for basic properties of time-inhomogeneous Poisson process.

We consider a N-year maturity TS which exchanges the i-th tenor LIBOR plus a spread

$B_{i,j,N}$ for the the j-th tenor LIBOR, where $i < j$ and $B_{i,j,N} > 0$. N, i and j are expressed in terms of year fractions. We assume that an arbitrageur follows the arbitrage strategy in Table 5.1. The arbitrageur gains $B_{i,j,N} * i$ at the end of each i-th tenor. We propose that the expected loss due to refinancing given a liquidity shock explains why the arbitrage strategy breaks down. Hence we impose the no-arbitrage condition that the expected loss offsets the expected gain. To gain more model tractability, we make several simplifying assumptions,

1) The TS is perfectly collateralized with zero threshold, which means the posted collateral must be 100% of the contract's mark-to-market value. The amount of collateral is continuously adjusted with zero minimum transfer amount (MTA) [2]. Because daily margin call is quite common in the market, continuous adjustment should reasonably well approximate the actual practice (see Fujii et al. 2009).

2) The first jump of the Poisson process can occur within any shorter tenor of the swap and there can be at most one jump within each shorter tenor. Upon the first liquidity shock, the arbitrageur is unable to roll over the shorter tenor loan and has to refinance until the end of the associated longer tenor. The instantaneous loss rate due to refinancing is $\pi(t)$. The arbitrageur then shuts down the borrowing, the lending and the TS in the arbitrage strategy at the end of the longer tenor within which the first jump occurs. To illustrate this assumption, we use a 3M vs 12M TS for 12 months. If the first liquidity shock occurs between initiation and 3 months, the borrowing and lending can only be

[2] The smallest amount of value that is allowable for transfer as collateral. This is the lower threshold below which the collateral transfer is more costly than the benefits.

shut down at the end of 12 months. The arbitrageur also unwinds the TS at the end of 12 months.

3) We assume remaining risks are negligible for the arbitrageur, including the default risk of the longer tenor lending and the mark–to–market value of the TS at the termination time.

Based on such simplifying assumptions, we calculate the present value (PV) of the expected gain and of the expected loss of the arbitrage strategy. We firstly examine the distribution of the first jump time τ. We assume that τ can occur within any shorter tenor. However, if τ arrives within the last shorter tenor for a given longer tenor, it is irrelevant because the arbitrageur can shut down the strategy at the end of the longer tenor without refinancing. Hence total number of relevant shorter tenors within which τ occurs is $\frac{N}{j} * (\frac{j}{i} - 1)$ and the PV of expected loss is expressed as

$$\begin{aligned} PV_{Loss} &= \sum_{k=1}^{K} \left(e^{\int_{T_k}^{T_{\eta(k)}} \pi(u)\,du} - 1 \right) P^Q(T_{k-1} < \tau \leq T_k) D^{OIS}(T_0, T_{\eta_{(k)}}) \\ &= \sum_{k=1}^{K} \left(e^{\int_{T_k}^{T_{\eta(k)}} \pi(u)\,du} - 1 \right) \left(e^{-\int_0^{T_{k-1}} \lambda(u)\,du} - e^{-\int_0^{T_k} \lambda(u)\,du} \right) D^{OIS}(T_0, T_{\eta_{(k)}}), \end{aligned} \tag{5.1}$$

where P^Q denotes the probability under the risk-neutral measure Q, $D^{OIS}(\cdot,\cdot)$ is the discount factor from the OIS curve. $K = \frac{N}{i}$ is the total number of shorter tenors until maturity and $T_k = k \cdot i$. η_k is expressed as

$$\eta_{(k)} = \min\left(m \,|\, T_k \leq T_{m \cdot n}\right) \cdot n, \tag{5.2}$$

where $n = \frac{j}{i}$. On the other hand, the PV of expected gain is

$$PV_{Gain} = \sum_{k=1}^{K} (B_{i,j,N} \cdot i) \left(e^{-\int_0^{T(\eta_{(k)}-n)} \lambda(u)\,du} \right) D^{OIS}(T_0, T_k). \tag{5.3}$$

The no–arbitrage condition is hence

$$\begin{aligned} & \sum_{k=1}^{K} \left(e^{\int_{T_k}^{T_{\eta_{(k)}}} \pi(u)\,du} - 1 \right) \left(e^{-\int_0^{T_{k-1}} \lambda(u)\,\mathrm{d}u} - e^{-\int_0^{T_k} \lambda(u)\,du} \right) D^{OIS}(T_0, T_{\eta_{(k)}}) \\ = \; & \sum_{k=1}^{K} (B_{i,j,N} * i) \left(e^{-\int_0^{T(\eta_{(k)}-n)} \lambda(u)\,du} \right) D^{OIS}(T_0, T_k). \end{aligned} \tag{5.4}$$

Given the OIS discount curve, we can use Eqn. (5.4) to calibrate the loss rate $\pi(t)$ and a risk–neutral intensity function $\lambda(t)$ to the selected set of tenor swaps. In credit risk literature (e.g. Schönbucher 2003) where the intensity model is used to calibrate credit spreads, joint calibration of the recovery rate and the deterministic intensity functions often produces unstable results. Hence the recovery rate, comparable to our $\pi(t)$, is often made constant and calibrated separately. We adopt this technique to calibrate a constant loss rate π and $\lambda(t)$.

To obtain an estimate of π, as the first step we assume $\lambda(t) = \overline{\lambda}$, where $\overline{\lambda}$ is a constant and jointly calibrate π and $\overline{\lambda}$. To do this we use the mean-squared deviation function to obtain the optimal fit by minimizing the function by varying π and $\overline{\lambda}$:

$$\sum_{i=1}^{N} \left(\frac{PV^i_{Loss}(\pi, \overline{\lambda})}{PV^i_{Gain}} - 1 \right)^2, \tag{5.5}$$

where N is the number of TSs used for the calibration. This measure uses relative deviations and hence is independent of the scale of individual PVs.

In the second step, we use the estimated π from step 1 as the input and calibrate time-dependent and piecewise constant $\lambda(t)$ to the same set of selected swaps. To achieve a perfect fit and impose minimal structure on the intensity curve, we use the bootstrap method to strip $\lambda(t)$ from observed spreads. The bootstrap procedure is as follows,

1) Tenor swaps are ordered in the appropriate order [3].

2) $\lambda(t)$ is piecewise constant. We first find λ_1 such that

$$PV^1_{Loss}(\pi, \lambda_1) = PV^1_{Gain}. \tag{5.6}$$

We then work iteratively to evolve the intensity curve. Eventually, given λ_1, ..., λ_{N-1} we find λ_N such that

[3] See details in Table 5.3 of subsection 5.3.2.

$$PV^N_{Loss}(\pi, \lambda_1, ..., \lambda_{N-1}; \lambda_N) = PV^N_{Gain}. \tag{5.7}$$

5.3 Data, Methodologies and Results

Because the set-up and implementation of the intensity model is currency independent, we collect USD data only. The calibration procedure is identical for currencies other than USD.

5.3.1 Construction of OIS Discount Factors

We use the standard bootstrap with interpolation method to construct the USD OIS discount factors required for both sides of Eqn. (5.4). To this end, we collect USD OIS rates available from Bloomberg. Maturities include 1-week (1W), 2W, 1M, 2M, 3M, 4M, 5M, 6M, 7M, 8M, 9M, 10M, 11M, 1Y, 15M, 18M, 21M, 2Y, 3Y, 4Y, 5Y and 10Y. The sample period starts from July 28th, 2008, since when the 10Y OIS rates are available, and ends at April 2nd, 2013. OIS rates beyond the 10Y maturity are only quoted from September 27th, 2011.

In order to extend the OIS curve to 30Y maturity, we use the USD Fed Funds (FF) basis swap quotes to approximate OIS rates (see, for example, Bloomberg 2011). FF basis swaps exchange the non-compounded daily weighted average of the overnight FF effective rate [4] for a 90-day period plus a spread and 3M USD LIBOR flat, with quar-

[4] FF rate is the interest rate at which depository institutions trade funds held at the U.S. Federal Reserve with each other. The weighted average of FF rate across all transactions is the FF effective rate.

terly payment frequency. On the other hand, two parties in an OIS agree to exchange the difference between interest accrued at the fixed rate and interest accrued at the daily compounded FF effective rate, with annual payment frequency. Although having different payment frequency and compounding conventions, both OIS and FF basis swaps are defined in terms of the daily reset FF effective rate, hence they are observables of the same underlying security.

By ignoring minor discrepancies such as compounding for weekends and holidays, Bloomberg (2011) proposes a quick approximation of OIS rates with IRS rates and FF basis swap spreads. Firstly, a fixed-floating FF swap can be set up by simultaneously entering an IRS and an FF basis swap. In the IRS, the fixed rate is received and the 3M LIBOR is paid. In the FF basis swap, the 3M LIBOR is received and FF rate plus the spread is paid. The net position is therefore IRS fixed rate vs daily average FF rate plus the spread. Based upon this setup, let S_N and FF_N denote the N-year IRS fixed rate and FF basis swp spread, the OIS rate OIS_{t_N} can be approximated as

$$OIS_{t_N} = \left(\left(1 + \frac{\widehat{OIS_{t_N}}}{360} \right)^{90} - 1 \right) \times 4, \tag{5.8}$$

where

$$\widehat{OIS_{t_N}} = \left(1 + \frac{r_Q - FF_N}{4} \right)^{4} - 1, \tag{5.9}$$

and

$$r_Q = \left(\left(1 + \frac{S_N \times \frac{360}{365}}{2} \right)^{\frac{2}{4}} - 1 \right) \times 4. \tag{5.10}$$

Eqn. (5.10) converts the semiannually paid IRS rate to quarterly paid (i.e. 3M LIBOR) rate and $\widehat{OIS_{t_N}}$ annualizes the quarterly paid FF effective rate. OIS_{t_N} then is the OIS rate with the daily compounding adjustment.

We then collect the IRS rates and FF basis spreads with corresponding maturities to approximate 12Y, 15Y, 20Y, 25Y and 30Y OIS rates. These maturities are chosen because they started to be quoted from September 27th, 2011. Because FF basis swaps are quoted from September 22nd, 2008, we approximate OIS rates from September 22nd, 2008 to September 26th, 2011. To evaluate how well this method performs, we compare actual quotes of OIS rates and approximated OIS rates from September 27th, 2011 to April 2nd, 2013. Table 5.2 shows that the approximated rates track the actual rates reasonably well.

Table 5.2: Average OIS Rate Approximation Errors

Maturity	12-year	15-year	20-year	25-year	30-year
Basis Points	0.71	0.94	1.17	0.98	0.70
Percentage	0.37%	0.43%	0.51%	0.40%	0.28%

We therefore have OIS rates with maturities from 1 week up to 30 years. Since OIS swaps have annual payment frequency, there is only one exchange of payments up to 1 year. Therefore to bootstrap the OIS curve up to 1 year, OIS rates are treated as deposit rates. With the day count convention of $Actual/365$, the OIS discount factors are calculated as

$$D^{OIS}(t_i) = \frac{1}{1 + \tau_i \cdot OIS(t_i)}, \tag{5.11}$$

where τ_i is the year fraction of maturity t_i. Similar to using IRS rates to construct the LIBOR discount curve, OIS discount curves from 1Y to 30Y are extracted from par OIS rates with the standard bootstrap method. Eqn. (5.12), (5.13) and (5.14) summarize this method:

$$OIS(t_N) \cdot \sum_{i=1}^{N} D_{t_i}^{OIS} + D_{t_N}^{OIS} = 1, \tag{5.12}$$

where N is the total number of payments. Discount factors $D_{t_N}^{OIS}$ are iteratively obtained with

$$D_{t_N}^{OIS} = \frac{1 - OIS(t_N) \cdot \sum_{i=1}^{N-1} D_{t_i}^{OIS}}{1 + OIS(t_N)}. \tag{5.13}$$

For maturities not quoted from Bloomberg, OIS rates are linearly interpolated with available quotes:

$$OIS_t = OIS_{t_i} + \left(\frac{t - t_i}{t_{i+1} - t_i} \right) \times (OIS_{t_{i+1}} - OIS_{t_i}), \quad t_i < t < t_{i+1}. \tag{5.14}$$

5.3.2 Bootstrap Liquidity Spreads

We bootstrap piecewise constant intensity λ_t to achieve perfect fits without imposing any functional form. We collect available TS data from Bloomberg, including 1M vs 3M, 3M vs 6M and 3M vs 12M swaps. We aim to include all TS instruments in order to capture as much market information as possible and evolve maturities up to 30 years. To illustrate our bootstrap approach, consider the 1M vs 3M swap with 3M maturity. Based upon the model assumptions, the first liquidity shock can arrive between 0 and 1 month, 1 and 2 months or 2 and 3 months. However, if the shock is between 2 and 3 months, it is irrelevant because the arbitrageur shuts down the strategy at the end of 3 months and does not need to refinance. Therefore, we assume a constant intensity between 0 and 2 months and use the 1M vs 3M swap with 3M maturity to calculate the intensity λ_1 with Eqn. (5.4). With λ_1, we are then able to calculate the constant intensity λ_2 between 2 and 3 months, by using the 3M vs 6M swap with 6M maturity. With this approach, we establish 36 piecewise constant intensities, which are summarized in Table 5.3.

In the bootstrap procedure, we exclude 3M v 12M swaps for two reasons. Firstly, 3M v 12M swaps quotes are only available from August 6th, 2009. Secondly, in our approach to extending bootstrap intervals, 3M v 12M swaps are redundant once we have used 3M v 6M swaps. Because 3M v 6M swaps have been quoted for a much longer period, we propose the quotes should be more consistent and reliable. Having established the bootstrap procedure, we use Eqn. (5.4) to calculate piecewise constant intensities. We start

Table 5.3: Bootstrap Piecewise Constant Intensities

Piecewise Constant λ_t	Bootstrap Interval	Tenor Swap
λ_1	0-2 months	1v3 3-month
λ_2	2-3 months	3v6 6-month
λ_3	3-5 months	1v3 6-month
λ_4	5-8 months	1v3 9-month
λ_5	8-9 months	3v6 1-year
λ_6	9-11 months	1v3 1-year
λ_7	11-15 months	3v6 18-month
λ_8	15-17 months	1v3 18-month
λ_9	17-21 months	3v6 2-year
λ_{10}	21-23 months	1v3 2-year
λ_{11}	23-33 months	3v6 3-year
λ_{12}	33-35 months	1v3 3-year
λ_{13}	35-45 months	3v6 4-year
λ_{14}	45-47 months	1v3 4-year
λ_{15}	47-57 months	3v6 5-year
λ_{16}	57-59 months	1v3 5-year
λ_{17}	59-69 months	3v6 6-year
λ_{18}	69-71 months	1v3 6-year
λ_{19}	71-81 months	3v6 7-year
λ_{20}	81-83 months	1v3 7-year
λ_{21}	83-93 months	3v6 8-year
λ_{22}	93-95 months	1v3 8-year
λ_{23}	95-105 months	3v6 9-year
λ_{24}	105-107 months	1v3 9-year
λ_{25}	107-117 months	3v6 10-year
λ_{26}	117-119 months	1v3 10-year
λ_{27}	119-141 months	3v6 12-year
λ_{28}	141-143 months	1v3 12-year
λ_{29}	143-177 months	3v6 15-year
λ_{30}	177-179 months	1v3 15-year
λ_{31}	179-237 months	3v6 20-year
λ_{32}	237-239 months	1v3 20-year
λ_{33}	239-297 months	3v6 25-year
λ_{34}	297-299 months	1v3 25-year
λ_{35}	299-357 months	3v6 30-year
λ_{36}	357-359 months	1v3 30-year

from the 1M v 3M with 3M maturity swap to calculate λ_1, then work iteratively to find λ_2, λ_3, ..., λ_{36}.

The bootstrap results demonstrate two problems. Firstly, the term structure of calibrated intensities severely oscillates. Secondly, many of the intensities are negative. Severe oscillations are an undesirable property for the term structure of intensities. Even worse, negative intensities invalidate the fundamental model assumption because $\lambda(t)$ must be a positively valued function. To illustrate, Figure 5.1 shows the bootstrap results on Oct 10th, 2008, with $\pi = 0.1$ which minimizes the deviation function (5.5). The order of intensities in Figure 5.1 follows the sequence of intensities constructed in Table 5.3.

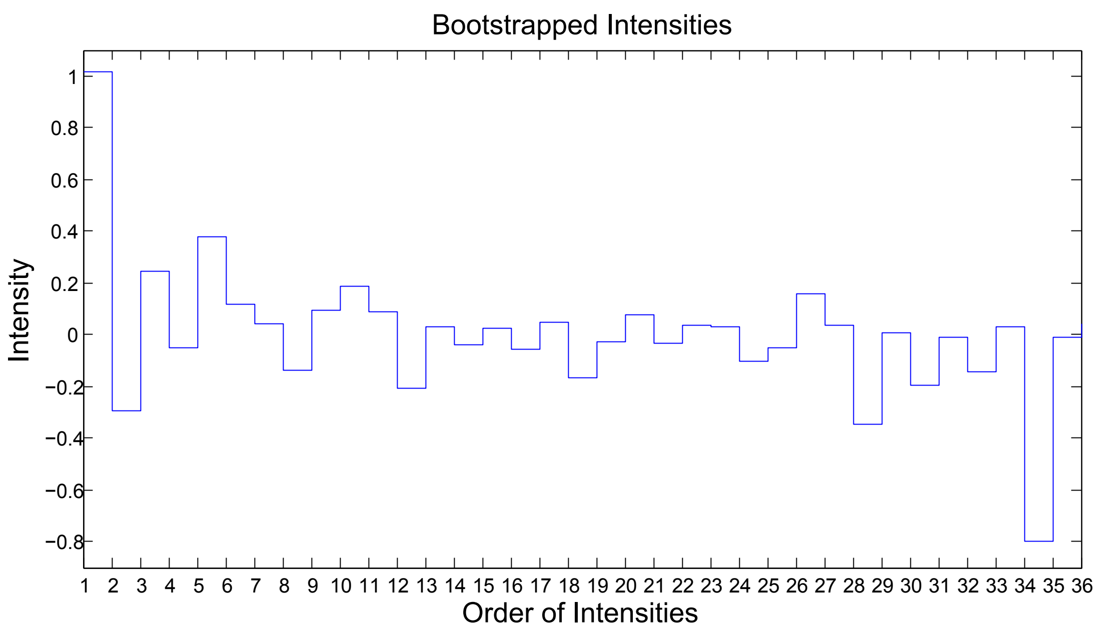


Figure 5.1: **Bootstrapped Piecewise Constant Intensities as at 10/10/2008.**

5.3.3 Analytical Analyses

To understand the cause of the problems shown in the bootstrap results, we perform an analytical analysis of the sample data. We conjecture that the oscillations and negative intensities result from the data. In the standard bootstrap of interest rate term structure with LIBOR/IRS data, in general the shape of the curve is monotonic or humped, without oscillations. The positivity of the curve is also almost always guaranteed. Compared to IRS, TSs, especially those of longer maturities, are only recently quoted. Table 5.4 summarizes the starting year of IRSs and TSs of various maturities,

Table 5.4: Starting Year of Swap Quotes from Bloomberg

Maturity	1-yr	5-yr	10-yr	15-yr	20-yr	25-yr	30-yr
IRS	1996	1988	1988	1994	1994	1999	1994
1M v 3M TS	1997	1997	1997	2008	2007	2008	2007
3M v 6M TS	1997	1997	1997	2008	2008	2008	2008

We find from Table 5.4 that TSs beyond 10Y maturity have been quoted for a much shorter period of time, compared to corresponding IRSs. In addition, since these instruments were introduced, financial markets have experienced turmoils such as the GFC and European sovereign-debt crisis. We hence suspect that the TS market is much less mature and consistent than the IRS market and the quotes may have caused the problems in the bootstrap results. To find out whether this is the case, we analyze respectively the shape of the term structure of the quoted spreads of 1M v 3M swaps and 3M v 6M swaps. To control for transaction costs, we also consider the bid-ask spread of the quotes. The analysis takes the following steps,

1) For each sample date, we extract the bid rate and the ask rate of the basis spread for each swap and list them in two rows. The ask rates are in the upper row and bid rates in the lower row. For each row, we order the rates in the ascending order of swap maturity. Hence a matrix S of 2 rows and 19 columns is formed for 1M v 3M swaps (17 columns for 3M v 6M swaps). Matrix elements are denoted as $S_{i,j}$, where i is the row index and j is the column index.

3) For 1M v 3M swaps, a matrix R of 2 rows and 19 columns (17 columns for 3M v 6M swaps) is used to record the results of the analysis. We initialize R by setting $R_{1,1} = S_{1,1}$ and $R_{2,1} = S_{2,1}$. For $i = 1$, we evolve the matrix R along the columns as follows:

If $R_{i,j} \geq S_{i,j+1} \geq S_{i+1,j+1}$, *then* $R_{i,j+1} = S_{i,j+1}$;

If $S_{i+1,j+1} < R_{i,j} < S_{i,j+1}$, *then* $R_{i,j+1} = R_{i,j}$;

Else, $R_{i,j+1} = S_{i+1,j+1}$.

For $i = 2$, the algorithm is:

If $R_{i,j} \geq S_{i-1,j+1} \geq S_{i,j+1}$, *then* $R_{i,j+1} = S_{i-1,j+1}$;

If $S_{i,j+1} < R_{i,j} < S_{i-1,j+1}$*, then* $R_{i,j+1} = R_{i,j}$;

Else, $R_{i,j+1} = S_{i,j+1}$.

The above algorithm is designed such that potential oscillations in the term structure of spreads are minimized. The rationale is that the actual transacted spread should always be bounded by the bid and the ask rate. Hence, in evolving the spread curve, we set the spread that spans a particular interval at the rate which minimises the change from the previous spread, with the constraint that the rate must be within the bid-ask bounds. As a result, for 1M v 3M swaps, we have two term structures of spreads on each sample date, one consists of $R_{1,1}, R_{1,2}, ..., R_{1,19}$ and the other consists of $R_{2,1}, R_{2,2}, ..., R_{2,19}$.

We then examine the shape of the spread curves resulting from the algorithm. If both term structures oscillate, we conclude that even after considering the bid-ask spread, the oscillations still persist on that sample date. By this criteria, we identify 90 days which shows oscillations for the 1M v 3M swaps and 68 days for the 3M v 6M swaps. After counting for overlapping days, there are 142 distinct days of oscillations. Table 5.5 shows a breakdown of these days by years.

Table 5.5: Days of Oscillations by Year

Year	2008	2009	2010	2011	2012	2013
Days	33	11	22	62	12	2

It is worth noting that the sample period is from September 22nd, 2008 to April 2nd, 2013. Hence for 2008 and 2013 we do not have a full year. However we can observe that a large number of oscillation days occurred in the last three months of 2008. The number of such days dropped significantly during 2009, but started to pick up in 2010 and intensified in 2011. Since 2012 such anomalies have stabilized and only occurred infrequently. Such an observation broadly corresponds to major financial market developments during the sample period. The late 2008 marked the peak the GFC. The European sovereign debt-crisis emerged in early 2010, intensified during 2011 and started to stabilize since mid-2012.

This finding lends support to our conjecture that during market turbulence, the quotes of TSs are inconsistent and the shape of the spread curve is not well-behaved. This could potentially be explained by the observation that TS market is less mature and developed, a problem that could be amplified during stressed market conditions.

The other analysis we perform on the sample data is related to the negative quotes. Because we propose that the transacted spread should be bounded by the bid and the ask rate, if the ask rate is negative, a negative transacted spread is necessarily implied. In principle, the arbitrage strategy in Table 5.1 can be reversed [5] to exploit the negative basis spread attached to the shorter tenor LIBOR. However, based on the model setup, a negative spread would imply a negative intensity or a negative loss rate or both. Hence model assumptions are violated. There are 74 days in our sample on which negative ask

[5] By reversing the strategy, the arbitrageur should borrow at 6M LIBOR and lend at 3M LIBOR. In the TS, the arbitrageur receives 6M LIBOR and pays the 3M LIBOR plus the negative spread.

rates are quoted. The distribution of these days are as follows,

Table 5.6: Days of Negative Spreads

Year	2008	2009	2010	2011	2012	2013
Days	39	29	6	0	0	0

From Table 5.6 we observe that the days with negative spreads are concentrated around the peak of the GFC. We surmise that inconsistent and less meaningful quotes may have resulted from large volatility and uncertainty associated with the market stresses. Taking into account overlapping days, we identify 192 distinct days with oscillations and/or negative spreads. In our subsequent analysis, we decide to exclude these days from the sample because these data lack meaningful behavior and/or conflict with the proposed model, and we justify this choice by the above argument that these days represent anomalies due to an immature market, which seem to be disappearing as the tenor swap market matures. As a result, the final sample period includes 787 trading days.

5.3.4 Global Optimization

Recognizing the problems with the bootstrap results and data issues, we instead calibrate model parameters with global optimization. By optimization, structures and constraints can be imposed to avoid oscillations and negative intensities. To achieve best fits, both intensities and loss rates are made time-dependent and piecewise constant.

The global optimization is implemented as follows. We denote $PV_{GainBid}$ and $PV_{GainAsk}$ respectively as the PV of the gains based on the bid rate and the ask rate of the TS spread. To fully take account of transaction costs in the arbitrage strategy proposed in Table 5.1, we subtract the LIBOR-LIBID spread (assumed to be fixed at 12.5 bps) from the bid rate of TS spread to calculate $PV_{GainBid}$. This is appropriate because when the arbitrageur lends fund, the deposit rate is the LIBID rate. Therefore the lower bound of the arbitrage profit is $PV_{GainBid}$. For each trading day, we then minimize the loss function G:

$$G = \sum_{i=1}^{N} [\max(PV^i_{Loss} - PV^i_{GainAsk}, 0) + \max(PV^i_{GainBid} - PV^i_{Loss}, 0)]^2, \tag{5.15}$$

where $N = 36$ is the number of swaps used in the global optimisation. Thus the loss function is chosen such that for a given set of parameters π_i and λ_i, the optimisation error is zero if the following condition is satisfied:

$$PV^i_{GainBid} \leq PV^i_{Loss} \leq PV^i_{GainAsk}. \tag{5.16}$$

In (5.16) we set $PV^i_{GainBid}$ as the lower bound and $PV^i_{GainAsk}$ as the upper bound for PV^i_{Loss}. If PV^i_{Loss} based upon the calibrated parameters falls within the bounds, we assume that the PV_{Gain} based on the actual transacted rate is matched and the error is set to zero. Therefore we only have positive error terms if PV^i_{Loss} is below the lower bound or above the upper bound.

In order to obtain sensible fits and avoid severe oscillations, we also impose a measure of smoothness on the optimization. We use the following smooth measure to penalize large oscillations of the intensities:

$$\begin{aligned} Smooth_\lambda &= \sum_{i=1}^{N-2} [(\lambda_{i+2} - \lambda_{i+1}) - (\lambda_{i+1} - \lambda_i)]^2 \\ &= \sum_{i=1}^{N-2} (\lambda_{i+2} + \lambda_i - 2\lambda_{i+1})^2. \end{aligned} \tag{5.17}$$

We therefore minimize the objective function - weighted sum of the loss function and the smoothness measure:

$$Scale_{loss} \cdot G + Scale_{smooth} \cdot \; Smooth_\lambda. \tag{5.18}$$

where $Scale_{loss}$ is the damping factor on the loss function and $Scale_{smooth}$ is the damping factor on the smoothness measure. The damping factors can be adjusted, depending on the main objective of the optimization. If the dominating objective is to minimize the fitting errors, $Scale_{loss}$ should be assigned a higher weight than $Scale_\lambda$. On the other hand, if the main objective is to have a smooth term structure of intensities without large fluctuations, $Scale_\lambda$ should be assigned a higher weight than $Scale_{loss}$.

To implement the global optimization scheme, we set initial values for intensities prior to optimisation as

$$\lambda_i = e^{(-0.1T_i)} \cdot \frac{Spread_1}{100}, \qquad \forall i \in [1, 2, ..., N], \tag{5.19}$$

and apply the constraint

$$0.00001 < \lambda_i < 0.99999, \qquad \forall i \in [1, 2, ..., N]. \tag{5.20}$$

The time-decay function in (5.19) is chosen because based upon the analysis of the sample data, the shape of the spread curve is monotonically decreasing on most of the trading days. T_i stands for the maturity of each bootstrap interval end. The weight factor of the decay function $\frac{Spread_1}{100}$ is used to assign different sets of initial intensities for each sample date, based upon the spread level of 1M v 3M TS with 3M maturity on that day. Eqn. (5.19) is used to both avoid oscillations and ensure smoothness of the calibrated intensities. The constraint in (5.20) is imposed to guarantee positivity of intensities, as well as prevent unusually high values.

We also set initial conditions for the loss rates. As we have no view on the shape of the loss rates, a constant is chosen as initial inputs for the optimization:

$$\pi_i = 0.01, \qquad \forall i \in [1, 2, ..., N], \tag{5.21}$$

Similarly, positive bounds are imposed:

$$0.0001 < \pi_i < 0.1, \qquad \forall i \in [1, 2, ..., N]. \tag{5.22}$$

5.3.5 Optimisation Results

We have three key results from the proposed global optimisation scheme, in relation to the fitting errors, intensities and loss rates.

1) On 247 sample days the fitting error is zero, i.e. the loss function G in Eqn. (5.15) is zero and the condition in (5.16) is satisfied. The bounds we set for the intensities and the loss rates are obeyed for all sample days.

2) The term structure of the calibrated intensities monotonically decreases on all sample days. This is expected given the initial condition we set for intensities in Eqn. (5.19).

3) The loss rate curve repeatedly oscillates on 111 sample days.

Figure 5.2, 5.3 and 5.4 are used to illustrate the optimisation results. In Figure 5.2 we observe that on January 27th, 2010, for all tenor swaps included in the sample, the PV of loss based on the calibrated intensities and loss rates is bounded by the PV of gains based on the bid rate of the spread (lower bound) and the ask rate of the spread (upper bound). Hence the fitting error is zero. In Figure 5.3 we see on April 20th, 2009 the calibrated piecewise constant intensities monotonically decrease. However, on the same day the loss rate curve in Figure 5.4 repeatedly oscillates.

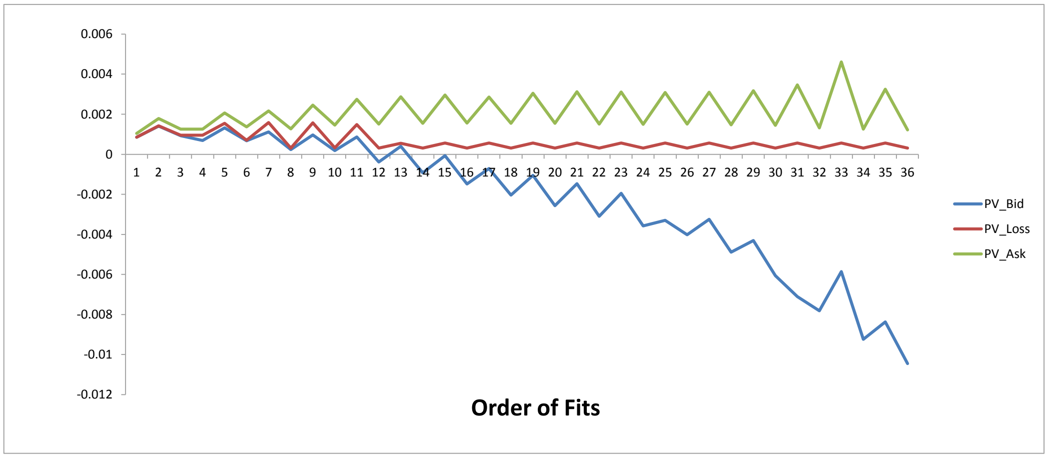


Figure 5.2: **Global optimisation Fits as at 27/01/2010.**

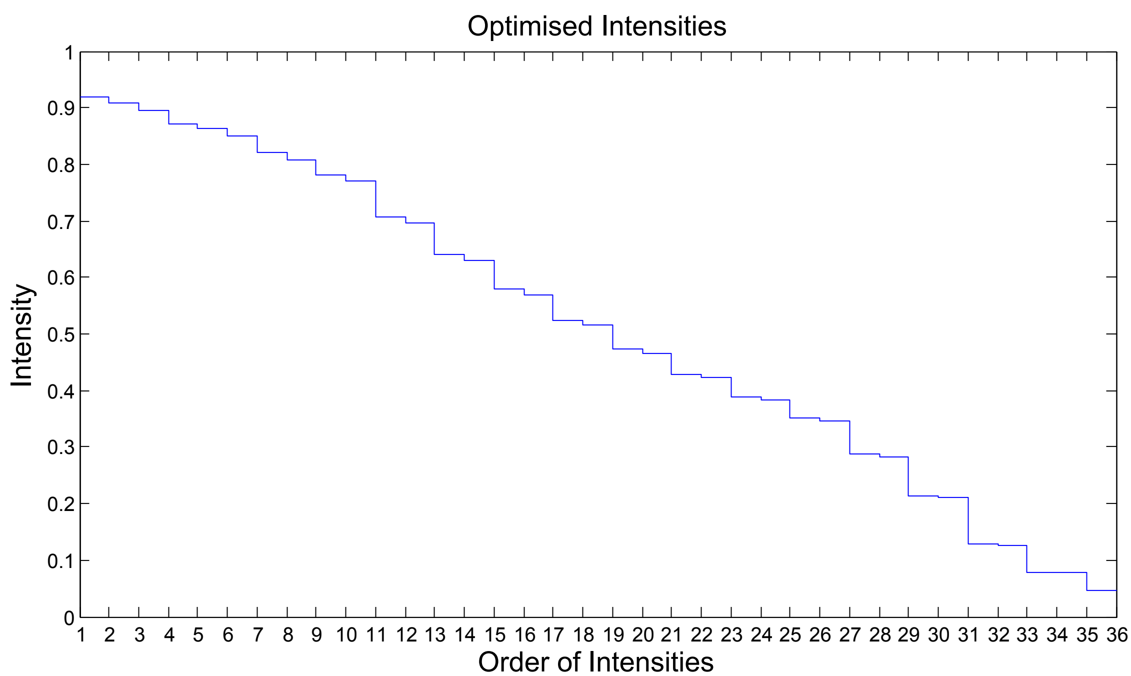


Figure 5.3: **Global optimisation Intensity Curve as at 20/04/2009.**

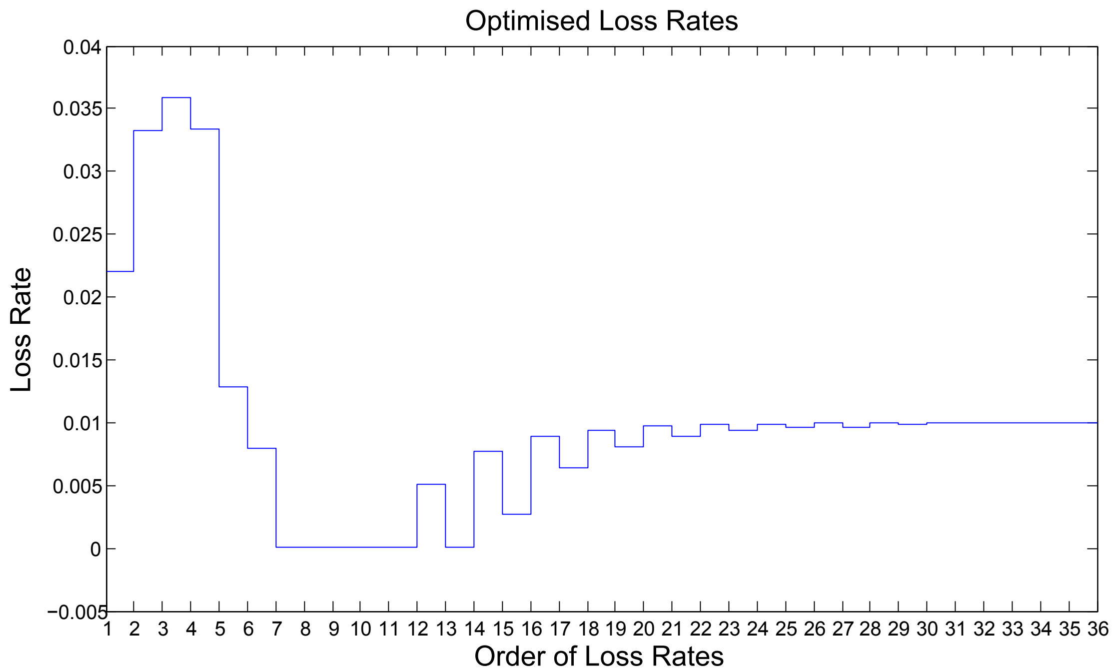


Figure 5.4: **Global optimisation Loss Rate Curve as at 20/04/2009.**

5.4 Conclusion

In this chapter we focus on the high-dimensional modelling problem, i.e. "multiple curve" approach by practitioners, existing in the single-currency TS market. In order to model the observed large spreads in market instruments, we propose an intensity-based model to describe the arrival time of liquidity shocks. Such an approach also helps to reduce the modelling dimension down to two economically fundamental variables: the intensity of liquidity shocks and loss due to refinancing in an illiquid market.

The analytical analyses of the problems in the bootstrap results show that the TS market, compared to the more established IRS market, is less mature and consistent. The immaturity and inconsistency of the market quotes is especially demonstrated during the market turbulence. We then propose a global optimisation scheme to calibrate time-dependent, piecewise constant model parameters. The optimisation results show the symptom of many degrees of freedom in this set-up. Though perfect fits are achieved and term structures of intensity of liquidity shock are mostly well-behaved, the loss rates repeatedly oscillate, leading us to address this by constructing a more parsimonious model in the next chapter.

Chapter 6

Parsimonious Modelling of Intensity and Loss Rate

6.1 Introduction

Our optimization results in Chapter 5 show symptoms of overparameterisation. Firstly both intensities and loss rates are made piecewise constant, deterministic functions of time, resulting in 72 degrees of freedom. Secondly, the loss rate curves show a lack of structure through repeated oscillations on 111 sample days.

A model with a large number of parameters, such as the one used in our optimization, is able to perfectly fit the observed data. However, it is less likely to explain well than a parsimonious model which assumes more smoothness. Furthermore, the fitting errors of the parsimonious model may represent an opportunity to study the systematic and idiosyncratic features of the data that the model fails to capture (see, for example, Nelson and Siegel 1987).

We are thus motivated to propose a parametrically parsimonious model for both intensities and loss rates, which allows us to capture a family of curve shapes. Nelson and Siegel (1987) proposed a model to fit the term structure of interest rates. The Nelson-Siegel model is consistent with a level-slope-curvature factor interpretation of the term structure (e.g, Litterman and Scheinkman 1991) and widely used in academia and practice. Nelson and Siegel (1987) models the instantaneous forward rate $f(\tau)$ as

$$f(\tau) = \beta_0 + \beta_1 e^{(-\tau/s)} + \beta_2(\tau/s)e^{(-\tau/s)}, \tag{6.1}$$

where τ is time to maturity and β_0, β_1, β_2 and s are constants to be estimated. The model hence consists of three factors: the constant β_0 represents the long term interest level, the exponential decay function $\beta_1 e^{(-\tau/s)}$ and a Laguerre function in the form of xe^{-x}. The role of the factors can be seen by examining the limiting behavior of time to maturity. If we let $\tau \to \infty$, the second and the third factor vanish and the long-term forward rate converges to β_0. As $\tau \to 0$, the Laguerre function vanishes and the forward rate converges to $\beta_0 + \beta_1$. Hence $-\beta_1$ measures the slope of the yield curve, where a positive (negative) β_1 represents a downward (upward) slope. Lastly, the Laguerre function represents the curvature of the yield curve and the shape parameter $s > 0$ determines the rate at which the slope and the curvature decay to zero. The location of the maximum (minimum) value of the curvature is determined by the value of s. Small (large) values of s correspond to rapid (slow) decay and therefore suitable for fitting curvatures at low (longer) maturities.

By pre-specifying a grid of shape parameters, Nelson and Siegel (1987) transformed the non-linear model in Eqn. (6.1) to a linear model and performed ordinary least squares (OLS) regressions for 37 data sets. The regression results explained a large fraction of the variations in the yields of treasury bills, with a median R^2 of 96%. Although the best-fitting shape parameter s varies for different data sets, by fixing s at its median value for all data sets only resulted in little loss of explanatory power. An important observation in Nelson and Siegel (1987) is that by plotting the time series of the estimated parameters, a breaking point October 1982 was identified, after which the importance of the curvature factor was evidently less. Since October 1982, both the magnitudes and variations of estimated β_1 and β_2 became much smaller and the yield curve shapes became simpler and more stable. The authors attributed such a structural break to the change of Federal Reserve monetary policy in October 1982, a switch from stabilizing the monetary aggregates to stabilizing interest rates. Nelson and Siegel argued that the market quotes may have become more accurate with more certainty in interest rates and resulted in simpler, lower-order yield curves. Such a structural break effect is particularly relevant with our study. It would be interesting to see if the intensity and loss rate curves become more stable during more recent periods than during the crises.

Therefore in this chapter we employ a Nelson-Siegel type model, which allows us to parsimoniously describe intensities and loss rates of the liquidity shock. To account for randomness, we also propose a preliminary stochastic model for these two parameters. The rest of the chapter proceeds as follows. In Section 2 we set up the Nelson-Siegel type model for intensities and loss rates. Section 3 proposes constraints and initial conditions for the optimisation scheme, which is used to calibrate the model parameters. The optimisation results are presented and discussed in Section 4. We then propose the preliminary stochastic model in Section 5 and conclude in Section 6.

6.2 Model Set-up

We propose a Nelson-Siegel type model for both the intensity $\lambda(t)$ and the instantaneous loss rates $\pi(t)$:

$$\lambda(\tau) = \lambda_0 + \lambda_1 e^{(-\tau/S_1)} + \lambda_2(\tau/S_1)e^{(-\tau/S_1)}, \tag{6.2}$$

$$\pi(\tau) = \pi_0 + \pi_1 e^{(-\tau/S_2)} + \pi_2(\tau/S_2)e^{(-\tau/S_2)}, \tag{6.3}$$

where τ is time to maturity. λ_0, λ_1, λ_2 (π_0, π_1, π_2) are respectively coefficient for the level, slope and curvature factor of the intensity (loss rate) and S_1 (S_2) is the shape parameter for the intensity (loss rate). Hence for each sample trading day there are 8 parameters to estimate, a drastic reduction compared to the global optimization in the previous chapter. In general researchers fix the shape parameter and estimate the linearized version of the Nelson-Siegel parameters. However, the linear regression method has been reported to behave erratically over time and have large variances. Annaert et al.(2012) showed that these problems result from multicollinearity. Alternatively, nonlinear optimization techniques can be used to estimate model parameters. The drawback of this approach is that the estimators are sensitive to the initial values used in the optimization (see Cairns and

Pritchard 2001). In the global optimisation scheme of Chapter 5 we deliberately account for transaction costs and fit the PV of losses with respect to two bounds, therefore we maintain this optimization approach and the associated conditions in Eqn. (5.15) and (5.16).

We integrate $\lambda(\tau)$ and $\pi(\tau)$ specified by Eqn. (6.2) and (6.3), the no-arbitrage condition in Eqn. (5.4) then becomes

$$\sum_{k=1}^{K}(e^{A}-1)(e^{-B}-e^{-C})D^{OIS}(T_0,T_{\eta(k)})=\sum_{k=1}^{K}(B_{i,j,N}\cdot i)e^{-D}D^{OIS}(T_0,T_k), \tag{6.4}$$

where A, B, C and D are respectively [1],

$$\begin{aligned}A&=\int_{T_k}^{T_{\eta(k)}}\pi(u)\,\mathrm{d}u\\&=(T_{\eta(k)}-T_k)\pi_0-\pi_1S_2\left(e^{(-T_{\eta(k)}/S_2)}-e^{(-T_k/S_2)}\right)\\&\quad-\pi_2\left(e^{(-T_{\eta(k)}/S_2)}(T_{\eta(k)}+S_2)-e^{(-T_k/S_2)}(T_k+S_2)\right),\end{aligned} \tag{6.5}$$

$$\begin{aligned}B&=\int_{0}^{T_{k-1}}\lambda(u)\,\mathrm{d}u\\&=T_{k-1}\lambda_0+(\lambda_1+\lambda_2)S_1(1-e^{(-T_{k-1}/S_1)})-\lambda_2T_{k-1}e^{(-T_{k-1}/S_1)},\end{aligned} \tag{6.6}$$

[1]See Appendix F for the proof.

$$\begin{aligned} C &= \int_0^{T_k} \lambda(u)\,\mathrm{d}u \\ &= T_k\lambda_0 + (\lambda_1+\lambda_2)S_1(1-e^{(-T_k/S_1)}) - \lambda_2 T_k e^{(-T_k/S_1)}, \end{aligned} \tag{6.7}$$

and

$$\begin{aligned} D &= \int_0^{T_{(\eta_{(k)}-n)}} \lambda(u)\,\mathrm{d}u \\ &= T_{(\eta_{(k)}-n)}\lambda_0 + (\lambda_1+\lambda_2)S_1(1-e^{(-T_{(\eta_{(k)}-n)}/S_1)}) - \lambda_2 T_{(\eta_{(k)}-n)} e^{(-T_{(\eta_{(k)}-n)}/S_1)}. \end{aligned} \tag{6.8}$$

Eqn. (6.4) is then used in the optimisation to calibrate the parameters for $\lambda(\tau)$ and $\pi(\tau)$.

6.3 Optimization Scheme

In this section we establish the constraints and initial values for the optimisation, which is used to calibrate parameters of λ and π.

6.3.1 Optimization Constraints

We establish parameter constraints for the nonlinear optimization scheme. For the intensity parameters, because λ_0 is the long-term level of intensity, we require that

$$0.000435 \le \lambda_0 \le 0.23872, \tag{6.9}$$

where the lower bound (resp. upper bound) is the minimum (resp. maximum) λ_0 calculated from the global optimization results in Chapter 5. The shape parameter S_1 is bounded by the maturities of tenor swap data and as such

$$0 \leq S_1 \leq 30. \tag{6.10}$$

The constraints for λ_1 and λ_2 are derived from the positivity of the model in Eqn. (6.2). Take the first derivative of Eqn. (6.2) with respect to τ we have

$$\lambda_\tau = \frac{-\lambda_1}{S_1} e^{(-\tau/S_1)} - \frac{\lambda_2}{S_1}(\tau/S_1)e^{(-\tau/S_1)} + \frac{\lambda_2}{S_1} e^{(\tau/S_1)}. \tag{6.11}$$

Let (6.10) equal to 0 we obtain

$$\tau = \left(\frac{\lambda_2 - \lambda_1}{\lambda_2}\right) S_1. \tag{6.12}$$

The second derivative of (6.10) with respect to τ is

$$\lambda_{\tau\tau} = \frac{1}{S_1^2}\left(\lambda_1 - 2\lambda_2 + \frac{\lambda_2 \tau}{S_1}\right) e^{(-\tau/S_1)}. \tag{6.13}$$

Substitute (6.11) into (6.12), the second derivative becomes

$$\lambda_{\tau\tau} = -\frac{\lambda_2}{S_1^2} e^{\left(\frac{\lambda_1-\lambda_2}{\lambda_2}\right)}. \tag{6.14}$$

Therefore the second derivative is positive if $\lambda_2 < 0$. It follows that the function $\lambda(\tau)$ has the local minimum at $\tau = (\frac{\lambda_2-\lambda_1}{\lambda_2})S_1$ and the function value is

$$\lambda(\tau) = \lambda_0 + \lambda_2 e^{\left(\frac{\lambda_1-\lambda_2}{\lambda_2}\right)}. \tag{6.15}$$

To ensure the positivity of the minimum, we examine the location of critical value of τ in (6.11). If $\tau = (\frac{\lambda_2-\lambda_1}{\lambda_2})S_1 < 0$, because $\lambda_2 < 0$, we must have $\lambda_2 > \lambda_1$. Then all is required is that $\lambda(0) = \lambda_0 + \lambda_1 > 0$, which ensures that $\lambda(\tau) > 0$ for all positive maturities. Therefore, in this case the constraints are

$$\lambda_2 < 0, \quad \lambda_2 > \lambda_1, \quad \lambda_0 + \lambda_1 > 0. \tag{6.16}$$

On the other hand, if $\tau = (\frac{\lambda_2-\lambda_1}{\lambda_2})S_1 \geq 0$, then $\lambda_1 \geq \lambda_2$. We require that $\lambda_0 + \lambda_2 e^{\left(\frac{\lambda_1-\lambda_2}{\lambda_2}\right)} > 0$, which leads to $\lambda_2 > \frac{-\lambda_0}{e^{\left(\frac{\lambda_1-\lambda_2}{\lambda_2}\right)}}$. Therefore the constraint for λ_2 is

$$\lambda_2 < \lambda_1, \quad \frac{-\lambda_0}{e^{\left(\frac{\lambda_1-\lambda_2}{\lambda_2}\right)}} < \lambda_2 < 0. \tag{6.17}$$

In a parallel fashion, the constraints of the loss rate parameters are set as follows,

$$0.0001 \leq \pi_0 \leq 0.02, \tag{6.18}$$

and

$$0 \leq S_2 \leq 30. \tag{6.19}$$

The constraints for π_1 and π_2 are

$$\pi_2 < 0, \quad \pi_2 > \pi_1, \quad \pi_0 + \pi_1 > 0, \tag{6.20}$$

or

$$\pi_2 < \pi_1, \quad \frac{-\pi_0}{e^{\left(\frac{\pi_1 - \pi_2}{\pi_2}\right)}} < \pi_2 < 0. \tag{6.21}$$

6.3.2 Initial Values

To ensure stability and smoothness in the estimated parameters, we use the estimated parameters on one day as the initial values for the next day. For the first sample date, 03/03/2009, the initial values are chosen and listed in Table 6.1.

Table 6.1: Initial Values for Optimisation as at 03/03/2009

Parameter	λ_0	λ_1	λ_2	S_1	π_0	π_1	π_2	S_2
Value	0.1789	0.6406	-0.35	2	0.01	0.0702	-0.06	2

Based upon the optimisation results in Chapter 5, we fix he value of λ_0 by the 30Y intensity. We then approximate $\lambda(0) = \lambda_0 + \lambda_1$ by the 2M intensity. Taking the difference of $\lambda(0)$ and λ_0, we obtain the initial value of λ_1. Initial values of π_0 and π_1 are chosen with the same procedure. We set the shape parameters S_1 and S_2 to be 2, which means at such initial values the location of the hump or trough of the Laguerre function $\beta_2(\frac{\tau}{s})e^{(-\frac{\tau}{s})}$ is at $\tau = 2$. Finally, we search over a grid of values for λ_2 and π_2 and choose the set of values, which in conjunction with other initial values, produces the least optimisation error.

6.4 Results

The calibration results of the model parameters are presented in Figures 6.1 and 6.2. In Figure 6.1 we observe that, except for the initial sample period when the market was still experiencing turmoils, the intensity parameters, λ_0, λ_1, λ_2 and S_1, show little time variations. In Figure 6.2 we have similar findings for the loss rate parameters S_2. On the other hand, π_0, π_1 and π_2 exhibit significant time variations. Different from the estimation of default risk, where the standard calibration to market instruments (e.g. CDS spreads) normally assumes a constant recovery rate and calibrates time-varying default intensities, our optimisation jointly calibrates loss rates and intensities. We therefore propose that the time variations of liquidity risk in our model are mainly captured by the loss rate parameters.

The calibrated model parameters fit 178 sample days perfectly, or a proportion of 22.61% of the whole sample period. It needs to be pointed out that, it is not the objective of the Nelson-Siegel type model to achieve perfect fits. Instead, we aim to identify the underlying relation of the model. Fitting errors may present an opportunity to examine the systematic and idiosyncratic features of the sample data. To this end, we study the distribution of the fitting errors across the sample period. Table 6.2 presents the distribution of the largest 10% fitting errors. The sample mean and standard deviation of fitting errors are summarized in Table 6.3.

Table 6.2: Distribution of the largest 20% of fitting errors

Year	2009	2010	2011	2012	2013
Days	78	0	0	1	0

Table 6.3: Sample Mean and Standard Deviations of Fitting Errors

Year	2009	2010	2011	2012	2013
mean	2.78E-06	3.92E-09	2.01E-08	1.87E-07	2.51E-09
standard deviation	3.96E-06	1.15E-08	5.39E-08	1.63E-07	2.36E-09

We see from Table 6.2 that the almost all large fitting errors are in the early sample period (i.e. 2009). Table 6.3 shows that since 2009, the fitting errors are characterized by both lower level and volatility, particularly in the most recent sample period (i.e. 2013). This lends support for our conjecture that the consistency of tenor swap market has improved since the crisis and liquidity risk in the market has gradually stabilized.

We also examine shapes of the intensity curve and the loss rate curve. As expected, both curves are well behaved without oscillations. On all sample days, both the intensity curve and the loss curve firstly decreases then increases. Figures 6.3 illustrates these curves on 03/03/2009.

Lastly, in Figure 6.2 we find that the upper bound (2%) of π_0 imposed for the optimisation is binding and hit on 36 sample days. We therefore increase the upper bound (to 5% and 7% respectively) and re-optimise. The results are summarized in Figures 6.4 and 6.5 (for 5%) and Figures 6.6 and 6.7 (for 7%). We see that the upper bound is only hit on 9 days (for 5%) and 1 day (for 7%) and such days all fall at the very beginning the sample period. We conclude that the boundary hitting is due to the heightened market stress and there is no need to further increase the upper bound of the long–term loss rate. The distribution of the fitting errors and curve shapes of intensity and loss rate are virtually unchanged after increasing the upper bound for π_0.

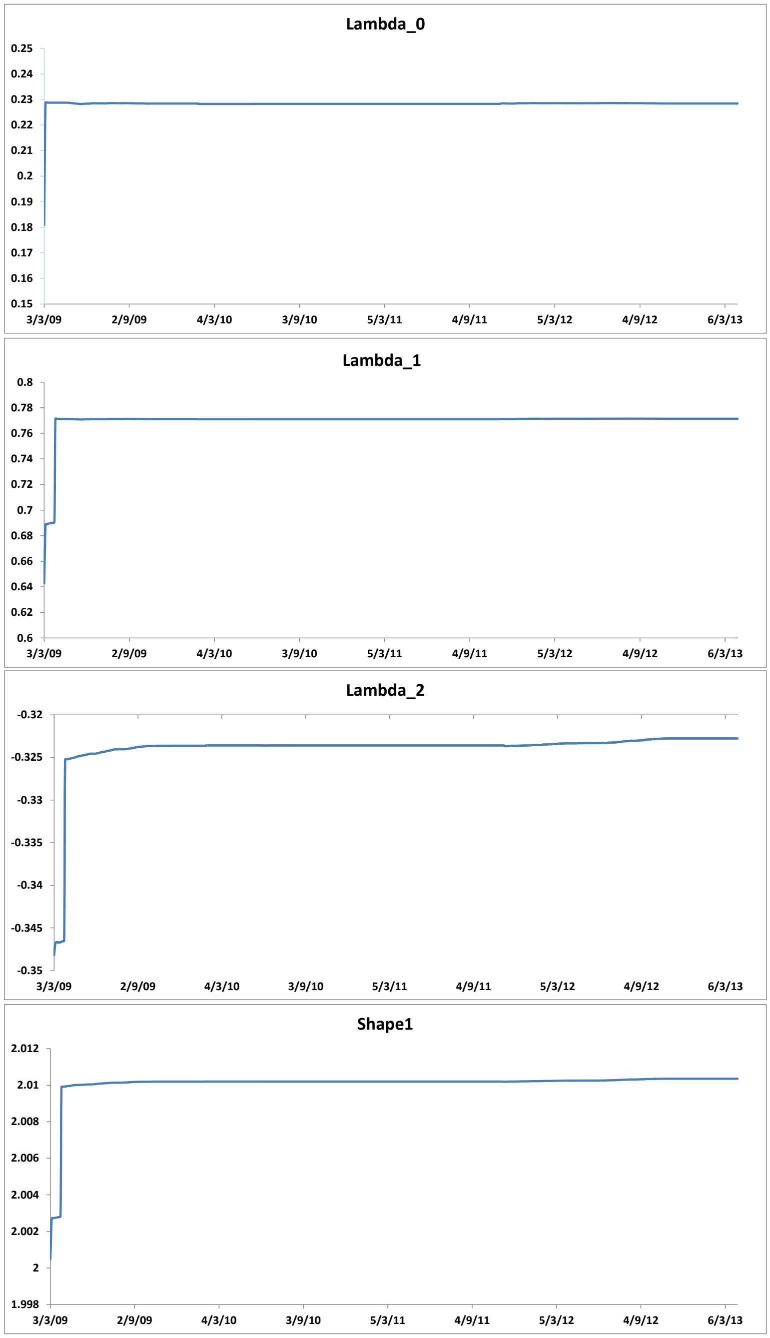


Figure 6.1: **Calibrated parameters of the Nelson-Siegel type model in Eqn. (6.2). Parameters include λ_0, λ_1, λ_2 and S_1. The upper bound of π_0 is 0.02.**

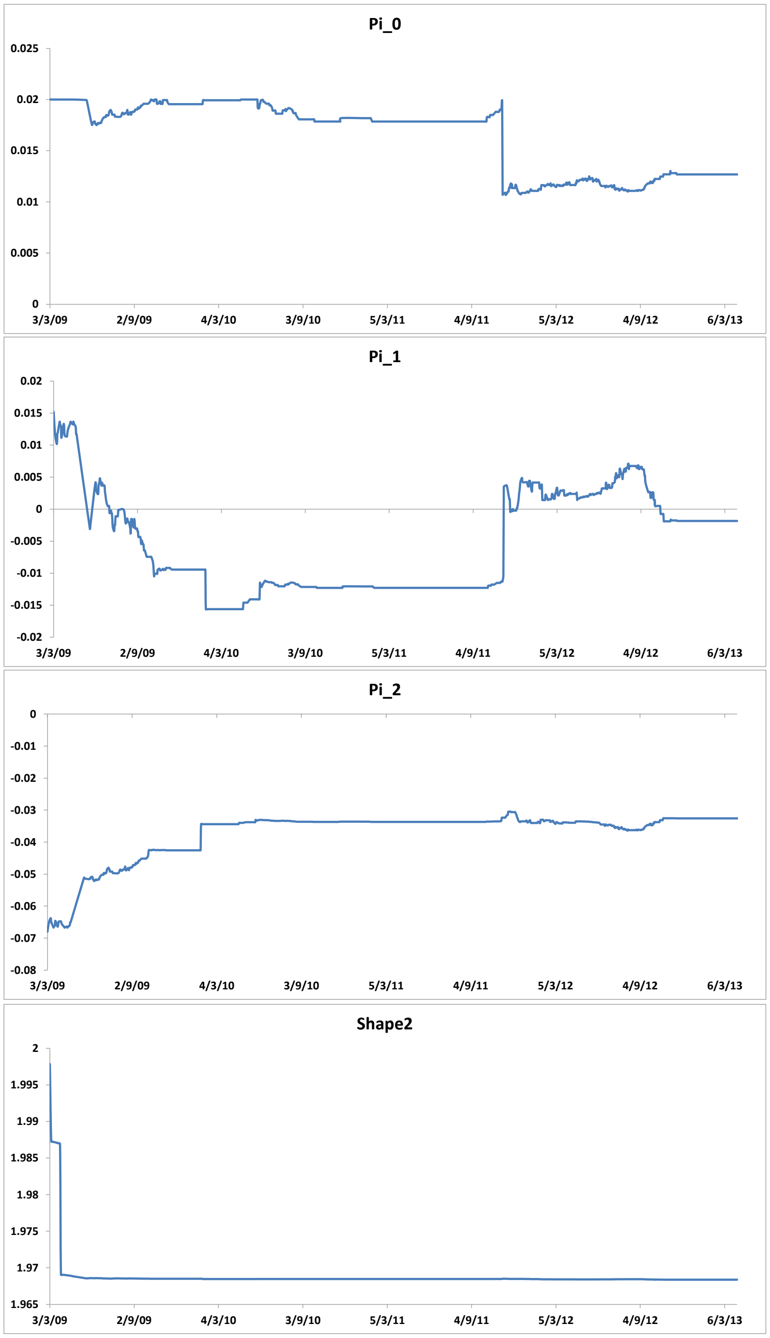


Figure 6.2: **Calibrated parameters of the Nelson-Siegel type model in Eqn. (6.3). Parameters include π_0, π_1, π_2 and S_2. The upper bound of π_0 is 0.02.**

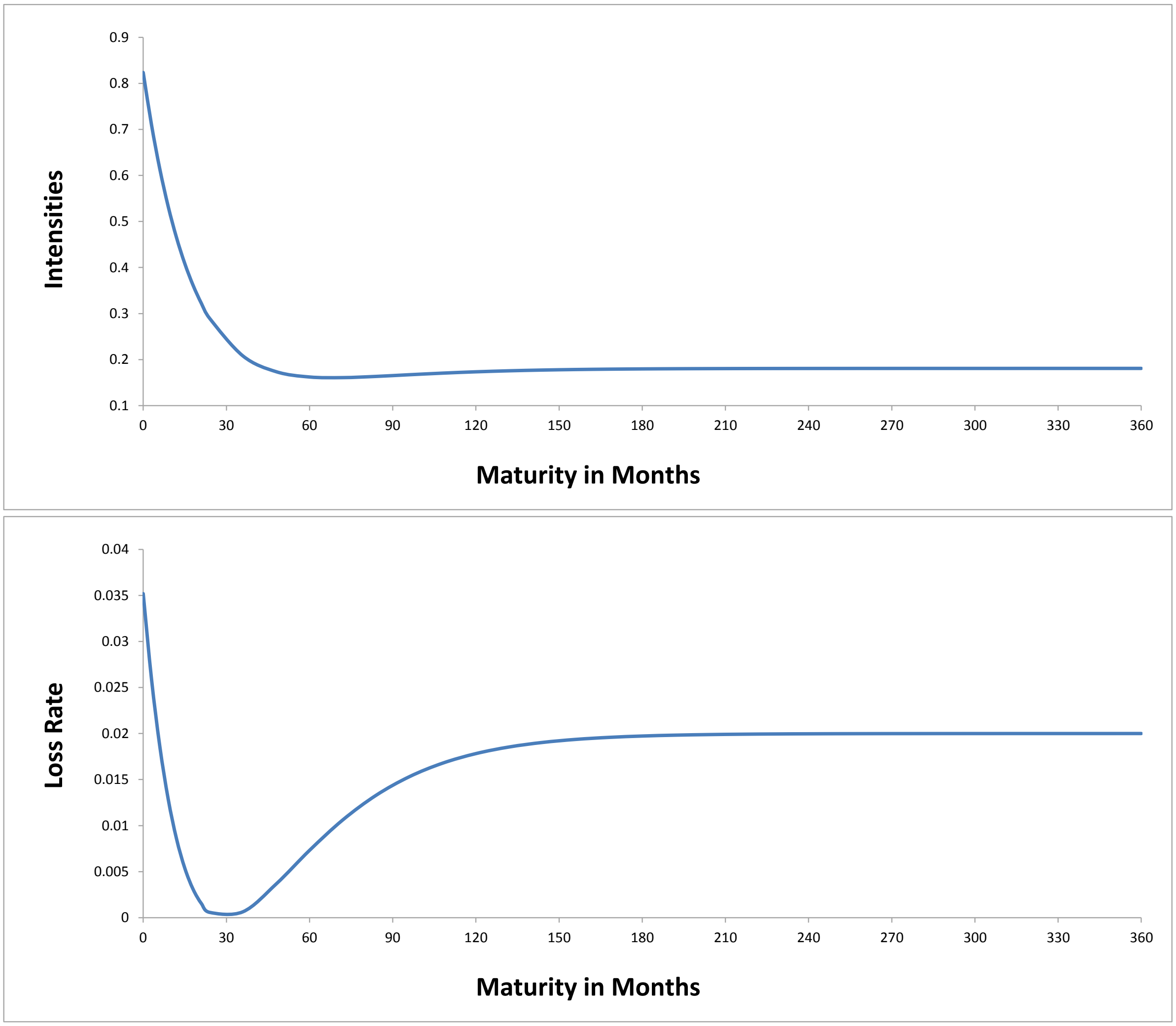


Figure 6.3: **Term Structure of the calibrated intensities and loss rates on 03/03/2009.**

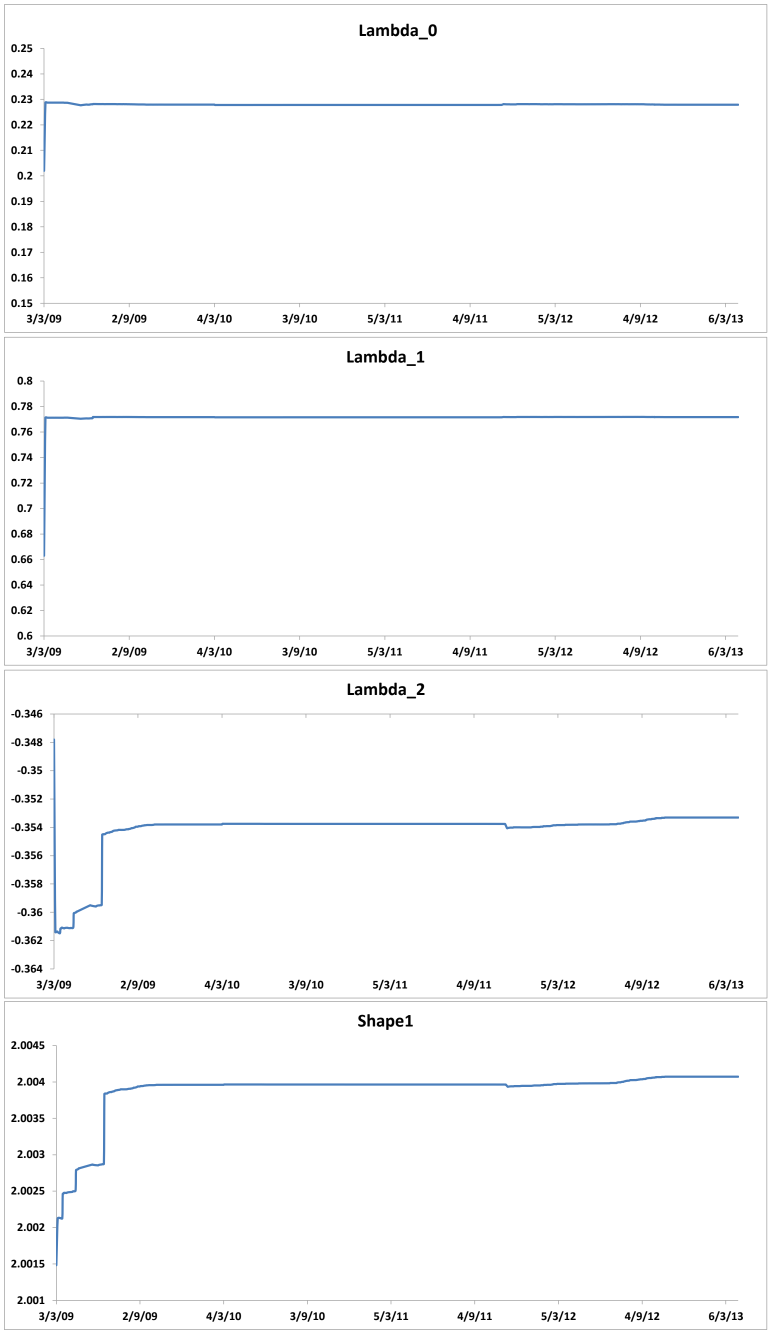


Figure 6.4: **Calibrated parameters of the Nelson-Siegel type model in Eqn. (6.2). Parameters include λ_0, λ_1, λ_2 and S_1. The upper bound of π_0 is 0.05.**

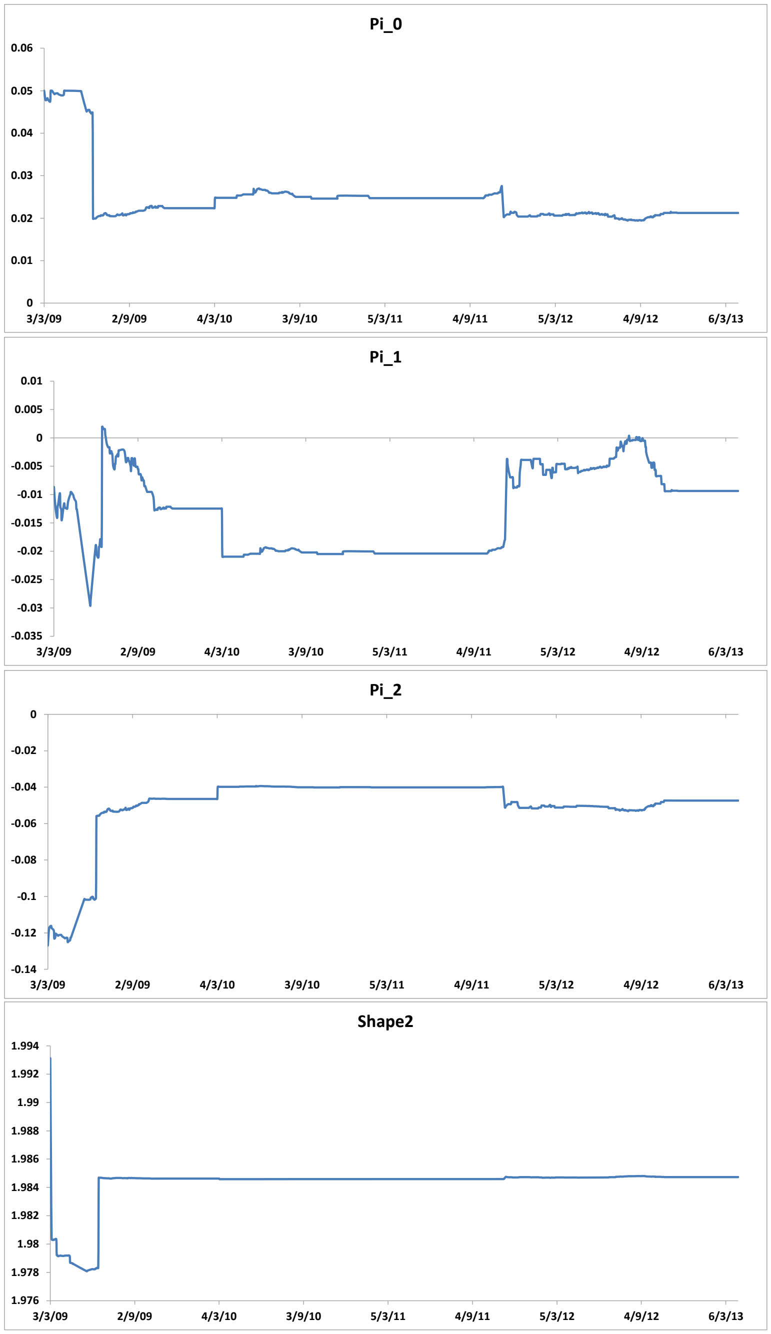


Figure 6.5: **Calibrated parameters of the Nelson-Siegel type model in Eqn. (6.3). Parameters include π_0, π_1, π_2 and S_2. The upper bound of π_0 is 0.05.**

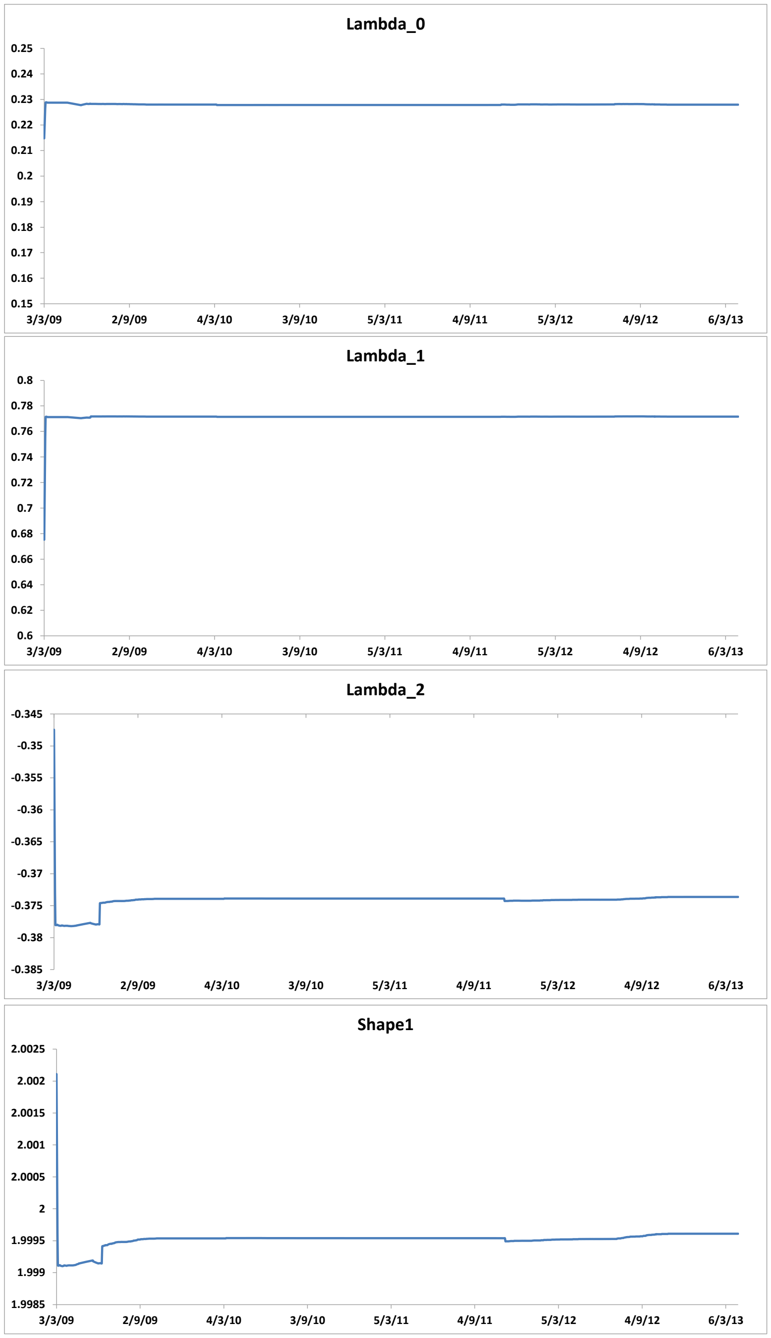


Figure 6.6: **Calibrated parameters of the Nelson-Siegel type model in Eqn. (6.2). Parameters include λ_0, λ_1, λ_2 and S_1. The upper bound of π_0 is 0.07.**

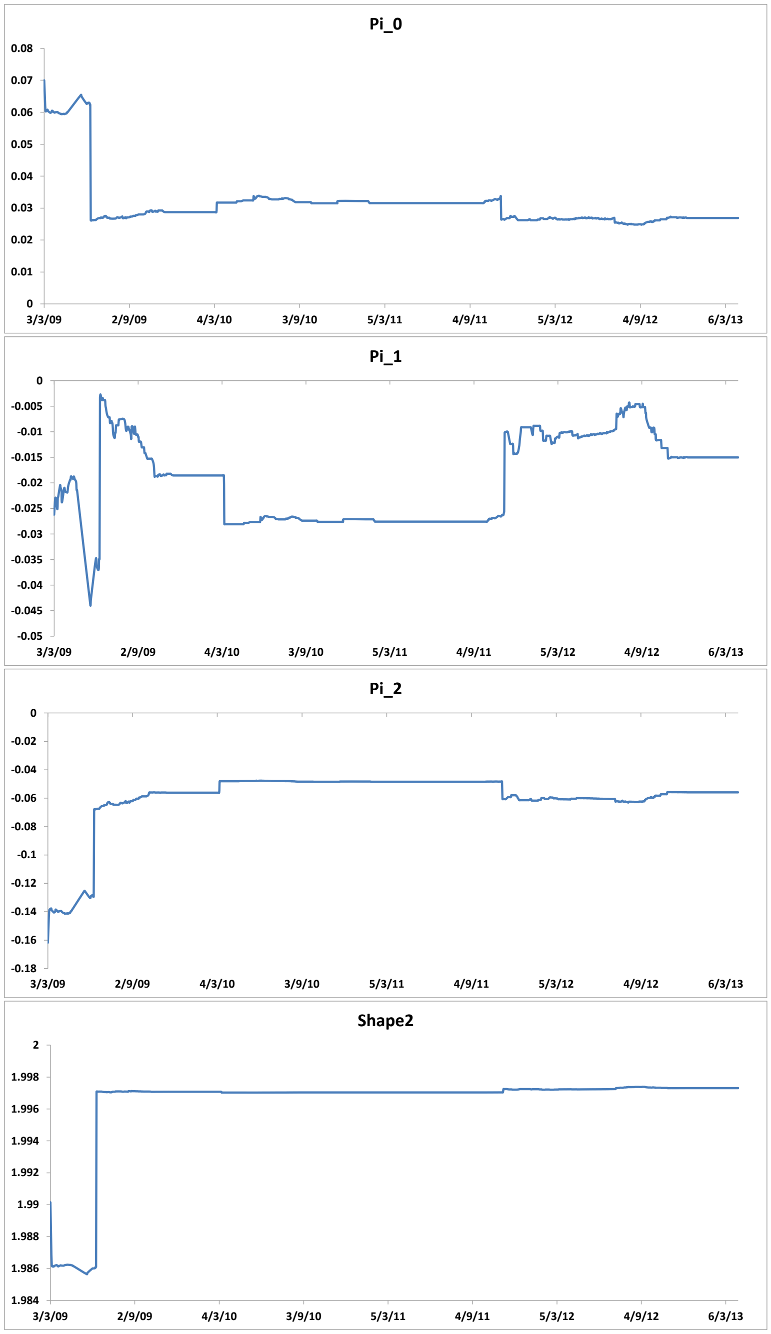


Figure 6.7: **Calibrated parameters of the Nelson-Siegel type model in Eqn. (6.3). Parameters include π_0, π_1, π_2 and S_2. The upper bound of π_0 is 0.07.**

6.5 A Stochastic Model

We have thus far modelled time-dependent, deterministic intensity and loss rate. To account for potential randomness, in this section we set up a preliminary stochastic model for these two parameters.

As in Chapter 5, we assume that when a liquidity shock occurs, the arbitrageur is unable to roll over the shorter tenor loan and has to refinance until the end of the associated longer tenor, shutting down the borrowing and lending in the arbitrage strategy at the end of the longer tenor within which the first jump occurs. However suppose now that the liquidity shock is triggered by a Cox process (Cox 1955)[2] with stochastic intensity λ. Assume that this process is independent of any interest rate dynamics and is given by a sum of d independent factors y_i, i.e.

$$\lambda(t) = \sum_{i=1}^{d} y_i(t), \tag{6.22}$$

where the y_i follows the Cox-Ingersoll-Ross (CIR) dynamics (Cox et al. 1985) under the pricing measure, i.e.

$$dy_i(t) = (\theta_i - a_i y_i(t))dt + \sigma_i \sqrt{y_i(t)} dW_i^{\lambda}(t), \tag{6.23}$$

where $dW_i^{\lambda}(t) \quad (i = 1, \cdots, d)$ are independent Wiener processes. The CIR-type model is chosen for its analytical tractability, as well as the guaranteed positivity of the modelled object, with the condition which ensures that the origin is inaccessible. In order to keep

[2]See Appendix G for a description of the Cox process, or doubly stochastic Poisson process.

the model analytically tractable we do not allow for time-dependent coefficients at this stage. Since each of the factors follow independent CIR-type dynamics, the sufficient condition for each factor to remain positive is $2\theta_i > \sigma_i^2, \forall i$, as discussed for the one-factor case in Cox et al. (1985).

For stochastic intensity λ, the LHS and RHS of the condition proposed in Eqn. (5.4) respectively becomes

$$\sum_{k=1}^{K}\left(e^{\int_{T_k}^{T_{\eta(k)}}\pi(u)\,\mathrm{d}u}-1\right)\left(E\left[e^{-\int_0^{T_{k-1}}\lambda(u)\,\mathrm{d}u}\right]-E\left[e^{-\int_0^{T_k}\lambda(u)\,\mathrm{d}u}\right]\right)D^{OIS}(T_0,T_{\eta_{(k)}}), \quad (6.24)$$

and

$$\sum_{k=1}^{K}(B_{i,j,N}*i)E\left[e^{-\int_0^{T_{(\eta_{(k)}-n)}}\lambda(u)\,\mathrm{d}u}\right]D^{OIS}(T_0,T_k). \quad (6.25)$$

The expectations under the pricing measure can be evaluated in the same manner as the multifactor CIR zero coupon bond price given in Chen and Scott (1995), i.e.

$$E\left[e^{-\int_t^T\lambda(u)\,\mathrm{d}u}\mid\mathcal{F}_t\right]=A(t,T)\cdot e^{-B(t,T)y(t)}, \quad (6.26)$$

with

$$y(t) = \begin{pmatrix} y_1(t) \\ \vdots \\ y_d(t) \end{pmatrix},$$

and $B(t,T)$ similarly a vector with components:

$$B_i(t,T) = 2w_i \left(e^{c_i(T-t)} - 1\right). \tag{6.27}$$

Furthermore,

$$A(t,T) = \prod_{i=1}^{d} A_i(t,T), \tag{6.28}$$

with

$$A_i(t,T) = \left(2c_i w_i e^{\frac{1}{2}(c_i+a_i)(T-t)}\right)^{(2\theta_i/\sigma_i^2)}. \tag{6.29}$$

The coefficients c_i and w_i are given by

$$w_i = \left((c_i + a_i)e^{c_i(T-t)} + c_i - a_i\right)^{-1}, \tag{6.30}$$

and

$$c_i = \sqrt{a_i^2 + 2\sigma_i^2}. \tag{6.31}$$

Similarly, in addition we may assume stochastic dynamics for the refinancing loss rate as well by setting π as a sum of $\tilde{d}$ independent factors z_i:

$$\pi(t) = \sum_{i=1}^{\tilde{d}} z_i(t), \tag{6.32}$$

with the stochastic process independent of interest rates and λ given by

$$dz_i(t) = (\xi_i - b_i z_i(t))dt + \gamma_i \sqrt{z_i(t)} dW_i^{\pi}(t), \tag{6.33}$$

where $dW_i^{\pi}(t) \quad (i = 1, \cdots, \tilde{d})$ are independent Wiener processes. We interpret the stochastic π as the instantaneous spread representing the cost of refinancing after a liquidity shock, meaning that if a liquidity shock occurs at time τ between T_{k-1} and T_k, the actual refinancing cost is represented by an implicit term structure of refinancing spreads. Denote this actual refinancing spread cost for the period from T_k to $T_{\eta_{(k)}}$ by $\tilde{\pi}(\tau)$, which is a continuously compounded, per annum rate. Then we model this by

$$e^{\tilde{\pi}(\tau)(T_{\eta_{(k)}} - T_k)} = \frac{E\left[e^{-\int_{\tau}^{T_k} \pi(s)\,\mathrm{d}s} \mid \mathcal{F}_{\tau}\right]}{E\left[e^{-\int_{\tau}^{T_{\eta_{(k)}}} \pi(s)\,\mathrm{d}s} \mid \mathcal{F}_{\tau}\right]}. \tag{6.34}$$

Thus quantity with a direct economic interpretation is $\tilde{\pi}(\tau)$, observable at the time τ of the liquidity shock, while the modelling of π as a multifactor CIR-type process serves to endow the $\tilde{\pi}(\tau)$ with a tractable stochastic dynamic; i.e. the term structure of incremental refinancing costs a the τ is given by the "discount factors":

$$E\left[e^{-\int_\tau^T \pi(s)\,\mathrm{d}s} \mid \mathcal{F}_\tau\right] = \tilde{A}(\tau,T)\cdot e^{-\tilde{B}(\tau,T)z(\tau)}, \tag{6.35}$$

with

$$z(\tau) = \begin{pmatrix} z_1(\tau) \\ \vdots \\ z_{\tilde{d}}(\tau) \end{pmatrix},$$

and $\tilde{B}(\tau,T)$ similarly a vector with components:

$$\tilde{B}(\tau,T) = 2\tilde{w}_i\left(e^{\tilde{c}_i(T-\tau)} - 1\right). \tag{6.36}$$

Furthermore,

$$\tilde{A}(\tau,T) = \prod_{i=1}^{\tilde{d}} \tilde{A}_i(\tau,T), \tag{6.37}$$

with

$$\tilde{A}_i(\tau,T) = \left(2\tilde{c}_i\tilde{w}_i e^{\frac{1}{2}(\tilde{c}_i+b_i)(T-\tau)}\right)^{(2\xi_i/\gamma_i^2)}. \tag{6.38}$$

The coefficients $\tilde{c}_i$ and $\tilde{w}_i$ are given by:

$$\tilde{w}_i = \left((\tilde{c}_i + b_i)e^{\tilde{c}_i(T-\tau)} + \tilde{c}_i - b_i\right)^{-1}, \tag{6.39}$$

and

$$\tilde{c}_i = \sqrt{b_i^2 + 2\gamma_i^2}. \tag{6.40}$$

Under the present independence assumptions, it therefore remains to calculate

$$\begin{aligned} E\left[e^{\tilde{\pi}(\tau)(T_{\eta(k)} - T_k)}\right] &= \frac{\tilde{A}(\tau, T_k)}{\tilde{A}(\tau, T_{\eta(k)})} E\left[e^{(\tilde{B}(\tau, T_{\eta(k)}) - \tilde{B}(\tau, T_k))z(\tau)}\right] \\ &= \frac{\tilde{A}(\tau, T_k)}{\tilde{A}(\tau, T_{\eta(k)})} \prod_{i=1}^{\tilde{d}} E\left[e^{(\tilde{B}_i(\tau, T_{\eta(k)}) - \tilde{B}_i(\tau, T_k))z_i(\tau)}\right]. \end{aligned} \tag{6.41}$$

If $\tilde{B}_i(\tau, T_{\eta(k)}) - \tilde{B}_i(\tau, T_k) < \frac{1}{2}$, we can apply Lemma 2.5 in Schlögl and Schlögl (2000) to obtain

$$E\left[e^{(\tilde{B}_i(\tau, T_{\eta(k)}) - \tilde{B}_i(\tau, T_k))z_i(\tau)}\right] = \frac{e^{(\zeta L/(1-2L))}}{(1-2L)^{\delta/2-1}}, \tag{6.42}$$

with,

$$L = \tilde{B}_i(\tau, T_{\eta(k)}) - \tilde{B}_i(\tau, T_k), \tag{6.43}$$

$$\zeta = \frac{4b_i e^{-b_i(\tau - T_0)}}{\gamma_i^2 (1 - e^{-b_i(\tau - T_0)})} z_i(T_0), \tag{6.44}$$

and

$$\delta = \frac{4\xi_i}{\gamma_i^2}, \tag{6.45}$$

where δ and ζ are respectively the degrees of freedom and the non-centrality parameter for the non-central chi-squared distribution function $\chi^2(\cdot, \delta, \zeta)$. The density function of $\chi^2(\cdot, \delta, \zeta)$ has the following representation (see, for example, Johnson and Kotz, 1970):

$$p_{\chi^2(\delta,\zeta)}(x) = \frac{e^{(-\frac{1}{2}(\zeta + x))}}{2^{\delta/2}} \sum_{j=0}^{\infty} \frac{\zeta^j}{2^{2j} j! \Gamma(\frac{\delta}{2} + j)} x^{(\delta/2 + j - 1)}. \tag{6.46}$$

The density function of z is

$$p_z(x) = c p_{\chi^2(\delta,\zeta)}(cx), \tag{6.47}$$

where

$$c = \frac{4b_i}{\gamma_i^2 (1 - e^{-b_i(\tau - T_0)})}. \tag{6.48}$$

If $\tilde{B}_i(\tau, T_{\eta_{(k)}}) - \tilde{B}_i(\tau, T_k) \geq \frac{1}{2}$, the calculation of this expectation is less tractable and we resort to numerical integration techniques.

We therefore propose that the stochastic models of λ and π can be used to estimate the parameters in Eqn. (6.23) and (6.33), with observed spreads in the tenor swap market.

6.6 Conclusion

In this chapter we address the issues which arise from the optimisation results in Chapter 5. We propose a Nelson-Siegel type model to parsimoniously capture the dynamics of intensity λ and loss rate π of the liquidity shock. The model is calibrated to the observed spreads in the tenor swap market. The results show good fits and well-behaved intensity and loss rate. An important finding is that the time variations of the liquidity risk are mainly captured by the loss rate parameters, while the intensity parameters show little variations over time. This contrasts with the standard default risk estimation, where time-variations of the default risk are captured by default intensities only. The results also demonstrate that since the turmoil of the GFC, the tenor swap market is in the process of maturing and stabilizing.

In order to account for potential randomness, as a preliminary step, we also set up stochastic CIR-type models for intensity and loss rate. We show that under certain conditions closed form solutions exist, which can be used to to tractably calibrate or estimate the model parameters.

Chapter 7

Conclusion

In this thesis, we investigated issues arising in the interest rate market since the GFC, specifically the large basis spreads quoted in single–currency basis swaps and cross–currency basis swaps. According to textbook theories, such anomalies present arbitrage opportunities, which should be efficiently exploited by market participants. Hence, the persistence of large spreads violates the textbook no–arbitrage conditions.

To explain and model such market changes, we proceeded in three stages. We firstly controlled for the effect of transaction costs in arbitrage strategies in the cross–currency market. The proposed no–arbitrage bounds for the basis spreads held consistently before the GFC. However, the violations of these bounds have been persistent since the crisis. Therefore, transaction costs are insufficient in explaining the large spreads. By textbook theory, this implies practical arbitrage profit. We drew upon recent empirical studies and attributed the persistent violations of no–arbitrage bounds to a new perception by the market of risks, in particular currency liquidity risk, involved in the execution of textbook "arbitrage" strategies.

As the second step, in order to identify the predictive power of liquidity risk for exchange rate movements, we used the violations of no–arbitrage bounds in the cross–currency market as a proxy for the market valuation of liquidity risk. FX market specific volatility proxies were also proposed. The empirical study of carry trade excess returns provided empirical support for a liquidity based model in explaining the UIP puzzle.

The third study of this thesis focused on the single–currency basis swap spreads and proposed a consistent framework to reconcile the differences between the classic "single curve" approach and practitioners' "multiple curve" approach. We used liquidity risk as the fundamental factor and set up an intensity–based model to describe the arrival time of liquidity shocks. The risk–neutral model parameters can be calibrated to quoted basis spreads. We also developed parsimonious modelling approaches for model parameters. The time–dependent, deterministic model was calibrated and the results showed that the tenor swap market is in the process of maturing since the turmoils of the GFC. Based on this analysis, we also set up a tractable stochastic model for liquidity risk.

There are various ways in which future research can build upon this thesis. Firstly, our studies concerning the cross–currency market (Chapter 3 and 4) can be extended to other currency pairs, particulary emerging market currencies, given their increasing importance in global economy. Secondly, our empirical studies use daily data. It would be interesting to see if results in this thesis hold for lower–frequency, such as weekly or quarterly data, or for high frequency intra–day data. Lastly, the parameters of the stochastic model developed in this thesis can be estimated, with either closed form solutions or numerical techniques, in order to examine its ability to fit observed basis spreads.

Appendix A

Proof of Equation (2.7)

We follow the theorem of general numéraire change proposed by Geman et al. (1995) to prove Eqn. (2.7).

Consider a probability space $(\Omega,\ \mathcal{F},\ \mathcal{F}_t(t \geq 0),\ P)$ which satisfies the usual hypotheses that the filtration $\mathcal{F}_t(0 \leq t \leq T)$ is right continuous with left limits and represents the uncertainty for a given economy for a time horizon $T > 0$. Ω is the set of possible states, the σ-field $\mathcal{F}$ represents the collection of sub-events of Ω. $\mathcal{F}_t$ is the information available at t and P is the probability measure that assigns probabilities to events in Ω. In the fundamental theorems of asset pricing, Harrison and Kreps (1979), Harrison and Pliska (1981, 1983) show the connection between the absence of arbitrage and the existence of the equivalent martingale measure[1] (or risk-neutral measure) Q, such that the discounted asset price between two payment dates is a martingale under probability

[1]Two probability measures P and Q on $(\Omega,\ \mathcal{F})$ are equivalent if P is absolutely continuous with respect to Q, and Q is absolutely continuous with respect to P. i.e. $P(A) = 0 \Leftrightarrow Q(A) = 0, \forall A \in \mathcal{F}$ (see, for example, Shreve 2008).

measure Q. Given the existence of an equivalent martingale measure, then there exists a unique no-arbitrage price for any attainable contingent claim[2]. When the market is complete, every contingent claim is attainable and the martingale measure is unique.

Geman et al. (1995) shows that though the use of risk-neutral probability measure is useful in pricing contingent claims in complete markets, Q is not always the most natural and efficient measure for the pricing of a given contingent claim. The change of measure technique is then proposed by Geman et al. (1995) is described as follows.

Definition. A numéraire is *any positive non-dividend-paying asset.*

Proposition. Assume there exists a numéraire $N(t)$ and a probability measure π equivalent to initial probability measure p, such that the price of any traded asset X (without intermediate payments) relative to $N(t)$ is a martingale under π,

$$\frac{X(t)}{N(t)} = E^{\pi}\left(\left.\frac{X(T)}{N(T)}\right| \mathcal{F}_t\right), \qquad 0 \leq t \leq T. \tag{A.1}$$

Let $U(t)$ be an arbitrary numéraire. Then there exists a probability measure Q^U, which is equivalent to p, such that the price of any attainable claim H normalized by U is a martingale under Q^U,

[2]A contingent claim C is a positively valued random variable which belongs to $L^2(\Omega, \mathcal{F}, P)$, i.e. C is square integrable with respect to P and $E^P(C^2) < \infty$. A trading strategy ϕ is self-financing if there is no exogenous infusion or withdrawal of cash and the value only changes due to changes in the prices of the assets in the strategy. C is said to be attainable if there exists some ϕ such that the value of ϕ at maturity is equal to C. C is said to be generated by ϕ.

$$\frac{H(t)}{U(t)} = E^{Q^U}\left(\left.\frac{H(T)}{U(T)}\right| \mathcal{F}_t\right), \qquad 0 \le t \le T. \tag{A.2}$$

The Radon-Nikodym derivative defining the measure Q^U is given by

$$\left.\frac{dQ^U}{d\pi}\right| \mathcal{F}_T = \frac{U_T N_0}{U_0 N_T}. \tag{A.3}$$

Following from the proposition by Geman et al. (1995), we assume the existence of an equivalent martingale measure (such as the risk-neutral measure Q), then the price of any asset relative to a numéraire is a martingale under the associated probability measure (see Brigo and Mercurio 2006).

We rewrite Eqn. (2.1) as

$$F(t; T_1, T_2)P(t, T_2) = \frac{1}{\tau(T_1, T_2)}(P(t, T_1) - P(t, T_2)), \tag{A.4}$$

where $F(t; T_1, T_2)P(t, T_2)$ is a portfolio of two zero coupon bonds, hence a portfolio of two assets. The zero coupon bond $P(t, T_2)$ is a strictly positive non-dividend-paying asset, hence it is a numéraire under the Q^{T_2} forward probability measure. The Radon-Nikodym derivative which changes measure Q to Q^{T_2} is given by

$$\left.\frac{dQ^{T_2}}{dQ}\right| \mathcal{F}_{T_2} = \frac{P(T_2, T_2)B(0)}{P(0, T_2)B(T_2)} = \frac{e^{-\int_0^{T_2} r(u)\,\mathrm{d}u}}{P(0, T_2)}, \tag{A.5}$$

where $B(t)$ is the numéraire under the risk-neutral measure Q. The *savings account* $B(t)$ accrues the instantaneous spot rate $r(t)$ from time 0 to t,

$$B(t) = e^{\int_0^t r(u)\,\mathrm{d}u}. \tag{A.6}$$

Hence,

$$F(t; T_1, T_2) = \frac{F(t; T_1, T_2) P(t, T_2)}{P(t, T_2)} = \frac{P(t, T_1) - P(t, T_2)}{\tau(T_1, T_2) P(t, T_2)} \tag{A.7}$$

must be a martingale under the T_2 forward measure, i.e.

$$E^{Q^{T_2}}[L(T_1, T_2) \mid \mathcal{F}_t] = E^{Q^{T_2}}(F(T_1; T_1, T_2) \mid \mathcal{F}_t) = F(t; T_1, T_2). \tag{A.8}$$

Appendix B

Proof of Equation (4.9)

From Eqn. (4.8)

$$d_2 = d_1 - \sigma\sqrt{\tau}, \tag{B.1}$$

$$\begin{aligned} d_2^2 &= d_1^2 - 2d_1\sigma\sqrt{\tau} + \sigma^2\tau \\ &= d_1^2 - 2ln[F(t,T)/K], \end{aligned} \tag{B.2}$$

$$\begin{aligned} n(d_2) &= \frac{1}{\sqrt{2\pi}} e^{[-d_2^2/2)]} \\ &= \frac{1}{\sqrt{2\pi}} e^{[-\frac{1}{2}d_1^2 + ln(F(t,T)/K)]} \\ &= \frac{1}{\sqrt{2\pi}} e^{[-\frac{1}{2}d_1^2]} e^{[ln(F(t,T)/K)]} \\ &= n(d_1)\frac{F(t,T)}{K}. \end{aligned} \tag{B.3}$$

The partial derivatives of $N(x)$ w.r.t F is

$$\frac{\partial N(x)}{\partial F} = n(x)\frac{\partial x}{\partial F}. \tag{B.4}$$

From (B.1) we see that

$$\frac{\partial d_1}{\partial F} = \frac{\partial d_2}{\partial F}. \tag{B.5}$$

Hence, for a call option V_c,

$$\begin{aligned}
\frac{\partial V_c}{\partial F} &= e^{-r_d\tau}N(d_1) + e^{-r_d\tau}F\frac{\partial N(d_1)}{\partial F} - Ke^{-r_d\tau}\frac{\partial N(d_2)}{\partial F} \\
&= e^{-r_d\tau}N(d_1) + e^{-r_d\tau}F\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial F} - Ke^{-r_d\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial F} \\
&= e^{-r_d\tau}N(d_1) + Fe^{-r_d\tau}n(d_1)\frac{\partial d_1}{\partial F} - Ke^{-r_d\tau}n(d_2)\frac{\partial d_2}{\partial F} \\
&= e^{-r_d\tau}N(d_1) + Fe^{-r_d\tau}n(d_1)\frac{\partial d_1}{\partial F} - Ke^{-r_d\tau}n(d_1)\frac{F}{K}\frac{\partial d_1}{\partial F} \\
&= e^{-r_d\tau}N(d_1).
\end{aligned} \tag{B.6}$$

Equation (4.9) is hence proved for $\omega = 1$. Put options ($\omega = -1$) can be proved similarly.

Appendix C

Principal Component Analysis

Principal component analysis (PCA) is a mathematical factor model that uses orthogonal transformation technique to convert possibly correlated variables into uncorrelated variables. The uncorrelated variables are the so called principal components, which are linear combinations of the original variables. The main objective of PCA is to reduce dimensionality to capture the most important influences from all variables simultaneously. Suppose the original correlated variables are X_1, X_2, $\cdots$, X_n, then the principal components can be denoted as

$$\begin{aligned} P_1 &= \alpha_{11}X_1 + \alpha_{12}X_2 + \cdots + \alpha_{1n}X_n \\ P_2 &= \alpha_{21}X_1 + \alpha_{22}X_2 + \cdots + \alpha_{2n}X_n \\ &\vdots \\ P_n &= \alpha_{n1}X_1 + \alpha_{n2}X_2 + \cdots + \alpha_{nn}X_n \end{aligned} \tag{C.1}$$

where α_{ij} are the factor loadings, which are the coefficients of the jth original variable in the ith principal component. If each original independent variable has m observations, then there will be m observations on each principal components. We also require that the square terms of the coefficients in each principal component sum to one

$$\begin{aligned}
\alpha_{11}^2 + \alpha_{12}^2 + \cdots + \alpha_{1n}^2 &= 1 \\
\alpha_{21}^2 + \alpha_{22}^2 + \cdots + \alpha_{2n}^2 &= 1 \\
\cdots \qquad \cdots \qquad &\cdots \\
\alpha_{n1}^2 + \alpha_{n2}^2 + \cdots + \alpha_{nn}^2 &= 1. \qquad \text{(C.2)}
\end{aligned}$$

The principal components are calculated based upon constrained optimization without assuming any particular distribution of the original variables. They are in fact the eigenvalues of the matrix $X'X$, where X is the matrix of original variable observations. $X'X$ is a $n \times n$ square matrix. Hence the matrix has n number of eigenvalues. If we denote these eigenvalues by $\lambda_i (i = 1, \cdots, n)$, the proportion of total variations in the observations of the original variables that can be explained by each principal component is

$$\phi_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}. \qquad \text{(C.3)}$$

The principal components are derived in a way such that the proportions ϕ_i are in the descending order. If the first k principal components ($0 < k < n$) are considered sufficient to explain the total variations in the matrix $X^{'}X$, then the remaining $n - k$ components are abandoned and the final regression model only includes the first k components. PCA is intended to maintain most of the useful information contained in the original variables. The benefit is the reduction of dimensionality and the removal of high correlations in the original variables, hence the ordinary least squares (OLS) estimators with PCA are more efficient.

Despite such benefits, PCA has limitations, in particular the lack of theoretical motivation and economic interpretation. A more technical introduction of PCA can be found in Alexander (2008).

Appendix D

Chow Breakpoint Test Results

We present in Table D.1 the Chow Breakpoint test results in Subsection 4.4.4 of Chapter 4. The break date is August 10th, 2007. The null hypothesis of the test is that there is no break at the specified break date.

Table D.1: Chow Breakpoint Test Results

Currency	JPY	EUR	AUD
F–statistic	3.8633	2.5146	0.8344
Prob. F	0.0000	0.0054	0.6148

Hence the null hypothesis is rejected at 1% significance level for JPY and EUR, but we fail to reject the null for AUD. We then choose March 18th, 2008 as the break point and test for AUD. The F–statistic is 2.7156 and the probability of the F–statistic is 0.0012. The break date is hence supported.

Appendix E

Time-Inhomogeneous Poisson Process

We follow Brigo and Mercurio (2006) to describe basic properties of the time-inhomogeneous Poisson process $N(t)$, with deterministic time-dependent intensity $\lambda(t)$.

$\lambda(t)$ is assumed to be a positive and piecewise, right continuous, function of process time t. $N(t)$ is a non-decreasing, integer-valued process with null initial condition $N(0) = 0$ and independent increments. The hazard function $\Gamma(t)$ is defined as the integral of the intensity,

$$\Gamma(t) = \int_0^t \lambda(u)\,\mathrm{d}u. \tag{E.1}$$

$N(t)$ can also be defined by the standard Poisson process $M(t)$ with constant intensity one

$$N_t = M_{\Gamma_t}. \tag{E.2}$$

Define τ as the first jump time of N_t, then $M(t)$ jumps the first time at Γ_τ. Because the first jump time of a standard Poisson process follows the standard exponential distribution we have

$$\Gamma_\tau =: \xi \sim \mathrm{Exp}(1). \tag{E.3}$$

Hence, the hazard function of $N(t)$ at the first jump time τ is a standard exponential random variable. Due to the memoryless property of the exponential distribution, Γ_τ is independent of previous processes in the given filtered probability space $(\Omega,\ \mathcal{F},\ \mathcal{F}_t(t \geq 0),\ P)$. A random variable known at t is measurable with respect to the filtration $\mathcal{F}_t$. By definition, the hazard function is a strictly increasing function of time. Hence the probability that the first jump has not occurred by t is

$$\begin{aligned} P(\tau > t) &= P(\Gamma_\tau > \Gamma_t) \\ &= P(\mathrm{Exp}(1) > \Gamma_t) \\ &= e^{-\Gamma_t} \\ &= e^{-\int_0^t \lambda(u)\,\mathrm{d}u}. \end{aligned} \tag{E.4}$$

Similarly, the probability that the first jump occurs between t_1 and t_2 is

$$\begin{aligned} P(t_1 < \tau < t_2) &= P(\Gamma_{t_1} < \Gamma_\tau < \Gamma_{t_2}) \\ &= P(\Gamma_{t_1} < \mathrm{Exp}(1) < \Gamma_{t_2}) \\ &= P(\mathrm{Exp}(1) > \Gamma_{t_1}) - P(\mathrm{Exp}(1) > \Gamma_{t_2}) \\ &= e^{-\Gamma_{t_1}} - e^{-\Gamma_{t_2}} \\ &= e^{-\int_0^{t_1} \lambda(u)\,\mathrm{d}u} - e^{-\int_0^{t_2} \lambda(u)\,\mathrm{d}u}. \end{aligned} \tag{E.5}$$

Lastly, the process $N(t)$ satisfies the Poisson Law,

$$P[N(T) - N(t) = n] = \frac{1}{n!} \left(\int_t^T \lambda(u)\,\mathrm{d}u \right)^n e^{-\int_t^T \lambda(u)\,\mathrm{d}u}. \tag{E.6}$$

Appendix F

Proof of Equation (6.5), (6.6), (6.7) and (6.8)

We prove Eqn. (6.5) as follows.

$$\begin{aligned}
\int_{T_k}^{T_{\eta(k)}} \pi(u)\,\mathrm{d}u &= \int_{T_k}^{T_{\eta(k)}} \left(\pi_0 + \pi_1 e^{(-u/S_2)} + \pi_2 (u/S_2) e^{(-u/S_2)}\right)\,\mathrm{d}u \\
&= (T_{\eta(k)} - T_k)\pi_0 + \pi_1 \int_{T_k}^{T_{\eta(k)}} e^{(-u/S_2)}\,\mathrm{d}u + \pi_2 \int_{T_k}^{T_{\eta(k)}} (u/S_2) e^{(-u/S_2)}\,\mathrm{d}u \\
&= (T_{\eta(k)} - T_k)\pi_0 - \pi_1 S_2 \left(e^{(-T_{\eta(k)}/S_2)} - e^{(-T_k/S_2)}\right) + \pi_2 \int_{T_k}^{T_{\eta(k)}} (u/S_2) e^{(-u/S_2)}\,\mathrm{d}u.
\end{aligned} \tag{F.1}$$

With integration by parts,

$$\begin{aligned}\pi_2 \int_{T_k}^{T_{\eta(k)}} (u/S_2) e^{(-u/S_2)} \,\mathrm{d}u &= (\pi_2/S_2) \int_{T_k}^{T_{\eta(k)}} u e^{(-u/S_2)} \,\mathrm{d}u \\ &= (\pi_2/S_2) \left(u \int_{T_k}^{T_{\eta(k)}} e^{(-u/S_2)} \,\mathrm{d}u + \int_{T_k}^{T_{\eta(k)}} S_2 e^{(-u/S_2)} \,\mathrm{d}u \right) \\ &= -\pi_2 \left(T_{\eta(k)} e^{(-T_{\eta(k)}/S_2)} - T_k e^{(-T_k/S_2)} \right) - \pi_2 S_2 \left(e^{(-T_{\eta(k)}/S_2)} - e^{(-T_k/S_2)} \right) \\ &= -\pi_2 \left(e^{(-T_{\eta(k)}/S_2)} (T_{\eta(k)} + S_2) - e^{(-T_k/S_2)} (T_k + S_2) \right). \qquad \text{(F.2)}\end{aligned}$$

Substitute (E.2) into (E.1) we get

$$\begin{aligned}\int_{T_k}^{T_{\eta(k)}} \pi(u) \,\mathrm{d}u = (T_{\eta(k)} - T_k)\pi_0 - \pi_1 S_2 \left(e^{(-T_{\eta(k)}/S_2)} - e^{(-T_k/S_2)} \right) \\ - \pi_2 \left(e^{(-T_{\eta(k)}/S_2)} (T_{\eta(k)} + S_2) - e^{(-T_k/S_2)} (T_k + S_2) \right). \qquad \text{(F.3)}\end{aligned}$$

Eqn. (6.5) is hence proved. Eqn. (6.6), (6.7) and (6.8) can be proved similarly.

Appendix G

Cox Process

We follow Duffie (2001) and give a brief description of the Cox process, or doubly stochastic Poisson process.

For a probability space $(\Omega,\ \mathcal{F},\ P)$, a given filtration $\mathcal{F}_t(t \geq 0)$ satisfies the usual conditions (i.e. right continuous with left limits). Let $N(t)$ be a Poisson process adapted to $\mathcal{F}_t$, i.e. $\mathcal{F}_t$ includes all information about $N(t)$ up to time t. Let the stochastic intensity of $N(t)$ be λ_t, which is a non-negative process adapted to $\mathcal{F}_t$ such that $\int_0^t \lambda_s \, \mathrm{d}s < \infty$ almost surely (nonexplosive).

Let $\mathcal{G}_t(t \geq 0)$ be an alternative filtration on the given probability space. $\mathcal{G}_t(t \geq 0)$ is constructed as the σ-algebra by $\mathcal{F}_t$ and $N(s)$, $0 \leq s \leq t$, hence $\mathcal{F}_t \subset \mathcal{G}_t$. $N(t)$ then is doubly stochastic, driven by $\mathcal{F}_t(t \geq 0)$. For all t and $\tau > t$, conditional upon the σ-algebra $\mathcal{G}_t \vee \mathcal{F}_\tau$ generated by $\mathcal{G}_t \cup \mathcal{F}_\tau$, $N(\tau) - N(t)$ has the Poisson distribution with parameter $\int_t^\tau \lambda_s \, \mathrm{d}s$.

The Poisson process $N(t)$ is hence doubly stochastic because not only the jump time is stochastic, the intensity, i.e. the probability of jumping, λ_t is also stochastic. Hence the Cox process is a two step stochastic process. $N(t)$ is generated as a Poisson process, conditional upon λ_t , which is itself stochastic. Finally, under standard independence assumptions it can be shown that the probability that the first jump time τ has not occurred by t is

$$P[\tau > t] = E\left[e^{-\int_0^t \lambda_u \,\mathrm{d}u}\right]. \tag{G.1}$$

which is completely analogous to the zero-coupon bond price given by a short rate model with λ replacing r.

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