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TIME-DELAY SYSTEMS: STABILITY, SLIDING MODE CONTROL AND STATE ESTIMATION

by

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Certificate of Authorship/Originality

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degree except as fully acknowledged within the text.

I also certify that this thesis has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis.

That Dinh Nguyen

ABSTRACT

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Time delays and external disturbances are unavoidable in many practical control systems such as robotic manipulators, aircraft, manufacturing and process control systems and it is often a source of instability or oscillation. This thesis is concerned with the stability, sliding mode control and state estimation problems of time-delay systems. Throughout the thesis, the Lyapunov-Krasovskii (L-K) method, in conjunction with the Linear Matrix Inequality (LMI) techniques is mainly used for analysis and design.

Firstly, a brief survey on recent developments of the L-K method for stability analysis, discrete-time sliding mode control design and linear functional observer design of time-delay systems, is presented. Then, the problem of exponential stability is addressed for a class of linear discrete-time systems with interval time-varying delay. Some improved delay-dependent stability conditions of linear discrete-time systems with interval time-varying delay are derived in terms of linear matrix inequalities.

Secondly, the problem of reachable set bounding, essential information for the control design, is tackled for linear systems with time-varying delay and bounded disturbances. Indeed, minimisation of the reachable set bound can generally result in a controller with a larger gain to achieve better performance for the uncertain dynamical system under control. Based on the L-K method, combined with the delay decomposition approach, sufficient conditions for the existence of ellipsoid-based bounds of reachable sets of a class of linear systems with interval time-varying delay and bounded disturbances, are derived in terms of matrix inequalities. To obtain a smaller bound, a new idea is proposed to minimise the projection distances of the ellipsoids on axes, with respect to various convergence rates, instead of minimising its radius with a single exponential rate. Therefore, the smallest possible bound can be obtained from the intersection of these ellipsoids.

This study also addresses the problem of robust sliding mode control for a class of linear discrete-time systems with time-varying delay and unmatched external disturbances. By using the L-K method, in combination with the delay decomposition technique and the reciprocally convex approach, new LMI-based conditions for the existence of a stable sliding surface are derived. These conditions can deal with the effects of time-varying delay and unmatched external disturbances while guaranteeing that all the state trajectories of the reduced-order system are exponentially convergent to a ball with a minimised radius. Robust discrete-time quasi-sliding mode control scheme is then proposed to drive the state trajectories of the closed-loop system towards the prescribed sliding surface in a finite time and maintain it there after subsequent time.

Finally, the state estimation problem is studied for the challenging case when both the system's output and input are subject to time delays. By using the information of the multiple delayed output and delayed input, a new minimal order observer is first proposed to estimate a linear state functional of the system. The existence conditions for such an observer are given to guarantee that the estimated state converges exponentially within an ϵ-bound of the original state. Based on the L-K method, sufficient conditions for ϵ-convergence of the observer error, are derived in terms of matrix inequalities. Design algorithms are introduced to illustrate the merit of the proposed approach.

From theoretical as well as practical perspectives, the obtained results in this thesis are beneficial to a broad range of applications in robotic manipulators, airport navigation, manufacturing, process control and in networked systems.

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Nomenclature and Notation

Throughout the thesis, capital letters denote matrices and lower-case alphabet and Greek letters denote column vectors and scalars, respectively and the following nomenclatures and notations are used:

- TDS: Time-delay system
- FWM: Free weighting matrix
- SMC: Sliding mode control
- VSS: Variable structure system
- CTSMC: Continuous-time sliding mode control
- DTSMC: Discrete-time sliding mode control
- QSM: Quasi-sliding mode
- QSMB: Quasi-sliding mode band
- L-K: Lyapunov-Krasovskii
- LQR: Linear quadratic regulator
- LMI: Linear matrix inequality
- LKF: Lyapunov-Krasovskii functional
- MAB: Maximum allowable bound
- PAM: Piecewise analysis method
- IQC: Integral quadratic constraint
- SISO Single input single output
- MIMO: Multi input multi output
- $\mathbb{R}$: Field of real numbers
- $\mathbb{R}^+$: Set of non-negative reals
- $\mathbb{Z}$: Set of all integer numbers
- $\mathbb{Z}^+$: Set of non-negative integer numbers
- $\mathbb{Z}[a, b] = \mathbb{Z} \cap [a, b]$

- $\mathbb{N}$: Set of all natural numbers
- $\mathbb{R}^n$: n-dimensional space
- $\mathbb{R}^{n\times m}$: Space of all matrices of $(n \times m)$-dimension
- $\mathcal{C}$: Continuous function
- A^T: Transpose of matrix A
- A^{-1}: Inverse of matrix A
- $A > B$: Inequality between real vectors or matrices are understood componentwise
- I_n: Identity matrix of dimension $n \times n$
- 0_n: Zero matrix of dimension $n \times n$
- $(*)$: in a matrix means the symmetric term
- $\lambda(A)$: Set of all eigenvalues of matrix A
- $\lambda_m(A)$: Smallest eigenvalue of matrix A
- $\lambda_M(A)$: Largest eigenvalue of matrix A
- diag$[A, B, C]$: Block diagonal matrix with diagonal entries A, B, C
- $\circ$: the Hadamard product, i.e., $(A \circ B)_{i,j} = A_{i,j}.B_{i,j}$
- $||.||$: Euclidean norm of a vector or spectral norm of a matrix