THREE ESSAYS ON MACROECONOMICS AND MONETARY ECONOMICS

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CERTIFICATE

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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To my parents, Fei and Yuejiao, my husband, Amos, and all of my siblings. I love you.

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Abstract

The thesis includes three essays on macroeconomics and monetary economics. The first essay is my work in macroeconomics, particularly Chinese economy. The rest two essays are my work in monetary economics, and also the focus of my doctoral thesis. After the Global Financial Crisis, the Quantitative Easing (or unconventional monetary policy) in the U.S., Japan and other advanced economies have inspired theoretical interest in the questions of how monetary policy can affect the interest rates of money and other assets, and further affect the real economy. In the second and third essays, I build micro-founded models with money and government bonds to address those questions.

The first essay studies the real exchange rate appreciation caused by non-traded factor misallocation in China. The departure of a factor in excess supply in the non-traded sector leads to a real exchange rate appreciation, in a setup that combines the canonical Lewis Model with a Balassa-Samuelson traded/non-traded dichotomy. China is an ideal case for non-traded factor appreciation, since it has not completed its structural transformation. My model identifies non-traded goods with rural output produced in the West of China, and traded goods with manufactures produced on the Eastern Seaboard using overseas capital. According to the model, China's real exchange rate should appreciate as the *hukou* system, which acts to trap labor in the rural West, is dismantled.

The second essay develops a micro-founded model of money and bonds to address effects of monetary policy on output and unemployment. The baseline model considers both money and short-term government bonds serving as media of exchange. We analyze the effects of conventional monetary policy when Central Bank conducts open market operations (OMOs) by adjusting short-term bonds holdings. Conventional monetary policy is effective only when the short-term interest rate is positive. Then we introduce long-term government bonds to address the effects of unconventional monetary policy, particularly when the short-term interest rate hits the zero-lower bound. Quantitative analysis shows that unconventional monetary policy can reduce unemployment only when the fraction of households holding the portfolio of money and bonds is not too big.

In the third essay, we extend standard models of monetary exchange to include, in addition to currency, government bonds. We then study monetary policy, including OMOs, under various assumptions about market structure, and about the liquidity of money and bonds – i.e., their acceptability or pledgeability as media of exchange or collateral. OMOs matter because the supply of liquid assets matters. Theory delivers sharp policy predictions. It can also generate novel phenomena, like negative nominal interest rates, endogenous market segmentation, and outcomes resembling liquidity traps. We also explore explanations for differences in the liquidity of money and bonds using information theory.

Key words: Non-traded Factor Appreciation; Search Theory; Liquidity; Open Market Operations

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1 Chapter 1
Non-traded Factor Appreciation in China

1.1 Introduction

Following 1978, China emerged from the Cultural Revolution into an era of double digit growth. Two contributing factors that have been put forward to explain this are China's real exchange rate, and, its dual (rural-urban) economy with the rural labor surplus (Economist, 2008).

China's exchange rate movement is important for the world economy. The attendant current account surpluses arguably influenced the international flow of funds prior to the GFC (Bagnai, 2009), and this, in turn, has spawned a very spirited debate about the appropriate combination expenditure switching and expenditure reduction policies in both China and the US (Cordon, 1994). Yet a competitive real exchange rate has coexisted with a seemingly endless supply of labor from its rural areas, raising questions about the possible connections between the two.

It is the purpose of this paper to explore these connections by combining two classic frameworks – the Lewis and Balassa-Samuelson models. The modeling strategy identifies non-traded goods with rural output produced in the West of China, and traded goods with manufactures produced on the Eastern Seaboard using overseas capital . According to our model, China's real exchange rate should appreciate as the hukou registration system, which acts to trap labor in the rural West, is dismantled.

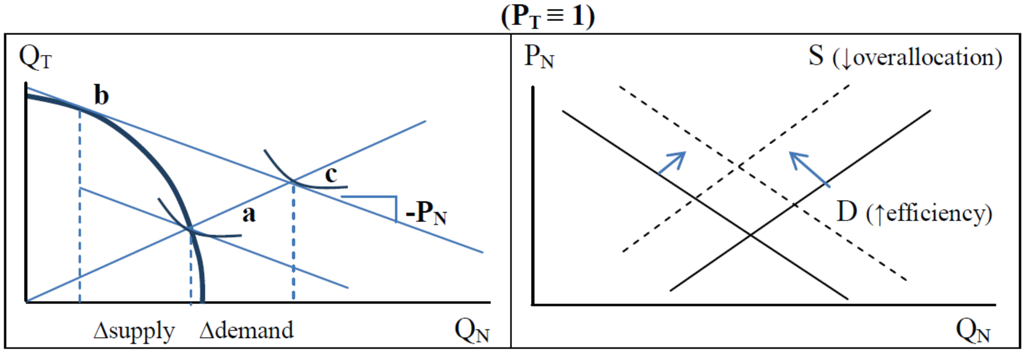
The rest of the paper is structured as follows. In Section 2 I define the non-traded factor appreciation (NTFA), and then ask if China could be a candidate for NTFA. Section 3 demonstrates NTFA in a stylized two-sector general equilibrium model and Section 4 represents the labour market of this model in a diagram reminiscent of the Specific Factors Model. Section 5 discusses the relationship between NTFA and the Balassa-Samuelson effect (Balassa, 1964, Samuelson, 1964) and concludes.

1.2 Non-traded Factor Appreciation and China

Firstly, I would like to define what is NTFA. It is about the exchange rate implication of factors being over-allocated to non-traded production. If an over-allocated factor leaves non-traded production, I argue that there will be a real appreciation. I call this phenomenon NTFA.

Figure 1 gives the intuition of NTFA for a two-good economy with a traded good (T) and non-traded good (N). In the left panel North-west movements along the production possibility frontier (PPF) occur as resources leave N to go to T, and demand exhibits constant expenditure shares for every level of income. The traded price is normalized to unity, so the real exchange rate is the non-traded price in Figure 1[1].

Figure 1: Real Exchange Rate Appreciation
($P_T \equiv 1$)



At point "a" on the left panel resources are over allocated to non-traded goods. Optimality requires that they leave non-traded production and enter traded production. As production moves to point "b" the budget set expands, and demand for both goods rises to point "c". The final P_N (not shown) will rise, responding to a fall in Q_N supply and an increase in Q_N demand, relative to

[1] Implicitly, the theoretical framework for Figure 1 is the one described in Section 1.3. Here I just want to use Figure 1 to show the main intuition of the model in Section 1.3.

“a”, where supply and demand were equal. The right panel tells the same story in partial equilibrium. The end of over-allocation implies a contraction in supply as factors leave the non-traded sector and an increase in demand as the economy becomes more efficient.

After defining NTFA, naturally I ask if China can be a candidate for NTFA. In what follows I outline a theory of over-allocation, and show how it might apply to China, since it is widely believed that China has a rural labour surplus. I also suspect that other important economies have over-allocated labour to non-traded production. For example, India's rural output is classified as non-traded, in Dumrongrittikul (2012).

In order to give content to the notion of over-allocation, I now review the Lewis model ((Lewis, 1954, Fei and Ranis, 1964, and Ranis and Fei, 1961) which remains the standard paradigm for discussing labour surpluses in developing countries (see the important review by Vines and Zeitlin, 2008, Fields, 2004, Kirkpatrick and Barrientos, 2004, and Temple, 2005).

A Lewis economy has two sectors: a traditional, overpopulated rural subsistence sector characterized by zero or low marginal labor productivity, and, a high-productivity modern urban industrial sector, to which labor from the subsistence sector is gradually transferred. The resultant expansion of modern-sector output (and employment) is assumed to continue until all surplus rural labor is in some sense absorbed in the modern industrial sector at *the Lewis Turning Point (LTP).*

The notion of the LTP is not pinned down precisely . Lewis himself defines it as the exhaustion of surplus labor in the traditional sector (Lewis, 1954). He even mentions a second turning point, which is reached when “the marginal product is the same in the capitalist and non-capitalist sectors, so that I have reached the neoclassical one-sector economy" (Lewis, 1972, pp.83).

This is nuanced further by Ranis and Fei (1961) who define three phases of transition. In the first phase, the marginal product of labour in the traditional sector is zero, so that the transfer of labor from the traditional sector to the modern sector does not lead to any reduction in the traditional sector's total output. The second phase, which they call the "shortage point", is ushered in when the marginal product of labour in the traditional sector becomes positive. All the while, in phases one and two, the wage rate in the traditional sector is an "institutional wage" equal to the average product of labour.

The third and final phase begins when marginal product catches up with the wage rate in the traditional sector, and thereafter the wage becomes the marginal product of labor. Ranis and Fei refer to this point as the "commercialization point" since the traditional sector can be said to have become commercialized. For Cobb-Douglas production, Ranis and Fei's commercialization point would never happen, since the average product is always above the marginal product.

In this paper I define the LTP as the moment when the average revenue product of labour (the "institutional wage") rises to the urban wage. Thereafter I assume rural workers are offered their marginal revenue product. As I shall see in Section 3 the main result of this paper applies across all of Ranis and Fei's phases. That is, the real exchange rate appreciates continually as over allocated labour leaves the traditional sector, whether it is paid its average or marginal product. It ceases, however, when the labour emigration stops.

So, returning to China, it is important for us to ask if the rural-to-urban labour flow is completed there. Empirical applications to China have sometimes claimed that it has (Cai and Du, 2011, Cai and Wang, 2008, Zhang, Yang and Wang, 2011). But Islam and Yokota (2008) estimate province-level rural production functions and find that the marginal product of labor is below the wage in the agricultural sector, which speaks of not-yet-completed transition.[2]

Although the empirical evidence is mixed, there is an important political-

[2]Furthermore, it is often asserted that China has a labour surplus (Economist, 2008),), which sits oddly with the claim that it has reached the LTP.

economy consideration which leads us to doubt that China has completed its transition. The Chinese government has both a strong incentive to understand the development process, and the power to shape the economy in significant ways. It has used its power to create the so called household registration (*hukou*) system, which has been a central instrument of the command economy since its inception in 1958 to prevent "undesirable" rural-to-urban migratory flows (Chan, 2010). The regulation decreed that all internal migrating be subject to approval by the relevant local government. Each person has a *hukou*, classified as "rural" or "urban", in a specific administrative unit. The *hukou* system limited the rural-urban labor mobility and also excluded rural population from access to state-provided goods, welfare, and entitlements.

Since 1978, China has begun to relax the *hukou* system. In the 1980s, a small number of rural workers were allowed to get *hukou* in towns if they could afford their own food[3] and also had fixed residences, stable jobs or ran their own business there. But this reform had a limited impact on labor mobility because it only focused on towns which did not provide many job opportunities for rural labor.

China began another round of *hukou* reforms aiming to expand small cities and towns around the mid-1990s. In 1997, the Ministry of Public Security of China (MPS) announced the *Pilot Plan on Household Registration System in Small Cities and Towns*, which permitted rural population who had stable jobs and fixed residences to have local urban *hukou* in these small cities. In 2001, this reform was expanded nationally and in 2003 China began experimenting with rural land reform and *hukou* deregulation - "crossing a river by feeling stones" - in Chengdu[4].

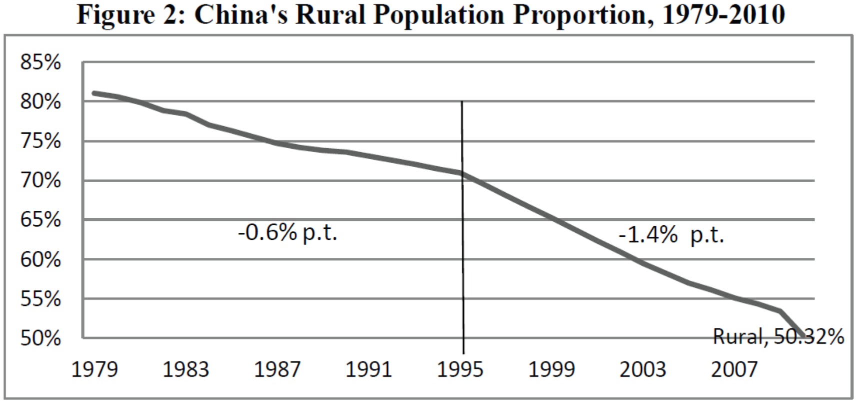
[3]In the plan-economy stage of China, urban hukou population was provided food by local governments in the forms of quota and subsidies.

[4]Chengdu is the capital city of Sichuan Province, which is a less-developed province in western China. However, as a "city", Chengdu covers nine urban districts, four small cities and six counties, which are a mix of urban and rural areas. Its total area is up to 12,100 square

All of the above reforms on the *hukou* system, particularly since the mid-1990s, are consistent with the trends shown in Figure 2. The average declining rate of rural population proportion during 1979-1995 is 0.6% while the declining rate following the mid-1990s reforms doubled to 1.4%.

Mai et al. (2009) analyzes the effects of the gradual dismantling of institutional barriers (mainly *hukou* system) to rural-urban labour migration in China, using a dynamic Computable General Equilibrium (CGE) framework. They find continued economic benefits of further migration, implying a current deadweight loss from misallocation labour. Furthermore, with half the population still in rural areas, and an urban-rural income gap that is high for its stage of development (Henderson, 2009), it is hard to avoid the conclusion that the authorities have slowed down labour movements, forestalling the LTP.

Figure 2: China's Rural Population Proportion, 1979-2010



Data source: China Statistical Yearbook 2010 and the Sixth National Population Census

Following the definition of NTFA, it is important to ascertain whether the output of the rural sector in China is primarily *non-traded*. There have been a number of recent studies using the Balassa-Samuelson traded/non-traded di-

meters. Chengdu's reforms cover six areas. First, its urban and rural areas are combined for planning purposes, in contrast to normal practice. Second, Chengdu has begun to identify farmers' property rights for housing and land. Third, Chengdu is establishing a cultivation-land protection fund. This fund buys social insurance for peasants who keep the cultivation land, to discourage them from selling it. Fourth, Chengdu provides public services to urban and rural areas equally. Fifth, Chengdu has created a unified administration for urban and rural areas. Finally, a democratic administration system operates at the village level. By the end of 2012, Chengdu will have dissolved its *hukou* system between its rural and urban areas.

chotomy for China (Guo, 2010, Lu and Han, 2006, Lu and Liu, 2007, Tang and Qian, 2007, Tyers and Zhang, 2011, and Wang and Yao, 2009) but most of these have confined the designation "non-traded" to services as a matter of definition.

However, there is evidence that supports the idea that China's rural sector is substantially non-traded. In particular, its input-output data exhibits a low tradability ratio. The tradability ratio is the ratio of exports and imports to gross output in this sector. De Gregorio et al.(1994) Ire the first to use a 10-percent cutoff for non-traded goods, which then became the conventional way of classifying nontraded/traded goods (Dumrongrittikul, 2012, Guo, 2010). Based on STAN OECD Input-output database for China, the tradability ratio for China's agricultural sector is 4%, 2%, 4% for mid-1990s, early 2000s and mid-2000s, respectively. This suggests China has indeed overallocated labor to non-traded production.

Naturally, the validity of NTFA does not depend on its applicability to a particular country, namely China, or to a particular factor, namely labour. And, as I said at the start of this section, our purpose is not to write an empirical paper. However, I have raised the spectre of NTFA for China's rural transformation because of the importance of the Chinese real exchange rate for the world economy. The attendant current account surpluses influenced the international flow of funds prior to the Great Recession (Bagnai, 2009) and, going forward, any framing of appropriate expenditure switching and expenditure reduction policies in the major world economies (Cordon, 1994) should take account of any prospect of a NTFA of China's real exchange rate.

Furthermore, since the exchange rate reform in the mid-1990s, China's real effective exchange rate has risen by around 30 percent (IMF, 2012), which is not inconsistent with a NTFA.

1.3 A General Equilibrium Model of NTFA

In this section I show that the intuition of Figure 1 survives in a stylized two-sector general equilibrium framework with mobile international capital. My key innovation is to assume that the urban manufacture sector produces traded goods while the rural sector produces non-traded goods. My model thus maps the Lewis rural/urban dichotomy onto the Balassa-Samuelson traded/non-traded dichotomy.

1.3.1 Production

Consider a dual (Lewis) economy with two sectors, a traded manufacture sector (T) and a nontraded rural sector (N). Output is Cobb-Douglas in both sectors. Rural land G (*Gen di*, in Chinese) is fixed. Total labor is normalized to unity, $L = L_T + L_N \equiv 1$, and the rural labor ratio is $l \equiv L_N/L \equiv L_N$.

$$Y_T = K^\alpha L_T^{1-\alpha} = K^\alpha (1-l)^{1-\alpha}, \tag{1.1}$$

$$Y_N = G^\beta L_N^{1-\beta} = G^\beta l^{1-\beta}. \tag{1.2}$$

In the traded sector, I assume that capital K is owned by overseas investors[5]. I normalize the price of traded goods so the real exchange rate $P_N/P_T = P_N$. I assume that both capital and labor are paid their marginal products in the traded sector,

$$\frac{\partial Y_T}{\partial K} = \alpha \left(\frac{K}{1-l}\right)^{\alpha-1} = \bar{r}, \tag{1.3}$$

$$\frac{\partial Y_T}{\partial L_T} = (1-\alpha) \left(\frac{K}{1-l}\right)^{\alpha} = W_T. \tag{1.4}$$

Capital supply is infinitely elastically at the world real rate of return $\bar{r}$, which fixes the capital labour ratio, K/L_T, in the traded sector. I use (1.3) to obtain this capital-labor ratio. Substitution of this ratio into (1.4) determines the

[5] Since the beginning of the reforms in 1978, and increasingly since the 1990s, Chinese capital, which is primarily located on the Eastern Seaboard, has been increasingly financed by foreigners.

(endogenous) wage in the traded sector.

$$\frac{K}{L_T} = \left(\frac{\alpha}{\bar{r}}\right)^{\frac{1}{1-\alpha}}, \tag{1.5}$$

$$W_T = (1-\alpha)\left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{1.6}$$

It is important to understand how the endogenous wage in the traded sector (and, when the Lewis transition is complete, the whole economy) is tied down by (1.6). From (1.5), any movement of workers from the rural sector to the traded sector (a rise in L_T) raises capital borrowings from overseas. That this must be so is evident from (1.3) and (1.4). Without the extra capital, extra workers arriving in the traded sector will reduce the capital-labour ratio in (1.3) and (1.4), lowering the marginal product of labour (see (1.4)) and raising the marginal product of capital (see (1.3)).

But profit maximizing firms notice that the last unit of borrowed capital becomes infra-marginal as workers arrive, and so they borrow more capital until the capital-labour ratio returns to its previous level, fixed by the world real rate of return on the right-hand-side of (1.3). At the conclusion of this process, wages in the traded sector, which had been subject to downward pressure from rural emigrants, are held up by the capital inflow in (1.4).

Thus the model exhibits an internationalized Lewis growth dynamic: emigrant rural labour drains into the urban sector to combine with foreign capital, fuelling economic growth.

In the nontraded sector, I distinguish two stages of Lewis-style transition. At *Stage I*, rural labor is paid its average product and as a consequence there is no output left for land. I define the payments to rural labor in this way because peasants' output has been shared in both traditional societies and in communist collectives, and paying the average product of labor is the simplest form of sharing.[6] As I noted earlier, it is also consistent with the "institutional

[6] China espoused egalitarianism in rural areas, with peasants "eating from the same big pot"

wage" of Ranis and Fei (1961). *At Stage II*, both rural labor and land are paid their marginal products.

At both stages, rural output is exhausted by the payments to factors, which allows us to substitute $P_N Y_N$ for the sum of payments to land and labour. I provide a detailed description of Stage I and Stage II presently. For now, I will use the fact that total rural income is always $P_N Y_N$ to derive an expression for non-traded demand.

1.3.2 Demand

I begin by characterizing demand for the non-traded good. I assume all consumers have taste parameter θ for nontradable goods, C_N. Denote Z as the nominal income of a representative agent. The consumer's budget constraint is,

$$Z = C_T + P_N C_N. \tag{1.7}$$

Then the optimization problem is as follows,

$$\max_{C_N, C_T} U(C_N, C_T) = C_N^{\theta} C_T^{1-\theta} \tag{1.8}$$
$$\text{s.t.: } Z = C_T + P_N C_N$$

The representative agent maximizes utility subject to the budget constraint[7]. The demand for nontraded goods can be obtained by the first order condition,

$$C_N = \frac{\theta Z}{P_N}. \tag{1.9}$$

If (1.9) is rewritten with P_N as the subject, it becomes a demand curve. It will become clear as I solve for the equilibrium that rural emigration increases

during 1952 to 1976 (The Cultural Revolution ended in 1976). In the post-Cultural-Revolution period, peasants still share output among extended family members.

[7] There may be heterogeneous agents, but if the utility, U, satisfies the Gorman Form (Varian, 1992), a representative consumer exists.

Z, so it becomes a shift parameter in (1.9) for the demand schedule in the right panel of Figure 1.

National income is the sum of payments to land and rural labor plus wages in the traded sector (the return to capital is paid overseas). In both Stage I and Stage II payments to land and rural labor sum to $P_N Y_N$, so I obtain,

$$Z = P_N Y_N + W_T(1 - l) \tag{1.10}$$

I substitute (1.10) into (1.9) to obtain,

$$C_N = \frac{\theta \left[P_N Y_N + W_T(1 - l)\right]}{P_N}. \tag{1.11}$$

1.3.3 Equilibrium

The non-traded price clears the market for non-traded goods so that the supply equals with the demand for non-traded goods, i.e.

$$Y_N = C_N. \tag{1.12}$$

Using (1.12) in (1.11) to eliminate C_N, I obtain an expression for nominal income in the non-traded sector in terms of wages in the traded sector which is then substituted into (10), giving,

$$Z = \frac{W_T(1 - l)}{1 - \theta}. \tag{1.13}$$

Thus rural emigration (a fall in l) increases nominal income Z, shifting out the demand curve (1.10) as required in the right panel of Figure 1. To find P_N, I substitute (1.12) into (1.11), using (1.2) for Y_N and (1.6) for W_T. Then I obtain,

$$P_N(l) = \psi \frac{1 - l}{l^{1-\beta}}, \tag{1.14}$$

$$\frac{\partial P_N}{\partial l} = -\psi \frac{l + (1 - \beta)(1 - l)}{l^{2-\beta}} < 0, \tag{1.15}$$

where $\psi = \theta\alpha^{\alpha/(1-\alpha)}(1-\alpha)/\left[G^{\beta}\bar{r}^{\alpha/(1-\alpha)}(1-\theta)\right]$. The expressions (1.14) and (1.15) give us the real exchange rate and its derivative in terms of the rural labor allocation. It is clear from (1.15) that a Lewis transition that results in a fall in l will appreciate the real exchange rate, which is the NTFA result.

It is also clear that any redistribution of rural income between labor and land that leaves total income as $P_N Y_N$ will not in itself effect the real exchange rate. However, it will be shown presently that the arrival of the LTP, which sees a transfer of income from labor to land, causes further labor re-allocation and this effects the real exchange rate through l in (1.14).

To that end, I now show how different rural labor payment regimes (i.e. being paid the average or marginal product) alter the labor allocation. This, in turn, determines the real exchange rate (1.14) at each stage of development (Stage I or Stage II).

1.3.4 Stage I of the Dual Economy

As discussed above, I take up the idea of an "institutional wage" equal to the average product of labor in Stage I (the pre-LTP stage). Rural labor is paid its average product, while rural land is not paid any return[8].

$$\frac{W_N}{P_N} \equiv \frac{Y_N}{l}. \tag{1.16}$$

I now show that (1.14) and (1.15) are still valid, because (1.10) holds in Stage I. I rewrite (1.16) as,

$$W_N l = P_N Y_N. \tag{1.17}$$

[8] This seems reasonable in China in theory because land is owned by "the people" and in practice because farmers only pay modest rent to the Chinese government.

This leads us directly to (1.10),

$$
\begin{aligned}
Z &= W_N l + W_T L_T, \\
&= P_N Y_N + W_T(1-l).
\end{aligned} \tag{1.18}
$$

Equilibrium is not attained in Stage I until the rural nominal wage (the value of average product of labor) equals the urban wage rate. The rural wage is given by substitution of (1.14)and (1.2) into (1.16),

$$
\begin{aligned}
W_N &= P_N \frac{Y_N}{l}, \\
&= \frac{\theta(1-\alpha)}{1-\theta}\left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-l}{l}.
\end{aligned} \tag{1.19}
$$

Since the equilibrium is given by $W_N = W_T$, where W_T is obtained from (1.6), then equilibrium occurs when,

$$
l = \theta. \tag{1.20}
$$

Equation (1.20) is also the condition for the economy to reach the LTP. After the LTP, rural labourers are offered their marginal products[9].

1.3.5 Stage II of the Dual Economy

At Stage II (the post-LTP stage), it is assumed that both rural labor and land are paid their marginal products. With Cobb-Douglas production, the average revenue product of labour exceeds the marginal revenue product of labour at every labour allocation, so if rural labour remains at (1.20) unemployment would emerge as labour demand shrinks. However, downward pressure in wages ensures that the allocation does not remain at (1.20). Extra workers leave until the

[9] Equation (20) gives a nice, albeit imprecise, interpretation of what it means for labour to be over allocated. The condition for this is that the share of non-traded labour - a supply side variable - exceeds the taste parameter for non-traded goods - a demand side variable. It is imprecise because the allocation of (20) is not first-best and more labour needs to leave the rural sector up to the point where workers in both sectors are paid their marginal products (equation (24) below).

allocation sets the marginal revenue products of urban and rural workers equal to each other. I define R_N to be the rental rate of rural land and Euler's theorem gives us,

$$\begin{aligned} P_N Y_N &= P_N \frac{\partial Y_N}{\partial G} G + P_N \frac{\partial Y_N}{\partial L_N} L_N \\ &= R_N G + W_N L_N. \end{aligned} \tag{1.21}$$

Total nominal income, Z, should be total wage income from two sectors plus the income from land (the return to capital is paid overseas). Again, I confirm (1.10) by recognizing (1.21) in the expression for total nominal income,

$$\begin{aligned} Z &= W_N L_N + W_T L_T + R_N G, \\ &= P_N Y_N + W_T (1 - l). \end{aligned}$$

Similarly, equilibrium of Stage II is given by $W_N = W_T$, where W_T is obtained from (1.6) again, but W_N is now the *marginal* product of labor,

$$\begin{aligned} W_N &= P_N \frac{\partial Y_N}{\partial L_N} \\ &= (1-\beta) \frac{\theta(1-\alpha)}{1-\theta} \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-l}{l} \end{aligned} \tag{1.22}$$

By $W_N = W_T$, I obtain the first-best rural labor allocation, l^*[10],

$$l^* = \frac{(1-\beta)\theta}{1-\beta\theta}. \tag{1.23}$$

I have shown that the real exchange rate is decreasing in the rural labour allocation (see (1.14) and (1.15)), and, in sections 1.3.4 and 1.3.5, that a real

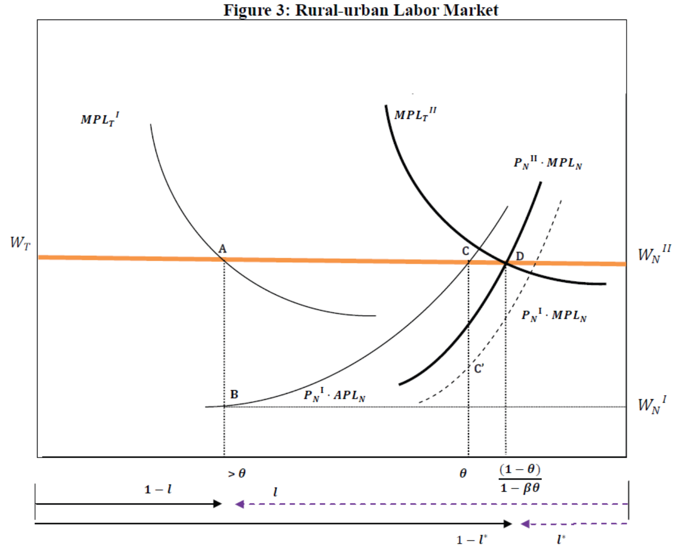
[10]It can be shown that allocating labour to equate the value of marginal products in both sectors maximizes social welfare, and is equivalent to the centralized solution of a planner who: (1) chooses labour in both sectors; (2) chooses overseas capital level; (3) divides national product among workers; (4) sets the price of non-traded goods to remove any queues or gluts; (5) assumes workers maximize their utility treating prices and income from the government as given. The derivation is available from the authors on request.

appreciation occurs over all the phases of development articulated by Lewis, Ranis and Fei[11].

1.4 NTFA in a Labour Market Diagram

Figure 3 illustrates the Lewis transition in terms of the labour market. As above, I assume that workers are paid their average product to begin with. Rural labour is read from right to left on the bottom axis, and traded labour left to right. From (1.15), I know that a fall in rural labour appreciates the exchange rate. Insofar as any transition in Figure 3 involves rural labour emigration, it implies a NTFA.

Figure 3: Rural-urban Labor Market



The economy commences with urban workers earning W_T at A and rural workers earning W_N^I at B. Superscripts I and II refer to Stages I and II. Their average revenue product is above their marginal revenue product, with the latter shown by the dashed line. This corresponds to the first two phases of Ranis and Fei (1961), since rural wages are above a low marginal product[12].

[11] As flagged earlier, so long as the output of the rural sector appears as domestic income, so that (1.10) holds, equations (1.2), (1.6) and (1.11)-(1.13) ensure (1.14) and (1.15) still hold. That is, NTFA is even robust to divisions of rural output that depart from those assumed in the narratives of Lewis/Ranis/Fei.

[12] Our modeling rules out the first phase of Ranis and Fei, where marginal product is literally

As rural workers emigrate to the traded sector, foreign capital meets them, pushing the marginal product of labour schedule rightwards. Rural wages rise along the chord BC until the labour allocation is at point C, which intersects with the shifted-out traded value of marginal product schedule (not shown). At that point there is an equilibrium, but not a utility maximizing one. The rural sector is then reformed (exogenously), perhaps with a desire to maximize utility, so that firms offer workers their marginal product. Unemployment emerges in the sector and workers leave rather than accept wages like C'.

As shown in Figure 1, the combination of declining supply of labour to rural output and increased income as the economy approaches the efficient labour allocation pushes up the real exchange rate. In Figure 3 it rises from P_N^I to P_N^{II}. The combination of a shift out in the value of marginal product of non-traded labour, due to a rise in P_N, and further capital inflow (pushing the value-of-marginal product of non-traded labour right) takes the economy to a final (utility maximizing) equilibrium at D, where all workers receive their value of marginal product, and rural workers receive W_N^{II}.

The non-traded labour allocation at C, namely θ, is taken from (20) by setting the value of *average* product in the non-traded sector equal to the value of *marginal* product in the traded sector. The allocation at D, namely $(1-\beta)\theta/(1-\beta\theta)$, is taken from (24) by setting both value of marginal products equal to each other.

Naturally, the extent, or even the existence, of NTFA in models of China will depend on a number of key modeling assumptions. Contrary to my stylized setup, the Chinese rural sector produces some traded goods and the urban sector has a sizeable share of non-traded services. To the extent that emigrating rural labour ends up in urban non-traded services, depressing their prices, the impact

zero. Marginal products are always positive for the Cobb-Douglas production function (see (2)), although they could be vanishingly small.Arguably, though, NTFA should still occur if the marginal product of labor in the rural sector were literally zero. This is because there would still be an increase in demand for the non-traded good arising from greater economic efficiencies.

on the overall index of P_N may be muted, or even negative.

Working against this, labour used to produce any *traded* rural output which is transferred to *traded* urban production may become more productive in its new location. This would amount to an improvement of traded sector productivity leading, in due course, to a Balassa-Samuelson appreciation.

Clearly much depends on the modelling assumptions made, though Mai et al. (2009) provide some support for NTFA. They consider a departure of 6.3 million workers from rural to urban employment, using a detailed sectoral model of China. Qualitatively, their analysis strikes a chord with mine. As workers arrive in the urban areas, the productivity of capital there goes up. This leads to a boom in both domestic investment (which Section 3 did not account for) and foreign sourced investment, financed through an open capital account. The boom in the urban industries, which have a production bias towards traded goods, pulls resources out of the non-traded goods sector. The fall in non-traded supply and the increased demand from a strong economy lifts the price of non-traded goods, appreciating the real exchange rate. Thus my prediction survives in a model where the rural and urban sectors are more realistically modeled[13].

1.5 Discussion and Conclusion

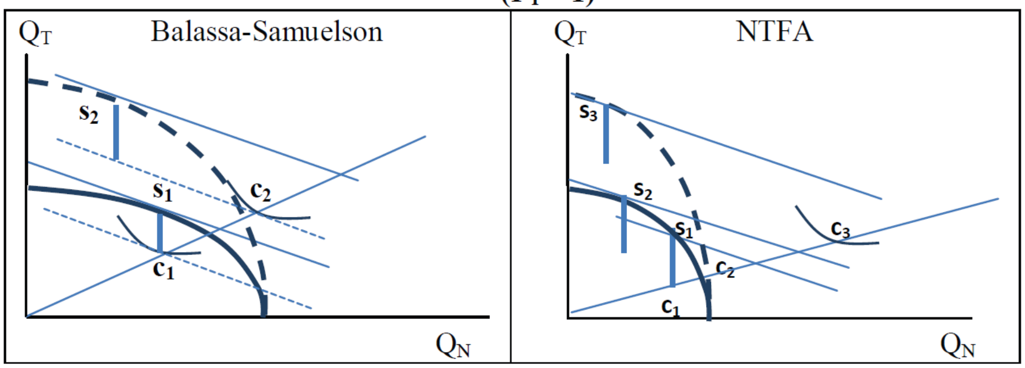
A real appreciation coinciding with the expansion of the traded sector is redolent of the famed Balassa-Samuelson effect (Balassa, 1964, Samuelson, 1964) where an increase in traded sector productivity appreciates the real exchange rate, and so I conclude by finding a connection between this effect and my stylized model.

Intuitively, the arrival of overseas capital in the urban-traded sector is like an increase in the traded sector productivity of labour. With no increase in land occurring (by assumption) "productivity" in the traded sector is rising faster than "productivity" in the non-traded sector.

[13] In Mai et al. (2009) (Figure 13, pp. 27), the appreciation of the real exchange rate is around 0.3 per cent.

Figure 4 clarifies the issues with respect to my model. Both panels show the excess demand for the non-traded good as the difference along the horizontal axis between consumption demand (c) and non-traded supply (s). They do not show the final equilibrium, after the non-traded price rises to clear the market.

Figure 4: Excess Demand for Non-traded Goods in Our Model
($P_T \equiv 1$)



In the left panel, production point s_1 lies above consumption point c_1 because a current account surplus (the line between the two) is required to service borrowed overseas capital.[14] Holding relative prices fixed, an increase in traded sector total-factor-productivity leads, via a Rybcyznski-type result, to a fall in non-traded goods supplied and an increase of traded goods supplied, at point s_2.[15] The current account surplus rises, because more machines are borrowed in response to an increase in the marginal product of capital, and consumption demand for non-traded goods rises along the income consumption curve to c_2. The excess demand for the non-traded good will result in a real appreciation (a rise in P_N) vertically aligning consumption and production in the final equilibrium (not shown).

In the right panel, I start the economy with non-traded equilibrium with s_1

[14] It is straightforward to confirm from Section 3 that $\bar{r}K$ equals the current account surplus, $Q_T - C_T$.

[15] In our model, we need a productivity shifter outside of (1). Technically, we would have to have one outside of (2) as well, since Balassa-Samuelson is about traded productivity rising relative to non-traded productivity, but to keep the diagram simple we set the increase in the latter to zero.

and c_1 vertically aligned. To keep the diagram simple I do not show the budget set lines for utility maximization, though I do show the consumption points and the income consumption curve. At points (c_1, s_1) there is no excess demand for non-traded goods but they have too much labour allocated for their production, implying that the budget constraint is not at a tangency to the PPF. I decompose the transition from points (c_1, s_1) to points (c_3, s_3) into two parts.

First, I conceptually fix capital, and labour moves to the traded sector taking us to production point s_2. This movement on its own is enough to ensure NTFA because excess demand opens up as supply falls to s_2 and demand rises North East along the income consumption curve to c_2. This part of the adjustment is not connected to a "productivity" explanation, however, and is distinct from the Balassa-Samuelson idea.

However, in the next part of the adjustment, capital is borrowed in response to the higher marginal product of capital (arising from the extra rural workers in the city). This is like a productivity expansion and so the transition from points (c_2, s_2) to points (c_3, s_3) exactly replicates the Balassa-Samuelson pattern in the left panel.

Thus both Balassa-Samuelson and NTFA are a result of excess demand for non-traded goods opening up as factors leave the non-traded sector. However, if I allow for foreign investment there is a second part of the adjustment. The overseas machines that greet the arriving rural workers create a Balassa-Samuelson "aftershock", which accentuates the gap between non-traded demand and supply. With this foreign borrowing, a Lewis transition implies a quasi-Balassa-Samuelson effect.

Making these connections is possible only because I have mapped the Lewis urban/rural dichotomy onto the Balassa-Samuelson traded/non-traded dichotomy. Both the Balassa-Samuelson and Lewis models have been used in studies of development, but to my knowledge the Balassa-Samuelson implications of a Lewis transition have not been explored. The reason, I suspect, is that the exchange rate implications of factor mobility have remained as isolated results, without being generalized into a principle like NTFA.

1.6 Special Acknowledgment

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2 Chapter 2
Liquidity, Monetary Policy and Unemployment

2.1 Introduction

This paper aims to address effects of unconventional monetary policy on macroeconomic performance such as output and unemployment. Conventional monetary policy in most of advanced economies targets short-term nominal interest rates by purchase and sale of short-term government bonds in open market operations (OMOs). The transmission mechanism and effects of such monetary policy have been extensively examined in the literature of monetary economics. The recent Global Financial Crisis (hereafter GFC) challenges the conduct of conventional monetary policy because the short-term nominal interest rate in the U.S. has been close to the zero lower bound(ZLB). In this case, the Fed is constrained in further lowering the short-term interest rate to stimulate the economy. Similar problems of conventional monetary policy have been observed in Japan and some European countries. In Japan, as early as in 1995, before the outbreak of 1997 Asian Financial Crisis, Bank of Japan cut the short-term interest rate almost to zero, which has lasted till the present. In UK and other countries in Euro zone, since the 2007-2008 GFC, central banks have also cut short-term interest rates to ZLB.

After hitting ZLB, central banks of the U.S., Japan, and some European countries all conduct unconventional monetary policy by either directly purchasing public or private assets in financial markets or expanding the holding of longer-term government bonds. The goal is to ease conditions of financial markets either directly or indirectly by putting downward pressure on longer-term interest rates, which will then support economic activities and job creation.

To understand the effects of unconventional monetary policy, we develop a general equilibrium model which features the coexistence of money and government bonds, and explicit goods and labor markets. Such a model is suitable to address how unconventional monetary policy affects macroeconomic performance such as output and unemployment. Our modeling of goods market follows the monetary search literature by specifying frictions that make money essential as a medium of exchange. The labor market follows the labor search literature to generate unemployment. Households act as buyers in the goods market and workers in the labor market. Firms are sellers in the goods market and employers in the labor market. Firms' profits from goods market trading directly affect firms' entry decision and hence the amount of vacancy in the labor market. The amount of unemployment directly affects the number of sellers in the goods market. In general, monetary policy in such models affects firms' profits from goods market trading, which will further influence the unemployment rate. Most of the existing papers that combine monetary search models with labor search models focus on the effects of inflation on unemployment. As money is often the only asset in those papers, those models cannot be used to address the effects of unconventional monetary policy.

In the baseline model, money and short-term government bonds coexist and can potentially be served as media of exchange during goods trading. Central Bank can conduct OMOs by adjusting its holding of short-term government bonds. We find that inflation still has a negative impact on unemployment, as previous findings such as Berentsen et al. (2011). The effect of OMOs is more interesting. Suppose Central Bank purchases short-term government bonds to inject money. This operation will increase demand for short-term bonds in the market and hence the interest rate of short-term bonds decreases. In the goods market, the decrease in the interest rate of short-term bonds benefits households who do not hold bonds and hurts households who hold bonds. We label the opposite effect of OMOs on households as a redistribution effect. Depending on the fraction of household that can hold bonds, firms' profits may increase or decrease. Therefore, unemployment may increase or decrease as a result of this OMOs.

We then extend the model to introduce long-term government bonds in addition to money and short-term government bonds. Long-term government bonds differ from short-term government bonds in that they are less liquid in goods market and hence offer a higher return than the latter. In this environment, Central Bank can conduct unconventional monetary policy by adjusting its holding of long-term government bonds[16]. Our model show that when the nominal interest rate of short-term bonds is close to zero, without changing the inflation rate, it is essential for Central Bank to use unconventional monetary policy to influence the economy. Suppose that Central Bank purchase long-term bonds to inject currency, which resembles Quantitative Easing (QE) conducted by central banks in the U.S. and other countries. We find that the redistribution effect is still critical in determining how such an unconventional monetary policy affects unemployment. That is, whether the unconventional monetary policy can reduce unemployment depends on the fraction of households who hold government bonds. Only when this fraction is not too big, unconventional monetary policy can reduce unemployment.

Our model is related to two lines of research in the literature of monetary theory with microfoundations. The first line of research integrates monetary search models with labour search models to study monetary policy and unemployment. Berentsen et al. (2011) examine effects of monetary policy on unemployment and

[16] As is mentioned before, central banks after the GFC also purchase large-scale private assets such as Mortgage-backed Securities(MBS) to conduct unconventional monetary policy. But large-scale purchases of long-term government bonds are an good approximation of unconventional monetary policy because they domintate the large-scale asset purchases by central banks in Japan and the U.S., etc. For example, the ratio of Treasury Bonds holdings is 61.1% of the total securities purchased by the Federal Reserve Bank of the U.S. in April, 2013 while the ratio of MBS is just 36.4%.

show that there is a positive relationship between inflation and unemployment in the long run. They provide a tractable framework where money and unemployment are both modeled with explicit microfoundations. However, as money is the only asset in their model, it is not applicable to address effects of unconventional monetary policy. Other papers built on Berentsen et al. (2011) include Gomis-Porqueras et al. (2013) and Bethune et al. (2014).

The second line of research involves explicit modeling of assets, liquidity and monetary policy. Along this line, there are too many papers, and recent surveys include Williamson and Wright (2010a,b), Nosal and Rocheteau (2011)and Lagos et al. (2014). Here we particularly list some literature who involves modeling liquidity, open market operations and unconventional monetary policy. Williamson (2012, 2013) build models with money and government bonds, and also include banking to address effects of both conventional and unconventional monetary policy. Rocheteau et al.(2014) uses the New Monetarist models with money and government bonds to study monetary policy, including OMOs, under various assumptions on market structure and the liquidity of money and bonds. There is difference in modeling OMOs between Williamson (2012) and Rocheteau et al.(2014). The former models OMOs by changing the ratio of currency in total government debt, which, in fact, involves changing the ratio of money to bonds. The latter claims the effects of OMOs is the same as only changing the outstanding stock of government bonds, given money is neutral. The approach of modeling OMOs in Williamson (2012) is followed by Mahmoudi (2013) and Wen(2013)[17].

[17] Mahmoudi (2013) builds a model with money, short-term and long-term government bonds, to show that the central bank can change the overall liquidity and welfare of the economy by changing the relative supply of assets with different liquidity characteristics. Wen (2013) provides a general equilibrium cash-in-advance model featuring government purchases of private debt, to study the efficiency of unconventional monetary policies. The main channel in the model is trade-off between the quantity and the quality of loans in the private debt market. The model predicts that unless private asset purchases are highly persistent and extremely large (on the order of more than 50% of annual GDP), money injections through LSAP cannot effectively boost aggregate output and employment even if in‡inflation is fully anchored and the real interest rate significantly reduced.

But there is no labour market in Williamson (2012, 2013), Rocheteau et al.(2014) or Mahmoudi (2013), therefore they do not address how unconventional monetary policy affects unemployment or job creation. Herrenbrueck (2013) studies QE and the liquidity channel of monetary policy, through a model with heterogenous households and frictional asset markets. It shows that central bank purchases of illiquid assets can reduce yields across the board and stimulate investment. Both Herrenbrueck (2013) and Wen (2013) address the effects of unconventional monetary policy on employment, but they involves employment through neoclassical production functions, and do not explicit model labor market or job creation as in my paper. Our paper is also related to Rocheteau and Rodriguez-lopez (2014), which develops a model of public and private provision of liquidity and study the relationship between liquidity and unemployment. Their model includes a Mortensen-Pissarides labor market and an over-the-counter market. However, private liquidity is generated from OTC Market instead of Central Bank. In addition, we follow Nosal and Rocheteau (2013) to model the difference in liquidity between short-term and long-term government bonds, which is similar to Kiyotaki and Moore (2012) in modeling the difference of liquidity between money and other assets.

The rest of the paper is organized as follows. Section 2 introduces Baseline Model, and Section 3 characterizes monetary equilibria and provide quantitative analysis. Section 4 extends Baseline Model by adding long-term government bonds. The extended model is then used to address effects of unconventional monetary policy. Quantitative analysis is also provided based on the extended model. The last section concludes.

2.2 Baseline Model

Baseline Model builds on Berentsen et al. (2011), which is based on Lagos and Wright (2005). Time is discrete and continues forever. There are three subperiods in each period: in the first subperiod, there is a labor market in the spirit of Mortensen and Pissarides (1994); in the second subperiod, there is a goods market in the spirit of Kiyotaki and Wright (1993); in the last subperiod, there is a Walrasian market in the spirt of Arrow-Debreu. We refer to these three markets as MP market, KW market and AD market hereafter. There are two types of agents, firms and households, indexed by f and h. The measure of firms is arbitrarily large, but not all firms are active. The measure of households is 1. In addition to firms and households, there exists a government who is a consolidated one of the fiscal authority and Central Bank. All government assets transactions take place in the AD market. Figure 2.1 shows the timeline of a representative period, with more details as follows.

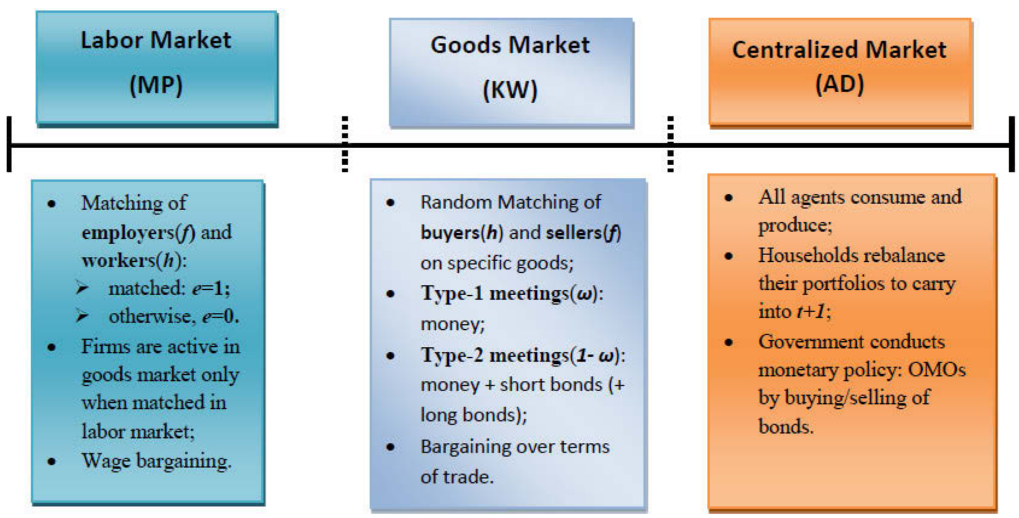

Figure 2.1: **Timeline of a Representative Period**

In the first subperiod, households and firms enter the labor market with different employment status. We use e to denote employment status: $e = 1$ if a household or a firm is matched and $e = 0$ otherwise. Households and firms match bilaterally to create a job. Once matched, output produced by the employed worker is y. The wage w paid to the worker is determined by generalized Nash bargaining. Unemployed workers receive unemployment benefits b in the subsequent AD market.

In the second subperiod, all households enter the KW market as buyers of a special good q. The utility from consuming q units of the special good is $\upsilon(q)$, where $\upsilon(0) = 0$, $\upsilon'(0) = \infty$ and $\upsilon'' < 0 < \upsilon'$. Only firms with output produced enter the KW market as sellers. Notice that if a firm is not matched in the previous MP market, it does not enter the KW market. The cost of producing q units of the special good in terms of y is $c(q)$, where $c(0) = 0$, $c' > 0$ and $c'' \geq 0$. Buyers and sellers are matched randomly and bilaterally according to the matching technology $\mathcal{M}(B, S)$, where B and S denote the measure of buyers and sellers, respectively. We assume that the matching technology is constant return to scale (CRS). The terms of trade are determined by generalized Nash bargaining in all meetings.

Suppose there are two types of meetings in the KW market: agents in type-1 meetings accept only money while agents in type-2 meetings accept money *and* bonds. Upon entering the KW market, a financial technology shock determines whether a household will be in type-1 or type-2 meetings in the KW market. The realization of the financial shock is i.i.d. across periods and households. With probability ω, a household is in a type-1 meeting and with the rest probability, the household is in a type-2 meeting. The roles of households and firms create the lack of double coincidence of wants problem. The fraction ω of the bilateral meetings (type-1) are anonymous, so that money is essential as a medium of exchange. The rest fraction $1 - \omega$ of meetings (type-2) gets involved with a communication technology which is costlessly available, and allows the buyer to transfer money and assets to the seller in a meeting. In Baseline Model, the only types of assets that coexists with money is short-term government bonds.

Here we could provide some explanation why we need two types of meetings in the KW market. Basically, we classify these two types to model households have access to different financial resources, in the model and in the real world as well.

Some households only have access to money, as in type-1 meetings, while others have access to a portfolio of money and assets (bonds), as in type-2 meetings. In fact, we can endogenize people's choices to type-1 and type-2 meetings, as in Rocheteau et al. (2014), or Chapter 3 of this thesis. But here we just assume that households go to type-1 or type-2 with some probabilities, because we want to focus on the main purpose of this paper, i.e., how (unconventional) monetary policy affects labor market.

All agents can enter the last subperiod, where a general good x is produced and traded in this competitive Walrasian market. They also receive wage income w (if employed in the previous MP market) or unemployment benefits b (if unemployed in the previous MP market), dividend income Δ from firms and government transfers T. For a household, the utility from consuming x units of the general good is x. If x is negative, it means that the household produces x. The production technology in the AD market is such that 1 unit of labor can produce 1 unit of x. Households rebalance their asset portfolios to carry into the next period. As in Berentsen et al. (2011), we adopt the following convention for measuring real balances. When a household brings in m_t dollars to the AD market in period t, we let $z_t = \phi_t m_t$, where ϕ_t reflects the value of money, or the inverse of the current price level. The household then take $z_{t+1} = \phi_t m_{t+1}$ out of that AD market. In the next AD market, the real value of the money is adjusted as $z_{t+1}\rho_{t+1}$, where $\rho_{t+1} = \phi_{t+1}/\phi_t$ converts z_{t+1}into units of the numeraire good x in that market.

Let (z_t, a_{st}) be the portfolio held by a household in period t, where z_t refers to the money holding in real terms and a_{st} refers to the assets holding, specifically referring to the short-term government bond holding (shown by the subscript "s"), in real terms. As in Williamson (2013), a short-term government bond, a_{st}, is issued by the government at a price p_{st} in the AD market of period t, and promises to pay one unit of money in AD of period $t + 1$.

We define value functions for the MP, KW and AD markets as $U_e^j(z_t, a_{st})$, $V_e^j(z_t, a_{st})$ and $W_e^j(z_t, a_{st})$, where $j \in \{h, f\}$ and $e \in (0, 1)$. From now on we focus my attention on steady states where all real variables are constant. So we can ignore the time subscript t in all of the variables. To distinguish variables in two sequential periods, for example, we use z and $\hat{z}$ to show real balances in the current period and next period.

2.2.1 Households

We begin with analyzing the value functions of households in each of the three subperiods. Without loss of generality, we start from the last subperiod, i.e., the AD market in the current period, and then the MP and KW market of next period. A household h entering the AD with employment status, e, a portfolio of money and bonds, (z, a_s) , chooses consumption x and the portfolio holding $(\hat{z}, \hat{a}_s)$ for the next period,

$$W_e^h(z, a_s) = \max_{x,\hat{z},\hat{a}_s} \{x + (1 - e)\chi + \beta U_e^h(\hat{z},\hat{a}_s)\},$$
$$\text{st. } x = ew + (1 - e)b + \Delta - T + z - \hat{z} + a_s - p_s\hat{a}_s.$$

where x is the general AD consumption good, χ is leisure, w is the wage determined in the previous MP market, b represents the unemployment benefit, Δ denotes dividend income distributed by firms, T is the lump-sum transfer by the government, and p_s is the discounted price of the short-term government bonds. These short-term government bonds mature in the next period AD market. A unit of the short-term government bonds is a promise of 1 unit of x at maturity. We can then express the gross real rate of return on these short-term bonds as

$$r_s = \frac{1}{p_s}. \tag{2.1}$$

Substituting x from the budget constraint, the household's value function becomes

$$W_e^h(z, a_s) = I_e + z + a_s + \max_{\hat{z},\hat{a}_s} \{-\hat{z} - p_s\hat{a}_s + \beta U_e^h(\hat{z},\hat{a}_s)\}, \tag{2.2}$$

where $I_e = ew + (1-e)(b+\chi) + \Delta - T$. As in Lagos and Wright (2005), quasi-linear preferences in the AD market implies that W_e^h is linear in z, a_s and I_e, and the choice of future real money and bond holdings, $(\hat{z}, \hat{a}_s)$, is independent of the current asset holding (z, a_s). Notice that $(\hat{z}, \hat{a}_s)$ may depend on e through U_e^h, although we will show later that $\partial U_e^h/\partial \hat{z}$, $\partial U_e^h/\partial \hat{a}_s$ and hence $(\hat{z}, \hat{a}_s)$ are independent of e. This implies that every household exits the AD with the same asset portfolio $(\hat{z}, \hat{a}_s)$.

For a household in the MP market of next period,

$$U_1^h(\hat{z}, \hat{a}_s) = \delta V_0^h(\hat{z}, \hat{a}_s) + (1-\delta)V_1^h(\hat{z}, \hat{a}_s), \tag{2.3}$$

$$U_0^h(\hat{z}, \hat{a}_s) = \lambda_h V_1^h(\hat{z}, \hat{a}_s) + (1-\lambda_h)V_0^h(\hat{z}, \hat{a}_s), \tag{2.4}$$

where δ is the exogenous job destruction rate and λ_h the endogenous job creation rate. The latter is determined by another CRS matching function $\mathcal{N}(u, v)$, where v denotes the amount of vacancies posted by firms. Let $\tau = v/u$ be the labor market tightness. We have $\lambda_h = \mathcal{N}(u, v)/u = \mathcal{N}(1, \tau)$. Once a household and a firm are matched, the wage rate is determined by the generalized Nash bargaining between the firm and the household, although the wage is paid in the next AD market.

In the KW market of next period, each buyer (household) is matched at random with a seller (firm). Recall that a financial technology shock is realized at the end of the AD market, and the realization of the shock is public information. We use superscripts "1" and "2" to denote variables associated with type-1 and type-2 meetings, respectively. The value function of a household upon entering the KW market is for $e = \{0, 1\}$

$$V_e^h = \omega V_e^{h1} + (1-\omega)V_e^{h2}. \tag{2.5}$$

In type-1 meetings,

$$V_e^{h1}(\hat{z}, \hat{a}_s) = \alpha_h \upsilon(q^1) + \alpha_h W_e^h[\rho(\hat{z} - d^1), \hat{a}_s] + (1-\alpha_h)W_e^h(\rho\hat{z}, \hat{a}_s),$$

where α_h is the probability of trade and (q^1, d^1) are the terms of trade in a type-1 meeting. The household uses d^1 units of real money balances to exchange q^1 units of the special good in the KW market. Using the linearity of W_e^h, we can rewrite V_e^{h1} as

$$V_e^{h1}(\hat{z}, \hat{a}_s) = \alpha_h[\upsilon(q^1) - \rho d^1] + W_{\hat{e}}^h(0,0) + \rho\hat{z} + \hat{a}_s. \tag{2.6}$$

In type-2 meetings,

$$V_e^{h2}(\hat{z}, \hat{a}_s) = \alpha_h \upsilon(q^2) + \alpha_h W_e^h[\rho(\hat{z} - d^2), (\hat{a}_s - \mu_s^2)] + (1-\alpha_h)W_e^h(\rho\hat{z}, \hat{a}_s),$$

where (q^2, d^2, μ_s^2) are the terms of trade in a type-2 meeting. In this type of meetings, the household can use both money and bonds to exchange the special goods. Here the terms of trade are such that the household uses d^2 units of real money balances and μ_s^2 units of bonds to exchange for q^2 units of the special good. Again, linearity of W_e^h implies that

$$V_e^{h2}(\hat{z}, \hat{a}_s) = \alpha_h[\upsilon(q^2) - \rho d^2 - \mu_s^2] + W_{\hat{e}}^h(0,0) + \rho\hat{z} + \hat{a}_s. \tag{2.7}$$

Substituting V_e^{h1} and V_e^{h2} from (2.6) and (2.7) into (2.5), we have

$$V_e^h(\hat{z}, \hat{a}_s) = \alpha_h S_h + W_{\hat{e}}^h(0,0) + \rho\hat{z} + \hat{a}_s, \tag{2.8}$$

where $S_h = \omega[\upsilon(q^1) - \rho d^1] + (1-\omega)[\upsilon(q^2) - \rho d^2 - \mu_s^2]$, and $\alpha_h = \mathcal{M}(B, S)/B$, is the probability of trade for a household. As $\mathcal{M}(B, S)$ is constant return to scale, we can rewrite α_h as $\alpha_h = \mathcal{M}(Q,1)/Q$, where $Q = B/S$ is the queue length or the market tightness. All households participate in the KW market, so $B = 1$; only firms with $e = 1$ enter the KW market, so $S = 1 - u$, where u denotes the rate of unemployment. Therefore, we have $\alpha_h = \mathcal{M}(1, 1-u)$.

Using the linearity of W_e^h, we substitute (2.8) into (2.3) and (2.4),

$$U_1^h(\hat{z},\hat{a}_s) = \alpha_h S_h + \rho\hat{z} + \hat{a}_s + \delta W_0^h(0,0) + (1-\delta)W_1^h(0,0),$$
$$U_0^h(\hat{z},\hat{a}_s) = \alpha_h S_h + \rho\hat{z} + \hat{a}_s + \lambda_h W_1^h(0,0) + (1-\lambda_h)W_0^h(0,0),$$

or,

$$U_e^h(\hat{z},\hat{a}_s) = \alpha_h S_h + \rho\hat{z} + \hat{a}_s + \mathbb{E}W_{\hat{e}}^h(0,0). \tag{2.9}$$

where $\mathbb{E}W_{\hat{e}}^h(0,0)$ is the expectation operator with respect to next period's employment status. We then substitute (2.9) into (2.2) to get

$$W_e^h(z,a_s) = I_e + z + a_s + \max_{\hat{z},\hat{a}_s}\ \{-\hat{z} + \beta\rho\hat{z} - p_s\hat{a}_s + \beta\hat{a}_s + \beta\alpha_h S_h\} + \beta\mathbb{E}W_{\hat{e}}^h(0,0). \tag{2.10}$$

From (2.10), the choice of $\hat{z}$ is independent of e, as well as I_e and z. Hence, every household takes the same portfolio of money and bonds out of the AD market.

2.2.2 Firms

The problem of firms follow the same timeline as that of households: start from the AD market in the current period, then the MP and KW market of next period. In the AD market, the portfolio decisions by firms are trivial. Firms would not carry any money or bonds out of the AD market since they would not use money or bonds in the subsequent MP or KW market. For a matched firm with inventory x, real money balances z and short-term bonds, a_s, the firms value function in the AD market is

$$W_1^f(x,z,a_s) = x + z + a_s - w + \beta U_1^f. \tag{2.11}$$

Depending on whether a firm is matched or vacant, the firm's value function in the MP market of next period is given by

$$U_1^f(\hat{z}, \hat{a}_s) = \delta V_0^f(\hat{z}, \hat{a}_s) + (1-\delta) V_1^f(\hat{z}, \hat{a}_s), \tag{2.12}$$
$$U_0^f(\hat{z}, \hat{a}_s) = \lambda_f V_1^f(\hat{z}, \hat{a}_s) + (1-\lambda_f) V_0^f(\hat{z}, \hat{a}_s).$$

where $\lambda_f = \mathcal{N}(u, v)/v = \mathcal{N}(1, \tau)/\tau$, is the endogenous rate at which matches are created. In the MP market, firms with $e = 0$ does not produce and they search for workers. Only with $e = 1$, firms enter the MP market with a matched worker to produce y units of output, which can be sold to households in the KW market. Notice that y is measured by the AD numeraire good.

In the KW market of next period, a firm may meet a household in a type-2 meeting or a type-1 meeting. So the firm's value function is

$$V_1^f(\hat{z}, \hat{a}_s) = \omega V_1^{f1}(\hat{z}, \hat{a}_s) + (1-\omega) V_1^{f2}(\hat{z}, \hat{a}_s). \tag{2.13}$$

where

$$V_1^{f1}(\hat{z}, \hat{a}_s) = \alpha_f W_1^{f1}[y - c(q^1), \rho d^1, 0] + (1-\alpha_f) W_1^{f1}(y, 0, 0), \tag{2.14}$$
$$V_1^{f2}(\hat{z}, \hat{a}_s) = \alpha_f W_1^{f2}[y - c(q^2), \rho d^2, \mu_s] + (1-\alpha_f) W_1^{f2}(y, 0, 0). \tag{2.15}$$

Here $\alpha_f = \mathcal{M}(B, S)/S$ is the firm's probability of trade. As in Berentsen et al. (2011), for $j = \{1, 2\}$, the firm's transformation technology in the KW market is such that $c(q^j)$ units of the AD goods are transferred into q^j units of the KW goods and $y - c(q^j)$ units of the AD goods are left over to be carried to the next AD market. Use (2.11) to rewrite (2.14) and (2.15),

$$V_1^{f1}(\hat{z}, \hat{a}_s) = \alpha_f[\rho d^1 - c(q^1)] + y - w + \beta U_1^f(\hat{z}, \hat{a}_s), \tag{2.16}$$
$$V_1^{f2}(\hat{z}, \hat{a}_s) = \alpha_f[\rho d^2 + \mu_s - c(q^2)] + y - w + \beta U_1^f(\hat{z}, \hat{a}_s). \tag{2.17}$$

Substituting (2.16) and (2.17) into (2.13), we obtain

$$V_1^f(\hat{z}, \hat{a}_s) = R - w + \beta U_1^f(\hat{z}, \hat{a}_s), \tag{2.18}$$

where $R \equiv \alpha_f\{\omega[\rho d^1 - c(q^1)] + (1-\omega)[\rho d^2 + \mu_s^2 - c(q^2)]\} + y$ is the expected revenue. Using (2.18) and (2.12), we can express V_1^f as

$$V_1^f(\hat{z}, \hat{a}_s) = \frac{R - w}{1 - \beta(1-\delta)}. \tag{2.19}$$

In addition, firms with $e = 0$ can choose to enter the market freely in the AD market of the current period by paying a cost of k in units of the AD good. These newly entered or unmatched (from the MP market of this period) firms can search for workers in the next MP market. Thus we have

$$W_0^f = \max\{0, -k + \beta\lambda_f V_1^f + \beta(1-\lambda_f)V_0^f\},$$

where $V_0^f = W_0^f = 0$ in equilibrium. It follows that $k = \beta\lambda_f V_1^f$. Combined with (2.19),

$$k = \frac{\beta\lambda_f(R - w)}{1 - \beta(1-\delta)}. \tag{2.20}$$

Recall that firms pay out profits as dividends in the AD market. The overall profit by all firms is $(1-u)(R-w) - vk$. For a household who owns shares of all firms, the dividend income $\Delta = (1-u)(R-w) - vk$.

2.2.3 Government

We assume that the government is a consolidated entity of the fiscal authority and Central Bank. We focus on its role as Central Bank and treat the fiscal authority as passive. The fiscal authority levies taxes just to support monetary policy. As in Williamson (2012), we assume the monetary authority commits to a policy such that the total stock of nominal money supply grows at a constant gross rate $1 + \pi = \phi_t/\phi_{t+1}$, and the ratio of currency to the total nominal government debt is a constant σ. That is,

$$\frac{M_{t+1}}{M_t} = 1+\pi, \tag{2.21}$$

$$M_t = \sigma(M_t + p_{st}A_{st}). \tag{2.22}$$

where M_t is the nominal money supply at period t, which is the nominal terms of aggregate real balances of money, z_t, and A_{st} denotes the short-term government bonds held by private sectors (i.e., households and firms) at period t, which is also the nominal term of aggregate"short-term bonds", a_{st}. Note that, adjusting values of σ is a representation of OMOs by Central Bank. In principal, $\sigma \in (-\infty, +\infty)$ is admissible, and if $\sigma < 0 (\sigma > 0)$ then the consolidated government is a net creditor (debtor). All government transactions take place in the AD market.

Since we focus on stationary equilibrium, we can ignore the time subscript here. Let $G = z + p_s a_s$ be the total value of government debt in the steady state. From (2.22), we have

$$z = \sigma(z + p_s a_s).$$

Monetary policy can be represented by adjusting (π, σ). We know r_s is the gross real interest rate on short-term government bonds. In monetary equilibrium, the no-arbitrage condition implies that r_s must not be lower than the rate of return on money and quasilinear utility implies that r_s cannot exceed the inverse of time preference. To summarize,

$$\frac{1}{1+\pi} \leq r_s \leq \frac{1}{\beta}. \tag{2.23}$$

2.3 Equilibrium

The terms of trade in three markets are determined as follows: agents are price takers in the AD market, and bargain over the terms of trade in the MP and KW markets. In the KW market, we can determine the values of (q^1, q^2) taking unemployment u as given. In the KW market, we take (q^1, q^2) as given and determine u. The interdependence between (q^1, q^2) and u establishes the link between the goods market and the labour market. In this section, we solve for equilibrium conditions in all markets. Together with the asset market clearing conditions, we define a stationary monetary equilibrium. Then we use the model to analyze the effects of monetary policy.

2.3.1 Goods Market Equilibrium

When a firm and a household meet in the KW market, the terms of trade are determined by the generalized Nash bargaining in both type-1 and type-2 meetings. Assume that the bargaining power of the household is θ. The terms of trade in these two types of meetings are determined as follows.

For type-1 meetings,

$$\max_{q^1, d^1} \; [\upsilon(q^1) - \rho d^1]^\theta [\rho d^1 - c(q^1)]^{1-\theta}, \tag{2.24}$$
$$\text{st. } d^1 \leq z \text{ and } c(q^1) \leq y,$$

where the constraints show that agents in type-1 meetings cannot leave with negative cash balances and inventories. The first term in (2.24) is the surplus of the household and the second term is the surplus of the firm using the linearity of W_e^j. As in Berentsen et al. (2011)[1], we assume that $c(q^1) \leq y$ is not binding. The solution to (2.24) is

$$q^1 = \begin{cases} q^{1*}, if \; d^1 < z, \\ g^{-1}(\rho z), \; if \; d^1 = z, \end{cases} \tag{2.25}$$

where q^{1*} is solved from $\upsilon'(q^{1*}) = g'(q^{1*})$. When $d^1 = z$, we have,

$$g(q^1) \equiv \frac{\theta c(q^1)\upsilon'(q^1) + (1-\theta)\upsilon(q^1)c'(q^1)}{\theta \upsilon'(q^1) + (1-\theta)c'(q^1)}. \tag{2.26}$$

Using the result from Lagos and Wright (2005) [10], we can prove that the solution to (2.24) is $d^1 = z$ and $q^1 = g^{-1}(\rho z)$ in a monetary equilibrium. Notice that $\partial q^1/\partial z = \rho/g'(q^1) > 0$, which means that with more money, the household in type-1 meetings will purchase more KW goods.

For type-2 meetings,

$$\max_{q^2,d^2,\mu_s^2} \left[\upsilon\left(q^2\right) - \rho d^2 - \mu_s^2\right]^\theta \left[\rho d^2 + \mu_s^2 - c\left(q^2\right)\right]^{1-\theta}, \tag{2.27}$$
$$\text{st. } d^2 \leq z,\ \mu_s^2 \leq a_s, \text{ and } c\left(q^2\right) \leq y.$$

Similarly, the constraints show that agents in type-2 meetings cannot leave with negative cash balances, assets holdings and inventories. The solution to (2.27) is

$$q^2 = \begin{cases} q^{2*}, if\ d^2 < z, \mu_s^2 < a_s, \\ g^{-1}(\rho z + a_s), if\ d^2 = z, \mu_s^2 = a_s, \end{cases}$$

where q^{2*} is solved from $\upsilon'(q^{2^*}) = g'(q^{2^*})$. When $d^2 = z$ and $\mu_s^2 = a_s$, we have

$$g(q^2) \equiv \frac{\theta c(q^2)\upsilon'(q^2) + (1-\theta)\upsilon(q^2)c'(q^2)}{\theta\upsilon'(q^2) + (1-\theta)c'(q^2)}. \tag{2.28}$$

As in type-1 meetings, the solution to (2.27) is $d^2 = z, \mu_s^2 = a_s$, and $q^2 = g^{-1}(\rho z + a_s)$ in a monetary equilibrium. Notice also that $\partial q^2/\partial z = \rho/g'(q^2) > 0, \partial q^2/\partial a_s = 1/g'(q^2) > 0$, which means that more money or bonds holding leads to more consumption for the household in a type-2 meeting. With the solutions for type-1 and type-2 meetings, we have,

$$g(q^1) = \rho z, \tag{2.29}$$
$$g(q^2) = \rho z + a_s. \tag{2.30}$$

With $g'(q^i) > 0(i = 1, 2)$, we can show that $q^2 \geq q^1$ from (2.29) and (2.30). That is, households in type-2 meetings consume no less than households in type-1 meetings.

Given the bargaining outcome, we can rewrite the choice of $\hat{z}, \hat{a}_s$ in (2.10) as

$$\max_{\hat{z},\hat{a}_s} \{-\hat{z} + \beta\rho\hat{z} - p_s\hat{a}_s + \beta\hat{a}_s + \beta\alpha_h[\omega(\upsilon(q^1) - \rho\hat{z})$$
$$+(1-\omega)(\upsilon(q^2) - \rho\hat{z} - \hat{a}_s)]\}, \tag{2.31}$$
$$\text{st. } \hat{z} \geq 0 \text{ and } \hat{a}_s \geq 0.$$

The first-order conditions for interior solutions are

$$\omega\frac{\upsilon'(q^1)}{g'(q^1)} + (1-\omega)\frac{\upsilon'(q^2)}{g'(q^2)} - 1 = \frac{\frac{1+\pi}{\beta} - 1}{M(1, 1-u)}, \tag{2.32}$$
$$\frac{\upsilon'(q^2)}{g'(q^2)} - 1 = \frac{\frac{1}{\beta r_s} - 1}{(1-\omega)M(1, 1-u)}. \tag{2.33}$$

where we use r_s to replace p_s by (2.1). Substituting $\upsilon'(q^2)/g'(q^2)$ in (2.33) into (2.32), we obtain

$$\frac{\upsilon'(q^1)}{g'(q^1)} - 1 = \frac{\frac{1+\pi}{\beta} - \frac{1}{\beta r_s}}{\omega M(1, 1-u)}. \tag{2.34}$$

The above results show that, in the KW market, (q^1, q^2) depend on u through the matching function. That is, more unemployment reduce the number of firms entering into the KW market and hence reduce the matching probability for households. This will affect equilibrium (q^1, q^2). In a monetary equilibrium, $\hat{z}$ is always positive. Notice that it is possible that depending on the rate of return of bonds, $\hat{a}_s$ takes on the corner solution, such as $\hat{a}_s = 0$ or $\hat{a}_s = \infty$. We will discuss these corner solutions in Section 3.3.

2.3.2 Labor Market Equilibrium

In the MP market, the wage rate is again determined by generalized Nash bargaining. Let η be the bargaining power of a firm. Following the same procedure as in Mortensen and Pissarides (1994), we can find

$$w = \frac{\eta[1-\beta(1-\delta)](b+\chi) + (1-\eta)[1-\beta(1-\delta-\lambda_h)]R}{1-\beta(1-\delta) + (1-\eta)\beta\lambda_h}. \tag{2.35}$$

Substituting (2.35) into (2.20), the free entry condition becomes,

$$k = \frac{\lambda_f \eta \{y - b - \chi + \alpha_f[\omega(\rho d^1 - c(q^1)) + (1-\omega)(\rho d^2 + \mu_s^2 - c(q^2))]\}}{r' + \delta + (1-\eta)\lambda_h}, \quad (2.36)$$

where $r' = (1-\beta)/\beta$. To simplify (2.36), use the steady state condition $(1-u)\delta = \mathcal{N}(u,v)$ to implicitly define $v = v(u)$, and write $\alpha_f = \mathcal{M}(1, 1-u)/(1-u)$, $\lambda_f = \mathcal{N}[u, v(u)]/v(u)$ and $\lambda_h = \mathcal{N}[u, v(u)]/u$. Together with the equilibrium conditions from the KW market, (2.36) becomes

$$k = \frac{\eta \frac{\mathcal{N}[u,v(u)]}{v(u)} \{y - b - \chi + \frac{\mathcal{M}(1,1-u)}{1-u}[\omega(g(q^1) - c(q^1)) + (1-\omega)(g(q^m) - c(q^2))]\}}{r' + \delta + (1-\eta)\frac{\mathcal{N}(u,v(u))}{u}}. \quad (2.37)$$

This is the equilibrium condition in the MP market, where u is determined through the matching function, given (q^1, q^2). This is similar to the MP curve in Berentsen et al. (2012) [1], but now the expected gain from trade in the KW market is the expected gain from both type-1 and type-2 meetings. In addition, we do not have the Hosios (1990) condition holding for labor market[18].

2.3.3 Equilibrium Allocation

After solving the equilibrium conditions in the KW and MP markets, we are ready to define general equilibrium allocation. Again, we focus on stationary equilibrium, in which real variables are constant over time.

Definition 1 *Given monetary policy parameters (π, σ), a stationary monetary equilibrium consists of $(z, a_s, q^1, q^2, r_s, u)$ such that (i) given u, (z, a_s, q^1, q^2) solves (2.31) where (z, a_s) satisfies (2.29) and (2.30);(ii) given (q^1, q^2), u satisfies (2.37); and (iii) the asset market clears,*

$$a_s = \frac{1-\sigma}{\sigma} r_s z. \quad (2.38)$$

[18] For the effects of inflation (we discuss it in Section 2.3.4), it means the effects of inflation are enlarged, without the Hosios condition holding in our model.

The existence of a monetary equilibrium requires that $1+\pi \geq \beta$, i.e., the nominal interest rate must be non-negative. We have established that $1/(1+\pi) \leq r_s \leq 1/\beta$. Similar to Williamson (2012)[19], we have four types of equilibrium depending on the equilibrium value of r_s.

Liquidity Trap Equilibrium When $1/(1+\pi) = r_s < 1/\beta$, the rate of return on money and bonds are equal (the nominal interest rate is zero). In this case, the economy is in a liquidity trap. Households would choose to hold only money because bonds cannot be used as medium of exchange during type-1 meetings in the KW market. Thus, we have $a_s = 0$, which implies that $q^1 = q^2$. Since $a_s = 0$, (2.33) doesn't hold but (2.32) still holds. Therefore, the equilibrium allocation (q^1, q^2, r_s) in the liquidity trap case is $r_s = 1/(1+\pi)$ and $q^1 = q^2 = q$ solving

$$\frac{v'(q)}{g'(q)} - 1 = \frac{\frac{1+\pi}{\beta} - 1}{\mathcal{M}(1, 1-u)}. \tag{2.39}$$

Correspondingly, the labour market equilibrium condition is reduced to

$$k = \frac{\eta \frac{\mathcal{N}[u, v(u)]}{v(u)} [y - b - \ell + \frac{\mathcal{M}(1,1-u)}{1-u}(g(q) - c(q))]}{r' + \delta + (1-\eta)\frac{\mathcal{N}(u, v(u))}{u}}. \tag{2.40}$$

Notice that the equilibrium allocation in the liquidity trap case is the same as the allocation in Berentsen et al. (2012). Here bonds are not used by households at all so that the economy resembles a pure monetary economy. From (2.38), $a_s = 0$ implies that $\sigma = 1$. Liquidity trap equilibrium exists if and only if (π, σ) is in the set

$$\{(\pi, \sigma) : \pi > \beta - 1, \sigma = 1\}.$$

Equilibrium with Plentiful Assets When $1/(1+\pi) < r_s = 1/\beta$, the rate of return on bonds equals to the inverse of time preference. With quasilinear utility in the AD market, it means that holding bonds is not costly. Households may choose to hold an infinite amount of bonds. We label this type of equilibrium as plentiful assets equilibrium. In this case, the equilibrium allocation (q^1, q^2, r_s) satisfies $r_s = 1/\beta$ and

$$\frac{\upsilon'(q^2)}{g'(q^2)} - 1 = 0, \tag{2.41}$$

$$\frac{\upsilon'(q^1)}{g'(q^1)} - 1 = \frac{\frac{1+\pi}{\beta} - 1}{\omega\mathcal{M}(1, 1-u)}, \tag{2.42}$$

where (2.41) and (2.42) are derived by substituting $r_s = 1/\beta$ into (2.33) and (2.34), respectively. From (2.41) and (2.42), one can show that $q^1 < q^2$ when $\upsilon'(q^j)/g'(q^j)$ is decreasing in q^j for $j \in \{1, 2\}$. Households in type-2 meetings always consume $q^2 = q^{2*}$. The labour market equilibrium condition remains the same as (2.37).

In order to have bonds being plentiful, we need

$$\rho z + a_s \geq g(q^{2*}). \tag{2.43}$$

When substituting (2.38) and $\rho = 1/(1+\pi)$ into (2.43), we reach

$$\sigma \leq \frac{1}{1 + \frac{1}{(1+\pi)r_s}[\frac{g(q^{2*})}{g(q_p^1)} - 1]},$$

where q_p^1 solves (2.42). Thus, plentiful assets equilibrium exists if and only if (π, σ) is in the set

$$\{(\pi, \sigma) : \pi > \beta - 1, 0 < \sigma \leq \frac{1}{1 + \frac{\beta}{1+\pi}[\frac{g(q^{2*})}{g(q_p^1)} - 1]}\}.$$

Equilibrium with Scarce Assets When $1/(1+\pi) < r_s < 1/\beta$, the nominal interest rate is positive and the return on bonds is less than the inverse of time preference. Households choose the optimal amount of bonds to hold. Compared to equilibrium with plentiful assets, we label this type of equilibrium as equilibrium with scarce assets. The equilibrium allocation (q^1, q^2, r_s, u) satisfies (2.33), (2.34), (2.37) and (2.38). Substituting (2.38) into (2.29) and (2.30), we can further express the equilibrium r_s as follows,

$$r_s = \frac{\sigma}{(1+\pi)(1-\sigma)}[\frac{g(q^2)}{g(q^1)} - 1],$$

where we can see how r_s is related to the monetary policy parameters (π, σ).

When $r_s < 1/\beta$, households in type-2 meetings consume less than q^{2*}. This implies that

$$g(q^2) = \rho z + a_s < g(q^{2*}).$$

We derive the necessary condition for equilibrium with scarce assets to exist as (π, σ) is in the set

$$\{(\pi, \sigma) : \pi > \beta - 1, \frac{1}{1 + \frac{\beta}{(1+\pi)}[\frac{g(q^{2*})}{g(q_p^1)} - 1]} < \sigma < 1\}. \tag{2.44}$$

Friedman Rule Equilibrium Lastly, when $1/(1+\pi) = r_s = \frac{1}{\beta}$, both money and bonds have the same rate of return, which equals the inverse of time preference. In this case, monetary policy is at the Friedman rule. The equilibrium allocation (q^2, q^1) is the same as (q^{2*}, q^{1*}) and the unemployment rate u is solved from (2.37). It is clear that the Friedman rule equilibrium exists if and only if $\pi = \beta - 1$.

2.3.4 Quantitative Analysis

Having defined monetary equilibrium in my baseline economy, we move to analyze the effects of monetary policy, i.e., changing (π, σ), on the equilibrium allocation and the unemployment rate. Changing π is equivalent to changing the inflation rate. When Central Bank adjusts the ratio of currency, σ, we refer to it as OMOs. Analytically, it is clear from the equilibrium conditions that OMOs have no real effect on the economy except in the equilibrium with scarce assets. Moreover, one can show that in liquidity trap equilibrium or equilibrium with plentiful assets, inflation distorts trading in the KW market and reduces firms' profits in the MP market. Similar to the findings in Berentsen et al. (2011), inflation leads to more unemployment. Only in equilibrium with scarce assets, the effects of inflation are less obvious. Therefore, we focus on equilibrium with scarce assets to further investigate the effects of monetary policy, and rely on quantitative analysis where we use commonly used functional forms of the utility function, cost function and matching functions.

In the KW market, we set the utility function as $\upsilon(q^j) = A(q^j)^\varepsilon/\varepsilon$ for $j = \{1, 2\}$ with $0 < \varepsilon < 1$, the cost function as $c(q^j) = q^j$ for $j = \{1, 2\}$, and the matching function in the KW market as $\mathcal{M}(B, S) = BS/(B+S)$. This matching function implies that $\alpha_h = (1 - u)/(2 - u)$ and $\alpha_f = 1/(2 - u)$. As for the MP market, we use a Cobb-Douglas matching function: $\mathcal{N}(u, v) = Zu^{1-\epsilon}v^\epsilon$, where $0 < \epsilon < 1$. In the numerical analysis, we truncate the matching probabilities $\lambda_h = \mathcal{N}(u, v)/u$ and $\lambda_f = \mathcal{N}(u, v)/v$ to ensure that they do not exceed 1.

For parameter values except π and σ, we adopt the same or similar values used in Berentsen et al. (2011). Since there is only one asset, money, in their model, their model does not have ω. We set ω to 0.5 as a benchmark value and explore the implications from different values of ω. Table 1 summarizes the parameter values that we use.

Parameter	Description	Value
β	discount factor	0.99
θ	KW household bargaining share	0.7
η	MP firm bargaining share	0.25
δ	MP job destruction rate	0.05
y	productivity	1
b	unemployment benefit	0.5
χ	leisure	0.48
k	MP entry cost	0.001
A	KW utility weight	1.01
ε	KW utility elasticity	0.8
Z	MP matching efficiency	0.36
ϵ	MP matching velasticity	0.28
ω	ratio of non-monitored meetings	0.5

Table 1: Parameter Values

Effects of Inflation We start with the effects of inflation by varying inflation from 1% to 30% and set the OMOs parameter as $\sigma = 0.5$. My numerical results show the effects of inflation on quantities of trading in both type-1 and type-2 meetings (q^1, q^2), the unemployment rate u, and the real rate of return on government bonds r_s in equilibrium with scarce assets.

Figure 2.2 shows that quantities of trading in both type-1 and type-2 meetings decrease when inflation increases. Notice that q^2 is always greater than q^1 since households can use both money and bonds to trade for goods in type-2 meetings. We can also see that the real interest rate of short-term bonds decreases when inflation increases. This is because when money becomes less valuable, bonds become more attractive so that households's demand for bonds increases. The higher demand for bonds leads to a higher price of bonds and hence a lower rate of return on bonds. As for the effect of inflation on unemployment, Figure 2 shows that the unemployment rate increases from about 6.0% to 7.4% when inflation increases from 1% to 30%. As higher inflation reduces both q^2 and q^1

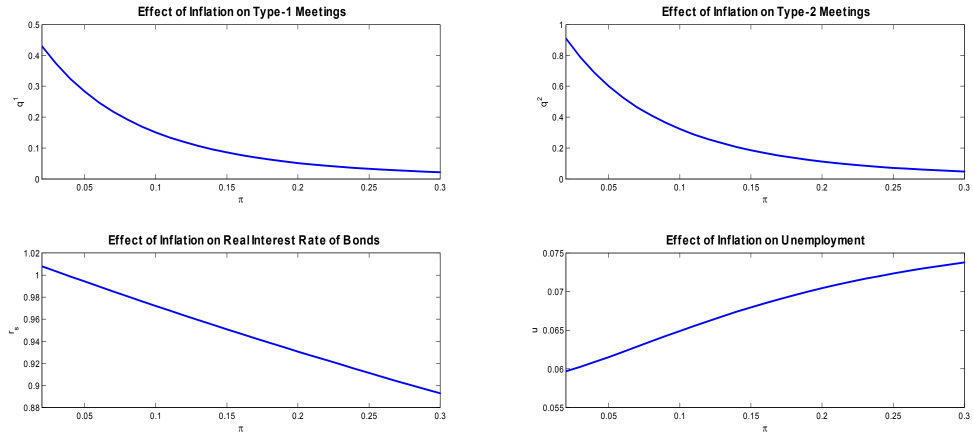

Figure 2.2: **Effects of Inflation**

in equilibrium with scarce assets, firms earn less profits from trading in the KW market. It implies that there would be less firms choosing to enter the market and eventually results in a higher unemployment rate.

Effects of OMOs To analyze the effects of OMOs, we fix the value of π and focus on the change in σ. We set the inflation rate as $\pi = 0.03$ and check the effects of σ increasing from 0.1 to 0.99, which corresponds to an easing monetary policy by purchasing short-term government bonds to inject money. Figure 2.3 show the impacts that σ increases only from 0.36 to 0.99 because we need to ensure that equilibrium with scarce assets exists based on the condition shown by (2.44). That is, only when σ is big enough, there exists equilibrium with scarce assets.

These figures show the effects of OMOs on (q^1, q^2, r_s, u). When Central Bank purchases short-term government bonds to inject money, it will increase the demand for short-term bonds, and hence increase the price of bonds but decrease the interest rate of short-term bonds, as is shown in Figure 2.3. As there is more money in the economy, quantity of trading in type-1 meetings q^1 increases , while quantity of trading in type-2 meetings q^2 decreases because the rate of return

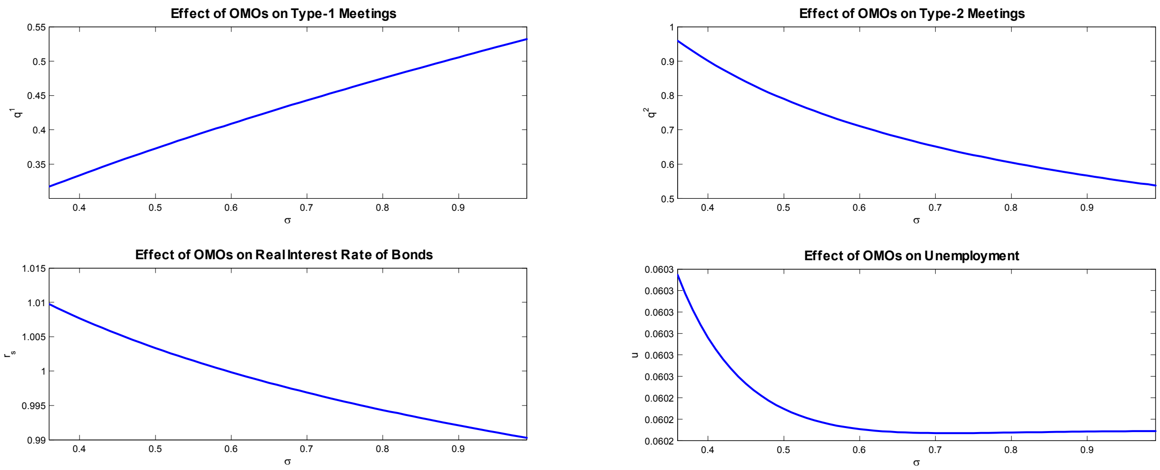

Figure 2.3: **Effects of OMOs**

on bonds is decreasing. As for the effect of OMOs on unemployment, Figure 2.3 shows that the unemployment rate decreases from about 6.03% to 6.02% when changing σ. Therefore, an easing monetary policy by purchase of short-term bonds to inject money has a slight positive impact on the labor market.

Redistribution Effect of OMOs The above analysis suggests that an easing monetary policy by OMOs has opposite effects on type-1 (increasing q^1) and type-2 meetings (decreasing q^2) by Figure 2.3. We label this as a redistribution effect. That is, an easing monetary policy by OMOs benefits type-1 meetings while it hurts type-2 meetings. Firms' expected profits are from both type-1 and type-2 meetings. The total effect of σ on firms' profits then depends on the ratio of type-1/type-2 meetings ω. In the following, we compare the effects of two different values of ω: (1) $\omega = 0.95$, which means that most meetings are type-1 in the KW market, and (2) $\omega = 0.2$, which means that most meetings are type-2 in the KW market.

Again fixing $\pi = 0.03$, we check the effects of σ increasing from 0.1 to 0.99. When $\omega = 0.95$, the effects of OMOs are shown in Figure 2.4. We can see that an easing monetary policy still benefits type-1 meetings and hurts type-2 meetings.

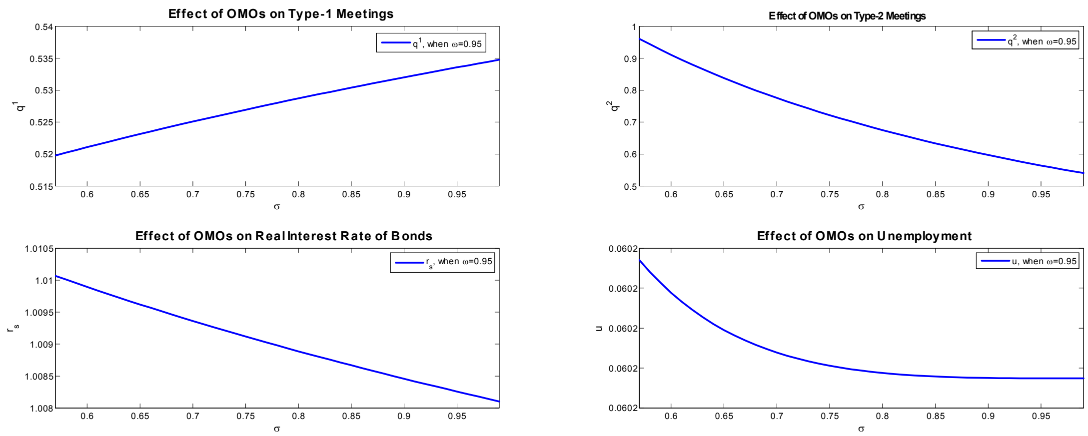

Figure 2.4: **Redistribution Effect of OMOs,** $\omega = 0.95$

However, since most meetings are type-1, very few households hold bonds. Then the rate of return on bonds decreases. And the positive effect of increasing σ on q^1 dominates, so firms' expected profits should increase. Therefore more firms enter the market and the unemployment decreases, although only slightly.

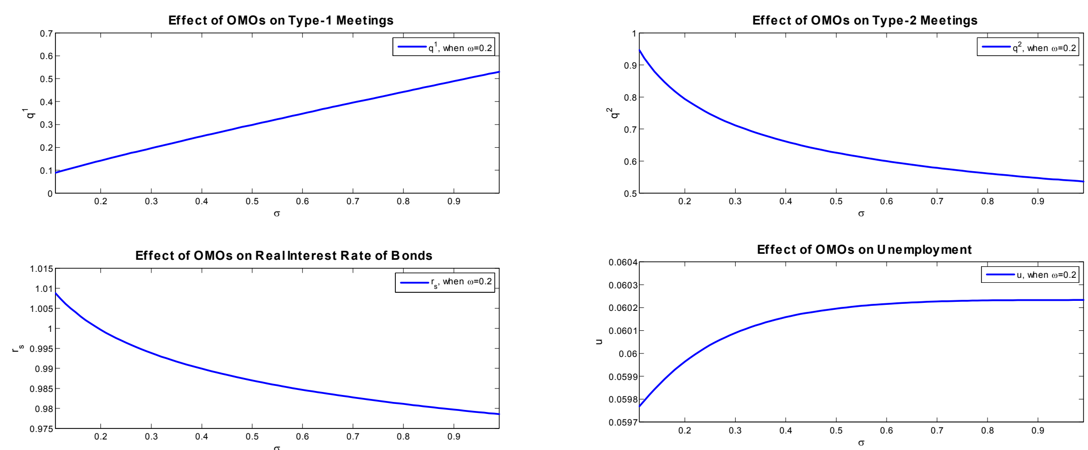

Figure 2.5: **Redistribution Effect of OMOs,** $\omega = 0.20$

When $\omega = 0.20$, the effects of increasing σ from 0.1 to 0.99 are shown in Figure 2.5. Comparing with the case $\omega = 0.95$, we can see that an easing monetary policy still benefits type-1 meetings and hurts type-2 meetings. The real interest rate again decreases. However, since there are more type-2 meetings, the negative effect of σ on q^2 begins to dominate in determining firms' expected profits. My numerical results suggest that firms profits decrease when we set $\omega = 0.20$ and increase σ from 0.1 to 0.99. Therefore, the unemployment rate *increases* from 5.98% to 6.02%. A higher σ can have a negative impact on the labor market when ω is low.

In addition, notice that the equilibrium existence condition in (2.44) implies that the valid boundary of σ varies when we change the value of ω. When $\omega = 0.95$, Figure 2.4 show that the valid boundary of σ is (0.57 : 0.99). And when $\omega = 0.2$, Figure 2.5 show that the valid boundary of σ is (0.11 : 0.99). It indicates that equilibrium with scarce assets is more likely to occur with a higher ratio of type-2 meetings, i.e., a smaller ω.

2.4 Extension: Adding Long-term Government Bonds

To address the recent financial crisis and the unconventional monetary policy responses of central banks, we extend the model to include one more type of assets: long-term government bonds. As is aforementioned, after the 2007-2008 GFC, central banks in the U.S. and other advanced economies have reached ZLB of the short-term interest rate. This ZLB restricts the ability of central banks to conduct monetary policy. In response, these central banks started to rely on unconventional monetary policy by large-scale purchases of long-term government bonds, and private assets such as mortgage-backed securities (MBS) and agency debts. The purpose of the unconventional monetary policy is to inject liquidity into the financial system and thus stimulate the economy.

To understand the effects of such an unconventional monetary policy, we introduce one more asset into Baseline Model: long-term government bonds, in addition to money and short-term government bonds.[19] we solve for monetary equilibrium and then examine the effects of OMOs on the economy's output and

[19] Alternatively, we can introduce some private assets to the baseline model so that one can examine the effects of the unconventional monetary policy as purchases of private assets.

unemployment. In particular, the unconventional monetary policy is modeled as OMOs that involve purchasing long-term government bonds to inject money.

2.4.1 Model

Based on my baseline model, now suppose the government can also issue long-term government bonds, in addition to money and short-term government bonds. These long-term bonds differ from short-term government bonds in two aspects. The first difference is their maturity. Similar to Williamson (2013), we assume that long-term government bonds are long-maturity bonds such as Consols. One unit of long-term government bond is a promise to pay one unit of money in every future AD market. Let $p_{\ell t}$ denote the discounted price of long-term government bonds in period t. In the steady state, the gross real rate of return on long-term bonds is r_ℓ is calculated as

$$r_\ell = \frac{p_\ell + 1}{p_\ell}, \tag{2.45}$$

where we drop the subscript t in $p_{\ell t}$ for steady state values. As in (2.23), $1/(1+\pi) \leq r_\ell \leq 1/\beta$ should also hold in monetary equilibrium. The second difference between short term government bonds and long term government bonds is their liquidity. We assume that long-term bonds can be used as a medium of exchange only in type-2 meetings. However, they are less liquid than short-term government bonds in the sense that when a household carries one unit of long-term government bonds, only a fraction of γ $(0 < \gamma < 1)$ can be used to purchase goods in the KW market.[20]

Formally, the value function (2.10) is updated as follows,

$$\begin{aligned} W_e^h(z, a_s, a_\ell) = I_e + z + a_s + (p_\ell + 1)a_\ell + \max_{\hat{z}, \hat{a_s}, \hat{a_\ell}} \{ -\hat{z} + \beta\rho\hat{z} - p_s\hat{a}_s + \beta\hat{a}_s - p_\ell\hat{a}_\ell \\ + \beta(p_\ell + 1)\hat{a}_\ell + \beta\alpha_h S_h'\} + \beta\mathbb{E}W_{\hat{e}}^h(0,0,0), \end{aligned} \tag{2.46}$$

[20] Nosal and Rocheteau (2013)[14] use a similar parameter to model different liquidity between money and assets. See also the discussion in Venkateswaran and Wright (2013).

where $I_e = ew + (1-e)(b+\chi) + \Delta - T$, $S_h' = \omega[\upsilon(q^1) - \rho d^1] + (1-\omega)[\upsilon(q^2) - \rho d^2 - \mu_s^2 - (p_\ell + 1)\mu_\ell^2]$, with (μ_s^2, μ_ℓ^2) denoting short-term bonds and long-term bonds transferred in type-2 meetings, respectively. For firms, R in (2.20) becomes

$$R \equiv \alpha_f\{\omega[\rho d^1 - c(q^1)] + (1-\omega)[\rho d^2 + \mu_s^2 + (p_\ell + 1)\mu_\ell^2 - c(q^2)]\} + y.$$

Adding long-term government bonds does not alter the generalized Nash bargaining outcome in type-1 meetings. The solution (q^1, d^1) still satisfies (2.25) and (2.26). However, in type-2 meetings, the generalized Nash bargaining is now

$$\max_{q^2, d^2, \mu_s^2, \mu_\ell^2} [\upsilon(q^2) - \rho d^2 - \mu_s^2 - (p_\ell + 1)\mu_\ell^2]^\theta [\rho d^2 + \mu_s^2 + (p_\ell + 1)\mu_\ell^2 - c(q^2)]^{1-\theta},$$
$$\text{st.} \quad d^2 \le z, \ \mu_s^2 \le a_s \text{ and } \mu_\ell^2 \le \gamma a_\ell \ .$$

Here $\mu_\ell^2 \le \gamma a_\ell$ reflects that households can spend only a fraction of γ of their long-term government bonds on the KW goods. The solution involves $d^2 = z$, $\mu_s^2 = a_s$, $\mu_\ell^2 = \gamma a_\ell$, and $q^2 = g^{-1}(\rho z + a_s + \gamma(p_\ell + 1)a_\ell)$, where $g(q^2)$ satisfies (2.28), but now,

$$g(q^2) = \rho z + a_s + \gamma(p_\ell + 1)a_\ell. \tag{2.47}$$

Notice we still have $\partial q^2/\partial z = \rho/g'(q^2) > 0$, $\partial q^2/\partial a_s = 1/g'(q^2) > 0$, but now $\partial q^2/\partial a_\ell = \gamma(p_\ell + 1)/g'(q^2) > 0$. It shows that, in type-2 meetings, with more assets, households can purchase more KW goods. Similar to Baseline Model, since $g'(q^j) > 0$ for $j \in \{1, 2\}$, we know that $q^2 \ge q^1$ from (2.29) and (2.47). Again, we need to discuss corner solutions in more detail in Section 4.3.

Given the bargaining solution, we can rewrite the choice of $(\hat{z}, \hat{a}_s, \hat{a}_\ell)$ in (2.46) as,

$$\max_{\hat{z}, \hat{a}_s, \hat{a}_\ell} \ \{-\hat{z} + \beta\rho\hat{z} - \hat{a}_s + \beta\hat{a}_s - p_\ell\hat{a}_\ell + \beta(p_\ell + 1)\hat{a}_\ell + \beta\alpha_h[\omega(\upsilon(q^1) - \rho\hat{z}) \tag{2.48}$$
$$+(1-\omega)(\upsilon(q^2) - \rho\hat{z} - \hat{a}_s - \gamma(p_\ell + 1)\hat{a}_\ell)]\}$$
$$\text{st.} \ \hat{z} \ge 0, \hat{a}_s \ge 0 \text{ and } \hat{a}_\ell \ge 0.$$

The first-order conditions for interior solutions are (2.32), (2.33) and

$$\frac{\upsilon'(q^2)}{g'(q^2)} - 1 = \frac{\frac{1}{\beta r_\ell} - 1}{\gamma(1-\omega)\mathcal{M}(1, 1-u)}. \tag{2.49}$$

where (p_s, p_ℓ) are substituted by (r_s, r_ℓ) using (2.1) and (2.45). From (2.33) and (2.49), we obtain,

$$r_\ell = \frac{r_s}{\gamma + (1-\gamma)\beta r_s}. \tag{2.50}$$

One can show that $r_\ell \geq r_s$ if and only if $r_s \leq 1/\beta$. Therefore, a strictly positive real term premium between long-term and short-term government bonds exists if and only if long-term bonds are less liquid than short-term bonds, i.e., $0 < \gamma < 1$, and short-term bonds are scarce, i.e., $r_s < 1/\beta$. Substituting $\upsilon'(q^2)/g'(q^2)$ in (2.33) into (2.32), we also reach (2.34). The interior solution of $(\hat{z}, \hat{a}_s, \hat{a}_\ell)$ implies that (q^1, q^2) is solved from (2.33) and (2.34). With money, short-term bonds and long-term bonds, there are more cases where $(\hat{a}_s, \hat{a}_\ell)$ may take corner solutions. We leave the discussion of these corner solutions to Section 4.3.

As for the labor market equilibrium condition, it is still shown by (2.37), where (q^1, q^2) are solved from goods market equilibrium condition.

2.4.2 Monetary Policy

With both short-term and long-term government bonds in the model, Central Bank can conduct not only conventional monetary policy, i.e., OMOs by purchase/sale of short-term bonds to adjust money supply, but also unconventional monetary policy, i.e., OMOs by purchase/sale of long-term government bonds to adjust money supply. We now introduce three parameters: σ_z,denoting the ratio of currency to the total nominal government debt; σ_s,denoting the ratio of short-term bonds to the total government debt; and σ_ℓ, denoting the ratio of long-term bonds to the total government debt. Thus, we have,

$$M_t = \sigma_z(M_t + p_{st}A_{st} + p_{\ell t}A_{\ell t}), \tag{2.51}$$

$$p_{st}A_{st} = \sigma_s(M_t + p_{st}A_{st} + p_{\ell t}A_{\ell t}), \tag{2.52}$$

$$p_{\ell t}A_{\ell t} = \sigma_\ell(M_t + p_{st}A_{st} + p_{\ell t}A_{\ell t}). \tag{2.53}$$

where $A_{\ell t}$ refers to long-term government bonds held by private sectors in period t. Notice that σ_z, σ_s and σ_ℓ are all parameters controlled by Central Bank. With (2.21) and the above (2.51), (2.52) and (2.53), the new monetary policy parameters are $(\rho,\, \sigma_z,\, \sigma_s, \sigma_\ell)$, where $\sigma_z + \sigma_s + \sigma_\ell = 1$.

Again we focus on the steady state, where the total government debt is $G' = z + p_s a_s + p_\ell a_\ell$ is constant. Replacing M_t, a_{st} and $A_{\ell t}$ with their real terms, and using (2.51), (2.52) and (2.53), we obtain

$$a_s = r_s \cdot \frac{\sigma_s}{\sigma_z} z, \tag{2.54}$$

$$a_\ell = (r_\ell - 1) \cdot \frac{\sigma_\ell}{\sigma_z} z. \tag{2.55}$$

2.4.3 Equilibrium

With equilibrium conditions in labor and goods market, and asset market clearing conditions, we are now ready to define general equilibrium.

Definition 2 *Given monetary policy parameters* (π, σ)*, a stationary monetary equilibrium consists of* $(z, a_s, a_\ell, q^1, q^2, r_s, r_\ell, u)$ *such that (i) given* u*,* $(z, a_s, a_\ell, q^1, q^2)$ *solves (2.48) where* (z, a_s, a_ℓ) *satisfies (2.29) and (2.47);(ii) given* (q^1, q^2)*,* u *satisfies (2.37); and (iii) asset markets clear and satisfy (2.54) and (2.55).*

The next step is to characterize different types of equilibrium, which depends critically on the relative return on currency and two types of bonds. Similar to the discussion in Baseline Model, we find that there could exist four types of equilibrium depending on monetary policy parameters.

Liquidity Trap Equilibrium When the real rate of return on currency is equal to the real rate of return on at least one kind of interest-bearing assets (a_s or a_ℓ, or both), but the real rate of returns on a_s and a_ℓ are less than the rate of time preference, we regard it as the liquidity trap case. And we have three sub-cases as follows.

Case 1: $1/(1+\pi) = r_s < r_\ell < 1/\beta$

When the real rate of return on money is equal to r_s but less than r_ℓ, households in type-2 meetings will just hold a portfolio of money and long-term bonds (z, a_ℓ), i.e., $a_s = 0$. Households would not hold short-term bonds since they offer the same return as money but money can used in both type-1 and type-2 meetings. With $a_s = 0$, (2.33) and (2.50) do not hold any more. If we substitute $\upsilon'(q^2)/g'(q^2)$ in (2.49) into (2.32), we obtain,

$$\frac{\upsilon'(q^1)}{g'(q^1)} - 1 = \frac{\frac{1+\pi}{\beta} - 1 - \frac{1}{\gamma}(\frac{1}{\beta r_\ell} - 1)}{\omega \mathcal{M}(1, 1-u)}. \tag{2.56}$$

The goods market equilibrium conditions are shown by (2.49) and (2.56). The assets market clearing conditions (2.54) and (2.55) imply

$$\sigma_z + \sigma_\ell = 1, \tag{2.57}$$

$$a_\ell = (r_\ell - 1) \cdot \frac{1 - \sigma_z}{\sigma_z} z, \tag{2.58}$$

$$r_\ell = \frac{\sigma_z}{\gamma(1+\pi)(1-\sigma_z)}[\frac{g(q^2)}{g(q^1)} - 1], \tag{2.59}$$

where we derive (2.59) using (2.29) and (2.47). The labor market equilibrium condition is still shown by (2.37), except that $g(q^2) = \rho z + \gamma a_\ell \cdot r_\ell/(r_\ell - 1)$, since households do not value short-term government bonds and hold only a portfolio of (z, a_ℓ).

The existence of liquidity trap equilibrium requires that $1/(1+\pi) = r_s < r_\ell < 1/\beta$. Following similar steps as in Baseline Model, the necessary condition that the liquidity trap equilibrium exists is that $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ is in the set

$$\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, \; \frac{1}{1 + \frac{\beta}{\gamma(1+\pi)} \left[\frac{g(q^{2*})}{g\left(q_p^1\right)} - 1 \right]} < \sigma_z < 1,$$
$$\sigma_s = 0, \, 0 < \sigma_\ell = 1 - \sigma_z < 1\},$$

where $\left(q^{2*}, q_p^1\right)$ are defined as before.

In addition, when we compare the current case of the extended model with different cases of Baseline Model, we can see that the current case is quite similar to equilibrium with scarce assets in Baseline Model. The only difference is that households hold a portfolio of (z, a_s) in Baseline Model while they holding a portfolio of (z, a_ℓ) in the current case.

Case 2: $1/(1+\pi) = r_\ell < r_s < 1/\beta$

When the real rate of return on money is equal to r_ℓ but less than r_s, households in type-2 meetings will just hold the portfolio of money and short-term bonds (z, a_s), i.e., $a_\ell = 0$, since money can be used in both type-1 and type-2 meetings, and short-term bonds are more liquid and also have higher real rate of returns than long-term bonds. With $a_\ell = 0$, (2.49) and (2.50) does not hold any more, and the goods market equilibrium conditions are shown by (2.33) and (2.34). The assets market clearing conditions (2.54) and (2.55) imply

$$\sigma_z + \sigma_s = 1, \tag{2.60}$$
$$a_s = r_s \, (\frac{1}{\sigma_z} - 1) z, \tag{2.61}$$
$$r_s = \frac{1 - \sigma_s}{\sigma_s (1+\pi)} [\frac{g(q^2)}{g(q^1)} - 1].$$

For liquidity trap equilibrium to exist, we also require $1/(1+\pi) = r_\ell < r_s < 1/\beta$.

The necessary condition for existence is $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ in the set

$$\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, \ \frac{1}{1 + \beta \left[g\left(q^{2*}\right) / g\left(q_p^1\right) - 1 \right] / (1 + \pi)} < \sigma_z < 1,$$
$$0 < \sigma_s = 1 - \sigma_z < 1, \sigma_\ell = 0\}.$$

The labor market equilibrium condition remains as (2.37) except that $g(q^2) = \rho z + a_s$, as households do not value long-term government bonds and hold a portfolio of (z, a_s). In addition, when comparing with Baseline Model, we can see the current case is exactly the same as equilibrium with scarce assets in Baseline Model.

Case 3: $1/(1 + \pi) = r_s = r_\ell < 1/\beta$

When the real rate of return on money is equal to r_s and r_ℓ, households will choose to hold only money in both type-1 and type-2 meetings since money is widely accepted. Thus, we have zero holding of short-term and long-term bonds, $a_s = a_\ell = 0$, which implies the trading quantities in type-1 and type-2 meetings should be the same, i.e., $q^1 = q^2$. This can also be derived formally from (2.29) and (2.47). With $a_s = a_\ell = 0$, (2.33) and (2.49) do not hold any more, but (2.32) still holds. If we let $r_s = r_\ell \equiv r$ and $q^1 = q^2 \equiv q$, from (2.32), we can obtain the same result as (2.39), which is the goods market equilibrium condition in the current case. On the other hand, from the assets market clearing conditions (2.54) and (2.55), we can obtain, $\sigma_z = 1, \sigma_s = \sigma_\ell = 0$. Thus, the equilibrium exists if and only if $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ is in the set

$$\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, \sigma_z = 1, \sigma_s = \sigma_\ell = 0\}.$$

The labor market equilibrium condition is again (2.40), since $q^1 = q^2 \equiv q$ in the current case. In addition, when comparing with Baseline Model, we can see the current case is the same as the liquidity trap equilibrium in Baseline Model.

Equilibrium with Plentiful Assets When the real rates of return on a_s or a_ℓ, or both of them, are equal to the rate of time preference, money is scarce relative to assets. Thus we call this as the equilibrium with plentiful assets. But when comparing the relative rate of return on assets with that of money, there are still three sub-cases to consider as follows.

Case 1: $1/(1+\pi) = r_s < r_\ell = 1/\beta$

When the real rate of return on money is equal to r_s, households in type-2 meetings will not hold any short-term bonds, i.e., $a_s = 0$. At the same time, r_ℓ is equal to the rate of time preference, therefore, money is scarce relative to long-term bonds. Thus, households in type-2 meetings will hold the portfolio of money and long-term bonds, (z, a_ℓ). When substituting $r_\ell = 1/\beta$ into (2.49) and (2.32) we can obtain the goods market equilibrium conditions shown by the same equations as in (2.41) and (2.42). The labor market equilibrium condition is shown by (2.37), with $g(q^2) = \rho z + \gamma a_\ell/(1-\beta)$, since households hold only a portfolio of (z, a_ℓ) and $r_\ell = 1/\beta$.

Substituting $a_s = 0$ into (2.54) and (2.55) yields the assets market clearing conditions (2.60) and (2.58). Similar to the case of plentiful assets in Baseline Model, we can show that monetary equilibrium exists if and only if $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ is in the set

$$\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, 0 < \sigma_z \leq \frac{1}{1 + \beta \left[g\left(q^{2*}\right) / g\left(q_p^1\right) - 1 \right] / [\gamma\left(1+\pi\right)]},$$
$$\sigma_s = 0 \text{ and } 0 < \sigma_\ell = 1 - \sigma_z < 1\}.$$

When comparing with Baseline Model, we can find the current case is similar to equilibrium with plentiful assets in Baseline Model, except households hold a different portfolio (z, a_ℓ) in the current case.

Case 2: $1/(1+\pi) = r_\ell < r_s = 1/\beta$

When the real rate of return on money is equal to r_ℓ, households in type-2 meetings will not hold any long-term bonds, i.e., $a_\ell = 0$. At the same time, the real rate of return on short-term bonds is equal to the rate of time preference, therefore, money is scarce relative to short-term bonds. Thus, households in type-2 meetings will hold the portfolio of money and short-term bonds, (z, a_s). When substituting $r_s = 1/\beta$ into (2.33) and (2.34). We can obtain the goods market equilibrium conditions shown by the same equations as in (2.41) and (2.42). The labour market equilibrium condition is shown by (2.37), with $g(q^2) = \rho z + a_s/(1-\beta)$, since households hold only a portfolio of (z, a_s) and $r_s = 1/\beta$.

Substituting $a_\ell = 0$ into (2.54) and (2.55) yields the assets market clearing conditions(2.57) and (2.61). Similar to the case of plentiful assets in Baseline Model, we can show that monetary equilibrium exists if and only if $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ is in the set

$$\Big\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, 0 < \sigma_z \leq \frac{1}{1 + \beta\left[g\left(q^{2*}\right)/g\left(q_p^1\right) - 1\right]/(1+\pi)},$$
$$0 < \sigma_s = 1 - \sigma_z < 1 \text{ and } \sigma_\ell = 0\Big\}.$$

In addition, when comparing with Baseline Model, we can find the current case is quite similar to equilibrium with plentiful assets in Baseline Model, except that households hold a different portfolio (z, a_ℓ) in the current case.

Case 3: $1/(1+\pi) < r_s = r_\ell = 1/\beta$

When the real rates of return on a_s and a_ℓ are both equal to the rate of time preference, and greater than that on money, households in type-2 meetings will choose to hold the portfolio of money and any mix of short-term bonds and long-term bonds. Here both short-term bonds and long-term bonds are costless to hold. As a result, households are indifferent between holding short-term bonds and long-term bonds even if long-term bonds are less liquid than short-term bonds. When substituting $r_s = 1/\beta$ into (2.32) and (2.33), we can obtain the goods market equilibrium conditions the same as in (2.41) and (2.42). The labor market equilibrium condition is shown by (2.37) with $g(q^2) = \rho z + a_s + \gamma r_\ell a_\ell/(r_\ell - 1)$. This type of monetary equilibrium exists if and only if $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ is in the set,

$$\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, 0 < \sigma_z < 1, 0 < \sigma_s,$$
$$0 < \sigma_\ell \text{ and } \frac{\sigma_s + \gamma\sigma_\ell}{1 - \sigma_s - \sigma_\ell} \geq \frac{\beta}{1+\pi}\left[\frac{g\left(q^{2*}\right)}{g\left(q_p^1\right)} - 1\right]\}.$$

When comparing with Baseline Model, we can find the current case is exactly the same as equilibrium with plentiful assets of Baseline Model.

Equilibrium with Scarce Assets This case occurs when $1/(1+\pi) < r_s < r_\ell < 1/\beta$. Households in type-2 meetings hold a portfolio of money, short-term bonds and long-term bonds, (z, a_s, a_ℓ). Substituting r_ℓ from (2.50), a_s and a_ℓ from (2.54) and (2.55) into (2.29) and (2.47), we get an implicit function of r_s as follows,

$$\sigma_s r_s + \frac{\sigma_\ell}{\beta(\frac{1}{\gamma} - 1) + \frac{1}{r_s}} = \frac{\sigma_z}{1+\pi}[\frac{g(q^2)}{g(q^1)} - 1]. \tag{2.62}$$

Thus the equilibrium (q^1, q^2, r_s, r_ℓ) is determined by (2.33), (2.50), (2.34) and (2.62). The labour market equilibrium condition is shown by (2.37) where $g(q^2) = \rho z + a_s + \gamma r_\ell a_\ell/(r_\ell - 1)$.

To ensure equilibrium with scarce assets to exist, we require $1/(1+\pi) < r_s < r_\ell < 1/\beta$. The necessary condition for existence of equilibrium is $(\pi, \sigma_z, \sigma_s, \sigma_\ell)$ is in the set

$$\{(\pi, \sigma_z, \sigma_s, \sigma_\ell) : \pi > \beta - 1, 0 < \sigma_z < 1, 0 < \sigma_s, \tag{2.63}$$
$$0 < \sigma_\ell \text{ and } \frac{\sigma_s + \gamma\sigma_\ell}{1 - \sigma_s - \sigma_\ell} < \frac{\beta}{1+\pi}\left[\frac{g\left(q^{2*}\right)}{g\left(q_p^1\right)} - 1\right]\}.$$

Friedman Rule Equilibrium Lastly, when $1/(1+\pi) = r_s = \frac{1}{\beta}$, both money and two types of bonds have the same rate of return, which equals the inverse of time preference. In this case, similar to Baseline Model, monetary policy is at the Friedman rule. The equilibrium allocation (q^2, q^1) is the same as (q^{2*}, q^{1*}) and the unemployment rate u is solved from (2.37). It is clear that the Friedman rule equilibrium exists if and only if $\pi = \beta - 1$.

2.4.4 Quantitative Analysis

Based on the extended model with long-term government bonds, we rely on quantitative analysis to examine how unconventional monetary policy affects the real economy. Recall that we view unconventional monetary policy as OMOs by purchase/sale of long-term government bonds to adjust money supply. The purchase of long-term government bonds corresponds to the large-scale purchase of long-term government bonds by central banks in the U.S. and other advanced economies after the GFC. Among different types of equilibria, unconventional monetary policy is feasible only when long-term bonds are valued by households, i.e., $a_\ell > 0$. In particular, we focus on Case 1 in Liquidity Trap Equilibrium and Equilibrium with Scarce Assets because these two type of equilibrium resembles two different stages of the economy in financial crisis. In equilibrium with scarce assets, both short-term bonds and long-term bonds offer positive rate of return that is not too high. Households hold a limited amount of assets. The economy is not very liquid. This scenario is more like the beginning of the financial crisis. Case 1 in Liquidity Trap Equilibrium resembles the real world when the short-term interest rate hits ZLB. In this case, there is no space for Central Bank to further lower the short-term interest rate to stimulate the economy. Central banks have to reply on unconventional monetary policy by purchase of long-term government bonds and private assets to directly adjust the long-term interest rate, and thus influence the macroeconomy. This scenario is more like the peak of the financial crisis.

In quantitative analysis, we adopt the same functional forms for the utility function, cost function and matching functions as in Section 3.4. We also use the same parameter values shown in Table 1, except that we add one more parameter, $\gamma = 0.5$, to measure the liquidity of long-term bonds. The quantitative results show how unconventional monetary policy affects the economy's output and unemployment in Equilibrium with Scarce Assets, and Case 1 of Liquidity Trap Equilibrium, respectively.

Unconventional Monetary Policy in Equilibrium with Scarce Assets
We first show the effects of unconventional monetary policy in the equilibrium with scarce assets. For simplicity, we fix the ratio of short-term bonds, $\sigma_s = 0.01$, and increase the ratio of currency, σ_z, from 0.1 to 0.99. This is equivalent to decrease the ratio of long-term bonds held by private sectors, given $\sigma_\ell = 1 - \sigma_s - \sigma_z$. This OMO is the purchase of long-term government bonds to inject currency by Central Bank.

Figure 2.6 shows that this type of unconventional monetary policy benefits type-1 meetings by increasing q^1, but hurts type-2 meetings by decreasing q^2. That is, the redistribution effect still exists under unconventional monetary policy. Both r_s and r_ℓ decrease because demand for short-term and long-term government bonds increase with the purchase of long-term bonds by Central Bank. As for the effect on unemployment, we can see that the unemployment rate decreases slightly from 6.03% to 6.02%. Based on my discussion about the redistribution effect, it means that firms' profits must increase as a result of this OMO and hence there is less unemployment. In addition, the equilibrium existence condition in (2.63) implies that the valid boundary of σ_z is $(0.22 : 0.99)$.

Note that we set the ratio of short-term government bonds, $\sigma_s = 0.01$. This is a good analogy to the real world because this ratio is very small in the total

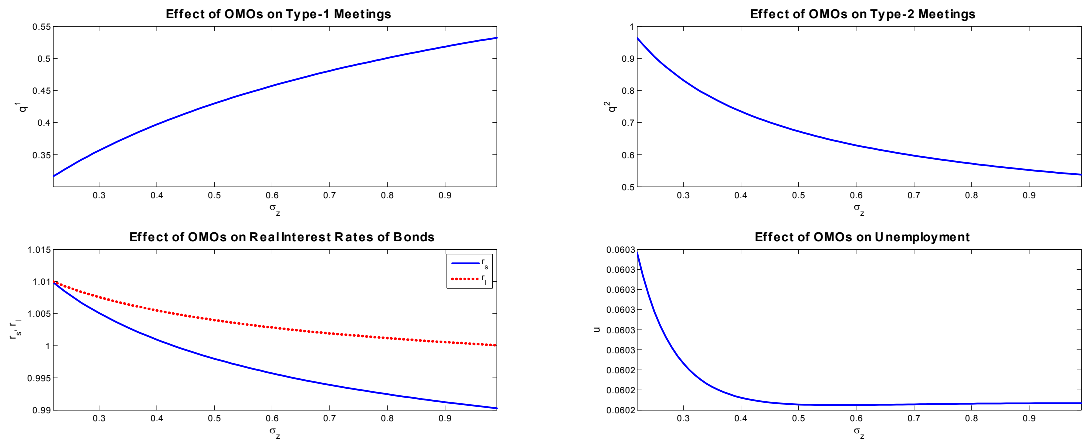

Figure 2.6: **General Effects of Unconventional Monetary Policy**

government debt, such as in the U.S. and other advanced countries after the GFC. In these countries, the conventional monetary policy by buying/selling short-term government bonds to target benchmark short-term interest rates is not valid any more when the short-term interest rate is close to the ZLB. Then central banks keep a very low fraction of short-term government bonds, and conduct unconventional monetary policy by large-scale purchase of long-term bonds and private assets.

Unconventional Monetary Policy in Liquidity Trap Equilibrium As is mentioned before, in my extended model, Case 1 of Liquidity Trap Equilibrium is a good analogy to ZLB faced by central banks in the U.S. and other advanced economies after the GFC. In Case 1, the nominal short-term interest rate is zero since the real interest rate of short-term bonds is equal with the rate of return on money.

Figure 2.7 shows the effects of unconventional monetary policy in Case 1, by increasing σ_z from 0.1 to 0.99. Again, this is equivalent to decreasing the ratio of long-term bonds held by private sectors, given $\sigma_\ell = 1 - \sigma_z$, which resembles the large-scale purchase of long-term government bonds. We still observe

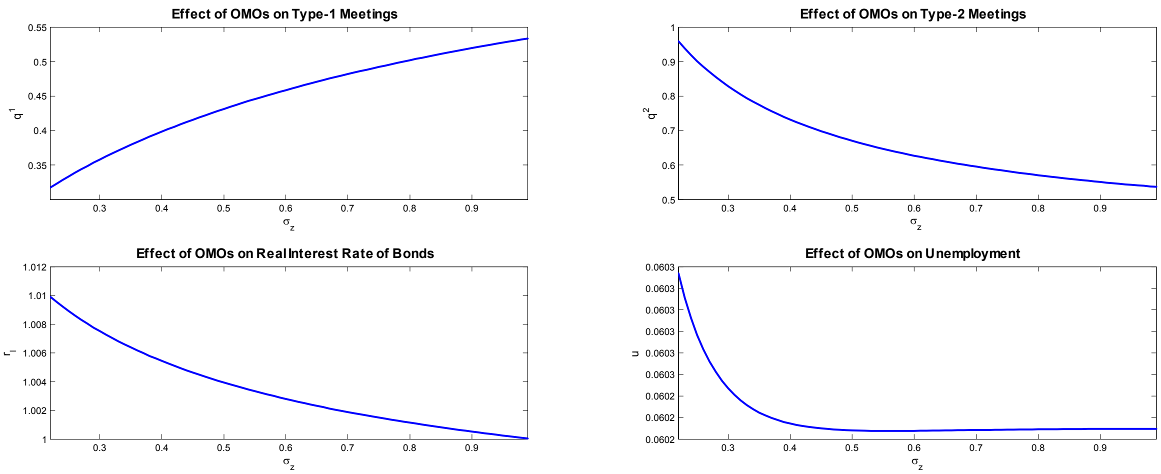

Figure 2.7: **Effects of Unconventional Monetary Policy in Liquidity Trap**

the redistribution effect of the OMOs. Increasing σ_z benefits type-1 meetings while it hurts type-2 meetings. And the real interest rate of long-term bonds, r_ℓ, decreases since the large-scale purchase of long-term bonds by Central Bank increases the demand of long-term bonds, and then the price. This is also consistent with the movement of yields on long-term government bonds since QE has been conducted in the U.S. and other advanced economies after the GFC. As for the effect of unconventional monetary policy on unemployment, Figure 2.7 shows that it decreases unemployment rate from 6.03% to 6.02%.

As is mentioned before, we know case 1 of Liquidity Trap Equilibrium in the extended model is quite similar to the equilibrium with scarce assets in Baseline Model, except that the portfolio holding of households is (z, a_ℓ), instead of (z, a_s). Therefore, when we further explore the redistribution effect of unconventional monetary policy in case 1, it is not surprising that we get similar results in the following two cases: (1) $\omega = 0.95$, type-1 meetings dominate the KW goods trading, and, (2) $\omega = 0.2$, type-2 meetings dominate the KW goods trading.

Figure 2.8 and 2.9 show that unconventional monetary policy has similar effects on type-1 trading, type-2 trading, real interest rate of long-term bonds,

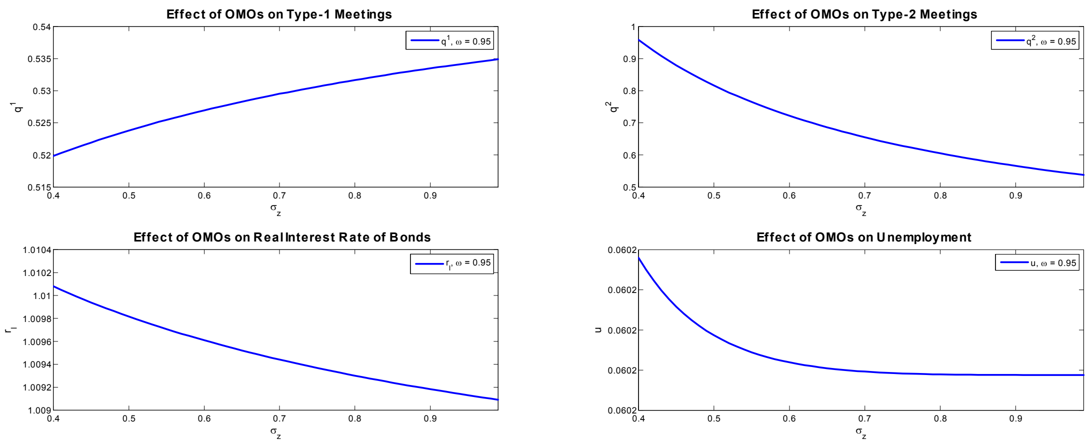

Figure 2.8: **Unconventional Monetary Policy in Liquidity Trap,** $\omega = 0.95$

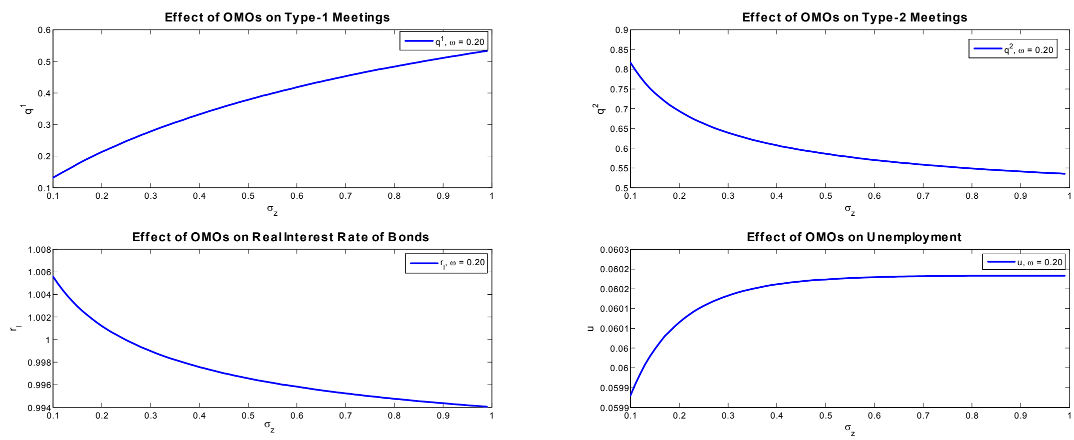

Figure 2.9: **Unconventional Monetary Policy in Liquidity Trap,** $\omega = 0.20$

but opposite effects on unemployment when comparing the unemployment rate at $\omega = 0.95$ with that at $\omega = 0.20$. Therefore, we can see that unconventional monetary policy can reduce unemployment at a minor level when the fraction of households that can hold government bonds is not too big.

Except the value of ω, we also check the robustness of other parameters, with σ_z still increasing from 0.1 to 0.99. There are robust results when we change values of the liquidity measurement of long-term government bonds, γ, bargaining power of households in goods trading, θ, and bargaining power of firms in the labor market, η.

The results show that, the bigger γ is, the closer the liquidity of long-term government bonds is to short-term government bonds in trading. Thus, the real interest rate of long-term bonds will be closer to that of short-term bonds. As for the effect on unemployment, the changes of γ do not really change the effect of an easing monetary policy on unemployment, comparing with the benchmark case setting $\gamma = 0.5$. Only when γ is really small, the unemployment rate may slightly decrease and then increase, but at a very minor level. The robustness of θ is shown by the results in which smaller θ leads to lower unemployment rate, but unemployment rate will increase when θ is really small, with an easing monetary policy. The economic intuition is not hard to understand: smaller θ means buyers (households) have less bargaining power in goods trading, which will increase matching in the KW market, and then have positive effect on the labor market. However, when the bargaining power of buyers becomes really small, buyers may not be willing to trade, and it may restrict the matching in the KW market, and finally have a negative effect on the labor market. As for the robustness of η, there is a threshold value, $\eta = 0.45$, approximately. When below this threshold value, the bigger η is, the lower the unemployment rate will be, and basically, an easing monetary policy increases unemployment. When above this threshold value, the unemployment rate does not vary much, at about the level of 4.76%, with an easing monetary policy. The economic intuition is that, when firms have bigger bargaining power in the labor market, it may increase matching with workers and decrease unemployment with an easing monetary policy. However, when the bargaining power of firms becomes big enough, an easing monetary policy almost has no effect on the labor market.

In general, my analysis shows that unconventional monetary policy can influence the macroeconomy through changing the interest rate on long-term government bonds. The easing monetary policy tends to increase q^1, but decrease q^2. Overall, firms' profits and their entry decisions depend on the value of ω.

As a result, the effect on unemployment of such a policy is ambiguous because changing the long-term interest rate may or may not increase firms' profits due to the redistribution effect.

2.5 Conclusion and Future Research

We build a model where money and assets coexist to examine the effects of monetary policy on the real economy. My model include explicit modeling of labor market, goods markets, and assets. In Baseline Model, money and short-term government bonds serve as media of exchange, and we show that monetary policy, including adjusting inflation and OMOs, has effects on macroeconomic activities. We then extend the model to add long-term government bonds serving as a medium of exchange for goods trading. Long-term government bonds differ from short-term government bonds in that they are less liquid in goods market and hence offer a higher return than short-term government bonds. The monetary authority can conduct unconventional monetary policy by adjusting its holding of long-term government bonds. We use the model to analyze how unconventional monetary policy affects output and unemployment. Quantitative analysis shows that unconventional monetary policy can have a positive impact on labor market performance only when the fraction of households holding government bonds is not too big.

It is interesting to address the linkage among liquidity, conventional/unconventional monetary policy and unemployment. We will further explore research in this area. Future research may include adding financial intermediaries (such as banks) to address the liquidity effect of unconventional monetary policy on financial institutions, and adding private assets purchase of central banks to show the comprehensive effects of unconventional monetary policy in the long run. Based on the current framework, we may also consider fiscal policy and the interaction between fiscal policy and monetary policy.

2.6 Special Acknowledgment

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3 Chapter 3 Open Market Operations

3.1 Introduction

Monetary policy is believed by many to have important effects on the economy. Broadly speaking, governments issue two categories of paper, fiat currency and bonds, although there are subcategories, including currencies of different denominations and real or nominal bonds with different maturities. Monetary policy consists of controlling the amounts of these objects outstanding, or their growth rates, often in an attempt to target nominal interest or inflation rates. There are different ways to change the supply of government-issued assets held by the public, including transfers and spending on goods or other assets. The traditional instrument used to alter the mix is to buy or sell bonds with cash – i.e., an *open market operation*, or OMO. This project studies the effects of these kinds of policies through the lens of the New Monetarist framework, which means, in particular, without recourse to sticky nominal prices.[21]

In this framework, at some points in time agents trade with each other in decentralized markets, as in search theory, while at other points in time they trade in more centralized markets, as in general equilibrium theory. When they trade with each other, frictions in the environment make it interesting to ask *how* they trade: Do they use barter, credit or media of exchange? If they use credit, is it unsecured or secured. What assets serve as media of exchange or constitute acceptable collateral? We spend some time analyzing why different assets, like currency or bonds, may be more or less acceptable as means of payment or pledgeable as collateral – i.e., why different assets may be more or less *liquid*.

[21] Recent expositions of this literature include Williamson and Wright (2010*a*,*b*), Wallace (2010), Nosal and Rocheteau (2011) and Lagos et al. (2014). Now, it is not that New Monetarist theory cannot accommodate (exogenous or endogenous) sticky prices: as discussed in those references, it can. It is rather that there is a belief that we do not need such devices for interesting analyses of monetary theory and policy.

Given this, we are especially interested in happens when we change policy, under various scenarios for the liquidity different assets, and for market structure, including random or directed search and bargaining or price posting.

One policy instrument is the growth rate of the money supply, which equals inflation in stationary equilibrium. As in any reasonable model, with flexible prices, while changing the level of the money supply is neutral changing the growth rate is not. Given this, an OMO is effectively the same as changing the outstanding stock of bonds, which may or may not affect total liquidity. While changing the bond supply may or may not affect real variables, accompanying changes in the money supply affect only prices. Theory delivers sharp predictions for these effects. It can also generate novel phenomena, like negative nominal interest rates, endogenous market segmentation, and outcomes resembling liquidity traps. In particular, OMOs that change the supply of bonds, through liquidity effects, sometimes reduce nominal bond returns and stimulate output (consumption or investment), although over some range we can fall into a trap, where further OMOs are neutral and nominal bond rates freeze at their lower bound, which may or may not be zero.

A zero lower bound almost always appears in theory, but negative nominal returns do arise in practice (*Wall Street Journal*, Aug. 10, 2012; *The Economist* July 14, 2014). Figure 3.1 shows German nominal bond yields are currently negative out to 3 years. The current nominal rate on reserves at the ECB is -0.1%. Other cases are discussed below. Of course, there have long been assets with negative nominal yields when those assets provide additional services, a leading example being traveller's checks, where the service is insurance against loss or theft. Here it is liquidity, not insurance, that is center stage, but the general idea is that once one explicitly models the role of assets in transactions it is not hard to get negative nominal rates. As regards liquidity traps, although discussions can be ambiguous if not downright mysterious to those less conversant in or enamoured with IS-LM, Wikipedia is as good a source as any when it describes situations where "injections of cash ... by a central bank fail to decrease interest rates and hence make monetary policy ineffective." Here, for some parameters, such situations can arise naturally, coinciding with especially low levels of economic activity.

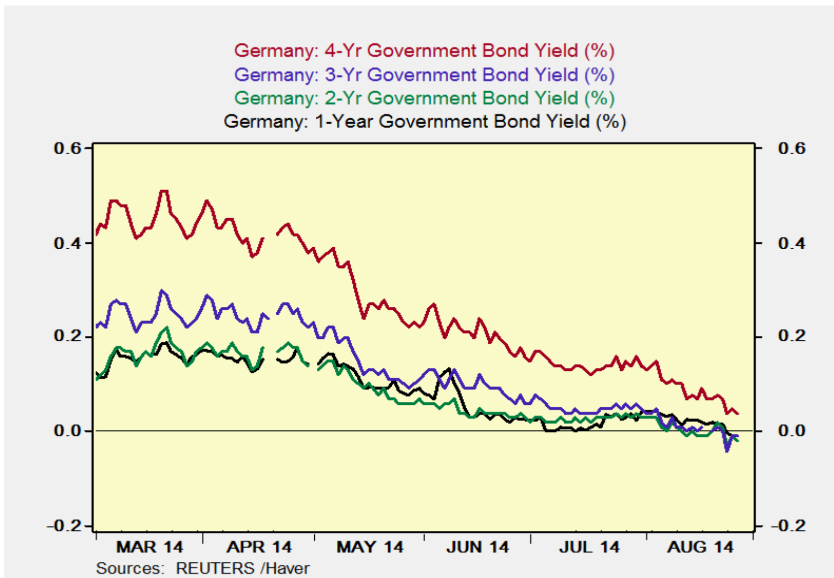

Figure 3.1: Recently negative nominal rates in Germany

It is by now well understood in modern monetary theory that search *per se* is not an essential ingredient (see the surveys listed in footnote 21), but it is used here for convenience, and because it delivers endogenous market segmentation. In particular, with directed search, buyers can choose to visit sellers that accept different payment instruments. Of course, it is also important to ask why sellers might treat different assets differently, and we address this based on information frictions. If some agents are less able to discern legitimate from fraudulent (e.g., counterfeit) versions of certain assets, the model generates outcomes where they might endogenously reject these assets outright, or accept them but only up to endogenous thresholds. Taken together, we think these results provide a relatively rigorous treatment of the effects of policy on the exchange process, where

OMOs may or may not matter, for reasons that are perhaps somewhat novel and certainly crystal clear.

The rest of the paper involves making the theory precise and proving these claims. Section 3.2 describes the environment – agents, preferences, etc. Section 3.3 studies the model assuming random search and bargaining under various scenarios for the liquidity of money and (real or nominal) bonds. Section 3.4 performs similar exercises assuming price posting and directed search rather than bargaining and random search, because we want to investigate the robustness of the results, and because it generates some additional phenomena of interest. Section 3.5 asks why sellers may treat assets differently based on information frictions, and in particular, we consider the possibility that some agents are not able to discern legitimate from fraudulent (e.g., counterfeit) assets. Section 3.6 concludes with a summary and discussion of potential directions for future research.

3.2 Environment

Time is discrete and continues forever. In each period two markets convene sequentially: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. Each period in the CM, a large number of infinitely-lived agents work, consume and adjust their portfolios. In the DM, some of these agents, called *sellers* and indexed by s, can produce but do not want to consume a different good, while others, called *buyers* and indexed by b, want to consume but cannot produce it. Generally, μ is the measure of buyers, and n is the ratio of the sellers to buyers in the DM, where they meet pairwise, with α the probability a buyer meets a seller, and hence α/n the probability a seller meets a buyer. At least in the benchmark model, the interesting analysis concerns buyers, who value assets for their liquidity, while sellers are relatively passive.

The period payoffs for buyers and sellers are

$$\mathcal{U}^b(q,x,\ell) = u(q) + U(x) - \ell \text{ and } \mathcal{U}^s(q,x,\ell) = -c(q) + U(x) - \ell, \tag{3.1}$$

where q is produced and consumed in the DM, x is the CM numeraire, and ℓ is labor supply. For sellers, $c(q)$ is a disutility cost of production. For buyers, one can interpret $u(q)$ as a utility function, or as a production function taking q as an input and delivering $x = u(q)$ as output that enters the next CM budget equation, given CM payoffs linear in numeraire, as shown below. The same equations therefore can be used to describe consumers acquiring output q or producers acquiring input q in DM transactions, which is relevant to the extent that liquidity considerations impinge on both households and firms. The constraints $x \geq 0$, $q \geq 0$ and $\ell \in [0, \hat{\ell}]$, where $\hat{\ell}$ is the CM time endowment, are assumed slack.

As usual, U, u and c are twice continuously differentiable with $U' > 0$, $u' > 0$, $c' > 0$, $U'' < 0$, $u'' < 0$ and $c'' \geq 0$. Also, $u(0) = c(0) = 0$, and there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q}) > 0$. Define the efficient q by $u'(q^*) = c'(q^*)$. Quasi-linear utility simplifies the analysis because it leads to a degenerate distribution of assets across agents of a given type in the DM, and because it makes CM payoffs linear in wealth.[22] Further concerning preferences there is a discount factor $\beta = 1/(1+r)$, $r > 0$, between the CM and DM, while any discounting between the DM and CM is subsumed in the notation in (3.1). Also, suppose that x and q are nonstorable to rule out direct barter, and that agents are anonymous in the DM to rule out pure (unsecured) credit. As is well understood, this can generate a role for assets in the facilitation of intertemporal exchange.

There are for now two assets that can potentially serve in this capacity: fiat money; and government bonds, like T-bills. Their supplies are A_m and A_b while their CM prices are ϕ_m and ϕ_b. We consider both real and nominal bonds, where

[22] As shown by Wong (2012), these same simplifications obtain for any $\tilde{U}(x, 1-\ell)$ as long as $\tilde{U}_{11}\tilde{U}_{22} = \tilde{U}_{12}^2$, which holds for quasi-linear $\tilde{U}$, but also for any $\tilde{U}$ that is homogeneous of degree 1, e.g., $\tilde{U} = x^a (1-\ell)^{1-a}$ or $\tilde{U} = [x^a + (1-\ell)^a]^{1/a}$.

the former (latter) are issued in one CM and pay 1 unit of numeraire (cash) in the next CM. The real stocks of money and bonds per buyer are denoted z_m and z_b. For money, $z_m = \phi_m A_m$; for nominal bonds $z_b = \phi_m A_b$; and for real bonds $z_b = A_b$. These assets are *partially liquid*, in the sense that they may or may not be accepted in DM meetings, or may be accepted only up to some limit. There are two interpretations. First, as in models following Kiyotaki-Wright (1989,1993), sellers may only accept some assets as media of exchange (immediate settlement). Second, as in models following Kiyotaki and Moore (1997,2005), they may only accept some assets as collateral securing promises of numeraire in the next CM (deferred settlement), with the idea being that buyers who renege on promises can be punished by having some of their assets seized. Everything below is independent of which interpretation one adopts.

There is additionally a third interpretation in terms of repurchase agreements: buyers give assets to sellers in DM exchange, then buy them back in the CM at a prearranged price. Of course, in theory it is not necessary to buy back the same assets, or to prearrange the price, when assets are fungible and traded in a frictionless market. Still, as in actual repo practice, assets in the model facilitate intertemporal trade. To be clear, the point here is not that there is anything deep about this discussion of money, collateralized credit and repos; the point is in fact the opposite, that different interpretations can be applied without changing the formal specification.

In any case, only a fraction $\chi_j \in [0,1]$ of asset j can be used in the DM, either as a payment instrument or collateral. Unless otherwise indicated, $\chi_m > 0$ so money can be valued, and $\chi_b > 0$ so the model does not reduce to the pure-currency economy in Lagos and Wright (2005). Under the deferred settlement interpretation, χ_j describes the haircut one takes when using z_j as collateral, often motivated by saying debtors can abscond with a fraction $1 - \chi_j$ of the asset in the out-of-equilibrium event of default. In Section 3.5 we show how to get $\chi_j < 1$ endogenously, whether assets are used as immediate or deferred settlement instruments, when a potential issue is counterfeiting (and note, e.g., that if we interpret money broadly to include demand deposits counterfeiting includes bad checks). It should be emphasized, however, that we do not need $\chi_j < 1$; most results go through at least qualitatively with $\chi_j = 1$, but there is no reason to restrict attention to a special specification at this point.

In the DM, α_m denotes the probability a buyer meets a seller that accepts only money; α_b the probability he meets one that accepts only bonds; and α_2 the probability he meets one that accepts both. One can think of these as products of the baseline arrival rate α and the probability the seller is of a certain type. Special cases include ones where all sellers accept cash, $\alpha_b = 0$; the assets are perfect substitutes, $\alpha_b = \alpha_m = 0$; and a pure-currency economy, $\alpha_b = \alpha_2 = 0$. Under the interpretation of deferred settlement, since agents renege iff a debt exceeds the value of the collateral, promises are constrained by asset holdings, just like immediate payments. Given this, to reduce notation, we usually frame the following discussion in terms of payment instruments rather than collateral, but it is good to keep in mind that it is basically a relabeling to switch between Kiyotaki-Moore credit and Kiyotaki-Wright money.[23]

We focus on stationarity equilibria, where $z_m = \phi_m A_m$ is constant, so the money growth rate π equals the inflation rate: $\phi_m/\phi_{m,+1} = A_{m,+1}/A_m = 1 + \pi$, where subscript $+1$ indicates next period. Stationarity also entails z_b constant, which means A_b is constant for real bonds, while the ratio $B = A_b/A_m$ is constant for nominal bonds. We restrict attention to $\pi > \beta - 1$, or the limit $\pi \to \beta - 1$, which is the Friedman rule (there is no monetary equilibrium with $\pi < \beta - 1$).

[23] One reason this is worth mentioning is that it helps motivate $\alpha_b > 0$: while there may be few retailers that take T-Bills and not cash, there are certainly agents, including financial institutions, that regularly use certain types bonds as collateral (more on this below). One disadvantage of using currency for some purpose may be that it is susceptible to loss or theft, as in He et al. (2005,2008) or Sanches and Williamson (2010).

The government budget constraint is

$$G + T - \pi\phi_m A_m + S = 0, \tag{3.2}$$

where the first term is their consumption of x, the second is a lump-sum transfer, the third is seigniorage, and the fourth is debt service. For real bonds, with A_b constant, $S = A_b(1-\phi_b)$. For nominal bonds, $S = z_b\left[\phi_m - \phi_b\left(1+\pi\right)\right]/\phi_b\left(1+\pi\right)$. Given other variables, we assume T adjusts to satisfy (3.2) each period.

It is important below to distinguish between different interest rates. Define the interest rate on an illiquid nominal bond – one that is never accepted in the DM – by the Fisher equation $1+\iota = \left(1+\pi\right)/\beta$, where $1/\beta = 1+r$ is the return on an illiquid real bond. Thus, $1+\iota$ is the amount of cash you would need in the next CM to make you indifferent to giving up a dollar today, while $1+r$ is the amount of x you would need in the next CM to make you indifferent to giving up a unit of x today. Whether or not these bonds are traded, or even exist, is irrelevant, since in any case we can price them. Denote the nominal rate on our partially-liquid government bonds by ρ. For nominal bonds, $1+\rho = \phi_m/\phi_b$. For real bonds, the nominal return means the amount of cash you can get in the next CM by investing a dollar in them today, $1+\rho = \phi_m/\phi_b\phi_{m,+1} = \left(1+\pi\right)/\phi_b$. Additionally, it also useful to define the spread $s = \left(\iota - \rho\right)/\left(1+\rho\right)$, as in Silveira and Wright (2010) or Rocheteau and Rodriguez-Lopez (2013).

The Fisher equation implies that the Friedman rule is $\iota = 0$, and that we can take either π or ι to be the policy instrument. We use ι as the policy choice, but one can take this to be simply short-hand notation for inflation, $\pi = \left(1+r\right)\left(1+\iota\right) - 1$. More interestingly, the cost of the liquidity services provided by z_m is ι, because rather than holding cash one could have invested in illiquid assets. Similarly, the cost of the liquidity provided by z_b is s, because the partially-liquid bond yields ρ while the illiquid bond yields ι. One way to see the connection is to write the Fisher equation and the definition of spread as

$$1 + \iota = (1 + \pi)(1 + r) \tag{3.3}$$

$$1 + \iota = (1 + \rho)(1 + s). \tag{3.4}$$

3.3 Random Search

Our first market structure involves search and bargaining, because it is easy, and common in the literature. Also, while little of substance depends on whether bonds are real or nominal, since some details differ, we consider them in turn.

3.3.1 Real Bonds

A buyers' state variable in the DM is his portfolio (z_m, z_b), while in the frictionless CM all that matters is the sum $z = z_m + z_b$. Let the CM and DM value functions be $W(z)$ and $V(z_m, z_b)$. Then

$$W(z) = \max_{x,\ell,\hat{z}_m,\hat{z}_b} \{U(x) - \ell + \beta V(\hat{z}_m, \hat{z}_b)\} \text{ st } x = z + \ell + T - (1+\pi)\hat{z}_m - \phi_b \hat{z}_b$$

where $\hat{z}_j$ is the real value of asset j taken out of the CM, and the real wage is 1 because we assume 1 unit of ℓ produces 1 unit of x. The relevant FOC's are $1+\pi = \beta V_1(\hat{z}_m, \hat{z}_b)$ and $\phi_b = \beta V_2(\hat{z}_m, \hat{z}_b)$. Also, $W'(z) = 1$, so as mentioned above one can interpret buyers' DM payoff $u(q)$ as either the utility from consuming q or the output of using it to produce numeraire for the next CM. Again, this is relevant if producers as well as consumers are subject to liquidity constraints; moreover, if q is investment and, as a straightforward generalization, x is produced using labor and capital, then DM outcomes affect CM employment.

There is a similar CM problem for sellers, but we can assume wlog they carry no assets (if assets are priced fundamentally they are indifferent to holding them; if assets bear a liquidity premium they strictly prefer to not hold them). So, to continue with buyers, in the DM

$$V(z_m, z_b) = W(z_m + z_b) + \alpha_m[u(q_m) - p_m] + \alpha_b[u(q_b) - p_b] + \alpha_2[u(q_2) - p_2]$$

where p_j are payments in type-j meetings and we use $W'(z) = 1$. Payments are constrained by $p_j \leq \bar{p}_j$, where $\bar{p}_j$ is the buyer's *liquidity position* in a type-j meeting: $\bar{p}_m = \chi_m z_m$, $\bar{p}_b = \chi_b z_b$ and $\bar{p}_2 = \chi_m z_m + \chi_b z_b$. Thus, only a fraction $\chi_j \in (0, 1]$ of z_j can be used, by assumption, but in Section 3.5 it is a result.[24]

The terms of trade are determined by bargaining: to get q buyers must pay $p = v(q)$, where $v(\cdot)$ depends on the solution concept. Kalai's (1977) proportional solution, e.g., is $v(q) = \theta c(q) + (1 - \theta) u(q)$, where θ is a buyer's bargaining power. Other than $v(0) = 0$ and $v'(q) > 0$, all we need is this: Let $p^* = v(q^*)$ be the payment required to get he efficient q. Then $p^* \leq \bar{p}_j \implies p_j = p^*$ and $q_j = q^*$, while $p^* > \bar{p}_j \implies p_j = \bar{p}_j$ and $q_j = v^{-1}(\bar{p}_j)$. This is satisfied by Nash and Kalai bargaining, although when we have to pick one, for examples, we usually use the Kalai since it has some attractive properties relative to Nash in these models (Aruoba et al. 2007). It is also satisfied by mechanisms like those in Hu et al. (2009), and by Walrasian pricing, which can be motivated by having agents meet multilaterally (Rocheteau and Wright 2005).

As is standard, $\iota > 0$ implies buyers pay all they can in type-m meetings and still cannot get q^* – i.e., $p_m = \chi_m z_m < p^*$. Since $\chi_m z_m < p^*$, in type-2 meetings buyers may as well pay all they can in cash before using bonds, because in these meetings agents are indifferent to any combination of z_m and z_b. Buyers also use all the bonds they can in type-2 meetings iff $\bar{p}_2 \leq p^*$, and use all they bonds they can in type-b meetings iff $\bar{p}_b \leq p^*$. It is clear that $p_2 \geq p_b$. What must be determined is whether: 1. $p_2 = \bar{p}_2$ and $p_b = \bar{p}_b$ (buyers constrained

[24] Under the interpretation of p as a promise to pay in the CM, we can add unsecured debt up to a limit $\bar{d}$, which may be exogenous, or endogenous as in Kehoe and Levine (1993). Then the bound on p_m, e.g., is $\bar{p}_m = \chi_m z_m + \bar{d}$. But this does not affect outcomes – as in Gu et al. (2014), if $\bar{d}$ changes, ϕ_m and z_m respond endogenously to keep $\bar{p}_m$ the same. Therefore, in monetary equilibrium, $\bar{d} = 0$ is not restrictive.

in all meetings); 2. $p_2 < \bar{p}_2$ and $p_b = \bar{p}_b$ (constrained in type-b but not type-2 meetings); or 3. $p_2 < \bar{p}_2$ and $p_b < \bar{p}_b$ (constrained in neither). We consider each case in turn, assuming throughout that a monetary equilibrium exists.[25]

Case 1: The interesting case has buyers are constrained in all meetings and

$$v(q_m) = \chi_m z_m,\ v(q_b) = \chi_b z_b, \text{ and } v(q_2) = \chi_m z_m + \chi_b z_b. \tag{3.5}$$

The Euler equations are derived by differentiating $V(z_m, z_b)$ using (3.5) and inserting the results into the FOC's for z_m and z_b:

$$1 + \pi = \beta \left[1 + \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2)\right] \tag{3.6}$$

$$\phi_b = \beta \left[1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)\right] \tag{3.7}$$

where $\lambda(q_j) \equiv u'(q_j)/v'(q_j) - 1$ is called the liquidity premium in a type-j meeting (equivalently, the Lagrange multiplier on the constraint $p_j \leq \bar{p}_j$). For instance, if $\chi_b = 0$ bonds have no liquidity and the are prices fundamentally, $\phi_b = 1/\beta$. Rearranging, we get

$$\iota = \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2) \tag{3.8}$$

$$s = \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2), \tag{3.9}$$

where the nominal rate on illiquid bonds ι and the spread s are defined above.[26]

[25] For the record, one can show $\alpha_m > 0$ implies monetary equilibrium exists iff $\iota < \bar{\iota}_m$, while $\alpha_m = 0 < \alpha_2$ implies monetary equilibrium exists iff $\chi_b A_b < p^*$ and $\iota < \bar{\iota}_2$, where $\bar{\iota}_m$ and $\bar{\iota}_2$ may or may not be finite. We skip the routine proof, but emphasize that $\alpha_m > 0$ is *not* needed for money to be valued. As long as $\alpha_2 > 0$, even if $\alpha_m = 0$, money can be valued if pledgeable bonds are not overly abundant.

[26] Condition (3.9) is reminiscent of the "convenience yield" notion in Krishnamurphy and Vissing-Jorgenson (2012), measured as the difference between yields on government and corporate bonds. One could say this "rationalizes" their reduced-form assumption of T-Bills in the utility function, although that is not necessarily our goal, any more than having (3.8) "rationalize" money in the utility function.

Recalling the nominal return on bonds is $1+\rho = (1+\pi)/\phi_b$, we get

$$\rho = \frac{\alpha_m \chi_m \lambda(q_m) - \alpha_b \chi_b \lambda(q_b) + (\chi_m - \chi_b)\alpha_2 \lambda(q_2)}{1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)}. \tag{3.10}$$

Notice $\rho < 0$ is possible, as sometimes seen in reality if not in standard theory. In contrast to illiquid bonds, for which the lower bound is $\iota = 0$, there are two ways to get $\rho < 0$ for liquid bonds: if $\chi_m = \chi_b$ then $\rho < 0$ iff $\alpha_m \lambda(q_m) < \alpha_b \lambda(q_b)$, because then z_b has a liquidity advantage over z_m on the extensive margin; and if $\alpha_m \lambda(q_m) = \alpha_b \lambda(q_b)$ then $\rho < 0$ iff $\chi_m < \chi_b$ as long as $\alpha_2 > 0$, because then z_b has an advantage on the intensive margin. This generalizes Williamson (2012), Dong and Xiao (2013) or Rocheteau and Rodriguez-Lopez (2014), where $\alpha_b = 0$ and $\chi_b = \chi_m = 1$, which imply $\rho \not< 0$. Whether or not our generalizations are realistic, this describes logically how to get negative nominal rates when T-Bills have some advantage over cash.

But, to be sure, according to *The Economist* (July 14, 2014), the logic may well be relevant: "Not all Treasury securities are equal; some are more attractive for repo financing than others. With less liquidity in the market, those desirable Treasuries can be hard to find: some short-term debt can trade on a negative yield because they are so sought after." Relatedly, according to the Swiss National Bank (2013): "With money market rates persistently low and Swiss franc liquidity still high, trading activity on the repo market remained very slight. However, activity on the secured money market did not grind to a complete halt, due to the demand for high-quality securities. The increased importance of these securities is reflected in the trades on the interbank repo market which were concluded at negative repo rates." This is consistent with our theory: traders want bonds for their liquidity and are willing to accept negative nominal yields to get them.[27]

[27] Aleks Berentsen further emphasized to us that interest rates on Swiss bonds are negative because they can be used as collateral in markets outside of Switzerland, where francs cannot. Hence, some (mainly foreign) banks use francs on reserve at the SNB to acquire Swiss bonds with negative returns to facilitate secured credit.

It is important to understand that $\rho < 0$ does not defy standard no-arbitrage conditions, because while individuals can issue bonds –i.e., borrow – they cannot guarantee claims against them will be liquid – i.e., circulate in the DM. This is similar to models where agents accept negative nominal returns on some assets, like traveller's checks or demand deposits, that are less susceptible than cash to loss and theft. That does not violate no-arbitrage if agents cannot guarantee the security of their paper in the DM, without incurring some cost, as is presumably incurred with traveller's checks and deposit banking. The model in He et al. (2008) where cash is subject to theft can deliver a negative lower bound for this reason. The model in Andolfatto (2013) can deliver a strictly positive lower bound, because imperfect commitment and monitoring hinder the ability to tax and hence to deflate. Here pure liquidity considerations make $\underline{\rho} \neq 0$.

Returning to theory, stationary monetary equilibrium is a list (q_m, q_b, q_2, z_m, s) solving (3.5)-(3.9) with $z_m > 0$. To characterize it, use (3.5) to rewrite (3.8) as

$$\iota = \alpha_m \chi_m L(\chi_m z_m) + \alpha_2 \chi_m L(\chi_m z_m + \chi_b z_b), \tag{3.11}$$

where $L(\cdot) \equiv \lambda \circ v^{-1}(\cdot)$. Given z_b, under standard assumptions (see fn. 25), a solution $z_m > 0$ to (3.11) exists, is generically unique and implies $L'(\cdot) < 0$ (Wright 2010). From z_m, (3.5) determines (q_m, q_b, q_2). Then (3.9) determines s, (3.10) determines ρ, etc. Clearly, a one-time change in A_m is neutral, because ϕ_m adjusts to leave $z_m = \phi_m A_m$ and other real variables the same. Hence, an OMO that swaps A_b for A_m has the same effect as simply changing A_b, given fiscal implications are offset by T. Our focus is thus on policies that change ι or z_b.

Letting $D_R \equiv \alpha_m \chi_m^2 L'(\chi_m z_m) + \alpha_2 \chi_m^2 L'(\chi_m z_m + \chi_b z_b) < 0$, we have $\partial z_m / \partial \iota = 1/D_R < 0$, so as usual a higher nominal rate on illiquid bonds (or inflation or money growth) reduces real balances. This hinders trade in monetary meetings,

$$\frac{\partial q_m}{\partial \iota} = \frac{\chi_m}{v'(q_m) D_R} < 0, \frac{\partial q_b}{\partial \iota} = 0, \text{ and } \frac{\partial q_2}{\partial \iota} = \frac{\chi_m}{v'(q_2) D_R} < 0.$$

In terms of financial variables,

$$\begin{aligned}
\frac{\partial s}{\partial \iota} &= \frac{\alpha_2 \chi_m \chi_b L'(\chi_m z_m + \chi_b z_b)}{D_R} > 0 \\
\frac{\partial \phi_b}{\partial \iota} &= \beta \frac{\alpha_2 \chi_m \chi_b L'(\chi_m z_m + \chi_b z_b)}{D_R} > 0 \\
\frac{\partial \rho}{\partial \iota} &= \frac{\alpha_m L'(\chi_m z_m) + \alpha_2 L'(\chi_m z_m + \chi_b z_b)[1 - (1+\rho)\chi_b/\chi_m]}{(1+s)[\alpha_m L'(\chi_m z_m) + \alpha_2 L'(\chi_m z_m + \chi_b z_b)]} \gtrless 0,
\end{aligned}$$

assuming $\alpha_2 > 0$; else the first two are 0, because then there is no substitution between z_m and z_b in any DM meeting. Given $\alpha_2 > 0$, higher ι increases s and ϕ_b as agents try to switch their portfolios out of cash and into bonds, but this can increase or decrease the bond return ρ (see Figure 3.2 below)

For an OMO that increases z_b,

$$\begin{aligned}
\frac{\partial z_m}{\partial z_b} &= -\frac{\alpha_2 \chi_m \chi_b L'(\chi_m z_m + \chi_b z_b)}{D_R} < 0 \\
\frac{\partial q_m}{\partial z_b} &= -\frac{\alpha_2 \chi_b L'(\chi_m z_m + \chi_b z_b)}{v'(q_m) D_R} < 0 \\
\frac{\partial q_b}{\partial z_b} &= \frac{\chi_b}{v'(q_b)} > 0 \\
\frac{\partial q_2}{\partial z_b} &= \frac{\alpha_m \chi_b L'(\chi_m z_m)}{v'(q_2) D_R} > 0,
\end{aligned}$$

assuming $\alpha_2 > 0$; else the first two are 0. Given $\alpha_2 > 0$, higher z_b decreases z_m and q_m because liquidity is less scarce in type-2 meetings, so agents economize on z_m, which comes back to haunt them in type-m meetings. Naturally, OMO's have different effects on different meetings, and what they do to average q is ambiguous. One can also check $\partial s/\partial z_b < 0$, $\partial \phi_b/\partial z_b < 0$ and $\partial \rho/\partial z_b > 0$, as higher z_b makes liquidity generally less scarce, assuming $\chi_b > 0$; else these effects all vanish.

While Case 1 is the most interesting, for completeness we report:

Case 2: Now buyers are unconstrained in type-2 meetings. The equilibrium conditions are similar, except $q_2 = q^*$, so $\lambda(q_2) = 0$ and z_m is determined as in a pure-currency economy. An increase in ι lowers z_m and q_m, does not affect q_b or q_2, and increases s as agents again try to shift from z_m to z_b. An increase in z_b does not affect z_m, q_m or q_2, increases q_b and decreases s. We can still have $\rho < 0$, as can be seen from (3.10), even with $q_2 = q^*$.

Case 3: If buyers are unconstrained in type-2 and type-b meetings, $q_b = q_2 = q^*$, so bonds provide no liquidity at the margin and $s = 0$. An increase in ι reduces z_m and q_m but otherwise affects nothing. An increase in z_b affects nothing.

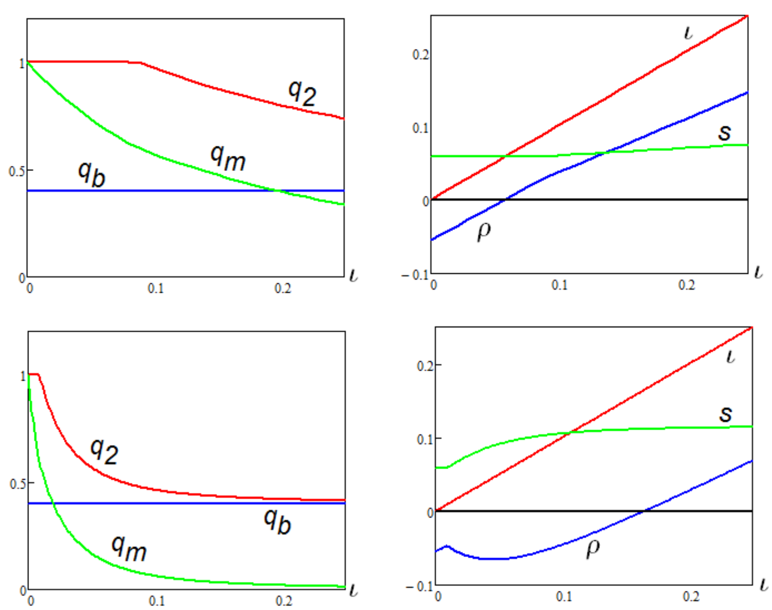

Figure 3.2: Effects of ι (nominal illiquid interest rate)

Which case obtains? If $\chi_b z_b \geq v(q^*)$ (bonds are abundant) we get Case 3. If $\chi_b z_b < v(q^*)$ (bonds are scarce) we get Case 2 when ι is small and Case 1 when ι is big. In Figure 3.2, drawn for $\chi_b z_b < v(q^*)$, Case 1 obtains iff ι is above the level where the curves kink. The left panels show quantities and the right financial variables, while the upper and lower panels use different χ_j.[28] Effects to notice

[28]Figure 3.2 uses $u(q) = 2\sqrt{q}$, $c(q) = q$, $v(q) = c(q)$ (buyer-take-all bargaining), $\beta = 0.95$,

are: q_m can be above or below q_b; ρ can be negative; and ρ can be nonmonotone in ι. Figure 3.3 shows the effects of z_b, where Case 1 obtains to the left of the point where q_2 kinks, Case 3 occurs to the right of the point where q_b kinks, and Case 2 occurs in between these points.

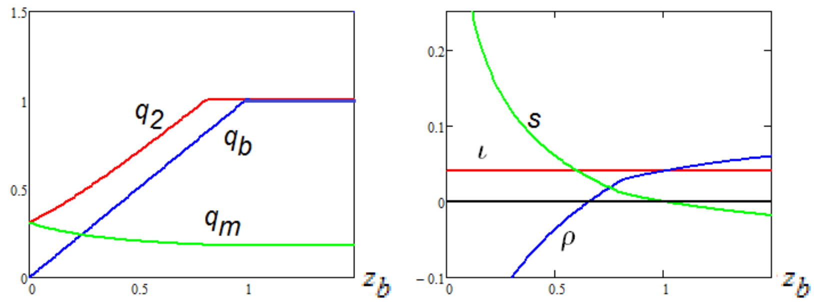

Figure 3.3: Effects of z_b (supply of liquid bonds)

Issuing currency and buying bonds – a traditional OMO depicted as a move to the left in Figure 3.3 – lowers ρ, and potentially increases q_m and decreases q_2 or q_b. It has an ambiguous effect on total DM output $\Sigma_j \alpha_j q_j$, which can be interpreted either as consumption or investment, depending on whether q is interpreted as an output or an input. Crucially, as emphasized earlier, this has nothing to do with increasing the money supply *per se*. That is neutral. The effects are due exclusively to decreasing the supply of liquid bonds, which stimulates demand for alternative sources of liquidity, and in particular real money balances. Printing money to finance bond purchases (or for any other reason) is irrelevant because, absent *ad hoc* restrictions on the ability of prices to adjust, it simply lowers ϕ_m so that z_m stays the same. Similarly, reducing z_b decreases ρ not by putting more currency in the hands of the public, but by raising the bond price ϕ_b and thus lowering the return. This is the New Monetarist anatomy of an OMO.

Policy can target the nominal T-bill rate ρ, although there can be a lower bound, which may be $\underline{\rho} = 0$ or $\underline{\rho} \neq 0$, as discussed in more detail below. The

$A_b = 0.4$, $\alpha_m = 0.3$, $\alpha_2 = 0.2$ and $\alpha_b = 0.1$. The upper panels use $\chi_m = \chi_b = 1$ and the lower panels $\chi_m = 0.1, \chi_b = 1$. Figure 3.3 below is similar, with $\chi_m = \chi_b = 1$ and $i = 0.04$.

target may be achievable through OMO's, or though induced adjustments in the nominal rate on illiquid bonds ι, which via the Fisher equation is tied to inflation and hence ultimately to monetary expansion.[29] We also remark that the above results are what one would expect if bonds were naively modeled as generic goods in the utility function, like apples: cutting the supply raises the price. This is dubbed naive because bonds are not goods; they are assets, valued for their returns plus their liquidity services, and the value of liquidity is not a primitive, the way the utility of apples might be. One ramification is that the value of the liquidity provided by bonds vanishes when ι is small (e.g., close to the Friedman rule). This suggests it is better to model liquidity explicitly than treat it like fruit juice. Now, some assets are somewhat like apples – e.g., apple trees – but to the extent that they provide liquidity services in addition to the utility one gets from their yield, it seems desirable to at least try to take this seriously.

3.3.2 Nominal Bonds

Now consider a nominal bond, issued in one CM and paying a dollar in the next CM, which implies $1+\rho=\phi_m/\phi_b$. For stationarity, let π be the growth rate of both assets, so $B=A_b/A_m$ is constant over time, as is z_m and $z_b=Bz_m$. The CM budget constraint becomes $x=z+\ell+T-(1+\pi)\hat{z}_m-(1+\pi)\hat{z}_b/(1+\rho)$, but otherwise the model is the same.

In Case 1, where buyers are constrained in all meetings, the terms of trade in type-b and type-2 meetings are now $v(q_b)=\phi_m\chi_b a_b=B\chi_b z_m$ and $v(q_2)=\phi_m(\chi_m a_m+\chi_b a_b)=(\chi_m+B\chi_b)z_m$. Also, the Euler equation for z_b has a ϕ_m on the RHS, but (3.8) and (3.9) are exactly the same. Similar to real bonds, letting $D_N\equiv\alpha_m\chi_m^2 L'(\chi_m z_m)+\alpha_2\chi_m(\chi_m+B\chi_b)L'(\chi_m z_m+B\chi_b z_m)<0$, we

[29] Policy can also try to target, say, both ρ and π, through a combination of instruments. While our view of policy is by design abstract, one can delve further into institutional details if so desired. See Berentsen and Monnet (2008), Berentsen and Waller (2011), Afonso and Lagos (2013), Bech and Monnet (2014), Berentsen et al. (2014) or Chiu and Monnet (2014).

have $\partial z_m/\partial\iota = 1/D_N < 0$. Moreover,

$$\frac{\partial q_m}{\partial\iota} = \frac{\chi_m}{v'(q_m)\,D_N} < 0,\ \frac{\partial q_b}{\partial\iota} = \frac{B\chi_b}{v'(q_b)\,D_N} < 0,\text{ and }\frac{\partial q_2}{\partial\iota} = \frac{\chi_m + B\chi_b}{v'(q_2)D_N} < 0.$$

One can also check other effects, like $\partial s/\partial\iota > 0$ if $\alpha_b > 0$. The only qualitative difference from real bonds is that now ι affects q_b.

For an OMO that increases the bond-money ratio B,

$$\begin{aligned}
\frac{\partial z_m}{\partial B} &= -\frac{\alpha_2\chi_m\chi_b z_m L'(\chi_m z_m + B\chi_b z_m)}{D_N} < 0\\
\frac{\partial q_m}{\partial B} &= -\frac{\alpha_2\chi_m^2\chi_b z_m L'(\chi_m z_m + B\chi_b z_m)}{v'(q_m)\,D_N} < 0\\
\frac{\partial q_b}{\partial B} &= \frac{\alpha_m\chi_m^2\chi_b z_m L'(\chi_m z_m) + \alpha_2\chi_m^2\chi_b z_m L'(\chi_m z_m + B\chi_b z_m)}{v'(q_b)\,D_N} > 0\\
\frac{\partial q_2}{\partial B} &= \frac{\alpha_m\chi_m^2\chi_b z_m L'(\chi_m z_m)}{v'(q_2)\,D_N} > 0,
\end{aligned}$$

and again one can derive the effects on ρ etc. This is all qualitatively the same real bonds. Cases 2 and 3 and the parameters for which each case obtains are also the same, and again $\rho < 0$ is possible, for similar reasons. Hence, there is little difference between real and nominal bonds: the analytic results are similarly sharp, and the economic implications are similarly clean.

3.3.3 Liquidity Trap

A recurring theme below is that there can emerge outcomes resembling a liquidity trap, where OMO's do not affect ρ, or the real allocation, which involves inefficiently low levels of economic activity. While Figure 3.3 shows that changes in z_b are neutral when bonds are abundant, that is because agents get satiated in the liquidity services they provide, and occurs when $q_b = q_2 = q^*$ are high, although q_m is low. To illustrate something altogether different, consider introducing heterogeneity: type j buyers have probabilities α_m^j, α_b^j and α_2^j of type-m, type-b and type-2 meetings.[30] Let μ_j be the measure of type-j, and consider real bonds (nominal bonds are similar). Suppose there is one type with $\alpha_m^j = \alpha_b^j = 0$ and $\alpha_2^j > 0$.

Suppose further that type-j holds both, $z_m^j > 0$ and $z_b^j > 0$. Then his Euler equations are

$$1 + \pi = \beta \left[1 + \alpha_2^j \chi_m \lambda(q_2^j)\right] \tag{3.12}$$

$$\phi_b = \beta \left[1 + \alpha_2^j \chi_b \lambda(q_2^j)\right] \tag{3.13}$$

Recall that the nominal return on bonds is $1 + \rho = (1 + \pi) / \phi_b$. Then it is immediate from (3.12)-(3.13) and the Fisher equation that $\rho = \underline{\rho}$, where

$$\underline{\rho} = \frac{(\chi_m - \chi_b)\, \iota}{\chi_m + \iota \chi_b}.$$

Hence the nominal T-bill rate is independent of the supply z_b. This provides a bound below which ρ cannot go, which can be positive, negative or 0. The reason is that for type j bonds and money are perfect substitutes: one unit of z_m in the DM always gets him the same q as χ_b/χ_m units of z_b. So $z_m^j > 0$ and $z_b^j > 0$ implies they must have the same return, after adjusting for pledgeability.

To further illustrate what can happen, suppose there are just two types: type-m buyers have $\alpha_b = \alpha_2 = 0$ and $\alpha_m > 0$; type-2 buyers have $\alpha_m = \alpha_b = 0$ and $\alpha_2 > 0$. It is clear that type-m hold money, $z_m^m > 0$, where the superscript indicates type, and we can assume wlog that they hold no bonds $z_b^m = 0$ – like sellers, they are indifferent to holding z_b when bonds are priced fundamentally and strictly prefer $z_b^m = 0$ when there is a liquidity premium. Thus, type-2 buyers hold all the bonds $z_b^2 = z_b/\mu_b > 0$ and maybe some cash $z_m^2 \geq 0$. For type-m

[30] The number of types does not matter here. Also, it is formally equivalent to have types determined randomly each period, as long as agents know their types before leaving the CM, so can tailor their portfolios appropriately. The idea is similar to Williamson (2012), but since his types are realized after the CM closes, they use banks to rebalance their portfolios – e.g., buyers that are more likely to have a type-m meeting take more cash out of the bank. While integrating banking into the theory is interesting, it is not necessary for the results here.

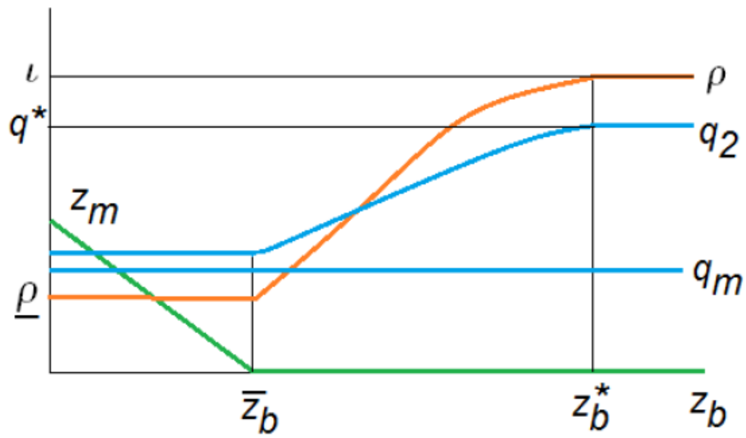

Figure 3.4: Random search bargaining equilibrium as a function of z_b

buyers, $\iota = \alpha_m \chi_m L(\chi_m z_m^m)$ determines real balances and $q_m = v^{-1}(\chi_m z_m^m)$, as in pure-currency economy, where there is no superscript on the q's since the subscript contains all the relevant information.

For type-2 buyers, there are three possibilities. If bonds are plentiful, in the sense $z_b \geq z_b^*$, where $\chi_b z_b^* / \mu_b = v(q^*)$, then $z_m^2 = 0$ and $q_2 = q^*$. If bonds are less plentiful, $z_b < z_b^*$, there are two cases. One has $z_m^2 = 0$ even though $q_2 < q^*$, and occurs if $z_b \geq \bar{z}_b$ where $\iota = \alpha_2 \chi_m L(\chi_b \bar{z}_b / \mu_b)$. In this case type-2 cannot get q^*, but he can get enough that it is not worth bearing the cost ι to top up his liquidity with cash. The other case has $z_m^2 > 0$, and occurs if bonds are relatively scarce, $z_b < \bar{z}_b$. In this case $\iota = \alpha_2 \chi_m L(\chi_m z_m^2)$, and this means total liquidity for type-2 is independent of z_b, because at the margin it is money that matters. As shown in Figure 3.4, z_b changes in z_b in $(0, \bar{z}_b)$ crowd out z_m one for one while nominal bond rate is stuck at the lower bound $\underline{\rho}$, which again can be positive, negative or 0.

Summarizing, type-m hold no bonds and get q_m. For type-2, as z_b increases in $(0, \bar{z}_b)$, z_m^2 is crowded out one for one until it hits 0 at $\bar{z}_b$. As z_b increases in $(\bar{z}_b, z_b^*)$, q_2 and ρ rise, so OMO's matter, until we hit z_b^* where type-2 are satiated in liquidity. In $(0, \bar{z}_b)$ we have a liquidity trap: swapping A_b for A_m affects ϕ_m

endogenously so that total liquidity stays the same, ρ is stuck at $\underline{\rho}$, and q is stuck at $\underline{q}$. As we said, in type-2 meetings z_m and z_b are perfect substitutes, so $z_m^2 > 0$ means they must have the same return, adjusting for pledgeability. This is similar to Wallace's (1981,1983) results in OLG economies, except (in our notation) he has $\alpha_m = \alpha_b = 0$ and $\alpha_2 = \chi_m = \chi_b = 1$, so money and bonds are always perfect substitutes. To clarify why there are two types here, we want some agents to go through different regimes as z_b increases, from using money and getting $q < q^*$, to not using money but still getting $q < q^*$, to eventually getting q^*. This is the role of type-2. But we do not want monetary equilibrium to collapse for $z_b \geq \bar{z}_b$, which is the role of type-m.

3.4 Directed Search

Now consider directed search and price posting. One reason is to check robustness. Another is that we want to communicate with some people predisposed to not like random search or bargaining.[31] And, it can deliver some interesting new phenomena. To clarify our method, we start with only money, then bring bonds back. Also, while pledgeability was an interesting part of the model in Section 3.3, for this extension we set $\chi_j = 1$.

3.4.1 Pure Currency

While there are different ways to do directed search, our approach appeals to agents called *market makers* that set up submarkets in the DM to attract buyers and sellers, who then meet bilaterally according to a standard matching technol-

[31] On random matching, Hahn (1987) opines "someone wishing to exchange his house goes to estate agents or advertises – he does not, like some *crazed-particle*, wait to bump into a buyer" (emphasis added). Similarly, Howitt (2005) says "when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer ... Few people would think of planning their economic lives on the basis of random encounters." On bargaining vs posting, Prescott (2005) says "the bilateral monopoly problem has been solved. There are stores that compete. I know where the drug store and the supermarket are, and I take their posted prices as given. If some supermarket offers the same quality of services and charges lower prices, I shop at that lower price supermarket." This vision of reality is reasonably close to the model presented below.

ogy.[32] Assuming constant returns, what matters is not the size of a submarket, but tightness, captured by the ratio of sellers to buyers σ. Let $\alpha(\sigma)$ be the probability a buyer meets a seller and $\alpha(\sigma)/\sigma$ the probability a seller meets a buyer, with the former increasing and the latter decreasing in σ. In the CM market makers post (q, z, σ) for the next DM, where traders commit to swapping q for z if they meet, and meetings are determined by σ.

Market makers design $(q, \hat{z}, \sigma)$ to maximize buyers' surplus subject to sellers getting a minimal surplus Π, or vice-versa, although in equilibrium surpluses are in fact dictated by the market. This problem reduces to

$$\max_{q,\hat{z},\sigma} \{\alpha(\sigma)[u(q) - \hat{z}] - \iota\hat{z}\} \text{ st } \frac{\alpha(\sigma)}{\sigma}[\hat{z} - c(q)] = \Pi, \tag{3.14}$$

which has a unique solution. Hence, all submarkets are the same – or, by constant returns, there can be just one. In either case, use the constraint to eliminate $\hat{z}$ and take FOC's wrt q and σ to get

$$\frac{u'(q)}{c'(q)} - 1 = \frac{\iota}{\alpha(\sigma)}, \tag{3.15}$$

$$\alpha'(\sigma)[u(q) - c(q)] = \Pi\left\{1 + \frac{\iota[1 - \varepsilon(\sigma)]}{\alpha(\sigma)}\right\}, \tag{3.16}$$

where $\varepsilon(\sigma) \equiv \sigma\alpha'(\sigma)/\alpha(\sigma) \in (0, 1)$. At this point, there are two approaches to closing the model.

First, suppose all agents participate in the DM for free, so that $\sigma = n$ (the equilibrium seller-buyer ratio in the representative submarket is given by the

[32] In Moen (1997) or Mortensen and Wright (2002), the motive of market makers is to charge entrance fees, which in equilibrium are 0. Often, it does not matter if instead of market makers sellers or buyers post (Faig and Huangfu 2007 give an exception, but it can be finessed as in Rocheteau and Wright 2005). Other money models with directed search are Lagos and Rocheteau (2005), Faig and Jerez (2006), Huangfu (2009), Dong (2011) and Dutu et al. (2011). None of these papers contains the analysis below.

population ratio). Then

$$\frac{\partial q}{\partial \iota} = \frac{c'}{\alpha u'' - (\alpha + \iota)\, c''} < 0, \text{ and } \frac{\partial q}{\partial n} = -\frac{\alpha'\,(u' - c')}{\alpha u'' - (\alpha + \iota)\, c''} > 0.$$

Also, at least if we assume that ε is constant, as in $\alpha(\sigma) = \sigma^{\varepsilon}$,

$$\begin{aligned}
\frac{\partial \hat{z}}{\partial \iota} &= \frac{\alpha \left\{u'c'\,[\alpha + \iota(1-\varepsilon)] - \varepsilon(1-\varepsilon)(u-c)\,[\alpha u'' - (\alpha+\iota)\, c'']\right\}}{[\alpha + \iota(1-\varepsilon)]^2\,[\alpha u'' - (\alpha+\iota)\, c'']} < 0 \\
\frac{\partial \hat{z}}{\partial n} &= \frac{\iota\alpha' \left\{\varepsilon(1-\varepsilon)\,(u-c)\,[\alpha u'' - (\alpha+\iota)\, c''] - u'c'\,[\alpha + \iota(1-\varepsilon)]\right\}}{[\alpha + \iota(1-\varepsilon)]^2\,[\alpha u'' - (\alpha+\iota)\, c'']} > 0.
\end{aligned}$$

The second approach gives one side, say sellers, a cost κ of entry. In equilibrium, assuming not all sellers participate, $\Pi = \kappa$ and σ is endogenous. Then

$$\frac{\partial q}{\partial \iota} = \frac{c'\alpha''(u-c)}{D} < 0, \text{ and } \frac{\partial q}{\partial \kappa} = -\frac{\alpha'[1 + \iota(1-\varepsilon)/\alpha](u'-c')}{D} < 0,$$

where $D = [\alpha u'' - (\alpha+\iota)\, c'']\,[\alpha''(u-c) + \iota\kappa(1-\varepsilon)\alpha'/\alpha^2] - \alpha'^2(u' - c')^2 > 0$ (while this is not true globally, in general, $D > 0$ in equilibrium by the SOC's; it is true globally close to the Freidman rule). Also, if ε is again constant,

$$\begin{aligned}
\frac{\partial \sigma}{\partial \iota} &= \frac{[\alpha u'' - (\alpha+\iota)c'']\kappa(1-\varepsilon)/\alpha - \alpha'(u'-c')c'}{D} < 0, \\
\frac{\partial \sigma}{\partial \kappa} &= \frac{[\alpha u'' - (\alpha+\iota)\, c''][1 + \iota(1-\varepsilon)/\alpha]}{D} < 0 \\
\frac{\partial \hat{z}}{\partial \iota} &= \frac{\kappa(1-\varepsilon)^2\,[\alpha u'' - (\alpha+\iota)\, c''] + c'^2\,[\alpha(u-c)\alpha'' - \iota\kappa(1-\varepsilon)\alpha']}{\alpha^2 D} < 0 \\
\frac{\partial \hat{z}}{\partial \kappa} &= -\frac{\iota\alpha' \left\{u'\,[\alpha + \iota(1-\varepsilon)] + \varepsilon(1-\varepsilon)c\,[\alpha u'' - (\alpha+\iota)\, c'']\right\}}{\alpha\,[\alpha + \iota(1-\varepsilon)]\, D} \gtrless 0.
\end{aligned}$$

In terms of efficiency, with or without entry, (3.15) implies $q = q^*$ iff $\iota = 0$. With entry, given iff $\iota = 0$, (3.16) implies $\alpha'(\sigma)[u(q) - c(q)] = \kappa$, which means $\sigma = \sigma^*$ where (q^*, σ^*) solves the planner's problem, $\max_{q,\sigma} \{\alpha\,(\sigma)\,[u\,(q) - c\,(q)] - \sigma\kappa\}$. By way of comparison, with bargaining as Section 3.3, but now additionally assuming $\kappa > 0$ so that not all sellers participate, we get

$$\frac{u'(q)}{v'(q)} - 1 = \frac{\iota}{\alpha(\sigma)} \tag{3.17}$$

$$\alpha(\sigma)\left[v(q) - c(q)\right] = \sigma\kappa. \tag{3.18}$$

In constrast to competitive search, with bargaining efficiency is not guaranteed. With Kalai bargaining, e.g., $q = q^*$ iff $\iota = 0$, and then $\sigma = \sigma^*$ iff $1 - \theta = \varepsilon(\sigma)$. The latter condition is the Hosios (1990) condition that says bargaining shares should equal the elasticity of matching wrt participation. Since directed search yields (q^*, σ^*) automatically at $\iota = 0$, it is sometimes said that it satisfies the Hosios condition endogenously.

To motivate the next part of the presentation, consider heterogeneity. With a fixed measure n_j of type j sellers per buyer, having fixed and variable costs κ_j and $c_j(q)$, equilibrium partitions into a submarket for each j. However, if any number of type j sellers can participate by paying κ_j, for generic parameters there is only one active submarket – e.g., if they all have the same $c(q)$, only those with lowest κ_j enter. If instead sellers are exogenously available, potentially multiple types participate – e.g., if the most efficient are few they are hard to meet, so equilibrium can allocate buyers to submarkets where they more easily find less efficient sellers. Thus, market tightness is endogenous even without entry, because equilibrium determines the measure of buyers that go to submarket j. We now pursue this type of model.

3.4.2 Money and Bonds

As in Section 3.3 there are in principle three submarkets – type-m, type-b and type-2 – but for simplicity here consider just two: *SM*, where only money is accepted; and *S2,* where both assets are accepted. As discussed above, with heterogenous sellers, rather than entry there is an exogenous measure n_j of type-j sellers, but tightness is still endogenous, since buyers go to submarket j with probability μ_j, and so $\sigma_j = n_j/\mu_j$. With two submarkets *SM* and *S2*, $\mu_m + \mu_2 = \mu$, where μ is assumed for now to be small enough that all buyers want to participate (see below). Also, as in Section 3.3.3, buyers going to *SM* bring only cash while buyers going to *S2* bring all the bonds, and maybe some cash. Thus, $\hat{z}^m_m > 0$ and $\hat{z}^m_b = 0$, while $\hat{z}^2_b > 0$ and $\hat{z}^2_m \geq 0$, where superscripts indicate the submarket in which buyers participate. Market clearing entails $\mu_m \hat{z}^m_m + \mu_2 \hat{z}^2_m = z_m$ and $\hat{z}^2_b = z_b$.

Since *SM* is the same as the pure-currency model, the same FOC's (3.17)-(3.18) determine (q_m, σ_m). In *S2*, the analog of (3.14) is

$$\max_{q_2, \hat{z}^2_m, \hat{z}^2_b, \sigma_2} \alpha(\sigma_2)\left[u(q_2) - \hat{z}^2_m - \hat{z}^2_b\right] - \iota \hat{z}^2_m - s\hat{z}^2_b \text{ st } \frac{\alpha(\sigma_2)}{\sigma_2}\left[\hat{z}^2_m + \hat{z}^2_b - c(q_2)\right] = \Pi_2.$$

Suppose first $\hat{z}^2_m > 0$. Since buyers going to *S2* bring both money and bonds, it must be that $\iota = s$. Hence, the problem only depends on $\hat{z}^2 = \hat{z}^2_m + \hat{z}^2_b$, making it the same as the *SM* problem. It follows that buyers are indifferent between the two submarkets iff $\Pi_2 = \Pi_m$. So, in equilibrium, $\Pi_2 = \Pi_m$, and hence $q_j = q$ and $\sigma_j = \sigma_2$ are the same in the two submarkets. While buyers in *S2* carry both assets and those in *SM* carry only cash, their total liquidity is the same: $\hat{z}^2_b + \hat{z}^2_m = \hat{z}^m_m = \hat{z}$, where

$$\hat{z} = \frac{\varepsilon(\sigma)\alpha(\sigma)}{\alpha + \iota\left[1 - \varepsilon(\sigma)\right]}u(q) + \frac{\left[1 - \varepsilon(\sigma)\right]\left[i + \alpha(\sigma)\right]}{\alpha + \iota\left[1 - \varepsilon(\sigma)\right]}c(q). \tag{3.19}$$

In this case $\partial q/\partial \iota < 0$, $\partial \hat{z}/\partial \iota < 0$, $\partial q/\partial n_j > 0$ and $\partial \hat{z}/\partial n_j > 0$ are identical to the pure-currency economy. Just like the case $\hat{z}^2_m > 0$ in Section 3.3.3, here OMO's are neutral: any change in z_b crowds out z_m with no impact on $\hat{z}$ or q.

Suppose instead $\hat{z}^2_m = 0$, which leads to

$$\frac{u'(q_2)}{c'(q_2)} = 1 + \frac{s}{\alpha(\sigma_2)} \tag{3.20}$$

$$\Pi_2\left[1 + \iota\frac{1 - \varepsilon_2(q_2)}{\alpha(\sigma_2)}\right] = \alpha'(\sigma_2)[u(q_2) - c(q_2)]. \tag{3.21}$$

Again there are two possibilities. One has z_b relatively abundant, so $q_2 = q^*$, and $s = 0$ because bonds have no liquidity value at the margin. Now, for buyers to be indifferent between the two markets, $s = 0 < \iota$ implies $\Pi_2 < \Pi_m$. From Rocheteau and Wright (2005, Lemma 5) σ is monotone in Π and hence $\sigma_2 < \sigma_m$. In this case, since bonds are not scarce, OMO's are again neutral. The remaining possibility has z_b less abundant, so $q_2 < q^*$ and $s \in (0, \iota)$, but again $\Pi_2 < \Pi_m$ and $\sigma_2 < \sigma_m$. In this situation OMO's matter: buyers in *S2* are constrained in the sense that $q_2 < q^*$ but still carry no cash, so changing z_b affects them.

As Figure 3.5 shows, for low z_b the two markets have the same (q, σ), and the economy is in a liquidity trap – OMO's do not affect ρ or the real allocation, which involves very low q. In this regime, total liquidity is independent of z_b again because at the margin it is money that matters, and increases in z_b crowd out z_m one for one. Moreover, the nominal rate on bonds is stuck at the lower bound, which here is $\underline{\rho} = 0$, given $\alpha_b = 0$ and $\chi_j = 1$. For intermediate z_b, both q_m and q_2 are increasing in z_b, so OMO's matter. They also affect tightness, with σ_m increasing and σ_2 decreasing in z_b. In contrast to Section 3.3, here OMO's affect all buyers. In particular, with random search, those who carry only money now have their q_m positively affected by increasing z_b. Moreover σ_m and hence α_m are positively, while σ_2 and hence α_2 are negatively, affected by z_b. This is due to directed search. In these models, buyers effectively choose to be type-m or type-2, and since they have to be indifferent, payoffs in the two submarkets are equated. Thus, there is more going on in Figure 3.5 than that in Figure 3.4.

As a special case, consider a Leontief matching function, $\alpha(\sigma) = \alpha_0 \min\{1, \sigma\}$, which can interpreted as not having search at all – instead we can simply say that the short side of the market always gets served while the long side gets rationed. Also, if we now assume the measure of buyers μ is large, then possibly not all participate in the DM: they participate in submarket j up the point where

$$\alpha_0 \min\{1, n_j\} \left[u(q_j) - \hat{z}_m^j - \hat{z}_b^j\right] - \iota \hat{z}_m^j - s \hat{z}_b^j = 0.$$

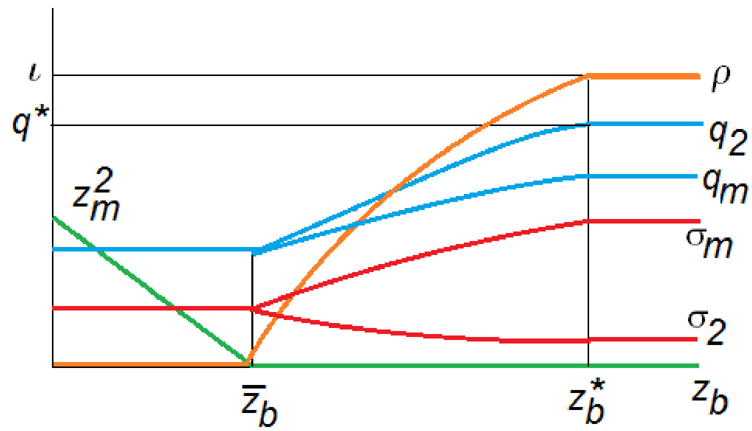

Figure 3.5: Competitive search (q_j, σ_j) as functions of z_b

If $\hat{z}_b^m = 0$ then $\sigma_m = \sigma_2 = 1$, so $\mu_j = n_j$ buyers go to each submarket, while the rest sit out. In *SM*, now $\alpha(\sigma_m) = \alpha_0$ and $z_m^m = \alpha_0 u(q_m)/(\iota + \alpha_0)$, due to Leontief matching. In *S2*, there are again two cases. One involves $z_m^b > 0$, $q_m = q_2 < q^*$, $s = \iota$ and occurs if bonds are scarce, $z_b/n_2 \le \alpha_0 u(q_m)/(\iota + \alpha_0)$. The other occurs if bonds are less scarce, and involves $\hat{z}_m^b = 0$, $z_b/n_2 = u(q_2)\alpha_0/(s + \alpha_0)$ and $s = \alpha_0 [u'(q_2)/c'(q_2) - 1] < \iota$. An increase in z_b raises q_2 and reduces s, but with this special matching technology q_m, σ_m and σ_2 do not depend on z_b.

3.4.3 Directed search with bargaining

For robustness, consider a blend of the previous models, with directed search but bargaining instead of posting. One can interpret this as a lack of commitment – agents know what payment instruments are accepted in the submarkets but the terms of trade cannot be guaranteed and will be negotiated in meetings, let us say using Kalai bargaining, $v(q_m) = \theta c(q_m) + (1-\theta)u(q_m)$. Now asset demands for buyers going to the two submarkets are

$$\iota = \alpha(\sigma_m)[\lambda(q_m) - 1] \text{ and } s = \alpha(\sigma_2)[\lambda(q_2) - 1]. \tag{3.22}$$

As usual there are various cases.

If the cost of holding bonds is less than the cost of holding money, $s < \iota$, then $z_m^2 = 0$. If $s = \iota$ then buyers are indifferent between money and bonds, so we can have $z_m^2 \geq 0$. If bonds are costly to hold, $s > 0$, then buyers only hold bonds that they intend to spend, which from market clearing implies $n_b z_b / \sigma_b = A_b$. If bonds are priced at their fundamental value then $s = 0$, $q_2 = q^*$ and $n_2 z_b / \sigma_2 \leq A_b$. Again buyers are indifferent between participating in the two markets. Hence,

$$-\iota z_m + \alpha(n_m) \left[u(q_m) - v(q_m) \right] = -s z_b + \alpha(n_2) \left[u(q_2) - v(q_2) \right], \tag{3.23}$$

where we use the fact that the payoff in *S2* is equal to the payoff if a buyer only holds bonds. Finally, the measure of buyers across markets is equal to μ,

$$\frac{n_m}{\sigma_m} + \frac{n_2}{\sigma_2} = \mu. \tag{3.24}$$

Suppose $\rho = 0$, $s = \iota$ and $z_m^2 \geq 0$. As buyers in *S2* are indifferent between holding money and bonds, (3.23) implies $\alpha(\sigma_2) = \alpha(\sigma_m)$, and 3.24) implies $\sigma_m = \sigma_2 = 1/\mu$. Then $q_m = q_2 = q$ where $\iota = \alpha(1) \left[\lambda(q) - 1 \right]$. As z_b and z_m are perfect substitutes in this case, the two submarkets are essentially identical. An increase in z_b does not affect output but simply crowds out real balances in *S2*. The equilibrium exists iff $\iota \leq \alpha(1)\theta/(1-\theta)$ and $z_b \leq \underline{z}_b = n_2 v(q)$. Bonds are scarce so buyers in the type-2 market compete with buyers in the type-m market for money, which drives the nominal interest rate on liquid bonds to $\rho = 0$.

Now suppose liquidity is abundant in *S2*, so $s = 0$. Then $q_2 = q^* > q_1$. A change in z_b is neutral. The distribution of buyers across submarkets solves

$$\alpha(\sigma_2) = \frac{-\iota v(q_m) + \alpha(\sigma_m) \left[u(q_m) - v(q_m) \right]}{u(q^*) - v(q^*)}.$$

Hence, $\sigma_2 \leq \sigma_m$ with a strict inequality if $\iota > 0$. In order for buyers to be indifferent, they must trade with a lower probability in *S2*. This equilibrium exists iff $z_b \geq z_b^* \equiv n_2 v(q^*)/\sigma_2$.

Finally consider an equilibrium with $0 < s < \iota$ and $z_m^2 = 0$. Then $v(q_2) = \sigma_2 A_b/n_2$ and buyer indifference implies

$$
\begin{aligned}
-\iota z_m + \alpha(\sigma_m)\left[u(q_m) - v(q_m)\right] &= -s z_b + \alpha(\sigma_2)\left[u(q_2) - v(q_2)\right] \\
&> -\iota z_m + \alpha(\sigma_2)\left[u(q_2) - v(q_2)\right].
\end{aligned}
$$

This implies $\alpha(\sigma_m) > \alpha(\sigma_2)$. Since bonds pay interest, they are less costly to hold than money and hence the probability of trade *SM* is larger than that in *S2*. From (3.22) and (**??**) $q_m < q_2$. The aggregate demand for bonds, $n_2 v(q_2)/\sigma_2$, increases as s decreases. Hence, an increase in z_b reduces s and σ_2 and raises σ_m, q_m and q_2. This equilibrium exists iff $z_b \in (\underline{z}_b, z_b^*)$. The outcome therefore looks qualitatively the same as In Figure 3.5.

3.5 Endogenous Liquidity

So far we have taken α_j and χ_j as exogenous. Can these arise as endogenous restrictions generated by the environment, including properties of assets and informational frictions? The literature has proposed different ways to answer this,[33] but several papers appeal to private information about the quality of assets. Lester et al. (2012) and references therein assume some agents cannot distinguish high- and low-quality versions of certain assets. If the latter have no value – e.g., they are pure counterfeit claims, or genuine claims to worthless assets – and can be produced on the spot for free, agents that cannot recognize certain them simply reject them outright. This allows the use of standard bargaining over assets that are recognized, since those that are not recognized are off the table, and explains how the α_j's can be rationalized. The same approach works here.

To see how it works, let n_m and n_2 be the measure of sellers that recognize

[33] See the surveys mentioned in fn. 1. One approach in, e.g., Zhu and Wallace (2007) and Nosal and Rocheteau (2013), has liquidity constraints emerging from pairwise-optimal trading mechanisms that treat assets asymmetrically. As Hu and Rocheteau (2013,2014) show, under some conditions this can be socially optimal. We instead take an information-based approach.

money and recognize both assets, and for simplicity consider the Leontief matching function discussed above. When buyers can produce counterfeits for free any time they wish, sellers never accept assets they cannot recognize. All sellers recognize money, they recognize and hence accept bonds iff they invest each period in information, or a technology, with seller-specific cost κ. Denote the distribution of costs across sellers by $F(\kappa)$, and let $\bar{\kappa}$ be the cost that makes a seller indifferent between investing or not, satisfying

$$-c(q_2) + \frac{\alpha_0 u(q_2)}{s+\alpha} - \bar{\kappa} = -c(q_m) + \frac{\alpha_0 u(q_m)}{\iota+\alpha}.$$

Then $n_2 = F(\bar{\kappa})$. Lester et al. (2012) show that it is easy to get multiple equilibria with n_2 big or small, fragile outcomes where small changes in parameters cause jumps in n_2, or hysteresis in liquidity, where once n_2 increases it is less likely to change back. However, it is not obvious the same effects would happen with competitive search, because market-makers can potentially coordinate agents' actions; we leave further exploration to future work.

A related approach is used in Rocheteau (2011), where pledgeability constraints emerged from adverse selection, and Li et al. (2012), where they emerge from moral hazard. Below we adopt the hidden-action formulation in Li et al. (2012) by assuming agents can produce counterfeit money or nominal bonds at costs proportional to their value, $\nu_m \phi_m$ and $\nu_b \phi_m$.[34] Different from Lester et al. (2012), all sellers are uninformed, and the decision to produce fraudulent assets is made in the CM, before agents meet in the DM, which implies it cannot be conditioned on the information of the seller. Thus, sellers that do not recognize an asset's quality may still accept it in the DM, although only up to some limit, endogenizing χ_j. Also, it is assumed that while fraudulent assets may trade in the DM, they are authenticated and confiscated in the following CM (an

[34]We also consider fixed costs but proportional costs simplify a few expressions. Also, while one can interpret this literally as counterfeiting, more generally the idea is that agents can take hidden actions to avoid transfering genuine value in DM payments.

assumption borrowed from Nosal and Wallace 2007). Hence, fraudulent assets have zero value to those accepting them in the DM. Given this, we start with a model where OMO's do not matter; then present a version where they do.

3.5.1 Endogenous Pledgeability

In order to use standard equilibrium concepts for signaling games with endogenous types, assume $\theta = 1$ (buyer-take-all bargaining in the DM). Moreover, for now suppose that all matches are type-2, $\alpha_2 = \alpha > 0 = \alpha_m = \alpha_b$, and consider nominal bonds (real bonds are similar). Let d_j denote a nominal asset transfer from buyer to seller. As in Li et al. (2012), the *resalability constraint* on d_j takes the form

$$(\phi_{j,-1} - \beta\phi_m)\, a_j + \beta\alpha_2\phi_m d_j \leq \nu_j \phi_m a_j. \tag{3.25}$$

The RHS is the cost of producing a_j fraudulent units of asset j; the LHS is the cost of investing in a_j genuine units, $(\phi_{j,-1} - \beta\phi_m)a_j$, plus the cost of transferring d_j to a seller, which occurs here with probability α_2.[35] Sellers rationally believe that buyers would not produce fraudulent assets when d_j is less than the upper bound implied by (3.25), depending on returns $\phi_{j,-1}/\phi_m$, trading frictions α_2, discounting β, and the cost ν_j. When $a_j > d_j$, there is asset retention, or over-collateralization, which can be a way to signal quality.

A buyer has several constraints in DM trade. Given $\theta = 1$, bargaining implies $p_2 = c(q_2)$, where $p_2 = \phi_m(d_m + d_b)$. However, even in type-2 meetings, what now matters is not just the total payment p_2, but the components, which in real terms are denoted $\tau_m = \phi_m d_m$ and $\tau_b = \phi_m d_b$. Feasibility requires $\tau_m \leq z_m$ and $\tau_b \leq z_b$.

[35] See the unpublished Appendix B, Equation (38), of Li et al. (2012). The resalability constraint is derived in the context of a model of securities backed with genuine and fraudulent bonds, where the cost of fraud has both a fixed and a proportional component. For now we set the fixed cost to 0.

Potentially more stringent in (3.25), which can be rearranged as $\tau_j \leq \chi_j z_j$ where

$$\chi_m = \frac{\nu_m - \beta\iota}{\alpha_2\beta} \text{ and } \chi_b = \frac{\nu_b - \beta s}{\alpha_2\beta} \tag{3.26}$$

This endogenizes the pledgeability parameters χ_m and χ_b, depending on the costs ν_m and ν_b, as well as the policy instrument ι and the endogenous spread s. As trading frictions are reduced – higher α_2 – pledgeability falls because fraud is more tempting when there are more opportunities to pass bad assets. If ι increases then χ_m falls because it more tempting to counterfeit money; similarly for s and χ_b. Hence, pledgeability in general is not invariant to policy or other changes in fundamentals.

Case 1: Suppose $\chi_j \in (0,1)$. Given all meetings are type-2, the analogs of (3.8) and (3.9) are

$$\iota = \frac{\nu_m - \beta\iota}{\beta}\lambda(q_2) \text{ and } s = \frac{\nu_b - \beta s}{\beta}\lambda(q_2), \tag{3.27}$$

where $\lambda(q) = u'(q)/c'(q) - 1$ since $\theta = 1$. Notice the arrival rate α_2 cancels with α_2 in the the denominator of (3.26). We determine q_2 from the first condition; it is decreasing in ι and increasing in ν_m, but independent of α or ν_b. We then solve the second condition for $s = \iota\nu_b/\nu_m$, which gives the spread in terms of policy and the relative cost of fraud. Also, the nominal rate on bonds is

$$\rho = \frac{(\nu_m - \nu_b)\,\iota}{\nu_m + \nu_b\iota}, \tag{3.28}$$

which once again can be negative.[36] Note that OMOs affect neither q_2 nor ρ here. Finally, to close this case, conditions on parameters that make $\chi_j \in (0,1)$

[36] Again the Fisher theory that ρ moves one-to-one with inflation fails – indeed, the elasticity of ρ wrt ι is $\nu_m/(\nu_m + \nu_b\iota)$. Also, if $\nu_b \to 0$ then bonds become illiquid and $\rho \to \iota$, and if $\nu_b \to \nu_m$ then $\iota \to 0$.

an equilibrium are

$$0 < \frac{\nu_m - \beta\iota}{\alpha\beta} < 1 \text{ and } 0 < \frac{\nu_b}{\nu_m}\left(\frac{\nu_m - \beta\iota}{\alpha\beta}\right) < 1.$$

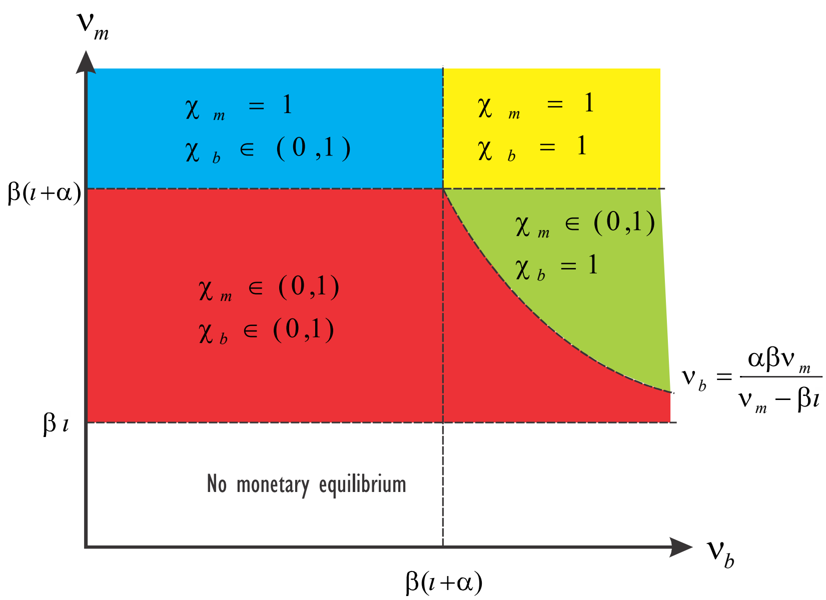

Figure 3.6: Types of equilibria under endogeneous pledgeability

Case 2: Suppose $\chi_m = 1$, which requires $\nu_m > (\iota + \alpha)\beta$, and $\chi_b \in [0, 1)$. Then (3.26) becomes

$$\iota = \alpha_2\lambda(q_2) \text{ and } s = \left(\frac{\nu_b - \beta s}{\beta}\right)\lambda(q_2). \tag{3.29}$$

This gives $s = \iota\nu_b/\beta(\alpha + \iota)$. Then q_2 is determined as in a pure currency economy from $\iota = \alpha_2\lambda(q_2)$. Now q increases with α_2, while s increases with ι and ν_b but decreases with α_2. In this case, the nominal rate on bonds is

$$\rho = \frac{(\alpha_2 + \iota)\beta - \nu_b}{(\alpha_2 + \iota)\beta + \iota\nu_b}\iota, \tag{3.30}$$

but again OMOs affect neither q nor ρ. Finally, the condition for $\chi_b \in [0, 1)$ to be an equilibrium is

$$\nu_b < (\alpha + \iota)\,\beta. \tag{3.31}$$

One can similarly consider the regime where $\chi_b = 1$ and $\chi_m \in [0,1)$, or the one with $\chi_m = \chi_b = 1$. Figure 3.6 displays where the different equilibria exist in $(\nu_b, \nu_m)-$space. This illustrates how pledgeability depends on trading frictions as measured by α, as well as policy as measured by ι. But in all of these equilibria OMOs do not matter.

3.5.2 Relevant OMOs

We now assume that ν_m is sufficiently large that money can be fully accepted in all matches. In contrast, bonds cannot be traded in a fraction α_m of matches, let say, because the technology to transfer the ownership of bonds is not available in those matches. Bonds can be traded in α_2 matches but they are still subject to the threat of counterfeiting. Assuming bonds are partially acceptable, the buyer's FOCs are (3.8) with $\chi_m = 1$ and (3.9) with $\chi_b = (\nu_b - \beta s)/\alpha_2\beta$, i.e.,

$$\iota = \alpha_m \lambda(q_m) + \alpha_2 \lambda(q_2) \tag{3.32}$$

$$s = \left(\frac{\nu_b - \beta s}{\beta}\right)\lambda(q_2). \tag{3.33}$$

From the seller's participations constraints, $c(q_m) = z_m$ and $c(q_2) = z_m + \tau_b$, the incentive-compatibility condition, $\tau_b = \chi_b z_b$, and the market-clearing condition, $z_b = A_b\phi_m = Bz_m$, we obtain

$$c(q_2) = \min\left\{c(q_m)\left[1 + \left(\frac{\nu_b - \beta s}{\alpha_2\beta}\right)B\right], c(q^*)\right\}.$$

Substituting s by its value given by (3.33), i.e.,

$$s = \frac{\nu_b \lambda(q_2)}{\beta\left[1 + \lambda(q_2)\right]},$$

we obtain

$$c(q_2) = \min \left\{ c(q_m) \left[1 + \frac{\nu_b B}{\alpha_2 \beta \left[1 + \lambda(q_2) \right]} \right], c(q^*) \right\}. \tag{3.34}$$

An equilibrium can be reduced to a pair (q_m, q_2) that solves (3.32) and (3.34) and the condition for the partial acceptability of bonds,

$$\frac{\nu_b}{\alpha_2 \beta \left[1 + \lambda(q_2) \right]} < 1.$$

In the case where bonds are fully accepted then $s = \alpha_2 \lambda(q_2)$ and $c(q_2) = \min \{ c(q_m) (1 + B), c(q^*) \}$. Condition (3.32) gives a negative relationship between q_m and q_2. This negative relationship captures the endogenous response of real balances to a change in the value of bonds. The relationship between q_2 and q_m as given by (3.34) can be non-monotonic. Intuitively, q_2 depends on the pledgeability of bonds, which depends on the spread s, which itself is a function of q_2. As q_2 increases, $\lambda(q_2)$ decreases, which reduces the spread, s, and makes bonds more pledgeable, which in turn tends to raise q_2. As a consequence, the steady state does not have to be unique.

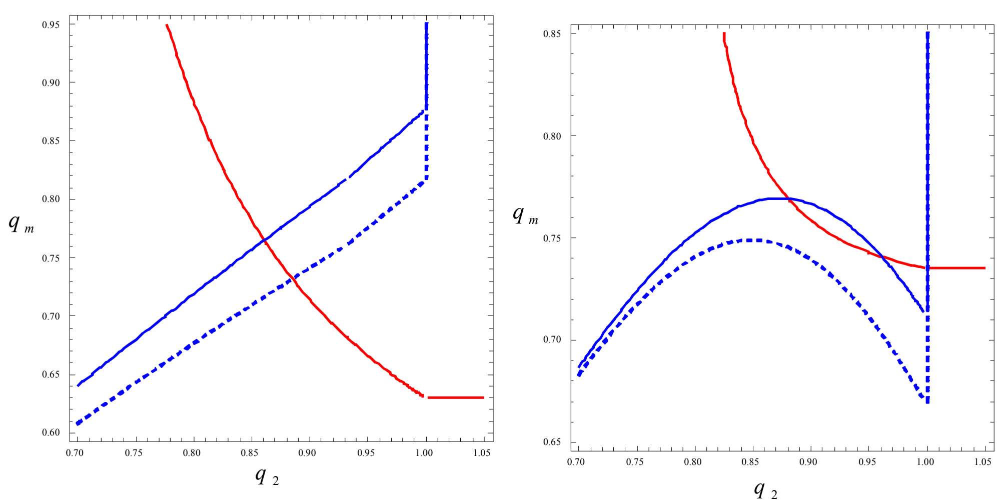

Figure 3.7: Equilibria with proportional cost of fraud

In Figure 3.7 we illustrate the determination of the equilibrium. The condition (3.32) is represented by the downward-sloping red curves while the condition

(3.34) is represented by blue curves. We adopt the following functional forms and parameter values: $c(q) = q^{1+\delta}/(1+\delta)$, $u(q) = q^{(1-a)}/(1-a)$, $\beta = 0.9$, $\nu_b = 1$. In the left panel $\iota = 0.1$, $a = 0.5$, $\delta = 1$, $\alpha_m = 0.1$, $\alpha_2 = 0.2$, $\nu_b = 0.2$, $B = 0.3$. For this parametrization the equilibrium is unique. An increase in B from 0.3 to 0.5 (the dashed blue curve) leads to an increase in q_2 but a decrease in q_m. In the right panel, $\iota = 3$, $a = 8$, $\delta = 1$, $\alpha_m = 0.2$, $\alpha_2 = 0.5$, $\nu_b = 0.4$, $B = 1.1$. In this case there is a multiplicity of steady-state equilibria. This multiplicity is new and it is due to the pecuniary externality working through the endogenous pledgeability constraints. Indeed, if agents believe that χ_b is small, then q_2 is small, $\lambda(q_2)$ is large, and the spread on bonds is large. But because bonds are costly to hold, the incentive to commit fraud is high, which makes χ_b small, in accordance with the initial belief. In our example there are two equilibria where bonds are partially acceptable and one equilibrium where bonds are fully acceptable and $q_2 = q^*$. An increase in the supply of bonds eliminates all equilibria except the one where bonds are fully acceptable.

3.5.3 Different Regimes

So far we have assumed that costs of fraud are proportional to the real value of the assets in order to provide microfoundations for the pledgeability constraints used in the literature. In order to illustrate the role played by this assumption, we now assume that the costs of producing fraudulent assets are fixed. For simplicity, only bonds can be counterfeited at a positive fixed cost $\nu_b > 0$, while fiat money is perfectly recognizable.[37] This setup can be justified by the observation that the transfer of bonds necessitates information about the identity and account of the bond holder, and identity theft is more prevalent than counterfeiting fiat currency.

[37] In Li et al. (2012) we discuss the general case where multiple assets are subject to counterfeiting in a two-period model.

From Li et al. (2012), the no-counterfeiting constraint (3.25) takes the form:

$$\left(\phi_{b,-1} - \beta\phi_m\right) d_b + \beta\alpha\phi_m d_b \leq \nu_b. \tag{3.35}$$

The left side is the cost from accumulating genuine bonds. The first term is the cost of holding genuine bonds. It is the difference between the price of newly issued bonds, $\phi_{b,-1}$, and the discounted price of the bonds at maturity, $\beta\phi_m$. The second term on the left side is the discounted value of the transfer of bonds should a trade occur in the following DM. The right side of the inequality is the fixed cost of accumulating fraudulent bonds. Because of the fixed cost of fraud, there is no signaling value from holding more assets than one spends and hence buyers will not hold more bonds than what they intend to spend, $a_b = d_b$, unless $s = 0$. The no-fraud constraint, (3.35), can be rearranged as

$$\tau_b \leq \frac{\nu_b}{\beta\left(s + \alpha\right)}, \tag{3.36}$$

where $\tau_b = \phi_m d_b$. The pledgeability constraint takes the form of an upper bound on the transfer of bonds in a match. In contrast to the previous section this bound is no longer proportional to the buyer's asset holdings. It is endogenous and it depends on the cost of fraud, ν_b, the acceptability of bonds, α, and the endogenous spread, $s = \left(\iota - \rho\right)/(1 + \rho)$. So if bonds are more costly to hold, because the nominal interest rate is low relative to ι, then the pledgeability constraint is more severe.

The buyer's problem is the standard complete-information problem subject to the pledgeability constraint, (3.36). In any monetary equilibrium the output traded in the DM solves (3.8) with $\alpha_2 = \alpha$ and $\chi_m = 1$, i.e.,

$$\iota = \alpha\lambda(q), \tag{3.37}$$

where $\lambda(q) = u'(q)/c'(q) - 1$. So in this setting open-market operations are neutral. We discuss next the determination of asset prices. Suppose first that the pledgeability constraint is not binding so that money and bonds are perfect substitutes as means of payment. It follows that $s = \iota$ and $\rho = 0$, the nominal rate is 0 so that money and bonds have the same rate of return. This "liquidity-trap" regime occurs when

$$\frac{B}{1+B} \leq \frac{\nu_b}{c(q)\beta\left(\iota+\alpha\right)}, \tag{3.38}$$

which is shown by Figure 3.8. So provided that bonds are sufficiently scarce, the nominal interest rate is 0. The value of money is

$$\phi_m = \frac{c(q)}{A_m + A_b}. \tag{3.39}$$

Moreover, a change in B has no effect on asset prices.

Consider next the regime where bonds are not costly to hold, $s = 0$ and $\rho = \iota$. There is no spread between the rate of return of liquid and illiquid bonds. The pledgeability constraint, (3.36), is binding since otherwise agents would trade q^* and fiat money would not be valued. So

$$\tau_b = \frac{\nu_b}{\beta\alpha} \tag{3.40}$$

$$z_m = c(q) - \frac{\nu_b}{\beta\alpha}. \tag{3.41}$$

From (3.41) aggregate real balances are independent of the supply of bonds but they depend negatively on the cost of fraud for bonds. This regime occurs when

$$\frac{B}{1+B} \geq \frac{\nu_b}{\beta\alpha c(q)}. \tag{3.42}$$

This regime occurs if the supply of bonds is sufficiently abundant.

Finally, there is a last regime where the pledgeability constraint, (3.36), binds

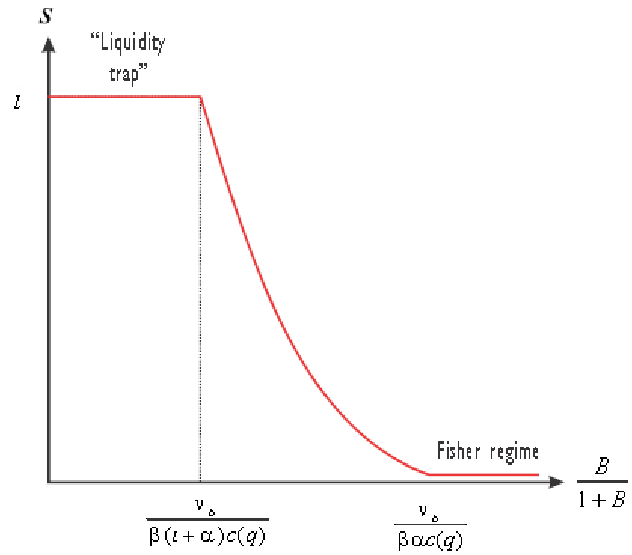

Figure 3.8: Spread and the Supply of Bonds

and bonds are costly to hold, $s > 0$, so that $d_b = A_b$. In this case

$$s = \frac{\nu_b}{\beta c(q)}\left(1 + \frac{1}{B}\right) - \alpha \tag{3.43}$$

$$z_m = \frac{c(q)}{1+B}. \tag{3.44}$$

From (3.43) an increase in the supply of bonds raises the nominal interest rate on bonds and reduces the spread with illiquid assets. From (3.44) it reduces the real value of money. This regime occurs when

$$\frac{\nu_b}{(\iota+\alpha)\,\beta c(q)} < \frac{B}{1+B} < \frac{\nu_b}{\alpha\beta c(q)}. \tag{3.45}$$

3.6 Conclusion

We build models with money and government bonds in the framework of New Monetarism, to study monetary policy, particularly OMOs. We start from the model with random search and bargaining in decentralized markets. Under various scenarios for the liquidity of money and bonds, different sellers may accept one or the other or both. We consider real and nominal bonds. We derive the effects of increases in the growth rate of the money supply, which is the same as inflation in stationary equilibrium. As is true in most reasonable models, changing the level of the money supply is neutral, even though changing the growth rate is not. Given this, an OMO is effectively the same as simply changing the stock of outstanding bonds: the change in the bond supply may or may not affect real variables, but the accompanying change in the money supply affects only nominal prices. The theory delivers sharp predictions for the effects of these policies. It also generates some novel phenomena, like the possibility of negative nominal interest rates on government bonds, and outcomes that resemble liquidity traps.

To check the robustness of the results, we then study the model with price posting and directed search, instead of bargaining and random search. Again, the theory delivers sharp predictions. It can also generate an endogenous partition of the market into submarkets where different instruments are used to facilitate exchange. Later we seek explanation why assets like money and bonds may have different liquidity based on information-based theory. This can generate endogenous outcomes where some sellers simply do not accept certain assets, or accept them only up to an endogenous threshold.

For future research, firstly we will continue the work on different liquidity of money and bonds in the directed search approach. We may also add private assets to the model, so that we can use the model to address unconventional monetary policy, particularly private assets purchase of the central banks since the Global Financial Crisis. Based on this framework, We may also further address the effects of unconventional monetary policy on the real economy (output and unemployment), which then echoes and complements my work in Chapter Two.

3.7 Special Acknowledgment

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