Structuring Cellular Automata Rule Space with Attractor Basins

Dave Burraston

Complex systems such as Cellular Automata (CA) produce global behaviour based on the interactions of simple units (cells). Their evolution is specified by local interaction rules that generate some form of ordered, complex or chaotic behaviour. This wide variety of behaviour represents an important generative tool for the artist. Chaotic behaviour dominates rule space, which has serious implications for the serendipitous use of these systems in artistic endeavour. A fresh insight into a recognised key problem, the structure of rule space, is presented based on empirical evidence. This provides a method for creating groups of rules with a broad range of behaviour for application within generative arts practice and will also be of interest to scientific practitioners.

1. Introduction

> On the phono-scales a common or garden F sharp gave a reading of 93 kilogrammes. It issued from a decidedly large tenor whose weight I took. (Erik Satie 1912)

The different classes of behaviour that CA produce, whether ordered, complex or chaotic, make them interesting to artists and scientists alike. They are fascinating objects, producing more pattern than a single human is capable of observing within their own lifetime. Stephen Wolfram has proposed twenty key problems in the theory of CA (Wolfram 1985), the seventh problem asks : *How is different behaviour distributed in the space of cellular automaton rules?* The structure of the elementary rule space was examined by Li and Packard, where their aim was to show inter and intra behaviour class connections (Li and Packard 1990). The approach taken in this paper and described in section 3 is different, providing fresh insight into rule space structure.

Within the domain of generative music access to a variety of behaviour is essential. CA have played a key part in generative music for many years (Burraston, Edmonds, Livingstone and Miranda 2004) (Burraston and Edmonds 2005). Reflective practice has also been utilised to investigate and describe generative music (Burraston and Edmonds 2004) (Burraston 2005a, 2005b & 2005c), and also a precursor to the work described in this paper (Burraston 2005d). Schon suggests research on *fundamental methods of inquiry and overarching theories* are a key part of reflective practice (Schon 2003). These methods and theories are the springboards for making sense of new situations. It has been suggested that rule choice is a fundamental process in generative music composition with CA (Burraston 2005a). Using recent theory as a foundation methodology, rule groups are created by considering the CA state space as a symbolic representation of rule space. A detailed example exposes the main process of creating a rule space structure. Applying this method to larger rule spaces demonstrates an ability to create reasonable diversity from such a random behaviour space. The approach taken in this paper may provide an interesting and alternative method of studying CA rule spaces in general.

2. Brief Overview of CA

Complex systems such as CA produce global behaviour based on the interactions of simple units. CA were conceived by Stanislaw Ulam and John von Neumann in an effort to study the process of reproduction and growths of form (Burks 1970). The concept of the universe as a type of CA computer was introduced by Konrad Zuse and termed *Calculating Space* (Zuse 1969). Here Zuse poses the controversial question : "Is nature digital, analog or hybrid?". Ed Fredkin, a long standing CA scientist, is convinced that the universe is digital (grainy) and has a developed his own "Digital Philosophy" termed "Finite Nature" (Fredkin 1992). Fredkin believes that the digital mechanics of the universe is much like a CA, deterministic in nature but computed with unknowable determinism. Space and time in this view are discrete quantities, everything is assumed to be grainy. Some important CA concepts are now reviewed to give a sufficient background for the purpose of this paper.

CA are dynamic systems in which space and time are discrete. They may have a number of dimensions, single linear arrays or two dimensional arrays of cells being the most common forms. The CA algorithm is a parallel process operating on this array of cells. Each cell can have one of a number of possible states. The simultaneous change of state of each cell is specified by a local transition rule. The local transition rule is applied to a specified neighbourhood around each cell. CA were classed by Stephen Wolfram with one of four behaviours (Wolfram 1984).

Class 1 : Patterns disappear with time or become fixed.
Class 2 : Patterns evolve to a fixed size with periodic structures cycling through a fixed number of states.
Class 3 : Patterns become chaotic.
Class 4 : Patterns grow into complex forms, exhibiting localized structures moving both spatially and temporally.

Other methods of behaviour classification have been devised, an example of six categories is given in (Li, Packard and Langton 1990). It is known that it is undecidable to assign a CA to a Wolfram class (McIntosh 1990) and there are several attempts at behaviour prediction parameters, five of which have been surveyed in (Oliveira, Oliveira and Omar 2001). The relatively rare complex behaviour of class 4 was suggetsed to occur at a "phase transition" between order (class 1 and 2) and chaos (class 3), termed the "edge of chaos" (Langton 1991). The concept of the "edge of chaos" and efficacy of Langton's "Lambda" prediction parameter has also been critically re-examined in (Mitchell, Crutchfield and Hraber 1993).

The global dynamics of CA (Wuensche & Lesser 1992) offers a different perspective based on the topology of attractor basins, rule symmetry categories and rule clustering. Wuensche's Discrete Dynamics Lab (DDLab) software allows for the exploration of global dynamics (Wuensche 2005), as well as many other important aspects of CA and related discrete networks. The number of cells in a CA is termed the system size and is represented by **L**. The number of state values is represented by **v** and the number of cells in the neighbourhood by **k**. This differs slightly from Wolfram's numbering scheme of **k** as the number of states and **r** as the neighbourhood radius (Wolfram 1984). Wuensche's numbering scheme allows for even numbers of cells within a neighbourhood. This is useful for considering CA as part of the wider field of random Boolean and intermediate networks.

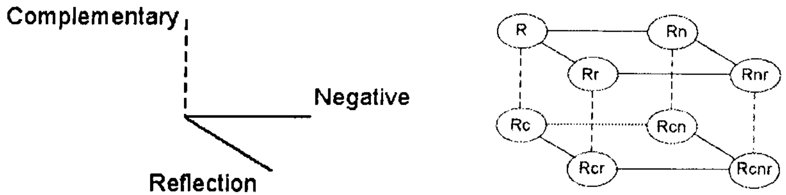

Figure 1 - Rule cluster axis (left) and layout (right)

The 1D v2k3 rules can be grouped into 88 equivalence rule classes, by **negative**, **reflective** and composite **negative reflective** transforms, with a maximum of four rules per equivalence class (Walker & Aadryan 1971). With the addition of a **complement** transform a total of 48 rule clusters can be formed from three basic rule table transformations (Wuensche & Lesser, 1992). The rule cluster axis is shown in Figure 1 (left) and the rule cluster layout is shown in Figure 1 (right). The lowest rule number (**R**) identifies a cluster and is always positioned in the top left corner. The negative (**Rn**) and reflection (**Rr**) transforms are identified along with a composite transform, the negative reflection (**Rnr**). The complement transform (**Rc**) also has corresponding negative (**Rcn**), reflection (**Rcr**) and negative reflection (**Rcnr**) transforms. Rule symmetry categories are formed by the reflection transformation. This groups rules with related symmetric properties in their spacetime behaviour into symmetric (**S**), semi-asymmetric (**SA**) and fully asymmetric rules (**FA**). The symmetry categories divide elementary rule space in the following proportions : S = 1/4, SA = 1/2 and FA = 1/4.

Wuensche has importantly shown that as the neighbourhood size k is increased beyond 5, the proportion of chaotic rules rises very sharply in a random sampling of rules (Wuensche 1997). This has serious implications for the serendipitous use of these systems in artistic endeavour. The magnitude of the numbers of rules is extremely large, Wentian Li (1989a) has commented on the 5 neighbour rules :

> "Even if we can produce a spatial-temporal pattern from each rule in 1 second, it is going to take about 138 years to run through all the rules. Considering the redundancy due to equivalence between rules upon 0-to-1 transformations, which cut the time by half, it still requires a solid 69 years."

Table 1 - Total number of CA rules for v2k2 to v2k7 of one dimension.

Rule type (vnkn)	**Total number of rules**
v2k2	2^(2^2) = 16
v2k3	2^(2^3) = 256 (the elementary rules)
v2k4	2^(2^4) = 65536
v2k5	2^(2^5) = 4,294,967,296
v2k6	2^(2^6) = 1.844674407370955e+19
v2k7	2^(2^7) = 3.402823669209385e+38

The total number of CA rules is a function of the number of states and the size of the neighbourhood. The v2k3 CA amount to a total of 256 rules, and are known as the elementary rules (Wolfram 1983). Table 1. shows a summary of the total number of rules for 1D binary CA with neighbourhoods of 2 to 7 cells. As the neighbourhood is increased there is astronomic increase in the total number of rules. To further compound matters the size of the digits specifying the transition rule number itself becomes very unwieldy. For example, say the left most digits of a 15 neighbour rule expressed as a hexadecimal number are :

c2dc0648578781fa0a07a05b40015c645e028d23a5bc64418d14019615c

the remaining digits for this rule number would take up 3 pages of A4!

This section will now review attractor basin theory to give sufficient background for the following sections. The state space of a CA consists of all possible global states. In a finite deterministic CA all state transitions must eventually repeat with period 1 or more. States are either part of an attractor cycle or lie on a transient leading to the attractor cycle. If a transient exists there will be states unreachable by any other states at the extremity. These extremities are called garden of Eden (**goE**) states. All transients leading to an attractor, and the

attractor cycle, is termed the basin of attraction (**boa**) of that individual attractor. An example basin of attraction is shown in Figure 2. State space for a particular CA rule and size is populated by one or more basins of attraction, termed the basin of attraction field. The boa field may contain equivalent basins or subtrees, where the states are rotationally symmetric.

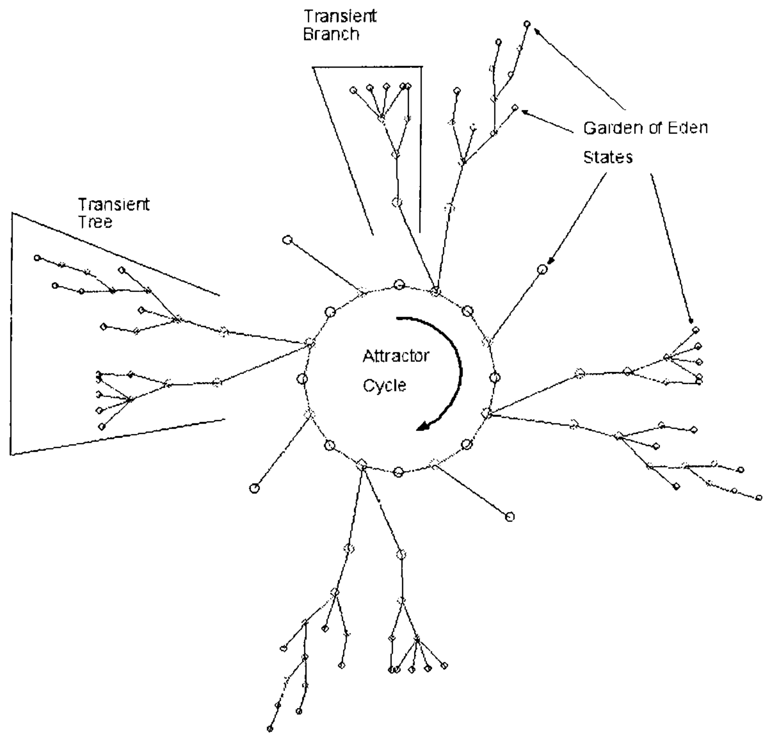

Figure 2 - Basin of attraction

Each elementary rule cluster has two boa fields, one for the top layer (**R**), and a different, but garden of Eden related field for the complementary bottom layer (**Rc**) if it exists. Rule clusters contain either 2, 4 or 8 different rules depending on whether the transformations result in the same rule number. For both layers the other transformed rules have identical boa measures and the states in spacetime are simply related by being negative (**Rn**), mirror image (**Rr**) or both (**Rnr**). For a deeper understanding of these concepts the reader is strongly advised to study the literature, in particular Wuensche and Lesser's book, now freely available on the internet (Wuensche and Lesser 1992).

3. Rule Space Structures from Attractor Basins

If we consider state space to be a symbolic representation of rule space, then a basin of attraction field can be interpreted as an example of the self-organisation of rule space. It follows that this organisation of rule space will reflect the properties of the chosen CA for a specific system size. The requirements for a one to one relationship between system size L and v2kn rule space is shown in Table 2.

Table 2 - System size (L) required to represent v2kn rule spaces.

k = 2	k = 3 (elementary)	k = 4	k = 5	... k
L = 2^2 = 4	L = 2^3 = 8	L = 2^4 = 16	L = 2^5 = 32	... L = 2^k

A rule containing a single boa field would be a good candidate for investigation, for the simple reason that all the states are contained in a single structure. The v2k3 rules 60 and 90 (and their *equivalence* related rules) are the only ones that exhibit this property for the required system sizes. In addition they are "Limited Preimage" or LP rules, giving the very important additional property of small and known number of braches in their attractor basins. Not all of the LP rules will give the same types of structure, for example some of them are chaotic rules. The boa field of rule 90 for a system size L = 4 is shown in Figure 3 (left). This structure has formed into three easily recognisable groups around a central period 1 attractor. A similar property is seen with rule 60, except here two groups are attached to the state immediately preceding the attractor, shown in Figure 3 (right).

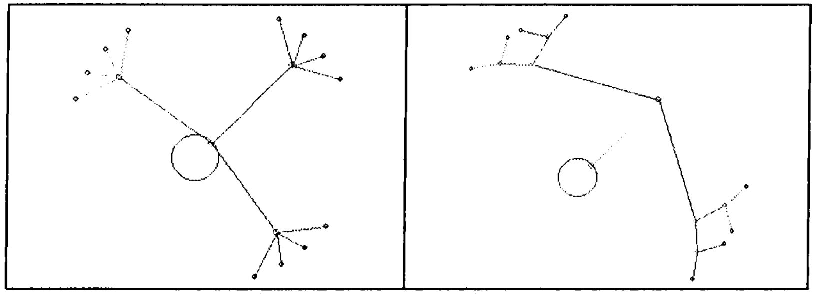

Figure 3 - Simple formations of groups of states : rule 90 (left) and rule 60 (right) with a system size L = 4.

From Table 2 a system size of L = 8 is required for a one to one relationship with elementary v2k3 rule space. The boa field of rule 90 at L =8 is shown in Figure 4, producing a similar structure to the L = 4 field. Three groups are again formed around a period 1 attractor. The decimal values of the states immediately preceding the attractor are 85, 170 and 255. These innocent looking numbers and the attractor itself are actually rule cluster pairs. The pair (0, 255) are a complete 2 rule cluster, and (85, 170) are the **reflection** half of the well known 4 rule cluster "left/right shift". DDLab can visualise state space as a matrix of values. This will be termed here as a rule space matrix to clarify that they are representing rule space in this paper. The rule space matrix for each tree is shown, Figure 4 top left, right and bottom right. Each matrix shows that the space is not randomly divided and each one appears to cover much of the area of the space.

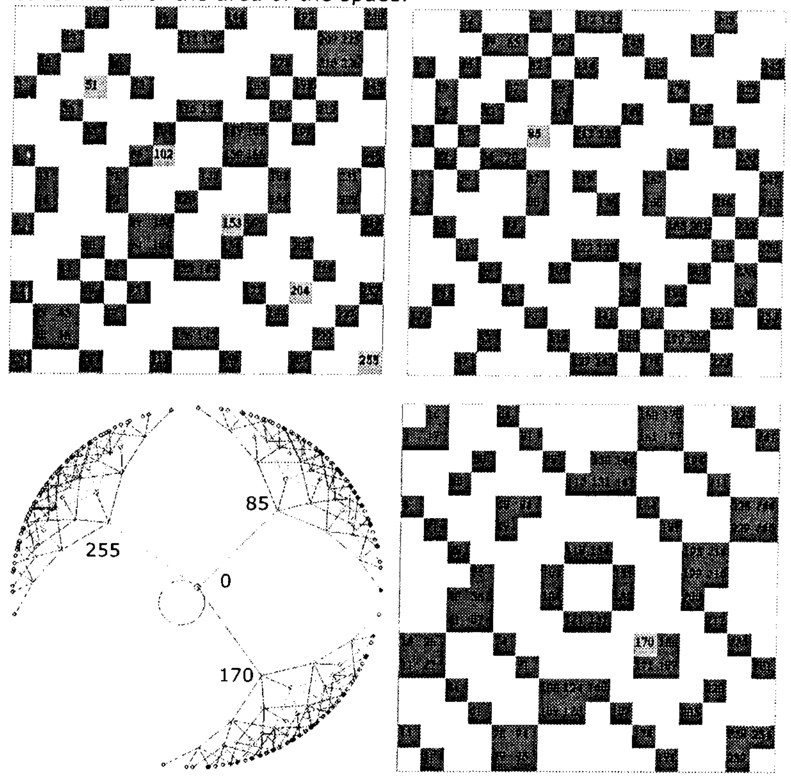

Figure 4 - The entire basin field for rule 90 L = 8 (bottom left) and rule space matrix for each tree (top left, right and bottom right).

A more detailed view of the attractor is shown in Figure 5 (left), depicting predecessor states up to two levels. The state values for the base of the three main groupings are shown again in decimal. The 16 rules seen here in the matrix diagonal of Figure 5 (right) are identical to the the 16 rule numbers identified as type (d) deterministic structure in Jen (1986), and also feature in Table 4.4a from the computational analysis by Voorhees (1996).

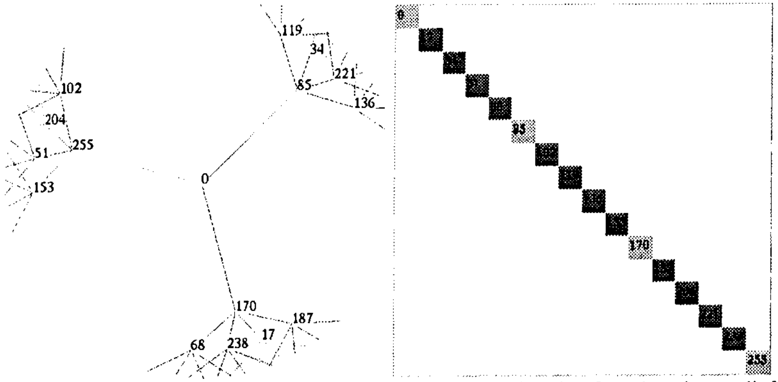

Figure 5 - Rule 90 L = 8, attractor and two levels of each subtree (left), represented as a rule space matrix (right).

From this group of 16 rules the remaining 12, three groups of four, can also be paired into rule cluster sections. This group of 16 rules will be called the *root cluster* of rule 90 and a visual representation is shown in Figure 6. The R and Rc cluster sections are indicated by a solid circle and rules not containing them are indicated by dashed circles. In practice I use the term *shadow* rules to describe cluster pairs that do not contain R & Rc. In the case of the shadow pair (102, 153) the cluster itself (R = 60) is collapsed and does not have any Rc. This cluster section (Rr & Rnr) is indicated by a vertical line between the rule numbers.

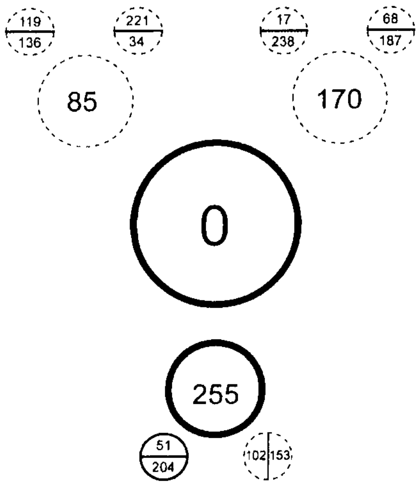

Figure 6 - The first 16 rules structured by rule 90 form a root cluster.

The pairing of rules 51 and 204 is significant, as these are the important rules of complement and identity. The shadow rules (119,136) and (221,34) are complimentary pairs from rule clusters 3 and 12 respectively. The same is true of the remaining pairs (17,238) and (68, 187). Symmetry categories in the root cluster are of the same proportion as the total number per category for the whole of v2k3 rule space : S = 1/4, SA = 1/2 and FA = 1/4.

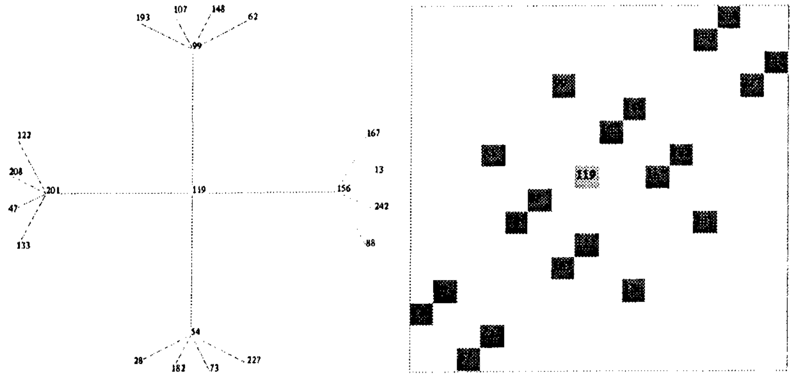

Figure - 7 Rule 90 L = 8, complete subtree of node 119 (left) and represented as a rule space matrix (right).

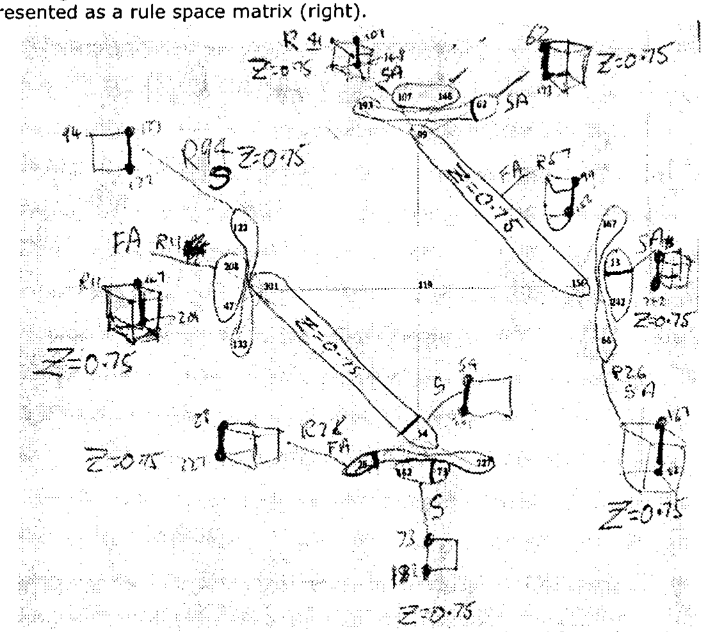

Figure 8 - Identifying the rule cluster sections by hand from the printout.

The remaining rule space is occupied by the outer 12 subtrees from the root cluster. An image for each subtree was created using DDLab and a printout obtained. Each subtree is of identical topology and contains 20 rules attached to the root rule. The subtree and rule space matrix for the root rule 119 is shown in Figure 7. The rule space matrix again does not appear randomly configured. Each of the 12 subtree page printouts was examined to see how the numbers related to the remaining rule clusters. In all cases these were organised by the global dynamics of rule 90 into pairs from a cluster, importantly all were complementary (or their collapsed equivalent) pairs. This was verified by drawing over each of the 12 pages to identify the main rule pairs as depicted in Wuensche and Lesser's atlas. The handrawn identifications for the subtree of 119 is shown in Figure 8.

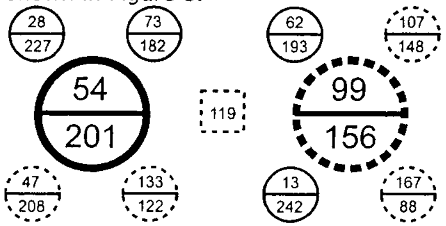

Figure 9 - The 10 rule pairings from root rule 119.

This group of 21 rules by rule 90 can be represented visually as shown in Figure 9. The diagram shows the main rules indicated by solid circles, the shadow rules by dashed circles and the root rule in the centre as a square. The larger circles are derived from the intermediate level above the root rule, the smaller circles from the goE nodes.

4. Results and Reflections

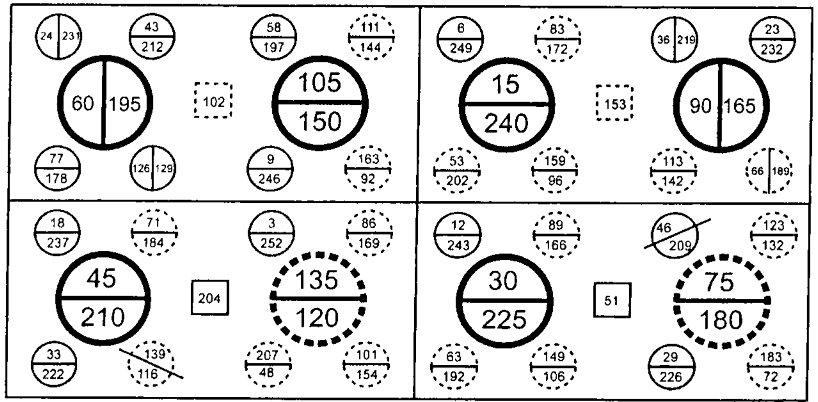

Figure 10 - The four rule groups from rule 255.

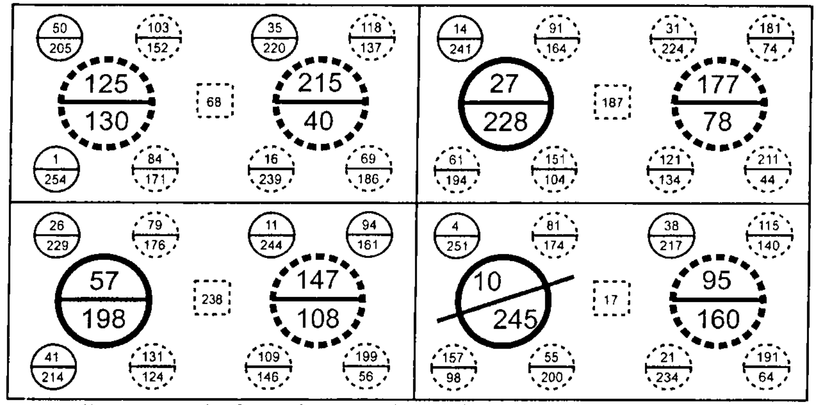

Figure 11 - The four rule groups from rule 170.

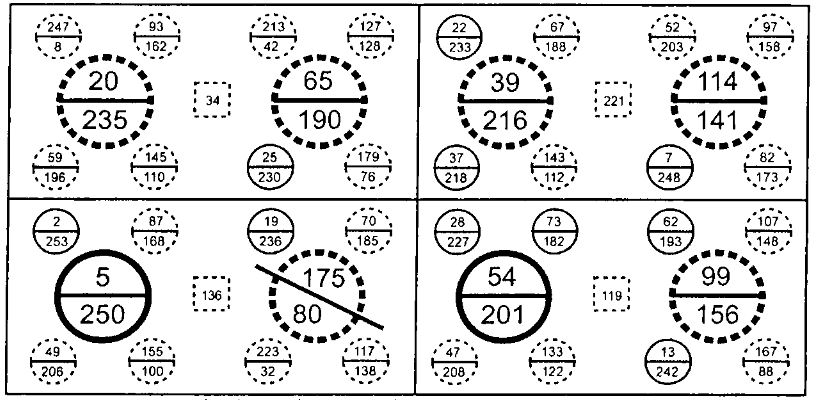

Figure 12 - The four rule groups from rule 85.

The remaining 11 hand drawn pages were also converted to this same diagrammatic form. The complete set of 12 groups of the elementary rule space is shown in Figures 10 to 12. Eight rules have collapsed compliments so these became paired with other rules in their cluster. These collapsed rules are indicated by an oblique or vertical line between the rule numbers indicating visually how the cluster has been paired. Significantly, the symmetry categories are again exactly proportioned (1/4, 1/2 & 1/4) in two ways : the 16 intermediate rules of each subtree (255, 170 and 85), and the 16 goE rules in each of the 12 groups.

Spacetime plots are a convenient way to visualise CA behaviour from a chosen starting state, different behaviours can be subjectively identified visually between plots. The cells are usually represented horizontally and time evolves vertically downwards in discrete steps. Example spacetime plots for the 20 rules from root rule 119 are shown in Figure 13. A wide variety of behaviour can be seen from the rules present in this group. The complex rule 193 is part of the well known equivalence class containing rules 110, 124 and 137) and is suggested by Wolfram to be the only v2k3 complex rule (Wolfram 2002).

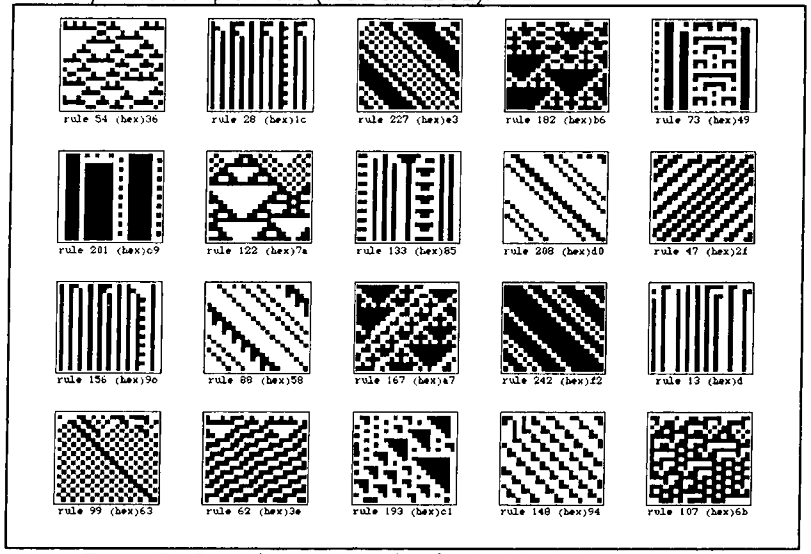

Figure 13 - Example spacetime plots for the 20 rules from root rule 119.

The applicability of this method to structure higher k rule spaces is undergoing current investigation and showing much promise. Detailed information is beyond the scope of this paper but some of the basic findings are now presented. When discussing rule spaces of higher k than the elementary v2k3 space it is more convenient to use hexadecimal to represent rule numbers. Both rules 60 and 90 preserve basin topology for each L size identified in Table 2. This allows for obtaining mixtures of higher k rules in easily accessible groups of e.g. 21, 85 or 341 in the case of rule 90. The complete basin structure of rule 90 for L = 16 and a v2k4 rule matrix from a partial subtree is shown in Figure 14 (top left and right). The rule matrix was obtained from the subtree of rule 9696 (hex) and contains 1365 rules. A partial view of the basin is shown in Figure 14 (bottom left and right), where the distribution of rules has similarities to the L=8 basin.

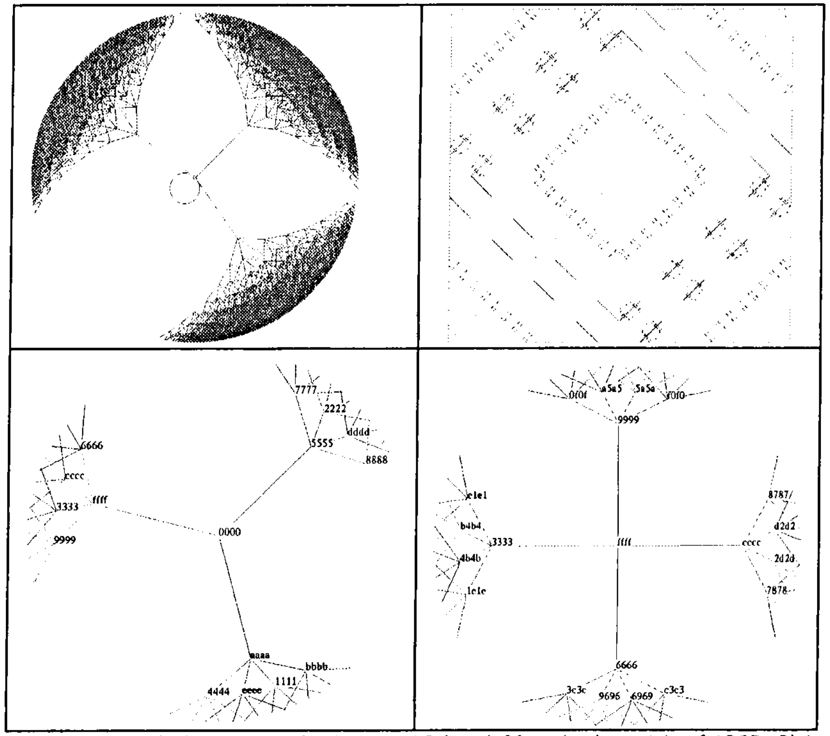

Figure 14 - Rule 90 attractor basin at L=16 (top left) and rule matrix of 1365 v2k4 rules (top right). Partial views coming up from the attractor (bottom left and right).

An example from v2k5 rule space, represented by rule 90 at L = 32 is shown in Figure 15. The rules are in 10 complimentary pairs, connected to rule cc6e5386. An entropy-density signature of the 21 v2k5 rules is shown in Figure 16 (left), indicating a diverse behaviour from the group. Classification of rules by entropy is presented in detail in (Wuensche 1997). Entropy measures the amount of disorder in the spacetime pattern, a high entropy value indicates chaos. All the rules were randomly seeded five times and each seed was evolved in a 150 cell system for approximately 1500 generations to create this signature. Example spacetime plots are shown depicting ordered, complex and chaotic behaviour from three of these rules. Filtering of spacetime patterns (Hanson and Crutchfield 1997, Wuensche 1999) would further assist in classification and is planned to be included in future work.

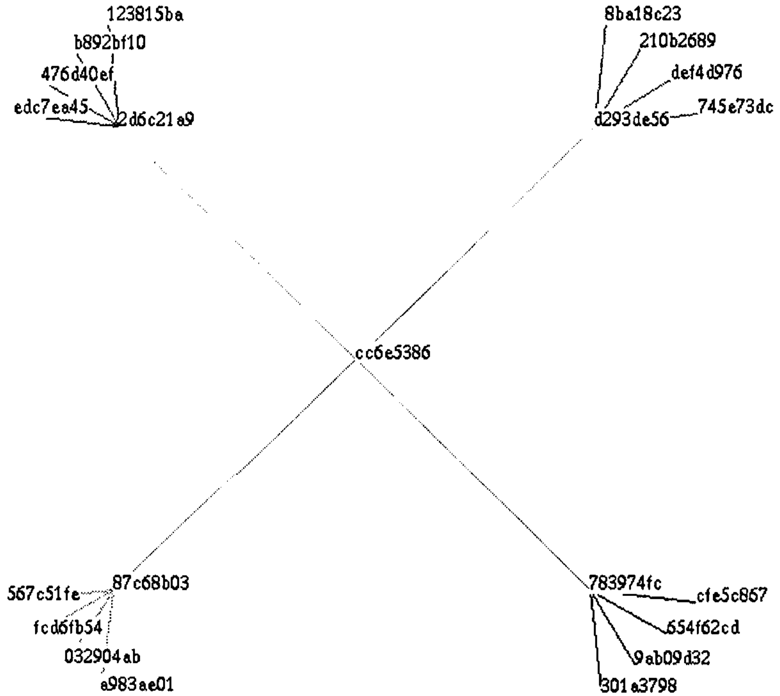

Figure 15 - Using rule 90 to create a v2k5 group, giving rules in 10 complementary pairs.

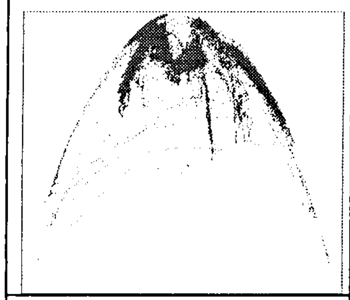	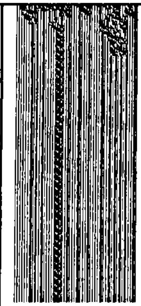	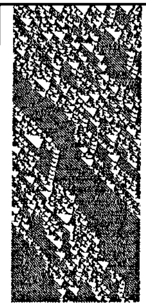	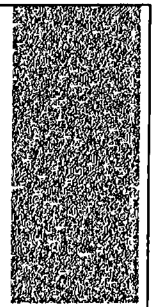
Entropy density signature of all 21 rules.	ordered : 783974fc	complex : 301a3798	chaotic : 123815ba

Figure 16 - Entropy (vertical) against density of 1's (horizontal), a signature showing a broad range of behaviour (left), and three spacetime examples.

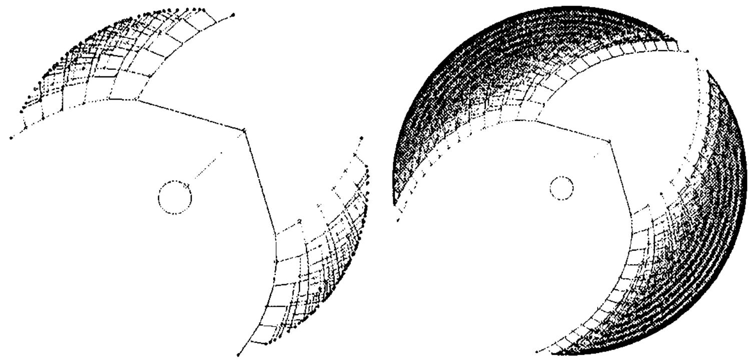

Figure 17 - Rule 60 at L = 8 (left) and L = 16 (right).

As a strategy for structuring rule space it is possible that this method could be used with some other v2k3 rules. Rule 60 is the only other v2k3 rule with a single basin rooted on attractor zero, if we disregard the equivalents. Basin fields are shown in Figure 17 for L = 8 (left) and L = 16 (right). At L = 8 the two main branches of rule 60 appear at 85 and 170. Examining these further shows the rule space is partitioned in a related way to rule 90, shown in Figure 18 (left). The state values are identical to rule 90 at L = 8 and the rule matrix in Figure 18 (right) is thus identical. After this point a different structuring of rule space occurs, which appears related to rule 90 in that complementary pairs are again formed.

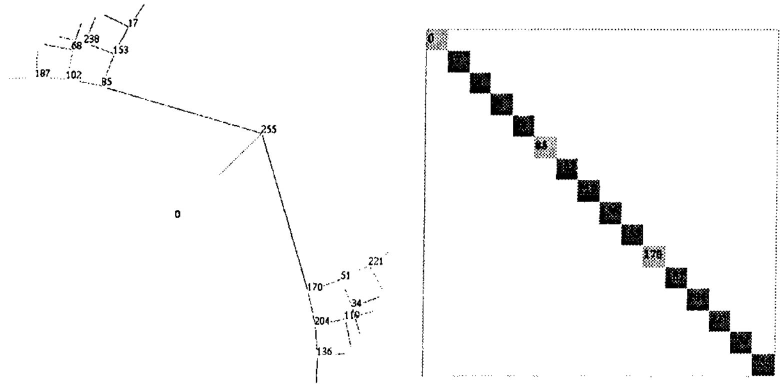

Figure 18 - Rule 60 at L = 8 attractor and two levels of each subtree (left), the rule space matrix (right) is identical to rule 90 at this point.

Binary 1D CA have rotation symmetries in their *state space* attractor basins, effectively allowing parts of the boa fields, e.g. equivalent subtrees, to be removed during analysis (Wuensche and Lesser 1992, Wuensche 1997). The L=8 rule 90 subtree equivalences are as follows (68, 34), (187,221), (17,136) and (238,119). Visual inspection confirms that all the cluster sections are in this case also *rule* equivalent, so effectively subtree 85 (55 in hex) = subtree 170 (aa in hex) removing approximately 1/3 of the total basin field. For subtree 255 (ff in hex) this is a special case as 51, 204, 102 and 153 are ALL rotationally symmetric as can be seen from their bit patterns. However, only one subtree can be removed because although this rotation symmetry exists over all these subtrees, it is in the context of *state space*. Rule space clustering does not include rotation symmetry of the rule table in the creation of a cluster. The subtrees with equivalent *rules* are 51 and 204 so it is possible to remove one of these, approximately reducing a further 1/12 of the the basin field. For v2k3 this leaves 7 groups of 21 rules occupying 7/12 of the total rule space and encompassing all 48 rule clusters (if we include the attractor). A general rule of thumb I use to obtain the total number of behaviour groups from rule 90 is

Total groups = ABS(((2^L)-1) * (7/12) / *groupsize*)

where L= 8 (or 16, 32 etc.) and *groupsize* = 21, 85, 341...

With v2k4 rule space and *groupsize* of 21 this would give a total of 1820 groups. With the same *groupsize* the v2k5 rule space has a total of 119,304,647 groups. This reduction and structuring of rule space by rule 90 importantly suggests that obtaining groups from larger rule spaces may be performed by traversing a limited part of the basin structure. The remaining rules of a particular cluster can be easily obtained by the rule axis transformations shown in Figure 1 if required in application. However, much less is known about these larger rules spaces and in the case of higher k it must be stressed that this is a useful "rule of thumb calculation", not a mathematical proof.

Deeper reflections on these results concerning rule space structure suggest many questions, particularly regarding the Lambda parameter and the "edge of chaos" phase transition debate. Lambda is simply a measure of the fraction of 1's in the rule table and a value of 0.5 is meant to imply chaos. This fails in elementary v2k3 space and two obvious deviants are the compliment/identity cluster (rule 51) and left/right shift cluster (rule 15) both which give fundamental ordered behaviour. Rules 60 and 90 are generally regarded as chaotic, but their global behaviour is ordered (null) when the system size L = 2^k. Mitchell and her colleagues object to parameters based on the "laws of motion", i.e. the rule table, suggesting they are not "appropriate loci of dynamic behaviour" (Mitchell et al 1993). This is exemplified by the shift in behaviour with system size when the rule remains fixed, as in rules 60 and 90. Li suggests that the "complex rules are scattered around some fractal-like set in the rule space" (Li 1989b). If this implies some form of self-similarity the basin of rule 90 certainly approaches this criterion.

However, the results presented show all the rules distributed in a self-similar manner based on symmetry category and complementary pairing, rather than complex behaviour.

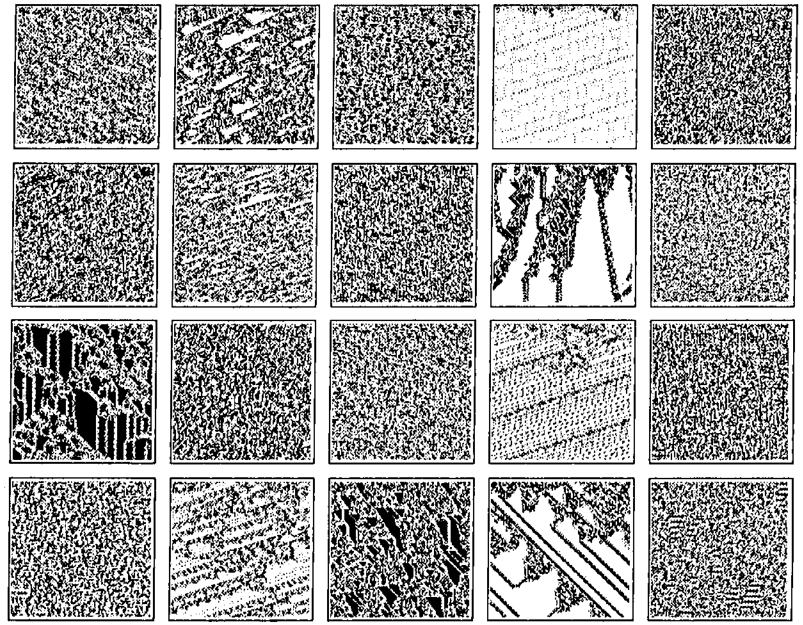

Figure 19 - Spacetime examples showing broad behaviour from seven neighbour rule space. The first column is generated from the intermediate nodes, the remaining four columns from goE nodes.

For the L=8 basin of rule 90, order from nullity begins at the attractor, traversing one level up we reach; compliment/identity, order with sections of clusters 3 and 12, and cluster sections of rule 60 giving a more chaotic rule. Traversing up the remaining two levels to the goE nodes we see the existence of order, chaos and complexity. Limited traversal of larger k rule spaces has confirmed that this structure is indeed repeated, with an increase in chaotic behaviour towards the goE nodes. In Figures 15 and 16 the complex rule is a goE node, the ordered rule is one level below, so the extremities of the space are not completely chaotic either. The spacetime examples in Figure 19 show a broad range of behaviour in the seven neighbour rule space and are derived from the goE extremity of rule 90 at L = 128. The first column is generated from the intermediate nodes, the remaining four columns are from the goE nodes. This suggests, in terms of the rule space structures created by rule 90, and possibly rule 60, that a clear phase transition does not exist when traversing from the attractor to the goE extremity. Will some mysterious single or multi valued parameter either derived from the rule table, or in conjunction with the data presented here, ever achieve a near perfect demarcation of rule space; maybe even with a clearly defined phase transition? In reality all attempts are destined and accepted to fall short in some manner due to undecidability. Perhaps a suitable name for the elusive mysterious parameter would be the "Minerva parameter", after the Roman goddess of crafts and wisdom, and wars, also said to be the inventor of musical instruments (Wikepedia 2005).

5. Conclusions and Future Work

The structure of rule space can be interpreted in more than one way and has been demonstrated in the context of cluster section groupings of varied behaviour. Attractor basins can provide a concrete form for rule space structures, exemplified using the well known elementary rules. This is a significant result because it demonstrates an elegant and natural structuring without recourse to a "law of motion" parameterisation of the space or Genetic Algorithm searching. Traversing the rule space structure and obtaining this mixture of behaviours further questions the validity of the edge of chaos phase transition hypothesis, especially when dealing with the smaller neighbourhood cases described.

The creation of this algorithmic method restores a serendipitous aspect to the use of CA in artistic application by assisting in the selection of rules. It is not necessary to understand the algorithm in order to make use of it. However, the art constructed by CA and human symbiosis should be viewed as a collaborative process by the human. This philosophy should encourage the artist to understand CA, even though the CA is unlikely to appreciate art.

Obtaining a variety of *pattern for free* is a challenge for both artist and scientist because of the large proportion of chaotic behaviour. This is not to deny the usefulness of chaos, merely an attempt to restore some balance between order and disorder. The approach presented is from an "artists perspective" based on binary state CA of a single dimension with small neighbourhood sizes. Detailed analysis of these results may provide further new insights into the wilderness of rule space. Some future work is suggested :

- More detailed analysis of rule 60 and 90 attractor basins
- Produce structures of the three neighbourhood binary rule space with other attractor basins
- Extend this work to produce structures of the four neighbourhood binary rule space
- Investigate applicability in rule spaces with increased numbers of dimensions, neighbours and states

- Investigate these structures to identify how the smaller neighbour subspaces are embedded.
- Is there a basin field that structures elementary rule space in distinct basins as a) the 88 equivalence classes b) the 48 clusters?
- Investigate if a parametric approach to behaviour classification is possible with this method, and demonstrate if a phase transition can be observed.
- Investigate applicability with random Boolean and intermediate network architectures

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