Lie Symmetry Methods for Multidimensional Linear Parabolic PDEs and Diffusions

A thesis submitted to the University of Technology, Sydney in fulfilment of the requirements of the degree of Doctor of Philosophy

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November, 2010

Certificate of Authorship/ Originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text. I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Acknowledgements

This work was carried out at the Department of Mathematics, University of Technology Sydney. My gratitude to the Australian Research Council, St George Bank, Alex Novikov and the Faculty of Science for their financial assistance. I would like to thank my family for their support over the years. I am most grateful to my supervisors, Professor Eckhard Platen and Doctor Mark Craddock. I owe special gratitude to Mark Craddock for his close supervision and support throughout my studies. Without him this thesis would not be possible.

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