Effective Methods for Human-Robot-Environment Interaction by means of Haptic Robotics

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Declaration of Authorship

- I, Pholchai Chotiprayanakul, certify that the thesis titled, "Effective Methods for Human-Robot-Environment Interaction by means of Haptic Robotics" and the work presented in this thesis are my own. I confirm that:
 - This work was done while in candidature for the degree of doctor of philosophy at the University of Technology, Sydney.
 - This thesis has not previously been submitted for a degree or any other qualification at any other institution.
 - The thesis is based on work done by myself and where I have consulted the published work of others, this is always clearly attributed.
 - All information sources and literature used are indicated in the thesis.

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Abstract

Industrial robots have been widely used to perform well-defined repetitive tasks in carefully constructed simple environments such as manufacturing factories. The futuristic vision of industrial robots is to operate in complex, unstructured and unknown (or partially known) environments, to assist human workers in undertaking hazardous tasks such as sandblasting in steel bridge maintenance. Autonomous operation of industrial robots in such environments is ideal, but semi-autonomous or manual operation with human interaction is a practical solution because it utilises human intelligence and experience combined with the power and accuracy of an industrial robot. To achieve the human interaction operation, there are several challenges that need to be addressed: environmental awareness, effective robot-environment interaction and human-robot interaction

This thesis aims to develop methodologies that enable natural and efficient Human-Robot-Environment Interaction (HREI) and apply them in a steel bridge maintenance robotic system. Three research issues are addressed: Robot-Environment-Interaction (REI), haptic device and robot interface and intuitive human-robot interaction.

To enable efficient robot-environment interaction, a potential field-based Virtual Force Field (VF²) approach has been investigated. The VF² approach includes an Attractive Force (AF) method and a force control algorithm for robot motion control, and a 3D Virtual Force Field (3D-VF²) method for real-time collision avoidance. Results obtained from simulation, experiments in a laboratory setup and field test have verified and validated these methods.

A haptic device-robot interface has been developed for providing intuitive humanrobot interaction. Haptic devices are normally small compared to industrial robots. Thus, the workspace of a haptic device is much smaller than the workspace of a big industrial manipulator. A novel workspace mapping method, which includes drifting control, scaling control and edge motion control, has been investigated for mapping a small haptic workspace to the large workspace of manipulator with the aim of providing natural kinesthetic feedback to an operator and smooth control of robot operation. A haptic force control approach has also been studied for transferring the virtual contact force (between the robot and the environment) and the inertia of the manipulator to the operator's hand through a force feedback function.

Human factors have significant effect on the performance of haptic-based humanrobot interaction. An eXtended Hand Movement (XHM) model for eye-guided hand movement has been investigated in this thesis with the aim of providing natural and comfortable interaction between a human operator and a robot, and improving the operational performance. The model has been studied for increasing the speed of the manipulator while maintaining the control accuracy. This model is applied into a robotic system and it has been verified by various experiments.

These theoretical methods and algorithms have been successfully implemented in a steel bridge maintenance robotic system, and tested in both laboratory and a bridge maintenance site located in Sydney.

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Nomenclature

General Formatting Style

<i>f</i> ()	A scalar valued function
f ()	A vector function
F()	A matrix function
$\{\ldots\}$	A set of
$[\ldots]^{\mathrm{T}}$	Transpose
	Absolute value
	Vector length

minmin{...} The objective function of two parameters minimization

```
Product sign
            A scalar variable
                                          - lower case and italic
X
            A scalar variable
                                          -upper case and italic
            A vector or a single-dimensional matrix
X
                                          - lower case and bold
X
            A matrix or a set
                                          - upper case and bold
P(Z \leq a)
                   Cumulative probability function
                   Complementary cumulative probability function
P(Z \ge a)
P(b \le Z \le a) Cumulative point-to-point probability function
```

General Referencing with Subscript and Superscript

x A scalar variable

- Front superscript is reference coordinate

- Rear subscript is transforming coordinate

x A scalar variable

- Front superscript is reference coordinate
- Rear first subscript is numerical index, customized abbreviation or description
- Rear second subscript is transforming coordinate

X or x A matrix

- Front superscript is reference coordinate
- Rear subscript is transforming coordinate

X or x A matrix

- Front superscript is reference coordinate
- Rear first subscript is numerical index, customized abbreviation or description
- Rear second subscript is transforming coordinate

X A set (does not have front superscript)

- Rear first subscript is customized abbreviation or description
- Rear second subscript is index of set's member

Local and Global Variables

i,j	Index of a set or to refer to a count
x,y,z	Axes of Cartesian coordinate
\Re^3	3DOF Cartesian space
\mathfrak{R}^6	6DOF Cartesian space
$\Re_{\mathbf{q}}^{7}$	7DOF manipulator's joint space

Specific Symbol Usage

A homogeneous transformation matrix (4 by 4) -- T A rotation matrix (3 by 3) \mathbf{R} A directional vector or a normal vector (3 by 1) n ··· **p** ... A position vector (3 by 1) - P A configuration vector (6 by 1), a combination of [n , p] components of n n_x , n_v , n_z components of p p_x, p_y, p_z F... A 6DOF spatial force f... A 3DOF linear force, a component of F A 3DOF angular moment, a component of F σ $f_{\dots x}, f_{\dots y}, f_{\dots z}$ scalar components of f... scalar components of o... $\sigma_{...x}, \sigma_{...y}, \sigma_{...z}$ A joint position matrix (7 by 1) q A position of joint i, a component in q q_i joint 1 to 6 are angle (degree) joint 7 is length (m) No. Mass-inertia matrix of the robot links Damping coefficient matrix of the robot joints B Stiffness coefficient matrix of the robot joints Γ A torque-force matrix defining in joint space (7 by 1) Joint torque-force of a joint i, a component in Γ τ_i - joint 1 to 6 are torque (N.m) joint 7 is linear force (N) T att Attractive torque-force matrix Γ_{rep_i}

Repulsive torque-force matrix on a link in joint i coordinate

 0 \mathbf{P}_{e} Configuration of the end-effector of a manipulator 0 **n**_e A component of ⁰P_e $^{0}\mathbf{p}_{e}$ A component of ⁰P 0 P. Configuration of target/attractive point for the manipulator's endeffector A component of ⁰P, °n, $^{0}\mathbf{p}_{t}$ A component of ⁰P, \mathbf{F}_{att} A6DOF spatial attractive force for the manipulator's end-effector A 3DOF attractive force (linear force), a component of \mathbf{F}_{att} \mathbf{f}_{att} A 3DOF attractive moment (angular moment), a component of \mathbf{F}_{att} σ_{att} components of \mathbf{f}_{att} fatt x, fatty, fatt z components of σ_{att} $\sigma_{att x}, \sigma_{attv}, \sigma_{att z}$ Katt 1 Coefficient of a linear attractive force's amplitude Katt 2 Coefficient of an angular attractive moment's amplitude A small non-zero positive constant (in this research ε_0 = 0.0001) A constant for defining a transient state of the attractive force K_{as} Attractive force function δ_a Pob Obstacle point cloud set 0 **p**_{ob i} The i^{th} member in the obstacle point cloud set Number of points in the obstacle point cloud set m inner force field D_{min} outer force field D_{max} Erdistance between D_{min} and D_{max} The radius of D_{min} r_c

Ending point 1 of the centre line of a link in joint i coordinate

° **p**1,

⁰**p**2, Ending point 2 of the centre line of a link in joint i coordinate 0 p3_i The nearest point on the centre line of a link on joint i coordinate A parameter of a centre line of a link function for defining ⁰ p3, 0 **p** $_{obs}$ The point in the point cloud set that is the closest point to a manipulator link at ⁰ p3, The shortest distance measured from ${}^{0}\mathbf{p}_{ob}$ to ${}^{0}\mathbf{p}3_{i}$ ds. The shortest distance from the point ${}^{0}\mathbf{p}_{obs}$ to the surface of D_{min} d_0 \mathbf{F}_{rep_i} A spatial repulsive force on a link of a manipulator in joint i coordinate \mathbf{f}_{rep} A repulsive force on a link of a manipulator in joint i coordinate, a component of \mathbf{F}_{rep_i} $f_{rep_{x_i}}, f_{rep_{y_i}}, f_{rep_{z_i}}$ components of \mathbf{f}_{rep_i} δ_r Repulsive force function K_f A coefficient of the amplitude of a repulsive force Force transformation matrix in joint i coordinate H, $^{0}\mathbf{p}_{f}$ Position vector of a force jth component of H h, Configuration matrix of joint i coordinate L, A rotation skew-symmetric matrix (6 by 6) M A skew-symmetric matrix (3 by 3) $S(\Delta p)$

 $\Delta p_x, \Delta p_y, \Delta p_z$ components of $\Delta \mathbf{p}$

A differential vector between ${}^{\circ}\mathbf{p}_{i}$ and ${}^{0}\mathbf{p}_{f}$

 \mathbf{F}_h A haptic force

 $\Delta \mathbf{p}$

 k_{vir} Coefficient of virtual spring stiffness

⁰ p _a	Position vector of the haptic cursor on the world coordinate
⁰ p _h	Position vector of the haptic cursor on the haptic workspace coordinate
⁰ p _w	Position vector of the haptic workspace origin on the world coordinate
$^{\circ}$ $\dot{\mathbf{p}}_{h}$	Velocity vector of the haptic cursor on the haptic workspace coordinate
° p ′ _w	Drifting velocity of the haptic workspace origin on the world coordinate
k_{sc}	Workspace scaling coefficient
\dot{k}_{sc}	Workspace scaling rate (first order differentiate)
k_{sp}	Positive scaling rate of scaling control
k_{sn}	Negative scaling rate of scaling control
k_{dp}	The positive drifting rate of drifting control
k_{em}	Drifting rate of edge motion control
v_{ht}	Haptic cursor speed threshold
k_e	Elasticity coefficient of a 3D-VF ² (virtual skin)
$\mu_{\it cap}$	Energy absorbability of a 3D-VF ²
$\mu_{\scriptscriptstyle in}$	Potential energy from the attractive force
φ	Probability density
O_{acc}	Pointing accuracy
Z_{ac}	Coefficient of the pointing accuracy
σ	Standard deviation of robot end-effector in a pointing task
σ^2	Variance of robot end-effector in a pointing task
IP	Index of Performance
Ø	Path-track width
\emptyset_{obj}	Dimension of a task plane
\emptyset_{scn}	Dimension of a computer screen
d_v	A view distance between a camera screen and an object plane

α_v	A camera's field of view
G_e	A control display scale
G_h	A control movement scale
G_x	A control scale
MT	Hand movement time
а	Operator's start/stop movement time
b	Operator's hand movement speed
v_a	A haptic cursor speed from hand movement model
k_{pw}	Coefficient of power expense in haptic interface