

Modelling Default Correlations in a Two-Firm Model with Dynamic Leverage Ratios

A Thesis Submitted for the Degree of
Doctor of Philosophy

by

Ming Xi Huang

B.Sc. (The Chinese University of Hong Kong, Hong Kong)

M.Sc. (The Chinese University of Hong Kong, Hong Kong)

MingXi.Huang@student.uts.edu.au

in

The School of Finance and Economics

University of Technology, Sydney

PO Box 123 Broadway

NSW 2007, Australia

August 11, 2009

CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate

Date

Acknowledgments

I would like to thank my supervisor, Professor Carl Chiarella for his guidance, advice and assistance throughout my PhD studies, and for his encouragement and support to me to participate in international conferences, seminars and workshops that have been an important part of the building up of my research experience. I would also like to thank my co-supervisor, Associate Professor Xue-Zhong (Tony) He and my alternate supervisor, Associate Professor Erik Schlogl, for their additional supervision and their insightful comments and suggestions.

I would especially thank Professor Chi-Fai Lo (Institute of Theoretical Physics and Department of Physics, The Chinese University of Hong Kong), for raising the initial motivation of this research topic and providing much useful advice on several technical aspects of this thesis. Special thanks are also due to Dr. Hing Hung (formerly of the School of Finance and Economics, UTS and now at the Commonwealth Bank of Australia), who shared with me a great deal of useful knowledge in implementation methodologies. I would also like to thank all staff and academic colleagues of the School of Finance and Economics at UTS, for their friendship and assistance over the last four years.

Additionally, the financial assistance from various scholarships and conference grants from the University of Technology, Sydney, the Faculty of Business, the Quantitative Finance Research Centre and the School of Finance and Economics are all greatly appreciated.

Last but not least, I would like to thank my parents, sisters and my husband for their endless love, support and encouragement over the years of my studies.

Contents

Abstract	vi
Chapter 1. Introduction	0
Chapter 2. Literature Review	3
Chapter 3. Framework of the Two-Firm Model	9
3.1. The One-Firm Model	9
3.2. The One-Firm Model with Time-Dependent Parameters	17
3.3. Framework of the Two-Firm Model	25
3.4. Default Correlations	29
3.5. Overview	31
Chapter 4. The Method of Images: Methodology and Implementation	32
4.1. The Method of Images in the 2-D Situation	32
4.2. Method of Images for Constant Coefficients at Certain Non-Zero Values of ρ_{12}	37
4.3. Method of Images for Time Varying Coefficients at Certain Non-Zero Values of ρ_{12}	39
4.4. Numerical Implementation	43
4.5. Overview	47
Chapter 5. Numerical Approaches	48
5.1. Alternating Direction Implicit Method	49
5.2. A Monte Carlo Simulation Scheme	53
5.3. Accuracy	56
5.4. Overview	59
Chapter 6. The Two-Firm Model under Geometric Brownian Motions	60
6.1. Choice of Parameters	60
6.2. The Impact of Geometric Brownian Motions	61
6.3. The Price of Credit Linked Notes	75

6.4. Overview	77
Chapter 7. Two-Firm Model with Mean-Reverting Processes	79
7.1. The Two-Firm Framework in the Case of Mean-Reverting Leverage Ratios	80
7.2. Solution of the CLN Partial Differential Equation by the Method of Images	83
7.3. Numerical Implementation for the Case of Mean-Reversion	89
7.4. A Monte Carlo Simulation Scheme	91
7.5. Accuracy	92
7.6. The Impact of Mean-Reverting Processes for Leverage Ratios on JSP & DC	94
7.7. Overview	97
Chapter 8. One-Firm Dynamic Leverage Ratio Model with Jumps	99
8.1. The Framework	100
8.2. Monte Carlo Simulations	102
8.3. Choice of Parameters	104
8.4. The Impact of Jump Risks on Default Probabilities	107
8.5. Calibration of the Average Jump Size to the Historical Data	109
8.6. Overview	113
Chapter 9. The Two-Firm Model with Jumps	115
9.1. A Monte Carlo Simulation Scheme to Calculate JSP under Jump-Diffusion Dynamics	117
9.2. Choice of Parameters	119
9.3. The Impact of Jump Risk on the Two-Firm model	120
9.4. Overview	127
Chapter 10. Comparison of the Two-Firm Model for Different Processes	129
10.1. The Impact of Different Dynamic Leverage Ratio Processes on Default Probabilities	129
10.2. The Impact of Different Dynamic Leverage Ratio Processes on Joint Survival Probabilities	131
10.3. The Impact of Different Dynamic Leverage Ratio Processes on Default Correlations	132
10.4. Overview	133

Chapter 11. Conclusions	135
11.1. Summary	135
11.2. Topics for Future Research	138
Appendix A. Applying the Separation of Variables to the PDE for the Corporate Bond Price	140
Appendix B. Transformation to the Heat Equation with Time Independent Coefficients	142
Appendix C. Transformation to the Heat Equation with Time Dependent Parameters	144
Appendix D. Derivation of Differential Equation Satisfied by a Credit-Linked Note	149
Appendix E. Applying the Separation of Variables to the PDE for the CLN	152
Appendix F. The Number of Images and the Correlation Coefficient ρ_{12}	155
Appendix G. Transformation of the PDE (4.30) to the 2-D Heat Equation in the Case of Constant Coefficients	161
Appendix H. The Derivation of the PDE (4.44) in the Case of Time Varying Barriers	164
Appendix I. Expressing the JSP in terms of the Bivariate Normal Distribution	167
Appendix J. The Operator $e^{x \frac{\partial}{\partial x}}$	171
Appendix K. The Transformation of the PDE for the CLN in the Mean-Reverting Case	174
Appendix L. Derivation of the PDE with Time Varying Barriers in the Mean-Reverting Case	176
Bibliography	179

Abstract

Default correlations have been an important research area in credit risk analysis. This thesis aims to extend the one-firm structural model of default to the two-firm situation for valuing default correlations. In the structural approach, default happens when the firm value falls below a default threshold. In the fundamental model of Merton (1974), the default threshold is simply the face value of the bond. Collin-Dufresne & Goldstein (2001) related the default threshold to the firm's debts and modelled it as mean-reverting to a constant long-term target. Hui et al. (2006) generalized the Collin-Dufresne & Goldstein (2001) model to consider the default threshold as stochastic and the long-term target as time-dependent. In these models, the corporate bond price is a function of the leverage ratio - a ratio of the firm's debt to its asset value. For this combined measure of the firm's default risk, Hui et al. (2007) proposed a dynamic leverage ratio model, where default happens when the leverage ratio falls below a certain level.

The aim of this thesis is to extend the one-firm dynamic leverage ratio model of Hui et al. (2007) to incorporate the default risk of two firms and interest rate risk. The model will be based on the consideration of a financial instrument (a credit linked note) that is exposed to the default risk of the two firms. Initially, the dynamic leverage ratios will be assumed to follow geometric Brownian motions and the stochastic interest rate assumed to follow a Vasicek (1977) process. The pricing problem will then be reduced to that of solving the first-passage-time problem that plays an important part in the valuation of default correlations.

In order to study the impact of the capital structures of firms on default correlations, the two-firm model is extended by allowing the dynamic leverage ratios to follow mean-reverting processes, so as to capture the behavior of firms when they adjust their capital structures to a long-term target. Then in order to capture the effect of external shocks on default correlations, the model is further extended to consider the situation in which the dynamic leverage ratios follow jump-diffusion processes. Finally, the numerical results of default correlations based on the two-firm model are studied and compared when the firm's leverage ratios follow these three types of processes.

The thesis concludes by pointing to some future research directions. These includes further development of the method of images approach for the solution of the first passage time problem to the time varying coefficients case by use of the multi-stage approximation. Development of approximate analytical methods to extend the range of applicability of the method of images approach. Extension of Fortet's integral equation approach for the solution of first passage time problem to

the two-dimensional situation. The estimation and calibration of leverage ratio models, including estimation of market prices of risk.

The main contributions of the thesis are:

- The setting up the two firm leverage ratio framework for evaluation of default correlations.
- The extension of the method of images approach to the two-dimensional situation for solving the first passage time problem with constant coefficients and the time varying barrier approach for time-dependent coefficients.
- Extension of the leverage ratio framework to incorporate jumps in both the one and two firm cases.
- A comparative study of the impact on default correlations and joint survival probabilities of the different types of processes for the leverage ratio dynamics.