Modelling Default Correlations in a Two-Firm Model with Dynamic Leverage Ratios

A Thesis Submitted for the Degree of

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by

Ming Xi Huang

B.Sc. (The Chinese University of Hong Kong, Hong Kong)

M.Sc. (The Chinese University of Hong Kong, Hong Kong)

MingXi.Huang@student.uts.edu.au

in

The School of Finance and Economics

University of Technology, Sydney

PO Box 123 Broadway

NSW 2007, Australia

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate . . . . . .

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Contents

Abstract

Default correlations have been an important research area in credit risk analysis. This thesis aims to extend the one-firm structural model of default to the two-firm situation for valuing default correlations. In the structural approach, default happens when the firm value falls below a default threshold. In the fundamental model of Merton (1974), the default threshold is simply the face value of the bond. Collin-Dufresne & Goldstein (2001) related the default threshold to the firm's debts and modelled it as mean-reverting to a constant long-term target. Hui et al. (2006) generalized the Collin-Dufresne & Goldstein (2001) model to consider the default threshold as stochastic and the long-term target as time-dependent. In these models, the corporate bond price is a function of the leverage ratio - a ratio of the firm's debt to its asset value. For this combined measure of the firm's default risk, Hui et al. (2007) proposed a dynamic leverage ratio model, where default happens when the leverage ratio falls below a certain level.

The aim of this thesis is to extend the one-firm dynamic leverage ratio model of Hui et al. (2007) to incorporate the default risk of two firms and interest rate risk. The model will be based on the consideration of a financial instrument (a credit linked note) that is exposed to the default risk of the two firms. Initially, the dynamic leverage ratios will be assumed to follow geometric Brownian motions and the stochastic interest rate assumed to follow a Vasicek (1977) process. The pricing problem will then be reduced to that of solving the first-passage-time problem that plays an important part in the valuation of default correlations.

In order to study the impact of the capital structures of firms on default correlations, the two-firm model is extended by allowing the dynamic leverage ratios to follow mean-reverting processes, so as to capture the behavior of firms when they adjust their capital structures to a long-term target. Then in order to capture the effect of external shocks on default correlations, the model is further extended to consider the situation in which the dynamic leverage ratios follow jump-diffusion processes. Finally, the numerical results of default correlations based on the two-firm model are studied and compared when the firm's leverage ratios follow these three types of processes.

The thesis concludes by pointing to some future research directions. These includes further development of the method of images approach for the solution of the first passage time problem to the time varying coefficients case by use of the multi-stage approximation. Development of approximate analytical methods to extend the range of applicability of the method of images approach. Extension of Fortet's integral equation approach for the solution of first passage time problem to

the two-dimensional situation. The estimation and calibration of leverage ratio models, including estimation of market prices of risk.

The main contributions of the thesis are:

- The setting up the two firm leverage ratio framework for evaluation of default correlations.
- The extension of the method of images approach to the two-dimensional situation for solving the first passage time problem with constant coefficients and the time varying barrier approach for time-dependent coefficients.
- Extension of the leverage ratio framework to incorporate jumps in both the one and two firm cases.
- A comparative study of the impact on default correlations and joint survival probabilities of the different types of processes for the leverage ratio dynamics.