Brand Performance Volatility from Marketing Spending

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ABSTRACT

While volatile marketing spending, as opposed to even-level spending, may improve a brand's financial performance, it can also increase the volatility of performance, which is not a desirable outcome. This paper analyzes how revenue and cash-flow volatility are influenced by own and competitive marketing spending volatility, by the level of marketing spending, by the responsiveness to own marketing spending, and by competitive response. From market response theory, the authors derive propositions about the influence of these variables on revenue and cash-flow volatility. In addition, they extend the Dorfman-Steiner theorem to derive the optimal level and volatility of expenditures if volatility effects are taken into account.

Based on a large sample of 99 pharmaceutical brands in four clinical categories and four European countries, the authors test for the empirical relevance of the propositions and assess the magnitude of the different sources of marketing-induced performance volatility. The authors find broad support for the predicted volatility effects. Volatility elasticities are significant and may be as large as 1.10 for cash-flow variance with respect to marketing responsiveness.

The findings imply that common volatility-increasing marketing practices such as price promotions or volatile advertising plans may be effective at the top-line, but could turn out to be ineffective after all costs are taken into account. Optimal marketing volatility needs to tradeoff sales effectiveness and extra costs due to marketing volatility.

Keywords: Revenue/Cash-Flow Volatility, Marketing Volatility, Econometric Models, Marketing Metrics

Introduction

In order to enhance sales impact, marketing practitioners often deploy their resources in spending bursts, i.e. regimes characterized by on-again, off-again marketing actions, including advertising campaigns, sales promotions and new-product launches. Inasfar as the volatility in such marketing activities causes demand/revenues and cash flows to become more volatile, it may have unintended negative consequences for the firm. Such effects may occur because managers from different departments do not fully appreciate the nature of demand volatility and interpret demand shifts differently.

Consider the following example from the computer industry (Hanssens 1998). The marketing manager of a manufacturer brand orders a sales promotion to stimulate lackluster demand. The dealer interprets the temporary sales lift as a true marketplace demand shift and increases orders to boost his inventory. The manufacturer's supply chain manager notices the sharp increase in orders and projects that the manufacturer will quickly run out of stock. Consequently, he adjusts production plans to avoid potential stockouts. Because the promotion-induced shift in sales was only of a temporary nature, the firm may now face additional warehousing and related costs. Similar examples have been described for other firms such as automobile manufacturers (Gottfredson and Aspinall 2005).

The increased demand volatility at the retail level leads to the well-known bullwhip effect (e.g. Lee, Padmanabhan & Whang 1997), i.e. increasing demand volatility in the supply chain from downstream echelons (retail) to upstream echelons (manufacturing). The effect occurs because information transferred in the form of orders among members of a supply chain tends to be distorted and may mislead upstream firms in their inventory and production decisions. Lee, Padmanabhan & Whang (1997) showed that this effect is not due to a behavioral anomaly, but results from rational and optimizing behavior of economic agents in a supply chain. Since the effect amplifies as one moves upstream in the supply chain, the volatility of orders or production becomes larger than that of sales or demand caused by end customers, with serious cost implications. Excess raw materials cost, additional manufacturing expenses, excess warehousing, and additional transportation costs may result in excess cost that may be as large as 25% (Lee, Padmanabhan, and Whang 1997). As a consequence, a marketing policy that stimulates not only the level of sales but also its variance may increase these costs.

In addition, demand volatility creates challenges for the management of limited resources such as labor force, machine equipment, and storage capacity. Opportunity costs arise due to unused capacities in periods of lower demand. Extra costs result from the overuse of resources due to equipment wearout, overwork of labor force, extra compensation for overtime, etc. In particular, employees in departments associated with sales, customer service and order fulfillment are directly impacted by demand fluctuations. Severe volatility may necessitate frequent hiring, firing and re-hiring of employees, which is costly due to training and severance pay. This will incur productivity losses due to employee idleness in trough periods, and supplementary costs (e.g., payment for overtime) in peak periods.

In addition to these observable economic effects of demand volatility, there are also motivational consequences. According to expectancy theory in organizational behavior (e.g., Steel and König 2006), worker motivation and morale will be lower when employees fail to perceive a linkage between their personal effort and the firm's performance. Thus if recurring marketing-induced volatility in the firm's revenue streams cannot be remedied, non-marketing employee motivation and loyalty will suffer, which can be costly for a firm. Additionally, if sales volatility results in either over-shooting or under-shooting of company revenue targets, that will adversely affect the compensation of sales people and executives in the firm (Misra and Nair 2011). Likewise, sales volatility may harm the relationship between manufacturers and retailers because it makes the order planning process more difficult and may force the manufacturer to make tough choices if orders exceed supply (Adelman and Mersereau 2013).

Marketing volatility may also increase cash-flow volatility, leading to higher opportunity costs and greater financing costs. Consider an advertising plan with alternating periods of high and low activity, which results in demand and cash-flow peaks and troughs. The negative consequences of such cash-flow volatility have been well recognized in the finance literature. In particular, a greater variability of cash flows forces management to hold larger cash reserves (Opler et al., 1999). In the Appendix, we demonstrate how a more volatile spending plan of a given advertising budget leads to extra financing costs because it requires mobilizing more capital compared to an even-spending plan. The illustrative example shows that, even though revenues and cash flows are higher under the volatile spending plan, the extra costs may outweigh the sales advantage.

However, it should be noted that volatility is not bad per se. If it is driven by an upward sales trend, for example, then it might even be desirable. The unexpected variation around the forecasted trend line is the kind of volatility that is undesirable. By better understanding the potential marketing sources of such volatility, decision makers can reduce the uncertainty around their sales and cash flow predictions.

Marketing literature. Revenue or cash-flow volatility has traditionally not been of major concern to marketers. As long as marketing managers are unaware of the potential negative effects of their marketing policies, they have no incentive to reduce the resulting revenue and cash-flow volatility. Thus, there may be a potential conflict between sales-impact maximization (a typical marketing objective) and stable revenue and cash-flow generation (typical operations and financial management objectives). The marketing literature, however, is virtually silent about the potential performance volatility induced by marketing-mix activities. To our knowledge, only two empirical studies have addressed the relationship between marketing-mix activities and revenue/cash-flow volatility to date. Raju (1992) examines the drivers of category sales variability and finds that the magnitude of discounts is positively associated with sales volatility. Vakratsas (2008) shows that marketing-mix variables including price, advertising, and distribution affect market-share volatility. Given that volatility in sales and cash flows may have significant, unfavorable side effects, however, we need a deeper understanding of how marketing activities drive these performance volatilities.

Contributions. This study focuses on the brand level and examines the effects of the *volatility* of *marketing expenditures*, the *level* of *marketing expenditures*, and *customer responsiveness* to *marketing expenditures,* both theoretically and empirically. Some of these relations are relatively transparent; for example more volatile spending and higher responsiveness should translate into higher revenue and cash-flow volatility because of the functional relationship between sales and marketing spending. However, Raju's (1992) finding that a higher frequency of promotional actions leads to *lower* sales volatility is counter to this intuition. In our theoretical analysis, we show that the intuition is accurate for the single-firm scenario but not necessarily true for a competitive scenario. Depending on the structure and intensity of competitive interaction, theory predicts the reverse outcome found by Raju (1992). Other effects, such as the impact of the spending level on cash-flow volatility, are not easy to predict without a deeper theoretical understanding. In addition, we develop results on the optimal spending level and the optimal spending volatility under Nash competition that extend the well-established Dorfman-Steiner theorem to the volatility case. Thus, our study provides the first in-depth theoretical analysis of the volatility effects of marketing spending policy under competition.

We test the predictions from theory with a large dataset of 99 pharmaceutical brands from four European countries and four categories. The pharmaceutical industry is especially relevant because marketing expenditures are substantial and show high volatility. Our empirical analysis makes two contributions. First, it informs whether the predicted volatility effects hold under real market conditions. Second, it enables us to quantify the magnitude of these effects. For this reason, we obtain elasticity estimates for the volatility relations. Decision makers would only care about the effects if they were of practical significance.

The remainder of the article is organized as follows. We first develop propositions about the effects of marketing spending on brand performance volatility. Next, we describe our research methodology to measure the effects in an empirical study. We present empirical results and discuss the theoretical and managerial implications of our findings. The article concludes with a synthesis of the findings, limitations and suggestions for future research.

Theory

Our conceptual development is rooted in market response theory. We start from the premise that sales follow a concave relationship with marketing expenditures. A concave response function is theoretically attractive because it implies diminishing returns, which are a prerequisite for marketing budget optimization. It is by far the most frequent type of response function encountered in empirical research (Hanssens, Parsons, and Schultz 2001). Since a concave log-log response model also turns out to best represent our data, our theory development is fully consistent with the subsequent empirical analysis. Finally, the results may be generalized to other types of response, such as an S-shaped or a differential stimulus response. Assuming rational, profit-maximizing behavior, budgets only vary within the concave zone of these functions, which is the only assumption we make.

By varying conditions such as responsiveness to marketing, we derive propositions on our focal volatility variables. Specifically, we consider two measures of volatility: the *variance* and the *range* (i.e. the difference between maximum and minimum values) of marketing expenditures, revenues, and cash flows. Variance is a common measure of variability and we will focus on this variable to derive our propositions. Range is another useful metric of volatility, which is often used in the finance literature (e.g., Alizadeh, Brandt and Diebold 2002).

Impact of Own Marketing on Performance Volatility

We start the discussion of volatility effects with the impact of own marketing spending behavior on the volatility of revenues, followed by its effects on the volatility of cash flows. Our general argument is that the volatility, the average level and the sales responsiveness of marketing expenditures together affect the volatility of revenues and cash flows. By sales responsiveness we mean the lift in sales that can be associated with an increase in marketing expenditures. It is measured by the slope parameter of the response function.

In the theoretical analysis, we assume that both own marketing and competitive marketing expenditures influence sales. The impact of competitive marketing on sales is measured by its cross-effect. Because of potential competitive interactions, there is a connection between own marketing expenditures and competitive expenditures that needs to be reflected in the volatility analysis. The correlation between own and competitive expenditures is the observable outcome of this interaction. In addition, we assume that the volatility of own marketing expenditures may have an effect on sales. This effect models the potential benefits of volatile marketing expenditures.

Definitions and Assumptions. Let $Q[MKT, CMKT, Var(MKT)]$ measure unit sales that depends on own marketing expenditures, MKT, the cumulative marketing expenditures by competitors, $CMKT$, and the variance of own marketing expenditures, $Var(MKT)$. Q is a nonlinear, twice differentiable function with $Q'(MKT) > 0$ and $Q''(MKT) < 0$. Q' measures the marginal own demand effect with respect to *MKT*. Assuming profit maximization together with S-shaped response functions, as an example, implies that firms operate in the concave part of the response function. Hence, our assumption about $Q''(MKT)$ still holds. Q'_c captures the cross-effect of competitive expenditures, *CMKT*, on demand. This effect may be substitutive ($Q'_c < 0$) or market expanding ($Q'_c > 0$). Let $\varepsilon = Q' \cdot MKT/Q$ denote the elasticity of sales with respect to own marketing expenditures and $\varepsilon_c = Q'_c \cdot CMKT/Q$ be the cross-elasticity with respect to competitive expenditures.

$Q'_{Var}\left[Var\left(MKT\right)\right] \geq 0$ measures the effect of expenditure volatility on sales. The marginal effect can be positive or zero depending on the type of response function assumed. The literature on advertising pulsing proposes various demand specifications (e.g., S-shaped market response, differential stimulus) that give rise to a positive effect of volatile marketing spending on sales (e.g., Simon 1982; Freimer and Horsky 2012). Our specification is very general in that we do not make any assumption about the specific demand conditions that lead to higher sales from expenditure volatility. We assume diminishing returns to scale, i.e. $Q''_{Var}\left[Var\left(MKT\right)\right] < 0$, if the marginal effect is strictly positive.

Using a linear Taylor series approximation with mean expenditures levels μ and μ_c for own and competitive expenditures, respectively, and θ for an arbitrary variance level of own marketing expenditures as expansion points gives:

$$\begin{aligned} Q\left[MKT, CMKT, Var\left(MKT\right)\right] \cong\; & Q\left[\mu, \mu_c, \theta\right] + Q'\left(\mu\right)MKT - Q'\left(\mu\right)\mu + Q'_c\left(\mu_c\right)CMKT \\ & - Q'_c\left(\mu_c\right)\mu_c + Q'_{Var}\left(\theta\right)Var\left(MKT\right) - Q'_{Var}\left(\theta\right)\theta \end{aligned} \tag{1}$$

Revenues, *RV*, and cash flows, *CF*, are given by the following expressions

$$RV = P \cdot Q\left[MKT, CMKT, Var\left(MKT\right)\right] \tag{2}$$

$$CF = \left(P - C\right) \cdot Q\left[MKT, CMKT, Var\left(MKT\right)\right] - MKT\ , \tag{3}$$

where *P* measures unit price and *C* denotes unit cost. From Equation (1) together with (2) and (3), we obtain the variance of revenues

$$\begin{aligned} Var\left[RV\left(MKT, CMKT\right)\right] \cong\; & P^2\left[Q'\left(\mu\right)\right]^2 Var\left(MKT\right) + P^2\left[Q'_c\left(\mu_c\right)\right]^2 Var\left(CMKT\right) \\ & + 2P^2 \rho\, Q'\left(\mu\right) Q'_c\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}}, \end{aligned} \tag{4}$$

and the variance of cash flows

$$
\begin{aligned}
Var\left[CF\left(MKT,CMKT\right)\right] \cong \left[\left(P-C\right)Q'\left(\mu\right)-1\right]^2 Var\left(MKT\right)+\left(P-C\right)^2\left[Q_c'\left(\mu_c\right)\right]^2 Var\left(CMKT\right) \\
+2\left(P-C\right)^2 \rho Q'\left(\mu\right)Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}},
\end{aligned}
\tag{5}
$$

where ρ measures the correlation between own and competitive marketing expenditures. Note that, while marketing expenditure volatility may impact sales, it has no relevance for deriving the variance equations above. The variances of revenues, cash flows, and marketing expenditures are based on the same time span. Hence, there is no variation in $Var(MKT)$.

From Dorfman and Steiner (1954), we know that the profit-maximizing marketing budget must satisfy the first-order condition $MKT^* = \varepsilon^*\left(P-C\right)Q^*$, where the asterisk indicates that variables are at their optimum. This relation also holds in a competitive Nash equilibrium (Fischer et al. 2011), where ε^* and Q^* reflect equilibrium values and depend on equilibrium competitive expenditures as defined in (1). We will use μ^*, the optimal equilibrium mean expenditure level, as a useful reference point in the subsequent analysis. Let us also introduce $\tilde{\mu}$, the near-optimal expenditure level that is derived from current parameter values according to

$$
\tilde{\mu} = \varepsilon\left(P-C\right)Q \tag{6}
$$

Fischer et al. (2011) show that, using this relation as a periodic rule to determine the optimal budget under Nash competition, $\tilde{\mu}$ quickly converges to the true optimum. In addition, we use the coefficient of variation as a normalized measure of the volatility of own and competitive marketing expenditures. They are defined as $CV = SD(MKT)/\mu$ and $CV_c = SD(CMKT)/\mu_c$, where CV denotes the coefficient of variation and SD is the standard deviation.

Finally, we assume that unit profit contribution and mean expenditure levels for own and competitive marketing are always strictly positive, i.e. $(P\text{-}C), \mu, \mu_c > 0$, and therefore $Var(CMKT) > 0$ and $Var(MKT) > 0$. We also assume $Q'(MKT) \neq 0$ and $Q_c'\left(CMKT\right) \neq 0$.

Effects on Revenue Volatility. We derive the following propositions on revenue volatility.

PROPOSITION 1A. *Ceteris paribus, a higher variance of own expenditures increases the variance of revenues if* $\varepsilon CV > -\rho\varepsilon_c CV_c$.

PROPOSITION 1B. *Ceteris paribus, a higher mean level of own expenditures decreases the variance of revenues if* $\varepsilon CV > -\rho\varepsilon_c CV_c$.

PROPOSITION 1C. *Ceteris paribus, a higher marketing responsiveness increases the variance of revenues if* $\varepsilon CV > -\rho\varepsilon_c CV_c$.

PROOFS. See Appendix. □

Apparently, the postulated effects of revenue volatility depend on the condition that $\varepsilon CV > -\rho \varepsilon_c CV_c$. At first glance, this result may appear surprising, as revenues always increase with own expenditures, albeit at a decreasing rate. Hence, we would expect that higher own expenditure variance also always translates into higher revenue variance, consistent with the monotonic shape of the response function. In fact, the intuition is not wrong if we consider a situation without competition. Then, $\rho = 0$, and we have the condition $\varepsilon CV > 0$, which is always satisfied since ε and $CV > 0$. Under competition, however, revenues are also affected by competitive actions. Higher expenditure volatility, as an example, does not necessarily increase the volatility of revenues but may in fact *decrease* it. Whether this situation arises depends on the type and intensity of competitive interaction.

We note that there is always a positive effect on revenue volatility if competitive behavior is accommodating ($\rho < 0$) and cross-effects are substitutive ($\varepsilon_c < 0$), or if competitive behavior is retaliatory (counteractive) ($\rho > 0$) and cross-effects are market-expanding ($\varepsilon_c > 0$). The reality in many competitive markets, however, is that cross-effects are substitutive ($\varepsilon_c < 0$) and competitive interaction is retaliatory ($\rho > 0$). A (counterintuitive) negative effect on revenue volatility does occur in that situation if $-\rho \varepsilon_c CV_c > \varepsilon CV$. This inequality implies that the demand-effective volatility of competitive expenditures, as represented by $\varepsilon_c CV_c$, must be higher than the demand-effective volatility of own expenditures, εCV. It increases with the strength of the cross-effect and the normalized variance of competitive expenditures. In addition, own and competitive expenditures must be positively correlated. This competitive interaction is the reason why a higher variance in own expenditures entails a competitive reaction that may overcompensate the volatility induced by own expenditure volatility.

Effects on Cash-Flow Volatility. The results on revenue volatility cannot be automatically transferred to cash-flow volatility since an increase (decrease) in revenues is also associated with an increase (decrease) in costs.

PROPOSITION 2A. *Ceteris paribus, a higher variance of own expenditures increases the variance of cash flows if* $\left(\frac{\tilde{\mu} - \mu}{\tilde{\mu}}\right)^2 > -\rho \frac{\varepsilon_c CV_c}{\varepsilon CV}$.

PROOF. See Appendix. □

Cash-flow volatility always increases if $\rho = 0$, i.e., if there is no competitive interaction. Consistent with the effect on revenue volatility, however, a positive effect on cash-flow volatility is not universally guaranteed under regular competitive conditions ($\rho > 0$ and $\varepsilon_c < 0$).

COROLLARY 1. *Under regular competitive conditions (ρ > 0 and ε_c < 0), there is always an expenditure level close enough to the optimal Dorfman-Steiner level when higher variance of own expenditures leads to a lower variance of cash flows.*

PROOF. See Appendix. □

This result can be explained intuitively from the flat maximum principle (e.g. Tull et al. 1986), i.e. we know that the cash-flow curve is flat around the optimum. A large variation of marketing expenditures is associated with only a small variation in cash flows. While cash-flow variance always increases if competitors do not react, retaliatory behavior and substitutive effects can overcompensate changes in cash flows if they are small, as is the case around the maximum. As a result, cash-flow variance decreases.

PROPOSITION 2B. *Ceteris paribus, the variance of cash flows follows a U-shape with higher mean levels of marketing expenditures if $\varepsilon CV > -\rho \varepsilon_c CV_c$.*

PROOF. See Appendix. □

Mathematically, this proposition implies that the first derivative of Equation (5) has a root, which defines the minimum of cash-flow variance. In contrast to the variance of revenues, the relationship between the variance of cash flows and the mean expenditure level is no longer monotonic. The following corollary characterizes this relationship more precisely.

COROLLARY 2. *Under regular competitive conditions (ρ > 0 and ε_c < 0), the variance of cash flows starts to increase at a level lower than the optimal Dorfman-Steiner level.*

PROOF. See Appendix. □

Interestingly, under regular competitive conditions, the optimal Dorfman-Steiner level of marketing expenditures is associated with lower variance in revenues but higher variance of cash flows, compared to a lower expenditure level. Note that the Dorfman-Steiner theorem ignores the effects of expenditure volatility on sales and costs. We extend this theorem later and derive a different optimal mean expenditure level. Corollary 2 still holds under these conditions.

PROPOSITION 2C. *Ceteris paribus, a higher marketing responsiveness increases the variance of cash flows if $\mu < \tilde{\mu}\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV$ and $\varepsilon CV > -\rho \varepsilon_c CV_c$. For $\mu > \tilde{\mu}\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV$, the variance of cash flows decreases with a higher marketing responsiveness.*

PROOF. See Appendix. □

Proposition 2C states that the effect of an increased marketing responsiveness on cash-flow volatility depends on the level of marketing expenditures. In fact, this interaction effect with the level of marketing expenditures is non-monotonic. While cash-flow volatility generally increases with higher marketing responsiveness, this relation turns into the opposite at a point close to the optimal expenditure level. One explanation for this effect is that every additional dollar spent beyond the optimal level incurs a loss. The loss, however, is less the greater the responsiveness of demand, i.e. the cash-flow function is less steep. Therefore, (negative) cash flows vary to a lesser extent with expenditures beyond the profit-maximizing level if sales responsiveness is larger.

Impact of Competitive Marketing and Interaction on Performance Volatility

We now turn our focus to two effects that arise from competitive interaction. Specifically, we consider the impact of the volatility of competitive expenditures and the correlation between own and competitive expenditures on revenue and cash-flow volatility.

Competitive-expenditure volatility. The effects of competitive-expenditure variance are the same on revenue and cash-flow variance. The conditions for the direction of the effects, however, are different depending on the type of cross-effect. Specifically, we specify the following conditions under which propositions 3A and 3B hold:

$$\textit{If } \varepsilon_c < 0 \textit{ and } \varepsilon_c CV_c < -\rho\,\varepsilon\, CV \textit{, then} \tag{7a}$$

$$\textit{If } \varepsilon_c > 0 \textit{ and } \varepsilon_c CV_c > -\rho\,\varepsilon\, CV \textit{, then} \tag{7b}$$

PROPOSITION 3A. *Ceteris paribus, a higher variance of competitive expenditures increases the variance of own revenues.*

PROPOSITION 3B. *Ceteris paribus, a higher variance of competitive expenditures increases the variance of own cash flows.*

PROOFS. See Appendix. □

The effects of competitive-expenditure volatility are symmetric to the effect of own-expenditure volatility on revenue volatility (see proposition 1A again). Whether the variance of revenues and cash flows increases with higher competitive-expenditure variance depends on the strengths of demand-effective volatilities and the type and intensity of competitive interaction. If there is no interaction, i.e. $\rho = 0$, we have the apparent result that volatility in our focal variables always increases. It does not depend on the direction of the cross-effect because variance itself has no directional meaning. The picture changes when we consider a situation with competitive interaction. Under regular competitive conditions ($\rho > 0$ and $\varepsilon_c < 0$), both sides of inequality (7a) are positive. It is not guaranteed that this inequality always holds. Hence, there may be conditions when a greater variance in competitive expenditures in fact *decreases* the variance of own revenues and cash flows, which is counterintuitive but a direct implication of propositions 3A and 3B.

How can we explain this finding? If own effect and (substitutive) cross-effect are in opposition to each other, a retaliatory firm behavior may result in a competitive reaction that overcompensates the volatility induced by competitive-expenditure volatility. Such an outcome is more likely to occur if the cross-effect is small relative to the own effect and if competitive interaction is strong (i.e., $\rho \to 1$). To see this, reverse the inequality condition (7a) and rearrange it to $|\varepsilon_C|/\varepsilon < \rho\, CV/CV_C$. A smaller ratio $|\varepsilon_C| / \varepsilon$ and a larger ρ are more likely to satisfy this inequality.

Competitive interaction. The propositions on the effects of the correlation between own and competitive marketing expenditures on revenue and cash-flow volatility are identical.

PROPOSITION 4A. *Ceteris paribus, a stronger (positive) correlation between own and competitive marketing expenditures increases the variance of revenues if* $\varepsilon_c > 0$. *The variance of revenues decreases if* $\varepsilon_c < 0$.

PROPOSITION 4B. *Ceteris paribus, a stronger (positive) correlation between own and competitive marketing expenditures increases the variance of cash flows if* $\varepsilon_c > 0$. *The variance of cash flows decreases if* $\varepsilon_c < 0$.

PROOFS. See Appendix. □

Our last propositions state that an increase (decrease) in retaliatory (accommodating) competitive behavior ($d\rho > 0$) increases the variance in revenues and cash flows if cross-effects are market-expanding. It decreases volatilities of the focal variables if cross-effects are substitutive. These results follow directly from the properties of the response function. Substitutive competitive expenditures, for example, reduce own sales and therefore compensate an increase in sales due to larger own expenditures. If competitive expenditures follow own expenditures more closely, i.e. ρ is higher, the compensation effect is greater and variance in sales declines.

Summary of Propositions on Brand Performance Volatility

Table 1 summarizes our propositions on brand performance variance. These propositions characterize the performance volatility effects under general conditions, i.e. we do not make any specific assumption about the structure of demand, competition, or rational firm behavior. Our theoretical analysis reveals a number of important, sometimes counterintuitive insights. First, we note that the variance, the level, and the sales responsiveness of own expenditures do impact the volatility of revenues and cash flows. Second, the effects are different for revenue and cash-flow volatility. Since marketing expenditures both positively and negatively affect cash flows, the relationship with the variance of cash flows is often non-monotonic. Third, while the direction of the volatility effects is usually well defined for a situation without competitive interaction, it is not at all clear under competitive conditions, producing sometimes counterintuitive results. Generally, under regular competitive conditions with substitutive cross-effects and retaliatory firm behavior, the direction of volatility effects depends on the magnitude of demand effects and the intensity of competition. Particularly, a larger variance in own and competitive marketing expenditures may lead to less volatile revenues and/or cash flows. This result is consistent with the observation that market shares are surprisingly stable in many FMCG markets despite the heavy intensity of promotional and advertising activities.

== Insert Table 1 about here ==

Optimal Mean Expenditure Level and Volatility

We now extend our analytical model to derive general optimality conditions that account for performance volatility effects. Following our previous analysis, we adopt a theoretical modeling approach. Our theoretical normative model represents the set of assumptions that we used to describe the marketing environment at the outset and identifies conditions under which the objective function is optimized. We assume that the firm decides about its optimal marketing spending policy, which we characterize in terms of its mean, μ, and variance, σ^2_{MKT} [$\triangleq$ *Var*(*MKT*)]. While our theoretical model may not inform about the exact structure of the optimal spending plan, it does not require specifying a particular demand function and thus allows for truly generalizable results about the optimality conditions.

Consistent with our introductory discussion of the cost implications of performance volatility, we introduce *w*, which measures the cost of one additional unit of revenue variance, and *r*, which measures the financing cost of one additional unit of cash-flow variance. We assume *w* and *r* to be constant. Thus, *w* and *r* measure the marginal cost of revenue and cash-flow volatility.

Assume management wants to maximize profit Π for its brand and sets the marketing budget independently of its competitors by taking the competitor budgets as given (Nash competition):

$$\max_{\mu,\sigma^2_{MKT}} \quad \Pi = \left(P-C\right)Q\left(\mu,\sigma^2_{MKT},CMKT\right)-\mu-w\sigma^2_{RV}-r\sigma^2_{CF}-f\ , \tag{8}$$

where σ^2_{RV} and σ^2_{CF} denote variance in revenues and cash flows, respectively, and *f* measures fixed cost. The following first-order conditions need to be satisfied in a competitive equilibrium (see Appendix):

$$\frac{\partial \Pi}{\partial \mu} = \left(P-C\right)\frac{\partial Q}{\partial \mu} - 1 - \left[w + rm\left(m + \rho_{Q,MKT}\frac{\sigma_{MKT}}{\sigma_{RV}}\right)\right]\frac{\partial \sigma^2_{RV}}{\partial \mu} = 0\ , \tag{9a}$$

$$\frac{\partial \Pi}{\partial \sigma^2_{MKT}} = \left(P-C\right)\frac{\partial Q}{\partial \sigma^2_{MKT}} - \left(w + rm^2\right)\frac{\partial \sigma^2_{RV}}{\partial \sigma^2_{MKT}} - r\left(1 - m\rho_{Q,MKT}\frac{\sigma_{RV}}{\sigma_{MKT}}\right) = 0\ , \tag{9b}$$

where m measures the profit margin [in percent], $\rho_{Q,MKT}$ represents the correlation between unit sales and own marketing expenditures, and all other terms are defined as earlier.

Note that $MKT^* = \mu^* = \varepsilon^*_{Q,\mu}\left(P-C\right)Q^*$ defines the classical Dorfman-Steiner (DS) solution for the optimal marketing expenditure level, where the asterisk means that variables are at their optimum. Since the classical theorem does not consider the effects of expenditure volatility, there are no results on that decision variable. Based on the conditions (9a) and (9b), we can characterize the optimal mean expenditure level and variance relative to the DS result. In the following, we assume $w > 0$ and $r = 0$ when deriving these optimality results. This assumption is not very restrictive and helps to isolate the differences with respect to the DS solution. Indeed, compared with the increased cost due to revenue volatility, which may be as large as 25% according to Lee, Padmanabhan, and Whang (1997), the pure additional financing cost due to cash-flow volatility is negligible.[1] The essential insights do not change if we relax this assumption. For the optimal mean expenditure level under volatile marketing spending, we obtain the following general result:

THEOREM 1.
$$\underbrace{\frac{\mu^*}{\left(P-C\right)Q^*} = \varepsilon^*_{Q,\mu}}_{\text{Dorfman-Steiner solution}} \underbrace{- \varepsilon^*_{Q,\sigma^2_{MKT}}\left(\frac{\varepsilon^*_{\sigma^2_{RV},\mu}}{\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}}\right)}_{\text{Volatility markup}}.$$

PROOF. See Appendix. ☐

The term $\varepsilon^*_{Q,\sigma^2_{MKT}}$ measures the elasticity of sales w.r.t. expenditure volatility, $\varepsilon^*_{\sigma^2_{RV},\mu}$ represents the elasticity of revenue volatility w.r.t. expenditure level and $\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}$ measures the elasticity of revenue volatility w.r.t. expenditure volatility. $\varepsilon^*_{Q,\sigma^2_{MKT}}$ is always greater than zero because $Q'\left(\sigma^2_{MKT}\right) > 0$. From propositions 1A and 1B, it follows that $\varepsilon^*_{\sigma^2_{RV},\mu} \big/ \varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}} < 0$. Hence, we derive the following proposition.

PROPOSITION 5A. *Provided the impact of variance of own marketing expenditures on brand sales is positive, the optimal mean expenditure level is always higher than the optimal Dorfman-Steiner level if the firm follows a volatile marketing expenditure policy.*

PROOF. As shown above. ☐

THEOREM 1 is a generalization of the DS theorem that takes the effects of volatile marketing spending, e.g., advertising pulsing, into account. If expenditure volatility has no effect on sales,

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[1] Our illustrative example in the Appendix implies that r ≈ .001 US$. In addition, treasury management may try to lower this cost even further by diversification. Given the optimal expenditure variance for each brand, management could coordinate the expenditure plans for the brands in a way that overall cash-flow volatility is reduced.

$\varepsilon^*_{Q,\sigma^2_{MKT}} = 0$, the expression reduces to the classical DS solution. If $Q'\left(\sigma^2_{MKT}\right) > 0$ the optimal budget is always higher. Various performance volatility effects define the magnitude of the increase. Ceteris paribus, the increase is higher when sales respond strongly to expenditure volatility. The increase is smaller if this volatility translates into higher revenue volatility. This influence, however, is alleviated by the responsiveness of revenue volatility to the expenditure level. Interestingly, the marginal cost of revenue volatility does not play a role in determining the optimal budget according to THEOREM 1.

To summarize, if expenditure volatility does have an incremental sales effect, marketing management should operate at a *higher spend* level compared to a situation where it does not have an incremental effect. Hence, despite the additional volatility cost, the increase in sales effectiveness due to expenditure volatility is synergistic. This result is mainly driven by the fact that a higher expenditure level attenuates the cost increase from expenditure volatility, $\varepsilon^*_{\sigma^2_{RV},\mu} < 0$ (see proposition 1B).

For the optimal level of expenditure volatility, we obtain the following theorem and proposition:

THEOREM 2. $$\left(\sigma^2_{MKT}\right)^* = \begin{cases} \dfrac{\left(P-C\right)Q^*}{w}\left(\dfrac{\varepsilon^*_{Q,\sigma^2_{MKT}}}{\partial\sigma^2_{RV}/\partial\sigma^2_{MKT}}\right) & \text{if } \partial\sigma^2_{RV}/\partial\sigma^2_{MKT} > 0 \\ 0 & \text{else.} \end{cases}$$

PROPOSITION 5B. *Provided the impact of own marketing expenditures on brand sales is positive, a volatile expenditure policy is always optimal if there is no competitive interaction among firms. In all other cases, a volatile marketing expenditure policy is only optimal if* $\dfrac{\sigma^*_{MKT}}{\sigma^*_{CMKT}} > -\rho^* \dfrac{\varepsilon^*_C}{\varepsilon^*}\dfrac{\mu^*}{\mu^*_C}$.

PROOF. Note the variance of marketing expenditures in THEOREM 2 is strictly positive if $\partial\sigma^2_{RV}/\partial\sigma^2_{MKT} > 0$. This implies that the inequality condition in proposition 1A must hold. Rearranging terms leads to the condition as stated in proposition 5B. If there is no competitive interaction among firms, then $\rho = 0$ and therefore the condition is always satisfied. ☐

Proposition 5B highlights that a volatile marketing expenditure policy is not optimal under all circumstances, even though sales may respond strongly to expenditure volatility. Provided expenditure volatility positively impacts sales, firms should always employ a volatile policy if there is no competitive interaction. But if they actively compete with each other, the resulting equilibrium budgets and expenditure volatilities need to satisfy the condition in 5B. Note that this condition is equivalent to the condition in proposition 1A. This, in turn, implies that the counterintuitive negative effect of own expenditure volatility on revenue volatility cannot occur in a market where firms follow a rational Nash behavior. We discuss further implications of optimal behavior for the performance volatility effects in more detail in the next section.

THEOREM 2 also shows that the optimal variance in own marketing expenditures increases with its relative impact on sales, but it decreases in the marginal cost of revenue volatility and the marginal effect of expenditure volatility on revenue volatility. It emphasizes our core message: volatile marketing spending may offer an opportunity to increase sales effectiveness. However, it is also important to consider the extra costs of such behavior, which have typically been ignored.

Effects on Brand Performance Volatility under Rational Firm Behavior

We now revisit the brand performance volatility effects of Table 1 by assuming that firms follow a rational, competitive Nash behavior (proofs are provided in the Appendix). Table 2 shows that the volatility effects can be quite different from those derived under general conditions (see Table 1 again). Higher own-expenditure variance and marketing responsiveness *always* increase revenue volatility. A higher mean level of expenditures *always* lowers revenue volatility. The counterintuitive finding that greater own-expenditure volatility and higher responsiveness may reduce the variance of revenues is therefore *not consistent* with rational firm behavior.

We also note there are fewer restrictions on the relations between expenditure level and responsiveness, respectively, and cash-flow variance. The relation between cash-flow variance and expenditure level always follows a U-shape. The direction of the effect of responsiveness on cash-flow variance depends on the expenditure level and is consistent with an inverted U-shape. The variance of cash flows increases with higher responsiveness for lower expenditure levels. However, the direction of the effect changes for higher expenditure levels, such as those beyond the optimal level, because higher responsiveness shields against losses from overspending. Finally, we note that, consistent with Table 1, larger expenditure volatility usually increases volatility in cash flows, but not for all expenditure levels. Corollary 1 still holds under rational firm behavior, i.e. cash-flow variance declines if the expenditure level is very close to its optimal level.

== Insert Table 2 about here ==

Consistent with the results under general conditions, our competitor behavior variables impact the variances of revenues and cash flows in the same direction. A stronger correlation of own and competitive expenditures decreases (increases) brand performance volatility if the cross-effect is negative (positive). The direction of the effect of competitive expenditure volatility also depends on the sign of the cross-effect. In addition, we need to consider the type of competitive interaction. Consistent with intuition, brand performance volatility always increases for positively (negatively) correlated own and competitive expenditures and market-expanding (substitutive) cross-effects and if expenditures are not correlated at all. For substitutive (market-expanding) cross-effects and retaliatory (accommodating) competitive interaction, however, performance volatility may *increase or decrease* with higher competitive expenditure volatility. Whether a negative volatility effect exists depends on the magnitude of demand effects and the intensity of competitive interaction. It is more likely for relatively small, substitutive cross-effects and strong competitive interaction. Most importantly, this result is consistent with rational firm behavior. In the subsequent empirical analysis, we predict and find a negative impact of competitive-expenditure volatility on performance volatility.

Extension to Dynamic Sales Effects

Brand Performance Volatility Effects. We have considered only static problems so far. However, marketing expenditures frequently involve carryover effects. The Nerlove-Arrow (1962) model provides a parsimonious but powerful way to model marketing dynamics. Let S denote the brand's own marketing stock and S_C the competitive marketing stock, respectively, and let sales be expressed in terms of these stock variables, i.e. $Q[S, S_C, Var(MKT)]$. The marketing stock in period t evolves according to the process:

$$S_t = \lambda S_{t-1} + MKT_t, \ \text{ with } 0 \le \lambda \le 1, \tag{10}$$

where λ measures the carryover coefficient and all other terms are defined as earlier. We assume the same process for competitive expenditures, though the carryover coefficient might be different. It is straightforward to show that the structure of the variance equations (4) and (5) does not change. The only difference is that variances, means, and responsiveness parameters now refer to marketing stocks instead of expenditures. For this reason, all propositions and corollaries derived earlier still hold; they are just expressed in stock quantities. Most importantly, they also hold with respect to expenditures because the mean, the variance, and the responsiveness of a marketing stock are only a scaled version of the respective expenditure quantities (see Appendix for the proofs). For example, consider the variance of own marketing stock:

$$Var\left(S\right) = \frac{1}{1+\lambda^2 - 2\lambda\rho_{AR(1)}} Var\left(MKT\right), \tag{11}$$

where $\rho_{AR(1)}$ denotes the autocorrelation coefficient of the stock variable.

Optimal marketing spending. It can also be shown that the propositions on the optimal levels of marketing expenditures and volatility do not change under the assumption of a dynamic sales response function. Given the process of goodwill accumulation and depreciation, we assume that the firm maximizes the discounted profit under Nash competition. The optimal policy can be found by applying the calculus of variations to solve the dynamic optimization problem (see Appendix).

While Theorem 2 does not change, the optimal level of marketing expenditures needs to satisfy the following condition (extended form of Theorem 1):

$$\mu^*_{long-term} = \underbrace{\frac{(P-C)Q^*\varepsilon^*_{Q,MKT}}{\phi+d}}_{\text{Dynamic Dorfman-Steiner solution}} - \underbrace{\frac{\varepsilon^*_{\sigma^2_{RV},MKT}(P-C)Q^*\varepsilon^*_{Q,\sigma^2_{MKT}}}{(\phi+d)\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}}}_{\text{Dynamic volatility markup}}, \tag{12}$$

where ϕ measures the decay coefficient of the differential equation for the marketing stock and d is the discount rate. If $\phi = 1$ (there is no marketing carryover) and $d = 0$ (no discounting), expression (12) reduces to Theorem 1, the optimal expenditure level of the static case. An extension to the Dorfman-Steiner solution under dynamic profit maximization is given by the first term. The second term measures the markup if we take the effect of expenditure volatility on sales into account ($\varepsilon^*_{Q,\sigma^2_{MKT}} > 0$). Again, the markup results in a larger optimal budget under volatility consideration compared to the Dorfman-Steiner solution. Hence, our major result from the static case extends to the dynamic case.

Data

We use data from several pharmaceutical markets to test our propositions and estimate the magnitude of the performance volatility effects under real market conditions. Data on prescription drugs from two therapeutic areas (cardio-vascular and gastro-intestinal) that cover four product categories are available. Two categories, calcium channel blockers and ACE inhibitors, comprise drugs for the treatment of cardio-vascular diseases. Drugs in the two other categories, H2 antagonists and proton pump inhibitors, are used in gastro-intestinal therapies. These four categories are among the largest prescription-drug categories. They differ in their therapeutic principles to treat diseases like hypertension or acid related gastro-intestinal disorders. Data, collected by IMS Health, are available on a quarterly basis for a time period of 10 years (1987-1996) covering the growth and maturity phases of the analyzed categories. They include unit sales (normalized over different application forms of the drug and transformed into daily dosages by a brand-specific dosage factor), revenues, and aggregate marketing expenditures on detailing, journal advertising and other communications media. Detailing has the lion's share in expenditures with more than 90%. Monetary values are in 1996 US$ and have been deflated by country-specific consumption price indices. The data cover four European countries: France, Germany, Italy, and the UK, and comprise sixteen product markets (4 categories × 4 countries). We analyze data on 99 brands, which were marketed by 26 pharmaceutical firms.

Table 3 shows the descriptive statistics of the variables used in the estimation equations. We provide variable correlations in the Appendix. Revenues average about $9.2 million per quarter, cash flows are about \$5.0 million, and average marketing spending amounts to about \$1.0 million. There is also considerable variation in the data across brands and time, as indicated by the standard deviations and the volatility measures in Table 3. Volatility is particularly high with respect to marketing spending. Moving variance is about \$151.1 million (or \$0.4 million in terms of standard deviation) and moving range is about \$0.8 million, virtually as high as the mean spending. We report on the operationalization of these variables subsequently. Plots of marketing spending over time (not shown) reveal substantial volatility for many brands in our sample.

=== Insert Table 3 about here ===

A groupwise analysis provides first evidence on the validity of our theoretical findings (see Table 4). For this purpose, we build two groups of brands with either low or high values for our volatility driver variables. A brand is assigned to the "low" ("high") group if the value for the respective variable is below (above) the sample average. T-tests on the difference between group means show that the variance of revenues and cash flows differs significantly ($p < .05$) between the two groups for all but one variable (level of expenditures). The differences are consistent with our results from the theoretical analysis.

=== Insert Table 4 about here ===

Methodology

In order to test our propositions 1A to 4A and quantify the magnitude of performance volatility effects under real market conditions, we estimate two types of models: (1) a brand sales model and (2) a volatility model. The brand sales model is an auxiliary model that provides input for the volatility model, which we eventually use to test our propositions.

Step 1. We apply the brand sales model to our sample and estimate sales effects of own and competitive marketing expenditures. Together with other sample characteristics these sales effects help predict the performance volatility effects. In principle, we could use the calibrated brand sales model to test our propositions. However, this test is not very powerful, as using the estimated response coefficients results in predictions for volatility effects that are subject to large standard errors.

Step 2. We therefore set up volatility models for revenues and cash flows that directly measure the postulated performance volatility effects. Specifically, we regress both revenue and cash-flow volatility on our focal predictor variables such as marketing-expenditure volatility. The estimated response coefficients from these models provide the basis for testing our propositions. Estimation results from the first step are incorporated into the volatility models in two ways. First, the responsiveness estimates are used as predictor variables. Second, we use the brand sales model to remove the effects of exogenous factors such as seasonality and trend from the brand sales time series. Such factors are outside the control of management and are therefore not relevant for the study of marketing spending impact on volatility. Brand expenditures are not subject to trend or seasonality, as revealed by specification tests.

Market Response Model

Specification. Following recent research on pharmaceuticals (e.g., Fischer and Albers 2010), we specify a log-log sales response model for each of the two therapeutic areas (cardio-vascular drugs and gastro-intestinal drugs). Let sales of drug $i \in I_k$, with I_k as country-specific index set, in country $k \in K$, with K=4, and in period $t \in T_i$, with T_i as brand-specific index set, be defined as follows:

$$\begin{aligned} \ln Q_{ikt} &= \alpha_{0ik} + \alpha_{1ik} \ln MKT_{ikt} + \alpha_{2ik} \ln MKT_{ikt-1} + \alpha_{3k} \ln CMKT_{ikt} + \alpha_{4k} \ln CMKT_{ikt-1} \\ &+ \alpha_{5k} \ln GDP_{kt} + \alpha_{6k} ET_{ikt} + \sum\nolimits_{l=1}^{K} \sum\nolimits_{h=1}^{H-1} \beta_{lh} SD_{ht} \times CTY_{lk} + u_{ikt}, \\ &\text{with } u_{ikt} = \varphi_1 u_{ikt-1} + \varphi_2 u_{ikt-2} + \eta_{ikt}, \text{ and } \eta_{ikt} \text{ i.i.d. } N\left(0, \sigma_\eta^2\right), \end{aligned} \tag{13}$$

where GDP measures the gross domestic product, ET denotes the elapsed time since launch of the brand, SD is a quarterly seasonal dummy variable, CTY is a country dummy variable, and all other terms are defined as earlier. The disturbance term u shows an autoregressive structure of second order, where φ is an autocorrelation coefficient, and η is a white-noise error term with zero mean and variance σ_η^2. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are parameter vectors to be estimated.

We tested several alternative response models such as a linear model and a semi-log model. We also estimated an S-shaped model that allows for saturation and extended our log-log model by a differential stimulus variable that captures any extra demand lift due to expenditure volatility (Simon 1982). Based on the Schwartz Information Criterion and Davidson-McKinnon comparative test (Greene 2006), we find that specification (10) best represents our data.

Our brand sales model includes variables that are relevant to the international markets over the ten-year sample period. Specifically, it incorporates own and competitive marketing expenditures, including lagged effects. To account for substitution effects across categories within a therapeutic area, we treat brands from other categories as competitors. The coefficients associated with previous quarter's own and competitive marketing expenditures capture lagged effects. This is consistent with prior findings that pharmaceutical marketing effects unfold over six months (Mizik and Jacobson 2004). In addition, seasonal dummies are used to capture sales dynamics, a trend variable (elapsed time since launch of a brand) is added to control for life-cycle effects, a country's gross domestic product (GDP) is used as a proxy of the overall economic condition of a country, and finally the autoregressive error term is added to capture inertia in sales (Hanssens et al., 2001).[2] We account for brand heterogeneity in demand, e.g., quality, brand equity, order of entry, by estimating brand-specific fixed effects. *Distribution* and *price are not relevant* variables in our context. In the European countries covered by our data, pharmacies are required to list every approved drug, resulting in 100% distribution for the drugs in our sample. Prices were highly regulated during the observation period and therefore not used as a tactical marketing instrument. There is only meaningful cross-sectional variation in prices that is captured by the brand-specific fixed effects.

Estimation and Endogeneity Issues. We estimate the brand sales model (13) with generalized least squares (GLS) to account for the specific error structure. We also test whether marketing expenditures can be treated as exogenous variables. If not, estimates will be biased and alternative estimators such as instrumental variables (IV) estimators should be employed. The drawback of the IV estimator is that it yields less efficient results and thus reduces the power of our tests.

The endogeneity of marketing expenditures could have several sources. A main source is the allocation of scarce marketing resources across brands at the portfolio level. Larger and more responsive brands tend to attract more marketing resources. In our empirical design, we effectively control for this endogeneity source by specifying brand-specific fixed effects. Since this may not be sufficient we also apply the Hausman-Wu test to our brand sales model. The test requires the use of instrumental variables. We considered cost-side instruments but were not able to obtain data for our observation period dating back 25 years. Following Azoulay (2002), we use the cumulative expenditures on a brand in countries other than the focal country as an instrument that identifies the potentially endogenous expenditure variable. Brand expenditures across countries are correlated because of allocation decisions by the firm. But expenditures in one country should not impact the demand for a drug in a different country.

The validity of an instrument rests on the assumption that it is strongly correlated with the endogenous variable but not with the error term. We check for this in various ways. First, R^2 for the first-stage regressions is high (on average, $R^2 > .40$) and the F-value exceeds the threshold of 10 in 7 of 8 markets suggesting that our instrument is strong (Greene 2006). Second, we acknowledge that our identifying assumption only holds if there are no common demand shocks for a brand across countries. The introduction of competitive brands could be a source for such a demand shock. During our observation period 1987-1996, firms usually used a waterfall strategy and introduced new drugs country by country with substantial delays. Hence, common demand shocks are unlikely to result from this

[2] We are aware of other dynamic specifications of pharmaceutical response models, such as the use of a lagged sales variable (e.g., Fischer and Albers 2010). For this specific dataset of the period 1987-1996, however, specification tests indicated that Equation (13) best represents the expenditure dynamics compared to other specifications (see Appendix for details). Note also that (13) and a model with lagged sales variables, such as the Koyck model, are closely related to each other (see Hanssens, Parsons, and Schultz 2001, Chapter 4).

source. In addition, we applied the Box-Jenkins procedure to decompose brand demand in other countries into its time-series components and isolate demand shocks. Neither the cross-country correlations of these shocks nor the correlations with the endogenous variables and instruments were significant. Third, since we can never be sure whether the exclusion restriction holds, we apply the procedure by Conley, Hansen, and Rossi (2012) to check for the sensitivity of IV estimation results when this restriction is relaxed. We find highly stable estimates for a wide range of relaxations, which strengthens our confidence in the validity of the chosen instrument.

Based on the results of the Hausman-Wu test, we cannot reject the assumption of exogenous marketing expenditures in any of the 8 markets. Hence, we apply GLS to the data to avoid a loss in efficiency that would result from using IV estimation. Note that the same applies to the volatility models because endogeneity there arises only from endogeneity in brand sales models.

Volatility Models

Structural Equations. Let *V*(*REV*) denote the volatility of revenues measured in terms of variance or range, respectively, *V*(*MKT*) represent the volatility of own marketing expenditures, *A*(*MKT*) be the average level of own marketing expenditures, *V*(*CMKT*) denote the volatility of competitive marketing expenditures, *CORR* represent the correlation between own and competitive marketing expenditures, *RESP* denote total marketing responsiveness (= $\alpha_{1ik} + \alpha_{2ik}$), **X** denote a vector including the remaining variables of the brand sales model as specified in Equation (13) (i.e., brand-fixed effects to control for order of entry, quality, etc., trend, seasonality, and *GDP* as a surrogate for general demand), $\boldsymbol{\gamma}$ be a parameter vector to be estimated, and ν be an error term with variance ξ. Omitting brand, country, and time subscripts for the moment, we specify the revenue volatility model as follows:

$$V\left(REV\right)=\gamma_0 V\left(MKT\right)^{\gamma_1} A\left(MKT\right)^{\gamma_2} V\left(CMKT\right)^{\gamma_3} Exp\left(\gamma_4 CORR+\gamma_5 RESP+\mathbf{X}\boldsymbol{\gamma}+\nu\right), \quad \text{with } \nu \sim N\left(0,\xi\right). \tag{14}$$

We assume the relationship between revenue volatility and its drivers to be multiplicative. Thus the variables interact with each other, consistent with the results from the theoretical discussion. The correlation between own and competitive marketing expenditures and the estimated marketing responsiveness parameter appear as part of an exponential function because they may become negative. The parameters $\gamma_{1\text{-}3}$ can be directly interpreted as elasticities and facilitate the comparison of volatility drivers. We subsequently describe how we transform the dataset to remove the *X*-variables and unobserved effects that are reflected by a brand-specific constant, which are not the focus in this study.

Since cash flows are constructed from revenues and costs, revenue volatility enters the cash-flow volatility equation:

$$V\left(CF\right)=\delta_0 V\left(REV\right)^{\delta_1} V\left(MKT\right)^{\delta_2} A\left(MKT\right)^{\delta_3} \operatorname{Exp}\left[\delta_4 A\left(MKT\right)+\upsilon\right], \quad \text{with } \upsilon \sim N\left(0,\psi\right), \tag{15}$$

where $V(CF)$ denotes the volatility of cash flows, $\boldsymbol{\delta}$ is a parameter vector to be estimated, and υ represents an error term with variance ψ. The effects of competitive-marketing-expenditure volatility, competitive reaction, marketing responsiveness, and X-variables on cash-flow volatility are mediated through revenue volatility. In addition, revenue volatility mediates the impact of own expenditures. Since own expenditures also enter the cash-flow equation as cost, we expect an additional direct effect on cash-flow volatility. Finally, note that specification (15) allows for a U-shaped influence of the level of marketing expenditures on cash-flow volatility, consistent with our proposition 2B. This situation occurs if $\delta_3<0$ and $\delta_4>0$. We further allow the error terms to be correlated across the two equations (14) and (15).

Data Transformation. By using the estimates of the brand sales model, we remove the effects of exogenous market factors such as seasonality, trend, and overall economic condition (measured by the GDP), and derive an adjusted unit-sales time-series for each brand. We multiply the unit sales with the brand's unit price and arrive at adjusted brand revenues. We then multiply the adjusted revenues by a cash contribution margin of 85% that is typical for original prescription drugs. From these gross cash flows we subtract the marketing expenditures and arrive at the final variable of adjusted brand cash flows.

The volatility of the adjusted revenues and cash flows is measured by the variance or range of these quantities over a time period of 8 quarters. Consequently, we use the first two available years of sales for each brand as an initialization period. We compute the volatility measure of the subsequent period by dropping the first period and including the information of the following period. We continue until the end of the brand-specific time series and thus obtain a time series of moving volatility measures of adjusted revenues and cash flows (moving-window analysis). This procedure is also applied to compute moving volatilities for own and competitive marketing expenditures and the moving average of own marketing expenditures. We denote moving volatilities with *MV* and moving averages with *MA*.

The application of moving-window analysis is well established in the accounting literature (e.g., Kothari 2001) and is justified for two reasons. First, it increases sample size and therefore improves the power of statistical tests. Note that observations are inevitably lost due to the calculation of the volatility measures. Second, it accounts for possible dynamic effects. Capital markets research has shown that it often takes some time until economic effects have fully materialized in earnings volatility.

Estimation Equations. The use of moving windows is helpful to increase the power of statistical tests due to the increase in degrees of freedom, but it is also likely to generate serially correlated errors in the time series of adjusted revenues and cash flows. We therefore transform expressions (14) and (15) into a series of relative differences. By taking the total differentials of the log-transformed equations (14) and (15), we obtain (see Appendix for details):

$$\frac{\Delta MV\left(AREV\right)_{ikt}}{MV\left(AREV\right)_{ikt-1}}=\gamma_1\frac{\Delta MV\left(MKT\right)_{ikt}}{MV\left(MKT\right)_{ikt-1}}+\gamma_2\frac{\Delta MA\left(MKT\right)_{ikt}}{MA\left(MKT\right)_{ikt-1}}+\gamma_3\frac{\Delta MV\left(CMKT\right)_{ikt}}{MV\left(CMKT\right)_{ikt-1}}+ \gamma_4\Delta MA\left(CORR\right)_{ikt}+\Delta\nu_{ikt}, \tag{16}$$

$$\frac{\Delta MV\left(ACF\right)_{ikt}}{MV\left(ACF\right)_{ikt-1}}=\delta_1\frac{\Delta MV\left(AREV\right)_{ikt}}{MV\left(AREV\right)_{ikt-1}}+\delta_2\frac{\Delta MV\left(MKT\right)_{ikt}}{MV\left(MKT\right)_{ikt-1}}+\delta_3\frac{\Delta MA\left(MKT\right)_{ikt}}{MA\left(MKT\right)_{ikt-1}}+ \delta_4\Delta MA\left(MKT\right)_{ikt}+\Delta\upsilon_{ikt}, \tag{17}$$

where,

$MV(AREV)_{ikt}$	=	Moving volatility of adjusted revenues of brand *i* in country *k* and period *t*
$MV(MKT)_{ikt}$	=	Moving volatility of marketing expenditures of brand *i* in country *k* and period *t*
$MA(MKT)_{ikt}$	=	Moving average of marketing expenditures of brand *i* in country *k* and period *t*
$MV(CMKT)_{ikt}$	=	Moving volatility of marketing expenditures of brand *i*'s competitors in country *k* and period *t*
$MA(CORR)_{ikt}$	=	Moving average correlation between own and competitive marketing expenditures of brand *i* in country *k* and period *t*
$MV(ACF)_{ikt}$	=	Moving volatility of adjusted cash flows of brand *i* in country *k* and period *t*
Δ	=	First-difference operator.

Equations (16) and (17) represent the original equations (14) and (15) in terms of relative differences. Unlike absolute differences, this representation not only removes brand-specific fixed effects and reduces serial correlation, but also controls for brand-size effects. For example, bigger brands are expected to have larger absolute changes in revenues, cash flows and marketing spending.

Equations (16) and (17) establish an equation system with possibly correlated errors across equations. Revenue volatility is the only endogenous variable occurring at the right hand side of Equation (17). Thus, the system is recursive and GLS, which allows for cross-equation error correlation, provides efficient estimates (Zellner 1962). Since first differencing may not completely remove serial correlation, we also allow for equation-specific autocorrelation coefficients in the variance-covariance matrix.

The first-differencing procedure eliminates the time-invariant marketing responsiveness variable that is part of the revenue volatility model (14). To measure its influence, we linearize (14) first via log-transformation and then build a cross-sectional regression model by obtaining averages of all time-varying variables. The resulting equation can be estimated with Ordinary Least Squares (OLS). However, the marketing-responsiveness parameters of the first stage are measured with sampling error that vanishes in the limit. As a consequence, OLS estimates from the second stage regression will be consistent but their standard errors may be biased (Murphy and Topel 1985). Following Nijs, Srinivasan, and Pauwels (2007), we obtain corrected standard errors by a bootstrapping procedure with 10,000 replications. First differencing also eliminates brand-specific factors such as quality that may explain different volatility levels among brands. Note that, together with the procedure to adjust revenues, we have therefore completely removed the impact of the *X*-variables of Equation (14) in our final estimation equations.

Results

Brand Sales Model

The log-log brand sales model describes sales evolution in the markets very well. The average total marketing elasticity equals .10. If weighted by relative standard errors to account for estimation uncertainty it is .19, which is well in line with recently reported results (e.g., Fischer and Albers 2010). Albeit small, the impact of competitive marketing activities is negative, with a mean value of -.01. In general, there is substantial variation in the marketing responsiveness estimates, which we use as a predictor in our volatility models. Particularly, there are several brands/markets that face market-expanding cross-effects. Recall that we use the total effect, which is the sum of current and lagged marketing responsiveness.

Volatility Models

Table 5 shows the estimation results for the revenue and cash-flow volatility models by using either (adjusted) variance or range as dependent variable. Our focal predictor variables explain a substantial part of variance in observed (i.e. unadjusted) revenue and cash-flow volatility in estimation and holdout samples, underlining the relevance of marketing activities for performance volatility. To form holdout samples we excluded the last four quarters (20% of total cases) in the first-difference models and the last 20 brands (20% of cases) in the cross-sectional model.

=== Insert Table 5 about here ===

In the following discussion we focus on variance as a volatility measure and on the results from first-difference models. Since the effect of marketing responsiveness, which does not vary within but across brands, can only be estimated by a cross-sectional model, we also report on the results of the cross-sectional regression model. This model includes time-invariant control variables, such as order of entry, quality, average price, and average time in market. These controls, however, do not add explanatory power to the model ($F_{4, 89}$=.158, p>.10). We note that, due to the missing time variation and the substantially lower number of observations in this model, the effects for the time-varying variables should be interpreted with caution.

According to our propositions 1-4, the direction of volatility effects depends on estimated demand parameters (own and cross-effects) and the correlation and volatility of own and competitive marketing expenditures. We use the sample means of these quantities, together with the general conditions in Table 1, to make predictions about the direction of the effects. These predictions hold for the average brand in our sample. They may be different for a specific brand depending on its set of parameter values.

We first discuss estimates from the revenue volatility model and then turn to the cash-flow volatility model. The volatility of marketing expenditures, measured by their variance, increases the volatility of revenues and supports our first prediction, with an estimated elasticity of .273 ($p<.05$).

The first-difference model also supports our second prediction on the influence of the level of marketing expenditures on revenue volatility; but the coefficient is not significant at $p<.05$. We obtain a significant negative effect from the cross-sectional regression (-1.99, $p<.05$). Note that this variable has been divided by average brand unit sales in order to control for brand-size effects. The effect comes out stronger in a pure cross-sectional regression.

Marketing responsiveness drives revenue volatility (8.11, $p<.05$), supporting our third prediction. The associated elasticity of .811 (=8.11×.10) is substantial. The correlation of own and competitive marketing expenditures shows a significant negative effect on revenue volatility (-.262, $p<.05$). Since the average cross-effect is small but negative, i.e. $\varepsilon_C < 0$, this finding is consistent with our prediction (see Table 1 again).

We find evidence for a negative effect of the volatility of competitive marketing expenditures on revenue volatility. The effect, however, is only marginally significant in the cross-sectional regression (-.222, $p<.10$). This result may seem counterintuitive, but is fully consistent with our theoretical analysis under both general conditions and rational firm behavior. Since we have a substitutive cross-effect ($\varepsilon_C < 0$), on average, a negative performance volatility effect arises if $\varepsilon_C CV_C > -\rho\varepsilon CV$. The estimated average own and cross-effects in our sample are -.01 and .10. Using these values and further sample information from Table 3, we verify that $-.01\times.303 > -.35\times.10\times.369$.

As expected, revenue volatility is an important driver of cash-flow volatility, with an elasticity of 1.36 ($p<.05$). Its lower boundary value is the squared profit margin, which would be achieved if cash flows consisted only of revenues multiplied by the profit margin. The direct effect of the volatility of marketing expenditures is positive and significant, with a value of .535 ($p<.05$). This coefficient represents the volatility effect due to the cost component of marketing expenditures. In order to fully evaluate the predicted effect of expenditure volatility on cash-flow volatility, we need to consider the total effect.

Table 6 displays the total effects in terms of elasticity, which facilitates the interpretation and comparison of the magnitude of effects. The total effect of expenditure volatility on cash-flow volatility amounts to .906 (=1.36×.273+.535; p<.05). Hence, we find strong support for our prediction. Interestingly, this elasticity is more than three times higher than that for revenue volatility. We also find strong support for the expected U-shaped influence of the level of marketing expenditures on cash flows (-2.38, p<.05 and .003, p<.05; see table 6). The direction of the influence of marketing responsiveness on cash-flow volatility is also consistent with our prediction. Its elasticity is high, with a value of 1.10 (p<.05). The volatility effect of the volatility of competitive marketing expenditures is not significant, which may be due to the fact that the estimated cross-effects are rather small and not uniform in sign across all categories. We find, however, support for the expected cash-flow volatility effect of the correlation of own and competitive marketing expenditures, though the associated elasticity is modest (-.123; p<.05).

=== Insert Table 6 about here ===

Both Tables 5 and 6 also show the results for models when we take range instead of variance as a volatility measure. Overall, the results are consistent with the results using variance as a volatility measure.

Robustness of Findings

We performed several analyses to verify the robustness of these results. First, we varied the window of the volatility measures. Instead of 8 quarters we computed volatility measures based on 4 quarters and 12 quarters. The results were similar but model fit deteriorated, underlining that the 8-quarter window is the best choice for our dataset. Second, we created volatility variables that do not overlap over time periods. For example, the first observation of an 8-quarter-based variance variable includes the first 8 quarters, the second observation is based on the subsequent 8 quarters, and so forth. This procedure reduces the sample size to only 292 observations. The results did not change materially, though the standard errors increased. Third, we used the original instead of adjusted time-series for revenues and cash flows to compute volatility measures based on 8-quarter windows. The results are in line with the results from using adjusted time series. However, the standard errors are higher, which is likely due to the increased noise from exogenous market factors. Fourth, we calculated revenue-volatility elasticities based on the estimated demand parameters and Equation (4), i.e. without estimating the separate Equation (16). It turns out that these elasticity estimates are associated with relatively high standard errors. The results are basically the same as those obtained from (16) and shown in Table 6. None of the differences is statistically significant and probably due to the estimation error. Finally, we verified whether the results are influenced by collinearity. The condition indices of the models were well below the critical value of 30 (Greene 2006).

Discussion

Our findings contribute to the advancement of knowledge in marketing as well as general management. Volatility in brand revenues and cash flows has been overlooked in marketing for a long time. However, performance volatility may have substantial negative consequences for the firm, due to excess cost associated with the bullwhip effect or higher capital cost from holding larger cash reserves. Our study is the first to describe marketing's potential to drive performance volatility in an analytic way. We do so by relying on extant market response theory, which allows us to make the formal connection between marketing spending, marketing responsiveness and revenue and cash-flow volatility.

The empirical application on a large dataset from the pharmaceutical industry supports the implications derived from the theoretical analysis. We find broad support for the expected effects under real market conditions. Furthermore, the volatility effects are substantial in a managerial sense, as reflected by the elasticity magnitudes.

While these empirics support our propositions in one important sector of the global economy, replication in other industries would be needed in order to formulate empirical generalizations. We conjecture that the product and competitive setting will have a strong impact on the results. For example, some sectors rely on virtually continuous marketing pressure in order to protect a brand's share of voice and achieve the brand's sales goals, while other sectors have more sporadic marketing spending, e.g. on the occasion of new-product launches. All else equal, we would expect the volatility effects to be stronger in the second scenario.

Managerial Implications

Our study provides insights that invite marketing decision makers to think differently about the consequences of their actions. First, our analysis suggests that higher marketing spending volatility usually leads to a higher volatility of revenues as well as cash flows. The empirical results show that the effects are substantial, and thus should not be neglected. Marketing managers who decide on the timing of media plans, promotion plans, product launches, etc. should be aware that their marketing decisions can influence the volatility of both their top-line and bottom-line performance. Since marketing expenditure costs grow faster than revenues, due to diminishing returns, their impact on cash-flow volatility is larger than on revenue volatility. Second, stronger market response parameters also translate into higher volatility of revenues. Thus, on the one hand, larger response parameters are good news for the marketing manager because his/her expenditures produce higher sales. On the other hand, higher responsiveness has a dark side since it makes revenues and cash flows more volatile, even if spending volatility itself does not change. Third, we find that a higher mean level of marketing expenditures *reduces* revenue volatility, holding spending volatility constant. Higher spending also decreases the cash-flow volatility for typical non-monotonic cash-flow distributions up to a certain level. Finally, the optimal budget under a volatile marketing policy should be higher than the optimal budget under an even-spending policy, provided that marketing volatility does have an additional effect on sales.

Can we derive general managerial recommendations from our study? Setting the optimal levels of marketing expenditures and volatility requires estimating the incremental cost and sales arising from larger revenue and cash-flow volatility (see Theorems 1 and 2 again). This information may not be readily available for various reasons. In such situations, our theoretical and empirical results point to a few general recommendations, which we summarize below.

First, some marketing tactics, such as promotions and advertising campaigns, are used frequently and involve a volatile deployment of the marketing budget. Sometimes these tactics improve a brand's top-line results, sometimes they do not, but in either case, we expect them to have an effect on the volatility of both revenues and cash flows. Since volatility may incur significant additional costs, even revenue-effective volatile marketing tactics may turn out to be harmful to the bottom line. This creates a managerial tradeoff. If the effect of marketing volatility on the level of revenues/cash flows is questionable and cannot be quantified at all, there is no need to increase marketing volatility, and in fact it should be avoided. If the effect on sales is supposedly high, managers need to find the right balance between that positive impact and its negative side effect and may use Theorem 2 as a reference.

Second, and similarly, different brands have different levels of marketing spending and our results show that those with higher spending *levels* enjoy protection against performance volatility, especially cash-flow volatility, so long as their expenditures are economically reasonable, i.e. they are not too far beyond the optimal levels. Since deviations from the optimal budget level do not harm profits too much (as per the flat maximum principle), it seems reasonable to overspend rather than underspend. This is also supported by Theorem 1 to benefit from a potential sales impact of marketing volatility.

Limitations and Future Research

Our research is subject to limitations that may stimulate future research. First, we have quantified the magnitude of volatility drivers in 8 prescription-drug markets. It would be interesting to extend this analysis to other industries. Second, revenue and cash-flow volatility may arise, not only from marketing spending behavior, but also from specific marketing-mix activities such as promotions and new-product introductions. The analytical models to analyze the effects of such activities may be different. Third, our analytical model provides general results on performance volatility effects and optimal mean expenditures and volatility. It would be interesting to develop a decision model that produces more specific insights into optimal volatile marketing policies. A key challenge for such a model is to estimate the cost of revenue volatility, such as those arising from the bullwhip effect. Another challenge is to correctly specify and estimate the demand model that suggests marketing volatility as an optimal policy.

We hope our study will stimulate future research on the relationship between marketing mix variables, performance volatility and its financial consequences from diverse perspectives. Such integration will enable higher-quality resource allocation decisions, for the benefit of the enterprise.

References

Adelman, Daniel and Adam J. Mersereau (2013), “Dynamic Capacity Allocation to Customers Who Remember Past Service,” *Management Science*, 59 (3), 592-612.

Alizadeh, S., M.W. Brandt, and F. X. Diebold (2002) “Range-Based Estimation of Stochastic Volatility Models,” *Journal of Finance*, 57 (3) 1047-1091.

Azoulay, Pierre (2002), “Do Pharmaceutical Sales Respond to Scientific Evidence,” *Journal of Economics & Management Strategy*, 11(4), 551-594.

Conley, Timothy G, Christian B. Hansen, and Peter E. Rossi (2012), “Plausibly Exogenous,” *Review of Economics and Statistics*, 94(1), 260-272.

Dorfman, R. and P. O. Steiner (1954), „Optimal Advertising and Optimal Quality,“ *American Economic Review*, 44 (December), 826-836.

Fischer, Marc and Sönke Albers (2010), “Patient- or Physician-Oriented Marketing: What Drives Primary Demand for Prescription Drugs?,” *Journal of Marketing Research*, 47 (1), 103-121.

Fischer, Marc, Sönke Albers, Nils Wagner, and Monika Frie (2011), "Dynamic Marketing Budget Allocation across Countries, Products, and Marketing Activities,“ *Marketing Science*, 30 (4), 568-585.

Freimer, Marshall and Dan Horsky (2012), “Periodic Advertising Pulsing in a Competitive Market,” *Marketing Science*, 31 (4), 637-648.

Gottfredsen, Mark and Keith Aspinall (2005), “Innovation vs. Complexity, What is Too Much of a Good Thing?,” *Harvard Business Review*, November, 62-71.

Greene, William H. (2006), *Econometric Analysis*. 6th ed., Upper Saddle River, NJ: Prentice Hall.

Hanssens, Dominique M. (1998), “Order Forecasts, Retail Sales, and the Marketing Mix for Consumer Durables,” *Journal of Forecasting*, 17, 327-346.

Hanssens, Dominique M., Leonard J. Parsons, and Randall L. Schultz (2001), *Market Response Models, Econometric and Time Series Analysis*. 2nd ed., Kluwer Academic Publishers: Boston.

Kothari, S.P. (2001), “Capital Markets Research in Accounting,” *Journal of Accounting and Economics*, 31, 105-231.

Lee, Hau L., V. Padmanabhan, and Seungjin Whang (1997). Information Distortion in a Supply Chain: the Bullwhip Effect. *Management Science*, 43 (4), 546-558.

Misra, Sanjog and Harikesh Nair (2011), “A Structural Model of Sales-Force Compensation Dynamics: Estimation and Field Implementation,” *Quantitative Marketing and Economics*, 9(3), 211-225.

Mizik, Natalie and Robert Jacobson (2004), “Are Physicians 'Easy Marks'?: Quantifying the Effects of Detailing and Sampling on New Prescriptions,” *Management Science*, 50 (12), 1704-1715.

Murphy, Kevin M. and Robert H. Topel (1985), “Estimation and Inference in Two-Step Econometric Models, *Journal of Business and Economic Statistics*, 3, 370-379.

Nerlove, Marc and Kenneth J. Arrow (1962), “Optimal Advertising Policy under Dynamic Conditions, *Economica*, 29 (114, May), 129-142.

Nijs, Vincent, Shuba Srinivasan, and Koen Pauwels (2007), “Retail-Price Drivers and Retailer Profits,” *Marketing Science*, 26 (4), 473-487.

Opler, Tim, Lee Pinkowitz, René Stulz, and Rohan Williamson (1999), “The Determinants and Implications of Corporate Cash Holdings,” *Journal of Financial Economics*, 52, 3-46.

Raju, Jagmohan S. (1992), “The Effect of Price Promotions on Variability in Product Category Sales,” *Marketing Science*, 11, 207-220.

Simon, Hermann (1982), "ADPULS: An Advertising Model with Wearout and Pulsation," *Journal of Marketing Research*, 19, 352-363.

Steel, P. and C. J. König (2006). Integrating Theories of Motivation. *Academy of Management Review, 31*(4), 889-913.

Tull, Donald S., Van R. Wood, Dale Duhan, Tom Gillpatick, Kim R. Robertson und James G. Helgeson (1986), "'Leveraged' Decision Making in Advertising: The Flat Maximum Principle and Its Implications," *Journal of Marketing Research*, 23, 25-32.

Vakratsas, Demetrios (2008), “The Effects of Advertising, Prices and Distribution on Market Share Volatility,” *European Journal of Operational Research*, 187 (1), 283-293.

Zellner, Anton (1962), “An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias,” *Journal of the American Statistical Association*, 57, 348-368.

TABLE 1

Brand Performance Volatility Effects under General Conditions

	Effect on brand performance volatility	
Due to…	Variance of revenues	Variance of cash flows
Higher variance of marketing expenditures	Positive if $\varepsilon CV > -\rho \varepsilon_c CV_c$. Negative else.	Positive if $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 > -\rho \frac{\varepsilon_c CV_c}{\varepsilon CV}$. Negative else.
Higher level of marketing expenditures	Negative if $\varepsilon CV > -\rho \varepsilon_c CV_c$. Positive else.	First negative, then positive (U-form) if $\varepsilon CV > -\rho \varepsilon_c CV_c$. Positive else.
Higher marketing responsiveness	Positive if $\varepsilon CV > -\rho \varepsilon_c CV_c$. Negative else.	Positive if $\varepsilon CV > -\rho \varepsilon_c CV_c$ and $\mu < \tilde{\mu}\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV$. Negative if $\mu > \tilde{\mu}\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV$.
	Variance of revenues and cash flows for $\varepsilon_c < 0$	Variance of revenues and cash flows for $\varepsilon_c > 0$
Higher variance of competitive marketing expenditures	Positive if $\varepsilon_c CV_c < -\rho \varepsilon CV$. Negative else.	Positive if $\varepsilon_c CV_c > -\rho \varepsilon CV$. Negative else.
Stronger (positive) correlation between own and competitive marketing expenditures[1)]	Always negative.	Always positive.

[1)] We note that in reality, competitive reaction occurs with a certain time lag that may lead to divergent correlation structures. With quarterly data as ours, however, this effect vanishes and we should observe a positive correlation if expenditures are synchronized.

TABLE 2

Brand Performance Volatility Effects under Rational Firm Behavior

Due to…	*Effect on brand performance volatility* Variance of revenues	Variance of cash flows
Higher variance of marketing expenditures	Always positive.	Positive if $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 > -\rho\frac{\varepsilon_c CV_c}{\varepsilon CV}$. Corollary 1 holds.
Higher level of marketing expenditures	Always negative.	Always first negative, then positive (U-form). Corollary 2 holds.
Higher marketing responsiveness	Always positive.	Always positive for low expenditure levels and negative for high levels (inverted U-form in expenditure level).

		Variance of revenues and cash flows for $\varepsilon_c < 0$	Variance of revenues and cash flows for $\varepsilon_c > 0$
Higher variance of competitive marketing expenditures	$\rho = 0$	Always positive.	Always positive.
	$\rho < 0$	Always positive.	Positive or negative.
	$\rho > 0$	Positive or negative.	Always positive.
Stronger (positive) correlation between own and competitive marketing expenditures[1)]		Always negative.	Always positive.

[1)] We note that in reality, competitive reaction occurs with a certain time lag that may lead to divergent correlation structures. With quarterly data as ours, however, this effect vanishes and we should observe a positive correlation if expenditures are synchronized.

TABLE 3

Descriptive Statistics (Period = Quarter)

Level variables	*Mean*	*Std. dev.*	*Volatility variables*	*Mean*	*Std. dev.*
Unit sales in daily dosages (,000)	17,817	20,392	Moving variance of adjusted revenues in th. US$	7,524,430	40,764,000
Revenues in US$ (,000)	9,342	10,400	Moving variance of adjusted cash flows in th. US$	3,310,330	17,186,000
Cash flows in US$(,000)	5,022	6,385	Moving variance of marketing expenditures in th. US$	151,134	347,868
Marketing expenditures in US$ (,000)	1,053	872	Moving variance of competitive marketing expenditures in th. US$	2,300,460	2,525,510
Competitive marketing expenditures in US$ (,000)	5,008	3,390	Moving range of adjusted revenues in th. US$	3,754	6,749
Moving average of marketing expenditures in US$ (,000)	960	732	Moving range of adjusted cash flows in th. US$	2,692	4,383
Moving average correlation between own and competitive marketing expenditures	0.35	0.40	Moving range of marketing expenditures in th. US$	854	756
			Moving range of competitive marketing expenditures in th. US$	3,675	2,322

Notes: All variables before log-transformation that is used in estimation. All values in 1996 dollars deflated by country-specific consumption price index.

TABLE 4

T-test Results on Differences between Group Means in US$ (,000)

		Group Means for Variance of Revenues			*Group Means for Variance of Cash Flows*		
	Expected difference	Low[1)]	High[1)]	Difference (p-value)	Low[1)]	High[1)]	Difference (p-value)
Variance of marketing expenditures	Low < High	1,672,517	4,331,894	.000	707,447	2,099,758	.000
Level of marketing expenditures[2)]	Low > High[3)]	2,455,130	1,934,147	.107	1,037,391	1,007,937	.830
Marketing responsiveness	Low < High	1,839,206	3,708,319	.000	876,018	1,513,152	.001
Variance of competitive marketing expenditures	Low > High[4)]	1,997,245	1,304,307	.000	907,069	660,641	.004
Correlation between own and competitive marketing expenditures	Low > High[5)]	3,131,701	1,587,563	.000	1,447,002	684,286	.000

Notes: Test for difference between group means is based on two-sided t-tests that corrects for unequal group variances if necessary.

[1)] Brands are assigned to the „Low“ („High“) group if their mean for the respective predictor variable, e.g., variance of marketing expenditures, is below (above) the sample mean. Reported cell values reflect the group mean of the respective criterion variable, e.g., variance of revenues.

[2)] Level of marketing expenditures was divided by the mean level of unit sales for a brand to account for brand size effects.

[3)] Because the relationship between cash-flow variance and level of marketing expenditures is non-monotonic (inverted U-shape), we cannot make a prediction. We rather expect no difference between group means.

[4)] Following propositions 3A and 3B, the predicted sign depends on sample characteristics such as the relation between own and cross-effects, which were estimated.

[5)] Following propositions 4A and 4B, the predicted sign requires that cross-effects are substitutive, which is consistent with our estimate from the brand sales model.

TABLE 5

Estimation Results for the Volatility Models

		Revenue volatility				*Cash-flow volatility*	
		First-difference model		Cross-sectional model		First-difference model	
	Expected sign	*Dependent variable*					
		Variance	Range	Variance	Range	Variance	Range
Constant		-		-11.120 (9.70)	-6.537 (4.883)	-	
Volatility of revenues	+	-		-		1.359 (.029)***	1.155 (.018)***
Volatility of marketing expenditures	+	.273 (.024)***	.101 (.024)***	1.926 (.699)***	2.214 (.819)***	.535 (.032)***	.227 (.023)***
Level of marketing expenditures	–	-.245 (.237)	.139 (.076)	-1.988[1)] (.665)***	-1.008[1)] (.327)***	-2.042 (.369)***	-.307 (.079)***
Exp(Level of marketing expenditures)	+	-		-		.003 (.001)***	.001 (.000)***
Volatility of competitive marketing expenditures	–	-.006 (.023)	.018 (.020)	-.222 (.143)*	-.449 (.285)*	-	
Correlation between own and competitive marketing expenditures	–	-.262 (.095)***	-.066 (.026)**	-2.337 (3.19)	-1.019 (1.481)	-	
Marketing responsiveness	+	-		8.110 (4.75)**	3.974 (2.078)**	-	
Variance explained in estimation/holdout samples[2)]		.245/.202	.339/.215	.689/.209	.726/.277	.724/.585	.771/.721
Total no. of observations		2,104		99		2,104	

Notes: Standard errors in parentheses. One-sided t-test applies to unidirectional expectations, two-sided t-tests otherwise. *** p < .01; ** p < .05; * p < .10

1) Level of marketing expenditures was divided by the mean level of unit sales for a brand to account for brand size effects.

2) Variance in log-transformed focal volatility variable explained by predictor variables. Estimation sample includes 80%, holdout sample 20% of cases.

TABLE 6

Total Effects in Terms of Elasticity (When Applicable)

	Expected sign	*Revenue Volatility*		*Cash-flow volatility*	
Dependent variable variance					
Variance of marketing expenditures	+	.273	(.024)***	.906	(.070)***
Level of marketing expenditures[1)]	–	-.245	(.237)	-2.375	(.490)***
Exp(Level of marketing expenditures)[1)]	+	-		.003	(.001)***
Marketing responsiveness[2)]	+	.811	(.474)**	1.102	(.645)**
Variance of competitive marketing expenditures	–	-.006	(.023)	-.009	(.031)
Correlation between own and competitive marketing expenditures[2)]	–	-.090	(.033)***	-.123	(.045)***
Dependent variable range					
Range of marketing expenditures	+	.101	(.024)***	.344	(.051)***
Level of marketing expenditures[1)]	–	.139	(.076)	-.147	(.118)
Exp(Level of marketing expenditures)[1)]	+			.001	(.000)***
Marketing responsiveness[2)]	+	.397	(.208)**	.459	(.240)**
Range of competitive marketing expenditures	–	.018	(.020)	.021	(.023)
Correlation between own and competitive marketing expenditures[2)]	–	-.023	(.009)***	-.026	(.010)***

Notes: (Approximated) standard errors in parentheses. Results are based on first difference models except for marketing responsiveness, which are based on cross-sectional models. *** $p < .01$; ** $p < .05$

1) For cash-flow volatility, results reflect parameters of a non-monotonic function, not elasticities.

2) Elasticities are not constant and are evaluated at sample means for responsiveness and expenditure correlations, respectively.

Technical Appendix

Illustrative Example of Financial Cost of Spending Volatility

This section illustrates the additional financial burden that may arise from a volatile advertising strategy. Consider a brand manager with a budget of $400,000 to be spent over the fiscal year (see Table A1). Under even spending we assume s/he invests $100,000 in marketing activities every quarter. Under the alternative volatile strategy, we assume $200,000 is spent in the first and third quarters, and zero is spent in the remaining two. Following the market response literature, we incorporate a carryover effect of marketing, i.e. sales do not drop immediately to their base level when expenditures are reduced to zero.

The upper panel of Table A1 shows the statement of cash flows associated with the two spending strategies. Column 3 and 4 present the incremental revenues, net of costs of goods sold, which accrue from marketing expenditures.[3] Consistent with the idea of the differential stimulus effect (Hanssens and Levien 1983; Simon 1982), incremental revenues are higher under alternating spending levels. The last two columns show the incremental cash flows (net revenues minus marketing expenditures). We assume that volatile spending generates cash flows that are 5% higher than those from even spending.[4]

Normally, the comparison of the two alternative spending schedules would stop at this point. However, the alternatives involve quite different levels of volatility of incremental revenues and cash flows, as shown in the last two rows of the upper panel. Volatility can be expressed as the range of monetary quantities or their standard deviation. By definition, it is zero for even spending but reaches a remarkable level under volatile spending. The financial side effect associated with volatile spending is demonstrated in the lower panel of Table A1.

Columns 1 and 2 show how cash flows accumulate over time until they reach their year-end total of $80,000 and $84,000, respectively. The accrual of cash flows is booked at the end of a quarter, for example, $20,000 is booked under even spending at the end of the first quarter. In order to realize these cash flows, however, capital must be provided at the beginning of the quarter. Columns 3 and 4 list the required level of cash holdings. It equals the size of marketing expenditures in the first quarter but decreases in subsequent quarters due to the incremental cash flows generated by marketing in previous periods. These required cash holdings are not costless, as investors expect their invested capital to generate at least a certain rate of return. Since the volatile spending pattern creates negative cash flows in some quarters, more capital is locked up over time. Compared with even spending, these higher capital

[3] The figures are generated by a typical market response function that accounts for carryover and differential stimulus effects. Marketing spending elasticity under even spending is assumed to be .30.

[4] Dubé, Hitsch, and Manchanda (2005) and Mahajan and Muller (1986) report gains in cash profits due to volatile spending are between 1% and 5%.

needs incur additional financing costs as shown in the last two columns. Assuming annual capital cost of 15% or 3.8% per quarter, these financing costs make the volatile spending a less attractive policy. Cash flows net of financing cost now amount to $70,000 for even spending, but only $68,000 for the volatile spending strategy.

Table A1

An Example of the Financial Costs of Spending Volatility

Statement of cash flows (quarterly) in Thousand US$

	Marketing expenditures		Incremental revenues net of cost of goods due to marketing expenditures		Incremental cash flows due to marketing expenditures	
	Even spending	Volatile spending	Even spending	Volatile spending	Even spending	Volatile spending
Quarter 1	100	200	120	150	20	-50
Quarter 2	100	0	120	92	20	92
Quarter 3	100	200	120	150	20	-50
Quarter 4	100	0	120	92	20	92
Total	400	400	480	484	80	84
Std. dev.	0	100	0	29	0	71
Range[1)]	0	200	0	58	0	142

Cash balance sheet (quarterly) in Thousand US$

	Cumulated cash flows due to marketing expenditures		Required cash holdings		Financing costs for capital lockup (3.8% per quarter)	
	Even spending	Volatile spending	Even spending	Volatile spending	Even spending	Volatile spending
Quarter 1	20	-50	100	200	4	8
Quarter 2	40	42	80	50	3	2
Quarter 3	60	-8	60	158	2	6
Quarter 4	80	84	40	8	1	0
Total			280	416	10	16

[1)] Range = Maximum expenditure – Minimum expenditure

Proofs of Propositions of Brand Performance Volatility Effects

PROOF OF PROPOSITION 1A. We need to show that $\frac{\partial Var\left[RV\left(MKT, CMKT\right)\right]}{\partial Var\left(MKT\right)} > 0$. Taking the first derivative of Equation (4) w.r.t. $Var(MKT)$ and setting > 0 gives,

$$P^2\left[Q'(\mu)\right]^2 + P^2\rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var\left(CMKT\right)}{Var\left(MKT\right)}\right]^{\frac{1}{2}} > 0, \tag{18}$$

which we divide by $P^2Q'(\mu)$ and rearrange to

$$Q'(\mu)\sqrt{Var\left(MKT\right)} > -\rho Q_c'(\mu_c)\sqrt{Var\left(CMKT\right)}. \tag{19}$$

Recall our expressions for elasticities, $\varepsilon = Q' \cdot \mu/Q$ and $\varepsilon_c = Q_c' \cdot \mu_c/Q$, and coefficients of variation, $CV = SD(MKT)/\mu$ and $CV_c = SD(CMKT)/\mu_c$. Dividing (19) by Q and expanding the l.hs. with μ and the r.h.s. with μ_c produces

$$\varepsilon\, CV > -\rho\, \varepsilon_c\, CV_c\ ,$$

which is equivalent to the condition in proposition 1A. ☐

PROOF OF PROPOSITION 1B. We need to show that $\frac{\partial Var\left[RV\left(MKT, CMKT\right)\right]}{\partial \mu} < 0$.

Taking the first derivative of Equation (4) w.r.t. μ and setting < 0 gives,

$$2P^2Q''(\mu)Q'(\mu)Var\left(MKT\right) + 2P^2\rho Q''(\mu)Q_c'(\mu_c)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}} < 0. \tag{20}$$

Note that $Q''(\mu) < 0$. Dividing (20) by $2P^2Q''(\mu)Var(MKT)$ and rearranging the result gives

$$Q'(\mu)\sqrt{Var\left(MKT\right)} > -\rho Q'(\mu_c)\sqrt{Var\left(CMKT\right)}, \tag{21}$$

which is equivalent to expression (19). As done before, we can transform this expression to

$$\varepsilon\, CV > -\rho\, \varepsilon_c\, CV_c\ ,$$

which is equivalent to the condition in proposition 1B. ☐

PROOF OF PROPOSITION 1C. We need to show that $\frac{\partial Var\left[RV\left(MKT, CMKT\right)\right]}{\partial Q'(\mu)} > 0$.

Taking the first derivative of Equation (4) w.r.t. $Q'(\mu)$ and setting > 0 gives,

$$2P^2Q'(\mu)Var(MKT)+2P^2\rho Q_c'(\mu_c)\left[Var(MKT)Var(CMKT)\right]^{\frac{1}{2}}>0, \tag{22}$$

which we divide by $P^2Q'(\mu)[Var(MKT)]^{1/2}$ and rearrange to

$$Q'(\mu)\sqrt{Var(MKT)}>-\rho Q_c'(\mu_c)\sqrt{Var(CMKT)}, \tag{23}$$

which is equivalent to expression (19). As done before, we can transform this expression to

$$\varepsilon CV>-\rho\,\varepsilon_c\,CV_c\ ,$$

which is equivalent to the condition in proposition 1C. □

PROOF OF PROPOSITION 2A. We need to show that $\dfrac{\partial Var\left[CF(MKT,CMKT)\right]}{\partial Var(MKT)}>0$. Taking the first derivative of Equation (5) w.r.t. $Var(MKT)$ and setting > 0 gives,

$$\left[(P-C)Q'(\mu)-1\right]^2+(P-C)^2\rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var(CMKT)}{Var(MKT)}\right]^{\frac{1}{2}}>0. \tag{24}$$

Selectively, we expand terms of (24) with Q, μ, and μ_c, respectively, to obtain

$$\frac{\left[\varepsilon(P-C)Q-\mu\right]^2}{\mu^2}>-(P-C)^2\rho\varepsilon\varepsilon_c\frac{Q^2}{\mu\mu_c}\left[\frac{Var(CMKT)}{Var(MKT)}\right]^{\frac{1}{2}}. \tag{25}$$

Dividing this expression by $\dfrac{\varepsilon^2(P-C)^2Q^2}{\mu^2}$ gives,

$$\frac{\left[\varepsilon(P-C)Q-\mu\right]^2}{\varepsilon^2(P-C)^2Q^2}>-\rho\frac{\varepsilon_c}{\varepsilon}\frac{\mu}{\mu_c}\left[\frac{Var(CMKT)}{Var(MKT)}\right]^{\frac{1}{2}}. \tag{26}$$

Substituting for the near-optimal expenditure level $\tilde{\mu}$ according to Equation (6) and the coefficients of variation, CV and CV_c, we can write for (26)

$$\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2>-\rho\frac{\varepsilon_c CV_c}{\varepsilon CV},$$

which is equivalent to the condition in proposition 2A. □

PROOF OF COROLLARY 1. We need to show that $\left(\dfrac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2<k,$ where $k=-\rho\dfrac{\varepsilon_c CV_c}{\varepsilon CV}$. From ε_c < 0 and ρ, ε, CV, CV_c > 0, it follows k > 0. Assume expenditures are set at the optimal level, i.e. $\mu=\mu^*$.

Since $\mu^{*}=\varepsilon^{*}\left(P-C\right)Q^{*}$, it follows with (6) that $\tilde{\mu}=\mu^{*}=\varepsilon^{*}\left(P-C\right)Q^{*}$. Let $\kappa\in\mathbb{R}$ be an arbitrary constant that measures how close actual expenditures are to the optimal level: $\mu=\left|\mu^{*}-\kappa\right|$. For $\kappa\to 0$, $\mu\to\mu^{*}$ and $\tilde{\mu}\to\mu^{*}$. As a consequence, $\left|\tilde{\mu}-\mu\right|\to 0$ and therefore, $\left.\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^{2}\right|_{\kappa\to 0}=0$. Hence, there is a κ small enough to satisfy $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^{2}<k$. □

PROOF OF PROPOSITION 2B. This proposition implies that the first derivative of (5) has a root. Hence,

$$
\begin{aligned}
\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial\mu} &= 0 \\
&= 2\left(P-C\right)Q''(\mu)\left[\left(P-C\right)Q'(\mu)-1\right]Var\left(MKT\right) \\
&\quad +2\left(P-C\right)^{2}\rho Q''(\mu)Q_{c}'(\mu_{c})\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}}.
\end{aligned}
\tag{27}
$$

Dividing (27) by $Q''<0$, expanding Q' and Q_{c}' with Q, μ, and μ_c, respectively, and substituting terms for elasticities, coefficients of variation and $\tilde{\mu}$ from (6), we solve for the root

$$\mu_{0}=\tilde{\mu}\left(\frac{\varepsilon CV+\rho\varepsilon_{c}CV_{c}}{\varepsilon CV}\right). \tag{28}$$

Because $\varepsilon CV>-\rho\varepsilon_{c}CV_{c}$, the root is defined for positive mean expenditure levels. For a U-shaped relation, we must show that $\left.\partial Var\left[CF\left(MKT,CMKT\right)\right]/\partial\mu\right|_{\mu<\mu_{0}}<0$ and $\left.\partial Var\left[CF\left(MKT,CMKT\right)\right]/\partial\mu\right|_{\mu>\mu_{0}}>0$. Rewriting Equation (27) by substituting terms for elasticities, coefficients of variation and $\tilde{\mu}$, these inequalities imply $\tilde{\mu}\left(\frac{\varepsilon CV+\rho\varepsilon_{c}CV_{c}}{\varepsilon CV}\right)-\mu>0$ and $\tilde{\mu}\left(\frac{\varepsilon CV+\rho\varepsilon_{c}CV_{c}}{\varepsilon CV}\right)-\mu<0$, respectively. Let $\mu=z\mu_{0}$, with $z>0$. Note that $\mu<\mu_0$ for $z<1$ and $\mu>\mu_0$ for $z>1$. Substituting μ_0 for (28), we easily verify that $\left.\partial Var\left[CF\left(MKT\right)\right]/\partial\mu\right|_{z<1}<0$ and $\left.\partial Var\left[CF\left(MKT\right)\right]/\partial\mu\right|_{z>1}>0$.

PROOF OF COROLLARY 2. Set $\mu = \mu^*$, which implies with (6) that $\tilde{\mu} = \mu^*$. Because $Q_c' < 0$ and $\rho > 0$, $\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV < 1$. Then, result (28) implies that $\mu_0 < \mu^*$. □

PROOF OF PROPOSITION 2C. The derivative of (5) w.r.t. $Q'(\mu)$ is given by

$$\begin{aligned}\frac{\partial Var\left[CF\left(MKT, CMKT\right)\right]}{\partial Q'(\mu)} &= 2\left(P-C\right)\left[\left(P-C\right)Q'(\mu)-1\right]Var\left(MKT\right) \\ &\quad +2\left(P-C\right)^2 \rho Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}}.\end{aligned} \tag{29}$$

Note that Equation (27), the derivative of (5) w.r.t μ, equals Equation (29) scaled by $Q''(\mu)$. Hence, both derivatives have the same root, as given by Equation (28). However, because $Q'' < 0$, the inequality conditions for $\partial Var\left[CF\left(MKT, CMKT\right)\right] / \partial Q'(\mu)$ at $\mu < \mu_0$ and $\mu > \mu_0$, respectively, are reversed. Following the chain of proof for proposition 2B, we can prove that these inequalities hold. Specifically,

$$\partial Var\left[CF\left(MKT, CMKT\right)\right] / \partial Q'(\mu)\Big|_{\mu<\mu_0} > 0 \Rightarrow \mu < \tilde{\mu}\left(\frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV}\right), \text{ with } \varepsilon CV > -\rho\varepsilon_c CV_c$$

and

$$\partial Var\left[CF\left(MKT, CMKT\right)\right] / \partial Q'(\mu)\Big|_{\mu>\mu_0} < 0 \Rightarrow \mu > \tilde{\mu}\left(\frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV}\right),$$

which is equivalent to the conditions in proposition 2C. □

PROOF OF PROPOSITION 3A. We need to show that $\dfrac{\partial Var\left[RV\left(MKT, CMKT\right)\right]}{\partial Var\left(CMKT\right)} > 0$. Taking the first derivative of Equation (4) w.r.t. $Var(CMKT)$ and setting > 0 gives,

$$P^2\left[Q_c'\left(\mu_c\right)\right]^2 + P^2 \rho Q'(\mu) Q_c'\left(\mu_c\right)\left[\frac{Var\left(MKT\right)}{Var\left(CMKT\right)}\right]^{\frac{1}{2}} > 0,$$

which we divide by P^2 to obtain

$$\left[Q_c'\left(\mu_c\right)\right]^2 + \rho Q'(\mu) Q_c'\left(\mu_c\right)\left[\frac{Var\left(MKT\right)}{Var\left(CMKT\right)}\right]^{\frac{1}{2}} > 0. \tag{30}$$

Consider first a substitutive cross-effect, i.e. $Q_c'\left(\mu_c\right) < 0$ and therefore $\varepsilon_c < 0$. Dividing (30) by $Q_c'\left(\mu_c\right)$ and rearranging the result gives,

$$Q_c'(\mu_c)\sqrt{Var(CMKT)} < -\rho Q'(\mu)\sqrt{Var(MKT)}\,. \tag{31}$$

Dividing (31) by Q and expanding the l.hs. with μ_c and the r.h.s. μ, we can rewrite and simplify the inequality to

$$\varepsilon_c CV_c < -\rho\varepsilon CV \quad ,$$

which proves condition (7a) for proposition 3A.

Consider now a market-expanding cross-effect, i.e. $Q_c'(\mu_c) > 0$ and therefore $\varepsilon_c > 0$. Divide (30) by $Q_c'(\mu_c)$ and rearrange the result to

$$Q_c'(\mu_c)\sqrt{Var(CMKT)} > -\rho Q'(\mu)\sqrt{Var(MKT)}\,. \tag{32}$$

Again, we can rewrite this expression in terms of elasticities and coefficients of variation

$$\varepsilon_c CV_c > -\rho\varepsilon CV \quad ,$$

which proves condition (7b) for proposition 3A. □

PROOF OF PROPOSITION 3B. We need to show that $\dfrac{\partial Var\left[CF(MKT, CMKT)\right]}{\partial Var(CMKT)} > 0$. Taking the first derivative of Equation (5) w.r.t. $Var(CMKT)$ and setting > 0 gives,

$$(P-C)^2\left[Q_c'(\mu_c)\right]^2 + (P-C)^2\rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var(MKT)}{Var(CMKT)}\right]^{\frac{1}{2}} > 0,$$

which we divide by $(P\text{-}C)^2$ to obtain

$$\left[Q_c'(\mu_c)\right]^2 + \rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var(MKT)}{Var(CMKT)}\right]^{\frac{1}{2}} > 0\,. \tag{33}$$

Note that this inequality is identical to inequality (30). With inequalities (31) and (32), we already showed that the conditions (7a) and (7b) satisfy inequality (33). □

PROOF OF PROPOSITION 4A. The first derivative of Equation (4) w.r.t. ρ is given by

$$\frac{\partial Var\left[RV(MKT, CMKT)\right]}{\partial\rho} = 2P^2 Q'(\mu)Q_c'(\mu_c)\left[Var(MKT)Var(CMKT)\right]^{\frac{1}{2}} \tag{34}$$

Again, we can rewrite this expression in terms of elasticities and coefficients of variation

$$\frac{\partial Var\left[RV(MKT, CMKT)\right]}{\partial\rho} = 2(PQ)^2\,\varepsilon CV\varepsilon_c CV_c\,. \tag{35}$$

Since P, Q, ε, CV, and CV_c are always strictly positive, it is easy to show that

$$\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial\rho}>0 \quad \text{iff} \quad \varepsilon_c>0$$

and

$$\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial\rho}<0 \quad \text{iff} \quad \varepsilon_c<0. \quad \square$$

PROOF OF PROPOSITION 4B. The first derivative of Equation (5) w.r.t. ρ is given by

$$\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial\rho}=2\left(P-C\right)^2 Q'\left(\mu\right)Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}},$$

which can be rewritten as

$$\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial\rho}=2\left(P-Q\right)^2 \varepsilon CV\varepsilon_c CV_c. \tag{36}$$

Note that $(P\text{-}C)^2$ is always strictly positive. Following the chain of proof for proposition 4A, it is easy to show that proposition 4B also holds. ☐

Proofs of Brand Performance Volatility Effects under Rational Firm Behavior

Table 2 summarizes the brand performance volatility effects that follow from our propositions if we assume that firms follow a rational, competitive Nash behavior. Note that Theorem 2 requires $\partial\sigma^2_{RV}/\partial\sigma^2_{MKT}>0$ for any optimal, volatile marketing policy. Consistent with proposition 1A, this implies that $\varepsilon CV>-\rho\varepsilon_C CV_C$ always holds. In the following, we frequently refer to this result to prove the statements in Table 2.

Impact of Own Marketing on Performance Volatility

PROOF OF EFFECTS ON VARIANCE OF REVENUES. Since $\varepsilon CV>-\rho\varepsilon_C CV_C$ the statements in Table 2 directly follow from propositions 1A, 1B, and 1C. ☐

PROOF OF THE EFFECT OF EXPENDITURE VARIANCE ON VARIANCE OF CASH FLOWS. From $\varepsilon CV>-\rho\varepsilon_C CV_C$, it follows $-\rho\frac{\varepsilon_C CV_C}{\varepsilon CV}<1$. Proposition 2A states that cash-flow variance increases with expenditure variance if $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 > -\rho\frac{\varepsilon_C CV_C}{\varepsilon CV}$. Hence, a higher variance of own marketing expenditures always increases the variance of cash flows if $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 \geq 1$. However, there may exist a budget that satisfies $0 \leq \left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 \leq 1$, which proves the result in Table 2. Since the result in Table 2 is fully consistent with proposition 2A, the proof for corollary 1 given above applies here, too. Thus, corollary 1 holds under the assumption of rational firm behavior. □

PROOF OF THE EFFECT OF LEVEL OF EXPENDITURES ON VARIANCE OF CASH FLOWS. Since $\varepsilon CV > -\rho\varepsilon_C CV_C$ the statement in Table 2 directly follows from proposition 2B. For this reason, the proof of corollary 2 also applies here. Thus, corollary 2 holds under the assumption of rational firm behavior. □

PROOF OF THE EFFECT OF RESPONSIVENESS ON VARIANCE OF CASH FLOWS. Since $\varepsilon CV > -\rho\varepsilon_C CV_C$ the statement in Table 2 directly follows from proposition 2C. □

Impact of Competitive Marketing on Performance Volatility

PROOF OF THE EFFECT OF COMPETITIVE EXPENDITURE VARIANCE ON VARIANCE OF REVENUES AND CASH FLOWS. Consider first the case of substitutive cross-effects, i.e. $\varepsilon_C < 0$. To demonstrate a positive effect of competitive expenditure variance on variance of revenues and cash flows, we need to show that the following inequalities are both satisfied:

$$\varepsilon_C CV_C < -\rho\varepsilon CV \text{ (see condition 7a)} \tag{37}$$

and

$$\varepsilon CV > -\rho\varepsilon_C CV_C . \tag{38}$$

Because ε, CV, and $CV_C > 0$, these inequalities are always satisfied for $\rho \leq 0$. For $\rho > 0$, we rearrange (37) to $-\frac{1}{\rho}\varepsilon_C CV_C > \varepsilon CV$. Then

$$-\frac{1}{\rho}\varepsilon_C CV_C > -\rho\varepsilon_C CV_C \tag{39}$$

must hold to satisfy (38). Recall that $\varepsilon_C < 0$ and $\rho > 0$. (39) can be simplified to $-1/\rho < -\rho$ and therefore $\rho^2 < 1$, which is always satisfied because $0 < \rho \leq 1$. Hence, a positive effect is consistent with $\rho > 0$. However, we can easily choose reasonable parameter values that are also consistent with a negative effect. Here, inequalites

$$\varepsilon_C CV_C > -\rho\varepsilon CV \tag{40}$$

and (38) must both be satisfied. Assume $\varepsilon = .19$, $\varepsilon_C = -.05$, $\rho = .35$, $CV = .365$, $CV_C = .303$ and insert these parameter values into (40) and (38) to demonstrate that the inequalities are satisfied. ☐

Consider now the case of market-expanding cross-effects, i.e. $\varepsilon_C > 0$. For a positive effect of competitive expenditure variance on variance of revenues and cash flows, inequality (38) together with

$$\varepsilon_C CV_C > -\rho\varepsilon CV \text{ (see condition 7b)} \tag{41}$$

must be satisfied. Again, it is straightforward to show that these inequalities are always satisfied for $\rho \geq 0$ because ε, CV, and $CV_C > 0$. For $\rho < 0$, we rearrange (41) to $-\frac{1}{\rho}\varepsilon_C CV_C < \varepsilon CV$. From this, we conclude that

$$-\frac{1}{\rho}\varepsilon_C CV_C > -\rho\varepsilon_C CV_C \tag{42}$$

satisfies (38). Recall that $\varepsilon_C > 0$ and $\rho < 0$. (42) can be simplified to $-1/\rho > -\rho$ and therefore $|\rho|^2 < 1$, which is always satisfied because $-1 \leq \rho < 0$. Hence, a positive effect is consistent with $\rho < 0$. But again, we can easily choose reasonable parameter values that are consistent with a negative effect. For example, the parameter values $\varepsilon = .19$, $\varepsilon_C = .03$, $\rho = -.35$, $CV = .365$, $CV_C = .303$ satisfy both (38) and the reverse of inequality (41). ☐

PROOF OF THE EFFECT OF EXPENDITURE CORRELATION ON VARIANCE OF REVENUES AND CASH FLOWS. The relationship between expenditure correlation and variance of revenues and cash flows depends on the sign of the cross-effect. The results in Table 2 are fully consistent with the results in Table 1 that are derived from propositions 4A and 4B. We have shown above that these propositions hold. ☐

Proofs of Theorems on Optimal Expenditure Behavior

Proof of Theorem 1

Expression (8) states the maximization problem that we want to solve. Define $m = \left(P - C\right) / P$ that measures the profit margin in percent. We can then write for the cash flows

$$CF = m \cdot PQ - MKT \tag{43}$$

and derive their variance

$$\sigma_{CF}^2 = m^2 P^2 \sigma_Q^2 + \sigma_{MKT}^2 - 2mPCov\left(Q, MKT\right). \tag{44}$$

Note that $Cov\left(Q, MKT\right) = \rho_{Q,MKT} \sqrt{\sigma_Q^2 \sigma_{MKT}^2}$, where $\rho_{Q,MKT}$ measures the correlation between unit sales and marketing expenditures. Using this relation and $\sigma_{RV}^2 = P^2 \sigma_Q^2$, we can write for (44)

$$\sigma_{CF}^2 = m^2 \sigma_{RV}^2 + \sigma_{MKT}^2 - 2mP\rho_{Q,MKT} \sqrt{\sigma_Q^2 \sigma_{MKT}^2}. \tag{45}$$

Insert this expression into the profit function (8) to obtain:

$$\max_{\mu, \sigma_{MKT}^2} \Pi = \left(P - C\right) Q\left(\mu, \sigma_{MKT}^2, CMKT\right) - \mu - \left(w + rm^2\right)\sigma_{RV}^2 - r\left(\sigma_{MKT}^2 + 2rm\rho_{Q,MKT}\sqrt{\sigma_Q^2 \sigma_{MKT}^2}\right) - f. \tag{46}$$

For this maximization problem, we derive the first-order conditions (9a) and (9b) that each competitor has to satisfy under Nash competition.

Assuming $r = 0$, the first-order conditions reduce to

$$\frac{\partial \Pi}{\partial \mu} = \left(P - C\right)\frac{\partial Q}{\partial \mu} - 1 - w\frac{\partial \sigma_{RV}^2}{\partial \mu} = 0, \tag{47a}$$

$$\frac{\partial \Pi}{\partial \sigma_{MKT}^2} = \left(P - C\right)\frac{\partial Q}{\partial \sigma_{MKT}^2} - w\frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} = 0. \tag{47b}$$

Multiply (47a) with μ and expand terms with Q and σ_{RV}^2, respectively, to obtain

$$\frac{\partial \Pi}{\partial \mu} = \left(P - C\right)\varepsilon_{Q,\mu} Q - \mu - w\varepsilon_{\sigma_{RV}^2,\mu} \sigma_{RV}^2 = 0, \tag{48a}$$

where $\varepsilon_{Q,\mu} = \frac{\partial Q}{\partial \mu}\frac{\mu}{Q}$ and $\varepsilon_{\sigma_{RV}^2,\mu} = \frac{\partial \sigma_{RV}^2}{\partial \mu}\frac{\mu}{\sigma_{RV}^2}$. Similarly, we multiply (47b) with σ_{MKT}^2 and expand terms with Q and σ_{RV}^2, respectively, and obtain

$$\frac{\partial \Pi}{\partial \sigma_{MKT}^2} = \left(P - C\right)\varepsilon_{Q,\sigma_{MKT}^2} Q - w\varepsilon_{\sigma_{RV}^2,\sigma_{MKT}^2} \sigma_{RV}^2 = 0, \tag{48b}$$

where $\varepsilon_{Q,\sigma^2_{MKT}} = \dfrac{\partial Q}{\partial \sigma^2_{MKT}} \dfrac{\sigma^2_{MKT}}{Q}$ and $\varepsilon_{\sigma^2_{RV},\sigma^2_{MKT}} = \dfrac{\partial \sigma^2_{RV}}{\partial \sigma^2_{MKT}} \dfrac{\sigma^2_{MKT}}{\sigma^2_{RV}}$.

Setting (48a) = (48b), we can solve for the optimal expenditure level

$$\mu^* = \left(P-C\right)Q^*\varepsilon^*_{Q,\mu} - \frac{\varepsilon^*_{\sigma^2_{RV},\mu}\left(P-C\right)Q^*\varepsilon^*_{Q,\sigma^2_{MKT}}}{\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}}, \tag{49}$$

where the asterisk means that variables are at their optimum. Rearranging terms shows that (49) is equivalent to Theorem 1. ☐

Proof of Theorem 2

Solve expression (48a) for the revenue volatility

$$\sigma^2_{RV} = \frac{\left(P-C\right)\varepsilon_{Q,\mu} - \mu}{w\varepsilon_{\sigma^2_{RV},\mu}} \tag{50}$$

and substitute for the optimal expenditure level (49) to obtain the revenue volatility at optimum

$$\left(\sigma^2_{RV}\right)^* = \frac{\left(P-C\right)Q^*\varepsilon^*_{Q,\sigma^2_{MKT}}}{w\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}}. \tag{51}$$

Substitute $\varepsilon_{\sigma^2_{RV},\sigma^2_{MKT}}$ for $\dfrac{\partial \sigma^2_{RV}}{\partial \sigma^2_{MKT}} \dfrac{\sigma^2_{MKT}}{\sigma^2_{RV}}$ in (51) and solve for the optimal variance of marketing expenditures

$$\left(\sigma^2_{MKT}\right)^* = \frac{\left(P-C\right)Q^*}{w}\left(\frac{\varepsilon^*_{Q,\sigma^2_{MKT}}}{\partial \sigma^2_{RV} \big/ \partial \sigma^2_{MKT}}\right). \tag{52}$$

Since variances are always positive, $\partial \sigma^2_{RV} \big/ \partial \sigma^2_{MKT} > 0$ must hold for (52), which is consistent with Theorem 2. ☐

Proofs for Dynamic Model Extensions

Proofs of Propositions of Brand Performance Volatility Effects

Note that the structure of the general response function (1) and thus the variance functions of revenues and cash flows does not change if we substitute own and competitive marketing expenditures, *MKT* and *CMKT*, for their stock variables S and S_C, respectively. As a result, all brand performance volatility effects for the dynamic model are identical to their counterparts that we derived and proved for the static model. The only difference is that the variance, the mean, and the responsiveness are expressed in terms of marketing stocks instead of marketing expenditures.

To prove that all propositions and corollaries also hold with respect to marketing expenditures, it is sufficient to show that the variance, the mean, and the responsiveness of expenditures differ from their stock equivalents only by a scaling factor. All results hold for competitive marketing stock and expenditures, as well.

MEAN. Taking the expectation of the marketing stock equation (10) gives

$$E\left(S_t\right)=\lambda E\left(S_{t-1}\right)+E\left(MKT\right). \tag{53}$$

Since $\lambda < 1$, the mean $E(S_t)$ is identical for all values of t. The above expression can be rearranged to

$$E\left(S\right)=\frac{\mu}{1-\lambda}, \tag{54}$$

where μ denotes the mean of own marketing expenditures. This result shows that the mean of own marketing stock is identical with the mean of its expenditures up to the scale 1/1-λ. □

VARIANCE. We arrange (10) to

$$S_t-\lambda S_{t-1}=MKT_t$$

and take the variance of both sides

$$Var\left(S_t\right)+\lambda^2 Var\left(S_{t-1}\right)-2\lambda\rho_{AR1}SD\left(S_t\right)SD\left(S_{t-1}\right)=Var\left(MKT_t\right), \tag{55}$$

where ρ_{AR1} measures the autocorrelation of the stock and all other terms are defined as earlier. Since $SD(S_t) - SD(S_{t-1}) \rightarrow 0$ for longer time-series of S, we can use $SD(S_t) \cong SD(S_{t-1})$ and simplify (55) to

$$Var\left(S\right)=\frac{1}{1+\lambda^2-2\lambda\rho_{AR(1)}}Var\left(MKT\right). \tag{56}$$

The denominator of (56) is a constant and always greater than zero. To see that note that the following inequality must be met

$$\frac{1+\lambda^2}{2\lambda}>\rho_{AR1}.$$

Since ρ_{AR1} cannot take on values greater than 1 we can rearrange this inequality to $\left(1-\lambda^2\right)>0$, which is always true. □

RESPONSIVENESS. From the econometric literature on distributed-lag models (Hanssens, Parsons, and Schultz 2001), it is well established that short-term and long-term sales effects relate to each other as follows

$$Q_S'(\cdot)=\frac{Q'(\cdot)}{1-\lambda}. \tag{57}$$

Hence, sales responsiveness w.r.t. stock is identical to the sales responsiveness w.r.t. expenditures up to the scale of 1/1-λ. □

In addition, the conditions that are part of the propositions and summarized in Table 1 and 2 only change by a scaling factor. They can be rewritten by simply substituting

$$\varepsilon CV \ \text{ for } \ \varepsilon CV\frac{1}{\sqrt{1+\lambda^2-2\lambda\rho_{AR1,S}}},$$

$$\varepsilon_C CV_C \ \text{ for } \ \varepsilon_C CV_C\frac{1}{\sqrt{1+\lambda_C^2-2\lambda_C\rho_{AR1,S_C}}},$$

and

$$\rho \ \text{ for } \ \rho_{S,S_C},$$

where λ_C denotes the carryover coefficient of competitive marketing expenditures and ρ_{S,S_C} measures the correlation between own marketing stock and competitive marketing stock. □

Proofs of Theorems on Optimal Expenditure Behavior

EXTENSION OF THEOREM 1. The firm faces the following dynamic profit maximization problem

$$\max_{MKT,\sigma^2_{MKT}} \int_0^\infty e^{-dt}\left[\left(P-C\right)Q\left(S,S_C,\sigma^2_{MKT}\right)-MKT-w\sigma^2_{RV}-r\sigma^2_{CF}-f\right]dt \tag{58}$$

subject to

$$\frac{dS}{dt}=-\phi S+MKT, \tag{59}$$

with

$$MKT\geq 0,\ \ \sigma^2_{MKT}\geq 0,\ \ S\geq 0,\ \ \text{and}\ \ S(0)=S_0. \tag{60}$$

Equation (59) describes the dynamic process for the evolution of the marketing stock (Nerlove and Arrow 1962), where ϕ is the depreciation rate of the stock. Note that time enters the objective function (58) explicitly through the discount term e^{-dt}, with discount rate d and $0 < d < \infty$. Since the discount rate is a constant the problem is quasiautonomous. Given the infinite time horizon, the solution to $S(t)$ is likely to tend to a steady state in the long run (Kamien and Schwartz 1991, 95f).

Using (59) and assuming again $r = 0$, we can restate the maximization problem as follows

$$\max\int_0^\infty e^{-dt}\left[\left(P-C\right)Q\left(S,S_C,\sigma^2_{MKT}\right)-\frac{dS}{dt}-\phi S-w\sigma^2_{RV}-f\right]dt. \tag{61}$$

The first-order conditions are given by the Euler equation

$$\frac{\partial F}{\partial S}\left[t,S^*(t),\frac{dS^*(t)}{dt},\left(\sigma^2_{MKT}\right)^*\right]-\frac{d}{dt}\left[\frac{\partial}{\partial(dS/dt)}F\left(t,S^*(t),\frac{dS^*(t)}{dt},\left(\sigma^2_{MKT}\right)^*\right)\right]=0 \qquad (62a)$$

$$\frac{\partial F}{\partial \sigma^2_{MKT}}=0, \qquad (62b)$$

where $F\left(t,S^*(t),dS^*(t)/dt,\left(\sigma^2_{MKT}\right)^*\right)$ is the integrand and the asterisk indicates that variable values are at their optimum. Under Nash competition, each competitor has to satisfy these first-order conditions. The needed derivatives are given by

$$\frac{\partial F}{\partial S}=e^{-dt}\left[(P-C)\frac{\partial Q}{\partial S}-\phi-w\frac{\partial \sigma^2_{RV}}{\partial S}\right], \qquad (63a)$$

$$\frac{\partial F}{\partial(dS/dt)}=-e^{-dt}, \qquad (63b)$$

$$\frac{d}{dt}\frac{\partial F}{\partial(dS/dt)}=de^{-dt}\ . \qquad (63c)$$

By using (63a) and (63c), we can write for (62a) and (62b)

$$(P-C)\frac{\partial Q}{\partial S}-\phi-w\frac{\partial \sigma^2_{RV}}{\partial S}-d=0, \qquad (64a)$$

$$(P-C)\frac{\partial Q}{\partial \sigma^2_{MKT}}-w\frac{\partial \sigma^2_{RV}}{\partial \sigma^2_{MKT}}=0. \qquad (64b)$$

Consider first expenditure volatility, which appears only in the integrand, and first-order condition (64b). Multiply the expression with σ^2_{MKT} and expand the first term with Q to obtain

$$(P-C)Q\varepsilon_{Q,\sigma^2_{MKT}}-w\frac{\partial \sigma^2_{RV}}{\partial \sigma^2_{MKT}}\left(\sigma^2_{MKT}\right)=0.$$

Solve the equation for $\left(\sigma^2_{MKT}\right)^*$ to obtain

$$\left(\sigma^2_{MKT}\right)^*=\frac{(P-C)Q^*}{w}\left(\frac{\varepsilon^*_{Q,\sigma^2_{MKT}}}{\partial \sigma^2_{RV}/\partial \sigma^2_{MKT}}\right), \qquad (65)$$

with $\partial \sigma^2_{RV}/\partial \sigma^2_{MKT}>0$. Solution (65) is consistent with Theorem 2. □

THEOREM 1. Multiply (64a) with S and expand terms with Q and σ^2_{RV}, respectively, to obtain

$$\left(P-C\right)Q\varepsilon_{Q,S}-w\varepsilon_{\sigma^2_{RV},S}\sigma^2_{RV}-\left(\phi+d\right)S=0, \tag{66}$$

where $\varepsilon_{Q,S}=\dfrac{\partial Q}{\partial S}\dfrac{S}{Q}$ and $\varepsilon_{\sigma^2_{RV},S}=\dfrac{\partial\sigma^2_{RV}}{\partial S}\dfrac{S}{\sigma^2_{RV}}$. Similarly, multiply (64b) with σ^2_{MKT} and expand terms with Q and σ^2_{RV}, respectively, and obtain

$$\left(P-C\right)Q\varepsilon_{Q,\sigma^2_{MKT}}-w\varepsilon_{\sigma^2_{RV},\sigma^2_{MKT}}\sigma^2_{RV}=0, \tag{67}$$

where $\varepsilon_{Q,\sigma^2_{MKT}}=\dfrac{\partial Q}{\partial\sigma^2_{MKT}}\dfrac{\sigma^2_{MKT}}{Q}$ and $\varepsilon_{\sigma^2_{RV},\sigma^2_{MKT}}=\dfrac{\partial\sigma^2_{RV}}{\partial\sigma^2_{MKT}}\dfrac{\sigma^2_{MKT}}{\sigma^2_{RV}}$.

Rearrange (66) and (67) to

$$\frac{\left(P-C\right)Q\varepsilon_{Q,S}-\left(\phi+d\right)S}{\varepsilon_{\sigma^2_{RV},S}}=w\sigma^2_{RV}\ , \tag{68}$$

$$\frac{\left(P-C\right)Q\varepsilon_{Q,\sigma^2_{MKT}}}{\varepsilon_{\sigma^2_{RV},\sigma^2_{MKT}}}=w\sigma^2_{RV}. \tag{69}$$

Setting (68) = (69), we can solve for the optimal stock

$$S^*=\frac{\left(P-C\right)Q^*\varepsilon^*_{Q,S}}{\phi+d}-\frac{\varepsilon^*_{\sigma^2_{Rv},S}\left(P-C\right)Q^*\varepsilon^*_{Q,\sigma^2_{MKT}}}{\left(\phi+d\right)\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}}. \tag{70}$$

The solution establishes a global maximum because the integrand $F\left(t,S^*\left(t\right),dS^*\left(t\right)/dt,\left(\sigma^2_{MKT}\right)^*\right)$ is concave in $S^*(t)$ and $dS^*(t)/dt$. In addition, we do not need a necessary transversality condition in infinite horizon problems (Kamien and Schwartz 1991, 95f).

Given $\phi S=MKT-dS/dt$, we multiply (70) with ϕ to obtain

$$MKT^*=\frac{\left(P-C\right)Q^*\varepsilon^*_{Q,MKT}}{\phi+d}-\frac{\varepsilon^*_{\sigma^2_{RV,MKT}}\left(P-C\right)Q^*\varepsilon^*_{Q,\sigma^2_{MKT}}}{\left(\phi+d\right)\varepsilon^*_{\sigma^2_{RV},\sigma^2_{MKT}}}+\frac{dS^*}{dt}, \tag{71}$$

where $\varepsilon^2_{Q,MKT}=\phi\varepsilon^2_{Q,S}$ and $\varepsilon_{\sigma^2_{RV},MKT}=\phi\varepsilon_{\sigma^2_{RV},S}$. Since the optimal solution is a steady-state solution in an autonomous problem with infinite horizon, dS^*/dt measures the deviations of optimal marketing expenditures around their long-term equilibrium level, i.e. around the optimal mean expenditure level $\mu^*_{long-term}$. The optimal variance of marketing expenditures and thus dS^*/dt is given by expression (65).

Since $dS^*/dt = 0$ for the optimal mean expenditure level, $\mu^*_{long-term}$, expression (71) reduces to the extended form of Theorem 1 in (12). □

Derivation of Estimation Equations

In this section, we derive the estimation equations (16) and (17) from equations (14) and (15). We start with a log transformation of equations (14) and (15),

$$\begin{aligned}\ln V(REV) = \ln\gamma_0 + \gamma_1 \ln V(MKT) + \gamma_2 \ln A(MKT) + \gamma_3 \ln V(CMKT) + \gamma_4 CORR + \\ \gamma_5 RESP + \mathbf{X}\gamma + \nu,\end{aligned} \tag{72}$$

$$\ln V(CF) = \ln\delta_0 + \delta_1 \ln V(REV) + \delta_2 \ln V(MKT) + \delta_3 \ln A(MKT) + \delta_4 A(MKT) + \upsilon. \tag{73}$$

Then we take the total differentials to obtain,

$$\begin{aligned}\frac{1}{V(REV)} dV(REV) &= \frac{\gamma_1}{V(MKT)} dV(MKT) + \frac{\gamma_2}{A(MKT)} dA(MKT) \\ &+ \frac{\gamma_3}{V(CMKT)} dV(CMKT) + \gamma_4 dCORR + \gamma_5 dRESP + \mathbf{dX\gamma} + d\nu,\end{aligned} \tag{74}$$

$$\begin{aligned}\frac{1}{V(CF)} dV(CF) &= \frac{\delta_1}{V(REV)} dV(REV) + \frac{\delta_2}{V(MKT)} dV(MKT) + \frac{\delta_3}{A(MKT)} dA(MKT) \\ &+ \delta_4 dA(MKT) + d\upsilon,\end{aligned} \tag{75}$$

which define the structural form of our estimation equations (16) and (17).

Alternative Model Estimation Results

In this section, we provide a summary of results of alternative brand sales model estimations and the correlation matrix of variables.

Note that the estimation of our suggested brand sales model (Equation 13) is based on a fixed-effects model with brand-specific effects for own and competitive marketing expenditures. Estimation of the alternative partial adjustment and Koyck specifications requires transforming the data into first differences. These models include the lagged dependent variable among predictor variables whose effect can only be consistently estimated in a first-difference panel data model (Greene 2006). Since the lagged difference of sales is still correlated with the differenced error term it needs to be instrumented. Values for sales lagged by two or three periods, respectively, provide valid instruments (Greene 2006). However, they further reduce the sample size and may turn out to be weak instruments. Finally, we note that because of the inherent differences between model specifications comparisons of model fit in terms of Pseudo R^2 (for first differences of the dependent variable) are limited.

TABLE A2

Overview of Estimation Results of Alternative Brand Sales Model Specifications

France	*Gastro-intestinal categories*					*Cardio-vascular categories*				
	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)
Average effect of own marketing expenditures	.14	.07	.13	.20	.16	.02	.04	.01	.03	.01
Average effect of competitive marketing expenditures	.11	.06	.04	.05	.05	-.04	.03	.02	-.02	-.03
Effect of lagged sales	n.a.	.66* (.31)	.68* (.31)	.32** (.11)	.35** (.10)	n.a.	.59** (.10)	.59** (.10)	.49** (.06)	.45** (.06)
Pseudo-R^2	.87	.39	.39	.37	.39	.90	.33	.34	.47	.46

Germany	*Gastro-intestinal categories*					*Cardio-vascular categories*				
	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)
Average effect of own marketing expenditures	.16	.15	.06	.17	.16	.00	-.01	-.01	.00	.02
Average effect of competitive marketing expenditures	-.01	.03	.01	.02	.02	.02	.03	.02	.00	.04
Effect of lagged sales	n.a.	.69** (.19)	.74** (.19)	.34* (.15)	.39** (.14)	n.a.	.65** (.11)	.65** (.10)	.50** (.06)	-.29 (.51)
Pseudo-R^2	.45	.08	.03	.45	.20	.91	.14	.15	.48	.45

Table A2 (continued)

Italy	*Gastro-intestinal categories*					*Cardio-vascular categories*				
	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)
Average effect of own marketing expenditures	.03	.02	.08	.01	.00	.08	.16	.12	.12	.13
Average effect of competitive marketing expenditures	-.01	-.09	-.06	-.07	-.06	-.02	.08	.09	.03	.04
Effect of lagged sales	n.a.	.12 (.42)	.20 (.28)	.04 (.33)	.17 (.19)	n.a.	.10 (.31)	.11 (.31)	-.13 (.39)	.26** (.05)
Pseudo-R^2	.60	.22	.37	.32	.31	.93	.15	.15	.27	.24
UK	*Gastro-intestinal categories*					*Cardio-vascular categories*				
	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)	Eq. 13	Partial (a)	Partial (b)	Koyck (a)	Koyck (b)
Average effect of own marketing expenditures	.03	.01	-.05	.03	.00	.00	.03	.04	.02	.02
Average effect of competitive marketing expenditures	-.01	.03	.03	-.02	-.01	.00	.01	-.03	.00	-.01
Effect of lagged sales	n.a.	.85** (.18)	.85** (.17)	.52** (.10)	.51** (.11)	n.a.	.43** (.08)	.43** (.09)	.52** (.06)	.52** (.04)
Pseudo-R^2	.62	.20	.29	.69	.68	.89	.60	.60	.60	.64

Notes: Standard errors in parentheses. Two-sided t-test applies. ** $p < .01$; * $p < .05$. Pseudo R^2 is the squared correlation between predicted and actual values of the dependent variable. Standard R^2 is not defined for GLS and IV estimation approaches, which were used for estimating the models.
The estimates of effects for own and competitive marketing expenditures represent average total effects across brands. Therefore, standard errors are not presented here.

Model specifications:
- Eq. 13 = Brand sales model with serial error correlation structure as of Equation (13)
- Partial (a) = Partial adjustment model with current + lagged own and competitive marketing effects (no serial error correlation).
- Partial (b) = Partial adjustment model with only current own and competitive marketing effects (no serial error correlation).
- Koyck (a) = Koyck model with current + lagged own and competitive marketing effects (serial error correlation).
- Koyck (b) = Koyck model with only current own and competitive marketing effects (serial error correlation).

TABLE A3

Correlation Matrix (Moving Variances and Moving Averages based on 8 Quarter Window)

	Variance of Revenues	Variance of Cash Flows	Variance of marketing expenditures	Level of marketing expenditures	Marketing responsiveness	Variance of competitive marketing expenditures	Correlation between own and competitive marketing expenditures
Variance of Revenues	1						
Variance of Cash Flows	.965***	1					
Variance of marketing expenditures	.238***	.383***	1				
Level of marketing expenditures[1)]	.344***,[1)]	.384***,[1)]	.603***,[1)]	1			
Marketing responsiveness	.072***	.062***	.050**	.085***	1		
Variance of competitive marketing expenditures	-.045**	-.03	.064***	.168***	-.139***	1	
Correlation between own and competitive marketing expenditures	-.086***	-.115***	-.065***	-.051**	.130***	.014	1

Notes: *** $p < .01$; ** $p < .05$; * $p < .10$

[1)] Note that this variable is not corrected for brand size. Larger brands have higher expenditure levels that involve larger variance.

References

Dubé, Jean-Pierre, Günter J. Hitsch, and Puneet Manchanda (2005), "An Empirical Model of Advertising Dynamics," *Quantitative Marketing and Economics*, 3, 107-144.

Greene, William H. (2006), *Econometric Analysis*. 6th ed., Upper Saddle River, NJ: Prentice Hall.

Fischer, Marc and Sönke Albers (2010), "Patient- or Physician-Oriented Marketing: What Drives Primary Demand for Prescription Drugs?," *Journal of Marketing Research*, 47 (1), 103-121.

Hanssens, Dominique M. and Henry A. Levien (1983), "An Econometric Study of Recruitment Marketing in the U.S. Navy," *Management Science*, 29, 1167-1184.

Hanssens, Dominique M., Leonard J. Parsons, and Randall L. Schultz (2001), *Market Response Models, Econometric and Time Series Analysis.* 2nd ed., Boston et al.: Kluwer Academic Publisher.

Kamien, Morton I. and Nancy L. Schwartz (1991), *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. 2nd ed. North-Holland: Elsevier.

Mahajan, Vijay and Eitan Muller (1986), "Advertising Pulsing Policies for Generating Awareness for New Products," *Marketing Science*, 5, 89-106.

Simon, Hermann (1982), "ADPULS: An Advertising Model with Wearout and Pulsation," *Journal of Marketing Research*, 19, 352-363.