ANALYTICAL AND NUMERICAL METHODS FOR DETECTION OF A CHANGE IN DISTRIBUTIONS

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I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Dedication

To Dad & Mom

and Family

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List of Notation and Abbreviations

Notation and abbreviations are used in this thesis are:

ARL	Average Run Length
AD	Average Delay
EWMA	Exponentially Weighted Moving Average
CUSUM	Cumulative Sum
MCA	Markov Chain Approach
IE	Integral Equations
MC	Monte Carlo Simulations
H	control limit/boundary
θ	moment of change-point
τ	stopping time/alarm time
α_0	in-control parameter
α	out-of-control parameter
λ	weighted smoothing parameter in EWMA statistic, $\lambda \in (0, 1)$
χ	overshoot
φ	characteristic function
ψ	cumulant generating function (logarithm of the moment generating function)

List of Publications and Conference Presentations

Parts of the work presented in this thesis have been previously published as:

1. Sukparungsee, Saowanit, & Novikov, Alexander. 2006. On EWMA Procedure for Detection of a Change in Observations via Martingale Approach. In *Proceedings of the KMITL International Conference on Science and Applied Science, 8-10 March,* Bangkok, Thailand (KSAS 2006).

2. Sukparungsee, Saowanit, & Novikov, Alexander. 2006. On EWMA Procedure for Detection of a Change in Observations via Martingale Approach. *KMITL Science Journal; An International Journal of Science and Applied Science*, 6(2a): 373-380.

3. Sukparungsee, Saowanit, & Novikov, Alexander. 2007. Analytical Approximations for Average Run Lengths in EWMA Charts in Case of Light-tailed Distributions. In *Proceedings of International Conference of Mathematical Sciences, 28-29 November,* Bangi-Putrajaya, Malaysia (ICMS 2007).

4. Sukparungsee, Saowanit, & Novikov, Alexander. 2008. Analytical Approximations for Detection of a Change-Point in Case of Light-tailed Distributions. *Journal of Quality Measurement and Analysis*, 4(2): 49-56.

Abstract

This thesis aims to derive analytical approximations and numerical algorithms for analysis and design of control charts used for detection of changes in distributions. In particular, we present a new analytical approach for evaluating of characteristics of the "Exponentially Weighted Moving Average (EWMA)" procedure in the case of Gaussian and non-Gaussian distributions with light-tails.

The main characteristics of a control chart are the mean of false alarm time or, Average Run Length (ARL), and the mean of delay for true alarm time or, Average Delay time (AD). ARL should be sufficiently large when the process is in-control and AD should be small when the process is out-of-control. Traditional methods for numerical evaluation ARL and AD are the Monte Carlo simulations (MC), Markov Chain Approach (MCA) and Integral Equations method (IE). These methods have the following essential drawbacks: the crude MC is very time consuming and difficult to use for finding optimal designs; MCA requires matrix inversions and, in general, is slowly convergent; IE requires intensive programming even for the case of Gaussian observations.

In this thesis we develop an approach based on a combination of the martingale technique and Monte Carlo simulations. With the use of a popular symbolic/numerical software Mathematica®, this new approach allows to obtain accurate procedures for finding the optimal weights, alarm boundaries and approximations ARL and AD for the case of Gaussian observations. Further, we show that our approach can be used also for non-Gaussian distributions with light-tails and, in particular, for Poisson and Bernoulli distributions.