Analysing Consolidation Data to Optimise Elastic Visco-plastic Model Parameters for Soft Clay

A thesis in fulfilment of the requirement for the award of the degree

Doctor of Philosophy

from

University of Technology, Sydney (UTS)

By

THU MINH LE, BEng (1st class Hons, UTS)

School of Civil and Environmental Engineering, Faculty of Engineering and Information Technology
2015
CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Thu Minh Le
February 2015
ABSTRACT

Analysing the behaviour of soft soils under embankments is a significant challenging task for geotechnical engineers. By having more insight into long term soil behaviour and understanding the key parameters influencing the results, there will be more chance to strategically plan and utilise the soft ground for construction purposes. The time-dependent behaviour of soft soils, especially the ground settlements under structural and non-structural loading, is considered as a significant issue, which has been studied for many decades. Prediction of creep settlement of soft soils is a challenging task, as a very long period of time counted in years is involved. Many theories have been proposed along with a large number of laboratory and field measurements in order to provide more precise knowledge of the time-dependent viscous behaviour of soft soils. However, there are still some disagreements between theoretical and practical studies, which may keep the accuracy of the predictions questionable.

Among the great number of developed models for soft soils, the elastic visco-plastic model with the non-linear creep function is considered as an effective method to describe the long-term stress-strain behaviour of soft soils. However, the difficulties to determine the model parameters limit the application of the model in practice. Since the relationship between the effective stress and strain during the dissipation of the excess pore water pressure cannot be identified easily, in the current practice the creep strain limit $\varepsilon_{im}^{vp}$ and the creep coefficient $\psi_o/V$ to form the creep function are determined based on the curve fitting of the experimental data after the end of the primary consolidation. As a result, the number of data points available for the curve fitting is limited, and the extremely long tests are required. Moreover, in the conventional procedure for the ease of the curve fitting, the time parameter $t_o$ in the elastic visco-plastic, which is the time value of the reference time line in the space of $\varepsilon-log(\sigma')$, has been assumed as the time at the end of primary consolidation process. Hence, based on this assumption of $t_o$, the reference time line would include viscous strain, which is contradict to the definition of a viscous free reference time line. Thus, the value of $t_o$ influences not only the reference time line parameters, but also the parameters of the creep function. Additionally, the conventional determination approach for the model parameters is influenced by the thickness of the soil sample. Hence, the model parameters obtained by the conventional method may not be unique.
As a result, the main objective of this research project is to propose a numerical solution to determine the model parameters for the elastic visco–plastic model adopting the trust–region reflective least square algorithm. The trust-region reflective least square algorithm is an advanced optimisation method for the non-linear equation system. A Crank–Nicolson finite difference scheme is applied to solve the coupled partial differential equations in order to simulate one-dimensional stress-strain behaviour of soft soil with different boundary conditions. The proposed method can adopt the experimental data during the dissipation of the excess pore water pressure to determine all the model parameters simultaneously.

In this thesis, a series of laboratory experiments were conducted at the UTS soil laboratory using two sizes of hydraulic consolidation Rowe cell setups. A 29.5 mm thick soil sample of a kaolinite mixture was tested and adopted to determine the model parameters, while an experimental result of a thicker soil sample (i.e. 140.5 mm thick) was compared with the predictions using the optimised model parameters. The Rowe cell setups can measure the volume change, the vertical settlement and the excess pore water pressure continuously. Especially, the large Rowe cell setup to conduct the test on the 140.5 mm thick soil sample was modified to measure the excess pore water pressure at different depth and different distances to the centre line at the base. Moreover, other four validation exercises including two laboratory–based case studies and two field–based case studies were included to verify the ability of the proposed method to analyse the time-dependent behaviour of soft soils.

The developed method can be considered as a simple, practical and accurate solution for the model parameter determination. The optimised model parameters allow the predictions of settlement to be in good agreement with the measurements, while the predictions of the excess pore water pressure are reasonably close to the measurement. Additionally, the variations of the creep strain limit, the creep coefficient and the creep strain rate during the dissipation of the excess pore water pressure can be observed. Moreover, the unusual increase of the excess pore water pressure in the early stages of loading can be also predicted. The numerical analysis applying the proposed method is able to illustrate the influence of the soil layer thickness on the time–dependent stress-strain behaviour of soft soil. The proposed approach can be adopted to back calculate the elastic visco-plastic model parameters for real case in the field utilising time-dependent settlement and excess pore water pressure measurements.
To my father, Pha Van Le, my mother, The Thanh T. Truong, my brother, Tri Minh Le and my husband, Thanh Tien Nguyen, whose love, encouragement and support have been always with me through this journey.
ACKNOWLEDGEMENT

I have discovered that researching and writing this thesis is both challenging and pleasant experience in my life. My PhD journey would not have been completed without accompany and supports from various groups of staff and friends. It is my pleasure to express my gratitude for all of them.

Firstly, I would like to express my greatest appreciation to my principal supervisor, Dr. Behzad Fatahi, and my co-supervisor, A/Prof. Hadi Khabbaz, who provide me this opportunity, encourage, assist and support me through my study. Their guidance, their time, ideas and support have inspired and motivated me in many aspects of my PhD study as well as my life.

I would like to thank Antonio Reyno, and other staff members in the university laboratories for their assistances and supports in my laboratory experiments. I am grateful to Babak Azari, Lam Nguyen and Liem Ho which have helped me to prepare and operate the equipments for my laboratory tests. I also appreciate the general assistance from Phyllis Agius and Van Le during my study.

The research effort in this thesis was carried out during 2010-2014 in the Faculty of Engineering and Information Technology of University of Technology, Sydney. It has been supported by the Australian Postgraduate Awards from the Australian Government in three and a half years. The financial supports from the university and the faculty during my study are gratefully acknowledged.

Thanks are extended to my friends and fellow friends at University of Technology, Sydney, whose accompany and helps have made my study life more enjoyable. For my geotechnical group members, particularly Babak Arazi, Behnam Fatahi, Ali Parsa-Pajouh, Lam Nguyen, Aslan Sadeghi Hokmabadi, Liem Ho, and Quoc Van Nguyen.

Lastly, I would like to express my love and appreciation to my family for their love, encouragement and support. My parents endlessly love me and always encourage me to study and follow my pursuits. My parents are not only my family, but also my lifetime teachers and my idols. I am also grateful to my brother for his presence at Sydney to bring me sense of family, while my parents are still in Vietnam. Additionally, I am indebted to my grandmother, my aunty and her family here for their caring and essential supports from the first day of my arrival in Sydney and now. And for my loving, supportive and patient husband Thanh, I appreciate his love and faithful support during the final stages of my PhD study.
LIST OF PUBLICATIONS

❖ Published/Accepted Journal Papers

❖ Submitted Journal Papers
Trust-region Reflective Optimisation to Obtain Soil Visco-plastic Properties, *Engineering Computations*

❖ Conference Papers


TABLE OF CONTENTS

ABSTRACT ............................................................................................................................................ III

ACKNOWLEDGEMENT .................................................................................................................... VI

LIST OF PUBLICATIONS ................................................................................................................ VII

TABLE OF CONTENTS .................................................................................................................... IX

LIST OF TABLES ............................................................................................................................ XIII

LIST OF FIGURES .......................................................................................................................... XV

LIST OF NOTATIONS ..................................................................................................................... XXV

CHAPTER 1

INTRODUCTION ................................................................................................................................. 1-1

1.1 OVERVIEW ............................................................................................................................... 1-1

1.2 STATEMENT OF PROBLEM ................................................................................................. 1-3

1.3 OBJECTIVES AND SCOPES OF RESEARCH ..................................................................... 1-6

1.4 ORGANISATION OF THE THESIS ....................................................................................... 1-7

CHAPTER 2

LITERATURE REVIEW ..................................................................................................................... 2-1

2.1 INTRODUCTION ....................................................................................................................... 2-1

2.2 PROBLEMS ASSOCIATED WITH SOFT SOILS ................................................................. 2-2

2.2.1 Description of soft soils ..................................................................................................... 2-2

2.2.2 Problems associated with soft soils .................................................................................. 2-6

2.3 CREEP MECHANISMS ............................................................................................................ 2-7

2.3.1 Creep due to the breakdown of inter–particle bonds ....................................................... 2-7

2.3.2 Creep due to jumping of molecule bonds ......................................................................... 2-9

2.3.3 Creep due to sliding among particles .............................................................................. 2-11

2.3.4 Creep due to water flows in a double pore system .......................................................... 2-12

2.3.5 Creep due to the structural viscosity .............................................................................. 2-14

2.3.6 Discussion ......................................................................................................................... 2-17

2.4 TIME–DEPENDENT STRESS – STRAIN BEHAVIOUR OF SOFT SOILS .............................. 2-20

2.4.1 Time effects ....................................................................................................................... 2-20

2.4.2 Strain rate effects .............................................................................................................. 2-23
2.4.3 Stress effects ................................................................. 2-27
2.4.4 Other influencing factors .............................................. 2-29
2.4.5 Stress relaxation .......................................................... 2-31
2.4.6 Undrained creep ........................................................... 2-31

2.5 HYPOTHESES A AND B ...................................................... 2-33
2.5.1 The suppositions .......................................................... 2-35
2.5.2 Discussion on the uniqueness concept of the end of primary consolidation void ratio 2-38
2.5.3 Laboratory study on soil samples with different thicknesses ........................................... 2-40

2.6 PREDICTION APPROACHES FOR THE LONG-TERM SETTLEMENT OF SOFT SOILS .... 2-45
2.6.1 Approach of coupling the consolidation theory and a constant $C_d$ ................. 2-45
2.6.2 The concept of unique $C_d/C_v$ and $e_{EOV}$ .................................................. 2-49
2.6.3 General stress – strain – strain rate models ....................................................... 2-49
2.6.3.1 Isotach models ................................................................................................................ 2-49
2.6.3.2 Time resistance concept ................................................................................................. 2-55
2.6.3.3 Other models ..................................................................................................................... 2-58

2.7 SUMMARY ............................................................................... 2-61

CHAPTER 3

NUMERICAL SOLUTION FOR ELASTIC VISCO–PLASTIC MODEL AND
MODEL PARAMETER DETERMINATION ......................................................... 3-1

3.1 INTRODUCTION ............................................................................. 3-1

3.2 ELASTIC VISCO–PLASTIC MODEL .................................................. 3-1
3.2.1 Time–line concept ................................................................. 3-1
3.2.2 Governing equations .............................................................. 3-6
3.2.3 Conventional procedure for the model parameter determination ............. 3-8
3.2.3.1 Instant time-line parameters ($\psi/V$ and $\epsilon_{r0}^p$) ................................................. 3-9
3.2.3.2 Creep function parameters ($\psi/V$, $\epsilon_{r0}^p$, and \(t_c\)) .............................................. 3-10
3.2.3.3 Reference time-line parameters ($\lambda/V$, $\epsilon_{r0}^{op}$, and $\sigma_{r0}$) ......................... 3-17
3.2.3.4 Numerical analysis for an example embankment on Ottawa clay ............... 3-21

3.3 COUPLED EQUATIONS OF THE EVP MODEL AND THE CONSOLIDATION THEORY .... 3-27
3.3.1 Coupled governing equations for a constant external loading ................. 3-27
3.3.2 Crank-Nicolson finite difference solution ............................................. 3-29
3.3.3 Time-dependent loading ........................................................................ 3-36

3.4 MODEL PARAMETER DETERMINATION ........................................... 3-37
3.4.1 General .................................................................................. 3-37
3.4.2 Least squares algorithm ......................................................... 3-37
3.4.3 Trust–region reflective algorithm ................................................................. 3-38
3.4.4 Parameters determination solution ............................................................... 3-41
3.5 SUMMARY ........................................................................................................ 3-45

CHAPTER 4

LABORATORY STUDY TO VERIFY THE PROPOSED NUMERICAL APPROACH ........................................... 4-1

4.1 GENERAL .......................................................................................................... 4-1
4.2 SMALL ROWE CELL EXPERIMENT AND PARAMETER DETERMINATION ............. 4-1
4.2.1 Test setup and experimental procedure ......................................................... 4-2
4.2.1.1 Soil sample preparation ................................................................................ 4-2
4.2.1.2 Experiment on thin KBS sample ................................................................. 4-4
4.2.1.3 EVP model parameter determination ......................................................... 4-10
4.3 LARGE ROWE CELL EXPERIMENT AND VERIFICATION EXERCISE ............ 4-16
4.3.1 Experimental setup and testing procedure ...................................................... 4-16
4.3.2 Experimental results and verification exercise ................................................ 4-21
4.4 DISCUSSION .................................................................................................... 4-26
4.5 SUMMARY ........................................................................................................ 4-34

CHAPTER 5

LABORATORY-BASED CASE STUDIES FOR FURTHER VALIDATION .......... 5-1

5.1 GENERAL .......................................................................................................... 5-1
5.2 CASE STUDY 1: HONG KONG MARINE CLAY, HONG KONG ......................... 5-2
5.2.1 Soil properties .............................................................................................. 5-2
5.2.2 Numerical simulation ................................................................................... 5-2
5.2.3 Results and discussion .................................................................................. 5-8
5.3 CASE STUDY 2: DRAMMEN CLAY, NORWAY ............................................... 5-25
5.3.1 Soil properties .............................................................................................. 5-25
5.3.2 Numerical simulation ................................................................................... 5-26
5.3.3 Results and discussions ................................................................................ 5-29
5.4 SUMMARY ........................................................................................................ 5-50

CHAPTER 6

FURTHER VERIFICATION EXERCISES–FIELD CASE STUDIES .............. 6-1

6.1 INTRODUCTION ............................................................................................... 6-1
6.2 CASE STUDY 1: VÄSBY TEST FILL ................................................................. 6-1
6.2.1 Project and site description .......................................................................... 6-1
6.2.2 Subsoil profile and soil properties ................................................................. 6-4
6.2.3 Numerical simulation and analysis ............................................................... 6-5
  6.2.3.1 Model parameter determination .............................................................. 6-5
  6.2.3.2 Numerical modelling of Väsby test fill ...................................................... 6-13
6.2.4 Results and discussion .............................................................................. 6-20
  6.2.4.1 Settlement predictions ............................................................................ 6-20
  6.2.4.2 Excess pore water pressure ................................................................. 6-23
  6.2.4.3 Creep compression properties .............................................................. 6-26
6.3 CASE STUDY 2: SKÅ-EDEBY TEST FILL ...................................................... 6-34
  6.3.1 Project and site description ...................................................................... 6-34
  6.3.2 Subsoil profile and soil properties ............................................................ 6-37
  6.3.3 Numerical simulation .............................................................................. 6-39
    6.3.3.1 Model parameter determination .......................................................... 6-39
    6.3.3.2 Numerical modelling for Skå Edeby test fill ........................................ 6-52
  6.3.4 Results and discussion ............................................................................ 6-58
    6.3.4.1 Settlement predictions ........................................................................ 6-58
    6.3.4.2 Excess pore water pressure ................................................................. 6-62
    6.3.4.3 Creep compression properties .............................................................. 6-66
6.4 SUMMARY ..................................................................................................... 6-73

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS ..................................................... 7-1

7.1 SUMMARY .................................................................................................... 7-1
7.2 CONCLUSIONS ............................................................................................. 7-3
7.3 RECOMMENDATIONS FOR FUTURE RESEARCH .................................... 7-7

REFERENCES ....................................................................................................... Ref-1
LIST OF TABLES

Table 2.1 Flocculation and dispersion of Clays (after Mitchell 1956) ........................................2-4
Table 2.2 Consistency of fine grained soils by consistency index and undrained shear strength .................................................................2-5
Table 2.3 Types of peds and pores (after Matsuo & Kamon 1977) .........................2-13
Table 2.4 Main creep mechanisms in soft soils ..........................................................2-18
Table 2.5 Secondary compression based on $C_\alpha$ ........................................2-48
Table 2.6 Summary of existing model category (modified after Leroueil 1985) ........2-61
Table 2.7 Comparison of some constitutive models for time-dependent behaviour of soft soils ..................................................................................................................2-63
Table 3.1 Influences of $t_0$ on $\psi_0/V$ and $e_{\text{im}}^{\text{yp}}$ of Ottawa clay ......................3-11
Table 3.2 Options to determine the reference time-line parameters ......................3-23
Table 3.3 The EVP model parameters by different options .....................................3-23
Table 4.1 Properties of Q38 kaolinite and Active Bond 23 bentonite .................4-2
Table 4.2 Initial information adopted in the optimisation procedure to determine the model parameters for KBS mixture ........................................4-11
Table 4.3 The optimised model parameters and soil permeability properties .........4-12
Table 4.4 Details of loading stages using LRC apparatus ........................................4-20
Table 4.5 Initial input information for LRC simulation ............................................4-20
Table 5.1 Initial information adopted in the optimisation procedure to determine the model parameters for Hong Kong marine clay .........................................5-3
Table 5.2 Summary of the adopted model parameters for Hong Kong marine clay ....5-6
Table 5.3 Initial information of four specimens of Drammen clay (after Berre & Iversen 1972) ..................................................................................................................5-27
Table 5.4 Model parameters for Drammen clay obtained adopting the proposed TRRLS approach .......................................................................................5-29
Table 6.1 Initial information adopted for the EVP model parameter determination for Värsby clay .................................................................................................6-9
Table 6.2 The optimised model parameters and soil permeability properties for Värsby clay ........................................................................................................6-12
Table 6.3 Initial values of $m_v$ and $c_v$ reported by Chang (1969) and calculated coefficient of permeability $k$ based on $m_v$ and $c_v$ ............................................................ 6-14

Table 6.4 Soil properties of Samples A and B of Skå-Edeby clay ......................... 6-43

Table 6.5 Initial information adopted for the EVP model parameter determination for Skå-Edeby Sample A of Skå-Edeby clay ...................................................... 6-44

Table 6.6 The optimised model parameters and soil permeability properties of Sample A of Skå-Edeby clay ......................................................................................... 6-45

Table 6.7 Initial information adopted for the EVP model parameter determination for Sample B of Skå-Edeby clay ......................................................................................... 6-49

Table 6.8 The optimised model parameters and soil permeability properties of for Sample B of Skå-Edeby clay ......................................................................................... 6-49
LIST OF FIGURES

Figure 1.1 Port of Brisbane Expansion project
(http://www.portstrategy.com/__data/assets/image/0003/531534/Port-of-Brisbane-aerial-2010.jpg) ................................................................................ 1-2

Figure 1.2 Port of Botany Bay Extension

Figure 1.3 Typical settlement curve ............................................................................... 1-4

Figure 2.1 (a) Water molecule and (b) Attraction of water molecules on the surface of clay particle (modified after Ranjan & Rao 2007) ........................................... 2-3

Figure 2.2 A schematic view of a soil element of soil in macroscopic scale .......... 2-8

Figure 2.3 Schematic representation of an ion surrounded by water molecules, and the energy barrier which it must surmount in moving between equilibrium positions (after Low 1962) ................................................................. 2-10

Figure 2.4 Contact mechanism used for individual particles in numerical model: (a) Normal force mechanism, and (b) Tangential force mechanism (after Kuhn & Mitchell 1993) ................................................................................................ 2-12

Figure 2.5 Schematic concept of clay structure (modified after Zeevart 1986) ........ 2-13

Figure 2.6 A schematic view of clay–water system ..................................................... 2-15

Figure 2.7 Schematic clay-water system in (a) free water flows through the pore system from the beginning of the whole compression process, (b) after free water flowed out of the pores ................................................................. 2-20

Figure 2.8 (a-b) Creep stages and strain rates in creep tests performed by triaxial apparatus and (c-d) Compression stages and strain rates in step load tests by oedometer apparatus (after Augustesen et al. 2004) ......................... 2-22

Figure 2.9 Laboratory stress – strain curves of Berthieville test fill samples at different depths (after Kabbaj et al. 1988) ................................................................................. 2-23

Figure 2.10 Constant rate of strain (CRS) tests (a) the variation of strain with time at different strain rates, and (b) stress – strain relationship at different strain rates (after Augustesen et al. 2004) ............................................................. 2-24

Figure 2.11 Constant rate of strain tests on (a) Bastican clay and (b) St. Cesaire clay (after Lerouiel et al. 1985) ........................................................................................................ 2-25
Figure 2.12 EOP e–log $\sigma'$ curves from CRS and incremental loading (IL) oedometer tests of Berthierville clay (after Mesri & Feng, 1986) .......................................................... 2-26

Figure 2.13 Ranges of strain rates in the laboratory tests and in-situ (after Leroueil 2006) ......................................................................................................................... 2-27

Figure 2.14 (a) Types of compression curves dependent on the stress level (after Leroueil et al. 1985) and (b) the corresponding strain rate (after Augustesen et al. 2004) ............................................................................................................... 2-28

Figure 2.15 Several single loading tests on Batiscan clay indicating the effects of stress increments on the variation of strain-time behaviour (after Leroueil et al. 1985) ......................................................................................................................... 2-29

Figure 2.16 Stress – strain curves of Berthierville clay at different strain rates and different temperatures tested by Boudali et al. 1994 (after Leroueil 1996) 2-30

Figure 2.17 The variations of the axial strains and excess pore water pressure with time during undrained triaxial creep tests on (a-b) K$_{oc}$-consolidated Wenzhou marine clay and (b-c) isotropically consolidated Wenzhou marine clay (after Wang & Yin 2012) ............................................................................................................ 2-32

Figure 2.18 Void ratio versus time under the applied stress from $\sigma'_{cl}$ to $\sigma'_{cf}$ .......... 2-35

Figure 2.19 Void ratio versus time of thin and thick samples based on Hypothesis A 2-36

Figure 2.20 Void ratio versus time of thin and thick samples based on Hypothesis B 2-37

Figure 2.21 Void ratio versus effective stress at the end of primary consolidation (after Jamiolkowski et al. 1985) ........................................................................................................ 2-37

Figure 2.22 Vertical strains of soil samples of different thicknesses (after Aboshi 1995) ......................................................................................................................... 2-41

Figure 2.23 Stress – strain curves obtained in laboratory and observed in situ between 4.25m and 7.29m of Vasby test fill (Leroueil & Kabbaj (1987) ................. 2-44

Figure 2.24 Void ratio-effective stress relationship during the consolidation process (after Taylor & Merchant, 1940) ........................................................................ 2-46

Figure 2.25 Definition of instant compression and delayed compression compared to the primary and secondary compression (after Bjerrum 1967) (a) the change in effective stress, and (b) compression versus time ................................................. 2-51

Figure 2.26 Time-line system of Bjerrum (after Bjerrum 1967) .............................. 2-51
Figure 2.27 Time resistance for a load step in an oedometer test (a) the variation of the excess pore water pressure or strain with time and (b) the variation of time resistance R with time (after Janbu 1969) .................................................................2-56

Figure 2.28 Idealised stress – strain curve from an oedometer test with the division of strain increments into an elastic and creep component. Normal consolidation (NC) line has time value of 1 day ($\tau_c + t' = 1$ day on NC) (after Vermeer & Neher 1999) .............................................................................................................2-57

Figure 2.29 Creep behaviour obtained in the time resistance concept (after Vermeer & Neher 1999) .............................................................................................................2-57

Figure 3.1 The time-line system proposed by Yin (1990) in (a) natural scale and (b) in logarithm scale of effective stress (after Yin 1990) .................................................................3-5

Figure 3.2 Illustration of the reference time-line and the limit time-line........................3-6

Figure 3.3 Compression curves of Ottawa marine clay (after Crawford 1964) (a) vertical strain versus time and (b) vertical strain versus vertical effective stress ........3-8

Figure 3.4 Curve fitting for the instant time-line .................................................................3-9

Figure 3.5 Linear curve fitting for the creep function parameters applying for Stages 5-6-7 for $t_o = 45$ min ........................................................................................................3-12

Figure 3.6 Linear curve fitting for the creep function parameters applying for Stages 5-6-7 for $t_o = 55$ min ........................................................................................................3-12

Figure 3.7 Linear curve fitting for the creep function parameters applying for Stages 5-6-7 for $t_o = 40$ min ........................................................................................................3-13

Figure 3.8 Relationship between the effective stress and (a) the creep coefficient $\psi_o/V$ and (b) the creep strain limit $e_{im}^{vp}$ for $t_o = 45$ min .........................................................3-14

Figure 3.9 Relationship between the effective stress and (a) the creep coefficient $\psi_o/V$ and (b) the creep strain limit $e_{im}^{vp}$ for $t_o = 55$ min .........................................................3-15

Figure 3.10 Relationship between the effective stress and (a) the creep coefficient $\psi_o/V$ and (b) the creep strain limit $e_{im}^{vp}$ for $t_o = 40$ min .........................................................3-16

Figure 3.11 Schematic time-line concept for the model parameter determination ....3-18

Figure 3.12 Time-line system of Ottawa clay adopting $t_o = 45$ min .........................3-20

Figure 3.13 Time-line system of Ottawa clay adopting $t_o = 55$ min .........................3-20

Figure 3.14 Time-line system of Ottawa clay adopting $t_o = 40$ min .........................3-21

Figure 3.15 Soil profile adopted in the example calculation ........................................3-24
Figure 3.16 Variations of the initial overburden effective stress, final effective stress and the vertical applied pressure with depth under the example embankment ...3-25
Figure 3.17 Settlement prediction for Ottawa clay under the embankment adopting three sets of model parameters in Table 3.3 ...........................................................3-26
Figure 3.18 Variations of the excess pore water pressure with depth at (a) 25 years and (b) 50 years after construction of Ottawa clay under the embankment adopting three sets of model parameters in Table 3.3 ..................................................3-27
Figure 3.19 Calculation grid and the boundary conditions of the Crank-Nicolson finite difference solution for the partial differential equations for one way drainage condition .................................................................3-30
Figure 3.20 Calculation grid and the boundary conditions of the Crank-Nicolson finite difference solution for the partial differential equations for two way drainage condition ..................................................................................3-30
Figure 3.21 Flowchart for solving the coupled equations of the EVP model and the consolidation theory.................................................................3-35
Figure 3.22 Time-dependent loading .........................................................................................................................3-36
Figure 3.23 Flowchart of the trust-region reflective method .................................................................3-40
Figure 3.24 Flowchart for calculating \( u(x) \) ........................................................................................................3-44
Figure 4.1 KBS mixture (a) dry state of each material and (b) wet mixture ..................4-3
Figure 4.2 Schematic diagram of the small Rowe cell .................................................................4-4
Figure 4.3 Small Rowe cell system connection .........................................................................................4-5
Figure 4.4 Small Rowe cell set up in the laboratory ........................................................................4-6
Figure 4.5 Assembling soil sample in the small Rowe cell (a) soil sample assembled in the cell and covered by a filter paper, (b) porous plate placed on the soil sample surface and covered by water, and (c) the soil sample at the end of the test...4-7
Figure 4.6 Initial loading stages of Rowe cell consolidation test of 29.5 mm thick KBS .................................................................................................................................................4-9
Figure 4.7 Unloading stages of Rowe cell consolidation test of 29.5 mm thick KBS...4-9
Figure 4.8 Reloading stages of Rowe cell consolidation test of 29.5 mm thick KBS 4-10
Figure 4.9 Calculation grid for the simulation of SRC sample.................................................................4-12
Figure 4.10 Relation of void ratio and coefficient of permeability ..................4-13
Figure 4.11 Predictions and measurements of the SRC sample (a) The average vertical strain and (b) The excess pore water pressure at the impervious base ........4-15
Figure 4.12 Illustration of the large hydraulic consolidation cell (LRC) apparatus

Figure 4.13 Large Rowe cell set up in the laboratory

Figure 4.14 Large Rowe cell system connection

Figure 4.15 Calculation grid for the numerical simulation of the LRC sample

Figure 4.16 Prediction of the average vertical strain of the thick sample (LRC)

Figure 4.17 Excess pore water pressure of LRC during (a) 25 kPa – 50 kPa, and (b) 50 kPa – 100 kPa loading stages

Figure 4.18 Excess pore water pressure of LRC during (a) 100 kPa – 200 kPa and (b) 200 kPa – 400 kPa loading stages

Figure 4.19 Time-line system for the KBS mixture

Figure 4.20 Variation of the average creep strain limit $\varepsilon_{lm}^{sp}$ with time of (a) SRC and (b) LRC

Figure 4.21 Variation of the average creep coefficient $\psi \alpha \mathcal{N}$ with time of (a) SRC and (b) LRC

Figure 4.22 Variation of the average creep parameter $\psi / V$ with time of (a) SRC and (b) LRC

Figure 4.23 Variation of the average creep strain rate $\ddot{\varepsilon}_{z}^{sp}$ with time of (a) SRC and (b) LRC

Figure 4.24 Average vertical strain versus average effective stress

Figure 5.1 Experimental loading stages of the Hong Kong marine clay (after Yin 1999) (a) loading stages, (b) unloading and (c) reloading

Figure 5.2 Relationship between the void ratio and coefficient of permeability calculated based on $c_{v}$ and $m_{v}$ of the consolidation results from Yin (1999)

Figure 5.3 Predicted time dependent vertical strain under (a) 100 kPa and 200 kPa, (b) 400 kPa and 800 kPa vertical stresses using the parameters by the conventional method and the optimised parameters using the proposed approach

Figure 5.4 Predictions of (a) the average strain of various thickness soil layers and (b) the excess pore water pressure at the base of soil layers adopting model parameters obtained from the proposed approach and the conventional approach

Figure 5.5 Influence of soil thickness on the average vertical strain at 90%, 95% and 98% dissipation of excess pore water pressure at the impervious boundary
Figure 5.6 Average creep coefficient ($\psi_c/V$) predicted by (a) the proposed approach and (b) the conventional approach .......................................................... 5-14
Figure 5.7 Average creep strain limit ($e^{vp}_{lim}$) predicted by (a) the proposed approach and (b) the conventional approach .......................................................... 5-15
Figure 5.8 Average creep parameter ($\psi/V$) predicted by (a) the proposed approach and (b) the conventional approach .......................................................... 5-17
Figure 5.9 Average creep strain rate ($\dot{e}_z^{vp}$) predicted by (a) the proposed approach and (b) the conventional approach .......................................................... 5-19
Figure 5.10 Vertical stress – strain relationship predicted by (a) the proposed approach and (b) the conventional approach .......................................................... 5-20
Figure 5.11 Isochrones of the variation of the normalised excess pore water pressure with depth after (a) 1 min, and (b) 1 hour .......................................................... 5-22
Figure 5.12 Isochrones of the variation of the normalised excess pore water pressure with depth after (a) 1 day, and (b) 1 year .......................................................... 5-23
Figure 5.13 Isochrones of the variation of the normalised excess pore water pressure with depth after (a) 2 years, and (b) 10 years .......................................................... 5-24
Figure 5.14 Stress – strain curves of oedometer soil sample of different heights (after Berre & Iversen 1972) .......................................................... 5-26
Figure 5.15 Void ratio – permeability relationship for Drammen clay .......................................................... 5-28
Figure 5.16 Comparison of the prediction and the measured data for Test A (0.0188 m): (a) the average vertical strain, and (b) the excess pore water pressure at the base for Drammen clay .......................................................... 5-30
Figure 5.17 Comparison of the prediction and the measured data for Test B (0.075 m): (a) the average vertical strain and (b) the excess pore water pressure at the base for Drammen clay .......................................................... 5-33
Figure 5.18 Comparison of the prediction and the measured data for Test C (0.150 m): (a) the average vertical strain and (b) the excess pore water pressure at the base for Drammen clay .......................................................... 5-34
Figure 5.19 Comparison of the prediction average vertical strain (%), and the measured data for Test D (0.450 m) .......................................................... 5-35
Figure 5.20 Comparison of the prediction and the measured data for Test D (0.450 m): (a) the excess pore water pressures of Increment 4, and (b) the excess pore water pressures of Increment 5 for Drammen clay .......................................................... 5-36
Figure 5.21 Isochrones of the normalised excess pore water pressure with depth of four different thickness soil samples of Drammen clay after 1 minute for (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-38

Figure 5.22 Isochrones of the variation of the excess pore water pressure with depth of four different thickness soil samples of Drammen clay during after 60 minutes for (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-39

Figure 5.23 Isochrones of the variation of the excess pore water pressure with depth of four different thickness soil samples of Drammen clay during after 1440 minutes (1 day) for (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-40

Figure 5.24 Isochrones of the variation of the vertical strain with depth of four different thickness soil samples of Drammen clay during after 1 minute for (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-41

Figure 5.25 Isochrones of the variation of the vertical strain with depth of four different thickness soil samples of Drammen clay during after 60 minutes for (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-42

Figure 5.26 Isochrones of the variation of the vertical strain with depth of four different thickness soil samples of Drammen clay after 1440 minutes (1 day) for (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-43

Figure 5.27 Variation of the average creep coefficient $\psi_o/V$ of (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-47

Figure 5.28 Variations of the average creep parameters ($\psi/V$) of (a) Increment 4 and (b) Increment 5 of Drammen clay soil samples ........................................................................................................ 5-48

Figure 5.29 Variation of the average creep strain rate $\dot{\varepsilon}_{\text{CP}}$ with time during (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-49

Figure 5.30 Variation of the coefficient of permeability at the impervious base of the soil specimens during (a) Increment 4 and (b) Increment 5 ........................................................................................................ 5-50

Figure 6.1 Location of the test fields at Lilla Mällösa and Skå-Edeby (courtesy of Google Map 2014) ........................................................................................................ 6-2

Figure 6.2 Test fill locations at Väsby, Sweden (after Chang 1981) ........................................................................................................ 6-3

Figure 6.3 Subsoil profile of the Väsby test fill (after Chang 1969) ........................................................................................................ 6-6

Figure 6.4 Soil properties of the Väsby subsoil profile (after Chang 1969) ........................................................................................................ 6-7

Figure 6.5 Incremental oedometer test result of Väsby clay at 5 m depth (after Chang 1969) ........................................................................................................ 6-8
Figure 6.6 Vertical effective stress and vertical strain relationship of Väsby laboratory soil sample .................................................................6-8
Figure 6.7 Variation between the void ratio and the coefficient of permeability based on the compression results reported by Chang (1969) for Väsby clay ..........6-10
Figure 6.8 Settlement prediction adopting the TRRLS approach comparing with laboratory measurement from Chang (1969) for Väsby clay ..........6-12
Figure 6.9 Soil properties adopted in the numerical modelling for Väsby test fill (a) Initial void ratio, (b) Unit weight, (c) Coefficient of permeability and (d) Permeability change index (after Chang 1969, Larsson & Marsson 2003) ...6-17
Figure 6.10 Geotechnical profile adopted in the numerical modelling for Väsby test fill (a) Initial effective stress, final effective stress and preconsolidation pressure, (b) Overconsolidated ratio and (c) Applied stress distribution (after Chang 1969, Larsson & Mattsson 2003) .......................................................................6-18
Figure 6.11 Initial creep compression properties (a) Creep coefficient $\psi_V/V$, (b) Creep strain limit $\varepsilon_{lm}^{vp}$ and (c) Creep parameter $\psi/V$ and (d) the initial vertical strain obtained based on Equation (6.5) for Väsby test fill ........................................6-19
Figure 6.12 Predictions of the vertical settlement at several depths beneath the test fill in the linear scale (note: model predictions are shown in solid lines) ..........................6-21
Figure 6.13 Predictions of the vertical settlement at several depths beneath the test fill in the logarithmic scale (note: model predictions are shown in solid lines) .......6-22
Figure 6.14 Variations of the vertical settlements with depth at different time points 6-22
Figure 6.15 Prediction of excess pore water pressure after 21 years from the end of construction in 1968 .................................................................................6-24
Figure 6.16 Prediction of excess pore water pressure after 32 years from the end of construction in 1979 .................................................................6-25
Figure 6.17 Prediction of excess pore water pressure after 55 years from the end of construction in 2002 ....................................................................................6-25
Figure 6.18 Predicted and measured permeabilities in natural ground and below the Väsby test fill at Lilla Mellösa ..............................................................................6-26
Figure 6.19 Variations of (a) Creep coefficient $\psi_V/V$ and (b) Creep strain limit $\varepsilon_{lm}^{vp}$ with depth in 1968, 1979, 2002 and 2014 for Väsby site .........................6-30
Figure 6.20 Variations of (a) Creep parameter $\psi/V$ and (b) Creep strain rate $\varepsilon_z^{vp}$ with depth in 1968, 1979, 2002 and 2014 for Väsby site ........................................6-31
Figure 6.21 Predictions of (a) the creep coefficient $\psi / V$ and (b) the creep strain limit $\varepsilon_{lm}^{vp}$ with time at different depths ................................................................. 6-32

Figure 6.22 Predictions of (a) the creep parameter $\psi / V$ and (b) the creep strain rate $\dot{\varepsilon}_{z}^{vp}$ with time at different depths ........................................................................ 6-33

Figure 6.23 Test fills at the Skå-Edeby site (after Larsson & Mattsson 2003) .................. 6-35

Figure 6.24 Instrumentation layout of Area IV (without drains) at the Skå-Edeby site (after Hansbo 1960) ........................................................................................................ 6-36

Figure 6.25 Schematic subsoil profile under the test fill Area IV at Skå-Edeby ............ 6-36

Figure 6.26 Soil properties of the Skå-Edeby subsoil profile (after Hansbo 1960 and Larsson & Mattsson 2003) ...................................................................................... 6-40

Figure 6.27 Oedometer test results of Samples A and B of Skå-Edeby clay adopted for the model parameter determination (from Hansbo 1960) ................................. 6-42

Figure 6.28 Incremental oedometer test results on samples obtained at Area IV at the Skå-Edeby test fill (after Hansbo 1960) .............................................................................. 6-43

Figure 6.29 Stress – strain relationship of Sample A of Skå-Edeby clay ................... 6-45

Figure 6.30 Settlement prediction adopting the TRRLS approach for Sample A of Skå-Edeby clay ........................................................................................................ 6-46

Figure 6.31 Excess pore water pressure prediction adopting the TRRLS approach for Sample A of Skå-Edeby clay at loading stages (a) 80.42 kPa, (b) 166.71 kPa and (c) 338.38 kPa ........................................................................................................ 6-47

Figure 6.32 Stress – strain relationship of Sample B of Skå-Edeby clay ......................... 6-50

Figure 6.33 Settlement prediction adopting the TRRLS approach for Sample B of Skå-Edeby clay ........................................................................................................ 6-50

Figure 6.34 Excess pore water pressure prediction adopting the TRRLS approach for Sample B of Skå-Edeby clay at loading stages (a) 80.42 kPa, (b) 158.71 kPa and (c) 261.84 kPa ........................................................................................................ 6-51

Figure 6.35 Soil properties adopted in the numerical modelling for Skå-Edeby test fill. 6-55

Figure 6.36 Geotechnical profile adopted in the numerical modelling for Skå-Edeby test fill .................................................................................................................. 6-56

Figure 6.37 Initial creep compression properties (a) Creep coefficient $\psi / V$, (b) Creep strain limit $\varepsilon_{lm}^{vp}$ and (c) Creep parameter $\psi / V$ for Skå-Edeby site .................. 6-57
Figure 6.38 Vertical settlement predictions at several depths beneath the test fill at Skå-Edeby site........................................................................................................6-59

Figure 6.39 Vertical settlement predictions at several depths beneath the test fill in the logarithmic scale of time at Skå-Edeby site.................................................................6-59

Figure 6.40 Variations of vertical settlements with depth at several time points (a) 1 year, 5 years, and 10 years, and (b) 24 years and 45 years at Skå-Edeby site...6-61

Figure 6.41 Prediction of excess pore water pressure after 14 years from the end of construction in 1971 at Skå-Edeby site.................................................................6-64

Figure 6.42 Prediction of excess pore water pressure after 25 years from the end of construction in 1982 at Skå-Edeby site.................................................................6-64

Figure 6.43 Prediction of excess pore water pressure after 45 years from the end of construction in 2002 at Skå-Edeby site.................................................................6-65

Figure 6.44 Predictions and measurements of coefficient of permeability under the test fill at Skå-Edeby site..................................................................................6-65

Figure 6.45 Variations of (a) Creep coefficient $\psi_0/V$ and (b) Creep strain limit $e_{\text{lim}}^{\text{VP}}$ with depth at CY11, CY22, CY45 and CY57 for Skå-Edeby site.................................6-69

Figure 6.46 Variations of (a) Creep parameter $\psi/V$ and (b) Creep strain rate $e^{\text{VP}}_z$ with depth at CY11, CY22, CY45 and CY57 for Skå-Edeby site.................................6-70

Figure 6.47 Variation of the creep coefficient ($\psi_0/V$) with time at (a) $z = H_o/4$ and $H_o/2$ and (b) $z = 3H_o/4$ for the Skå-Edeby test fill .................................................6-71

Figure 6.48 The variation of the creep strain limit $e_{\text{lim}}^{\text{VP}}$ with time at $z = H_o/4$, $H_o/2$ and $3H_o/4$ at the Skå-Edeby test fill.................................................................6-72

Figure 6.49 Variation of the creep parameter ($\psi/V$) with time at $z = H_o/4$, $H_o/2$ and $3H_o/4$ at the Skå-Edeby test fill.................................................................6-72

Figure 6.50 Variation of the creep strain rate $e^{\text{VP}}_z$ with time at $z = H_o/4$, $H_o/2$ and $3H_o/4$ at the Skå-Edeby test fill.................................................................6-73
# LIST OF NOTATIONS

<table>
<thead>
<tr>
<th>English letters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Model parameter in Singh and Mitchell (1968)</td>
</tr>
<tr>
<td>$a$</td>
<td>Function coefficient of creep strain limit</td>
</tr>
<tr>
<td>$a'$</td>
<td>Relation of instantaneous compression and effective stress in Taylor &amp; Merchant (1940) concept</td>
</tr>
<tr>
<td>$a_f$</td>
<td>Relation between void ratio and effective stress in Taylor &amp; Merchant (1940) concept</td>
</tr>
<tr>
<td>$B$</td>
<td>Strip width</td>
</tr>
<tr>
<td>$BP$</td>
<td>Back pressure</td>
</tr>
<tr>
<td>$b$</td>
<td>Function coefficient of creep strain limit</td>
</tr>
<tr>
<td>$b$</td>
<td>Haft of a strip width</td>
</tr>
<tr>
<td>$CB$</td>
<td>Control box of IVC</td>
</tr>
<tr>
<td>$CGT$</td>
<td>Controlled gradient test</td>
</tr>
<tr>
<td>$CL$</td>
<td>Centre line</td>
</tr>
<tr>
<td>$CP$</td>
<td>Cell pressure</td>
</tr>
<tr>
<td>$CRS$</td>
<td>Constant rate of strain</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Compression index</td>
</tr>
<tr>
<td>$C_{ijkl}$</td>
<td>Elastic matrix</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Recompression index</td>
</tr>
<tr>
<td>$C_a$</td>
<td>Coefficient of secondary compression</td>
</tr>
<tr>
<td>$C_{\text{max}}$</td>
<td>Positive constant in Karim et al. (2010)</td>
</tr>
<tr>
<td>$C^*$</td>
<td>Non constant creep coefficient in Karim et al. (2010)</td>
</tr>
<tr>
<td>$C_{ae}$</td>
<td>Compressibility ratio</td>
</tr>
<tr>
<td>$C_{ae}$</td>
<td>Coefficient of secondary compression based on void ratio</td>
</tr>
<tr>
<td>$C_{ae}$</td>
<td>Coefficient of secondary compression based on vertical strain</td>
</tr>
<tr>
<td>$C$</td>
<td>Function coefficient of creep coefficient</td>
</tr>
<tr>
<td>$c$</td>
<td>Coefficient of permeability change index</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Coefficient of consolidation</td>
</tr>
<tr>
<td>$(c_v)_{(i,j)}$</td>
<td>Coefficient of consolidation at coordinator (i,j)</td>
</tr>
<tr>
<td>$D$</td>
<td>Diagonal scaling matrix</td>
</tr>
<tr>
<td>$DL$</td>
<td>Data logger</td>
</tr>
<tr>
<td>$d$</td>
<td>Function coefficient of creep coefficient</td>
</tr>
<tr>
<td>$EOP$</td>
<td>End of primary consolidation</td>
</tr>
<tr>
<td>$EVP$</td>
<td>Elastic visco-plastic</td>
</tr>
<tr>
<td>$E_{ac}$</td>
<td>Activation energy</td>
</tr>
<tr>
<td>$e$</td>
<td>Void ratio</td>
</tr>
<tr>
<td>$e_0$</td>
<td>Initial void ratio</td>
</tr>
<tr>
<td>$e_{EOP}$</td>
<td>Void ratio at the end of primary consolidation</td>
</tr>
<tr>
<td>$\dot{e}$</td>
<td>Rate of change in void ratio</td>
</tr>
<tr>
<td>$\dot{e}_{gr}$</td>
<td>Change of void ratio with respect of the effective stress at an instant time t</td>
</tr>
<tr>
<td>$\dot{e}_{r}$</td>
<td>Change of void ratio with time at a constant effective stress</td>
</tr>
<tr>
<td>$F(x)$</td>
<td>Vector valued function having the $i^{th}$ component equal to $f_i(x)$</td>
</tr>
<tr>
<td>$f_n$</td>
<td>Normal force</td>
</tr>
<tr>
<td>$f_t$</td>
<td>Tangential force</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>Objective function of optimisation procedure</td>
</tr>
<tr>
<td>$f_i(x)$</td>
<td>Function value at time i</td>
</tr>
<tr>
<td>$G_s$</td>
<td>Specific gravity</td>
</tr>
<tr>
<td>$g$</td>
<td>Potential function</td>
</tr>
<tr>
<td>$g$</td>
<td>Gradient of $f(x)$ for the current $x$</td>
</tr>
<tr>
<td>$g(e_z, \sigma_z')$</td>
<td>Creep strain rate</td>
</tr>
<tr>
<td>$(g(e_z, \sigma_z'))_{(i,j)}$</td>
<td>Creep strain rate at coordinator (i,j)</td>
</tr>
<tr>
<td>$H$</td>
<td>Maximum drainage distance</td>
</tr>
</tbody>
</table>
$H$ Symmetric matrix of second derivatives in trust-region algorithm
$H_0$ Initial soil layer thickness
$H_{EOP}$ Soil thickness at the end of primary consolidation
$h_z$ Soil depth
KBS Kaolinite – bentonite – fine sand mixture
$K_r$ Lateral earth pressure at rest
$k$ Coefficient of vertical permeability at coordinator (i,j)
$(k)_{(i,j)}$ Coefficient of vertical permeability
$k_o$ Initial coefficient of vertical permeability
$I_z$ Influence factor
IL Incremental loading
IVC Infinite volume controller
$I$ Jacobian of $F$
LL Liquid limit
LPDT Linear potentiometer displacement transducer
LRC Large Rowe cell
$MSL_{24}$ Multiple stage loading with increments every 24 hours
$MSL_p$ Multiple stage loading with increments at the end of primary consolidation
$m$ Model parameter in Singh & Mitchell (1968)
$m_v$ Coefficient of volume compressibility
$(m_v)_{(i,j)}$ Coefficient of volume compressibility at coordinator (i,j)
$N$ Trust region of current point $x$
$N$ Positive constant in Karim et al. (2010)
$N$ Specific volume of a soil normally isotropic consolidated at ln$p'$ value of zero
NC Normally consolidated
$N_{SPT}$ SPT blow count
OC Overconsolidated
OCR Overconsolidation ratio
PC Computer
PI Plasticity index
$PVC_p$ Primary pressure/volume controller
$PVC_s$ Secondary pressure/volume controller
PWP Pore water pressure
PWPT Pore water pressure transducer
$p'$ Mean effective stress
$p_e'$ Equivalent pressure
$p_{el}$ Creep exclusion preconsolidation pressure in Karim et al. (2010)
$p_o'$ Creep inclusive preconsolidation pressure in Karim et al. (2010)
$q$ Deviator stress at time $t$
$q_i$ Deviator stress level in Singh & Mitchell (1968)
$q_o$ Initial deviator stress
$q_o$ Uniform applied stress caused by test fill
$q(s)$ Approximation function of objective function $f(x)$
$R$ Time resistance
$R_t$ Time resistance after end of primary consolidation
$R'$ Coefficient of determination
$r_s$ Creep resistance
SM Settlement marker
SPT Standard penetration test
SRC Small Rowe cell
$S$ Two–dimensional subspace of $s$
$S_c$ Secondary compression or creep compression
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_p$</td>
<td>Primary compression</td>
</tr>
<tr>
<td>$S_t, S_j$</td>
<td>Surface settlement at time $t$</td>
</tr>
<tr>
<td>$S_u$</td>
<td>Undrained shear strength</td>
</tr>
<tr>
<td>$s$</td>
<td>Trial step of $x$</td>
</tr>
<tr>
<td>$\dot{s}$</td>
<td>Sliding velocity</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature</td>
</tr>
<tr>
<td>$T_r$</td>
<td>Unitless time factor</td>
</tr>
<tr>
<td>TRRLS</td>
<td>Trust-region reflective least squares</td>
</tr>
<tr>
<td>$t$</td>
<td>Elapsed loading time</td>
</tr>
<tr>
<td>$t'$</td>
<td>Difference between $t$ and $t_e$</td>
</tr>
<tr>
<td>$t_o$</td>
<td>Time parameter</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Time at conventional end of primary consolidation</td>
</tr>
<tr>
<td>$t_i$</td>
<td>Time corresponding to the instant time-line in Garlanger (1972) or reference time in Singh &amp; Mitchell (1968)</td>
</tr>
<tr>
<td>$t_e$</td>
<td>Equivalent time</td>
</tr>
<tr>
<td>$t_r$</td>
<td>Extrapolated time corresponding to $R = 0$</td>
</tr>
<tr>
<td>$t_{tot}$</td>
<td>Total loading time</td>
</tr>
<tr>
<td>$t_{vp}$</td>
<td>Visco-plastic (creep) time</td>
</tr>
<tr>
<td>$t_{EC}$</td>
<td>Construction time</td>
</tr>
<tr>
<td>$t_{EOP}$</td>
<td>Time at the end of primary consolidation</td>
</tr>
<tr>
<td>$U$</td>
<td>Degree of consolidation</td>
</tr>
<tr>
<td>$u_{(i,j)}$</td>
<td>Excess pore water pressure at coordinator $(i,j)$</td>
</tr>
<tr>
<td>$u_0$</td>
<td>Initial excess pore water pressure in Taylor &amp; Merchant (1940)</td>
</tr>
<tr>
<td>$u_0$</td>
<td>Hydrostatic pore water pressure, initial equilibrium water pressure</td>
</tr>
<tr>
<td>$u_e$</td>
<td>Excess pore water pressure</td>
</tr>
<tr>
<td>$u_{di}$</td>
<td>Initial excess pore water pressure ($=\Delta \sigma$)</td>
</tr>
<tr>
<td>$u_x$</td>
<td>Excess pore water pressure at time $t$ in Taylor &amp; Merchant (1940)</td>
</tr>
<tr>
<td>$u(x)$</td>
<td>Total squares of difference between measured and predicted values</td>
</tr>
<tr>
<td>$V$</td>
<td>Specific volume corresponding to $e_o$</td>
</tr>
<tr>
<td>$v$</td>
<td>Specific volume of a soil at the normal stress $p'$</td>
</tr>
<tr>
<td>$w_o$</td>
<td>Initial water content</td>
</tr>
<tr>
<td>$w, w_i$</td>
<td>Water content</td>
</tr>
<tr>
<td>$x$</td>
<td>Vector of variable</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Measured data point at time $i$</td>
</tr>
<tr>
<td>$z$</td>
<td>Soil depth</td>
</tr>
<tr>
<td>$z_i$</td>
<td>Compression per unit of layer thickness</td>
</tr>
</tbody>
</table>

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>Trust region radius $&gt; 0$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time step</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>Space step</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Relationship between the logarithm of the preconsolidation pressure and the logarithm of the strain rate</td>
</tr>
<tr>
<td>$\bar{\alpha}$</td>
<td>Model parameter in Singh &amp; Mitchell (1968)</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Immediate settlement per unit of thickness and unit of load</td>
</tr>
<tr>
<td>$\alpha_e$</td>
<td>Rate of secondary compression per unit thickness and load unit</td>
</tr>
<tr>
<td>$\Delta \sigma$</td>
<td>Load increment</td>
</tr>
<tr>
<td>$\Delta \sigma_e$</td>
<td>Applied stress increment</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Fluidity parameter</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Unit weight of soil</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Unit weight of water</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Vertical strain</td>
</tr>
</tbody>
</table>
Initial vertical strain

Vertical strain at coordinator (i,j)

Average vertical strain at time t

Vertical elastic strain

Vertical elastic strain at $\sigma_z = \sigma'_u$

Elastic plastic strain at $\sigma'_z = \sigma'_{z0}$

Vertical plastic strain

Vertical reference strain at $\sigma'_z$

Vertical reference strain at $\sigma'_{z0}$

Creep strain limit

Creep strain limit at coordinator (i,j)

Visco-plastic (creep) strain

Strain rate

Creep strain rate

Elastic strain rate

Visco-plastic strain rate

Vertical visco-plastic (creep) strain rate

End of primary consolidation strain rate

Vertical plastic strain rate

Elastic stiffness

Slope of the normal isotropic consolidation line in the isotropic normal compression

Elastic plastic stiffness

Coefficient of secondary compression by Taylor & Merchant (1940)

Modified creep parameter

Coefficient of viscosity

Total vertical stress

Stress state

Preconsolidation pressure before loading

Preconsolidation pressure

Vertical effective stress

Vertical effective stress at coordinator (i,j)

Unit vertical effective stress (i.e. 1 kPa)

Initial vertical effective stress

EVP model parameter, vertical effective stress corresponding to $\varepsilon_{z0}^{ep}$

Final vertical effective stress

Stress rate

Rate of change in effective stress

Time parameter = 1 day

Difference between $t_c$ and $t_r$

Creep coefficient

Creep parameter
CHAPTER 1

INTRODUCTION

1.1 Overview

As the social and economical development of the world keep advancing along with the increase in population, the availability of appropriate sites with proper soil for construction becomes increasingly meagre. Soft soils are more predominant around coastal areas or river sides, where the demand for construction is usually higher. Various projects in Australia have been constructed on unsuitable ground such as the Ballina Bypass (part of the Pacific Highway upgrade in New South Wales), Port of Brisbane Expansion, and Port Botany Expansion (Botany Bay, Sydney). The Ballina Bypass project has been undertaken to upgrade a 12 km four lane road between Newcastle and the Queensland border. Most of the road is on coastal soil including soft clays of various thicknesses. Two trial embankments were constructed in 1998 to investigate the soil behaviour in that area, and it is reported that soft soils exhibit large creep deformation (Huang et al. 2006).

In the Port of Brisbane Expansion project, the land expansion of 235 hectares of a sub-tidal area is a reclaimed site underlaid by 9 m of soft dredged materials overlying 30m deep soft to firm seabed clays (Ameratunga et al. 2010). Due to the deep soft soil deposit (30 m), long term settlement and the secondary consolidation component are critical factors to be dealt with. Ground improvement techniques such as surcharge with wick drains and vacuum preloading were adopted in this project to reduce the post construction settlement as the clay deposit becomes overconsolidated after improvement. As discussed by Ameratunga et al. (2010), the amount of over–consolidation to be targeted in the design was a difficult question for the designer because of uncertainties in the secondary consolidation behaviour.
Port Botany Expansion is an important infrastructure development in New South Wales, Australia. This project to extend the existing port at Botany Bay reclaimed an area of 63 hectares for developing a new container terminal. The major structure of the project is an 1850 m long quay wall for the berth, which comprises more than 200 precast counter-fort units of 20 m height with the weight of 640 tonnes per unit. The geological study of this area shows that the site rests on about 15 m layer of dense sand over a deep layer of marine clays (Mcllquham 2010).

In Melbourne, creep settlement has been a challenging issue for several projects carried out on the areas resting over Coode Island Silt. The soil is classified as soft silty clay with high compressibility and low permeability (Ervin 1992). The soil produces excessive settlement, even though the applied pressure was in overconsolidation range. Therefore, settlement predictions for Coode Island silt were unreliable and still present a challenge for geotechnical design and construction (Bouazza et al. 2004).
These projects are just a small number of existing developments carried out on inappropriate ground for construction in Australia. These emphasise the importance of advanced research on soft soil engineering, particularly soil deformation. Soft soils consist of clay and silt with remarkable amount of pore water, and they may undergo extraordinary settlements under the effect of the surcharge loads. Therefore, knowledge of how to identify soft soil and its engineering properties and characteristics is critical. This will assist geotechnical engineers to identify if ground improvement is viable, and which methods would be most appropriate for the site. Predictions of consolidation and creep compression are vital in soft soils and the long term deformation must be emphasised in engineering design and practice. It should be noted that the soil creep mechanism is the foundation to develop constitutive models for soft soil behaviour, particularly for long term settlement.

1.2 Statement of problem

The term of secondary compression has become a significant characteristic of soft soil behaviour. Meanwhile, creep is the term firstly applied to mention the viscous behaviour of other materials such as metals and polymers. For all materials, creep is generally defined as time–dependent deformation under a constant applied stress (Abe et al. 2008; Brown 1999). After the theory of consolidation of Terzaghi (1923),
Buisman (1936) placed the foundation for the research on secondary compression of soft soils based on the observation of the long–term laboratory tests on clay and peat samples and the settlement of embankments on those soils. Secondary compression was initially defined as the compression under a constant effective stress and occurs after the dissipation of the excess pore water pressure completes as shown in Figure 1.3. Meanwhile, creep compression has recently been accepted to occur concurrently with the compression induced by the dissipation of the excess pore water pressure. Therefore, the term of secondary compression may cause the confusion on the reference point of the time–dependent compression process. Creep compression is accepted as a general term to mention about the time–dependent compression under a constant effective stress distinguished from the time-independent compression induced by the change of the effective stress rather than primary consolidation and secondary compression.

![Figure 1.3 Typical settlement curve](image)

Research in several decades on the creep compression of soft soils has shown that creep compression increases the resistance of the soil structure against further compression (Bjerrum 1967; Taylor 1942). The creep compression causes not only excessive settlement of soft soil under an applied stress, but also impacts on other soil properties such as the preconsolidation pressure. On the other hand, the time–dependent compression is observed to be mainly influenced by time, strain rate and stress rate.

Most of the models have predicted the linear increase of creep compression with the logarithmic of time by a constant coefficient of secondary compression ($C_D$) as shown in Figure 1.3. However, the observations in both laboratory tests and field case studies (e.g. Berre & Iversen 1972; Bjerrum 1967; Leroueil et al. 1985) suggested the
rate of creep compression decreases with time. Therefore, the assumption of a constant \( C_a \) may not describe the time–dependent behaviour of soft soil correctly, and results in overestimating the compression of soils.

Among a remarkable number of constitutive models for the time-dependent behaviour of soft soils, the elastic visco–plastic (EVP) model proposed by Yin & Graham (1989) is simple but yet practical for the numerical simulation of long-term settlement of soft soils. The EVP model developed based on the time-line concept of Bjerrum (1967) can simulate the stress–strain–time behaviour of soft soils under different stress – strain conditions such as creep, relaxation, constant rate of strain and constant rate of stress. The time-line concept of the EVP model includes the instant time-line representing the elastic behaviour, the reference time-line simulating the elastic plastic behaviour, and several time-lines with different equivalent time and creep strain rates. The visco–plastic behaviour of soft soils in the original EVP model is described by the linear logarithmic function which results in the infinitive creep strain as time approaches to infinity.

Therefore, Yin (1999) proposed a non–linear creep function with the introduction of the stress–dependent creep strain limit \( \varepsilon_{im}^{vp} \) and creep coefficient \( \psi_o/V \). Adopting the non–linear function, the creep strain at a particular effective stress will approach a certain value at the time approaching infinity. Thus, the non–linear function can overcome the limitation of the linear logarithmic visco–plastic function. Generally, the non–linear creep model can simulate the long–term settlement of soft soils more accurately. However, the model parameter determination procedure proposed for the non-linear creep parameters \( (\varepsilon_{im}^{vp} \text{ and } \psi_o/V) \) expose several limitations which can narrow the applications of the EVP model.

As explained by Yin (1990), the EVP model parameters can be obtained by curve fitting the laboratory measurements of the standard oedometer tests. There are no special tests required for the model parameter determination. However, in order to estimate the parameters of the non–linear creep function, longer loading durations are required to obtain more relevant data points after the excess pore water pressure is negligible. Moreover, the value of the time parameter \( t_o \) is assumed in advance to be equal to the time at the end of primary consolidation. Since the compression induced by creep and the dissipation of the excess pore water pressure cannot easily be separated during the dissipation process, a value of \( t_o \) within the dissipation time is hard to
determine. However, the time at the end of primary consolidation ($t_{EOP}$) is not a unique soil parameter, since $t_{EOP}$ can vary for each loading stage in the case of the incremental loading tests, as well as change with the soil sample thickness and the drainage conditions. $t_o$ is not only the time value of the reference time-line, but also influences other model parameters especially the non–linear creep parameters. Therefore, $t_o = t_{EOP}$ may lead to a non–unique set of the EVP model parameters. Moreover, as a result of $t_o = t_{EOP}$, the reference time-line is not truly viscous free line, since the reference time-line may include some viscous strain due to the value of $t_o$.

The conventional model parameter determination procedure proposed by Yin (1999) cannot be simply applied. Therefore, Yin et al. (2002) suggested adopting constant values of $\varepsilon_{im}^{vp}$ and $\psi_o/V$ for a soil instead of stress-dependent parameters as proposed in Yin (1999). The variable $\varepsilon_{im}^{vp}$ was assumed to be equal to the strain value at zero void ratio ($\varepsilon_{im}^{vp} = \frac{e_o}{1+e_o}$) and $\psi_o/V$ is obtained by curve fitting (Yin et al. 2002). The assumptions of $\varepsilon_{im}^{vp}$ and $\psi_o/V$ can simplify the model parameter determination procedure, but increase the uncertainty of the parameter determination, since the choice of a loading stage should be made to define the corresponding values of $t_o$ and $\psi_o/V$. In other words, different loading stages would provide different values for $t_o$ and $\psi_o/V$, consequently different reference time-line parameters.

1.3 Objectives and scopes of research

The ultimate objective of this study is to propose an advanced model parameter determination method (optimisation method), which can overcome the limitations of the conventional approach. The proposed optimisation method can predict all elastic visco–plastic model parameters simultaneously. The proposed method is not a graphical solution by curve fitting consolidation data, but a numerical solution combining the advanced optimisation algorithms with the finite difference solutions for the simulation of time–dependent stress – strain behaviour of soft soils.

The study consists of the following steps:

- Developing the numerical code for the Crank–Nicolson finite difference solution for the partial differential equations coupling the non-linear elastic visco–plastic model and the one-dimensional consolidation theory to predict the time–dependent settlement and the excess pore water pressure response,
Developing the numerical solution for the model parameter determination using the trust-region reflective least squares algorithm incorporated in the Crank–Nicolson finite difference solution,

Conducting laboratory experiments to validate the proposed solution for the model parameter determination, and

Simulating several case studies including laboratory-based and field case studies for verification.

The Crank–Nicolson finite difference solution is coded to simulate the time-dependent settlement and the variation of the excess pore water pressure with time for a soil layer. The finite difference solution is applied for a one-dimensional stress-strain condition with different options for the drainage boundaries (i.e. one–way drainage or two–way drainage).

The proposed solution for the model parameter determination is designed to utilise available consolidation data including the measurements during the dissipation of the excess pore water pressure. Consequently, a numerical code is written to combine the numerical optimisation solution with the Crank–Nicolson finite difference solution. Thanks to the Crank–Nicolson finite difference solution, the variation of the settlement and excess pore water pressure with time are predicted, and used to optimise a set of the EVP model parameters by comparing the predictions and the measurements. Moreover, since consolidation data during the dissipation of the excess pore water pressure is adopted for the model parameter determination, the value of time parameter $t_o$ can be adopted as 1 (unit of time) to overcome the uncertainty of the time at the end of primary consolidation.

The detailed objectives of this study can be outlined as follows:

- Providing a simple but yet precise and practical numerical solution for estimating the model parameters,
- Evaluating the efficiency of the proposed method,
- Investigating the effectiveness of the non-linear creep function in the laboratory and field conditions

1.4 Organisation of the thesis

This thesis comprises of seven chapters, which are structured as follows:
In Chapter 1, a brief introduction about the importance of the time-dependent stress-strain behaviour of soft soils is presented along with the objectives and scope of this study.

Chapter 2 presents a comprehensive literature review on soft soils and associated aspects including a study on creep mechanisms. Additionally, this chapter discusses the time-dependent stress–strain behaviour of soft soils and the influencing factors. In this chapter, a discussion on the suppositions of Hypotheses A and B related to the creep contribution during the dissipation of the excess pore water pressure for soil layers of different thicknesses is provided. Importantly, this chapter also reviews several existing models to simulate the time-dependent behaviour of soft soils.

In Chapter 3, the detailed description of the elastic visco–plastic (EVP) model with the non-linear creep function is provided along with an example of the conventional model parameter determination method. In addition, the proposed solution for the model parameter determination is provided in details.

Chapter 4 is to provide a validation exercise for the proposed method for the model parameter determination based on the laboratory experiments. Incremental consolidation tests were conducted for two samples of different thicknesses of a mixture of kaolinite, bentonite and sand by using the hydraulic consolidation Rowe cell systems available at UTS soil laboratory. Consolidation data of a thin sample with a thickness of 29.5 mm was adopted to obtain the set of the model parameters applying the proposed method. Consequently, the set of model parameters is applied to predict the settlement and excess pore water pressure for a thicker soil sample with a thickness of 140.5 mm from the same material. The predictions of the settlement and the excess pore water pressure are verified against the laboratory measurements of the thick sample.

Chapters 5 and 6 present further verification exercises using laboratory-based and field case studies, respectively. In Chapter 5, there are two laboratory-based case studies on the Hong Kong marine clay (Hong Kong) and Drammen clay (Norway). In the case of the Hong Kong marine clay, the proposed solution is applied to define a set of the model parameters using the available laboratory test results. The predictions of the vertical strains using the optimised model parameters are compared with the predictions obtained using the similar model
parameters by the conventional determination procedure. Furthermore, the predictions of the settlement and the excess pore water pressure are simulated for five soil layers of different thicknesses using the model parameters obtained by the proposed and conventional approaches. Differences between the results of two determination methods are compared and discussed. In the case of Drammen clay, the consolidation results of the thinnest soil sample of 0.02 m thickness were adopted to determine the EVP model parameters applying the the proposed method. The model parameters then were applied to simulate the compression and the excess pore water pressure response of three thicker soil samples. The predictions were compared with the laboratory measurements in order to evaluate the proposed method.

- Chapter 6 provides the numerical simulations of two field case studies in Sweden including the Väsby square test fill and the Skå–Edeby circular test fill. Applying the proposed solution, the EVP model parameters for the subsoils of the case studies are obtained using the available consolidation data reported in the literature. The EVP model parameters are used to simulate the time–dependent settlement and the excess pore water pressure in 67 years for the Väsby test fill and in 57 years for the Skå–Edeby test fill. The predictions obtained using the optimised model parameters are compared to the actual field measurements reported in the literature.

- Finally, in Chapter 7, the summary of the thesis is provided along with the conclusions and the recommendations for further research.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction
Time dependent behaviour of soils has been a contentious topic for many researchers for several decades in the field of geotechnical engineering. Although Terzaghi’s classical theory of one–dimensional consolidation can provide a reasonable estimation of the settlement induced by the hydrodynamic effects, the true settlement is believed to continue in a long period after the end of pore water pressure dissipation period. The term “creep” or “secondary compression” has been adopted to describe the settlement, or so-called volume change, under a constant effective stress (Bjerrum 1967). In general, the real behaviour of soils depends on various factors such as the soil composition, the clay mineralogy, the moisture content, and the stress – strain relationship (Feda 1992). However, interactions of various factors influencing the behaviour of soils, especially the consolidation process, remain unclear. It is important to understand profoundly the mechanism of the soil deformation because further numerical modellings are constructed based on the assumptions associated with the nature of the soil deformation. Different mechanisms can lead to different solutions for predicting the soil behaviour. The conventional or simplified methods may not be appropriate to answer the complicated questions regarding soil behaviour such as the relationship between stress – strain – strain rate, and the effect of temperature, even though they can provide reasonable estimation of soil settlement in some simple conditions. In this chapter, after a brief review of description of soft soils and the associated problems, various mechanisms proposed by geotechnical investigators to describe soil creep phenomenon are explained in details. Then, the discussion of the proposed mechanisms in regard to soil macroscopic and microscopic properties is presented. Additionally, the existing approaches to predict the long-term settlements of
soft soils are classified into four categories including (i) a constant coefficient of secondary compression $C_d$, (ii) the uniqueness concept of $C_d/C_c$ and the end of primary consolidation void ratio and effective stress relationship, (iii) the isotach concept, and (iv) other approaches. The merits as well as the weaknesses of the approaches are discussed in this chapter.

2.2 Problems associated with soft soils

Soft soils are one of problematic soils which exhibit high compressibility, low shear strength, low bearing capacity as well as high shrink-swell potential, and usually low permeability. The engineering properties of soft soils cause many technical and construction challenges for design engineers and contractors. This section provides a general description of soft soils along with their associated problems.

2.2.1 Description of soft soils

Soft soils usually contain significant amount of clay or silty particles. The presence of clay particles in soils is one of the main reasons for soil creep deformation. Therefore, it is essential to review basic knowledge of clay mineralogy, which has significant impact on the engineering properties of clay soils. The clay particles have different shapes such as bards, plates and sheets and are composed of complex silicates of aluminium, magnesium, and ion. According to Das (2008), the two basic units of clay minerals are silica tetrahedron and alumina octahedron, forming respectively silica and alumina sheets. Different structures of the stacked combinations of the basic sheet structures with different types of bonding between the sheets result in different types of clay minerals. The three main clay minerals are kaolinite, illite and montmorillonite.

According to Das (2008), clay particles have a net negative charge on the surface, and present both negative and positive charges on the edge. As a result of balancing the negative charge, the positive charged ions or cations in the pore water are attracted to the surface of the particles. These ions are defined as the exchangeable ions such as $\text{Al}^{3+}$, $\text{Ca}^{2+}$, $\text{Mg}^{2+}$, $\text{NH}_4^+$, $\text{K}^+$ and $\text{Na}^+$. Due to the attractive and repulsive forces between ions and clay surface charge, the ion concentration of cations is inversely proportional to the distance from the surface of the particles, while the ion concentration of anions increases with the distance. A water molecule is known as a dipole with positive and negative charges at its two ends, and water molecules can be electrically attracted to the clay particles in three ways (i) the positive ends of dipoles are attracted to the negative
charged surface of clay particles, (ii) the negative charged ends of dipoles are attracted to cations around the clay surfaces, and these cations are attracted to the surfaces of clay particles, and (iii) hydrogen bonding is formed between the oxygen atoms in water molecules and oxygen atoms on clay surfaces by sharing the hydrogen atoms of water molecules. The illustration of attraction patterns of water molecules on clay particles is shown in Figure 2.1. The layer of floating cations and anions on clay particle surface is termed as double layer (Mitchell & Soga 2005). Double layer affects the range of interaction forces including repulsive and attractive forces between clay particles, and is affected mainly by the concentration and valence of cations (Craig 2004). Adsorbed water is the innermost part of the double layer which is strongly held on the clay particle surface. In addition, absorbed water molecules are believed to be able to move freely parallel to the clay particle surface, but their movement is restricted on the perpendicular direction to the surface.

![Figure 2.1 (a) Water molecule and (b) Attraction of water molecules on the surface of clay particle (modified after Ranjan & Rao 2007)](attachment:figure21.png)

Inter–particle forces between the clay particles are also important for elucidation of clay mineralogy. The net force among particles is determined based on the difference between two opposite forces (repulsive and attractive forces) with respect to the distance of particles. According to Mitchell (1956), the inter–particle forces between clay particles affect the arrangement of clay particles, particle orientation or the relative position between adjacent particles, all of which are defined as the fabric of clay. The
fabric of clay has significant impact on the engineering properties of that clay (Mesri & Olson 1971; Mitchell 1956; Rao & Mathew 1995)). Table 2.1 summarises the characteristics of two distinct clay fabric arrangements which are flocculation and dispersion. Moreover, for a given mass at any compression pressure, clay in dispersed state has a smaller volume than clay in flocculated state, because the parallel arrangement of particles in dispersed state is more oriented than the random arrangement in flocculated state. At the same preconsolidation pressure, under the effect of a same increment of stress, there is more relative movement between clay particles in flocculated state than in dispersed state. The movement of clay particles tends to change the particle arrangement from random to parallel orientation.

**Table 2.1 Flocculation and dispersion of Clays (after Mitchell 1956)**

<table>
<thead>
<tr>
<th>Flocculation</th>
<th>Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Net force between clay particles is attraction.</td>
<td>• Net force between clay particles is repulsion.</td>
</tr>
<tr>
<td>• Flocculation caused by the attraction between the positive charged edge of a particle to the negative charged surface of its adjacent particle</td>
<td>• Dispersion caused by the repulsion between the negative charged surfaces of adjacent particles.</td>
</tr>
<tr>
<td>• The system of clay particles in random arrangement.</td>
<td>• The system of clay particles in parallel arrangement.</td>
</tr>
</tbody>
</table>

Based on the experiments on the fabrics of 14 undisturbed and remoulded clays, Mitchell (1956) investigated the effects of the clay fabric on secondary compression or creep. Undisturbed clay has more random particle arrangement than remoulded clay. Therefore, the orientation change happens more in undisturbed clay than in remoulded clay, while remoulded clay has more packed state, leading to less relative movement (shifting) between clay particles. As a result, the secondary compression ratio of undisturbed clay is higher than that of remoulded clay (Mitchell 1956). In brief, clay mineralogy including mineral composition, clay water system and electrostatic forces within clay system altogether result in the distinctive properties of clay in comparison to other soils, and effects of clay mineralogy on creep behaviour of clay are significant.

Soft soils can be near-normally consolidated clays, clayey silts, and peat which exhibit the high degree of compressibility (Look 2007). Soft soils can also be identified
based on the consistency known as the degree of firmness. The consistency of clays is
classified from ‘very soft” to “hard” based on the consistency index and/or the
undrained shear strength (Table 2.2). The consistency index is determined based on the
liquid limit (LL) and plasticity index (PI).

Consistency index = \( \frac{\text{LL} - w_c}{\text{PI}} \)  \hspace{1cm} (2.1)

where, \( w_c \) is the water content of a soil.

Table 2.2 Consistency of fine grained soils by consistency index and undrained shear strength

<table>
<thead>
<tr>
<th>Description</th>
<th>Consistency Index (Reeves et al. 2006)</th>
<th>Approximate undrained shear strength (kPa) (Reeves et al. 2006)</th>
<th>Undrained shear strength in the intact state (kPa) (AS 1726 1993; Leroueil et al. 1990)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard</td>
<td>&gt; 300</td>
<td>&gt; 200</td>
<td></td>
</tr>
<tr>
<td>Very stiff</td>
<td>&gt; 1</td>
<td>150 – 300</td>
<td>100 – 200</td>
</tr>
<tr>
<td>Stiff</td>
<td>0.75 – 1</td>
<td>75 – 150</td>
<td>50 – 100</td>
</tr>
<tr>
<td>Firm</td>
<td>0.5 – 0.75</td>
<td>40 – 75</td>
<td>25 – 50</td>
</tr>
<tr>
<td>Soft</td>
<td>&lt; 0.5</td>
<td>20 – 40</td>
<td>12 – 25</td>
</tr>
<tr>
<td>Very soft</td>
<td>&lt; 20</td>
<td>&lt; 12</td>
<td></td>
</tr>
</tbody>
</table>

However, this classification method is not as common as the classification using
the undrained shear strength (Reeves et al. 2006). The undrained shear strength of clay
soils can be determined by following tests: field vane, laboratory vane, cone penetration
test, undrained triaxial tests with isotropic consolidation (CIU), and other laboratory
tests (Leroueil et al. 1990). Due to the type of testing methods, sampling methods and
other ambient conditions, there are disparities in the values of undrained shear strength
of the soils provided from two different sources. Soft clays with no coarse grains can
have undrained shear strength of about 12 to 25kPa (Leroueil et al. 1990); whereas,
Reeves et al. (2006) showed the approximate undrained shear strength of soft fine-
grained soils is in the range from 20 to 40kPa. Within very soft soil, “extremely soft
soil” or “super soft soil” is defined as clay soil exposing excessive high water content
and low shear strength value (i.e. <1 kPa) which is unable to determine by the
conventional method (Fakher et al. 1999). For example, Brazilian extremely soft soils in Rio de Janeiro (Brazil) has the water content around 60% - 500%, SPT blow counts $N_{SPT} < 1$, the undrained shear strength $S_u < 12$ kPa, and the design strength as low as 3 kPa (Riccio et al. 2013).

2.2.2 Problems associated with soft soils

According to the approximate undrained shear strength of soft soils in Table 2.2, soft soils exhibit low bearing capacity. The low shear strength of mud, clays and silts can make the difficulties in ensuring the stability of embankments and excavation slopes. In addition, stability problems may rise due to deep excavations in soft clays.

The low permeability of soft soils can cause drainage problems, time dependent deformation and consequently failure as a result of low rate of shear strength gain. As a consequence of low hydraulic conductivity (permeability), the excess pore pressure changes can take a long time to dissipate during and after loading. The coefficient of permeability of clay soils ranges from $10^{-9}$ to $10^{-11}$ m/s, thus clay soils may be considered as impervious materials. It was proved that the consolidation of soft clays significantly depends on void ratio, the depth of clay layer, and the permeability of clay, and can occur over more than one year. The special feature of soft soils is the high compressibility. For example, normally consolidated clays may compress ten times more than normally consolidated sands (Vermeer & Neher 1999).

The stresses applied to compressible foundation soils by embankment structures produce large strain. In the cases of hydraulic structures constructed on soft soils such as dykes and dams, the foundation settlements can be counted in several meters (Leroueil et al. 1990). Because the typical hydraulic conductivity value of saturated clayey soils (less than $10^{-8}$ m/s) is significantly smaller than of sands, the excess pore water pressure generated by loading takes a long time to dissipate. As a result, the consolidation in the clays continues after the immediate (elastic) settlement. This leads to the consolidation settlement of clay deposits several times larger than the elastic settlement. Moreover, the differential settlement due to the variable occurrence of compressible soft clays/silts/peat could cause structural cracking. The leaning tower of Pisa (Italy) whose construction began in 1173 suffers from the typical problem of differential settlement. The foundation of the tower consists of a 3 m bed of silty sand that is underlain by 30 m of soft compressible clay on a deposit of sand. The tower tilted
by the end of 1178, and its function was damaged by differential settlement with the average settlement about 3.25 m (Burland et al. 2009).

2.3 Creep mechanisms

After Terzaghi’s outstanding theory of one dimensional consolidation of soils in 1923 that explains the rate of excess pore water pressure dissipation, it has been observed from laboratory results and field observations that settlement continues even after complete dissipation of pore water pressures. In order to distinguish the two components of the compression, the term ‘primary consolidation’ is used to describe the time dependent process due to the dissipation of the excess pore water pressure by the expulsion of water from the voids, and transferring loads from the pore water to the soil particles. On the other hand, creep or so-called secondary compression is generally defined as the deformation under a constant effective stress (Bjerrum 1967; Taylor & Merchant 1940). It is necessary to exclude creep phenomenon from the deformation under constant load because the effective stresses can be variable under a constant load. Research on the long-term settlement of soils has become important and been developed for many decades. However, there has been no unified approval to explain the mechanism of creep deformation resulting in different schools of thought and consequently various methods to predict soil settlement. This section attempts to make a comprehensive explanation for the mechanism of creep for clayey soils based on various existing relevant studies such as Barden (1969), Mitchell & Soga (2005), Mitchell et al. (1968), Murayama & Shibata (1961), Navarro & Alonso (2001), Taylor & Merchant (1940), and others. Although some of the explained mechanisms can be used to describe the creep behaviour of silt and granular soils, this section focuses on the creep behaviour of soft clays and the primary factors.

2.3.1 Creep due to the breakdown of inter–particle bonds

Soil is considered as a complex structure because of its heterogeneous compositions when compared to other materials such as metal or glass. In the macroscopic point of view, an element of clayey soils, as shown in Figure 2.2, contains clay particles, coarse grain particles and water. Water in the macroscopic view is defined as free water which flows due to the hydraulic gradient. According to Taylor & Merchant (1940) and Terzaghi (1941), the compression processes including both primary and secondary (or creep) settlement are explained based on the transfer of stress.
and the rearrangement of soil particles. Under the effect of the applied stress, free water flows out of the soil element, resulting in the rearrangement of the soil structure. It may lead to an increase in the solid–to–solid contacts in the soil. The flow of free water may take a period of time to be over, and is controlled by the soil permeability; therefore, the primary consolidation is a time dependent process. The primary consolidation increases the contact between soil particles, and also decreases the voids between particles. Since the spacing between the particles reduces, soil particles transform to more packed state. The contacts between particles gradually increase, causing the increase in the effective stress with time as a result of the total stress transferring from pore water to the contacts between particles. At the contacts between particles, particularly between clay particles, different types of bonding, and inter–particle forces such as the primary valence bonds between particles, van der Waals forces, hydrogen bonds, bonds by sorbed cations, attraction forces between particles with different charge, and cementation bonds, occur (Yong et al. 2010).

However, the bonds between particles may be broken or destroyed by the increase in the effective stress. The breakdown of bonds between soil particles may cause further rearrangement of soil particles, consequently further settlement or compression called creep. The breakdown of inter–particle bonds is considered as a mechanism of creep deformation proposed by Taylor & Merchant (1940), Terzaghi (1941), and accepted by Crooks et al. (1984), Gibson & Lo (1961), Mesri (1973, 2003), and Mesri & Godlewski (1977).

![Schematic view of a soil element of soil in macroscopic scale](image)

*Figure 2.2 A schematic view of a soil element of soil in macroscopic scale*

There are various causes of the breakdown of the inter–particle bonds. It can be induced by the relative movements of particles with respect to each other due to the
shear displacement or the change in particle spacings induced by the change in the net
inter–particle forces (Mesri 1973). Bolt (1956) considered the deformation of a natural
soil induced by the combinations of both mechanical and chemical factors considering
the soil composition including both coarse grained and fine grained particles with
different compressibility.

2.3.2 Creep due to jumping of molecule bonds
Christensen & Wu (1964), Kwok & Bolton (2010), Mitchell (1964), and Murayama &
Shibata (1961) explained the creep mechanism based on the theory of rate process. The
creep deformation is caused by the movement of the atoms and molecules to a new
equilibrium position under the effect of constant stress. Because the movement of atoms
and molecules (called flow unit) relative to each other is resisted by virtual energy
barriers, a sufficient activated energy is required to conquer the barriers. Creep in clay is
the displacement of oxygen atoms, which is seen as the flow units, within the contact
surface between clay mineral particles (Kuhn & Mitchell 1992). In fact, the flow units
do not remain static, but dynamically vibrate with a certain frequency. Considering that
creep is defined as a rate process, Mitchell et al. (1968) suggested that the activation
energy, $E_{\text{act}}$, not only depends on the deviatoric stress but also depends on elapsed time
of creep.

The concept of activation energy is explained in detail by Low (1962) as he
examined the influence of the absorbed water on the exchangeable ion movement. In
Figure 2.3, there is the schematic illustration for the concept of activate energy which
includes a close packed arrangement of an ion (gray filled circle) surrounded by water
molecules (white filled circles). If the ion moves from one position to another (such as
from a to b), it must break the bonds with all adjacent molecules with the charge on the
clay surface and push back the molecules in front of it to replace to that space. The
whole process of ion movement from a to b requires a sum of different energies for
different bonds, which is named the activation energy, $E_a$. 

2-9
According to Mitchell et al. (1968), the deformation of soils is evaluated based on the activation energy and the number of inter–particle bonds per unit area. The inter-particle bonding is referred to solid–to–solid bonds (such as between soil particles), or mineral–to–mineral, or mineral to mineral through the interlayer adsorbed layer, which is temperature dependent.

From the results of several triaxial creep tests with different temperature conditions and shear stress levels, Mitchell et al. (1968) concluded that soil creep is a thermally activated process, as the logarithm of creep strain rate divided by the absolute temperature $T$, $\log\left(\frac{\dot{e}}{T}\right)$, is inversely proportional to the reciprocal of the absolute temperature $T$ $(1/T)$. In general, the increase in the temperature may induce the increase in the creep rate. However, Mesri (1973) suggested that the effect of temperature on the creep rate is insignificant compared to the influence of other factors such as the precompression and the sustained loading. In terms of the effects of shear stress on creep, Mitchell et al. (1968) observed the linear relationship between the logarithm of creep strain rate and the deviatoric stress obtained from several undrained creep tests. In addition, based on shear creep tests, creep behaviour is shown to depend on the applied shear stress (Singh & Mitchell 1968; Yin & Graham 1999). As reported by Vermeer and Neher (1999), there are three typical types of creep behaviour under a constant shear stress. For deviatoric stresses about 30% of the shear strength, creep strain is small, and creep strain rate decreases and approaches zero after a period of time. For higher deviatoric stresses up to 70% of the shear strength, creep strain develops with a constant...
strain rate. For deviatoric stresses greater than 70% of the shear strength, the creep strain rate is accelerated, and may cause soil failure.

2.3.3 Creep due to sliding among particles

Grim (1962) explained creep as the reduction in volume mainly due to slipping between soil grains, while primary consolidation is caused by squeezing out water with insignificant slippage between grains. When water content reduces during primary consolidation, the bonding forces between the particles increase with the decrease in the spacing between particles. The increase in bonding forces causes the increase in the frictional resistance of particles against slipping relative to each other. Hence, the reduction in the volume during creep takes place slowly. Gupta (1964) explained that creep is caused by the relative sliding movement of clay particles under an external load. However, the relative movement between the clay particles is delayed by the bonds of water molecules existing in adsorbed water layer, and can gradually cause the change in the orientation of soil particles.

Kuhn & Mitchell (1993) proposed a new concept for creep compression which is due to a sliding movement between the particles. This concept is similar to the mechanism of the deformation of inter–particle bonds as discussed in Section 2.3.1. All types of soils including wet and dry materials appear to have inter–particle bonds between solid–solid contacts, at which the resistance to shearing deformation exists when the soil is subject to an application of pressure. This proposed mechanism was expected to describe soil creep generally under the effects of shear stress and temperature. As a result of the nature of viscous friction, the sliding movement is caused by the tangential component of the contact forces between soil particles, \( f_t \). The deformation is proposed by the relationship between the sliding velocity \( \dot{s} \), the sliding force and the friction ratio between the tangential force and the normal force \( f_t/f_n \). The mechanism is derived by the principle of rate process theory of Mitchell et al. (1968). The mechanisms of normal force and tangential force at the contact of two individual particles are shown in Figure 2.4.

The creep deformation is described by a system including a linear string which is represented for the normal force in Figure 2.4(a) and a combination of linear string and a dashpot in series which is represented for the tangential force illustrated by the dashpot in Figure 2.4(b). As the model displays similarity to the creep behaviour of soils, it is suggested that the sliding mechanism can be applied for all soils.
2.3.4 Creep due to water flows in a double pore system

Creep, on the other hand, is suggested to result from the transfer of pore water from micropores to macropores. This mechanism is explained based on the assumption of double levels of soil structure, microstructure and macrostructure, which was originally proposed by de Jong & Verruijt (1965) and pursued by many researchers including Berry & Poskitt (1972), Mitchell & Soga (2005), Navarro & Alonso (2001), Wang & Xu (2007), and Zeevaart (1986). According to the clay mineralogy, single particles consist of stacked layers of structural sheets of clay minerals. The microstructure unit of clay consists of several single particles packed together by inter–particle forces and bonds. The arrangement of single particles in a microstructure unit produces amount of pores which is called microspores. The microstructure units are also known by terms as flocs, clusters or peds. The macrostructure of clay is formed by the aggregation of several microstructure units, and the pores between microstructure units are called macropores (Yong et al. 2010).

Meanwhile, Zeevaart (1986) described two structures differently. The primary structure consists of a continuous skeleton formed by coarse grains and large pores of gravitational water, while the secondary structure is formed by clay clusters aggregated around the primary structures. Micropores are the pores existing within clay clusters, and water inside micropores which is formed by double layer water has different viscosity from water in pores within the primary structure. Figure 2.5 shows the illustration of two structures of Zeevaart (1986).
Table 2.3 Types of peds and pores (after Matsuo & Kamon 1977)

<table>
<thead>
<tr>
<th>Type of ped</th>
<th>Type of pore</th>
<th>Recognition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macroped (&gt;50μm)</td>
<td>Macropore (&gt;10 μm)</td>
<td>By naked eye and spy glass</td>
</tr>
<tr>
<td>Mesoped (2 ~ 50μm)</td>
<td>Mesopore (1~10 μm)</td>
<td>By optical microscope</td>
</tr>
<tr>
<td>Microped (0.1~ 2μm)</td>
<td>Micropore (0.01~1 μm)</td>
<td>By scanning electron micropore</td>
</tr>
<tr>
<td>Submicroped (&lt;0.1μm)</td>
<td>Submicropore (&lt;100Å)</td>
<td>By transmission electron microscope</td>
</tr>
</tbody>
</table>

Under the application of pressure, water inside the micropores of the clay clusters will be expelled out of the clay clusters moving to the larger pore (or macropores). The water flow may cause the deformation inside the clay clusters such as the reduction in spacing between clay minerals or the relative movement between the particles inside the clusters. Creep compression is explained as the reduction in micropore water and the relative reduction in pores in the microstructure. Navarro & Alonso (2001) explained the transfer of water from micropores to macropores induced by the difference of chemical potentials of water between microstructural and macrostructural water which causes the mass transfer in order to reach the equilibrium state. According to the quantitative analysis of exchangeable cations in pore fluid, Akagi (1994) concluded that the mechanism of the consolidation is based on the drainage of pore fluids and the
deformation of pores. The primary consolidation is due to the drainage of pore fluid in the macropores between the grains of clay particles (called “peds”), and creep (or “secondary consolidation” adopted by Akagi (1994) is the result of the delayed deformation of micropores within the clay particles induced by the viscous flow of pore fluid existing in the micropores. The definition of peds and pores is provided in Table 2.3. Moreover, according to Akagi (1994), the compression of soft clay during the constant rate of strain loading is explained as the deformation of macropores between macropeds.

2.3.5 Creep due to the structural viscosity

In general, viscosity is defined as the resistance of fluid to flow or deform under the applied stress. Fredlund & Rahardjo (1993) defined viscosity as the frictional drag of one plate of fluid sliding over another platelet. Taylor (1942) believed there was plastic structural resistance to compression within clay structure due to double layer surrounding the surface of clay particles, which was termed as structural viscosity. Structural viscosity has significant impact on soil creep. This theory is supported by Barden (1969), Bjerrum (1967), Christie & Tonks (1985), Graham & Yin (2001) Garlanger (1972) among others. According to several researches on clay mineralogy, the viscosity is imposed by the absorbed water layer around particles, and may induce the plastic resistance against the relative movement between the clay particles. As mentioned in the previous section, the absorbed water layer contains different cations and anions, which makes the water layer exhibit electrochemical properties far different from the normal water or free flowing water (Grim 1968; Reeves et al. 2006; Sridharan 2001). According to Guven (1992), water system absorbed within microstructure of clay is formed by three types which are (i) the water adsorbed on the internal surfaces of clay mineral, (ii) the water layer held between clay minerals (double layer water), and (iii) the capillary water held in pores between the clay particles. Figure 2.6 shows a schematic view of soil water system. Creep compression may be a result of the deformation of clay clusters or microstructure units due to the flow of viscous water. Based on the observation of Winterkorn (1943), the water layer which is held directly on the clay mineral surfaces can be considered as in a solid state, and the state of the water will be changed with respect to the distance from the clay mineral surface. Moreover, Grim (1968) concluded that the viscosity of double layer water increases
with the proximity to the clay surfaces, and the viscosity of double layer water should be higher than that of free water.

![Figure 2.6 A schematic view of clay–water system](image)

Terzaghi (1941) also considered that the viscosity of adsorbed water causes the decrease in speed of the rearrangement of soil structure. Bjerrum (1967) divided the consolidation process of soils into instant and delayed compression instead of primary and secondary compression, because he believed it is not possible to separate these two processes. An instant compression occurs simultaneously with the increase of the effective stress, and causes the reduction in void ratio until to reach the new equilibrium state to support the overburden pressure. Besides, delayed compression is the volume change under constant effective stress. According to Bjerrum (1967), delayed compression causes the reduction in water content, resulting in the rearrangement of soil structure which is more stable to resist against the further compression due to the increase in the number of contacts between clay particles. Moreover, the viscosity of pore water was considered as the factor causing the delay in the pore water pressure dissipation. Garlanger (1972), Yin & Graham (1989) are the followers of this concept.

Leonards & Girault (1961) investigated the effect of pore fluid on secondary compression (creep) by comparing compression of the soil with pore fluid of water and carbon tetrachloride (CCl₄). They found that the electrochemical distribution near clay particles due to the orientation of polar molecules within double layer cannot be seen as the influencing factor in any mechanism causing creep deformation. However, the effect of the physico-chemical property of pore fluid was observed to be significant on the rate of secondary compression. The experimental result shows clearly that void ratios at the
same effective stress were different for two different pore fluids in the range of normal consolidation. However, Leonards & Girault (1961) had no explanation for the reduction, but emphasised that the difference of the preconsolidation pressure between the two curves may be due to the change in inter–particle forces resulting from the secondary compressions.

A number of researchers believed that the secondary compression of soils depends on the electrochemical nature of clay minerals. Sridharan & Rao (1973, 1979) concluded that electrical attraction and repulsion forces in the soil structure affect the strength of clay particles, which controls creep compression, while viscosity of pore fluid was suggested to affect only the coefficient of secondary compression.

As a result of the experimental tests, Sridharan & Rao (1979) supported the conclusion of Leonards & Girault (1961) that the viscosity of pore fluid has insignificant effect on the secondary compression, while the dielectric of pore fluid has notable impacts on the variation of attractive forces and repulsive force. As a result, the effective stress and shear strength of the soil are also influenced. According to Yin (2003), creep mainly results from the combination of two processes including (a) viscous flow of adsorbed water in double layers on clay particles and (b) viscous adjustment of clay structure (plate structure) to reach a new equilibrium to balance with the external effective stresses. Therefore, creep occurs as long as the effective stress exists in the soil, and creep is not related to free pore water whose flow is controlled by the hydraulic gradient.

Feda (1992) explained that as temperature increases, the inter–particle bonds between soil particles weaken, causing the increase of deformations within the soil skeleton. Temperature variations can also affect the viscosity of adsorbed water. According to Grim (1968), micropore water and adsorbed water can be completely removed by heating just above room temperature. Temperatures in excess of 100ºC can completely remove interlayer water in montmorillonite minerals. By heating over 400ºC, the clay mineral structure can be altered or destroyed. Graham et al. (2001) carried out drained isotropic compression tests on reconstituted illite at 28ºC, 65ºC and 100ºC, and suggested that temperature changes may cause the irreversible volume changes in soils. However, the response of volume changes induced by temperature variations is very much dependent on the soil type.
2.3.6 Discussion

The nature of creep compression is still a challenging topic to study even though it has been investigated for many decades. A general definition of creep deformation of soils is accepted to be the deformation or settlement of soils under a constant effective stress. Therefore, there are many proposed theories to elucidate the mechanisms of creep. In the previous sections, five proposed mechanisms by different researchers are critically reviewed as summarised in Table 2.4. They are (a) the breakdown of inter–particle bonds in the soil structure, (b) sliding between particles, (c) water flows in two systems (theory of double porosity), (d) deformation due to the structural viscosity, and (e) deformation due to the jumping of molecule bonds (theory of rate process). Various mechanisms exhibit some similarities. For instance, the creep mechanisms explained by the double pore structures and the structural viscosity focus on the clay mineralogy, and the influencing factors are adsorbed water system, the viscosity and the physico–chemical properties of the clay water system. The combination of these two mechanisms can provide a more comprehensive explanation of creep in microscopic scale. The jumping of molecule bonds can be used to explain the viscous flow of micropore water within the microstructure of soils because double layer water and adsorbed water layer contain complex electro–chemical properties controlled by different types of bonds and forces such as primary valence bonds, van der Waals forces and hydrogen bonds.

Therefore, in spite of the variety of reasons for creep, the mechanism of creep compression can be a combination of various processes, and can be categorised into macroscopic and microscopic aspects. In the macroscopic view, creep deformation is a result of the soil structural rearrangement or adjustment to reach a new equilibrium under an applied stress. The deformation can be induced by the breakdown of the bonds between the soil particles and/or the relative movement of particles. In the microscopic level, creep deformation is explained as the deformation of microstructure (clay minerals and absorbed water layer) due to the drainage of pore fluid in micropores, or due to the structural viscosity of pore fluids. In the microscopic level, the inter–particle forces between clay particles and the electro-chemical properties of double layer water are the important factors influencing the deformation. It is believed that the combination of these two structure levels (i.e. macroscopic and microscopic) can be adopted to explain the nature of soil creep, particularly for clay soils. However, in practice or the
field scale, the composition of the natural soil is complex and inherently heterogeneous. Therefore, it may be unreliable to explain the behaviour of soils established by the numerical expressions, which are developed based on the micro-elements with assumption of homogenous materials.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Main factors/Focus</th>
<th>Introduced or supported by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakdown of inter–particle bonds</td>
<td>Relative movement of particles, soil structure rearrangement</td>
<td>Taylor &amp; Merchant (1940), Terzaghi (1941), and accepted by Gibson &amp; Lo (1961), Mesri (1973, 2003), Mesri &amp; Godlewski (1977), and Crooks et al. (1984).</td>
</tr>
<tr>
<td>Sliding particles</td>
<td>Activate energy, contact forces</td>
<td>Grim (1962), Gupta (Gupta 1964), Kuhn &amp; Mitchell (1993)</td>
</tr>
</tbody>
</table>

Moreover, the volume of solid in a soil element can have a minimum value, if there is no void existing within that soil element. According to Mitchell (1956), for a particular soil, void ratio can reach to a minimum value regardless of the pressure or initial orientation. However, because of the irregular shapes of soil particles, the relatively minute gaps still exist between particles, so that the minimum void ratio cannot be zero. That also means the soil structure cannot deform infinitely. Based on observation of compression of soils, the relationship between the compressions with logarithmic of time was seen to vary not linearly (Bjerrum 1967; Leroueil et al. 1985). Yin (1999) also believed that there exists a finite creep limit strain which is stress dependent. Thus, there is a finite final strain, meaning that the compression of the soils...
under a particular applied pressure must cease after a finite period that can be counted in years or decades. The deformation may cease when there is no void and fluid inside the clay particles, or under the final effective stress that the ultimate equilibrium inside the soil structure is reached. The equilibrium state under an applied stress can be reached when free water still exists with soil structure in the cases of low level of applied stress. The higher the applied pressures, the lesser pore water exists at the equilibrium state. It is due to the solid contacts between soil particles are stronger to support the pressures than pore water. Additionally, it could be the greatest limit strain, which is similar to the upper bound creep limit strain corresponded with zero void ratio defined by Yin et al. (2002). Figure 2.7 illustrates a schematic clay water system in two stages (a) free water flows through the voids between soil particles during the compression process, and (b) a system after free water squeezed out completely. As mentioned the creep process may cease after some times; thus, the final soil element at its greatest limit state could be similar to the schematic view shown in Figure 2.7(b). Therefore, it is assumed that when creep deformation reaches its greatest limit state, the soil system would involve (i) no void allows for water flows (ii) no free water within the soil, (iii) the whole soil in solid phase and (iv) coarse grained particles are rearranged and form a close packed system with clay particles. The third condition is suggested based on the assumption that the innermost adsorbed water layer which is held strongly to the clay mineral surfaces cannot be removed under the normal ambient temperature and loading conditions. Thus, that layer of water is treated as solid phase (Grim 1962, Navarro and Alonso 2001).

Creep settlement in soft soils remains a notable and challenging issue for geotechnical engineers. In order to improve the ground for future development, various ground improvement techniques such as preloading with vertical drains, vacuum assisted preloading, and deep soil mixing have been applied to consolidate the ground, improve the soil bearing capacity and thus, reduce the risk associated with the long-term behaviour of the soft deposit. The combination of vacuum and wick drains applied in the Port Brisbane Expanion project (Ameratunga et al. 2010), vacuum and surcharge preloading in combination with prefabricated vertical drains at Ballina Bypass project (Indraratna 2010), deep soil mixing method considered for Coode Island Silt (Bouazza et al. 2004) are some examples.
Reduction in the creep settlement after the ground improvement can be still explained using the above mentioned mechanisms. For instance, preloading method in conjunction with vertical drains, which is one of the most popular techniques applied effectively to improve soft soils, aims to accelerate the consolidation process by excessive applied stress and horizontal and vertical drainages. Water flow through voids inside the soil speeds up by horizontal and vertical drainage paths. In addition, the applied pressure induced by preloading can cause water flow by hydraulic gradient, and later cause the reduction in voids or pores within soil skeleton. Thus, the soil becomes more compressed, and improved. Therefore, when the preloading exceeds the design load, the secondary compression is greatly reduced and consequently the post construction settlement can be minimised (Wong 2006).

2.4 Time–dependent stress – strain behaviour of soft soils

Creep has been an important contributor to the time–dependent characteristics of soft soils. This section highlights effects of various factors on the time–dependent settlement of soft soils, such as time effects, strain rate effects and the stress dependency. Additionally, the differences of the compression behaviour of soft soils in the laboratory and field conditions are also discussed.

2.4.1 Time effects

Since settlement of soft soils is a time-dependent process, time is one of the important influencing factors on the long–term behaviour of soft soils. As a stress increment is applied on a soft soil layer, the compression rate is initially controlled by the dissipation of the excess pore water pressure (Rajot 1992). After the effective stress increases to an
almost constant value, the compression induced by creep continues to infinite time. Augustesen et al. (2004) distinguished the differences of the creep compression and strain rate effects through the creep tests conducted in the triaxial apparatus (Figure 2.8(a) & (b)) and conventional step loading tests using oedometer set-up (Figure 2.8(c) & (d)).

Creep tests under a constant effective stress carried out by triaxial setups indicate three stages of creep including primary, secondary and tertiary creep (Figure 2.8a). While the primary stage is referred to the transient creep corresponding to the decreasing strain rate, the secondary and tertiary creep stages are the stationary creep with constant creep strain rate and the acceleration creep with the increasing strain rate, respectively (Augustesen et al. 2004). The corresponding creep strain rate of three creep stages with time are shown in Figure 2.8(b). The tertiary creep phase eventually results in the creep failure or creep fracture of soils, which can only be observed by triaxial creep tests (Augustesen et al. 2004).

On the other hand, the three compression stages in Figure 2.8(c) are different from the three creep stages in Figure 2.8(a). It should be noted that the compression stages obtained by the standard oedometer tests are different from the creep stages of the triaxial creep tests due to the development of the effective stress during the primary compression stage in the standard oedometer tests. On the other hand, the effective stress maintains constant through the triaxial creep tests. The compression during the primary stage in Figure 2.8(c) varies with different strain rates, since the compression process is controlled by the dissipation of the excess pore water pressure (i.e. the effective stress does not stay constant, but increases with time). In contrast, the primary creep compression in Figure 2.8(a) is induced by creep under a constant effective stress only. The strain rate in Figure 2.8(d) corresponding to the compression stages in Figure 2.8(c), continuously decreases with time, different from the variation pattern reported in Figure 2.8(b).
Figure 2.8 (a-b) Creep stages and strain rates in creep tests performed by triaxial apparatus and (c-d) Compression stages and strain rates in step load tests by oedometer apparatus (after Augustesen et al. 2004)

Kabbaj et al. (1988) performed several multiple stage loading tests on different soils such as Vasby (Sweden) clay, Gloucester clay, St. Alban clay and Berthiverville (Canada) clay in order to investigate the differences between the in-situ and laboratory stress – strain relationships. In the laboratory, two types of the multiple stage loading tests were carried out, including (1) the stress increment applied at the end of the primary consolidation (EOP) (when the excess pore water pressure is approximately zero) and (2) the stress increment applied every 24 hours. Consequently, Figure 2.9 shows the time effects on the stress – strain curves in the laboratory for two Berthiverville clay samples taken from different depths. The thickness of the soil sample was 19 mm, and the stress increment ratio was 0.5 (Kabbaj et al. 1988). The results in Figure 2.9 show that under the same loading conditions, the compressions of 24–hour tests are more than the compressions of the EOP tests. Moreover, both of the laboratory curves, especially the EOP curves underestimate the in-situ performance. The 24–hour curves involved more sustained loading time compared to the EOP curves. Thus, the compression induced by creep is also more in the 24–hour tests. The similar results of the effects of sustained loading time on the compressibility were observed for
other clayey soils such as Väsby clay (Chang 1969), Skå-Edeby clay (Hansbo 1960), and Yokohama mud (Imai & Tang 1992).

2.4.2 Strain rate effects

Besides the time effects, the strain rate is another significant factor influencing the time–dependent compressibility of soft soils which has been studied by several researchers such as Crawford (1964), Jia et al. (2010), Lerouile et al. (1985), Mesri & Feng (1986) and Suklje (1957). Suklje (1957) emphasised the effects of consolidation rate on the stress – strain relationship when proposing the isotach concept incorporating the unique relationship between the effective stresses, void ratios and consolidation rate.

Constant rate of strain (CRS) consolidation tests have been adopted broadly in order to understand the stress – strain behaviour of soils, especially clay soils. Figure 2.10 illustrates the stress – strain – time relationship at different strain rates. In general, a larger strain rate results in a higher effective stress at a certain strain. Thus, the stiffness of the soil increases with the increase of the strain rate (Augustesen et al. 2004). Lerouile et al. (1985) suggested based on the results of various oedometers tests including the CRS tests there is the unique relationship among stress – strain – strain rate. This unique relationship is described as similar as the isotach concept of Suklje (1957). Figure 2.11 presents the CRS test results for Bastiscan clay and St. Cesaire clay. Each curve in Figure 2.11 represents for a unique relationship for stress – strain – strain rate. The influence of the strain rate on the preconsolidation pressure is also obviously observed. The higher strain rate resulted in the higher preconsolidation pressure in both two soil samples.
The relationship between stress – strain obtained by a CRS test is more direct and accurate, since the stress – strain curve of soil can be measured continuously (Feng 1991). CRS tests compared with standard oedometer tests have some advantages with respect to the required testing duration. The CRS test can be automatically operated, and the testing duration is usually about 2 days. However, the main challenge of CRS consolidation tests is the choice of the imposed strain rate to obtain the design curve. It should be emphasised that if CRS test is conducted to obtain the information of EOP stress – strain relationship (as well as the coefficient of permeability and coefficient of consolidation of soils), the imposed strain rate consequently should be approximately selected to produce the data corresponding to the convention primary consolidation curve. In other words, the adopted strain rate should produce insignificant pore water pressure furthest from the drainage boundary (Mesri & Feng 1992). Various researchers have proposed different expressions to determine the appropriate strain rate to be used in CRS test (e.g. Armour Jr & Drnevich 1986; Crawford 1964; Gorman et al. 1978; Mesri & Feng 1986; Smith & Wahls 1969).

\[ \varepsilon = c_3 \quad c_3 > c_2 > c_1 \]
\[ \varepsilon = c_2 \]
\[ \varepsilon = c_1 \]

*Figure 2.10 Constant rate of strain (CRS) tests (a) the variation of strain with time at different strain rates, and (b) stress – strain relationship at different strain rates (after Augustesen et al. 2004)*

For example, Mesri & Feng (1986) suggested an equation to define the EOP strain rate for a soil to be used in CRS test as follows:

\[ \dot{\varepsilon}_p = \frac{k_{vo}}{2^{e} c^2 c_k H^2} \frac{\sigma'_{pc} c_g}{\gamma_w c_c} \]  \hspace{1cm} (2.2)

where \( k_{vo} \) is the initial coefficient of vertical permeability, \( c_k = \Delta e/(\Delta \log k) \) with \( e \) void ratio and \( k \) vertical permeability coefficient, \( H \) is the maximum drainage distance, \( \gamma_w \) is the unit weight of water (9.81 kN/m\(^3\)), and \( \sigma'_{pc} \) is the preconsolidation pressure corresponding to the EOP \( e – \log \sigma'_z \).
The input parameters for Equation (2.2) may be obtained from the results of incremental loading (IL) oedometer tests on standard size samples (H = 20 mm). Mesri & Feng (1986) applied the EOP strain rate to obtain the EOP compression curve, and recommended that CRS test should be performed using the EOP strain rate obtained by Equation (2.2) to obtain \(e - \log \sigma'_z\) corresponding to EOP curves obtained from incremental loading oedometer tests. Figure 2.12 shows the comparison of experimental results obtained from incremental loading oedometer tests and CRS test with the applied...
strain rate computed by Equation (2.2). However, the procedures to determine the input parameters such as the ratio of $C_c/c_k$ for CRS tests were unclearly provided, as $C_c$ can vary with the applied stress. In Equation (2.2), as the thickness of a soil layer increases, the value of $\dot{\varepsilon}_p$ decreases. The remaining question is that whether the results of CRS tests adopting different strain rates corresponding to different sample thicknesses and/or drainage conditions will produce a unique $(e-\log \sigma'_{EOP})$ or not.

The strain rate effects observed in Figure 2.11 are similar to the time effects as shown in Figure 2.9. Comparing Figure 2.9 and 2.11 and as expected, the compression curves obtained from longer sustained loading time correspond to lower strain rates. As a result, the compressibility of the in–situ soil layer may be different from the behaviour of soft soils in the laboratory. Leroueil (2006) suggested the range of strain rates of different soils in in-situ along with the strain rates in the different laboratory tests. The strain rates in the laboratory tests are higher than the in–situ strain rates. Additionally, the strain rates corresponding to the 24–hour incremental loading tests are closer to the strain rates in the field in comparison with the strain rates obtained from other tests as shown in Figure 2.13.

![Figure 2.12 EOP e–log $\sigma'_v$ curves from CRS and incremental loading (IL) oedometer tests of Berthierville clay (after Mesri & Feng, 1986)](image)

- IL - 8
- IL - 3
- EOP test
- CRS

- $H = 19.05\text{mm}$
- $k_o = 4.1 \times 10^{-8}\text{mm/s}$
- $C_c/c_k = 1.8$
- $\sigma'_p = 55.06\text{kPa (IL)}$
- $C_d/C_c=0.044$
- $\dot{\varepsilon}_i$ (CRS) = $8 \times 10^{-7}/\text{s}$

Berthierville clay

Figure 2.12 EOP e–log $\sigma'_v$ curves from CRS and incremental loading (IL) oedometer tests of Berthierville clay (after Mesri & Feng, 1986)
2.4.3 Stress effects

According to Leroueil et al. (1985), the non-linear variations of strains with time as well as the stress level for clay soils is generalised and illustrated in Figure 2.14 based on the long term creep tests. Three types of time–depending compression curves are identified depending on the applied stress level. In the case of the overconsolidated soils, while the final effective stress is less than the preconsolidation pressure (Type (i) in Figure 2.14(a)), the compression is not significant, and the slope of compression continuously increases after the EOP. The corresponding slope of strain rate for Type (i) in Figure 2.14(b) decreases linearly with time. When the effective stress is approximately equal to the preconsolidation pressure (Type (ii)), the slope of the compression after the EOP exceeds the corresponding value for in Type (i). The similar compression curve is obtained, when the effective stress increases from the overconsolidated range to the normally consolidated range. The curve of Type (iii) is the common response of soft soils, as the effective stress is within the normally consolidated range. The compression is excessive during the dissipation of the excess pore water pressure, and the strain rate after the EOP keeps decreasing with time. The corresponding variations of logarithm of strain rate with logarithm of time in Figure 2.14(b) indicate the difference between the overconsolidated and normally consolidated soils. The curve of Type (ii) shows the delay in strain rate for some times between the decreasing processes which is supposed

Note:
- CGT: controlled gradient test
- CRS: constant rate of strain
- MSLp: multiple stage loading with increments at the end of primary consolidation
- MSL24: multiple stage loading with increments every 24 hours

Figure 2.13 Ranges of strain rates in the laboratory tests and in-situ (after Leroueil 2006)
as the transition between the overconsolidated to the normally consolidated stages (Kabbaj et al. 1986).

![Diagram of compression curves and strain rate](image)

**Figure 2.14 (a)** Types of compression curves dependent on the stress level (after Leroueil et al. 1985) and (b) the corresponding strain rate (after Augustesen et al. 2004)

For example, Figure 2.15 illustrates the compression of several samples of Berthierville clay under different single stage creep tests. The initial effective stress is about 65 kPa, and the stress increments vary from 2 kPa to 86 kPa in order to observe the influence of the stress level on the compressibility. For the small stress increments (i.e. the final stresses of 67 kPa and 78 kPa), the slope of the curves keeps increasing with time. In the case of the final stresses of 90 kPa to 109 kPa, the compression curves are similar to Type (ii) in Figure 2.14. The compression slopes under those stresses increase much more than the cases of 67 kPa and 78 kPa. The reversed S curves of the
compression are more evident in the last 4 stress increments, supposing that they are within the normally consolidated range.

Figure 2.15 Several single loading tests on Batiscan clay indicating the effects of stress increments on the variation of strain-time behaviour (after Leroueil et al. 1985)

2.4.4 Other influencing factors

Soil disturbance is one of the important factors influencing the accuracy of the predictions (Chang 1969; Leroueil & Kabbaj 1987; Tanaka 2008). Soil sampling and preparation can cause destruction within soil structure, and consequently the different behaviours for the soil in laboratory experiments and the in situ conditions. Due to some extent of soil disturbance, the preconsolidation pressure can be underestimated, and the change of void ratio at the in-situ preconsolidation pressure may be overestimated. Thus, the compressibility obtained from the standard consolidation test can be larger than the actual value, resulting in inaccurate interpretation of results (Tanaka 2008; Wesley 2010).

According to Chang (1969), in the case of Väsby clay (Sweden), the coefficient of consolidation for a sample taken from 3.5 m depth and tested in an 80 mm diameter sample was about 2.3 times higher than the values obtained from 50 mm diameter samples. Leroueil & Kabbaj (1987) considered the increases of the compressibility ratio \( C' = \Delta \varepsilon / \Delta \log \sigma'_z \) and the preconsolidation pressure \( \sigma'_pc \) when comparing the results tested on the samples taken by the Swedish standard piston sampler and the Laval 200 mm diameter sampler. The sample taken by the Swedish standard sample exhibited
lower values of compressibility and the preconsolidation pressures ($C_{\alpha} = 0.47$ and $\sigma'_{pc} = 23\text{kPa}$) compared to the values obtained by the sample taken by the Laval sampler ($C_{\alpha} = 0.74$ and $\sigma'_{pc} = 35\text{kPa}$).

Temperature is another factor able to influence the compression behaviour, especially the preconsolidation pressure (Abuel-Naga et al. 2006; Boudali et al. 1994; Campanella & Mitchell 1968; Sultan et al. 2002). Figure 2.16 shows the typical effects of temperature on the preconsolidation pressure. The preconsolidation pressure tends to decrease with the increase of the temperature. The stress at the yield points is reported to decrease as the temperature increases. Heating may cause the change in the soil structure, thickness and the viscosity of the adsorbed water layer. The clay minerals and the void water may be expanded due to heating. Moreover, temperature also changes the net repulsive forces within the mineralogical structure of the soil, resulting in the change in the support for the applied stresses through the net forces of the soil in the microscale (Graham et al. 2001; Lingnau et al. 1995). The similar effects of the temperature on the preconsolidation pressure and compressibility were obtained based on the test results on other clayey soils such as Bangkok clay (Abuel-Naga et al. 2006), Backebol clay (Tidfors 1987) and Boom clay (Sultan et al. 2002).

![Stress–strain curves of Berthierville clay at different strain rates and different temperatures tested by Boudali et al. 1994 (after Leroueil 1996)](image-url)

Figure 2.16 Stress – strain curves of Berthierville clay at different strain rates and different temperatures tested by Boudali et al. 1994 (after Leroueil 1996)
2.4.5 Stress relaxation

The time–dependent behaviour of soft soil also occurs in another process called stress relaxation in which the stress decreases at a constant value of strain (Lacerda & Houston 1973; Murayama & Shibata 1964). Stress relaxation tests have been usually carried out by triaxial apparatus in order to observe the change of the deviator stress ($q$) under constant strains (e.g. Drumright & Nelson 1985; Lacerda & Houston 1973; Silvestri et al. 1988; Yin & Cheng 2006). During the stress relaxation tests, the cell pressure and axial deformation are maintained constant at a certain strain rate value, while the variations of the deviator stress ($q$) and the excess pore water pressure ($u_e$) are monitored (Sheahan et al. 1994). The test was carried out until the deviator stress and the excess pore water pressure were stabilised with negligible changes in the values. Silvestri et al. (1988) reported the deviator stress reached the stabilised value less than 1 day for the relaxation tests of the Louiseville clay, while other relaxation test results indicate longer test duration varying up to 7 days (Sheahan et al. 1994; Yin et al. 2014).

Based on the observation and analysis of several triaxial stress relaxation test results, Lacerda & Houston (1973) concluded that the normalised deviator stress ($q/q_o$), in which $q$ and $q_o$ are the deviator stresses at time $t$ and at the beginning of the test, respectively decreases linearly with the increase of the logarithm of time. Furthermore, the strain rate corresponding at the beginning of the test influences on the time at which the relaxation begins. The slower initial strain rate corresponds to the longer delayed time to start the relaxation process (Lacerda & Houston 1973). Yin & Cheng (2006) also observed the similar initial strain-rate dependency on the variation of the effective stress for Hong Kong marine clay. According to Yin & Cheng (2006), the higher initial strain rate at the beginning of relaxation results in the significant decrease of the effective stress. The similar observation was reported in Graham et al. (1983) and Fodil et al. (1997). Additionally, although the relaxation tests were mostly carried out under undrained conditions and the deviator stress decreased with time, the excess pore water pressure during the test was reported to remain almost constant (Lacerda & Houston 1973; Murayama & Shibata 1961; Silvestri et al. 1988; Yin & Cheng 2006).

2.4.6 Undrained creep

Drained and undrained creep tests are two types of creep tests carried out under triaxial conditions. In the drained creep tests, the mean effective stress $p'$ and the deviator stress $q$ are maintained constant to obtain the constant effective stress. On constrast to the
drained tests, as the drainage path is closed during the undrained creep tests, the pore water pressure increases with the decrease of the mean effective stress $p'$ and the constant deviator stress $q$. Consequently, the axial strain increases with the decreasing of the strain rate. An equilibrium state is achieved, when the creep strain rate approaches zero value at a certain time (Augustesen et al. 2004; Vermeer & Neher 1999). Since the excess pore water pressure may generate during the construction in field situations, the soil deposits may deform under the undrained conditions. Therefore, the undrained creep tests become important to investigate, as the undrained shear strength may significantly reduce under undrained creep (Holzer et al. 1973; Oka et al. 2002; Singh & Mitchell 1968; Wang & Yin 2012).

Figure 2.17 presents the undrained creep test results on Wenzhou marine clay consolidated under $K_o$ and isotropical stress conditions. The variations of the axial strains and the excess pore water pressure with time at different deviator stress levels in Figure 2.17 are typical results obtained from undrained creep tests as reported in the literature (e.g. Arulanandan et al. 1971; Holzer et al. 1973; Sekiguchi 1984). The rate of the increase of the axial strain and the excess pore water pressure is influenced by the deviator stress level. As observed in Figure 2.17, the higher deviator stress level results
in the faster increase of the axial strain as well as the evolution of the excess pore water pressure (Wang & Yin 2012).

2.5 Hypotheses A and B

Buisman (1936) carried out long-term oedometer tests on clay and peat, and observed the settlement of a road embankment and a levee approximately in two years. He concluded that settlement increases linearly with logarithm of time after the end of loading. The settlement per unit of thickness was assumed to be proportional to the applied load, and expressed as the following equation,

\[ z_t = \Delta \sigma (\alpha_p + \alpha_s \log_{10} t) \]  

(2.3)

where, \( z_t \) is the compression per unit of layer thickness, \( \Delta \sigma \) is the load increment, \( t \) is time unit corresponding to the moment of load application, \( \alpha_p \) is the immediate settlement per unit of thickness and unit of load, and \( \alpha_s \) is the rate of secondary compression per unit thickness and load unit. The coefficient \( \alpha_s \) was suggested to be approximately proportional to the applied load.

Furthermore, according to Buisman (1936) observation, it was concluded that the total time dependent deformation (secondary compression) of a single loading is equal to the sum of the compressions induced by each loading increment of that load. Buisman (1936) conclusions have significant impacts on later research on secondary compression or creep. The logarithmic relationship between the settlement and loading time is still the guidance for constitutive modelling of creep compression.

According to Taylor & Merchant (1940), the general constitutive equation for one-dimensional compression which includes both primary consolidation and secondary compressions is as follows:

\[ e = e (\sigma'_z, t) \]  

(2.4)

In Equation (2.4), void ratio \( e \) is considered as a function of effective stress \( \sigma'_z \) and loading time \( t \).

Partial derivatives of Equations (2.4) with respect to \( \sigma'_z \) and \( t \) can be presented as follows:

\[ \dot{\epsilon}_\sigma (\sigma'_z, t) = \frac{\partial e}{\partial \sigma'_z} \left( \frac{\partial e}{\partial t} \right) \]  

(2.5a)

\[ \dot{e}_t (\sigma'_z, t) = \frac{\partial e}{\partial t} \]  

(2.5b)
Equation (2.5a) indicates variations of the void ratio with respect of the effective stress at an instant time $t$ (or constant $t$), while Equation (2.4b) presents the change of void ratio with time ($t$) at a constant effective stress $\sigma_z'$. The total derivative of void ratio with respect to time can be written as follows:

$$\frac{de}{dt} = \left(\frac{\partial e}{\partial \sigma_z'}\right)_t \frac{d\sigma_z'}{dt} + \left(\frac{\partial e}{\partial t}\right)_{\sigma_z'}$$

(2.6)

where, $\left(\frac{\partial e}{\partial \sigma_z'}\right)_t \frac{d\sigma_z'}{dt}$ is the decrease in the void ratio associated with the increase of effective stress with time, and $\left(\frac{\partial e}{\partial t}\right)_{\sigma_z'}$ is the decrease in the void ratio with time under a constant effective stress. Equation (2.6) shows the overall dependency of void ratio with time via indirect dependencies of partial derivatives.

In general, Equation (2.6) can be integrated in order to obtain the change in the void ratio as a function of time (i.e. $(\Delta e)_t$):

$$(\Delta e)_t = \int_0^t \left[ \left(\frac{\partial e}{\partial \sigma_z'}\right)_t \frac{d\sigma_z'}{dt} + \left(\frac{\partial e}{\partial t}\right)_{\sigma_z'} \right] dt$$

(2.7)

However, the main challenge to solve Equation (2.7) is that a unique function $e(\sigma_z', t)$ representing the relations between void ratio ($e$), vertical effective stress ($\sigma_z'$) and time $t$ is required. Besides, representation of the relation between stress – void ratio (or strain) – time is difficult, especially while significant excess pore water pressure exists in the soil, as well as when the majority of the excess pore water pressure has been dissipated.

Ladd et al. (1977) and Jamiolkowski et al. (1985) suggested two hypotheses A and B to address this key challenge based on the effect of the soil layer thickness on the compression of the soil particularly when the excess pore water pressure is dissipating. According to Hypothesis A, the different relations to express $e(\sigma_z', t)$ during and after the dissipation of the excess pore water pressure should be used. On the other hand, Hypothesis B assumes that a unique relationship for $e(\sigma_z', t)$ exists that can be used at any time irrespective of the excess pore water pressure. In general, assuming the same soil properties and initial conditions of void ratio and stresses, as well as the loading increment, the difference between these two hypotheses is the difference in the void ratios at the end of primary consolidation for the thin and thick samples because the value of void ratio at the end of primary consolidation can give an indication of the compression process while excess pore water pressure is dissipating. The details of Hypotheses A and B are provided in the following sections.
2.5.1 The suppositions

As illustrated in Figure 2.18, the whole consolidation process can be divided into two stages, primary consolidation stage \( \left( \frac{d\sigma_z'}{dt} \neq 0 \right) \) and secondary compression \( \left( \frac{d\sigma_z}{dt} \approx 0 \right) \). These two stages are conceptually separated at the time, \( t_{EOP} \), at which the dissipation of the pore water pressure is deemed to be completed. In the other words, the time \( t_{EOP} \) is defined as the duration of the dissipation of the excess pore water pressure or the primary consolidation time for an incremental loading case, during which the effective stress keeps changing, (i.e. \( \frac{d\sigma_z}{dt} \neq 0 \)). Therefore, Equation (2.7) can be rewritten as Equation (2.8). The first term in Equation (2.8) is to calculated \( e_{EOP} \), while the second term is used to calculate the compression under an almost constant effective stress.

\[
(\Delta e)_t = \int_0^{t_{EOP}} \left( \frac{\partial e}{\partial \sigma_z'} \frac{d\sigma_z'}{dt} + \frac{\partial e}{\partial \sigma_z} \frac{d\sigma_z}{dt} \right) dt + \int_{t_{EOP}}^t \frac{\partial e}{\partial \sigma_z} \sigma_z' \, dt \quad \text{when} \quad t \geq t_{EOP}
\]

\[\text{(2.8)}\]

![Figure 2.18 Void ratio versus time under the applied stress from } \sigma_z' \text{ to } \sigma_{zf} \]

According to Hypothesis A, experiencing the identical stress conditions such as the initial effective stress \( (\sigma_z') \) and the stress increment \( (\Delta \sigma_z) \) and the same void ratio \( e_o \) (or strain \( e_e \)), the value of \( e_{EOP} \) is unique regardless of the soil sample thickness, drainage conditions as well as the loading time. In other words, although the duration to reach the end of primary consolidation \( (t_{EOP}) \) for the thick sample is longer than that for the thin sample (which is approximately proportional to the thickness squared), the void ratios \( e_{EOP} \) of the thin or thick samples are the same as shown in Figure 2.19.
As a result, creep whether or not occurring during the dissipation of the excess pore water pressure has no influence on $e_{EOP}$. In Figure 2.19, according to supposition of Hypothesis A, after the end of primary consolidation, the compression curves of thin and thick samples will continues parallel to each other (i.e. $\delta \neq 0$).

On contradiction to Hypothesis A, Hypothesis B believes that since creep occurs during the dissipation of excess pore water pressure, longer $t_{EOP}$ results in more compression (i.e. smaller $e_{EOP}$ due to a larger change of void ratio) under a particular effective stress. As a result, the void ratio at $t_{EOP}$ for an applied stress increment depends on the thickness of the soil layer, the drainage conditions and the loading duration time. As illustrated in Figure 2.20, the $e_{EOP}$ of the thicker soil sample tends to be smaller than $e_{EOP}$ of the thin soil sample due to its longer drainage path and longer $t_{EOP}$. Referring to Equation (2.8), as the loading time increases ($t_{EOP}$ increases), the integration will be larger and consequently the void ratio change at $t_{EOP}$ will be greater. Figure 2.20 not only illustrates the supposition of Hypothesis B but also the model obtained from the experimental observation carried out by Aboshi (1973). Ladd et al. (1977) suggested that after the end of primary consolidation, the curves of compression with time for both thin and thick samples merge together. In this case, the value of $\delta$ in Figure 2.19 is equal to zero. However, based on the laboratory observation of Aboshi (1973), after the end of
primary consolidation the curves continue parallel to each other (i.e. \( \delta \neq 0 \)). In other words, Figure 2.20 shows that according to Hypothesis B, thin and thick soil samples have different values of the end of primary consolidation void ratio \( e_{EOP} \). Additionally, after the end of primary consolidation, there are two suggestions: (1) two curves will merge together (i.e. \( \delta = 0 \)), and (2) the curves continue parallel to each other (i.e. \( \delta \neq 0 \)).

![Figure 2.20 Void ratio versus time of thin and thick samples based on Hypothesis B](image)

Note: Both of samples have the same identical conditions (initial void ratio, initial stress condition)

- Hypothesis B:
  - Creep contributes to the value of \( e_{EOP} \)
  - \( (e_{EOP})_{\text{thin}} > (e_{EOP})_{\text{thick}} \)
  - Thick curve merges to thin curve after the thick sample reaches EOP.

- Hypothesis A:
  - EOP for all soil layer thickness
  - EOP of a thin sample (thickness: \( H_1 \))
  - EOP of a thick sample (\( H_2 = 5H_1 \))
  - EOP of a thick sample (\( H_3 = 100H_1 \))

![Figure 2.21 Void ratio versus effective stress at the end of primary consolidation (after Jamiolkowski et al. 1985)](image)

Effective stress \( (\sigma'_e) \) (log scale)
Figure 2.21 illustrates the contrast of Hypotheses A and B based on the compression curves at the end of the dissipation of the excess pore water pressure (i.e. at EOP). The unique $e_{EOP}$ based on Hypothesis A results in a unique value of the preconsolidation pressure ($\sigma'_pc$) for a particular soil. On the other hand, according to the propositions of Hypothesis B, the compression induced by creep is accumulated as the thickness increases, resulting in the EOP stress – strain (or $e_{EOP}$) curve shifting downward, and the reduction of preconsolidation pressure ($\sigma'_pc$) as shown in Figure 2.21.

The key distinction between Hypotheses A and B is the influence of the soil sample thickness on the total settlement. The influence of the soil sample thickness can also be presented via the drainage path and $t_{EOP}$. The soil sample thickness significantly affects the predictions of the settlement, and other properties of the soils such as the preconsolidation pressure as shown in Figure 2.21.

2.5.2 Discussion on the uniqueness concept of the end of primary consolidation void ratio

According to Mesri and his co-authors (e.g. Feng 1991; Mesri 2003; Mesri & Choi 1985b; Mesri & Rokhsar 1974), an interrelationship between the variation of the void ratio with stress $\left(\frac{\partial e}{\partial \sigma'}_{zt}\right)$ and the variation of the void ratio with time $\left(\frac{\partial e}{\partial t}\right)_{\sigma'_z}$ exists during the excess pore water pressure dissipation. Thus, this interrelationship may result in the unique $e_{EOP}$ for a particular soil which is independent of the soil layer thickness and drainage conditions. The first term of Equation (2.8) is defined as a constant, and becomes an input to calculate the long-term settlement of the soil. However, the possibility of the interrelationship as well as the uniqueness concept of $e_{EOP}$ has remained questionable, because $e_{EOP}$ has not been evaluated directly by mathematics via the terms $\left(\frac{\partial e}{\partial \sigma'}_{zt}\right)$ and $\left(\frac{\partial e}{\partial t}\right)_{\sigma'_z}$. In other words, no formulations have been proposed for Equation (2.6) to prove the possible interrelationship to support this hypothesis. Due to the difficulty to evaluate the uniqueness concept mathematically, Mesri and his co-authors (Mesri & Choi 1985b, Feng 1991) carried out a number of laboratory experiments in order to persuade that a unique $(e-\sigma')_{EOP}$ is possible and practical to predict the field settlement. The uniqueness concept proved based on the laboratory results by Mesri & Choi (1985b) has been accepted by Jamiołkowski et al.
(1985), Feng (1991), and Li et al. (2004), but also challenged by various researchers such as Degago et al. (2009), Kabbaj et al. (1988), Leroueil & Kabbaj (1987), Leroueil et al. (1985), and Yin & Graham (1989).

Feng (1991) concluded the unique EOP $e$-$\log\sigma'$ relationship based on isotropic loading test results of several soil samples of different thicknesses (0.025 m, 0.051 m, 0.127 m and 0.508 m) of different soils (e.g. Batiscan clay, St. Hilaire clay, Berthierville clay and Väsby clay) is independent on $t_{EOP}$. Moreover, Feng (1991) also attempted to prove the interrelationship of the terms $\left(\frac{\partial e}{\partial \sigma'_z}\right)_t$ and $\left(\frac{\partial e}{\partial t}\right)_{\sigma'_z}$ during the primary consolidation by interpreting the measured axial compression and volume change data obtained from the isotropic consolidation results of the 0.508m thick soil samples. The 0.508m thick sample was subdivided into 20 sub–layers, the values of $\left(\frac{\partial e}{\partial \sigma'_z}\right)_t$ and $\left(\frac{\partial e}{\partial t}\right)_{\sigma'_z}$ were evaluated based on the slope of the interpreted $e$–$\sigma'_z$ and $e$–$\log t$ curves at the $(e, \sigma'_z)$ and $(e, t)$ state points of a sub–layer. The local void ratio and local volumetric strain of a sub-layer were not measured, but obtained based on interpreting methods based on the total axial compression and total volume change of the 0.508m thick sample (Feng 1991) Both of $\left(\frac{\partial e}{\partial \sigma'_z}\right)_t$ and $\left(\frac{\partial e}{\partial t}\right)_{\sigma'_z}$ were observed to be influenced by the distance from the drainage boundary, but also result in the same amount of the EOP void ratio (Feng 1991). However, the chosen criterion to determine the end of primary consolidation has been challenged by Degago et al. (2011; 2009). Thus, the conclusion of the uniqueness concept remains questionable along with the interrelationship of $\left(\frac{\partial e}{\partial \sigma'_z}\right)_t$ and $\left(\frac{\partial e}{\partial t}\right)_{\sigma'_z}$ during the dissipation of the excess pore pressure.

The concept of the unique $e_{EOP}$ is challenged through the definition of the end of primary consolidation or the end of the dissipation of the excess pore water pressure. Theoretically, the end of primary consolidation is when the dissipation of excess pore water pressure completes, or when the excess pore water pressure, $(u_e)$, is equal to zero resulting in $t_{EOP} = \infty$. Traditionally, the value of $t_{EOP}$ and the corresponding void ratio $e_{EOP}$ could be estimated by the Casagrande logarithmic method, and Taylor square root method or can be more accurately determined directly by measuring the pore water pressure. According to Crawford (1964), based on the measurement of excess pore water pressures, the end of primary consolidation determined by pore water pressure
indication occurs earlier than that determined by determining the inflection point by Casagrande’s method from the curves.

Mesri & Choi (1985b), using pressure transducers to measure pore water pressures, decided to define the void ratio and effective stress at the end of primary consolidation when 98% of the dissipation of pore water pressure has occurred. Feng (1991) assumed that when excess pore water pressure drops to 1 kPa or less, the end of primary consolidation is reached. However, the definition of a small value of excess pore water pressure can influence the true value of \( e_{EOP} \) and the value of \( t_{EOP} \). Moreover, the excess pore water pressure varies not only with time, but also with depth. The excess pore water pressure within the soil layers closer to the drainage boundary dissipates faster than other layers further away from the drainage boundary. Therefore, adopting \( t_{EOP} \) based on the excess pore water pressure, measured at a single location, could mislead to incorrect results to prove the uniqueness concept. For example, Degago et al. (2009) considered a consistent criterion for the definition of the end of primary because the excess pore water pressure \( (u_e) \) can be generated as a result of creep. They have drawn attentions for the final value and the degree of dissipation of the excess pore water pressure when analysing the EOP test results between thin and thick specimens. At the end of the primary consolidation, the difference of the degree of dissipation can lead to the variation of the final value of excess pore water pressures of both thin and thick specimens. Moreover, in theory of one-dimensional consolidation, when the degree of consolidation \( (U) \) reaches 100%, \( T_v \) approaches infinity, consequently \( t_{EOP} \) goes to infinity. Therefore, theoretically the primary consolidation should not be separated from the compression induced by creep, or the term \( t_{EOP} \) could be meaningless.

Due to lack of consistent theoretical development for the settlement of soils including the primary consolidation and creep, the effect of the soil sample thickness as well as the verification for the concept of unique \( e_{EOP} \) have been attempted by both laboratory experiments and field observations.

2.5.3 Laboratory study on soil samples with different thicknesses
Aboshi (1973) performed significant tests in order to investigate the scale effect on the consolidation of soils. Large–scale consolidation tests were carried out for the specimens of 0.4 m and 1 m thick, while the three smaller specimens (0.02 m, 0.048 m, and 0.2 m thick) were tested in the laboratory. The soil material was muddy clay
excavated from a reclamation site in Hiroshima Bay, and stored in a trench of 15 \text{ m} \times 10 \text{ m} \times 2 \text{ m} covered by a thin layer of sand, and resettled for 6.5 years to obtained homogeneous soil samples (Aboshi 1995, 2004). All specimens were precompressed by preconsolidation pressure ($\sigma'_\text{pc}$), then subjected to a step loading from 20 kPa to 78 kPa over a long period. According to test results (Figure 2.22), Aboshi (1973, 1995) concluded that $e_{\text{EOP}}$ (corresponding to $e_{\text{EOP}}$) gradually increases with the specimen thickness, while the secondary compression curves are parallel. In addition, it was concluded that the coefficient of consolidation ($c_v$) increases with the thickness, the gradient of secondary compression decreases gradually. Based on the measured data, the value of $C_a$ of 1 m thick sample decreased from 0.0084 to 0.0034 in 10 years. However, Mesri & Choi (1985b), and Imai (1995) have challenged conclusions of Aboshi (1973) claiming that there was lack of information about the initial conditions of the tested samples.

Aboshi et al. (1981) also performed an interconnected oedometer test called separate type consolidation tests for Fukuyama clay. There were five oedometers connected in a series and loaded hydraulically under a uniform pressure. The excess pore water pressures were observed at the base of each sub–specimen. As observed from the test results, the strain of each sub specimen at EOP was not uniform, but varied with respect to the distance from the drainage surface. There was no criterion to
determine the EOP reported in Aboshi et al. (1981). However, based on the reported measurement of the excess pore water pressure at the EOP, the EOP might be defined at 90% dissipation of the excess pore water pressure at the impervious base of the bottom sub-specimen. Additionally, at EOP, an amount of residual pore water pressure (i.e. about 10% of the excess pore water pressure) at the undrained surface was observed. According to the test result, Aboshi et al. (1981) concluded that there was no unique relationship of stress–strain–time in clays, as each sub-specimen had its own strain curve different from the others. It was suggested that clays have no unique relationship of stress – strain curve, and the settlements of soil layers vary based on the distance from the drainage boundary. However, Imai (1995) questioned the test results since they were obtained from single step loading as well as the identical initial void ratio of sub–specimens.

Li et al. (2004) performed one-dimensional tests on clay samples with three different thicknesses of 10 mm, 35 mm and 60 mm in order to examine the scale and time effects on the consolidation. In this test, the soil specimens were reconstituted for making the identical initial conditions before single loading test. Based on the test results, creep was confirmed to occur during the primary consolidation, and the settlement curves are parallel after the end of primary consolidation. However, the void ratios at the end of primary consolidation are not much different for different thicknesses. However, Li et al. (2004) stated that the validity of the test results is limited, since it is difficult to make identical soil samples with different drainage distances. Additionally, the thicknesses of soil samples are not much different to draw a comprehensive conclusion.

To further evaluate and confirm the unique concept of Mesri & Choi (1985b), Feng (1991) in his thesis conducted various EOP tests on different thick specimens from 25.4 mm (1 in), 50.8 mm (2 in), 127 mm (5 in) to 508 mm (20 in) of different soils including Berthierville clay, Batiscan clay and St-Hilaire clay. The tests were carried out in triaxial equipment under isotropic compressions. The details of the soil properties and test procedures can be found in Feng (1991). Even though the tests were carried out with 4 different thicknesses, the results of isotropic consolidation tests on specimens of 127 mm and 508 mm thick are the most valuable to investigate of the effect of drainage conditions on consolidation of soils. According to the presented results, Feng (1991)
concluded that the uniqueness concept of the EOP void ratio–effective stress relationship is valid and independent on the maximum drainage paths.

However, Degago et al. (2009) have reinterpreted the test results of Feng (1991), and questioned the possibility of the uniqueness concept (Hypothesis A). As discussed in their paper (Degago et al., 2009), the excess pore water pressures at the EOP for thin and thick specimens are not consistent with each other. For example, for Batiscan clay, the excess pore water pressure dissipation percentage at EOP for thin (125 mm) and thick (508 mm) specimens are 99.5% (0.1 kPa) and 96.2% (0.8 kPa), respectively. For St Hilaire clay, they were 97.2% (1.0kPa) and 93.6% (2.2kPa) for thin and thick specimens respectively. Therefore, Degago et al. (2009) reinterpreted the test data to get the same final excess pore water pressure or the same degree of excess pore water pressure dissipation. As a result, Degago et al. (2009) concluded that the EOP void ratios are different with different thicknesses; thus, the EOP relationship of void ratio and effective stress \((e-\sigma'_E)_{EOP}\) is not unique.

According to Mesri & Choi (1985a), considering a unique \((e-\sigma'_E)_{EOP}\) relationship, the prediction of the settlement can be simplified, but yet provide good agreement with the field measurement, while incorporating with constant \(C_d/C_c\) approach. However, the efficiency of their approach and the settlement analysis were questioned and challenged by various researchers such as Chang (1987), Degago et al. (2009), Leroueil & Kabbaj (1987), Schiffman et al. (1987), and Tavenas (1987). Figure 2.22 shows the difference between the EOP compression curve reported by Mesri & Choi (1985a) and the EOP compression curve obtained by Laval sampler, and the in situ behaviour of embankment on Väsby clay reported by Leroueil & Kabbaj (1987). Laval sampler is considered to provide higher quality of soil samples. It was observed that the EOP compression curve could underestimate the real behaviour of soil in situ. Additionally, other researchers found that the prediction approach by a set of unique parameters recommended by Mersi and his co–workers was insufficient to predict the settlement and pore water pressure in situ correctly, since it neglects the strain rate effect, thickness of soil layers, drainage conditions and the soil behaviour during the primary consolidation process (Leroueil & Kabbaj 1987; Tavenas 1987). According to Leroueil (2006), the relationship between the logarithm of the preconsolidation pressure and the logarithm of the strain rate \((\alpha = \Delta \log \sigma'_p / \Delta \log \dot{e}_z\), which is equivalent to \(C_d/C_c\), can be considered a constant within a range of strain rates in the laboratory. However, the value
of $\alpha$ or $C_d/C_c$ would decrease with the strain rate in the field condition. Therefore, the assumption of a constant $C_d/C_c$ for field prediction may result in unrealistic prediction of long-term deformations.

![Stress-strain curves](image)

*Figure 2.23 Stress–strain curves obtained in laboratory and observed in situ between 4.25m and 7.29m of Vasby test fill (Leroueil & Kabbaj (1987))*

Nearly one century has passed since Terzaghi proposed the theory of consolidation of soils. However, the true time dependent behaviour of soils has remained uncertain and no unique theory exists. In the extensive knowledge available in the literature, time dependent behaviour of cohesive soils has been explained by two main Hypotheses A and B. In some cases, Hypothesis A can be reasonably applied. However, in practical point of view, due to the complication of deep subsoil layers in the field, the application of Hypothesis A can be aggressive to provide appropriate prediction. On the other hands, Hypothesis B which has been accepted widely can be the foundation to analyse the time dependent behaviour of soft soils accurately (Degago et al. 2009; Imai & Tang 1992; Kabbaj et al. 1988; Murakami 1988; Yin & Graham 1989).
2.6 Prediction approaches for the long-term settlement of soft soils

As aforementioned, there is no unique theory for the time dependent behaviour of soft soils including consolidation compression and creep settlement. However, since the compression induced by creep has been considered in the soft soil analysis, the theoretical and practical methods can be divided into three main approaches to predict the long-term behaviour of soft soils, as follows:

(i) Simple approach for settlement estimation based on the theory of one-dimensional consolidation proposed Terzaghi (1923) with a constant coefficient of secondary compression $C_D$.

(ii) Uniqueness approach including the concept of a unique $e_EOP$ and a unique ratio $C_d/C_c$ while both $C_d$ and $C_c$ change with time and stress.

(iii) General stress – strain – strain rate models

The simple approach (i) adopts the assumption that the creep compression occurs after the compression induced by the dissipation process, while the uniqueness approach (ii) supposes that the creep compression occurs concurrently with the consolidation compression. However, since the compression induced by creep and the compression induced the consolidation process cannot be separated during the dissipation of the excess pore water pressure, the uniqueness concept (ii) suggests that $e_{EOP}$ including both the creep compression and the consolidation compression is a unique value, based on which the total compression after the dissipation process can be calculated. On the other hand, the approach (iii) includes more complicated models which have been proposed based on the relationship between stress – strain – strain rate and occasionally stress rate. This section is to explain and discuss the advantages and disadvantages of each approach along with their remarkable related models.

2.6.1 Approach of coupling the consolidation theory and a constant $C_a$

Taylor & Merchant (1940) developed the theory of consolidation of Terzaghi (1923) by including the creep effects. They suggested that the compression of soil elements is a function of effective stress and time (Equation 2.7). In order to solve Equation (2.7), they proposed two Equations (2.9) and (2.10). According to Equation (2.9), the instantaneous compression (O–G in Figure 2.24) is proportional to the effective stress, represented by a slope “$a'$” in Figure 2.24. Due to the hydrodynamic retardation, for a stress increment from $\sigma'_z1$ to $\sigma'_z2$, the curve of void ratio and effective stress is O–C–B.
The final total compression for a particular stress increment was assumed to be unique, but during the stress increment, the delayed compression causes the increase in the void ratio along with the instantaneous compression which corresponds to the void ratio difference for points E and C. C–D path is the undeveloped secondary compression under a constant effective stress of $\sigma'_z$. The secondary compression was assumed to develop at the rate which is proportional to the undeveloped secondary compression presented by C–D. Furthermore, the rate of time dependent viscous compression (under constant effective stress) is expressed in Equation (2.10).

\[
\left( \frac{\partial e}{\partial \sigma} \right)_t = -a' \tag{2.9}
\]

\[
\left( \frac{\partial e}{\partial t} \right)_{\sigma'_z} = -\mu (a_f (\sigma'_z - \sigma'_{z1}) + (e - e_1)) \tag{2.10}
\]

where, $\mu$ is a constant which was named as the coefficient of secondary compression by Taylor & Merchant (1940).

Figure 2.24 Void ratio-effective stress relationship during the consolidation process (after Taylor & Merchant, 1940)

Therefore, substituting Equations (2.9) and (2.10) into Equation (2.7) yields the rate of change in void ratio for a soil element as shown in Equation (2.11). Equation (2.11) shows that creep deformation (as the delayed compression) occurs simultaneously with the instantaneous compression or the primary consolidation.
The general consolidation equation was developed by combining Equation (2.11) with the flow equations (Darcy’s equation), while adopting the simplifying assumptions of Terzaghi’s theory, as shown in Equation (2.12).

\[
\frac{de}{dt} = -a_f \frac{d\sigma'}{dt} - \mu (a_f (\sigma' \mathcal{Z} - \sigma' z_1) + (e - e_l)) \tag{2.11}
\]

Equation (2.12) was a very first approach to include creep along with the consolidation theory, which has been the foundation for the study on creep contribution on the settlement prediction. However, this model is yet difficult to illustrate the consolidation accurately (Rajot 1992), since there is no clear method to estimate \( \mu \).

After Taylor & Merchant (1940), secondary compression or creep has been studied extensively. The expression of the coefficient of secondary compression (\( C_D \)) has been defined differently depending on the terms by which the compression is computed (i.e. vertical strain or void ratio). Equation (2.12) is the expression to define the coefficient of secondary compression based on the vertical strain (Lambe & Whitman 1969; Raj 2008), while Equation (2.13) calculates the coefficient of secondary compression based on the void ratio (Budhu 2010; Das 2008; Mitchell & Soga 2005; Ranjan & Rao 2007). Most of the expressions are subject to obtain \( C_D \) based on the settlement after the dissipation of the excess pore water pressure.

\[
C_{ae} = \frac{\Delta H}{H_o \Delta \log (t)} \text{ or } C_{ae} = \frac{dH}{H_o d \log (t)} \tag{2.13}
\]

where, \( H_o \) is the initial soil sample thickness, and \( t \) is the elapsed time corresponding to the change of the soil thickness \( \Delta H \).

\[
C_{ae} = \frac{\Delta e}{\Delta \log (t)} \text{ or } C_{ae} = \frac{-de}{d \log (t)} \tag{2.14}
\]

where, \( \Delta e \) or \( de \) is the change in void ratio under a constant effective stress during the elapsed time \( t \).

While there is the relation between the void ratio and strain \( (\Delta H/H_o = \Delta e/(1+e_o)) \), the correlation between \( C_{ae} \) and \( C_{ae} \) is expressed as follows:

\[
C_{ae} = \frac{C_{ae}}{(1+e_o)} \tag{2.15}
\]

where \( e_o \) is the initial void ratio.

For simplification, based on the definition of \( C_{ae} \), the secondary compression (or creep compression) is estimated separately and after the completion of primary compression \( S_p \).
\[ S_p = H_o \frac{c_c}{1 + e_o} \Delta \log \sigma'_z \] (2.16)

where, \( C_c = \frac{d e}{d(\log \sigma'_z)} \) is the compressibility index.

Several equations have been developed to estimate the compression induced by creep (or secondary compression) as presented in Table 2.5. Equation (2.17) is preferable, since the coefficient of secondary compression enables to convert between void ratio and strain. Equation (2.18) involves both the initial thickness and \( e_{EOP} \), which conflicts with the calculation of the change in void ratio. Equation (2.19) is more reasonable as Equation (2.17), since the values of \( e_{EOP} \), and adopted thickness \( H_{EOP} \) are corresponding to the time \( t_{EOP} \), but it is rarely adopted since \( H_{EOP} \) is not a certain known value compared to \( H_o \).

<table>
<thead>
<tr>
<th>Equation</th>
<th>References</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ S_c = H_o \frac{C_{ae}}{1 + e_o} \Delta \log t ] [ S_c = H_o C_{ae} \Delta \log t ] (2.17)</td>
<td>McCarthy (2002), Raj (2008)</td>
<td>Based on the conversion between ( e ) and ( e_o ), ( S_c ) can be determined based on either ( C_{ae} ) or ( C_{de} ).</td>
</tr>
<tr>
<td>[ S_c = H_o \frac{C_{ae}}{1 + e_{EOP}} \Delta \log t ] (2.18)</td>
<td>Das (2008), Holtz et al. (2011), Ranjan &amp; Rao (2007)</td>
<td>The void ratio at the end of primary consolidation ( e_{EOP} ), is adopted instead of ( e_o ).</td>
</tr>
<tr>
<td>[ S_c = H_{EOP} \frac{C_{ae}}{1 + e_{EOP}} \log \left( \frac{t}{t_{EOP}} \right) ] (2.19)</td>
<td>Sivakugan &amp; Das (2009)</td>
<td>( H_{EOP} ) is the thickness of the soil layer at ( t_{EOP} ).</td>
</tr>
</tbody>
</table>

In fact, the approach of estimating secondary compression based on a constant \( C_{ae} \) or \( C_{ae} \) is a useful method for rough estimation, due to its simplicity for calculation. However, the accuracy of this method for prediction of field situations is questionable, since creep is estimated separately from the primary compression. Therefore, the real time-dependent behaviour of soft soils cannot be captured. Applying this approach, the compression induced by creep can continue to infinity, as the time approaches infinity due to a constant \( C_{ae} \) or \( C_{ae} \) adopted.
2.6.2 The concept of unique $C_d/C_c$ and $e_{EOP}$

Mesri and his co-workers (e.g. Mesri & Choi 1985b; Mesri & Godlewski 1977; Mesri & Rokhsar 1974) conducted significant studies concerning the relationship between the compression index $C_c$ and the coefficient of secondary compression $C_d$. Mesri (1973) found that high value of compressibility during the primary consolidation ($C_c$) results in high compressibility during the secondary phase. Mesri & Godlewski (1977) proposed the unique relationship between $C_d$ and $C_c$ during the secondary phase, which is considered as the central of their method for calculating the secondary compression. According to their studies, the ratio of $C_d/C_c$ for any soils is a constant, even though $C_d$ and $C_c$ individually can vary with time or vary with effective stresses, respectively. Moreover, this unique ratio can be applied for both overconsolidated and normally consolidated stress range, during compression or recompression.

The combination of the unique creep ratio $C_d/C_c$ and unique EOP curve is suggested to be a reliable and simple method to estimate the settlement of soils (Mesri and Choi, 1985). The total vertical strain can be computed during the secondary compression phase as the following equation.

$$
(\varepsilon_z)_t = (\varepsilon_z)_{EOP} + \frac{C_c}{1+e_{EOP}} \frac{C_d}{C_c} \log \frac{t}{t_{EOP}} \quad (2.20)
$$

Even though this method can be considered as very simple and easy approach in practice/design to apply, it has been challenged by various researchers such as Aboshi (2004), Degago et al. (2011), Kabbaj et al. (1988), Leroueil et al. (1988; 1985), and Yin & Graham (1989). As mentioned in Section 2.5.1, the EOP stress – strain curves in the laboratory were reported to be different from the curves of the field condition. Moreover, the values of $e_{EOP}$ and $t_{EOP}$ were sensitive to the method to determine the end of the primary consolidation. Based on this approach, the effect of creep on the time-dependent behaviour of the soil during the excess pore water pressure dissipation is neglected and cannot be assessed accurately.

2.6.3 General stress – strain – strain rate models

2.6.3.1 Isotach models

Suklje (1957) firstly proposed the isotach concept presenting the unique relationship between stress, void ratio and strain rate based on the incremental oedometer tests on a lacustrine clay sample. The isotach concept composes of a system of compression curves in the space of $e$–$\sigma'_z$, in which each compression curve represents a constant
consolidation rate (i.e. the rate of change in void ratio). The isotach concept of Suklje (1957) has emphasised the contribution of creep during the dissipation of the excess pore water pressure, as well as the effects of soil sample thickness, soil permeability and drainage conditions on the time–dependent behaviour of soils (Claesson 2003; Watabe et al. 2012).

Bjerrum (1967) provided a detailed study about clay in Drammen, Norway. As a result of the study, he believed that the conventional separation of compression into primary consolidation and secondary compression based on the dissipation of excess pore water pressure (A–C–D in Figure 2.14) is unsuitable to describe the soil behaviour considering the effect of effective stresses. It may be due to two compression processes occurring simultaneously during the dissipation of the excess pore water pressure. Therefore, the compressions should be divided into “instant” and “delayed” components (AB and BC as seen in Figure 2.25). “Instant compression” is the volume change induced by the increase in the effective stress, and “delayed compression” is the deformation under a constant effective stress. The hydrodynamic retardation causes correlation between these two components. Moreover, he found that delayed compression induces reverse resistance against further compression by observing the increase in the critical pressure (comparable to preconsolidation pressure) after the loading was sustained longer than the standard load duration.

According to Bjerrum (1967), the relationship between compression, applied stress and time can be represented by a system of lines. Each line corresponds to the void ratio for different values of effective overburden stress for a particular sustained loading time. It can be observed that each line represents a unique relationship between void ratio, stress and time (Figure 2.26). However, no detailed procedure or method was provided to identify or to formulate the time-line system.
Definition of instant compression and delayed compression compared to the primary and secondary compression (after Bjerrum 1967) (a) the change in effective stress, and (b) compression versus time.

Vertical effective stress ($\sigma'_z$) in log scale

Figure 2.26 Time-line system of Bjerrum (after Bjerrum 1967)
The isotach concept of Sukjie (1957) along with the definition of instant and delayed compression of Bjerrum (1967) have become the foundation for many researchers who have persuaded the effects of time and strain rate on the long-term behaviour of soft soils (e.g. Degago et al. 2011; Garlanger 1972; Imai et al. 2003; Leroueil et al. 1985; Rajot 1992; Watabe et al. 2012; Yin et al. 1994).

Based on the idea of time-line concept, Garlanger (1972) proposed a mathematical model for Bjerrum’s concept. The general equation to calculate the compression at any time $t$ is provided as follows. Equation (2.21) is applicable when the applied stress is more than the critical pressure $\sigma'_{pc}$.

$$\Delta e = C_r \log \frac{\sigma'_{pc}}{\sigma'_{zi}} + C_c \log \frac{\sigma'_{zf}}{\sigma'_{pc}} + C_d \log \frac{t_{i+t}}{t_i}$$

where, $\Delta e$ is the change in void ratio, $C_r$ is the slope on $e$–$\log(\sigma'_z)$ diagram of the compression line from $\sigma'_{zi}$ to $\sigma'_{pc}$, $C_c$ is the slope of the instant compression line, $C_d$ is the slope of the $e$–$\log(t)$ curve, and $t_i$ is the time corresponding to the instant compression line. As the definition of the instant compression line was defined in accordance with the concept of Bjerrum as shown in Figure 2.26, $t_i$ can be equal to the duration of standard consolidation tests (i.e. 24 hours), while $t$ is the sustained loading time.

Based on Equation (2.21), the compressibility $\left(\frac{\partial e}{\partial \sigma'_z}\right)_t$ and the creep rate $\left(\frac{\partial e}{\partial t}\right)_\sigma'_z$ can be obtained as the derivatives of Equation (2.21) with respective to time and stress, respectively. As a result, the compressibility and the creep rate are functions of void ratio and stress. Coupling the compressibility and the creep rate functions with the theory of consolidation, as reported by Garlanger (1972) the mathematical model resulted in a reliable predictions as verified with the observed values reported by Berre & Iversen (1972). However, the general procedure to construct the instant compression line was not provided. The instant compression line in Garlanger’s model as well as Bjerrum’s concept has a slope of $C_c$ which may include some amount of creep deformation. The work of Garlanger (1972) has been adopted to develop by many subsequent researchers such as Christie & Tonks (1985), Larsson (1986), and Yin & Graham (1989).

Christie & Tonks (1985) defined a time-line as the combination of the instant plastic compression and delayed compression, and adopted the space of $\log(e)$–$\log(\sigma'_z)$.
to describe the time-line system. Also, Yin (1990) adopted the concept of time-line system from Bjerrum (1967). According to Yin (1990), time-lines are the lines which have the same values of “equivalent time” \( t_e \) which will be explained later in this study, and \( t_e \) values are not equal to the loading duration which is different from the concept of Bjerrum (1967), Christie & Tonks (1985) and Garlanger (1972). Additionally, each time-line is related to a unique creep strain rate, defined by an equivalent time \( t_e \). In Yin (1990) the time-line system includes an instant time-line, a reference time-line, and a set of equivalent lines.

In Yin (1990), the creep compression was expressed as the visco-plastic behaviour which is described by a linear logarithmic function as shown in Equation (2.22). In Equation (2.22), the model parameter \( \left( \frac{\psi}{V} \right) \), compatible with \( C_{ov}/\ln10 \), is adopted as a constant, while \( t_e \) and \( t_0 \) are the equivalent time and the time parameter (also the time value of the reference time-line), respectively. Due to the linear logarithmic function, the creep compression based on Equation (2.22) continues to infinity with time.

\[
\varepsilon^{vp}_x = \frac{\psi}{V} \ln \frac{t_e+t_0}{t_0} \quad (2.22)
\]

Therefore, Yin (1999) improved the elastic visco–plastic (EVP) model with a non-linear creep function, which produces a limit strain for time approaching infinity (Equation 2.23). The time-line system will be described in detailed in Chapter 3. The EVP model is simple but yet practical to describe the time dependent behaviour of soft soils appropriately. The non–linear creep parameter \( \left( \frac{\psi}{V} \right) \) in Equation (2.23) in the new creep function of Yin (1999) is not only time dependent, but also stress dependent. It is different from the conventional coefficient of secondary compression, which is assumed a constant value for a particular soil. Additionally, the creep strain limit \( \varepsilon^{vp}_{lim} \), which is the maximum creep strain achievable as time approaching infinity was introduced in the non–linear creep function. The creep strain limit may decrease with the increase of the effective stress (Yin 1999), although it may be considered as a constant (corresponding to zero void ratio for simplicity).

\[
\varepsilon^{vp}_x = \frac{\psi_{o,v}}{1+\varepsilon^{vp}_x \ln \frac{t_e+t_0}{t_0}} \ln \frac{t_e+t_0}{t_0} \quad (2.23)
\]

\[
\frac{\psi}{V} = \frac{\psi_{o,v}}{1+\varepsilon^{vp}_x \ln \frac{t_e+t_0}{t_0}} \quad (2.24)
\]
where, $\varepsilon_{vp}$ is the creep strain, $\psi_e/V$ and $\varepsilon_{im}$ are the creep coefficient and creep strain limit, respectively, $t_e$ and $t_o$ are the equivalent time and the reference time, respectively.

The non-linear creep function can overcome the limitation of the linear logarithmic function of the original creep function, since the strain will approach a limit at time of infinity. The decrease in creep strain rate has been noticed by several researchers such as Bjerrum (1967), Larsson (1986), and Rajot (1992). Thus, the non-linear creep function can be captured the decrease of the strain rate with time. However, the difficulty of determining the model parameters exposes the limitation for the model application. The detailed discussion on the limitation of this model is presented in Chapter 3.

Karim et al. (2010) also developed a non–constant creep coefficient $C_a^*$ adopting the concept of Bjerrum (1967) related to the quasi-preconsolidation pressure for the normally consolidated clays (Equation 2.25). The proposed creep coefficient was implemented in the modified model of Kutter & Sathialingam (1992) for the numerical simulation.

$$C_a^* = C_{amax} \exp[-N(\bar{p}_e - p_L)]$$

where, $C_{amax}$ and $N$ are the positive constants; $\bar{p}_e$ and $p_L$ are the creep inclusive and creep exclusive preconsolidation pressures calculated based on the model of Kutter & Sathialingam (1992) and the Modified Cam Clay (MCC) models. The constants $C_{amax}$ and $N$ are determined by fitting the function to the oedometer test data available after the end of primary consolidation plotted in $C_a^* - (\bar{p}_e - p_L)$ space. The predictions using the varied values of $C_a^*$ to predict the long term settlement under the centre of an embankment were in good agreement with the measurements. However, the employment of long term creep test data after the end of primary consolidation for determining $C_a^*$ may require a significant number of long term tests in laboratory. Moreover, the newly introduced parameters including $\bar{p}_e$ and $p_L$ are not convenient for application, while the procedure to estimate these parameters is complex.

Applying the isotach concept, Watabe et al. (2008) developed a model based on the relationship between the preconsolidation pressure and strain rate and a compression curve, which were determined based on the creep tests and constant rate of strain tests, respectively on the Osaka Clay. Additionally, the similar tests were carried out on several other clay samples in order to compare the results with the isotach parameters estimated for the Osaka Clay. As a result, the isotach properties of the Osaka Clays can
be applied for other clays. They also suggested that the value of $C_d/C_c$ is not constant but decreases with the visco-plastic strain rate, which is consistent with the study of Lerouile (2006).

### 2.6.3.2 Time resistance concept

Janbu (1969) proposed the time resistance concept based on the resistance concept which has been widely used in engineering field. All media acquire resistance against the change of existing equilibrium conditions due to an applied force. The resistance is defined as the ratio of the incremental action over the incremental response (Janbu 1985). Applying this concept to the soil deformation, the external action is elapsed time, while the internal response is strain. Thus, the time resistance as depicted in Figure 2.27 is the tangent to the strain–time curve ($R = dt/d\dot{e}$). In Figure 2.27, initially the $R$-$t$ curve is parabolic up to time $t_c$, since this period is corresponding to the conventional primary consolidation. After $t_c$, the relationship between the resistance parameter $R$ with time is linear, and the value of $R_s$ for $t \geq t_c$ is calculated by Equation (2.26). Janbu (1969) separated the compression process into three sections including primary, linked and secondary processes. The secondary process starts immediately after the dissipation of excess pore water pressure completes. As shown in Figure 2.27, the linked process occurs between primary and secondary processes.

$$R_s = r_s(t - t_r)$$  \hspace{1cm} (2.26)

where, $r_s$ defined as the creep resistance number is the slope of the $R$-$t$ curve as shown in Figure 2.27, $t_r$ is the extrapolated time of the linear line corresponding to $R = 0$, and $t$ is the time since the beginning of loading.

Based on the time resistance definition, creep strain is estimated as follows:

$$\varepsilon_s = \frac{1}{r_s} \ln \left( \frac{t-t_r}{(t_c-t_r)} \right)$$  \hspace{1cm} (2.27)

The time resistance concept has been adopted by several researchers (Havel 2004; Svano et al. 1991; Vermeer & Neher 1999) since the first proposal by Janbu (1969). Vermeer & Neher (1999) developed a soft soil creep model considering the logarithmic law for creep components and applying the framework of Modified Cam Clay. The total strain includes the elastic and visco-plastic strains with the assumption of a constant creep rate with time. This creep model also adopts the time resistance concept for estimating the creep component of the settlement. Vermeer & Neher (1999) considered creep starts from $t_c$ rather than $t_p$ as shown in Figure 2.27, and applied the method of Janbu (1969).
to evaluate the values of parameters \( \tau_c \) and \( C \), in which \( C \) is equal to \( r_s \) and \( \tau_c = t_c - t_r \).

As a result, the total strain induced by the applied stress increment from \( \sigma'_{zi} \) to \( \sigma'_{zf} \) in which \( \sigma'_{zf} \geq \sigma_{pc} \) (preconsolidation pressure) can be estimated by the following equation:

\[
\varepsilon_z = \varepsilon_z^e + \varepsilon_z^c = \frac{c_r}{(1+e_0)\ln 10} \frac{\sigma'_{zf}}{\sigma'_{zi}} + \frac{c_rC_c}{(1+e_0)\ln 10} \frac{\sigma'_{pc}}{\sigma'_{po}} + C \ln \frac{t_{r+c}+t_t}{t_c} \tag{2.28}
\]

where \( \varepsilon_z^e \) and \( \varepsilon_z^c \) are the elastic and creep strain components of the total strain \( \varepsilon_z \); \( C_r \) and \( C_c \) are the conventional swelling and compression index, respectively; and \( l' = \tau - t_c \) with \( \tau = 1 \) day; \( \sigma'_{po} \) and \( \sigma'_{pc} \) are the preconsolidation pressure corresponding to the before–loading and the end of loading states, respectively. Figure 2.28 shows the schematic stress – strain curves summarising the calculation elements in Equation (2.28). The parameter \( C \) later denoted as \( \mu^* \) is a modified creep parameter which is assumed constant. The value of \( C \) can be estimated using the standard oedometer test results as shown in Figure 2.29. Comparing between Figure 2.29(b) and Figure 2.27(b), \( C \) is correlated with the resistance value \( r_s \) \((C = 1/r_s)\).

Figure 2.27 Time resistance for a load step in an oedometer test (a) the variation of the excess pore water pressure or strain with time and (b) the variation of time resistance \( R \) with time (after Janbu 1969)
Figure 2.28 Idealised stress – strain curve from an oedometer test with the division of strain increments into an elastic and creep component. Normal consolidation (NC) line has time value of 1 day ($\tau_c + t' = 1$ day on NC) (after Vermeer & Neher 1999)

According to Havel (2004), the creep resistance $r_s$ is stress-dependent. Based on the result of the oedometer test on Onsoy clay, he concluded that the creep resistance $r_s$ reduces significantly as the effective stress approaches the preconsolidation pressure ($\sigma'_p$ in Figure 2.28). Thus, the minimum value of $r_s$ corresponds to $\sigma'_p = \sigma'_p$. Thereafter, the creep resistance slightly increases as the effective stress increases (Havel 2004).

In the soft soil creep model developed based on the time resistance concept (Vermeer & Neher 1999), the modified creep parameter $\mu^*$ is adopted as a constant value similar to the conventional coefficient of secondary compression.
Singh & Mitchell (1968) proposed a general stress–strain–time equation (Equation 2.29) which can be applied for drained and undrained creep based on the rate process theory (mentioned in Section 2.3.2).

\[ \dot{\varepsilon} = A e^{\bar{q} \alpha} \left( \frac{t_i}{t} \right)^m \]  

(2.29)

where, \( A, \alpha, \) and \( m \) are the model parameters determined by creep tests, \( \bar{q} \) is the deviator stress level defined by the ratio of the creep stress \( q \) and the maximum strength \( q_{max} \) at the beginning of creep; \( t_i \) is the reference time.

According to Singh & Mitchell (1968), the value of \( m \) generally varies from 0.70 to 1.25 for different soils. Moreover, \( A, \alpha, \) and \( m \) are dependent on the initial soil fabric such as the particle size, shapes and arrangement. Moreover, integrating Equation (2.29) with \( m = 1 \) yields the linear relationship between the strain and the logarithm of time, while with \( m \neq 1 \), the integration generates the non-linear relationship between strain and time (Liingaard et al. 2004). Although Singh & Mitchell (1968)’s model formulated in Equation (2.29) can describe the drained and undrained creep, the model is limited to a single stress level for one-dimensional condition. The value of \( m \) is defined as a constant value, while different stress levels may result in different values of \( m \) (Liingaard et al. 2004).

Leroueil et al. (1985) and Kabbaj et al. (1986) proposed a unique relationship between the effective yield stress, strain and strain rate based on the results of constant rate of strain tests on normally consolidated clays. Kabbaj et al. (1986) suggested the linear relationship between the logarithm of the preconsolidation pressure \( (\sigma'_pc) \) and the logarithm of the plastic strain rate \( (\dot{\varepsilon}^p_z) \) (Equation 2.30) as well as the linear relationship between the plastic strain and the normalised effective stress (Equation 2.31).

\[ \ln \dot{\varepsilon}^p_z = A + B \ln \sigma'_pc \]  

(2.30)

\[ \ln \frac{\sigma'_z}{\sigma'_pc} = C + D \varepsilon^p_z \]  

(2.31)

where, \( A, B, C, \) and \( D \) are the experimental constants.

The proposed equations involve the effects of the strain rate on the preconsolidation pressure based on the observations of several test results. Since the model is rheological, its efficiency is questionable for the prediction of the long-term behaviour in field situations. Kabbaj et al. (1986) also stated that the model is limited to simulate the relaxation process, since the total strain cannot be separated into elastic and
plastic components. Moreover, the model is developed for the normally consolidated range, thus, the heavily overconsolidated behaviour of soils cannot be captured correctly.

In addition, there is a group of more general constitutive models which were developed to capture the general soil behaviours under any possible loading conditions. The exclusive discussions on the group of general constitutive models can be found in Liingaard et al. (2004) and Karim & Gnanendran (2014). Most of the models in this group are three dimensional stress – strain conditions. This section introduces briefly some remarkable general elastic visco–plastic models based on the over–stress theory of Perzyna (1963).

In the overstress theory of Perzyna (1963), the total strain rate is divided into two components of elastic and visco–plastic strain rates with the assumption of no viscous effects within the elastic region. The elastic strain rate \( \dot{\varepsilon}^e \) is assumed to follow the generalised Hook’s law, while the visco–plastic strain rate \( \dot{\varepsilon}^{vp} \) is based on the non–associated flow rule. The total strain rate \( \dot{\varepsilon} \) based on the overstress theory is expressed as in Equation (2.32). In Equation (2.32), the visco–plastic strain occurs as the current stress state is outside the yield surface; otherwise, the total strain includes only the elastic strains.

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{vp} = C_{ijkl} \dot{\varepsilon}_{ij} + \gamma \Phi(F) \frac{\partial g}{\partial \sigma'_{ij}} \text{ for } \sigma'_{ij} > f(\sigma'_{ij})
\]

\[
\dot{\varepsilon} = \dot{\varepsilon}^e = C_{ijkl} \dot{\varepsilon}_{ij} \text{ for } \sigma'_{ij} \leq f(\sigma'_{ij})
\]

(2.32)

where, \( C_{ijkl} \) is the elastic matrix which is a three-dimensional (3D) version of the elastic modulus \( E \), \( \Phi(F) \) is the 3D overstress function representing for the excess stress above the yield stress \( (\sigma' - \sigma'_{yield}) \), \( \gamma \) is the fluidity parameter, which is equal to the inverted value of coefficient of viscosity \( (\eta) \), \( g \) is the potential function and \( \sigma'_{ij} \) is the stress state.

Adachi & Okano (1974) proposed a general constitutive model for fully saturated normally consolidated clays based on the over–stress theory of Perzyna (1963) and the critical state theory of Schofield & Wroth (1968). Although it was claimed as the first model capturing the rate dependency to simulate the behaviour of clays, the model was not verified by test data (Karim & Gnanendran 2014). Thereafter, Adachi & Okano (1974) model has been developed further to be able to describe time–dependent behaviour of soils more efficiently (more details in Adachi & Oka 1982; Adachi et al. 1987; Oka 1981).
On the other hand, there are several constitutive models developed from the time-line concept of Bjerrum (1967) incorporating the over-stress theory of Perzyna (1963), and the modified Cam Clay framework (Roscoe & Burland 1968) to extend the available 1D applications to 3D model (e.g. Borja & Kavazanjian 1985; Kutter & Sathialingam 1992; Yin & Graham 1999; Yin et al. 2002).

Borja & Kavazanjian (1985) applied the modified Cam Clay framework to the concept of Bjerrum (1967) to develop the time dependent elasto-plastic models. The modified Cam Clay yield surface was employed to describe the time independent stress – strain behaviour. The time dependent strains including the elastic and plastic component were described by the model of Kavazanjian & Mitchell (1980). The creep compression was also included by applying the Taylor (1948) approach to obtain the volumetric creep strain, and Singh & Mitchell (1968) model to obtain the deviatoric creep strain. The complex model of Borja & Kavazanjian (1985) is suggested to predict the time dependent stress – strain behaviour of wet clays well. However, the model involves too many model parameters (i.e. 13 parameters) and has not been validated for long term laboratory test data.

Yin & Graham (1999) and Yin et al. (2002) extended their linear and non-linear elastic visco–plastic models from 1D to 3D based on the modified Cam Clay and the over-stress theory of Perzyna (1963). The extended linear EVP model of Yin & Graham (1999) could simulate the acceleration of creep strain rates when the deviator stresses approach the strength envelope as well as simulate different load conditions such as unloading-reloading, relaxation and shearing rate. The limitation of the EVP model is related to the linear logarithmic function modelling creep compression (explained in Section 2.5.2.3.2 and Chapter 3). The limitation was overcome by the new non-linear creep function included in the extended EVP model in Yin et al. (2002). The 3D non-linear EVP model was extended based on the same manner of the linear EVP model, but included a new loading surface to describe the time dependent stress – strain behaviour of overconsolidated and normally consolidated clays. The validation was performed using the triaxial test data available for the soft Hong Kong marine clay and a mixture of kaoline and bentonite. However, the application for the field situation has not been verified for field situation when the initial conditions change with depth. Additionally, the difficulty of the model parameter determination for the non–linear creep function remains the limitation of the EVP model.
Table 2.6 Summary of existing model category (modified after Leroueil 1985)

<table>
<thead>
<tr>
<th>Model Category</th>
<th>Reference</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R (\sigma', e) = 0$</td>
<td>Terzaghi’s theory of consolidation (1941)</td>
<td>The relationship between effective stress and void ratio is independent of the time and unique. This approach shows the limitation since the void ratio is observed to vary under constant effective stress.</td>
</tr>
<tr>
<td>$R (\sigma', e, \sigma_z, \dot{e}) = 0$</td>
<td>Taylor &amp; Merchant (1940), Gibson &amp; Lo (1961), Yin &amp; Graham (1989), den Han (1996), Yin (1999), Zhou et al. (2005), Kimoto &amp; Oka (2005), Karim &amp; Gnanendran (2009),</td>
<td>Rate of change in void ratio is a function of effective stress, void ratio and rate of change in effective stress.</td>
</tr>
<tr>
<td>$R (\sigma', e, \dot{e}) = 0$</td>
<td>Singh &amp; Mitchell (1968), Kabbaj et al. (1986)</td>
<td>Void ratio is a function of effective stress and strain rate.</td>
</tr>
</tbody>
</table>

### 2.7 Summary

Consolidation and creep are significant in soft soils and must be taken into account in engineering design and practice. For quantifying the long term settlement of soil under applied loads, viscous creep is a key deformation mechanism. This chapter has provided several creep mechanisms based on different theories. As a summary, the clay mineralogy has an important role in the creep behaviour of soft soil. Most of recent general stress – strain constitutive models for soft soils were developed based on the concept of a unique relationship between stress – strain – strain rate. Table 2.6 summaries four categories of existing stress – strain – time models. In Table 2.6, $\sigma'_z$ is the vertical effective stress, $e$ is the void ratio, $t$ is the time, $\dot{e} = \partial e / \partial t$ is the rate of change in void ratio, and $\dot{\sigma}_z = \partial \sigma'_z / \partial t$ is the rate of change in the effective stress. In
terms of presentation and formulation, equation can be easily rewritten in terms of strain $(e)$ rather than void ratio $(e)$ and $\dot{e} = \frac{\partial e}{\partial t}$, is the strain rate.

Most of models in Table 2.6 were proposed for one-dimensional (1D) straining, and can be easily extended for three dimensional (3D) stress conditions by applying theory of visco-plasticity proposed by Perzyna (1963) and the framework of Modified Cam Clay. The existing models have focused on the influences of the soil layer thickness, the drainage conditions, loading time as well as other factors such as loading conditions and temperatures, on the long term behaviour of soft soils. Most of the models can provide a good agreement with the laboratory measurement, while the prediction for the actual field measurements requires more investigations. However, the complexity of the mathematical expressions and the difficulty of model parameter determination expose the limitations for most of the models. Table 2.7 shows that most of the existing models adopt constant value of the creep parameters when simulating the creep compression of soft soils. Additionally, the creep parameters as well as other model parameters are obtained by the curve fitting the test data. Especially for the creep parameters, the longer durations of loading increments are required, since the creep parameters are based on the data available after the end of the primary consolidation only. The difficulty to separate the creep compression and the compression induced by the dissipation of the excess pore water pressure is applied for all the models in Table 2.7.

Among the existing models, the elastic visco–plastic models with the non-linear creep function proposed by Yin (1999) is a remarkable model which can describe the time dependent behaviour of soft soils effectively. The model including the non-linear creep function can illustrate the decrease in creep compression with time to overcome the limit of logarithmic function. However, there are a number of limitations in the EVP model regarding the model parameter determination which may affect its widespread application. The limitations of the EVP model are explained in details in Chapter 3. Consequently, Chapter 3 expresses a new effective approach to overcome the difficulties of the conventional approach for model parameter determination.
## Table 2.7 Comparison of some constitutive models for time-dependent behaviour of soft soils

<table>
<thead>
<tr>
<th>Model</th>
<th>Compression components</th>
<th>Number of model parameters</th>
<th>Number of tests involved</th>
<th>Test type</th>
<th>Creep model parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buisman (1936)</td>
<td>Instant and delayed</td>
<td>2</td>
<td>1</td>
<td>IL oedometer test</td>
<td>( \alpha_t )</td>
</tr>
<tr>
<td>Taylor &amp; Merchant (1940)</td>
<td>Instant and delayed</td>
<td>3</td>
<td>1</td>
<td>IL oedometer test</td>
<td>( \mu )</td>
</tr>
<tr>
<td>Koppejan (1948)</td>
<td>Instant and delayed</td>
<td>2</td>
<td>1</td>
<td>IL oedometer test</td>
<td>( C_s )</td>
</tr>
<tr>
<td>Garlanger (1972)</td>
<td>Instant and delayed</td>
<td>4</td>
<td>1</td>
<td>IL oedometer test</td>
<td>( C_a )</td>
</tr>
<tr>
<td>Mesri &amp; Choi (1985a)</td>
<td>Primary and secondary</td>
<td>4</td>
<td>1</td>
<td>IL oedometer test</td>
<td>( C_aC_c )</td>
</tr>
<tr>
<td>Kabbaj et al. (1986)</td>
<td>Instant and delayed</td>
<td>5</td>
<td>1</td>
<td>CRS</td>
<td></td>
</tr>
<tr>
<td>Yin &amp; Graham (1989)</td>
<td>Instant and delayed</td>
<td>6</td>
<td>2</td>
<td>IL oedometer test + creep test</td>
<td>( \psi/V )</td>
</tr>
<tr>
<td>Rajot (1992)</td>
<td>Instant and delayed</td>
<td>5</td>
<td>1</td>
<td>IL oedometer test</td>
<td>( C_a )</td>
</tr>
<tr>
<td>Yin (1999)</td>
<td>Instant and delayed</td>
<td>7</td>
<td>2</td>
<td>IL oedometer test + long–term IL creep test</td>
<td>( \psi/V )</td>
</tr>
<tr>
<td>Vermeer &amp; Neher (1999)</td>
<td>Instant and delayed</td>
<td>5</td>
<td>2</td>
<td>IL oedometer test</td>
<td>( \mu^* )</td>
</tr>
<tr>
<td>Karim et al. (2010)</td>
<td>Instant and delayed</td>
<td>8</td>
<td>3</td>
<td>long–term IL oedometer tests</td>
<td></td>
</tr>
</tbody>
</table>

Note: IL: incremental loading
3 NUMERICAL SOLUTION FOR ELASTIC VISCO–PLASTIC MODEL AND MODEL PARAMETER DETERMINATION

3.1 Introduction
This chapter presents in detail the elastic visco–plastic (EVP) model with the non-linear creep function, originally proposed by Yin (1999). The EVP model with the non–linear creep function is a reasonably simple but yet reliable model to illustrate the time–dependent stress – strain behaviour of soft soils. However, determining the model parameters is a challenging task. The conventional procedure to evaluate the EVP model parameters includes many assumptions, which results in uncertainty and difficulty in determining the model parameters. In this chapter, the reference time-line is redefined by considering the reference time as a unit of time. Moreover, the EVP model is coupled with the theory of consolidation in order to evaluate the variation of the settlement and the excess pore water pressure with time for soft soils. The coupled equations of the EVP model and the consolidation theory will be solved by Crank–Nicolson finite difference scheme. Applying the Crank–Nicolson finite difference solution, a model parameter determination procedure is proposed to obtain all EVP model parameters simultaneously. The determination procedure adopts the trust–region reflective least squares algorithm, which also is included in this chapter.

3.2 Elastic visco–plastic model

3.2.1 Time–line concept
Yin (1990) adopted the time–line concept of Bjerrum (1967), and believed that time dependent compression is due to the viscous nature of soils. According to Yin (1990), creep of soils results from (a) the viscous expulsion of adsorbed water from double layers around clay particles and (b) viscous, irrecoverable adjustment of clay structure and deformation of clay particles. Adsorbed water is not flowed due to the hydraulic
gradients, but depends on the effective stress. When effective stresses increase, adsorbed water will move out slowly until the soil structure reaches a new equilibrium which is established by the adjustment of soil particles and internal forces (attractive and repulsion forces) to balance with the external applied stress.

The time-line system proposed by Yin (1990) includes an instant time-line, a reference time-line, a limit time-line and a system of parallel time-lines with different equivalent times $t_e$. Figure 3.1 illustrates the time-line concept of Yin (1990) plotted in the natural scale and the natural logarithmic scale of effective stress.

The instant time-line in the EVP model of Yin (1990) is a time-independent elastic compression line. The expression of the instant time-line is as Equation (3.1).

$$\varepsilon^e_z = \varepsilon^e_{zo} + \frac{\kappa}{V} \ln \frac{\sigma^e_z}{\sigma^e_{tu}}$$  (3.1)

where, $\sigma^e_{tu}$ is a unit stress, and and $\varepsilon^e_{zo}$ is the strain at $\sigma^e_z = \sigma^e_{tu}$, $\kappa/V$ is a model parameter used to describe the elastic stiffness of the soil in which $V$ is the specific volume $(1+e_s)$.

Yin (1990) defined the reference time-line as the elastic-plastic time-line of which the equivalent time $t_e$ is equal zero ($t_e = 0$). The reference time-line is a time-independent line where the viscosity of the soil is equal to zero (Yin & Graham 1994). Thus, by its definition, the reference line includes no creep, because creep is believed as a result of deformation due to structural viscosity. The reference time-line is expressed as follows:

$$\varepsilon^{ep}_z = \varepsilon^{ep}_{zo} + \frac{\lambda}{V} \ln \left( \frac{\sigma^{ep}_z}{\sigma^{ep}_{zo}} \right)$$  (3.2)

where, $\varepsilon^{ep}_{zo}$ is the strain at $\sigma^{ep}_z = \sigma^{ep}_{zo}$, $\sigma^{ep}_{zo}$ and $\lambda/V$ are the model parameters. $\lambda$ is used to define the elastic–plastic behaviour of the soil, and the term $\lambda/V$ is similar to that in Cam Clay models.

According to Yin (1990), an equivalent time is somehow similar to the equivalent pressure defined in the critical state soil mechanics. According to Wood (1990), the equivalent pressure $p^{ep}_e$ is the pressure which in the isotropic normal compression would give the soil its current specific volume.

$$p^{ep}_e = e^{\left(\frac{N-v}{\lambda}\right)} \quad \text{with} \quad v = N - \lambda \ln p'$$  (3.3)

where, $v$ is the specific volume of a soil at the normal stress $p'$; $N$ is the specific volume of a soil normally isotropic consolidated at $\ln p'$ value of zero; $\lambda$ is the slope of the normal isotropic consolidation line.
The equivalent time $t_e$ is referred to as the time of young clay should wait after instantaneous loading along the reference line ($\lambda$ line) to get to the required future state point condition.

Yin (1990) defined the equivalent time $t_e$ at a state point as the time which would be needed to creep from the reference line to that point under the same effective stress $\sigma'$. An equivalent time $t_e$ relates to a unique creep rate which is dependent on a state point ($\sigma', \varepsilon$). Equivalent times are negative above the reference line, and positive below the reference line as shown in Figure 3.1. The creep compression strain is described by the following equation.

$$\varepsilon^{vp}_z = \frac{\psi}{V} \ln \left( \frac{t_o + t_e}{t_o} \right) \quad \text{for} \quad -t_o < t_e < \infty \quad (3.4)$$

where, $\frac{\psi}{V}$ and $t_o$ are the model parameters. If $t_e = 0$, the creep strain is 0, which is corresponding to a strain point on the reference time-line by the definition of the reference time-line. Thus, $t_o$ is also the time value of the reference time-line.

Differentiating this equation provides the creep rate as follows:

$$\dot{\varepsilon}^{vp}_z = \frac{\psi}{V} \frac{1}{t_o + t_e} \quad (3.5)$$

This equation shows that the creep strain rate $\dot{\varepsilon}^{vp}_z$ is dependent on the equivalent time $t_e$. Every constant equivalent time-line $t_e$ is corresponding to a constant visco–plastic strain rate $\dot{\varepsilon}^{vp}_z$. The logarithmic creep function in Equation (3.4) will provide an infinite creep settlement, as the creep time approaches infinity. Moreover, as discussed in Section 2.3.6, the volume of a soil element may reach a minimum value at which there is no void within the soil element. Thus, the compression or strain cannot continue infinitely, but cease after reaching the ultimate equilibrium state. Therefore, Yin (1999) proposed a new creep function in order to describe the creep behaviour of soils properly. The non–linear creep function is expressed as follows.

$$\varepsilon^{vp}_z = \frac{\psi}{V} \ln \left( \frac{t_o + t_e}{t_o} \right) = \frac{\psi_o}{V} \ln \left( \frac{t_o + t_e}{t_o} \right) = \frac{\psi_o}{V} \ln \frac{t_o + t_e}{t_o} \quad (3.6)$$

where, $\frac{\psi}{V}$ and $t_o$ are the model parameters. If $t_e = 0$, the creep strain is 0, which is corresponding to a strain point on the reference time-line by the definition of the reference time-line. Thus, $t_o$ is also the time value of the reference time-line.

$$\dot{\varepsilon}^{vp}_z = \frac{\psi}{V} \frac{1}{t_o + t_e} \quad (3.7)$$

with $\varepsilon^{vp}_{lim}$ is the creep strain limit, and $\frac{\psi_o}{V}$, the initial creep parameter, is the initial value of $\frac{\psi}{V}$ at time $t_e = 0$.  

3-3
In Equations (3.6) and (3.7), \( \frac{\psi_0}{V} \), \( t_e \) and \( \varepsilon_{lm}^{vp} \) are three model parameters which can be determined from oedometer creep tests. In the original concept, the creep parameter \( \frac{\psi}{V} \) is a model constant. However, in this new function, \( \frac{\psi}{V} \) is a function of the equivalent time \( t_e \), while \( \frac{\psi_0}{V} \) and \( \varepsilon_{lm}^{vp} \) are effective stress and time dependent. Yin (1999) suggested the linear logarithmic functions to correlate these model parameters to the vertical effective stress, as follows:

\[
\varepsilon_{lm}^{vp} = a - b \ln \left( \frac{\sigma'_{z}}{\sigma_u'} \right) \tag{3.8}
\]

\[
\frac{\psi_0}{V} = c - d \ln \left( \frac{\sigma'_{z}}{\sigma_u'} \right) \tag{3.9}
\]

where, \( a, b, c, \) and \( d \) are the model parameters determined by the curve fitting method, and \( \sigma'_{z} \) and \( \sigma_u' \) are the effective stress and the unit effective stress (i.e. 1 kPa), respectively.

However, Yin et al. (2002) stated that the measurement of the limit strain is a difficult task, since it is impossible to carry out the tests to infinitive duration. Therefore, as the limit strain can be understood as the void within the soil approaches zero under stress at the infinitive time, the limit creep strain \( \varepsilon_{lm}^{vp} \) can be estimated based on the initial void ratio (i.e. \( \varepsilon_{lm}^{vp} \approx \frac{e_0}{1+e_0} \)).

Yin & Graham (1994) suggested there exists a unique limit time-line in the space of \((\sigma'_{z}, e_{z})\) for both viscous and non-viscous soils. They defined the limit time-line as the line has \( t_e = \infty \) and the creep strain rate of zero. The behaviour of soils beyond the limit time-line is time-independent. It is believed that the creep straining will finally terminate after very long time (infinity), when the soil particles occupy a fixed volume. However, the original logarithmic creep function cannot be used to find the limit time-line because as time goes to infinity, the settlement is infinite. As the results from long term creep tests, the creep strain curve with time is non-linear, which is assumed that the conventional coefficient of secondary compression \( C_a \) decreases with time. Therefore, the non-linear creep function proposed by Yin (1999) can capture this behaviour, and be used to define the limit time-line as shown in Figure 3.2. Under the effective stress \( \sigma'_{z} \), the reference strain (Point A) is calculated based on Equation (3.2), the creep strain limit as the distance from A to C is yielded using Equation (3.8). As a result, the limit strain at Point C is the sum of the reference strain and the creep strain limit.
Figure 3.1 The time-line system proposed by Yin (1990) in (a) natural scale and (b) in logarithm scale of effective stress (after Yin 1990)
3.2.2 Governing equations

In general, the vertical strain $\varepsilon_z$ at the effective stress $\sigma'_z$ is calculated based on Equation (3.10), in which the equivalent time $t_e$ is known.

$$
\varepsilon_z = \varepsilon_{zo}^r + \frac{\lambda}{V} \ln \frac{\sigma'_z}{\sigma'_{zo}} + \frac{\psi_o}{\psi_{lim}} \ln \frac{t_e}{t_0} = \varepsilon_z^r + \frac{\psi_o}{\psi_{lim}} \ln \frac{t_e}{t_0} \quad (3.10)
$$

where, $\varepsilon_{zo}^r$ is the reference strain corresponding to the vertical effective stress $\sigma'_{zo}$, and $\varepsilon_z^r = \varepsilon_{zo}^r + \frac{\lambda}{V} \ln \frac{\sigma'_z}{\sigma'_{zo}}$. As mentioned earlier, the equivalent time $t_e$ is defined as the duration needed to creep from the reference time-line to the current value of the vertical strain $\varepsilon_z$ under the constant effective stress $\sigma'_z$. Since $t_e$ is not real loading time, the value of $t_e$ can be determined as follows:

$$
t_e = t_o \exp \left( \frac{\psi_{lim}}{\psi_o/V} \frac{(\varepsilon_{zo}^r - \varepsilon_z^r) - (\varepsilon_{zo}^r - \varepsilon_{zo}^r)}{\ln \frac{t_e}{t_0}} \right) - t_0 \quad (3.11)
$$

The incremental strain $d\varepsilon_z$ induced by the incremental $d\sigma'_z$ is the sum of the incremental elastic strain $d\varepsilon_z^e$ and the incremental visco–plastic strain $d\varepsilon_z^{vp}$. The incremental elastic strain $d\varepsilon_z^e$ is the differentiation of elastic strain $\varepsilon_z^e$ with respective to $\sigma'_z$, while $d\varepsilon_z^{vp}$ is the product of the visco–plastic strain rate $\dot{\varepsilon}_z^{vp}$ with the change of time $dt$. Therefore, the incremental strain $d\varepsilon_z$ can be expressed as follows:
Based on Equation (3.12), the general stress – strain – strain rate–stress rate differential equation can be defined. If the stress-strain state point \((\sigma'_z, \varepsilon_z)\) is below the limit time-line (Point D in Figure 3.2), the soil is assumed to be under a time-independent behaviour, and the strain rate is determined by Equation (3.13). For \((\sigma'_z, \varepsilon_z)\) point above the limit time-line such as Point B in Figure 3.2, Equation (3.14) is used to evaluate the strain rate including the elastic and visco–plastic behaviours.

- For any \((\sigma'_z, \varepsilon_z)\) point below the limit time-line:

\[
\frac{\partial \varepsilon_z}{\partial t} = \frac{\kappa}{\sqrt{\sigma_z}} \left( \frac{\partial \sigma_z}{\partial t} \right)
\]

(3.13)

- For any \((\sigma'_z, \varepsilon_z)\) point above the limit time-line:

\[
\frac{\partial \varepsilon_z}{\partial t} = \frac{\kappa}{\sqrt{\sigma_z}} \left( \frac{\partial \sigma_z}{\partial t} \right) + \left( \frac{\psi}{V} \right) \left( 1 + \frac{\varepsilon^p_{zp}-\varepsilon_{zp}}{\varepsilon^p_{lp}} \right)^2 \exp \left( \frac{1}{\psi} \left( \frac{\varepsilon^p_{zp}-\varepsilon_{zp}}{\varepsilon^p_{lp}} \right) \right)
\]

(3.14)

The elastic visco-plastic model can be applied for various stress – strain conditions such as creep, relaxation, a constant rate of strain and a constant rate of stress (Yin & Graham 1989; Yin 1990). Moreover, the EVP model can be coupled with the theory of consolidation proposed by Terzaghi (1923) in order to estimate the settlement and the excess pore water pressure response simultaneously. Yin & Graham (1996) demonstrated that the coupled original EVP model with the consolidation theory can predict the settlement and the excess pore water pressure effectively, when they carried out the numerical study on Drammen Clay. The predictions of the settlement and the excess pore water pressure agree well with the measurements of several soil samples of different thicknesses reported by Berre & Iversen (1972). Moreover, the EVP model has capacity to extend from one-dimensional (1D) model to 3D model by applying the viscoplasticity theory of Perzyna (1963) (Yin et al. 2002). In general, the non–linear EVP model has many merits to describe the time–dependent behaviour of soft soil realistically. However, there exist difficulties and uncertainty exists in regard to the model parameter determination, resulting in the limitations for the use of the EVP model in practice. The conventional procedure for the model parameter determination is explained and discussed in the following section. It is followed by the new modification for the model parameters, as well as a method proposed to overcome the difficulties.
3.2.3 Conventional procedure for the model parameter determination

This section explains the conventional procedures for the EVP model parameter determination by gathering the descriptions in Yin & Graham (1994), Yin (1999), and Yin et al. (2002). The demonstration for the model parameter determination is carried out on the consolidation test data of a marine clay sample taken from a tunnel in the city of Ottawa reported by Crawford (1964). The clay sample, which had 65% clay sized...
particles, had the natural water content of 57%, the void ratio of 1.63, the plastic limit of 25%, the liquid limit of 54%, the sensitivity of 50, and the field vane strength about 49kPa (Crawford 1964). The clay sample was 20mm high, and carried out a multistage loading test with stress increments applied daily. The variation of the vertical strain with time, and the vertical stress-strain curve are plotted in Figures 3.3(a) & (b). The end of primary consolidation curve in Figure 3.3(b) was obtained based on the measurement of the excess pore water pressure reported by Crawford (1964).

3.2.3.1 Instant time-line parameters (κV and ε_0)

The elastic parameter κV can be obtained by curve fitting the loading stages in the overconsolidated range or the unloading–reloading data if available using Equation (3.1). Yin & Graham (1994) suggested that the overconsolidated range has insignificant creep involved. In this example, the first three loading stages (stages 1 to 3) are used to determine κV and ε_0, since the loading times for these stages were shorter and they are within the overconsolidated range as shown in Figure 3.3(b). Applying Equation (3.1) for the first three loading stages with σ’u = 1kPa, the fitted value of κV is 0.0027, while the parameter ε_0 is assumed to be zero due to its insignificant value. Figure 3.4 shows the measured data along with the best fitting curve. The fitting curve is considerably good with the coefficient of determination about 0.93.
3.2.3.2 Creep function parameters ($\psi_0/V$, $\varepsilon^{vp}_{im}$ and $t_o$)

In order to define the relationships of the creep parameters ($\psi_0/V$ and $\varepsilon^{vp}_{im}$) with effective stresses as shown in Equations (3.8) and (3.9), several stress increments with long creep durations are required (Yin 1999). Yin & Graham (1994) suggested that the loading increments in the normally consolidated range can provide clearly the creep behaviour than the increments in the overconsolidated range. The process to determine the parameters $\psi_0/V$, $\varepsilon^{vp}_{im}$, and $t_o$ starts with the selection of $t_o$. Firstly, a reference point is chosen to determine the value of $t_o$ in advance, which will be used to determine the reference time-line parameters in the next step. The value of $t_o$ is to define the start point of creep compression, which was suggested to be one of these options: (1) early in the loading increments, (2) at the end of primary consolidation, or (3) at the end of the loading increments (Yin & Graham 1989).

However, the value of $t_{EOP}$ has been usually adopted as $t_o$, when the excess pore water pressure has been dissipated almost completely, and the change in strain is due to creep only. After $t_o$ is adopted, the creep strains ($\varepsilon^{vp}_{z}$) of each stress increment are determined by subtracting the total strain ($\varepsilon_{z}$) to the strain at $t_{EOP}$ ($\varepsilon_{EOP}$), and similarly the creep times $t_{vp}$ are the differences between loading time $t$ and $t_{EOP}$. Then, the creep strains are used to fit to Equation (3.15) to determine the values of $\psi_0/V$ and $\varepsilon^{vp}_{im}$ for each stress increment. Equation (3.15) is converted from Equation (3.6) to yield a linear function as $y = ax + b$, in which $a$ and $b$ are the inverts of $\varepsilon^{vp}_{im}$ and $\psi_0/V$ respectively, are obtained by curve fitting. Finally, the relationships of these model parameters with effective stresses can be obtained by curve fitting the model parameters with the effective stresses.

$$\frac{\ln(t_o+t_{vp})}{\varepsilon^{vp}_{z}} = \frac{1}{\psi_0/V} + \frac{1}{\varepsilon^{vp}_{im}} \ln \left( \frac{t_o+t_{vp}}{t_o} \right)$$  \hspace{1cm} (3.15)

In this example, the end of the primary consolidation stress-strain of the loading Stage 5 is used as the reference point to define $t_o$, since this stage is in the range of the normally consolidated stresses. $t_{EOP}$ of this stage is 45min, and $\varepsilon_{EOP} = 9.11\%$. The creep strains of Stages from 5 to 7 are used to determine the creep parameters. According to the excess pore water pressure measurement, the dissipations of the excess pore water pressures almost completed at about 45 min, 55 min, and 40 min for Stages 5, 6 and 7 respectively (Crawford 1964). Therefore, referring Figure 3.3(a), each stress increment

3-10
had about 1 loading cycle, after the excess pore water pressure almost finished the dissipation process. The numbers of points after the EOP are reliable to define the creep parameters, even though the duration for creep was not too long. The fitting results are shown in Figure 3.5. The best fitting curves are in the form of $y = ax + b$, with the coefficient of determination ($R^2$) greater than 0.95. Figures 3.5 to 3.7 illustrate the fitting curves for Equation (3.14) adopting different values of $t_0$ for Stages 5 to 7. Based on the coefficients of the fitting curves, the values of $\psi/V$ and $\varepsilon_{lm}^{vp}$ for Stages 5, 6 and 7 adopting different values of $t_0$ are presented in Table 3.1.

It should be noted that the values of $\psi/V$ and $\varepsilon_{lm}^{vp}$ decrease with the increase of the effective stress. Thus, Figures 3.8 to 3.10 present the linear relationship between $\psi/V$ and $\varepsilon_{lm}^{vp}$ and the logarithmic of the effective stresses as suggested by Yin (1999). The linear logarithmic expression visibly presents well for the relationship between $\psi/V$ and the effective stress in Figures 3.8(a), 3.9(a) and 3.10(a) with $R^2 > 0.90$. The coefficient of determinations ($R^2$) of the fitting curves of the creep strain limit and the effective stress are about 0.83 – 0.84 in Figures 3.8(b), 3.9(b) and 3.10(b), which is reasonable, since the test duration may not be long enough to define the creep strain limit.

### Table 3.1 Influences of $t_0$ on $\psi/V$ and $\varepsilon_{lm}^{vp}$ of Ottawa clay

<table>
<thead>
<tr>
<th>$t_0$ (min) = $t_{EOP}$</th>
<th>Effective stress $\sigma'$ (kPa)</th>
<th>$\psi/V$</th>
<th>$\varepsilon_{lm}^{vp}$</th>
<th>$\psi/V$</th>
<th>$\varepsilon_{lm}^{vp}$</th>
<th>$\psi/V$</th>
<th>$\varepsilon_{lm}^{vp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>392.3 (Stage 5)</td>
<td>784.5 (Stage 6)</td>
<td>1569.1 (Stage 7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage 5</td>
<td>45</td>
<td>0.0170</td>
<td>0.111</td>
<td>0.0095</td>
<td>0.058</td>
<td>0.0069</td>
<td>0.051</td>
</tr>
<tr>
<td>Stage 6</td>
<td>55</td>
<td>0.0199</td>
<td>0.093</td>
<td>0.0111</td>
<td>0.049</td>
<td>0.0081</td>
<td>0.043</td>
</tr>
<tr>
<td>Stage 7</td>
<td>40</td>
<td>0.0155</td>
<td>0.126</td>
<td>0.0086</td>
<td>0.065</td>
<td>0.0063</td>
<td>0.058</td>
</tr>
</tbody>
</table>
Figure 3.5 Linear curve fitting for the creep function parameters applying for Stages 5-6-7 for $t_0 = 45$ min

Figure 3.6 Linear curve fitting for the creep function parameters applying for Stages 5-6-7 for $t_0 = 55$ min
Figure 3.7 Linear curve fitting for the creep function parameters applying for Stages 5-6-7 for $t_o = 40$ min

\[ y = 7.921x + 64.492 \quad R^2 = 0.979 \]

\[ y = 15.307x + 115.800 \quad R^2 = 0.985 \]

\[ y = 17.206x + 158.185 \quad R^2 = 0.950 \]
Figure 3.8 Relationship between the effective stress and (a) the creep coefficient \( \psi_o/V \) and (b) the creep strain limit \( \varepsilon_{lm}^{vp} \) for \( t_o = 45 \text{ min} \)
Figure 3.9 Relationship between the effective stress and (a) the creep coefficient $\psi_{o/V}$ and (b) the creep strain limit $\varepsilon_{lm}^{vp}$ for $t_o = 55$ min
Figure 3.10 Relationship between the effective stress and (a) the creep coefficient $\psi_o/V$ and (b) the creep strain limit $\varepsilon_{im}^{vp}$ for $t_o = 40$ min

\begin{align*}
\psi_o/V &= 0.0543 - 0.0066 \ln(\sigma'_z/\sigma'_a) \\
R^2 &= 0.92 \\
\varepsilon_{im}^{vp} &= 0.4108 - 0.0491 \ln(\sigma'_z/\sigma'_a) \\
R^2 &= 0.83
\end{align*}
3.2.3.3 Reference time-line parameters ($\lambda/V$, $\varepsilon_{zo}^{ep}$, and $\sigma'_z$)

The parameters $\lambda/V$, $\varepsilon_{zo}^{ep}$, and $\sigma'_z$ are used to define the reference time-line (Equation 3.2). The reference time-line represents the time-independent elastic plastic compression line, which is referred to “instantaneous” loading. Yin (1999) suggested the slope of the reference time-line ($\lambda/V$) can be the slope of the EOP compression curve. Another option to define the reference time-line parameters is to fit to Equation (3.2) with the reference point adopted during the determination of creep parameters. The stress-strain of the reference point can be taken as $\sigma'_z$ and $\varepsilon_{zo}^{ep}$, respectively. However, it is convenient to choose a point at which $\sigma'_z = \sigma'_z$ and the corresponding strain $\varepsilon_{zo}^{ep} = 0$ (Yin and Graham, 2003). In this case, the value of $\lambda/V$ is determined based on Equation (3.15).

$$
\varepsilon_{zo}^{ref} = \varepsilon_{zo}^{ep} + \frac{\lambda}{V} \ln \left( \frac{\sigma'_z}{\sigma'_z} \right) = \frac{\lambda}{V} \ln \left( \frac{\sigma'_z^{ref}}{\sigma'_z} \right)
$$

(3.16)

where, $\varepsilon_{zo}^{ref}$ and $\sigma'_z^{ref}$ are the known vertical strain and effective stress of the reference point, respectively.

Another loading stage such as from $\sigma'_z$ to $\sigma'_z$ in Figure 3.7 in the normally consolidated range is required to be determined the value of $\lambda/V$ and $\sigma'_z$. Based on Equation (3.10), the vertical strain at Point 2 under $\sigma'_z$ is calculated as in Equation (3.17). In Equation (3.17), there are three unknown including $\lambda/V$, $\sigma'_z$, and $t_{e2}$ required to be determined. If $t_{e2}$ is defined, Equations (3.16) and (3.17) can be used to obtain the rest two parameters $\lambda/V$ and $\sigma'_z$.

$$
\varepsilon_{z2} = \frac{\lambda}{V} \ln \frac{\sigma'_z}{\sigma'_z} + \frac{(\psi_{o}/V)_2}{t_{e2}+t_o} \ln \frac{t_{e2}+t_o}{t_o}
$$

(3.17)

The value of $t_{e2}$ can be determined via an intermediate point, Point 2’ in Figure 3.7. Point 2’ is the instant elastic point under the effect of the stress increment from $\sigma'_z$ to $\sigma'_z$, and the equivalent time-line going through Point 2’ has the equivalent time $t_e = t_{e2}'$ counted from the reference time-line. The vertical strain at Point 2’ can be obtained by two ways: (1) by applying the instant elastic equation from $\sigma'_z$ to $\sigma'_z$, or (2) by applying Equation (3.10).

$$
\varepsilon_{z2'} = \varepsilon_{z1} + \frac{\lambda}{V} \ln \frac{\sigma'_z}{\sigma'_z} = \frac{\lambda}{V} \ln \frac{\sigma'_z}{\sigma'_z} + \frac{(\psi_{o}/V)_2}{t_{e2}+t_o} \ln \frac{t_{e2}+t_o}{t_o}
$$

(3.18)
Derived from Equation (3.16), the value of \( t_{e2} \) is expressed as follows:

\[
t_{e2} = t_0 \exp \left( \frac{\epsilon_{1m2}^p}{\psi_0/V} \right) \left( \frac{\epsilon_1 + \frac{\kappa}{\varepsilon_1} \ln \sigma_2' / \sigma_1' + \frac{\lambda}{V} \ln \sigma_2' / \sigma_2'}{1 - \frac{1}{\epsilon_{1m2}^p} \left( \epsilon_1 + \frac{\kappa}{\varepsilon_1} \ln \sigma_2' / \sigma_1' + \frac{\lambda}{V} \ln \sigma_2' / \sigma_2' \right)} \right) - t_0 \quad (3.19)
\]

Since the compression from Point 1 to Point 2' is time-independent, the total loading time of the stress increment is from Point 2' to 2. Therefore, at Point 2, the equivalent time \( t_{e2} \) is computed as follows:

\[
t_{e2} = t_{e2'} + \Delta t = t_0 \exp \left( \frac{\epsilon_{1m2}^p}{\psi_0/V} \right) \left( \frac{\epsilon_1 + \frac{\kappa}{\varepsilon_1} \ln \sigma_2' / \sigma_1' + \frac{\lambda}{V} \ln \sigma_2' / \sigma_2'}{1 - \frac{1}{\epsilon_{1m2}^p} \left( \epsilon_1 + \frac{\kappa}{\varepsilon_1} \ln \sigma_2' / \sigma_1' + \frac{\lambda}{V} \ln \sigma_2' / \sigma_2' \right)} \right) - t_0 + \Delta t \quad (3.20)
\]

Substituting Equation (3.20) into Equation (3.17) yield in another equation of two unknowns, which is combined with Equation (3.16) to form a system of two equations of two unknowns \( \lambda/V \) and \( \sigma_2' \). Therefore, the model parameters \( \lambda/V \) and \( \sigma_2' \) can finally be determined.

A similar procedure is applied for the loading stage from Point 2 to Point 3. Equations (3.17) to (3.20) can be used to estimate the vertical strains at Points 3 and 3' \((\varepsilon_{z3}, \varepsilon_{z3'})\) and the equivalent times \( t_{e3} \) and \( t_{e3'} \). The number of stages adopted to
determine the reference time-line parameters depends on the available loading stages in the normally consolidated range. One stage is used to defined the reference point which stays on the reference time-line, and other normally consolidation loading stages are used as the fitting points. For example in Figure 3.11, the loading stages from Point 1 to Point 2 and from Point 2 to Point 3 are adopted to determine $\lambda/V$ and $\sigma'_z$.

Referring back to the example, the reference point chosen in the previous step is the EOP point of Stage 5 ($\varepsilon_{zp}^{EOP} = 9.11\%$, $\sigma'_z = 392.3\,kPa$). That means the reference time-line will go through this point. Thus, the vertical strain at this point is expressed as follows:

$$9.11\% = \frac{\lambda}{V} \ln \left( \frac{392.3}{\sigma'_z} \right) \quad (3.21)$$

Two additional loading stages used to determine $\lambda/V$ and $\sigma'_z$ are Stage 6 and Stage 7. Applying Equation (3.17) and Equation (3.20), with $\varepsilon_5 = 13.10\%$, $\sigma'_z = 392.3$ kPa, $\varepsilon_6 = 21.11\%$, $\sigma'_z = 784.5$ kPa, $\Delta t_6 = 1578$ min, $\kappa/V = 0.0027$, $(\psi_6/V)_6 = 0.0095$, $(\varepsilon_{vp})_6 = 0.058$ and $t_0 = 45$ min, the vertical strain at Point 6 is expressed as follows:

$$21.11\% = \frac{\lambda}{V} \ln \frac{784.5}{\sigma'_z} + \frac{0.0095}{1 + 0.0058 \ln \frac{t_{e6} + 45}{45}} \ln \frac{t_{e6} + 45}{45} \quad (3.22)$$

where,

$$t_{e6} = 45 \exp \left( \frac{0.058}{0.0095} \left( \frac{1.131 + 0.0082 \ln \frac{784.5}{\sigma'_z} \ln \frac{784.5}{\sigma'_z}}{1 - \frac{0.058}{0.0095} (0.131 + 0.0082 \ln \frac{784.5}{\sigma'_z} \ln \frac{784.5}{\sigma'_z})} \right) - 45 + 1578 \quad (3.23)$$

For Stage 7, $\varepsilon_6 = 21.11\%$, $\sigma'_z = 784.5$ kPa, $\varepsilon_7 = 27.62\%$, $\sigma'_z = 1569.1$ kPa, $\Delta t_7 = 4228$ min, $\kappa/V = 0.0027$, $(\psi_7/V)_7 = 0.0069$, $(\varepsilon_{vp})_7 = 0.051$ and $t_0 = 45$ min, the vertical strain at Point 7 is expressed as follows:

$$27.62\% = \frac{\lambda}{V} \ln \frac{1569.1}{\sigma'_z} + \frac{0.0069}{1 + 0.051 \ln \frac{t_{e7} + 45}{45}} \ln \frac{t_{e7} + 45}{45} \quad (3.24)$$

where,

$$t_{e7} = 45 \exp \left( \frac{0.051}{0.0069} \left( \frac{0.2111 + 0.0069 \ln \frac{1569.1}{\sigma'_z} \ln \frac{1569.1}{\sigma'_z}}{1 - \frac{0.051}{0.0069} (0.2111 + 0.0069 \ln \frac{1569.1}{\sigma'_z} \ln \frac{1569.1}{\sigma'_z})} \right) - 45 + 4228 \quad (3.25)$$

Based on Equations (3.21), (3.22) and (3.24), $\lambda/V = 0.1178$ and $\sigma'_z = 181.075$ kPa are yielded to provide the best fitting values for the equations. The final time-line systems for Ottawa clay adopting different values of $t_0$ are illustrated in Figures 3.12 to 3.14. It is emphasised that different values of $t_0$ lead to different time-line systems as shown in Figures 3.12 to 3.14. It is noted that the limit time-line is not parallel to the
reference time-line, since the creep strain limits decrease with the increase of the vertical effective stresses as observed in Figures 3.12 to 3.14. As denoted in Figures 3.12 to 3.14 as well as Figure 3.9, the slope of the measured curve and the slope of the end of primary consolidation curve (in Figure 3.9) are not constant, but decrease with the effective stresses. Meanwhile, in this EVP model, the value of $\lambda/V$ is adopted as a stress–independent parameter.

![Figure 3.12 Time-line system of Ottawa clay adopting $t_o = 45$ min](image1)

![Figure 3.13 Time-line system of Ottawa clay adopting $t_o = 55$ min](image2)
3.2.3.4 Numerical analysis for an example embankment on Ottawa clay

As suggested by Yin (1999), for the ease of calculations, the value of $t_o$ can be chosen in advance equal to the time at which the excess pore water pressure can be neglected (or at the end of primary consolidation). However, the time when the remaining excess pore water pressure is negligible (or is traditionally assumed the end of primary consolidation time) is significantly influenced by the sample thickness. Therefore, according to the procedure developed by Yin (1999) to determine the model parameters, the value of $t_o$ depends on the size of the sample and is not a unique model parameter. It means that adopting the procedure reported by Yin (1999), consolidation test results on identical soil samples but with different thicknesses (e.g. in a Rowe cell) would not result in the same (unique) set of parameters to be used in the design. The main reason for this shortcoming is that the procedure presented by Yin (1999) does not use the settlement data during the excess pore water pressure dissipation process to find the creep strain rate and the creep limit. Yin & Graham (1994) also mentioned that ignoring the pore water pressure response may lead to incorrect stress – strain behaviour during the early stages of consolidation. In addition, as mentioned by Mesri & Vardhanabhuti (2005), it is not the best practice to estimate the soil creep parameters based on the data available after full dissipation of the excess pore water pressure and then extrapolate the results to obtain the creep during the excess pore water pressure dissipation process.
Thus, the most realistic creep function should consider the settlement data during and after the dissipation process.

Although the nonlinear creep function proposed by Yin (1999) is one step closer to the real behaviour of soils, the determination of the parameters is a challenging task. It requires more creep data points for curve fitting to define the creep strain limit. Moreover, during the dissipation process, the excess pore water pressure varies with different rates. The relation between the effective stress and the strain during the dissipation process is complex and cannot be easily determined; hence, the data points during the dissipation of the excess pore water pressure are not incorporated for the creep parameters determination in the conventional procedure. However, the number of data points after the dissipation of the excess pore water pressure is usually limited for the creep parameter determination. Therefore, Yin (2002) suggested using the value of strain correlated to zero void ratio as the ultimate creep strain limit, and adopted one value of creep coefficient for a soil instead of a stress dependent function.

Adopting $t_o$ equal to the time to reach the end of primary consolidation implies that creep commences after a period of $t_o$. Therefore, for the laboratory soil samples creep cannot be calculated during the dissipation of the excess pore water pressures and the choice of $t_o$ may influence the values of creep strain limit ($\varepsilon_{im}^{vp}$) and the initial creep coefficient ($\psi_o/V$). Additionally, $t_o$ is also the time value of the reference time-line. As a result, the reference time-line may also include a significant viscous strain, when consolidation test results on thick samples are adopted. Furthermore, for multi-stage loading tests, the time to reach the end of dissipation of the excess pore water pressure varies with the applied stresses.

For example, in the above exercise, $t_{EOP}$ for Stages 5, 6 and 7 for Ottawa clay are 45 mins, 55 mins and 40 mins, respectively. Since $t_o$ is chosen as 45 min, the soil sample in Stage 6 had not reached its end of primary consolidation, and the sample in Stage 7 had already passed its EOP. Table 3.2 and Figure 3.9 show the influences of $t_o$ on $\psi/V$ and $\varepsilon_{im}^{vp}$, consequently on the relationships between these parameters and the effective stress, respectively. For a particular effective stress, the higher value of $t_o$ results in the greater values of $\psi/V$, while $\varepsilon_{im}^{vp}$ tends to be lesser corresponding to the greater $t_o$. It is obviously shown in Table 3.2, when $t_o$, adopted $t_{EOP}$ of Stage 6, is the highest value, the associated $\psi/V$ and $\varepsilon_{im}^{vp}$ for each stress stage are the greatest and lowest respectively, compared to the values obtained using other $t_o$ values. The reverse
results are observed for the case of the lowest \( t_0 = 40 \) min. As a result, the variations of the parameters with the effective stresses are also affected by the value of \( t_0 \) (Figures 3.8–3.10). Different fitting equations for the relationships \( \psi_o/V \) and \( \varepsilon_{\text{lm}}^{vp} \) with effective stresses are produced, when different \( t_0 \) are adopted. In general, \( t_0 \) plays an important role in the parameter determination process, since it has interrelationship with other parameters, and significantly influences their values.

\[
\begin{array}{cccc}
\hline
\text{Case 1} & 45 & \varepsilon_{\text{EOP5}} = 9.11\%, & \sigma'_5 = 392.3 \text{ kPa}, \\
 & & \varepsilon_5 = 21.11\%, & \sigma'_5 = 784.5 \text{ kPa}, \\
 & & \Delta t = 1579 \text{ min} \\
 & & \varepsilon_7 = 27.62\%, & \sigma'_7 = 1569.1 \text{ kPa}, \\
 & & \Delta t = 4228 \text{ min} \\
\hline
\text{Case 2} & 55 & \varepsilon_{\text{EOP6}} = 19.02\%, & \sigma'_6 = 784.5 \text{ kPa} \\
 & & \varepsilon_5 = 13.09\%, & \sigma'_5 = 392.3 \text{ kPa}, \\
 & & \Delta t = 1917 \text{ min} \\
 & & \varepsilon_7 = 27.62\%, & \sigma'_7 = 1569.1 \text{ kPa}, \\
 & & \Delta t = 4228 \text{ min} \\
\hline
\text{Case 3} & 40 & \varepsilon_{\text{EOP7}} = 25.7\%, & \sigma'_7 = 1569.1 \text{ kPa} \\
 & & \varepsilon_5 = 13.09\%, & \sigma'_5 = 392.3 \text{ kPa}, \\
 & & \Delta t = 1917 \text{ min} \\
 & & \varepsilon_6 = 21.11\%, & \sigma'_6 = 784.5 \text{ kPa}, \\
 & & \Delta t = 1579 \text{ min} \\
\hline
\end{array}
\]

Table 3.3 The EVP model parameters by different options

<table>
<thead>
<tr>
<th>( t_0 ) (min)</th>
<th>( \kappa V )</th>
<th>( \lambda/V )</th>
<th>( \sigma'_{zo} ) (kPa)</th>
<th>( \varepsilon_{\text{lm}}^{vp} )</th>
<th>( \psi/V )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>Case 1</td>
<td>45</td>
<td>0.0027</td>
<td>0.124</td>
<td>188.1</td>
<td>0.3603</td>
</tr>
<tr>
<td>Case 2</td>
<td>55</td>
<td>0.0027</td>
<td>0.121</td>
<td>163.0</td>
<td>0.3015</td>
</tr>
<tr>
<td>Case 3</td>
<td>40</td>
<td>0.0027</td>
<td>0.116</td>
<td>169.7</td>
<td>0.4108</td>
</tr>
</tbody>
</table>

As \( t_0 \) is the time value of the reference time-line, it not only has influence on \( \psi_o/V \) and \( \varepsilon_{\text{lm}}^{vp} \), but also on the reference time-line parameters. Generally, if creep parameters are kept constant, smaller values of \( t_0 \) shift the reference time-line to the right, resulting in greater values of \( \sigma'_{zo} \), while there may be slight changes in \( \lambda/V \). Thus, a roughly chosen \( t_0 \) is not a unique material parameter, and is significantly influenced by the
sample thickness. To observe the influence of \( t_o \) on the reference time-line evidently, the determination process are repeated with 3 cases listed in Table 3.3. Adopting the end of the primary consolidation data of three loading stages in the normally consolidated range (i.e. Stages 5, 6, and 7) as the reference point, three values of \( t_o \) can be used for 3 cases. As EOP point of a stage is used, the final stress – strain data points of other two loading stages are used as the fitting points. The creep strain limits and the creep coefficients which are associated with the effective stress of the fitting points and obtained using \( t_o = t_{EOP} \) of the reference point, are used to determine the corresponding \( \lambda/V \) and \( \sigma'_{20} \). As a result, Table 3.4 reports the obtained reference time-line parameters of each case. Evidently, \( \lambda/V \) and \( \sigma'_{20} \) are influenced by \( t_o \), since \( \lambda/V \) varies from 0.116 to 0.124, and the corresponding \( \sigma'_{20} \) varies 169.7 kPa to 188.1 kPa corresponding to \( t_o = 40 \) min to 55 min, respectively.

In order to examine the influence of the model parameters on the predictions of the settlement and the excess pore water pressure, an example calculation is performed adopting three sets of the EVP model parameters listed in Table 3.3. In Figure 3.14, an embankment of 2.5 m height is built on a 20 m thick normally consolidated Ottawa marine clay. The external applied pressure induced by the embankment on the soil layer is 50 kPa. In this example, the vertical distribution of the applied stress is uniform for the entire depth. The initial coefficient of permeability (\( k_o \)) is \( 1 \times 10^{-7} \) m/min, the initial void ratio (\( e_o \)) of 2.5 is applied for the entire soil profile. The coefficient of the permeability change index is adopted as \( 0.5 e_o \). The groundwater table is at the ground surface as shown in Figure 3.15. The sand layer at the bottom of the soil layer is treated as the drained surface. Therefore, the drainage condition applied in this example is two-way drainage.

![Figure 3.15 Soil profile adopted in the example calculation](image-url)
The initial overburden effective stress is the difference between the overburden total stress \( \sigma_z = \gamma_s h_z \) with \( \gamma_s \) is the unit weight of the soil and \( h_z \) is the soil depth) and the hydrostatic pore water pressure \( u_o = 9.81 h_z \). The final effective stress is the sum of the initial overburden effective stress and the vertical applied pressure. The investigation time in this example is 50 years. The numerical simulation is performed by the Crank–Nicolson finite difference solution for the coupled EVP model and the one-dimensional consolidation theory. The detail of the Crank–Nicolson solution is explained in Section 3.3. Adopting three sets of the model parameters in Table 3.3, the predictions of settlement and excess pore water pressure of Ottaway clay under the example embankment are presented in Figures 3.17 and 3.18.

In Figure 3.17, the influence of the model parameters on the settlement prediction is obvious, since three sets of model parameters in Table 3.3 provide three different settlement curves for the same soil under the same applied load. Additionally, the disparity between the curves tends to increase with time. Considering the influence of \( t_o \), a smaller value of \( t_o \) (e.g. 40 min curve in Figure 3.17) may predict more settlement compared to higher \( t_o \) (e.g. 45 min and 55 min curves). Even though the values of \( t_o \) of 40 min and 45 min are very close, the prediction curves of Case 1 and Case 3 are distinguishable as shown in Figure 3.17. In regard to the excess pore water pressure, the
predicted variations of the excess pore water pressure with depth are also influenced by the model parameters as observed in Figure 3.18. For Case 2 with the largest adopted $t_o$ (i.e. 55 min), the maximum predicted excess pore water pressure is less than the corresponding values predicted by smaller $t_o$. For example, 25 years after the construction (Figure 3.18a) the maximum excess pore water pressure in Case 2 is 45.11 kPa, compared to 49.43 kPa and 50.29 kPa in Case 1 and Case 3, respectively. After 50 years of construction (Figure 3.18b) the excess pore water pressure predicted in Case 2 is still lower than other two cases. Meanwhile, there is less difference between the predictions in Cases 1 and 3.

In general, the adopted $t_o$ influences the values of other EVP model parameters. Moreover, the different sets of the EVP model parameters cannot provide the same predictions of the settlement and the excess pore water pressure of a thick soil layer. The example of field conditions of an embankment on Ottawa clay has provided obviously the influence of the model parameters on the time–dependent stress – strain behaviour on soft soils as shown in Figures 3.17 and 3.18. The disparity between the predictions adopting different sets of the EVP model parameters results in the difficulty and uncertainty to decide the most precise set of the EVP model parameters for the numerical analysis.

![Settlement prediction](image)

*Figure 3.17 Settlement prediction for Ottawa clay under the embankment adopting three sets of model parameters in Table 3.3*
Figure 3.18 Variations of the excess pore water pressure with depth at (a) 25 years and (b) 50 years after construction of Ottawa clay under the embankment adopting three sets of model parameters in Table 3.3

3.3 Coupled equations of the EVP model and the consolidation theory

3.3.1 Coupled governing equations for a constant external loading

In order to obtain the coupled governing equations of the vertical strain and the excess pore water pressure dissipation, the following assumptions are made:

- Soil is fully saturated,
- Soil particles and water are incompressible,
- The water flow and compression are only in the vertical direction, and
- The continuity equation (Equation 3.23) should be satisfied.

\[
\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = -\frac{\partial \varepsilon_z}{\partial t} \tag{3.26}
\]

where, \( k \) is the coefficient of hydraulic conductivity, \( \gamma_w \) is the unit weight of water, \( u \) is the pore water pressure equal to the sum of the excess pore water pressure \( (u_e) \) and the initial equilibrium water pressure \( (u_o) \), \( z \) is the depth, \( \varepsilon_z \) is the vertical strain and \( t \) is the elapsed time.

Assuming \( u_o \) remains unchanged during the consolidation process, then \( \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 u_e}{\partial z^2} \). During the consolidation, the effective stress \( \sigma'_Z \) in Equation 3.13 and 3.14 is calculated as the difference between the total stress and the excess pore water pressure.
(\sigma'_z = \sigma_z - u_p). In this section, the external loading is constant with time, the total stress \(\sigma\) may vary with depth and maintains constant with time. Thus, \(\frac{\partial \sigma_z}{\partial t} = 0\) is applied for this section.

Substituting Equations (3.13) and (3.14) into Equation (3.26) yields the following expression:

- For any \((\sigma'_z, \varepsilon_z)\) point below the limit time-line:

\[
\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = -\frac{k}{v \sigma'_z} \left( \frac{\partial \sigma_z}{\partial t} - \frac{\partial u_e}{\partial t} \right)
\]

(3.27)

- For any \((\sigma'_z, \varepsilon_z)\) point above the limit time-line:

\[
\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = -\left( \frac{k}{v \sigma'_z} \left( \frac{\partial \sigma_z}{\partial t} - \frac{\partial u_e}{\partial t} \right) + \frac{\psi_o}{\psi_o} \left( 1 + \frac{\varepsilon'_z - \varepsilon_z}{\varepsilon_{lim}^p} \right)^2 \exp \left( \frac{1}{\psi_o/\psi} \left( \frac{\varepsilon'_z - \varepsilon_z}{\varepsilon_{lim}^p} \right) \right) \right)
\]

(3.28)

In the conventional consolidation equation, the coefficient of volume compressibility, \(m_v = \partial \varepsilon_z / \partial \sigma'_z\), and the consolidation coefficient, \(c_v = k/(m_v \gamma_w)\), are adopted as constant values. However, in this numerical solution, \(m_v\) and \(c_v\) are both depth and time dependent. Moreover, for a particular soil and saturating liquid, the coefficient of permeability is solely a function of the void ratio (Leroueil et al. 1990). Thus, considering the relation between the vertical strain and the void ratio (Equation (3.28)), the variation of the permeability of the soil \((k)\) with the strain level can be calculated using Equation (3.30).

\[
\varepsilon_v = (e_o - e)/(1 + e_o)
\]

(3.29)

\[
\varepsilon_z = -\frac{c_k}{1+e_o} \log \frac{k}{k_o}
\]

(3.30)

where, \(k_o\) is the initial permeability corresponding to the initial void ratio \(e_o\) (or at zero strain), \(\varepsilon_v\) is the vertical strain at a particular depth corresponding to the void ratio \(e\), and \(c_k\) is the permeability change index.

Substituting \(m_v\) and \(c_v\) into Equations (3.14), (3.27) and (3.28) yields to the coupled partial differential equations to evaluate the time dependent relationship between the vertical effective stress, excess pore water pressure and the vertical strain.

\[
\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{1}{m_v} g(\varepsilon_z, \sigma'_z)
\]

(3.31a)

\[
\frac{\partial \varepsilon_z}{\partial t} = -m_v \left( \frac{\partial u}{\partial t} \right) + g(\varepsilon_z, \sigma'_z)
\]

(3.31b)

where,
For any \((\sigma_z', \varepsilon)\) point below the limit time-line:
\[
g(\varepsilon_z, \sigma_z') = 0
\]

For any \((\sigma_z', \varepsilon)\) point above the limit time-line:
\[
g(\varepsilon_z, \sigma_z') = \frac{\psi_o}{v_t} \left(1 + \frac{\varepsilon_z - \varepsilon_{z'}}{\varepsilon_{im}}\right)^2 \exp\left(\frac{V}{\psi_o} \left(\frac{\varepsilon_z - \varepsilon_{z'}}{1 + \frac{\varepsilon_z - \varepsilon_{z'}}{\varepsilon_{im}}}\right)\right) (3.32)
\]

In brief, Equations (3.31a) and (3.31b) are the coupled partial differential equations to evaluate the time dependent stress – strain relationship of a soil layer under uniform vertical pressures. These equations are the combination of the elastic visco-plastic model with nonlinear creep function and the theory of consolidation. By solving these equations simultaneously, the time dependent strain (or the settlement of the soil layer) can be obtained. Besides, the variation of the excess pore water pressures can be evaluated not only with time, but also with depth. In order to solve these differential equations, a numerical solution using the Crank–Nicolson finite different scheme is described in detail in the following section.

### 3.3.2 Crank-Nicolson finite difference solution

Crank–Nicolson finite difference scheme is introduced to solve the partial differential Equations (3.28a) and (3.28b). The Crank–Nicolson calculation procedure is based on central difference approximate in space and the trapezoidal rule in time (Crank & Nicolson 1947). Crank–Nicolson method is fully implicit and unconditionally stable. The Crank-Nicolson finite difference scheme is adopted for the numerical simulation due to its minimal truncation errors compared to explicit and implicit finite difference schemes. Moreover, the Crank-Nicolson method ensures the stability and convergence without requirement for the time step adjustment (Chen et al. 2003; Grasselli & Pelinovsky 2008). In other words, the time step is not restricted, and can be larger compared to other explicit methods (Chen et al. 2003). Figures 3.10 and 3.11 show the general conception of the finite difference grids with the coordinator axes for depth and time for one-way drainage and two-way drainage conditions, respectively. The variation in the depth \(z\) is presented by the subscript \(i\) starting from 1 to \(n\), and the subscript \(j\) stands for the variation in the time coordinator. In other words, a coordinator \((i, j)\) includes the subscript \(i\) for depth followed by the subscript \(j\) for time. To acquire the numerical solutions, the partial differential equations are implemented into a
MATLAB code. It is noted that the coordinators begin with 1 instead of 0 for the programming simplification.

By applying the Crank–Nicolson scheme, the partial differential Equations (3.31a) and (3.31b) are derived as follows:

\[(c_p)_{(i,j)} \frac{1}{2(\Delta z)^2} \left( (u_{(i+1,j+1)} - 2u_{(i,j+1)} + u_{(i-1,j+1)}) - (u_{(i+1,j)} - 2u_{(i,j)} + u_{(i-1,j)}) \right) +
\]
\[ut_{i-1,j} = 1\Delta t u(i,j+1) - u(i,j) - 1 mvi,jgez,\sigma'z_{(i,i)} \]

\[(3.33a)\]

\[(\varepsilon_z)_{(i,j+1)} = (\varepsilon_z)_{(i,j)} - (m_v)_{(i,j)} (u_{(i,j+1)} - u_{(i,j)}) + \Delta t (g(\varepsilon_z, \sigma'z))_{(i,j)} \]

\[(3.33b)\]

where,
\[
(g(\varepsilon_x, \sigma'_x))_{(i,j)} = \frac{(\psi_p V_p)_{(i,j)}}{t_0} \left(1 + \frac{(\varepsilon_x)_{(i,j)} - (\varepsilon_x)_{(i,j)}}{(\varepsilon_x)_{(i,j)}} \right)^2 \exp \left(\frac{1}{(\psi_p)_{(i,j)}} \left(\frac{(\varepsilon_x)_{(i,j)} - (\varepsilon_x)_{(i,j)}}{(\varepsilon_x)_{(i,j)}} \right) \right)
\]

(3.33c)

and, \(i = 2, 3, \ldots n\), and \(j = 1, 2, 3 \ldots m-1\), and \(\Delta z\) and \(\Delta t\) are the increments of depth and time, respectively. \(\Delta z\) is kept constant, while \(\Delta t\) can change during the calculation process.

Defining \(r(i,j) = (c_p)_{(i,j)} \frac{\Delta t}{(\Delta z)^2}\) and substituting \(r(i,j)\) into Equation (3.33a) yields:

\[-0.5r(i,j)u_{(i-1,j+1)} + (1 + r(i,j))u_{(i,j+1)} - 0.5r(i,j)u_{(i+1,j+1)} = 0.5r(i,j)u_{(i-1,j)} + (1 - r(i,j))u_{(i,j)} + 0.5r(i,j)u_{(i,j+1)} + h_{(i,j)}\]  
(3.34a)

where, \(h_{(i,j)} = \Delta t \frac{1}{(m_p)_{(i,j)}} (g(\varepsilon_x, \sigma'_x))_{(i,j)}\)  
(3.34b)

For the one–way drainage condition as shown in Figure 3.20, the boundary conditions corresponding to the pervious top and impervious bottom layers are expressed as follows:

- At \(z = 0\), \(u_{(1,j)} = 0\) for \(j = 1\) to \(m\)  
  (3.35a)

- At \(z = H_o\), \(\frac{\partial u}{\partial z} = 0\). It is called derivative boundary condition or Neumann boundary condition (2011). Applying the central difference in space for the derivative equation can lead to a better accuracy for solution (Chen et al. 2003; Morton & Mayers 2005).

\[
\frac{u_{(n+1,j)} - u_{(n-1,j)}}{2\Delta z} = 0
\]

Thus, \(u_{(n+1,j)} = u_{(n-1,j)}\) for \(j = 1\) to \(m\)  
(3.35b)

In order to implement the boundary conditions into the Crank–Nicolson scheme, a fictitious layer is added below the undrained boundary. The fictitious layer has \(i\) equal to \(n+1\) as shown in Figure 3.20.

In the case of two–way drainage at the top and bottom surface as shown in Figure 3.21, the boundary conditions are as follows:

- At \(z = 0\), \(u_{(1,j)} = 0\) for \(j = 1\) to \(m\)  
  (3.36a)

- At \(z = H_o\), \(u_{(n,j)} = 0\) for \(j = 1\) to \(m\)  
  (3.36b)

By applying Crank–Nicolson procedure, the initial values including \(\sigma_{z(i,j)}\), \(u_{(i,j)}\) and \(\varepsilon_{(i,j)}\) are required, and the initial effective stresses are calculated using \(\sigma'_{z(i,j)} = \sigma_{z(i,j)}\)
In addition to the key Equations (3.33a-c), Equations (3.37a-e) are incorporated to compute the effective stress and time dependent parameters in the calculation process. Equations (3.37a) and (3.37b) define the stress dependent creep strain limit and the creep coefficient with respect to depth and time. In these two equations, \( a, \ b, \ c, \) and \( d \) are four constant model parameters obtained by curve fitting of the data points. Equations (3.37c), (3.37d) and (3.37e) determine the coefficient of volume compressibility \( m_v \), the permeability \( k \) and the consolidation coefficient \( c_v \) with respect to depth and time, respectively. It should be noted that the set coefficient of Equations (3.37a)-(3.37e) is recalculated at each time step.

\[
\begin{align*}
\varepsilon_{im}^{vp}(i,j) &= a - b \ln \left( \frac{\sigma'_{z}(i,j)}{\sigma'_{u}} \right) \\
\psi_o/V(i,j) &= c - d \ln \left( \frac{\sigma'_{z}(i,j)}{\sigma'_{u}} \right) \\
(m_v)(i,j) &= \frac{k}{v \left( \sigma'_{z}(i,j) \right)} \\
(k)(i,j) &= k_o \times 10^{-\left(\varepsilon_{z}(i,j)\left(1+\varepsilon_0\right)^{\varepsilon_k}\right)} \\
(c_v)(i,j) &= \frac{(k)(i,j)}{\gamma_w(m_v)(i,j)}
\end{align*}
\]

where, \( i = 1, 3, \ldots n \), and \( j = 1, 2, 3 \ldots m-1 \).

In the case of one-way drainage conditions, adopting the initial values, the boundary conditions (Equations 3.35a-b), Equation (3.34) is converted into a system of equations involving a tridiagonal square matrix (Equation 3.38) to solve for excess pore water pressure \( u_{(i,j+1)} \). The matrix system has \( n-1 \) equations to solve for \( n-1 \) unknown \( u_{(i,j+1)} \). For the ease of observation, \( r \) in the tridiagonal system stands for \( r_{(i,j)} \). Adopting the values of \( u_{(i,j+1)} \), Equation (3.33b) is used to calculate the vertical strains \( (\varepsilon_z)_{(i,j+1)} \).
Similar to the case of one-way drainage, a triagonal square matrix (Equation 3.39) is formed as applying the boundary conditions (Equations 3.36a & 3.36b). The matrix system has \( n-2 \) equations to solve for \( n-2 \) unknown \( u(i,j+1) \), since the excess pore water pressures at the drained top and bottom surface are zero. Consequently, Equation (3.33b) is used to calculate the vertical strains \( (\varepsilon_v)_{(i,j+1)} \), after Equation (3.39) is solved.

\[
\begin{bmatrix}
1+r & -0.5r & 0 & 0 & 0 \\
-0.5r & 1+r & -0.5r & 0 & 0 \\
0 & -0.5r & 1+r & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 1+r & -0.5r \\
0 & 0 & 0 & -r & 1+r
\end{bmatrix}
\begin{bmatrix}
u_2 \\
u_3 \\
u_4 \\
\vdots \\
u_{n-1} \\
u_n
\end{bmatrix} =
\begin{bmatrix}
1-r & 0.5r & 0 & 0 & 0 \\
0.5r & 1-r & 0.5r & 0 & 0 \\
0 & 0.5r & 1-r & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 1-r & 0.5r \\
0 & 0 & 0 & 0 & -0.5r
\end{bmatrix}
\begin{bmatrix}
u_2 \\
u_3 \\
u_4 \\
\vdots \\
u_{n-1} \\
u_n
\end{bmatrix}
\]

\( (3.38) \)

The ground surface settlement (at \( z = 0 \)) at time \( t \) is derived using Equation (3.40) adopting the vertical strains at time \( t \).
\[ S_t = \int_{z=0}^{z=H_o} (\varepsilon_z)_t dz \]  

(3.40)

Using an approximate solution for Equation (3.40), the ground surface settlement at time \( t \) \((S_t)\) and the average vertical strain at time \( t \) (or at each time step \( j \)) \((\varepsilon_{z,ave})_t\) are computed using Equations (3.41) and (3.42), respectively.

\[
S_t = S_j = \left[ 0.5 \left( \varepsilon_{z_{(1,j)}} + \varepsilon_{z_{(n,j)}} \right) + \sum_{i=2}^{j=n-1} (\varepsilon_{z_{(i,j)}}) \right] \Delta z
\]

(3.41)

\[
(\varepsilon_{z,ave})_t = (\varepsilon_{z,ave})_j = \frac{S_j}{H_o}
\]

(3.42)

where, \( H_o \) is the initial thickness of the soil layer.

In short, Figure 3.21 summarises the numerical solution for the coupled partial differential equations by a flow chart, which has been coded using MATLAB software for the ease and accuracy of calculation.
Enter the following initial information:
- Layer depth $H_o$
- Initial total stress $\sigma_o$
- Stress increment $\Delta \sigma$
- Initial pore water pressure $u_o$
- Initial Strain $e_0$
- Elapsed time $t_{el}$

$n = H_o / \Delta x + 1$
$m = t_{el} / \Delta t$

$t_j = 0, j = 1$
$i = 1 : n$

Calculate the following parameters based on the initial information:
- $(\sigma_{j})_{(i,j)} = \sigma_o + \Delta \sigma$
- $(u_{0j})_{(i,j)} = \Delta \sigma$
- $(\sigma'_{j})_{(i,j)} = (\sigma_{j})_{(i,j)} - (u_{0j})_{(i,j)}$
- $(e_{j})_{(i,j)} = e_{0j} - (e_{j})_{(i,j)}$
- $(e_{m})_{(i,j)} = \left( \psi_{j}^{i} / \theta_{j}^{i} \right)_{(i,j)}$
- $(g(e_{j}, \sigma'_{j}))_{(i,j)}$
- $(m_{j})_{(i,j)}, (k_{j})_{(i,j)}, (c_{j})_{(i,j)}$
- $S_{j} , (e_{z_{ave}})_{j}$

[Equations (3.37a) to (3.37e)]

Boundary conditions

One-way drainage from the top surface:
- $z = 0, u_{(0,i)} = 0$
- $z = H_o, u_{(n+1,i)} = u_{(n-1,i)}$

Two-way drainage:
- $z = 0, u_{(0,i)} = 0$
- $z = H_o, u_{(n,i)} = 0$

Update the following parameters:
- $(\sigma_{j+1})_{(i,j)} = (\sigma_{j})_{(i,j)}$
- $(\sigma'_{j+1})_{(i,j)} = (\sigma_{j})_{(i,j)} - (u_{0j})_{(i,j+1)}$
- $(e_{m})_{(i,j+1)} = \left( \psi_{j+1}^{i} / \theta_{j+1}^{i} \right)_{(i,j)}$
- $(g(e_{j}, \sigma'_{j}))_{(i,j+1)}$
- $(m_{j})_{(i,j)}, (k_{j})_{(i,j)}, (c_{j})_{(i,j)}$
- $S_{j+1} , (e_{z_{ave}})_{j+1}$

[Equations (3.37a) to (3.37e)]

Solve Equations (3.38) & (3.33b) simultaneously to obtain:
- $(u_{j+1})_{(i)}$
- $(e_{j+1})_{(i)}$
- $t_{j+1} = t_j + \Delta t$

Solve Equations (3.39) & (3.33b) simultaneously to obtain:
- $(u_{j+1})_{(i)}$
- $(e_{j+1})_{(i)}$
- $t_{j+1} = t_j + \Delta t$

Record the following information:
- $(u_{j})_{(i,j)}$
- $(e_{j})_{(i,j)}$
- $(e_{m})_{(i,j)}$
- $S_{j} , (e_{z_{ave}})_{j}$

for

$i = 1$ to $n$
$j = 1$ to $m$

Stop

Figure 3.21 Flowchart for solving the coupled equations of the EVP model and the consolidation theory
### 3.3.3 Time-dependent loading

The numerical solutions for the elastic visco–plastic model in Sections 3.3.1 and 3.3.2 are applied for instantaneous loading. In order to apply for the field case studies with the stage construction, the loading can be applied in steps, and the numerical solution for the partial differential equations to determine the settlement and the excess pore water pressure also included the time–dependent loading. Since \( \frac{\partial \sigma_z}{\partial t} \neq 0 \), Equations (3.28) are rewritten as follows:

\[
\begin{align*}
    c_v \frac{\partial^2 u}{\partial z^2} &= -\left( \frac{\partial \sigma_z}{\partial t} - \frac{\partial u}{\partial t} \right) - \frac{1}{m_v} g(\varepsilon_z, \sigma'_z) \quad (3.43a) \\
    \frac{\partial \varepsilon_z}{\partial t} &= m_v \left( \frac{\partial \sigma_z}{\partial t} - \frac{\partial u}{\partial t} \right) + g(\varepsilon_z, \sigma'_z) \quad (3.43b)
\end{align*}
\]

**Figure 3.22 Time-dependent loading**

The applied stress increases linearly with time to the target value within the construction time as shown in Figure 3.22. After the end of the construction (i.e. the target applied stress is reached), the applied stress maintains constant with time. After \( t_{EOC} \), the numerical solution for Equations 3.43(a-b) follows the explanation in Section 3.3.2.

During the construction time \( t_{EOC} \), applying Crank-Nicolson finite difference scheme, Equations 3.43(a-b) can be expanded as follows:

\[
\begin{align*}
    -0.5r_{(i,j)}u_{(i-1,j+1)} + (1 + r_{(i,j)})u_{(i,j+1)} - 0.5r_{(i,j)}u_{(i+1,j+1)} &= 0.5r_{(i,j)}u_{(i-1,j)} + (1 - r_{(i,j)})u_{(i,j)} + 0.5r_{(i,j)}u_{(i+1,j)} + h_{(i,j)} \quad (3.44a) \\
    \text{with}, \quad r_{(i,j)} &= (c_v)_{(i,j)} \frac{\Delta t}{2(\Delta z)^2}, \quad \text{and} \quad h_{(i,j)} = \Delta t \left( \frac{1}{m_v} \right) g(\varepsilon_z, \sigma'_z)_{(i,j)} + ((\sigma_z)^{i+1} - (\sigma_z)^{i}) \\
    (\varepsilon_z)_{(i,j+1)} &= (\varepsilon_z)_{(i,j)} + (m_v)_{(i,j)} \left[ (\sigma_z(i,j+1) - \sigma_z(i,j)) - \left( u_{(i,j+1)} - u_{(i,j)} \right) \right] + \Delta t \left( g(\varepsilon_z, \sigma'_z) \right)_{(i,j)} \quad (3.44b)
\end{align*}
\]
Depending on the boundary conditions, Equations (3.44a & 3.44b) can be solved based on the matrix in the form of Equation (3.38) or (3.39). The initial conditions and Equations (3.37) are applied as similarly as the case of a constant external loading in Section 3.3.2.

3.4 Model parameter determination

3.4.1 General
In this section, a solution is proposed to obtain all of the EVP model parameters simultaneously by employing the trust-region reflective least squares algorithm in combination with the finite difference solution explained in the previous section. The Crank–Nicolson finite difference solution is used to obtain the prediction of the vertical strains as well as the excess pore water pressures with time and depth by adopting a set of the EVP model parameters. The differences between the predictions and the measurements are calculated, and the squares of the differences are the objective to be optimised. The set of the model parameters are optimised until the minimum difference between the predictions and measurements is achieved. The optimisation process is operated adopting the trust-region reflective algorithm. Thus, the proposed method for the EVP model parameters is the conjunction of the application of the Crank–Nicolson finite difference solution, the least squares algorithm and the trust-region reflective method. This proposed approach is an advanced solution for the model parameter determination, since the input data for the optimisation include several loading stages with a great number of data points. The involvement of the finite difference solution for the prediction of vertical stress-strain allows the vertical strains predicted during the dissipation process. As a result, the consolidation data are utilised. Since several loading stages are employed in the optimisation process simultaneously, the influences of the fitting points or reference point (as discussed in Section 3.2.3) on the parameters can be avoided. Consequently, the optimised EVP model parameters can be considered as the unique material parameters.

3.4.2 Least squares algorithm
The proposed solution in this study adopts the least squares algorithm to obtain the optimised parameters by minimising the difference between the measured and calculated data. Least squares analysis is one of the advanced tools for finding the best fits to the measured data, when the number of data points is more than the unknown
parameters. The large scale nonlinear least squares problems have been studied extensively in various application fields, since the optimisation problems become more complex and involve many variables and non-linear equations (e.g. Gould et al. 2005; Sung et al. 2010; Yuan 2011). In this study, the proposed solution for the model parameter determination is based on nonlinear least squares fitting, incorporating an advanced optimisation procedure, using the trust–region reflective algorithm.

For the purpose of least square optimisation, an objective function \( f_i(x) \) is defined to calculate the average strain at each time point \( t_i \) with parameters \( x = [x_1, x_2... x_n] \) and the measured data points \((y_1), (y_2),..., (y_m)\) corresponding to time points from \( l \) to \( m \). The optimisation problem can be rewritten as follows:

\[
\min_x \sum_{i=1}^{m} (y_i - f_i(x))^2
\]

(3.45)

### 3.4.3 Trust–region reflective algorithm

Since the least square optimisation has become more popular for research in many different fields, various algorithms have been proposed to solve the optimisation problems such as Newton method, Gauss–Newton method, and Lavenberg–Marquardt method (Dennis & More 1977; Geletu 2007; Gould et al. 2005). The trust-region method is one of the most accurate optimisation methods for nonlinear problems (Conn et al. 2009; Wang et al. 2011). Along with the interior-reflective algorithm, the nonlinear least squares problem can be solved with bound constraints (Geletu 2007). The trust–region optimisation method incorporated in the interior reflective Newton algorithm proposed by Coleman & Li (1994, 1996) is a simple, but yet powerful approach to solve bound constrained nonlinear minimisation problems. For a brief description of this approach, assume \( f(x) \) is the function to be minimised with \( x \) as a vector. The value of \( x \) can be bounded by upper and lower constrains. The concept of the trust–region method is to approximate \( f(x) \) with a quadratic function \( q(s) \), which reflects the behaviour of function \( f(x) \) in a neighbourhood \( N \) around the current point \( x \). That neighbourhood \( N \) is called the trust region. The trust–region subproblem of the method is to compute a trial step \( s \) by minimising the area \( N \). If \( f(x+s) < f(x) \), the current point \( x \) is updated to be \( x+s \). This step is called successful, and the trust region can remain for the next step. Otherwise, the step is unsuccessful, and consequently \( x \) remains unchanged, and the region \( N \) will be reduced for the next step. Therefore, the challenging issue of the trust–region method is to solve its subproblem, computing the
quadratic function $q(s)$ and to define the trust–region $N$. The general trust–region subproblem can be presented as follows:

$$\min_s q(s) = \min_s \frac{1}{2} s^T H s + s^T g$$

such that $\|Ds\| \leq \Delta$ \hfill (3.46)

where, $g$ is the gradient of $f(x)$ for the current $x$, $H$ is a symmetric matrix of second derivatives, $D$ is a diagonal scaling matrix, $\Delta$ is the trust region radius $> 0$ and $\|\cdot\|$ is the second norm. Coleman & Li (1996) proposed the reflective Newton algorithm to minimise the quadratic function, i.e. to solve Equation (3.46). In particular, to solve Equation (3.46) more quickly, $s$ can be restricted in a two dimensional subspace $S$. The subspace $S$ is spanned by two vector directions $s_1$ (the direction of the gradient $g$), and $s_2$ (the approximate Newton direction or a negative direction of curvature, $s_2^T H s_2 < 0$).

The approximation model $q(s)$ for the objection function does not have to be necessarily a quadratic function. A linear approximation model can be adopted, since the linear model can ensure the global convergence to first-order criteria information. However, the convergence rate would also be linear, which is consequently too slow. Meanwhile, a quadratic model is more effective, since a quadratic model can allow global convergence to the second-order and produce faster local convergence (Conn et al. 2009; Floudas & Pardalos 2008). Moreover, a quadratic model can capture the curvature of the non-linear function which cannot be achieved by a linear function (Conn et al. 2009; Murty 2010). Therefore, the quadratic function implemented in the optimisation scheme in this study can handle the non-linear equations of the EVP model to produce the best optimized results for the model parameters.

Applying the trust–region method to a general nonlinear least squares problem,

$$\min_x \sum_{i=1}^n f_i^2(x) = \min_x \|F(x)\|_2^2$$ \hfill (3.47)

where, $F(x)$ is the vector valued function having the $i$th component equal to $f_i(x)$.

To define the subspace $S$ for the problem more efficiently, an approximate Newton direction $s_2$ can be found by solving the following problem:

$$\min_s \|Js + F\|_2^2$$ \hfill (3.48)

where, $J$ is the Jacobian of $F$. Equation (3.48) can be approximately solved by applying the preconditioned conjugate gradient method to its normal equations in each iteration as follows:

$$J^T J s = -f^T F$$ \hfill (3.49)
According to Strikwerda (2007), the preconditioned conjugate gradient method is an effective method to solve a large symmetric positive definite system of linear equations \( Ax - b = 0 \) with a large condition number. If matrix \( A \) has a large condition number \( \kappa(A) \), where \( \kappa(A) = \|A\|\|A^{-1}\| \), the slower the conjugate gradient method produces convergence. Thus, the concept of preconditioning is to reduce the condition number by transforming the original system \( Ax - b = 0 \) to \( M^{-1} (Ax - b) = 0 \), where \( M \), known as a preconditioner for \( A \), is a symmetric positive definite matrix.

![Figure 3.23 Flowchart of the trust-region reflective method](image)

After solving the problem given in Equation (3.49), the vector directions of the subspace \( S \) are determined, and consequently, the minimisation process can be accomplished. In summary, as presented by Geletu (2007) and Coleman & Li (1994, 1996), the trust – region reflective algorithm can briefly be described as follows:

**Step 1:** \( k = 0 \), choose an initial vector parameter \( x_0 \), trust – region size \( \Delta_k \in (0, \bar{\Delta}) \), \( 0 < \eta_1 < \eta_2 < 1 \), \( 0 < \gamma_1 < 1 < \gamma_2 \), tolerance \( \varepsilon > 0 \)

**Step 2:** Compute the two dimensional subspace \( S \) to find two subspace vectors \( s_1 \) and \( s_2 \).

**Step 3:** Solve Equation (3.41) to find the trial step \( s_k \)

**Step 4:** Compute \( r_k = \frac{f(x_k) - f(x_k + s_k)}{q(0) - q(s_k)} \)

If \( r_k \geq \eta_1 \), set \( x_{k+1} = x_k + s_k \)

3-40
otherwise, set \( x_{k+1} = x_k \)

**Step 5:** Adjust the trust region size \( \Delta_{k+1} \)

- \( \Delta_{k+1} \in (0, \gamma_1 \Delta_k) \) if \( r_k < \eta_1 \)
- \( \Delta_{k+1} \in (\gamma_1 \Delta_k, \Delta_k) \) if \( \eta_1 \leq r_k < \eta_2 \)
- \( \Delta_{k+1} \in (\Delta_k, \min(\gamma_2 \Delta_k, \Delta_k)) \) if \( r_k \geq \eta_2 \) and \( \| s_k \| = \Delta_k \)

**Step 6:** If \( \|g := \nabla f(x_k)\| \leq \varepsilon \), the solution is convergence, and then the algorithm terminates (\( \varepsilon \) is the required tolerance). Otherwise, set \( k = k + 1 \), and update \( q(s_k) \) and the trust region size \( \Delta_k \), then repeat Steps 2 to 6 until to reach a desired convergence.

The flowchart in Figure 3.23 illustrates the algorithm of the trust-region reflective method using for the optimisation. In order to apply for the model parameter determination, the trust-region reflective method is incorporated to the least squares method which introduces the objective function for the optimisation process. The application of the trust-region reflective least squares method for the model parameter determination process is presented in the following section.

### 3.4.4 Parameters determination solution

To apply the optimisation algorithm for the parameter determination in this study, the measured vertical strains with time \( (y_i) \), the predicted vertical strains \( f_i(x) \) with time obtained by the finite difference solution mentioned in Section 3.3.2, are required. Additionally, the objective function \( u(x) \) used to compute the absolute difference of the predicted and measured vertical strains, in which \( x \) is the vector of the required parameters, is introduced. The trust-region reflective algorithm used for optimisation requires the number of variables to be less than the number of equations (Coleman et al. 1999). The EVP model includes a non-linear creep function, and in combination with the consolidation theory, the partial differential equations are highly non-linear. Thus, obtaining an explicit solution of the parameters is difficult despite the large number of data points available. Moreover, the number of data points is generally more than the number of equations as well as the number of the model parameters. Therefore, the optimisation procedure is required to obtain the most accurate model parameters to simulate the stress-strain behaviour of the soft soils.

In general, the EVP model with non-linear creep function includes 8 key model parameters \( (\varepsilon_{zo}, k/V, \varepsilon^p_{zo}, \lambda/V, \sigma^e_{zo}, t_o, \psi_o/V, \text{and } \varepsilon^{vp}_{lm}) \). Particularly for the non-linear creep function, \( \varepsilon^{vp}_{lm} \) and \( \psi_o/V \) are two effective stress dependent functions as presented.
in Equations (3.8) and (3.9). Considering Hypothesis B, the smaller value of $t_o$ is preferred, as creep contribution is included from the very early stages of loading. Since the semi-logarithmic relationship between strain and time is used, $t_o = 0$ cannot be applied. Thus, in this study, $t_o = 1$ (unit of time) is recommended to include the creep contribution from a very early stage of loading. Adopting $t_o = 1$ is also to overcome the difficulty to define the reference time-line. The reference time-line is expected to be a viscous free line. The corresponding $\lambda/V$ and $\sigma'_{zo}$ can uniquely be obtained with $t_o = 1$. Thus, the reference time-line would include a negligible viscous strain. More accurate model parameters can be determined by adopting the consolidation data during the dissipation of the excess pore water pressure. As a result, the test duration to collect enough creep data points might be reduced. In addition, it is not the best practice to estimate the soil creep parameters based on the data available after full dissipation of the excess pore water pressure and then extrapolate the results to obtain the creep during the excess pore water pressure dissipation process (Mesri & Vardhanabhuti 2005). The most realistic creep function should consider the settlement data during and after the dissipation process. Thus, parameters $a$, $b$, $c$, and $d$ should be included as the model parameters to be determined as well. Among these model parameters, for the ease of determination, the values of $\varepsilon_{zo}^e$ and $\varepsilon_{zo}^r$ can be adopted as zero. Therefore, the required parameters include 7 EVP model parameters and 2 soil physical property parameters related to the soil permeability ($k_o$ and $c_k$).

$$ x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_9] = [a, b, c, d, \lambda/V, \sigma'_{zo}, \kappa/V, k_o, c_k] $$

and,

$$ u(x) = \sum_{i=1}^{m} (y_i - f(t_i, x))^2 $$

The user-defined function $f(t_i, x)$ is the coded function implemented in the finite difference solution described above to predict the average strain, and the excess pore water pressure of the soil with time using the set of parameters $x$. The measured data include several sets of vertical strain – time curves for different applied stresses for a particular soil. In Equation (3.51), $y_i$ is the average strain at a time point $t_i$ while $i$ can change from 1 to $m$. Here $m$ is the total number of available data points of all stress increments.

The required initial information of each stress increment (i.e. each loading stage) include soil properties (initial void ratio $e_o$, initial thickness $H_0$) and the loading conditions (initial vertical total stress $\sigma_v$, stress increment $\Delta \sigma$, and initial strain $\varepsilon_{zi}$).
Incorporating the initial trial set of parameters and initial conditions into the finite difference solution as described in Figure 3.12 yields the predicted sets of average vertical strain and time for each stress increment.

Assume there are \( n \) number of loading stages, where \( n \) is a positive integer, while each of the loading stages has \( v_k \) number of data points (e.g. stage \( n \) has \( v_n \) data points available). The subscript \( k \) varies from 1 to \( n \). Thus, the total number of data points \( (m) \) is as shown in Equation (3.52).

\[
m = \sum_{k=1}^{n} v_k
\]  

(3.52)

where, \( v_k \) are the data point numbers corresponding to loading stage \( k \), where \( k \) changes from 1 to \( n \).

For an individual loading stage (i.e. loading stage \( k \)), the squares of the differences between the predictions and measurements are calculated as follows:

\[
u_k(x) = \sum_{i=1}^{v_k} (y_i - f_i(x))^2 \quad \text{for } k = 1 \text{ to } n
\]  

(3.53)

Thus, Equation (3.53) can be restated as follows:

\[
u(x) = \sum_{k=1}^{n} \sum_{i=1}^{v_k} (y_i - f_i(x))^2
\]  

(3.54)

Figure 3.24 summaries the process to calculate the squares \( u(x) \). Applying the trust–region reflective least squares algorithm, the optimal set of parameters can be determined by solving the following optimisation problem (Equation (3.55)). \( u(x) \) is the objective function as \( f(x) \) in Section 3.4.3.

\[
\min_x u(x) = \min_x \sum_{k=1}^{n} \sum_{i=1}^{v_k} (y_i - f_i(x))^2
\]

(3.55)

The above mentioned algorithms, representing the trust–region reflective least squares, is implemented and coded in MATLAB using “lsqnonlin” function. In cooperation with the finite difference solution for the EVP model and the consolidation equations, the optimisation procedure can be used efficiently to obtain the required parameters all together. The optimum values of the parameters can be ensured by several termination tolerances and the number of iterations in the optimisation process. The termination tolerances include the minimum changes in the values of the variables (i.e. the model parameters) and the minimum changes in the value of the objective function \( f(x) \). Besides, the number of iterations can also be adjusted. In this study, the adopted tolerances for both variables and functions were \( 10^{-10} \) to ensure the accuracy of the optimised model parameters.
Regarding the model parameters, the effects of the initial conditions were examined to ensure the unique final set of parameters. The initial set of the model parameters was adopted to be close to the values of corresponding parameters obtained by the conventional approach. For example, the value of $\kappa V$ is the slope of the recompression line in the space of $\dot{\varepsilon}-\ln \sigma_z$, while $\lambda/V$ is the slope of the normally consolidated line. Additionally, the initial values of the model parameters keeps being updated until the change of the coefficient of the determination $R^2$ for each stress stage is less than 0.1%. To investigate sensitivity of the selection of the initial model parameters on the final optimised parameters, initial parameters were changed by 100%. However, the optimisation results were not changed implying independence of the
optimised parameters to the initial selection of parameters. Moreover, in the geotechnical practice, the model parameters have the range of values which are also adopted to avoid the irrelevant values as well as to reduce the computation time.

The trust-region reflective algorithm allows the constraints of the unknowns to be set in order to avoid unrealistic values. Therefore, upper bound and lower bound constraints can be introduced for each model parameter. In order to ensure that the global minimum of the objective function can be achieved, the bounded ranges of \( a, b, c, d, \kappa/V, \lambda/V \) are 0 to 10. It should be noted that the ratio of \( C_r/C_c \) (\( C_r \) and \( C_c \) are the recompression and compression indices, respectively), is mostly from 0.02 to 0.2, and some soft clay and silt deposits may have the ratio less than 0.02 (Terzaghi et al. 1996). Thus, the correlated \( (\kappa/V)/(\lambda/V) \) would be in the similar range. The value of \( \sigma'_{zo} \) is controlled by the reference time-line properties including \( t_0, \lambda/V \) and \( e_{zo}' \). Thus, the lower and upper bounds of \( \sigma'_{zo} \) are \( -\infty \) and \( +\infty \), respectively. For the soil permeability properties, the range of the coefficient of permeability can be adopted based on the estimation obtained from \( m_v \) and \( c_v \). Besides, the permeability change index \( c_k \) has the upper and lower bounds of 5 and 0, respectively. According to Berry & Wilkinson (1969), the typical value of \( C_v/c_k \) for most soft soils is within 0.5 to 2. Thus, the ratio of the optimised \( (\lambda/V)/c_k \) is suggested to be in the range from \( 0.5/(1+e_o) \times \ln10 \) to \( 2/(1+e_o) \times \ln10 \). In general, the optimised model parameters and permeability properties are evaluated to be within the appropriate ranges.

3.5 Summary

This chapter has provided an explanation and discussed on the non-linear elastic visco–plastic model. The elastic visco–plastic model with the non-linear creep function is a practical means to analyse the time–dependent behaviour of soft soils. Discussion on the difficulties and the uncertainty in determining the EVP model parameters has been presented in Section 3.2.3. A worked example concerning Ottawa clay has been provided to demonstrate the conventional procedure to obtain the model parameters. The predictions of the settlement and the excess pore water pressure of a 20 m thick soil layer of Ottawa clay under the 2.5 m high embankment are performed adopting three sets of model parameters. As a result, there were disparities between the predictions adopting different cases of model parameters. Therefore, the conventional approach for
the model parameter determination exhibits difficulty and uncertainty to apply for the precise numerical analysis.

As a result, in order to overcome the difficulty of determining the model parameters adopting the conventional approach, as well as enhance the merits of the EVP model, an approach for the model parameter determination has been proposed in Section 3.4. The proposed approach has employed the Crank–Nicolson finite difference solution for the stress – strain prediction in conjunction with the advanced method for optimisation called the trust–region reflective least squares algorithm. The Crank-Nicolson solution has several advantages including unconditional stability and less truncation error compared to explicit and implicit finite difference schemes. The trust-region reflective least square (TRRLS) algorithm is an advanced optimisation method for the non-linear function. Hence, the combination of the TRRLS method and the Crank-Nicolson solution for the numerical analysis adopting the EVP model forms a feasible method for the model parameter determination. All the EVP model parameters can be determined simultaneously with the adopted $t_0 = 1$ (unit of time), and the consolidation data during the dissipation of the excess pore water pressure can be utilised in the optimisation procedure. The the proposed model parameter determination procedure and the Crank-Nicolson finite difference solution for the numerical simulation are coded using MATLAB software. The verification exercises are provided in Chapters 4, 5 and 6 with several case studies including the applications for laboratory conditions as well as the field situations.
4.1 General
Chapter 3 has provided the proposed approach for the model parameter determination. The proposed approach is a combination of the finite difference solution for the coupled partial differential equations of the elastic visco–plastic model and the theory of consolidation and the advanced trust-region reflective least square algorithm for the optimisation solution. This chapter is to verify the proposed method with a case study. In order to examine and validate the ability of the developed optimisation method as the finite difference solution for the EVP model with a nonlinear creep function, laboratory experiments using two different size Rowe cell setups are carried out. A thin sample of clay (29.5 mm thick) is used to determine the set of model parameters, while the result from a thicker sample (140.5 mm thick) is used to validate the model and the numerical solution. The detailed experiments and the numerical analysis are provided along with the discussion on the results.

4.2 Small Rowe cell experiment and parameter determination
In this study, in order to apply and verify the developed method for the EVP model parameter determination, two hydraulic consolidation setups of different sizes were used to carry out two series of incremental loading tests on a reconstituted soil consisting of kaoline, bentonite and fine sand. A multistage loading test was carried out on a 29.5 mm thick soil sample in order to provide a set of compression data with time to determine the EVP model parameters utilising the proposed numerical method. The optimised model parameters then were used to predict the settlement as well as the excess pore water pressures with time for a 140.5 mm thick soil sample. The predictions of settlement and excess pore water pressures are compared to the laboratory measurements. This section is to provide the details of laboratory study using the small
hydraulic consolidation cell and the model parameter determination by applying the proposed method to the experimental results.

4.2.1 Test setup and experimental procedure

4.2.1.1 Soil sample preparation

In this study, a mixture of Q38 kaolinite, Active Bond 23 bentonite, and fine sand (KBS) was adopted to prepare a reconstituted sample. The liquid, plastic, and shrinkage limits of clay materials were determined based on Australian Standard AS. 1289.3.4.1. The results are given in Table 4.1. As observing by colour, kaolinite is in white colour, while bentonite has darker grey as shown in Figure 4.1(a).

| Table 4.1 Properties of Q38 kaolinite and Active Bond 23 bentonite |
|-----------------------------|-------------|-------------|-----------|
| Material                    | Liquid limit (%) | Plastic limit (%) | Linear shrinkage (%) | USCS |
| Q38 Kaolinite               | 50.5         | 29           | 9          | CL   |
| Active Bond 23 Bentonite    | 340          | 50           | 35         | CH   |

Another material in the mixture was uniformly graded sand (SP) added to simulate a real soil usually found on site. The fine sand has 60% passing grain diameter ($D_{60}$) of 0.55 mm, 30% passing grain diameter ($D_{30}$) of 0.4 and 10% passing grain diameter ($D_{10}$) of 0.24 mm. The portions of kaolinite, bentonite and fine sands in the mixture were 70% kaolinite, 15% bentonite, and 15% fine sand by the total dry mass. The mixture was adopted after several trials of different portions to achieve a high plasticity (CH) fine grained soil. The plastic and liquid limits of kaoline–bentonite–sand (KBS) mixture were 21.67% and 80.12%, respectively. The use of bentonite in the mixture is to increase the plasticity of the soil, since bentonite includes a high amount of montmorillonite which can hold large amount of absorbed water within its microscopic structure compared to kaolinite (Mitchell & Soga 2005). Montorillonite mineral also has higher liquid and plastic limits compared to kaolinite mineral. Thus, the properties of soft soils are mainly due to the presence of montorillonite. Therefore, the use of bentonite in KBS is to constitute a mixture of soft soils exhibiting the properties of real...
soft soils. Kaolinite, bentonite and fine sands were dry mixed with the design portions before mixing with water, which was 116% (i.e. 1.45 times of the liquid limit of KBS). Then, the wet mixture was stored in a closed container for 1 week to enhance the saturation before the tests. Figure 4.1 shows the dry state of kaolinite, bentonite and fine sand as well as the mixture after mixing with water. Two different sizes of KBS samples were used in this study. The thin sample of 29.5 mm height by 75 mm diameter was tested in a 75 mm diameter Rowe cell. The test setup and experimental procedure are explained in the following sections.

Figure 4.1 KBS mixture (a) dry state of each material and (b) wet mixture
4.2.1.2 Experiment on thin KBS sample

The concept of a Rowe cell test (originally developed by Rowe & Barden 1966) is that the test sample is loaded hydraulically by water pressure acting on a flexible diaphragm. The drainage condition can be controlled, and the pore water pressure can be measured along with the volume change and vertical displacement. According to Head (1998), Rowe cell consolidation test allows the back pressure application, while different drainage conditions including in both vertical and horizontal directions can be simulated. In addition, different stresses or strains can be applied as loadings, and the volume change and pore water pressure can be measured continuously.

The small scale Rowe cell (SRC), used for model parameter optimisation, has the internal diameter of 75.3 mm and height of 56 mm. The schematic illustration of SRC adopted in this study is shown in Figure 4.2. The maximum height of the soil sample that can be accommodated in this setup is 32 mm after excluding the thickness of the porous plates and filter papers. The cell body is clamped between the cell top and the cell base by four long tie–bolts.

![Figure 4.2 Schematic diagram of the small Rowe cell](Image)

GDS Rowe cell control system has been used in this case study. Two enterprise level pressure/volume controllers (PVC) were used to control back pressure and cell pressures as well as volume changes. The controllers have configurations of 1MP pressure and 200cc volume capacity. The pressure and volume measurements have
accuracy at 0.25% of the full range with 0.4% of the measured value and +/- 500mm back lash, respectively. The system also includes a 16bit data acquisition to collect data from the linear potentiometer displacement transducer (LPDT) for the axial displacement and the pore water pressure transducer (PWPT). The data acquisition is connected to the computer to record the collected data from 2 channels, which are connected to LPDT and PWPT. The PVCs for the cell and back pressure sources are connected to an overhead water tank filled with de–aired distilled water. Besides, the controllers are connected to the cell pressure or back pressure pipelines attached to the Rowe cell. The brief description of the system connection is presented in Figure 4.2. Figure 4.3 shows the set up of the Small Rowe cell test in the laboratory.

![Small Rowe cell system connection](image)

Figure 4.3 Small Rowe cell system connection
Due to the prepared mixture having high water content to satisfy the full saturation, the soil sample for the small Rowe cell is prepared inside the cell. Before carrying out the tests, the Rowe cell was assembled and the system was de-aired by flushing water through the drainage pipelines connecting the cell with the controllers, while allowing air trapped between the diaphragm and the cell to drain out.

The valves connected between the PVC (back pressure) and the PWPT, and between the PWPT and the cell are opened to flush air in the pipeline out. The base of the cell is covered by a thin de-aired water layer. Then, the valve between the cell and the PWPT is closed to de-air the air bleed plug of the PWPT. After finishing de-airing the PWPT, the valve between the PWPT and the PVC (back pressure) is closed and kept closed during the testing period to measure the pore water pressure at the base of the soil sample.

The cell body is placed on the cell base after covering a thin layer of grease around the inner surface of the cell to reduce the side friction. A saturated thin filter paper is placed inside the cell body on the cell base to prevent the soil particle to penetrate to the porous plate at the base. It is noted that all the porous plates are saturated with distilled water in an ultrasonic tray. Then, KBS mixture is poured inside the cell to the maximum height level for the soil sample. The mixture should be poured gradually and flatted to make a smooth surface. A saturated filter paper and a saturated porous plate are then placed on top of the soil sample as shown in Figure 4.5(a-b).
Figure 4.5 Assembling soil sample in the small Rowe cell (a) soil sample assembled in the cell and covered by a filter paper, (b) porous plate placed on the soil sample surface and covered by water, and (c) the soil sample at the end of the test

The cell is then overflowed by de–aired water to avoid air trap before placing the diaphragm inside. The drainage valve connecting the PVC (back pressure) to the cell top is opened to water and de–air the pipeline before placing the cell top on the cell body. The rim drain valve is also opened to drain excess water and air. After that, the diaphragm with the cell top is placed inside the cell body on top of the soil sample. The air bleed plug and the connection valve of the cell top are also opened to avoid air trap inside the diaphragm. Then, the cell top is tightened to the cell body by four long tie–bolts. Water is kept flowing out for some time before closing. Before starting the consolidation test, the air bleed plug and the rim drain valve are kept closed through the test.

After assembling the Rowe cell with the testing sample, soil was consolidated under the vertical effective stress of 5 kPa by maintaining the back and cell pressures equal to 5 kPa and 10 kPa respectively. Then, B–check stage was carried out to ensure the saturation. The back pressure was maintained at 5 kPa, while the cell pressure was increased by 40 kPa, from 10 kPa initial cell pressure to 50 kPa. The B-value is calculated as the ratio of the increase of the excess pore water pressure to the applied stress increment (Head 1998). For the KBS soil, the B–value of the thin SRC sample was 0.95. Since the soil had high water content, and B–value was reasonable, the
saturation stage to increase the saturation degree of the soil is skipped. Then, an incremental loading test was carried out with vertical one-way drainage condition. The base of the cell was the impervious boundary, while water could drain through the top porous plate and flowed back to the back pressure controller. The incremental loads were applied until reaching the stress level of 400 kPa. Then, the loading was reduced in stages to 25 kPa (Figure 4.7) and increased again in stages to 800 kPa (Figure 4.8). The variation of the vertical strains with time for the KBS soil sample in the small Rowe cell are presented in Figures 4.6 to 4.8. The loads were sustained for up to 21 days for the first round of loading as shown in Figure 4.6(a) to observe soil creep. The reloading stages from 50 kPa to 400 kPa were performed in 1 day, while the final reloading from 400 kPa to 800 kPa was carried out in 21 days (Figure 4.6). It should be noted that the desired applied stresses were achieved by the difference between the cell pressure and back pressure. During the test, the back pressure was maintained at a constant value of 50 kPa, while the cell pressure was increased instantaneously to the target value and maintained constant during the stage.

The preparation of the soil mixture includes the saturation process before assembling in the Rowe cells. The wet mixture was stored in a closed container for 1 week to enhance the saturation before the tests. Additionally, the use of bentonite in the KBS mixture can reduce the effect of sedimentation in the soil. Moreover, the measurement of moisture contents at different depths at the end of the test was in the range of 33% to 35% (the average moisture content was 33.33%). Therefore, a notable sedimentation during the consolidation process was not observed.
Figure 4.6 Initial loading stages of Rowe cell consolidation test of 29.5 mm thick KBS

Figure 4.7 Unloading stages of Rowe cell consolidation test of 29.5 mm thick KBS
Adopting the trust–region reflective least squares (TRRLS) algorithm for the parameter
determination described in detail in Chapter 3, four stages of effective stresses 50kPa,
200kPa, 400kPa and 800kPa were employed to determine all the EVP model parameters
as well as the soil permeability parameters ($k_o$ and $c_h$). It should be noted that $k_o$ and $c_h$
could be measured directly using falling head permeability tests for clay soil samples
consolidated using different effective stresses. Due to the high water content of KBS
sample, the initial stage exhibited excessive settlement, while the following loading
stages became more stable. The four stages of loading selected for model parameter
determination were in the range of normal consolidation, which is suitable for
evaluating the relation of the creep strain limit and creep coefficient with the effective
stresses (Yin 1999). Since the soil experiencing the applied stress increment within the
normally consolidated stress range may include more creep compression compared the
soil under the stress in the overconsolidated range. Thus, the stress stages within the
normal consolidation in the model parameter determination are more suitable to be
adopted to obtain the reference line parameters as well as visco–plastic parameters

Figure 4.8 Reloading stages of Rowe cell consolidation test of 29.5 mm thick KBS
compared with other overconsolidation stages. The finite difference solution embedded with the advanced TRRLS optimisation procedure is coded using MATLAB software. Figure 4.9 is the calculation grid for the numerical simulation for the SRC sample. The soil sample thickness is divided into 5 sub–layers, resulting to 6 calculation nodes (n = 6). While the sub–layer thickness Δz is constant, time step Δt is designed to increase as the program runs. Based on the initial information and soil properties, the model parameters obtained by the proposed method are presented in Table 4.2.

The model parameters obtained by the proposed optimisation method are presented in Table 4.3. The range of the permeability change index $c_k$ of various soft soils varies from 0.5 to 2 (Mesri & Choi 1985b). Additionally, it is widely suggested that $c_k = 0.5e_o$, where $e_o$ is the in-situ void ratio can be adopted during consolidation analysis (Leroueil et al. 1990; Tavenas et al. 1983). In this study, the optimised $c_k = 1.03$, and the soil permeability coefficient ($k_o = 10^{-6}$ m/min) have been in good agreement with the typical values recommended in the literature. Figure 4.10 shows the relation between the coefficient of permeability and void ratio. The measured points of the SRC and LRC were obtained using the relation between $c_v$ and $m_v$, while the predicted relationship is based on the values of $k_o$ and $c_k$ obtained along with other EVP model parameters through the optimisation procedure. The coefficient of determination ($R^2$) for the correlation between the measured data and the prediction for the coefficient of permeability is about 0.861. The optimised values of $k_o$, $c_k$ and $e_o$ were used with other EVP model parameters to examine the ability of the proposed method for model parameter determination.

Table 4.2 Initial information adopted in the optimisation procedure to determine the model parameters for KBS mixture

<table>
<thead>
<tr>
<th>Loading Stage</th>
<th>$H_o$ (m)</th>
<th>$e_o$ (%)</th>
<th>$\sigma_d$ (kPa)</th>
<th>$\sigma_{sf} = \sigma_d + \Delta\sigma$(kPa)</th>
<th>$u_{el} = \Delta\sigma$ (kPa)</th>
<th>$t_{tot}$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>29.5</td>
<td>34.271</td>
<td>25</td>
<td>50</td>
<td>25</td>
<td>21</td>
</tr>
<tr>
<td>200</td>
<td>29.5</td>
<td>40.915</td>
<td>50</td>
<td>200</td>
<td>150</td>
<td>21</td>
</tr>
<tr>
<td>400</td>
<td>29.5</td>
<td>53.492</td>
<td>200</td>
<td>400</td>
<td>200</td>
<td>21</td>
</tr>
<tr>
<td>800</td>
<td>29.5</td>
<td>58.373</td>
<td>400</td>
<td>800</td>
<td>400</td>
<td>21</td>
</tr>
</tbody>
</table>
Table 4.3 The optimised model parameters and soil permeability properties

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Instant time-line</th>
<th>Reference time-line</th>
<th>Equivalent time-line</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa/V$</td>
<td>0.008</td>
<td>$\lambda/V$</td>
<td>0.1045</td>
</tr>
<tr>
<td>$\sigma_{zo}$ (kPa)</td>
<td>2.546</td>
<td>$t_e$ (min)</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon_{lp}$</td>
<td>$a = 0.1549$</td>
<td>$b = 1 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>$\psi_0/V$</td>
<td>$c = 0.068$</td>
<td>$d = 0.0098$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.9 Calculation grid for the simulation of SRC sample
The EVP model parameters are obtained using four loading stages in the range of normally consolidation in order to include the elastic ($\kappa/V$) and plastic ($\lambda/V$) properties along with the creep functions simultaneously. The instant time-line parameter (i.e. $\kappa/V$) is included due to the creep contribution during the dissipation of the excess pore water pressure and the apparent preconsolidation pressure. For time-dependent soils, preconsolidation pressure is influenced by different factors including loading history, creep times and strain rate (Graham et al. 1983; Leroueil et al. 1985; Yin & Graham 1994). According to Leroueil & Vaughan (1990), as a normally consolidated soil exhibits creeping under a constant effective stress (e.g. $\sigma'_{z1}$), the decrease in void ratio can result in an apparent increase in preconsolidation pressure when the soil is subject to another stress increment from $\sigma'_{z1}$ to $\sigma'_{z2}$. Therefore, the soil experiences overconsolidation due to creep at every stress increment. Consequently, the time-independent elastic strain defined by the instant time-line parameter occurs due to the apparent preconsolidation pressure. The time-independent elastic behaviour is included in the response of the soil under the dissipation of excess pore water pressure (i.e. $\frac{\kappa}{V\sigma'_{z2}} \left( \frac{\partial \sigma_{z2}}{\partial t} - \frac{\partial u_{z2}}{\partial t} \right)$ in Equation 3.28). Moreover, since the time value $t_0$ of the reference time-line is adopted as a unit of time (creep contribution is considered from the very early stage of loading), the creep contribution to the total compression is more significant. Thus, the influence of creep on the preconsolidation pressure is more
obvious. Therefore, applying the TRRLS approach, all EVP model parameters can be obtained altogether by utilising several loading stages in spite of the absent of the unloading-reloading data.

The fitting curves of the average vertical strains of the thin sample (SRC) are in good agreement with the measurement as shown in Figure 4.11(a). The coefficients of determination of the predictions and measurements for the vertical strains are greater than 0.9. Additionally, Figure 4.11(a) shows the prediction of the variation of the excess pore water pressure at the base of SRC with time. It is observed that the maximum increase of the excess pore water pressure at the base is less than the applied stress increment. The maximum increase of the excess pore water pressure of SRC is about 93% of the applied stress increment, while in some cases the value drops to 75%. Since the soil has high initial water content and was stored submerged in water for one week before testing, the degree of saturation may not explain some of the observed pore water pressure responses.

The issue may be due to the stiffness of the pore water pressure measurement device (Robinson 1999; Whitman et al. 1961). The stiffness of the pore water pressure measurement system may allow a partial drainage of pore water from the base of the soil sample. Charlie (2000) discussed that during the consolidation test, water from the soil can flow into or out the measurement system, resulting in the change of the drainage condition of the impervious base of the soil sample. Additionally, the predicted dissipation of the excess pore water pressures exhibits a higher rate compared to the measurements. The difference between the calculated and measured values of the excess pore water pressure may also be caused by the blocked porous plate at the base and top of the soil sample. As the particle sizes of kaoline, bentonite and fine sand in the mixture are relatively small, the soil particles may gradually block the pores of the filter papers and the porous plates, resulting in the delay in pore water pressure response.

In order to evaluate and validate the developed optimisation method and validity of the obtained model parameters, a larger scale Rowe cell test was also carried out for the same soil mixture by using a 250 mm diameter Rowe cell. The experimental preparation and testing procedure adopting the large Rowe cell is explained in the following section.
Figure 4.11 Predictions and measurements of the SRC sample (a) The average vertical strain and (b) The excess pore water pressure at the impervious base
4.3 Large Rowe cell experiment and verification exercise

4.3.1 Experimental setup and testing procedure

The large hydraulic consolidation Rowe cell (LRC) setup used in this study has the similar configuration of the small Rowe cell described in the previous section. The “large” and “small” terms adopted in this study are used to distinguish two different sizes of the testing setups. The LRC setup also includes the cell top, cell body and the cell base. The internal diameter of the cell is 250 mm, and the height of the cell is 200 mm. The cell top of LRC includes a large rubber diaphragm used to apply the cell pressure (vertical stress) on the soil sample. Due to the deeper cell body compared to the SRC, the LRC can be modified to have several pore water pressure measurement points at different depths of the soil sample. As shown in Figure 4.12, there is 1 pore water pressure measurement point around the cell body (B1), as well as 2 measurement points at the base (A1 and A2). B1 is at 160 mm depth from the top of the cell body, and A2 is 75 mm far from A1 located at the centre of the base.

Each measurement point was equipped with a brass porous plate, which was maintained fully saturated in an ultrasonic cleaner before the test. This modification is to investigate the variation of the excess pore water pressure with depth, as well as at a distance from the centre line of the soil sample. At each pore water pressure measurement point, two drainage control valves connected to the pore water pressure transducer are used for the initial de-airing process as well as controlling the drainage conditions if necessary. All the PWPTs were connected to data loggers to record the measurement logged by GDSLAB software. In this study, the cell base acted as the impervious boundary of the soil sample. The cell base and the cell top were firmly attached to the cell body flange by 9 clamping bolts. O-rings were placed between the cell base and cell body flange, as well as between the cell flange and the cell base for the sealing purpose during the test.

With 200 mm high cell body and considering the required space for the rubber diaphragm, the maximum achievable height of the soil sample is about 150 mm. A 3 mm thick brass porous plate was placed between the soil sample and the diaphragm to uniformly distribute the applied stress on the soil surface. The porous plate was kept submerged in water until the setup was fully assembled to ensure saturation. The test carried out using LRC was under the free strain loading condition and vertical one-way
drainage from the soil surface. Water drained out of the soil sample surface through the porous plate travelled back to the PVC connected to the back pressure line.

Figure 4.12 Illustration of the large hydraulic consolidation cell (LRC) apparatus

Figure 4.13 Large Rowe cell set up in the laboratory
In this LRC setup, two enterprise level pressure/volume controllers (PVC) were employed for each of cell and back pressure/volume sources, because the volume capacity of one PVC was insufficient to supply the required volume of water. Therefore, for each of cell and back pressure sources, two PVCs were connected parallel with an infinite volume controller (IVC) which was connected directly to an overhead tank of de-aired water. One PVC is actuated as the primary controller, and the other PVC is the secondary controller which is activated whenever the primary controller runs out of water. Figure 4.14 shows the connection system of the Large Rowe cell, and Figure 4.13 gives the idea of the most recent Large Rowe cell set up in UTS soil laboratory. The Rowe cell in Figure 4.13 has been modified to connect more measurement points around the wall and the base of the Rowe cell. The Rowe cell in this case study just had one measurement point on the wall, and two measurement points on the base.

When the primary controller is active, the secondary controller keeps seeking the same current pressure level as the primary controller. Thus, the secondary controller is always ready to be switched with the primary controller. While the secondary controller is in charge, the primary controller is refilled and pressurised to be ready to switch with the secondary controller. The primary and secondary controllers are automatically
controlled by the infinite volume controller (IVC). The IVC includes one input connected to the water reservoir, one output connected to the pressure supply line on the cell, and two inputs connected to the primary and secondary PVCs. The adopted IVC automatically switches the valves every 5 to 10 seconds. GDSLAB software is used to control the whole system and collect information from the data loggers during the test. It should be noted that the de-aired water was provided from the water tank which was connected directly to the IVCs for each pressure line.

Before starting the test, the Rowe cell was required to be fully saturated. Firstly, the PVC and IVC controllers were de-aired by emptying and filling at least three times using the overhead water tank. The de-aired PVC and IVC controllers were isolated from the cell before de-airing the water line and other pressure transducers. The valves directly connecting the water tank to the back pressure line were opened to let water flow through the pipeline to the PWP transducers. All the valves connected between the cell and the PWP transducers were closed, while the valves connected between the PWP transducers and the IVC back pressure line were opened. The air bleed pluges on the transducers were opened gradually to de-air each PWP transducer. At each PWP transducer, after de-airing the valve connected from the transducer to the cell body was opened to de-air the pipeline connected between them. Water then flowed through the porous plate attached at the PWP measurement point of that PWP transducer in order to flush air out of the pipe and check if the porous plate was blocked. Then, the valves connected at two sides of that PWP transducer were closed before de-airing the next PWP transducer.

After all PWP transducers were de-aired and fulfilled with water, a thin saturated filter paper was inserted to avoid the soil particles block in the porous plates. In addition, a thin grease layer was applied around the cell wall interior to minimise the wall friction. In this case study, because the soil material was in slurry state, the material was poured gradually into the cell to the design height. Then, the soil sample was covered by a thin filter paper, and the sample height was measured again. The rest of the cell body was filled with distilled water before inserting the top porous plate. The top porous plate as shown in Figure 4.12 was placed between the soil sample and the cell diaphragm, and was saturated before installation. Due to excess water overflowed from the cell during assembling, a hollow base was placed underneath the cell as shown in Figure 4.14 to store overflowed water from the cell. The air bleed plug on the cell top
and the rim drain valve on the cell body flange were opened during assembling the cell top to flush air out the system as well as to avoid applying excessive initial water pressure on the soil sample. The cell top was secured on the cell body by clamping bolts. Water at this time gradually filled the diaphragm, and flushed air trapped out through the air bleed plug on the cell top.

Table 4.4 Details of loading stages using LRC apparatus

<table>
<thead>
<tr>
<th>Loading stage (kPa)</th>
<th>(H_0) (mm)</th>
<th>Back pressure (kPa)</th>
<th>Initial cell pressure (kPa)</th>
<th>Final cell pressure (kPa)</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>140.5</td>
<td>90</td>
<td>100</td>
<td>115</td>
<td>14</td>
</tr>
<tr>
<td>50</td>
<td>140.5</td>
<td>90</td>
<td>115</td>
<td>140</td>
<td>39</td>
</tr>
<tr>
<td>100</td>
<td>140.5</td>
<td>90</td>
<td>140</td>
<td>190</td>
<td>28</td>
</tr>
<tr>
<td>200</td>
<td>140.5</td>
<td>90</td>
<td>190</td>
<td>290</td>
<td>46</td>
</tr>
<tr>
<td>400</td>
<td>140.5</td>
<td>90</td>
<td>290</td>
<td>490</td>
<td>46</td>
</tr>
</tbody>
</table>

Table 4.5 Initial input information for LRC simulation

<table>
<thead>
<tr>
<th>Loading stage (kPa)</th>
<th>(\varepsilon_i) (%)</th>
<th>(\sigma'_{id}) (kPa)</th>
<th>(\sigma'_{id} + \Delta \sigma_i) (kPa)</th>
<th>(u_{ei} = \Delta \sigma_i) (kPa)</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>27.76</td>
<td>25</td>
<td>50</td>
<td>25</td>
<td>39</td>
</tr>
<tr>
<td>100</td>
<td>40.93</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>28</td>
</tr>
<tr>
<td>200</td>
<td>46.84</td>
<td>100</td>
<td>200</td>
<td>100</td>
<td>46</td>
</tr>
<tr>
<td>400</td>
<td>54.26</td>
<td>200</td>
<td>400</td>
<td>200</td>
<td>46</td>
</tr>
</tbody>
</table>

In this laboratory experiment using LRC, the soil material is also kaoline-bentonite-fine sand (KBS) mixture with the identical soil properties to the soil sample tested in SRC. After assembling LRC, soil was preconsolidated under the vertical effective stress of 10kPa by maintaining the back and cell pressures equal to 50 kPa and 60 kPa, respectively. After the initial pre-consolidation stage, the B–check stage was operated to ensure the saturation degree. With the applied stress increment of 35 kPa, the B-values were calculated as the ratio of the increases of excess pore water pressures
to the stress increment at the PWP measurement points. The B-values at A1, A2, and B1 were 0.983, 0.963 and 0.965, respectively. Due to the high B-values, the saturation stage was not required. Five loading stages were conducted in series by increasing the cell pressure and maintaining the constant back pressure, as similar as the procedure conducted in SRC. The details of the loading stages are presented in Table 4.4. The effective stress started from 10 kPa, since the preconsolidation effective stress was 10 kPa during the assembling process. The loading durations were planned to be 14 days for the first loading stage (i.e. 10 kPa – 25 kPa), and 28 days for other loading stages. However, during the test, due to the several power shutdowns occurred during weekends, the test durations had been extended as shown in Table 4.4.

The results of settlement and the variation of excess pore water pressures with time are presented in the following section along with the numerical predictions adopting the model parameters obtained by the proposed optimisation method as reported in Table 4.5.

### 4.3.2 Experimental results and verification exercise

Figures 4.15, 4.16 and 4.17 show the laboratory test results as well as the numerical predictions adopting the optimised model parameters obtained from SRC sample (Table 4.3). The initial conditions for each stage are summarised in Table 4.5. The simulation is carried out using the finite difference solution explained previously. In this numerical analysis, in comparison to the applied loads, the overburden effective stresses induced by the self weight of the soil are neglected due to the insignificant thickness of the soil sample. Thus, the effective stresses are calculated based on the external applied stresses only. For the finite difference calculation, the thickness of the soil sample was divided into 5 sub–layers as shown in Figure 4.15, and the drainage condition is one–way vertical drain from the top surface. In Figure 4.16, the predictions of the average vertical strains of the LRC sample are presented and compared to the laboratory measurements. The vertical strain predictions are in good agreement with the laboratory measurement with the coefficient of determination ($R^2$) greater than 0.93, as shown in Figure 4.16.

Corresponding to the predictions of the vertical strains, the variation of excess pore water pressures with time is simulated for each loading stage. Figures 4.17 and 4.18 show the predictions of the excess pore water pressures (EPWP) at the base (A1 and A2) and on the side (B1) of the soil sample. While the measured EPWP at A1 and A2 are almost identical, the measured EPWP at B1 is lower than the EPWPs at the base.
The EPWP at B1 may dissipate quicker than the EPWP at the base, since B1 is closer to the drainage surface. Comparing between the predictions and measurements, the prediction of the EPWP at the base is compared to the measured EPWPs at A1 and A2. In the case of B1, since the PWP transducer B1 is at the soil depth of 100.5 mm, two prediction curves at the depths of 112.4 mm and 84.3 mm (i.e. at calculation nodes \( i = 4 \) and 5) are used to compare to the measurements. In general, the predictions of the excess pore water pressures at the base have good agreement with the measurements, while the predictions of the excess pore water pressure at different depths slightly overestimate the measurement at B1.

In general, the prediction of the vertical strains and the excess pore water pressure for the LRC sample are in good agreement with the laboratory measurements. For the loading stage from 25 kPa to 50 kPa, the predicted EPWP keeps increasing greater than the applied stress increment of 25 kPa. In order to explain the increase of EPWP during the dissipation process, Yin & Graham (1996) suggested that the viscous compression (so called creep) may contribute to the variation of EPWP in the early stages of the dissipation. Due to the thickness of the soil sample in the LRC, the dissipation process is delayed, and the effective stress close to the base stays approximately constant, resulting in the compression induced by creep. Compression induced by creep can induce excess pore water pressure, and as a result, creep contributes to continuous delay in the dissipation or even increase in the excess pore water pressure in the early stages. In other words, as the EPWP near the impervious base remains unchanged, and the pore water near the base is prevented to flow out, the vertical strain induced by the dissipation remains almost constant. Thus, relaxation under a constant vertical strain occurs, leading to the decrease of the effective stress and the increase of the EPWP. In short, there are dual effects of creep and relaxation happening simultaneously at locations near the impervious base.

After the loading stage from 25 kPa – 50 kPa, the predicted and measured EPWPs behave normally with the dissipation delayed for about 100 minutes after loading. As the EPWP dissipation delays, the corresponding compression (Figure 4.16) also maintains steady until the dissipation facilitates. The length of the delayed period of the EPWP dissipation may be due to the drainage path. Comparing the results obtained from SRC and LRC, the EPWP dissipation of SRC is delayed for a shorter time, since the SRC sample has shorter drainage path. After the delayed time, the
dissipation rates of the excess pore water pressures change with various rates before approaching zero. Corresponding to the excess pore water pressure dissipation pattern, the compression also accelerates along with the dissipation of the EPWP after the delayed time.

Figure 4.15 Calculation grid for the numerical simulation of the LRC sample

Figure 4.16 Prediction of the average vertical strain of the thick sample (LRC)
Figure 4.17 Excess pore water pressure of LRC during (a) 25 kPa – 50 kPa, and (b) 50 kPa – 100 kPa loading stages
Figure 4.18 Excess pore water pressure of LRC during (a) 100 kPa – 200 kPa and (b) 200 kPa – 400 kPa loading stages
4.4 Discussion

This section is dealing with observing and discussing the variations of the creep function parameters with time. Considering the creep strain limit ($\varepsilon^{\text{vp}}_{\text{lim}}$) and the creep coefficient ($\psi/\nu$), the changes of the creep parameter ($\psi/\nu$) and creep strain rate with time are calculated and discussed.

Based on the obtained model parameters, the time-line system including the instant time-line, reference time-line and the limit time-line is illustrated in Figure 4.19. In Figure 4.19, the measured data of the small Rowe cell (SRC) and the large Rowe cell (LRC) at the end of each loading stage are presented. Referring to Figure 4.19, the first loading stages of both samples exhibited significant compression due to the initial high water content of the soil samples. The slurry state of the soil material resulted in extreme settlement even under small applied stress increments (20 kPa for SRC and 15 kPa for LRC). The vertical strains at the end of the first loading stage of the SRC and LRC were 34.27% and 27.7%, respectively. However, after the first loading stage, the curves of stress – strain of SRC and LRC are approximately parallel with the reference time-line. Additionally, since the LRC sample experienced longer loading durations compared with the SRC sample, the compression curve of the LRC is close to the curve of the SRC sample. The longer creep time may lead the LRC sample experiencing the similar strain rate and the equivalent time at the end of the loading stages as the SRC sample.

As the KBS mixture did not exhibit any loading history, and due to its initial slurry state, the value of $\sigma'_{20}$ corresponding to $t_0$ of 1 minute for the soil was small. If $t_0$ is higher than 1 min (for example at the end of primary consolidation), the value of $\sigma'_{20}$ is expected to reduce to a lower value. As shown in Table 4.3, the value of $b$ parameter related to the creep strain limit function is insignificant. Therefore, the creep strain limit of the KBS soil is considered as constant and stress independent. As a result, the limit time-line is almost parallel to the reference time-line in this particular case. On the other hand, the obtained values of $c$ and $d$ parameters corresponding to the creep coefficient indicate that $\psi_0$ is stress dependent. The variations of the creep coefficient and the creep strain limit vary with time corresponding to the variations of the effective stresses during the EPWP dissipation. Figure 4.19 also shows that the slope of the instant time-line ($\kappa/\nu$) is approximately parallel to the slope of unloading–reloading. The optimised
As stated in Chapter 3, the non-linear creep function includes two stress and time dependent elements, which are the creep strain limit \( \varepsilon_{lm}^{vp} \) and creep coefficient \( \psi_{o}/V \).

Applying the numerical analysis using the Crank-Nicolson finite difference solution, \( \varepsilon_{lm}^{vp} \) and \( \psi_{o}/V \) can be calculated for each time step at each depth node. Thus, the variation of those parameters with time can be examined. Figures 4.20 and 4.21 depict the variations of the average creep strain limit (\( \varepsilon_{lm}^{vp} \)) and the average creep coefficient (\( \psi_{o}/V \)) with time for four loading stages of each soil sample (i.e. SRC and LRC). As mentioned earlier, the parameter \( b \) corresponding to the function of \( \varepsilon_{lm}^{vp} \) is insignificant. Thus, Figures 4.20(a) & (b) show that the variations of the creep strain limits with time for SRC and LRC are minor, even though it is also shown that there are decreases of the creep strain limits with time. Additionally, the creep strain limit decreases, while the vertical effective stress increases. For example, for both SRC and LRC, the loading stage of 50 kPa has higher the creep strain limits compared to other stress increments. The higher stress stage exhibits the lower value of the creep strain limit. Since the creep strain limit is stress dependent only, \( \varepsilon_{lm}^{vp} \) stays constant after the target effective stress is reached. Furthermore, at the same stress level, there is no difference between the thick

Figure 4.19 Time-line system for the KBS mixture
and thin soil samples. In this case, since the changes of the creep strain limit are unremarkable, the creep strain limit can be reasonably considered constant for the soil.

Meanwhile, the changes of the average creep coefficient ($\psi_o/V$) with time are apparent in Figures 4.21(a) & (b) for both SRC and LRC. It is noted that the initial and final values of the average creep strain limit $\varepsilon_{lm}^{vp}$ and the creep coefficient ($\psi_o/V$) for each loading stage are independent on the thickness of the soil sample. However, the variations of those parameters during the dissipation of the excess pore water pressure are influenced by the soil sample thickness, and the variation pattern is comparable to the variation pattern of the excess pore water pressure (Figure 4.11b, 4.17 and 4.18). For the SRC sample, the excess pore water pressure dissipation completes earlier in comparison to the LRC sample. Thus, when the remaining excess pore water pressure is insignificant, the creep strain limits and creep coefficients remain constant, as the effective stress stays nearly constant.

The non–linear creep function was proposed to avoid the infinity compression of the soil. Thus, in the non–linear creep function, the creep parameter ($\psi/V$), which is comparable with the secondary compression coefficient $C_{ae}$, is calculated based on the creep strain limit $\varepsilon_{lm}^{vp}$, the creep coefficient $\psi_o/V$ and the equivalent time $t_e$. (Equation 3.7 in Chapter 3). Although the creep coefficient ($\psi_o/V$) remains constant after the EPWP dissipation completes, the creep parameter ($\psi/V$) continuously decreases with time. The creep parameter ($\psi/V$) is comparable with the secondary compression coefficient $C_{ae}$. In Figure 4.22(b), the thicker soil sample in LRC has lower initial creep parameters ($\psi/V$) except for the loading 25 kPa – 50 kPa as compared with the values of the SRC sample in Figure 4.22(a). For the SRC sample, the average creep parameters ($\psi/V$) increase to particular values, and consequently decrease with time (Figure 4.22a). Exceptionally, the loading stage of 400 kPa – 800 kPa has a different pattern, as the creep parameter keeps decreasing at various rates with time. Meanwhile, the average creep parameter of the LRC sample decreases with time for the loading stage of 25 kPa – 50 kPa, while the creep parameters of other loading stages increase, then decrease as similar as those of the SRC sample.
Figure 4.20 Variation of the average creep strain limit $\varepsilon_{im}^{vp}$ with time of (a) SRC and (b) LRC
Figure 4.21 Variation of the average creep coefficient $\psi_{c/V}$ with time of (a) SRC and (b) LRC

Figures 4.23(a) & (b) also show the variations of the average creep strain rate $\dot{\varepsilon}_{cp}$ of both SRC and LRC samples with time. The curves of the average creep strain rate are plotted in logarithmic scale for both axes in order to clearly observe the variation...
pattern. The average creep strain rates of most of the loading stages keep decreasing with time, except that of the loading stage of 400 kPa – 800 kPa of the SRC sample. In order to explain the change of the creep strain rate with time, the average effective stress and the average vertical strain relation of the soil samples is shown in Figure 4.24 along with the corresponding creep strain rate in Figures 4.23(a) & (b). Considering the stress – strain relation of the SRC sample (the solid curves in Figure 4.24), during a loading stage, the vertical strains increase with the increase of the effective stresses, and the curves tend to initially develop toward the reference time-line, then continue parallel to the reference time-line. When almost reaching the target effective stresses, the curves bend and increase vertically toward the limit time-line (i.e. vertical strains induced by creep under a constant effective stress). The creep line would be more obvious, if longer time is allowed for creep. Based on the time-line concept, each equivalent time-line has a unique creep strain rate, and the higher equivalent time $t_e$ is associated to the smaller creep strain rate (Yin & Graham 1989).

Consequently, as the effective stress increases, the equivalent time-line associated to that effective stress moves toward the reference time-line, resulting in the increase in the creep strain rate and the decrease in the equivalent time $t_e$. When the dissipation of the excess pore water pressure is almost completed, and the maximum effective stress of the stress increment is almost established, the creep strain rate gradually decreases. After the effective stress reaches the maximum value, and the compression is allowed to continue under the constant effective stress, the equivalent time $t_e$ increases and the equivalent time-line moves toward the limit time-line. As a result, the associated creep strain rate decreases, which explain the variations of the corresponding creep strain rates in Figures 4.22(a) & (b). The stress-strain behaviour of the LRC sample is alike to the behaviour of the SRC sample. It is noted that during the early stage for the load 25 kPa – 50 kPa for the LRC sample the effective stress decreases corresponding to the increase of the excess pore water pressure at the base in the early stages of loading, which was reported in Figure 4.17(a). Consequently, after decreasing to 1.5 kPa, the effective stress increases again to 50 kPa, since the EPWP dissipation proceeds. As a consequent of the variation of the effective stress, the creep strain rate of this loading stage decreases with the decrease of the effective stress, then increases again with the effective stress, and finally drops gradually as the maximum effective stress is achieved.
Figure 4.22 Variation of the average creep parameter $\psi/V$ with time of (a) SRC and (b) LRC
Figure 4.23 Variation of the average creep strain rate $\dot{\varepsilon}_z^{op}$ with time of (a) SRC and (b) LRC
4.5 Summary

The non-linear elastic visco-plastic (EVP) model can be coupled with the theory of one dimensional consolidation in order to improve the prediction of time dependent stress – strain behaviour of soft soils. In order to avoid difficulty of defining the time parameter \( t_o \), this parameter is adopted as a unit of time, while the stress and time dependent creep strain limits and creep coefficient are incorporated. Crank–Nicolson finite difference solution has been applied to solve the coupled equations of the EVP model and the consolidation theory. Moreover, a model parameter determination method is introduced to define several model parameters simultaneously. The developed method is based on the trust–region reflective least square algorithm in association with the finite difference solution. In order to validate the competence of the proposed method, Rowe cell test results from two soil samples with different thickness have been analysed.

The purpose of Rowe cells in this study is to increase the thickness of the soil samples. While the standard oedometer test cell can only accommodate the maximum sample thickness of 20 mm, the maximum soil sample thickness to be tested in the small Rowe cell in this study is 30 mm. Thanks to the large Rowe cell, the maximum soil sample thickness is 150 mm. In addition, the vertical displacement and excess pore water pressure can be continuously monitored using the computer system. The
measurement of the excess pore water pressure was used to evaluate the accuracy of the proposed method. The soil sample is the mixture of kaoline, bentonite and fine sand (KBS) with high water content prepared in the laboratory conditions. In this validation exercise, the proposed method has been employed to determine a set of model parameters obtained from the consolidation test results on the thin KBS soil sample (29.5 mm thick) using the experimental measurement of a multiple stage loading test. The obtained model parameters result in good agreement between the settlement prediction and laboratory measurements. Regarding the excess pore water pressure, the prediction is reasonably agreed with the laboratory measurement. Then, the settlement and the variation of excess pore water pressure of the 140.5 mm thick sample have been predicted using the finite difference solution and compare with the laboratory results obtained from a large Rowe cell. The findings of this study indicates that the thickness of the soil deposit can have a considerable influence on the long term settlement pattern as well as the response of the excess pore water pressure. It means that the thicker the soil sample, the more delayed time the dissipation of excess pore water pressure at the base endures. In other words, the more delayed compression takes place during the period of excess pore water pressure dissipation for the thicker specimen. Adopting the non–linear creep function in the EVP model, the creep strain limit \( \varepsilon_{im}^{vp} \), creep coefficient \( \psi_0/V \), and consequently the creep strain rate vary with the effective stresses and time. The vertical strains tend to approach the strain limit as the elapsed time continues toward infinity as a consequence of the non-linear creep strain function. Generally, the non–linear creep function can analyse the stress-strain behaviour more realistically, and the proposed method can be a practical tool to enhance the application of the non–linear EVP model. Moreover, the developed procedure can be applied to back calculate the elastic visco–plastic soil parameters utilising the field measurements for deep soft soil layers, while significant excess pore water pressure is remaining.
5.1 General

This chapter further examines the effectiveness of the proposed approach for the EVP model parameter determination. In Chapter 4, a verification exercise has been provided by simulating the laboratory experiments with two soil samples of different thicknesses conducted at UTS soil laboratory. In order to enhance the efficacy of the proposed approach, this chapter provides two validation exercises for two soft soils including Hong Kong marine clay and Drammen clay of Norway available from literature. The case study of Hong Kong marine clay is to compare the predictions using the parameters obtained by the conventional method and the proposed method. The procedure to obtain the conventional EVP model parameters for Hong Kong marine clay was well described in Yin (1999). Therefore, the conventional EVP model parameters are adopted to compare with the EVP model parameters optimised by the proposed method. The long–term settlements and the variation of the excess pore water pressures are predicted and compared for five different soil layers with thicknesses varying from 0.02 m to 5.12 m using two sets of the EVP model parameters. The second case study is to simulate the well-known oedometer tests of Berre & Iversen (1972) for different thickness soil samples of Drammen clay. The proposed method is applied to obtain the EVP model parameters and the prediction of settlements and the excess pore water pressures for four soil samples are provided and discussed. Additionally, this chapter also provides the improved understanding of the creep strain limit, the creep coefficient, the creep parameter as well as the creep strain rates with time and with the thickness of the soil samples. As a result, the elastic visco–plastic model parameters obtained by the proposed approach can be utilised for a reliable analysis for soft soils,
capturing the influence of the soil layer thickness as well as the time–dependent behaviours.

5.2 Case study 1: Hong Kong marine clay, Hong Kong

5.2.1 Soil properties

Yin (1999) presented a set of long term oedometer test results of Hong Kong marine deposit and proposed a systematic curve fitting procedure to obtain the nonlinear creep function parameters which is referred to as the conventional method. The soil material (a mixture of clay, silt and fine sand) was sieved and reconstituted to obtain consistent samples. As reported in Yin (1999), the specific gravity of the soil \( G_s \) was 2.66, and the initial water content \( w_o \), the plastic limit and liquid limit were 57.4%, 28.5% and 60.0%, respectively. Thus, the initial void ratio \( e_o = w_o G_s \) was calculated to be 1.53, assuming that the soil sample was fully saturated. Then, the soil sample was subject to multiple stage loading tests using Casagrande type oedometer setup. The applied stress started from 10 kPa, then increased to 400 kPa by the stress increment ratio \( \Delta \sigma \) of 1. Later, the soil was unloaded back to 50 kPa and reloaded to 800 kPa with the same stress increment ratio. The duration of each loading stage was about 1 day, except for 200 kPa and 800 kPa stages. The testing durations for 200 kPa and 800 kPa stages were about 33 days and 18 days, respectively. The experimental loading stages including loading, unloading and reloading are presented in Figure 5.1.

5.2.2 Numerical simulation

Adopting the trust–region reflective least squares (TRRLS) algorithm for the parameter determination described in detail in Chapter 3, four stages of effective stresses 100 kPa, 200 kPa, 400 kPa and 800 kPa were employed to determine all model parameters. Similar to Yin (1999), the above mentioned four loading stages are all in normally consolidated range to be used for creep model parameter determination. In this case study, the above four loading stages are used to determine not only the creep model parameters, but also the elastic \( \kappa / V \) and elastic – plastic \( \lambda / V \) and \( \sigma'_{zo} \) parameters. The required model parameters include six parameters of the EVP model, presented by the vector \( x \) as below:

\[
x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9] = [a, b, c, d, \lambda / V, \sigma'_{zo}, \kappa / V k_o c_k]
\] (5.1)

The adopted initial data of the four stages including the initial thickness \( H_o \), the initial vertical strain \( \varepsilon_{zi} \), the initial and final total stresses \( \sigma_{zi} \) and \( \sigma_{zf} \), the initial
excess pore water pressure \((u_e)\), and total loading time \((t_{tot})\) obtained from the laboratory measurements from Yin (1999) are summarised in Table 5.1. Considering the oedometer testing procedure, it can be assumed that the effect of external applied stress is immediate; thus, the initial total stress \(\sigma_f\) at \(t = 0\) is taken as the total final stress \(\sigma_f\). The drainage condition applied in this simulation is two–way drainage from the top and bottom pervious surfaces.

Table 5.1 Initial information adopted in the optimisation procedure to determine the model parameters for Hong Kong marine clay

<table>
<thead>
<tr>
<th>Loading Stage</th>
<th>(H_o) (m)</th>
<th>(\varepsilon_i) (%)</th>
<th>(\sigma_i) (kPa)</th>
<th>(\sigma_f = \frac{\sigma_i + \Delta \sigma_c}{(kPa)})</th>
<th>(u_e = \Delta \sigma_c) (kPa)</th>
<th>(t_{tot}) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100kPa</td>
<td>0.02</td>
<td>8.50</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>2936</td>
</tr>
<tr>
<td>200kPa</td>
<td>0.02</td>
<td>13.75</td>
<td>100</td>
<td>200</td>
<td>100</td>
<td>51248</td>
</tr>
<tr>
<td>400kPa</td>
<td>0.02</td>
<td>19.60</td>
<td>200</td>
<td>400</td>
<td>200</td>
<td>1505</td>
</tr>
<tr>
<td>800kPa</td>
<td>0.02</td>
<td>25.25</td>
<td>400</td>
<td>800</td>
<td>400</td>
<td>23754</td>
</tr>
</tbody>
</table>

The initial coefficient of permeability \((k_o)\) and the permeability change index \((c_k)\) were estimated using conventional approaches adopting consolidation test results \((k = c_v m_r Y_w)\). In Figure 5.2, the coefficients of permeability for the loading stages from 10 kPa to 400 kPa and the last stage of 800 kPa were obtained based on \(c_v\) and \(m_r\) calculated using the oedometer compression results. Considering that the permeability is a physical property of the soil, and for a particular soil and saturating liquid is solely a function of the void ratio (e.g. Carman 1956; Taylor 1948), a unique set of the permeability coefficient \((k_o)\) and the permeability change index \((c_k)\) can be determined. The best fitting curve in Figure 5.2 gives \(k_o = 3.812\times10^{-8}\) m/min and \(c_k = 0.61\) with \(e_o = 1.53\). The soil permeability property is adopted in both of the conventional approach and the proposed trust–region reflective least squares (TRRLS) approach. It should be noted that \(k_o\) and \(c_k\) can be measured directly using falling head permeability tests for clay soil samples consolidated using different effective stresses.
**Figure 5.1** Experimental loading stages of the Hong Kong marine clay (after Yin 1999)

(a) loading stages, (b) unloading and (c) reloading
Based on the initial information and soil properties, the model parameters obtained by the proposed method are presented in Table 5.2. In addition, the model parameters, obtained using Yin (1999) conventional approach, are also reported in Table 5.2. The slope of the instant time-line ($N/V$) obtained by the proposed method is lower than the conventional $N/V$ obtained by curve fitting the slope of the unloading-reloading stages. Since the unloading-reloading stages can include creep components which had been assumed to be insignificant (Yin 1990), the slope $N/V$ obtained by the conventional approach includes by creep. Consequently, the conventional $N/V$ is higher than the optimised $N/V$ which in the finite different solution is only included in the compression due to the change of the excess pore water pressure. In consideration of the reference time-line’s parameters, the optimised $\lambda/V$ and $\sigma'_{zo}$ are slightly higher than the values obtained by the conventional approach. The value of $\lambda/V$ was the slope of the end of primary consolidation compression (Yin 1999), and $\sigma'_{zo}$ is the corresponding effective stress to $\varepsilon_{zo} = 0$ in the space of $\varepsilon_e - \ln(\sigma')$. As explained in Chapter 3, the value of $t_o$ has significant influence on other EVP model parameters. The smaller $t_o$ causes a higher value of $\sigma'_{zo}$. Therefore, the reference time-line, which is drawn by the optimised parameters, shifts to the right of the reference time-line by the model parameters obtained by the conventional method.
Table 5.2 Summary of the adopted model parameters for Hong Kong marine clay

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Proposed TRRLS method</th>
<th>Yin (1999) approach (Conventional approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instant time-line</td>
<td>$\kappa/V$</td>
<td>0.0069</td>
</tr>
<tr>
<td>Reference time-line</td>
<td>$\lambda/V$</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>$\sigma'_{zo}$ (kPa)</td>
<td>30.883</td>
</tr>
<tr>
<td>$t_v$ (min)</td>
<td>1</td>
<td>49</td>
</tr>
<tr>
<td>Equivalent time-line</td>
<td>$\varepsilon_{in}^{vp}$</td>
<td>$a$ 0.0797</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$b$ 0.0074</td>
</tr>
<tr>
<td></td>
<td>$\psi_{zo}/V$</td>
<td>$c$ 0.0185</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d$ 0.0018</td>
</tr>
<tr>
<td>Soil permeability parameters</td>
<td>$e_o$</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>$k_o$ (m/min)</td>
<td>$3.812 \times 10^{-8}$</td>
</tr>
<tr>
<td></td>
<td>$c_k$</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Additionally, the optimised parameters for the non-linear creep function are higher than the parameters by the conventional method. It is reasonable since the parameters have been optimised using the data during the dissipation of the excess pore water pressures. The creep compression during the consolidation contributes to the value of the creep strain limit and the creep coefficients.

In order to evaluate the developed finite difference solutions incorporating the proposed parameter determination procedure, the time dependent vertical strains of four stress stages are predicted for both sets of parameters (i.e. the proposed TRRLS method and the conventional approach) and compared to laboratory measurements in Figure 5.3. It is observed that the reported coefficients of determination ($R^2$) are higher than 0.9 for both sets of parameters, thus both parameter determination methods are efficient to match the laboratory results. However, the predictions using the optimised EVP model parameters have higher $R^2$ compared with the predictions using the model parameters obtained from the conventional method. In Figure 5.3, adopting the optimised EVP model parameters, the predictions of the compression during the dissipation process of
the excess pore water pressure have better agreement with the laboratory measurements. Meanwhile, the model parameters from the conventional approach underestimate the compressions during the dissipation.

**Figure 5.3** Predicted time dependent vertical strain under (a) 100 kPa and 200 kPa, (b) 400 kPa and 800 kPa vertical stresses using the parameters by the conventional method and the optimised parameters using the proposed approach.
In general, the settlement predictions adopting the set of soil parameters obtained by the proposed approach are in a good agreement with the laboratory measurement. In the next step, the optimised model parameters using the proposed approach are used to predict the average strain and the excess pore water pressure for thicker soil deposits. The five different thicknesses ($H_o$) adopted are 0.02 m, 0.08 m, 0.32 m, 1.28 m and 5.12 m. The external applied stress increment is 50 kPa to increase the vertical effective stress from 50 kPa to 100 kPa. In this analysis, the soil layers are assumed uniform, and the clay layers of different thicknesses start with the same initial vertical strain and stress conditions. The soil deposits are assumed to be free drained at the top surface and impervious at the bottom. The Crank–Nicolson finite difference solution for the coupled consolidation equations as explained in Chapter 3 is adopted to analyse the soil surface settlement (or average strain) and the excess pore water pressure response at the base of the soil deposit. The analysis is carried out for four different thicknesses, and the results are compared with the laboratory specimen thickness of 0.02 m. The durations of loading time is 10 years for the specimens of the first three thicknesses, while the loading times for the 1.28 m and 5.12 m thick soil deposits are 50 years and 100 years in order to observe the reverse S shaped curve of the average strain with logarithm of time. The results and discussion are provided in the following section.

5.2.3 Results and discussion

Figure 5.4(a) shows the prediction of the average strain with time, while Figure 5.4(b) indicates the time variation of the excess pore water pressure at the impervious boundary of the soil layers. In this study, the influence of the soil layer thicknesses on the time depended settlement and the variation of the excess pore water pressure can be observed and discussed. As observed in Figure 5.4(a), the strain curves of five specimens are almost parallel during the dissipation process of the excess pore water pressure, while the strain curves shift to the right as the sample thickness increases. However, the average strain curves of thicker specimens (i.e. 0.08 m, 0.32 m, 1.28 m to 5.12 m) tends to merge to the settlement curve of the thinnest specimen (i.e. 0.02 m), when the excess pore water pressure at the base approaches zero. As denoted by Jamiolkowski et al. (1985), compression induced by creep is accumulated as the thickness increases, and the settlement curves of thicker soil layers will merge to the settlement curve of the thin specimen after a period of loading time.
Meanwhile, Figure 5.4(b) shows that the excess pore water pressure at the impervious base is influenced by the soil layer thickness. As the thickness of the soil layer increases, the curves of the excess pore water pressure at the base are shifted to the right with similar patterns. The dissipation of the excess pore water pressure varies with different rates during the consolidation process before approaching zero. Referring to Figure 5.4(b), the dissipation of the excess pore water pressure at the impervious base delays before proceeding with various rates, until approaching an almost constant value. It is noted that the amount of delayed time increases with the soil deposit thickness. As the drainage path for water flowing toward the drainage boundary is expected to increase with an increase in the deposit thickness, the required time for water travelling through the drainage path also increases. The delayed time observed in Figure 5.4(b) for the soil specimens of 0.02 m, 0.08 m, 0.32 m, 1.28 m and 5.12 m are approximately 1 min, 20 min, 200 min, 8000 min, and 10000 min, respectively. During the delayed time, the compression presented by the average strain also tends to be delayed as observed in Figure 5.4(a). After the delayed time, the dissipation rate of the excess pore water pressure at the base starts to increase with various rates before approaching zero. Corresponding to the excess pore water pressure dissipation pattern, the average vertical strain also increases significantly. When the dissipation process is almost completed, the compression continues mainly due to viscous behaviour of soil under an almost constant effective stress with a creep strain rate, reducing with time.

Figures 5.4(a) & (b) illustrate the time dependent stress – strain relationship known as Hypothesis B. The suppositions of Hypotheses A and B have been explained and discussed in Chapter 2. The main difference between Hypotheses A and B is associated with the effect of the sample thickness on the void ratio (strain) at the end of traditional primary consolidation stage. Hypothesis B (adopted in this study) believes that the void ratio (or strain) at the end of primary consolidation is influenced by the soil layer thickness or the drainage conditions, while Hypothesis A assumes the end of primary consolidation void ratio is unique and not a function of soil thickness.

Figure 5.5 shows the influence of the soil thickness on the average vertical strain, when 90%, 95% and 98% of the excess pore water pressures at the base of the soil specimens are dissipated. In the semi-logarithmic scale of Figure 5.5, the curves of the average strain at different degrees of dissipation are almost parallel, and the average strain at a particular degree of dissipation increases with the increase of the soil
thicknesses. For example, according to Figure 5.5, the prediction applying the optimised model parameters shows that at 90% dissipation the average strain increases by 3.8%, 6.3%, 8.1% and 9.5% for 0.08m, 0.32 m, 1.28 m and 5.12 m thick samples in comparison to 0.02 m thick sample, respectively. A similar trend of variation of the average strain with the increase of soil layer thickness is observed at 95% and 98% dissipation and also by the predictions adopting parameters obtained from the conventional approach. It is also noted that the difference between the average strains is more significant as the soil specimen thickness increases from 0.02 m to 0.08 m. As can be seen in Figure 5.5, adopting the model parameters obtained from the proposed approach, the differences of the average strain at 90%, 95%, and 98% dissipation between the 0.02 m and 0.08 m thick soil layers are 3.8%, 3.4% and 2.9%. The model parameters by the conventional approach estimate the vertical strains the end of primary consolidation higher than the predictions obtained by the proposed method. Thus, the differences of the vertical strains between the 0.02 m and 0.08 m thick soil layers by the conventional approach are 5.8%, 4.9% and 4% at 90%, 95% and 98% dissipations, respectively.

Figures 5.6(a) & (b) depicts the variations of the average creep coefficient \( \psi \) using the optimised model parameters and by the model parameters by the conventional method, while Figures 5.6(a) & (b) describes the variation of the average creep strain limit \( \epsilon_{\text{vp}} \) with time by the two approaches. As inferred from Figures 5.6(a) & (b) and 5.7(a) & (b), \( \psi \) and \( \epsilon_{\text{vp}} \) are not constant, and change during the excess pore water pressure dissipation process. The patterns of these two parameters (i.e. \( \epsilon_{\text{vp}} \) and \( \psi \)) with time are similar to the variation pattern of excess pore water pressure in Figure 5.4(b), as \( \epsilon_{\text{vp}} \) and \( \psi \) are directly related to the effective stress or indirectly to the excess pore water pressure. The average creep strain limit as well as the average creep coefficient decrease with the increase of the effective stress. Thus, during the dissipation of the excess pore water pressure; the effective stress keeps increasing, resulting in decreasing \( \epsilon_{\text{vp}} \) and \( \psi \). As soon as the dissipation completes, these model parameters reach the constant values, since the effective stress remains constant after increasing to the maximum value. The creep coefficient \( \psi \) remains constant, when the vertical effective stress is constant.

Comparing Figures 5.6 and 5.7 the average creep coefficients \( \psi \) obtained by the proposed method is higher (up to 35.82 %) than the values by the conventional
approach. Meanwhile, the average creep strain limit by the proposed method is up to 38.25% higher than the corresponding values by the conventional approach. It is due to the adoption of $t_o$ value and the involvement of the consolidation data in the optimisation procedure. The amount of creep strain limit by the conventional approach has been lessened, since $t_o$ was adopted as the time at the end of primary consolidation. With a smaller $t_o$ value, the proposed approach has captured the amount of creep compression during the dissipation of the excess pore water pressure. Thus, the resulted creep strain limits are higher.

In the non–linear creep function, the creep parameter ($\psi/V$), which is comparable with the conventional coefficient of secondary compression $C_{at}$, is expressed in terms of the creep strain limit and the creep coefficient as in Equation (3.7). In this case study, applying Equation (3.7), the average creep parameters varying with time for five different thickness layers are calculated and shown in Figures 5.7(a) & (b) using the model parameters obtained by the proposed approach and the conventional method.

Although both $e_{tm}^{pp}$ and $\psi/V$ decrease with the increase of the effective stress, the bell shaped pattern of the creep parameter ($\psi/V$) (Figures 5.7(a) & (b)) shows that the creep parameter ($\psi/V$) is not only effective stress dependent, but also time dependent. As the dissipation of the excess pore water pressure accelerates after the delayed time, the creep parameter ($\psi/V$) significantly increases. Once reaching the maximum value, the creep parameter ($\psi/V$) decreases gradually with time. Additionally, compared the values obtained by the proposed optimisation method and the conventional method, the proposed method also gives a lower creep parameter ($\psi/V$). The bell shaped curves are observed more clearly for the predictions by the proposed method, while the creep parameters by the conventional method increase insignificantly, especially for the soil layers higher 0.032 m.
Figure 5.4 Predictions of (a) the average strain of various thickness soil layers and (b) the excess pore water pressure at the base of soil layers adopting model parameters obtained from the proposed approach and the conventional approach.
Figure 5.5 Influence of soil thickness on the average vertical strain at 90%, 95% and 98% dissipation of excess pore water pressure at the impervious boundary.
Figure 5.6 Average creep coefficient ($\psi_o/V$) predicted by (a) the proposed approach and (b) the conventional approach.
Figure 5.7 Average creep strain limit ($\varepsilon_{im}^{vp}$) predicted by (a) the proposed approach and (b) the conventional approach.
Figure 5.8 also shows the influence of the soil layer thickness on the average creep parameter ($\psi/V$). The thicker soil layer tends to have a lower creep parameter. Comparing between the thinnest and thickest soil layers, the maximum $\psi/V$ of the 0.02 m thick layer are about 2.24 (Figure 5.8a) and 2.85 (Figure 5.8b) times higher than the maximum $\psi/V$ of the 5.12 m thick layer predicted by the proposed method and the conventional method, respectively. At the peak value of $\psi/V$, the corresponding values of $\psi/V$ and $\epsilon_{im}^{pp}$ tend to decrease against the increase of the soil layer according to Figures 5.6 and 5.7. Additionally, the value of the excess pore water pressure at the base of the soil layers corresponding to the peak value of $\psi/V$ also increases with the soil layer thickness (Figure 5.4b). Thus, the average remaining excess pore water pressure of the thin soil layer is expected to be lower than the corresponding value of the thick soil layer, and the average effective stress of the thin soil layer is consequently higher than the value of the thick soil layer.

The corresponding stress – strain points at the peak value of $\psi/V$ of the thin soil layer as the 0.02 m thick soil sample stays closer to the reference time-line. As the thickness of the soil layer increases, the distance from the stress – strain state points at the peak value of $\psi/V$ to the reference time-line increases. Based on the stress – strain point at the peak value of $\psi/V$, the thick soil layer has a higher value of the equivalent time $t_e$ compared to the thinner soil layer. As a combination of several influencing factors including $\psi/V$, $\epsilon_{im}^{pp}$, and $t_e$, the peak value of $\psi/V$ tends to decrease with the increase of the soil layer thickness. This trend of the influence of the soil layer thickness on the maximum value of $\psi/V$ is observed in both of the conventional method and the proposed method. Additionally, as observed in Figure 5.8, the curves tend to merge to one curve, when the effective stress approaches the target value. The trend pattern is more obvious for the thin soil layers of 0.02 m to 1.28 m. Since the thickest soil layer of 5.12 m has not established its target effective stress yet, the mergence of the curve cannot be clearly observed.
Figures 5.9(a) & (b) observes the variation of the average creep strain rate ($\dot{\epsilon}_{\text{avg}}$) with time. The creep strain rate is estimated based on Equation (3.5) in Chapter 3. In the finite difference solution for the coupled EVP model and the consolidation theory, the creep strain rate is also represented by the function $g(\epsilon, \sigma')$ (Equation 3.30c). The variation of the average creep strain rate with time is plotted in the full logarithmic space in order to observe the trend clearly. In comparison between the predictions of two methods, the disparities of the creep strain rates of the two methods are insignificant. The average creep strain rates are predicted to decrease with time with
different rates by both of the methods. Figures 5.9(a) & (b) also shows the influence of the soil layer thicknesses on the creep strain rate. As the thickness of the soil layer increases, the average creep strain rate decreases. As the thinnest soil layer, the 0.02 m thick soil layer has the average creep strain rate higher than the values of thicker soil samples as observed at the same loading time up to 400 min. The 0.08 m thick soil sample has the same initial creep strain rate as the 0.02 m thick sample for 2 min. It should be noted that during the dissipation of the excess pore water pressure, the average creep strain rates of five soil samples vary parallel to each other with time. Gradually, the average creep rates of five soil samples merge to the same curve as similarly observed in the variations of the average creep parameters in Figure 5.8.

Based on the time-line concept, each equivalent time-line has a unique creep strain rate, and the higher equivalent time $t_e$ is associated to the smaller creep strain rate (Yin & Graham 1994). The thicker soil sample requires higher equivalent time to reach the same vertical strain of the thinner soil sample. Thus, the strain rate of the thicker soil sample is smaller than the thinner soil sample at the same strain. Figures 5.10(a) & (b) can illustrate more obviously the progress of the vertical strain with the vertical stress under a stress increment.

Figures 5.10(a) & (b) show the relationship between the vertical stress-vertical strain during the stress increment from 50 kPa to 100 kPa predicted by the proposed method and the conventional method. The thinnest soil sample (0.02 m thick) shows an increase in the effective stress earlier compared to other soil samples. Therefore, the compression curve of the 0.02 m thick soil sample moves to the right faster than other soil thicknesses. Consequently, the equivalent times of the thinnest soil sample decrease more quickly, and resulting in the higher creep strain rates compared to other soil thicknesses. As the dissipation of excess pore water pressure almost completes, the effective stress nearly approaches the target value, and the compression curve proceeds under almost a constant effective stress. Thus, the compression curve moves downwards, with the increase in the equivalent time, resulting in reduction in the creep strain rate. Meanwhile, the behaviour of the thicker soil samples tends to be influenced by the soil thicknesses. The dissipation of the excess pore water pressure has been delayed for some time as shown in Figure 5.4(b). The thicker soil layer has lower creep parameters, and consequently a slower creep strain rate. Although the thicker soil layer has a lower creep strain rate, the required time to achieve insignificant excess pore
water pressure (known as the end of primary consolidation) is much higher than the corresponding required time for the thinner soil layer. As a result, the compression of the thicker soil layer at the end of the conventional primary consolidation (i.e. when the effective stress reaches an almost constant value) is greater than the compression of the thinner soil layer.

Figure 5.9 Average creep strain rate ($\dot{\varepsilon}_c^{pp}$) predicted by (a) the proposed approach and (b) the conventional approach
Figure 5.10 Vertical stress – strain relationship predicted by (a) the proposed approach and (b) the conventional approach
Moreover, Figures 5.11, 5.12 and 5.13 show the isochrones of the variation of excess pore water pressure with depth and time for five specimens with different thicknesses predicted by the optimised parameters and by the parameters by the conventional approach, respectively. The vertical axis is the normalised depth \( z/H_0 \), and the horizontal axis is the normalised excess pore water pressure \( u_e/\Delta \sigma \). According to Figure 5.11(a), after 1 minute, the excess pore water pressures at the bases of five soil layers are not dissipated yet. However, the excess pore water pressure at the upper half of the thinnest soil sample (0.02 m thick) started dissipating, while the excess pore water pressures at locations near the soil surface of other soil samples especially the thickest soil sample (5.12 m) delayed to dissipate.

According to Figure 5.11(b), after 60 minutes, the excess pore water pressure dissipation at the base of 0.02 m thick layer dissipated about 50% predicted. The dissipations of the excess pore water pressure at upper locations near the drained surface completed more than 60%. Meanwhile, the dissipation rates of thicker specimens are less than the dissipation rate of the thinnest (0.02 m thick) specimen. The dissipations of the excess pore water pressures at the base of the specimens thicker than 0.02 m have been delayed by well more than 60 minute (Figure 5.11b). For example after 1 day (Figure 5.12a), the excess pore water pressure at the base of the 1.28 m thick specimen starts dissipating, while the bottom half of the 5.12 m thick specimen exhibits delay of dissipation. On the other hand, after 1 day the thinner soil sample such as the 0.02 m and 0.08 m thick samples has almost been completed the dissipation process of the excess pore water pressure.

After one year (Figure 5.12b), the excess pore water pressures at the upper half of the 5.12 m thick layer proceed to dissipate, while the 1.28m thick sample completed about 60% dissipation, and other specimens settle under a constant effective stress, (or the excess pore water pressure \( \approx 0 \) kPa). The dissipation of the excess pore water pressure at the base of the thickest soil layer (5.12 m thick) delayed more than 2 years as shown in Figure 5.13(a), and after 10 years (Figure 5.13b), the 5.12 m thick soil layer exhibits about 64% and 72% considering the proposed TRRLS and the conventional approaches, respectively. As shown in Figure 5.4(b), the dissipation of the excess pore water pressure at the base of the 5.12 m thick sample required nearly 100 years to complete.
Figure 5.11 Isochrones of the variation of the normalised excess pore water pressure with depth after (a) 1 min, and (b) 1 hour
Figure 5.12 Isochrones of the variation of the normalised excess pore water pressure with depth after (a) 1 day, and (b) 1 year.
Figure 5.13 Isochrones of the variation of the normalised excess pore water pressure with depth after (a) 2 years, and (b) 10 years
Both the proposed TRRLS approach and the conventional method for the model parameter determination can predict the soft soil creep behaviour with a similar pattern, and capture the influence of the thickness of the soil layers on the dissipation process. However, some differences observed between the predictions obtained by two sets of model parameters may be associated with the model parameters. In other words, the selected value of $t_o$ influences the predictions. It can be noted that the proposed TRRLS approach has employed more data points including the points during the dissipation process to determine the model parameters compared to the conventional approach. Thus, the prediction of the compression and the excess pore water pressure during the dissipation process adopting TRRLS approach can be more accurate.

In brief, adopting the Crank-Nicolson finite difference solution and the model parameters obtained from TRRLS approach fitting the laboratory consolidation data can be used to predict reasonable patterns of the time dependent compression as well as the dissipation of the excess pore water pressure in soft soil deposits. Additionally, in Case Study 1, the compression (vertical strain) of the soil deposit for a given excess pore water pressure dissipation level increases with the increase of the soil thickness. As expected, the dissipation of the excess pore water pressure is delayed, as the thickness of the soil layer increases. In order to further evaluate the proposed approach adopting the proposed approach to obtain the EVP model parameters, Case Study 2 about Drammen clay is presented to predict the time dependent behaviour of the soft soil samples with different thicknesses.

5.3 Case study 2: Drammen clay, Norway

5.3.1 Soil properties

Berre & Iversen (1972) carried out a series of consolidation tests on Drammen clay specimens with four different thicknesses using modified oedometer apparatus recording the variation of the excess pore water pressure at the base with time. Drammen clay (Norwegian soft clay) was slightly overconsolidated, marine post-glacial clay obtained from depths of 5.2 m to 6.7 m in Drammen, Norway. The natural water contents were in range of 57% - 60%. The plastic and liquid limits were about 28% - 34% and 54% - 60%, respectively. The soil had approximately 45% content of particles less than 2μm. The heights of soil specimens tested were 0.0188 m, 0.0757 m, 0.15 m, and 0.45 m. The thickest specimen (0.45 m) was comprised of three 0.15 m high sub-
specimens connected in series with free drainage at the top of the topmost specimen, and no drainage at the base of the lowest specimen. The pore water pressure was measured at the base of each specimen. Multiple stage loading tests with five stress increments were applied to all specimens. The duration of the first three stages was 1 day, while the last two increments (Increments 4 and 5) had longer duration (5 days for each increment). Berre & Iversen (1972) presented the measurements of the vertical strain and the excess pore water pressure at the base of each specimen at two stress increments (Increments 4 and 5) as shown in Figure 5.14.

![Stress-strain curves of oedometer soil sample of different heights](image)

**Figure 5.14 Stress – strain curves of oedometer soil sample of different heights (after Berre & Iversen 1972)**

### 5.3.2 Numerical simulation

In this study, the test results from the last two increments (Increments 4 and 5) (see Table 5.3) for 0.0188 m thick specimen are used to determine the model parameters. Adopting the optimised model parameters, the average strains and the excess pore water pressures at the base of the thicker soil specimens are predicted to compare with the experimental measurements. The details of initial input information for the predictions are presented in Table 5.3. The initial void ratio $e_o$ of each soil specimen is derived from the initial water content $w_o$ assuming that soil specimens are fully saturated. According to Andersen et al. (1980), the specific gravity of Drammen clay is 2.76. The initial
strains summarised in Table 5.3 were obtained from the laboratory observations reported by Berre & Iversen (1972). It should be noted that the conventional approach proposed by Yin (1999) cannot be readily used to predict the non-linear creep strain rate and limit functions in this study adopting only two stress increments. However, Yin & Graham (1996) applied their original EVP model with linear creep function (constant creep parameter) for these two stress increments. Figure 5.15 shows the relationship between the void ratio and permeability based on the measured values of permeability and water content reported in Berre & Iversen (1972). As a result, the permeability change index $c_k$ obtained as the slope of a straight line in the space of $e - \log_{10}(k)$ is 0.859, while the initial permeability is $k_o = 1.73 \times 10^{-7}$ m/min.

<table>
<thead>
<tr>
<th>Test</th>
<th>Increment</th>
<th>$H_o$ (m)</th>
<th>$\varepsilon_i$ (%)</th>
<th>$\sigma'_d$ (kPa)</th>
<th>$\sigma'_d + \Delta\sigma_i$ (kPa)</th>
<th>$\nu_i$ (kPa)</th>
<th>$\varepsilon_o = 2.76w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>0.0188</td>
<td>2.33</td>
<td>55.3</td>
<td>92.5</td>
<td>37.2</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.0188</td>
<td>6.01</td>
<td>92.5</td>
<td>140.2</td>
<td>47.7</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>0.0757</td>
<td>1.19</td>
<td>55.9</td>
<td>93.3</td>
<td>37.4</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.0575</td>
<td>5.54</td>
<td>93.3</td>
<td>140.5</td>
<td>47.7</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>0.150</td>
<td>1.09</td>
<td>55.2</td>
<td>92.5</td>
<td>37.3</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.150</td>
<td>4.46</td>
<td>92.5</td>
<td>140.2</td>
<td>47.7</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$H_o/3$</td>
<td>1.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.45</td>
<td>2/3$H_o$</td>
<td>1.47</td>
<td>53.4</td>
<td>89.2</td>
<td>35.8</td>
</tr>
<tr>
<td></td>
<td>$H_o$</td>
<td>1.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_o/3$</td>
<td>6.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.45</td>
<td>2/3$H_o$</td>
<td>4.73</td>
<td>89.2</td>
<td>134.7</td>
<td>45.5</td>
</tr>
<tr>
<td></td>
<td>$H_o$</td>
<td>5.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The model parameters obtained by applying the proposed approach are presented in Table 5.4. The slope of the reference time-line ($\lambda/V$) obtained by the proposed approach is close to the reported value by Yin & Graham (1994) using the original EVP model. The slope of the instant time-line ($\kappa/V$) is about 10 times less than $\lambda/V$, which is higher than the value obtained in the original model. With $t_0 = 1$ min, the value of $\sigma'_{zo}$ is about 101.7 kPa, larger than the $\sigma'_{zo}$ of the original model (i.e. 79.2 kPa). The predictions for the thicker soil samples are provided and discussed in the following section.

Applying the optimised parameters, the predictions of the average vertical strains and the excess pore water pressure for Test A sample are illustrated in Figure 5.16. As the compression data of sample A was used for the model parameter optimisation, the measured data of Test A are fitted well by the predictions adopted the optimised EVP model parameters. The coefficients of determinations $R^2$ for the settlement predictions in Figure 5.16(a) are higher 0.95, showing the good agreement between the predicted and measured data. Additionally, the predictions of the excess pore water pressure in Figure 5.15(b) are also fairly close to the laboratory measurement.
As a result, the EVP model parameters in Table 5.4 are used to simulate the compression behaviour of three thicker samples of the same soils. The initial stress – strain conditions of Tests B, C, D samples are presented in Table 5.3. The simulation results and discussions are provided in the following section.

**Table 5.4 Model parameters for Drammen clay obtained adopting the proposed TRRLS approach**

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>The optimised parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instant time-line</td>
<td>$\kappa/V$ 0.0158</td>
</tr>
<tr>
<td>Reference time-line</td>
<td>$\lambda/V$ 0.17 101.71</td>
</tr>
<tr>
<td>$\sigma_{zo}'$ (kPa)</td>
<td>1</td>
</tr>
<tr>
<td>$t_o$ (min)</td>
<td>$a$ 0.6</td>
</tr>
<tr>
<td>Equivalent time-line</td>
<td>$b$ 1×10^{-6}</td>
</tr>
<tr>
<td>$\varepsilon_{im}$</td>
<td>$c$ 0.0134</td>
</tr>
<tr>
<td>$\psi_r/V$</td>
<td>$d$ 8.78×10^{-4}</td>
</tr>
</tbody>
</table>

### 5.3.3 Results and discussions

Figures 5.17–5.20 present the predictions of average vertical strain and the excess pore water pressure at the base of the four specimens. For the thickest specimen (0.45 m), the excess pore water pressures at three locations are presented in Figures 5.20(a) & (b). In general, the predictions for three specimens with different thicknesses are in good agreement with the laboratory measurements. As reported in Berre & Iversen (1972), the stress – strain states shift from overconsolidated to normally consolidated range during Increment 4 for all soil samples from Test A to Test D as shown in Figure 5.14. Additionally, the total number of data points in both increments of Test A sample used for the model parameter determination is 33 points including the points during the dissipation process of the excess pore water pressure. Thus, the proposed approach can employ effectively the consolidation data to determine the model parameters. The optimised parameters using Test A’s experimental data can predict the compression and the excess pore water pressures for other test samples accurately.
Figure 5.16 Comparison of the prediction and the measured data for Test A (0.0188 m): (a) the average vertical strain, and (b) the excess pore water pressure at the base for Drammen clay
As observed in Figures 5.17–5.20, the numerical predictions of excess pore water pressures and the average vertical strains are in good agreement with the laboratory measurements. The variation patterns of the excess pore water pressure are clearly obtainable to be influenced by the thickness of the soil specimen. Based on Figure 5.16(b) for the thinnest specimen (0.0188 m), the dissipation of the excess pore water pressure at the base occurs very rapidly in both increments (less than 10 minutes). As observed in Figures 5.17(b), 5.18(b), and 5.20 (a) & (b), for the thicker soil specimens, the dissipation process is inclined to delay some times before hastily accelerating, and the time delay increases with the thickness. It can be noted that a similar behaviour was observed in the predictions of excess pore water pressure in Case Study 1 (Figure 5.3b). It is noted that the initial measured values of excess pore water pressures reported in Berre and Iversen (1972) were lower than the applied stress increment for saturated soil, causing the disparities between the predicted and measured values in early stages as shown in Figures 5.16(b), 5.17(b), 5.18(b) and 5.20(a) & (b). The phenomenon that the excess pore water pressure in a consolidation test reaches its maximum value which is usually less than the applied stress increment, has been observed and discussed in several studies (Whitman et al. 1961, Crawford 1964, Sonpal and Katti 1973 and Robinson 1999). The stiffness of the pore water pressure measurement device was considered as the reason causing the difference between the maximum pore water pressure and the applied stress increment. The stiffness of the pore water pressure measurement system may allow a partial drainage of pore water from the base of the soil sample (Robinson 1999; Sonpal & Katti 1973; Whitman et al. 1961). Charlie (2000) discussed that during the consolidation test the water from the soil can flow into or out the measurement system, resulting in the change of the drainage condition of the impervious base of the soil sample.

Corresponding to the variation of the excess pore water pressure with time, the proposed finite difference solution adopting model parameters determined by TRRLS is able to predict the average strain of four tests with high coefficient of determination ($R^2$) in comparison to laboratory measurements as shown in Figures 5.16(a), 5.17(a), 5.18(a) and 5.19. The prediction of Increment 4 of Test C has lowest $R^2$ in comparison to the results of other tests. The finite difference solution combined with the proposed optimisation method overestimates the settlements of both increments for Test C. The
specimen of Test C is the thickest individual sample compared to the specimen of Test A and Test B, while the sample in Test D was made of three individual specimens connected in series. According to Berre and Iversen (1972), the effect of side friction of the sample of Test C was notable and reported about 5.4 – 10.1% for Increment 4 and 8.9 – 5.1% for Increment 5, which were measured at the beginning and at the end of the increment respectively. The side friction was considered relatively small, and not measured at all tests. Thus, the measured results were not corrected for side friction (Berre and Iversen 1972). Although the side friction suggested was not remarkable, the predictions may be influenced for Test C with highest side friction.

A remarkable influence of the soil layer thickness on the stress–strain–time relationship of soft soils is the increase of the excess pore water pressure during the dissipation process. In this case study, the increase of the excess pore water pressure during the dissipation is obviously observed in the prediction of the thickest sample (Test D) during Increment 5 (as referred Figures 5.20(a) & (b)). The measurements of the excess pore water pressures at the base of Test D sample in Increments 4 and 5 show the increase of the excess pore water pressure during the dissipation process, particularly during the delayed period and before the dissipation accelerates (Figures 5.20(a) & (b)). The prediction for Test D also shows the increase of the excess pore pressures during the dissipation in Increment 5. The contribution of viscous effect is suggested as the reason of this variation of the excess pore water pressure (Yin & Graham 1996; Yin & Zhu 1999). The increase of the soil layer thickness leads to the delay in both the compression and the dissipation of the excess pore water pressures near the impervious base. The delay in dissipation of the excess pore water pressure results in the delay in the increase of effective stress, consequently the compression induced by creep at constant effective stress. On the other hand, the creep compression reduces the drainage path of the void water, resulting in void water near the impervious base locally blocked to flow out. At those locations where void water is blocked, vertical strains remain almost unchanged, leading to relaxation effect. During relaxation, the strain keeps constant, and the effective stress decreases. Thus, the excess pore water pressure increases (Yin & Graham 1996). In short, there are dual effects including creep and relaxation occurring at location near the undrained base of the thick sample causing the increase of the excess pore water pressure during the dissipation. However, the increase of the excess pore water pressure is suggested to occur when the

5-32
effective stress sufficiently exceeds the previous preconsolidation pressure (Yin & Graham 1996).

Figure 5.17 Comparison of the prediction and the measured data for Test B (0.075 m): (a) the average vertical strain and (b) the excess pore water pressure at the base for Drammen clay
Figure 5.18 Comparison of the prediction and the measured data for Test C (0.150 m): (a) the average vertical strain and (b) the excess pore water pressure at the base for Drammen clay.
Figure 5.19 Comparison of the prediction average vertical strain (%), and the measured data for Test D (0.450 m)
Figure 5.20 Comparison of the prediction and the measured data for Test D (0.450 m): (a) the excess pore water pressures of Increment 4, and (b) the excess pore water pressures of Increment 5 for Drammen clay
In order to analyse the abnormal increase of the excess pore water pressure during the dissipation process, Figures 5.21 and 5.26 shows the isochrones of the normalised excess pore water pressures and vertical strains with depth after some periods such as 1 minute, 1 hour and 1 day of four different thickness samples of Test A to Test D. After 1 minute in Figure 5.20, the isochrones of the normalised excess pore pressures show the delay in the excess pore water pressure dissipation at locations near the impervious base of Tests B, C, and D samples. The delay in dissipation is observed in both Increments 4 and 5. After 1 hour (60 minutes) (Figure 6.21) the excess pore water pressure of the top half of Test D sample started to dissipate, while the bottom half near the impervious base of the sample maintained unchanged. It should be noted that the excess pore water pressure at the base of Test D sample (i.e. the base of D3) tends to increase, and reaches the maximum value after 200 minutes as detected in Figure 5.19(b). On the other hand, at the same time (i.e. 200 minutes) the excess pore water pressures of the thinnest sample (Test A) have reached the end of the dissipation process for both stress increments. Tests B and C have achieved the maximum degrees of dissipation of approximately 80% and 20% respectively for Increment 4, and about 50% and 10% respectively for Increment 5 after 200 minutes. After 1 day, Test D sample has proceeded up to 50% and 10% dissipation of the excess pore water pressures at the base of the sample during Increments 4 and 5 respectively. It can be seen that the thickness of the soil sample has influence on the dissipation rate. The thicker soil sample takes longer time to reach the same dissipation amount compared to the thinner soil sample.

Figures 5.25 to 5.27 present the corresponding variation of the vertical strains of four different thickness samples with depth at some particular times (1 minute, 1 hour, and 1 day). At the free drainage boundary, the vertical strains are much higher than at deeper locations, since the excess pore water pressures near the top surface dissipate more quickly. While the vertical strains of the thinner samples increase with depth and time, the vertical strains of the half depth near the impervious base of the thickest sample (Test D) remains almost unchanged after 60 minutes in Figure 5.25. As explained before, viscous effect results in the relaxation locally. Thus, the vertical strains are almost constant for some times, while the corresponding excess pore water pressures increase.
Figure 5.21 Isochrones of the normalised excess pore water pressure with depth of four different thickness soil samples of Drammen clay after 1 minute for (a) Increment 4 and (b) Increment 5
Figure 5.22 Isochrones of the variation of the excess pore water pressure with depth of four different thickness soil samples of Drammen clay during after 60 minutes for (a) Increment 4 and (b) Increment 5.
Figure 5.23 Isochrones of the variation of the excess pore water pressure with depth of four different thickness soil samples of Drammen clay during after 1440 minutes (1day) for (a) Increment 4 and (b) Increment 5.
Figure 5.24 Isochrones of the variation of the vertical strain with depth of four different thickness soil samples of Drammen clay during after 1 minute for (a) Increment 4 and (b) Increment 5.
Figure 5.25 Isochrones of the variation of the vertical strain with depth of four different thickness soil samples of Drammen clay during after 60 minutes for (a) Increment 4 and (b) Increment 5
Figure 5.26 Isochrones of the variation of the vertical strain with depth of four different thickness soil samples of Drammen clay after 1440 minutes (1 day) for (a) Increment 4 and (b) Increment 5.
Figure 5.27 reports the average creep coefficient \( \psi_o/V \) of Increments 4 and 5 of four samples of Tests A to D. As shown in Figure 5.27, the initial values of \( \psi_o/V \) of four samples are different, since the initial effective stress of four tests are slightly different. The patterns of the variation of \( \psi_o/V \) with time are similar to the patterns of \( \psi_o/V \) in Case Study 1 of Hong Kong marine clay. Since \( \psi_o/V \) decreases against the increase of the effective stress, \( \psi_o/V \) also decreases with time during the dissipation of the excess pore water pressure, and remains almost constant after the target effective stresses are reached. The thicker sample has a higher value of \( \psi_o/V \) at any point of time, since the effective stress increases much slower than the thinner samples. For example, for Increment 4 in Figure 5.27(a), \( \psi_o/V \) of the 0.02 m thick sample starts decreasing and keeps constant after 10 minutes, while \( \psi_o/V \) of the thicker soil samples take longer time to reach a constant value. The samples in Test B, C, and D as observed in Figures 5.17(b), 5.18(b), and 5.20(a)-(b) have not reached the end of the excess pore water pressure dissipation yet after 10 minutes, and their \( \psi_o/V \) continue to decrease. Additionally, \( \psi_o/V \) decreases as the effective stress increases; thus, \( \psi_o/V \) of Increment 4 is higher than \( \psi_o/V \) value of Increment 5. For Drammen clay, the value of the coefficient \( b \) of the relationship equation of the creep strain limit \( \varepsilon_{im}^{vp} \) with the effective stress (i.e. Equation 3.8) is negligibly small compared to the coefficient \( a \) reported in Table 5.4. Therefore, the creep strain limit \( \varepsilon_{im}^{vp} \) can be considered as constant for the reported case, since its variation with the effective stress is insignificant.

The variations of \( \psi_o/V \) and \( \varepsilon_{im}^{vp} \) with time result in the variation of the creep parameter \( \psi/V \). Figure 5.28 illustrates the change of the average creep parameters with time. The average creep parameters of four Drammen clay specimens tend to increase to the maximum value, and then start to decrease. The maximum values of \( \psi/V \) are observed to decrease against the increase of the soil sample thicknesses. For example, during Increment 4, for Test A sample (0.0188m thick), the maximum \( \psi/V \) increases 9.12% compared with the initial \( \psi/V \), while the relative changes of \( \psi/V \) from the initial values to the maximum values of Tests B, C, and D samples are 8.73%, 8.28% and 7.41%, respectively. During Increment 5, the increases of \( \psi/V \) from the initial values to the maximum values of Tests A, B, C, and D samples are 5.10%, 3.43%, 1.45% and 1.52% respectively. After increasing to the maximum values, the average creep parameters \( \psi/V \) tend to decrease with time, since \( \psi/V \) is not only stress dependent but
also time dependent. Recalling the patterns of $\psi/V$ of Hong Kong marine soil samples in Case Study 1, the similar pattern is observed in the case of the thinnest soil sample (i.e. 0.02 m thick), while $\psi/V$ of the thicker soil layers decrease with time with various rates.

Figure 5.29 shows the variation of the average creep strain rate of Increment 4 and Increment 5 for four different thickness soil samples in the full logarithmic space in order to denote the influence of the soil sample thickness. During Increment 4 (Figure 5.29a), the thickest soil sample (Test D) has the lowest average creep strain rate, and the average creep strain rate increases with the increases of the soil specimen thickness. It is also denoted that the average creep strain rates seem to maintain constant during the early stages of the stress increment. It also means that creep contributes insignificantly during the early period of Increment 4, which may be because Increment 4 is the transition from the overconsolidation range to the normal consolidation range as shown in Figure 5.14. Therefore, during the early times of the stress increment (Increment 4), the compression induced by the elastic behaviour is more dominant, compared with the compression induced by creep. Gradually, after the effective stresses increase and pass the preconsolidation pressure, the creep contribution becomes more remarkable.

Based on the variation of the average creep strain, the development of the compression curve (or stress – strain curve) can be explained. As the effective stress increases from the initial effective stress to the final effective stress, the creep strain rate increases. If the initial stress – strain state is above the limit time-line, the compression under the applied stress increment is due to a combination of elastic visco–plastic behaviour. Therefore, as the effective stress increases during the dissipation of the excess pore water pressure, the stress – strain state moves further away from the limit time-line and gets closer to the reference time-line. Consequently, the equivalent time $t_e$ decreases, and thus, the creep strain rate increases. As the effective stress becomes stabilised and almost constant, the compression develops under a constant effective stress, and tends to increase toward the limit time-line. Thus, the equivalent time $t_e$ increases, and the creep strain rate decreases. The creep strain rate tends to decrease with time until reaching zero at time of infinity. As the effective stresses are in the range of the normal consolidation, the average creep strain rates during Increment 5 of the four specimens decrease with time at different rates. The thickest soil sample (Test D) has the lowest average creep strain rate, while the thinnest sample (Test A) has the highest value. Therefore, similar to the creep strain limit, the creep coefficient and the
creep parameter, the average creep strain rate is influenced by the thickness of the soil sample.

Moreover, Figure 5.30 shows the coefficient of permeability $k$ at the impervious boundary of four tests varying with time subjected to loading Increments 4 and 5. The pattern of permeability variation with time at the base is correlated to the pattern of the variation of excess pore water pressure. The permeability coefficient at the base of thick samples in Tests B, C, and D stays constant during the period of the dissipation process. Soon after, the permeability decreases as the dissipation process progresses. In the case of 0.0188m thick sample, the permeability at the base reduces slightly during the dissipation of the excess pore water pressure, while the permeability decreases more significantly during the compression under a constant effective stress. Thus, it can be concluded that the viscous (creep) compression can significantly change the void ratio and consequently permeability of the soil. According to Figure 5.30(a) & (b), as the thickness of the sample increases, the reduction in the permeability with time is greater. It should be noted that as shown in Figure 5.30(b), the initial permeability of Test D of Increment 5 is lower than the respective value of Test C, because the average initial void ratio of Test D of Increment 5 is higher than the respective value for Test C.

By and large, the predictions in Case Study 2 show the feasibility of the finite difference solution adopting the proposed TRRLS approach to obtain the EVP model parameters to predict the time dependent compression and the excess pore water pressure. It should be noted that in the proposed approach the model parameters especially the creep strain limit ($\varepsilon_{\text{cr}}^\text{vp}$) and the creep coefficient ($\psi_\epsilon/V$) can be estimated with two stress increments within the normally consolidation range. The lesser number of available loading stages may limit the accuracy of the optimisation. On the other hand, the conventional approach requires at least three creep tests with long loading time to predict the non-linear relationship between $\varepsilon_{\text{cr}}^\text{vp}$ and $\psi_\epsilon/V$ with effective stresses (Yin 1999). By adopting the proposed TRRLS approach for the model parameter determination, the model parameters can be obtained by employing the data points during the pore pressure dissipation. Moreover, the variations of the excess pore water pressure with time can be predicted with good agreement with the measurements.
Figure 5.27 Variation of the average creep coefficient $\psi_o/V$ of (a) Increment 4 and (b) Increment 5
Figure 5.28 Variations of the average creep parameters ($\psi/V$) of (a) Increment 4 and (b) Increment 5 of Drammen clay soil samples
Figure 5.29 Variation of the average creep strain rate $\dot{\varepsilon}_z^{\text{av}}$ with time during (a) Increment 4 and (b) Increment 5.
Figure 5.30 Variation of the coefficient of permeability at the impervious base of the soil specimens during (a) Increment 4 and (b) Increment 5

5.4 Summary
In order to assess the competence of the finite difference solution and the proposed optimisation approach, two laboratory–based case studies have been analysed in this chapter. In Case Study 1, the proposed approach has been employed to determine a set of model parameters of Hong Kong marine clay adopting the experimental measurement of a multiple stage loading test. The model parameters, obtained by the proposed approach result in good agreement between the predictions and laboratory
measurements. Then, the settlement and the variation of excess pore water pressures of five soil specimens of different thicknesses have been calculated using the finite difference solution. It is observed that the thickness of the soil deposit has significant influence on the long term settlement pattern as well as the response of the excess pore water pressure. The thicker soil layer, the more delayed time the dissipation of excess pore water pressure at the base endures. That also means the more delayed compression takes place during the period of excess pore water pressure dissipation for the thicker specimens. Based on the predicted average strain of different soil sample thicknesses, the average vertical strain, when the excess pore water pressure dissipation process is almost completed, increases with an increase of the soil layer thickness.

Case study 2 revisits the well-known laboratory tests of different thickness samples of Drammen clay. The proposed approach has been employed for validation of the developed method. The proposed approach has been applied to obtain the model parameters of Drammen clay from the thinnest soil specimen, and the finite difference solution has been used to predict the settlement and excess pore water pressure of the thicker specimens. The calculated results of the settlement are in good agreement with the measurement, while the predictions of the excess pore water pressures are reasonable. Despite the discrepancies of initial values of excess pore water pressure, the variation patterns of the excess pore water pressure are well predicted. It can be concluded that the proposed approach allows the data collected during the excess pore water pressure dissipation to be utilised to determine the EVP model parameters capturing the elastic, plastic and viscous behaviours. The unit value of $r_o$ can be adopted to avoid the arbitrary selection of this model parameter. The settlement data obtained during the traditional primary consolidation stage (while the excess pore water pressure is being dissipated) can be utilised to obtain the visco–plastic model parameters. It should be noted that the capacity of the proposed approach combined with the finite difference solution can be verified further, while the variation of the initial void ratio and the initial effective stresses with depth are considered for thick in situ clay layers.
6.1 Introduction
Chapters 4 and 5 have provided three case studies to verify the usefulness of the proposed method. However, the simulation exercises were carried out for the laboratory experiments. The thicknesses of the soil samples of the kaolinite mixture in Chapter 3 and Drammen clay in Chapter 4 were not significant enough to attest the ability of the proposed approach for the model parameter determination. Therefore, Chapter 6 provides two case studies of the field measurements to confirm the validity of the proposed method. Two case studies include the Väsby and the Skå–Edephy test fills both in Sweden. The laboratory oedometer tests for the subsoil properties and the field measurements including the settlements and the excess pore water pressures have been provided in detail in literature. In this chapter, the elastic visco–plastic (EVP) model parameters are obtained by the proposed approach adopting available consolidation test results for both case studies. Then, the predictions of the settlements and the excess pore water pressures are compared with the long–term field observations. This chapter focuses on further verification, practicality and accuracy of the proposed method in conjunction with the non–linear EVP model to analyse the long-term field performances of embankments built on soft soil deposits.

6.2 Case study 1: Väsby test fill

6.2.1 Project and site description
In 1945, the Swedish Geotechnical Institute (SGI) carried out the field-loading test to study the long-term behaviour of Swedish clays in order to select a suitable site to develop a new airport close to Stockholm, Sweden. The Väsby site and the Halmsjön site were two sites to be considered. The Väsby site was located at a ground of Lilla
Mellosa that was 6.0 km northeast of the village of Upplands, Väsby, and about 30 km north of Stockholm on the east coast of Sweden (Chang 1969). Figure 6.1 shows the locations of the Väsby test fill at Lilla Mällösa as well as the Skå–Edeby test fills in Case study 2. The Väsby site was sometime called the Lilla Mellosa site in some contexts such as Larsson & Mattsson (2003). The Väsby test fills have been monitored in terms of the settlements and the changes of the excess pore water pressure for several decades since these test fills were constructed in 1945–1947. Chang (1969) compiled the earlier available data of the site to analyse the long-term settlement and pore pressures of the ground. After ten years, Chang (1981) updated the report again with the accumulated settlements, the new pore water pressure measurements and other laboratory studies. Later, Larsson & Mattsson (2003) provided a new report on the field observations of the test fill at Lilla Mellosa.

![Figure 6.1 Location of the test fields at Lilla Mällösa and Skå–Edeby (courtesy of Google Map 2014)](image)

There were three test fills constructed at the Väsby site. One test fill was constructed in 1946 consisting of paper drains, and two test fills without paper drains were constructed in 1947 and 1948 as shown in Figure 6.2. This case study is to analyse

6-2
the long-term behaviour of the test fill of 2.5 m height without the paper drains constructed in October 1947 (i.e. grey filled square in Figure 6.2).

The test fill analysed in this case study had the height of 2.5 m, the bottom dimensions of 30 m×30 m and slopes of 1(V):1.5(H). The test fill of gravel with a density of 1700 kg/m³ was constructed in 25 days. Based on the geometry of the fill, the applied stress at the ground surface was 40.6 kPa (Larrsson & Mattsson 2003).

In order to monitor the settlements and the pore water pressures, several settlement markers and piezometers were installed at different depth below the ground before the fills were placed. The settlement measurement markers included two different types for ground surface settlement and sub-ground settlement (Chang 1969). The settlement marker used to monitor the ground surface settlement consisted of a square steel plate with a vertical rod welded perpendicularly to the plate. The marker was placed directly on the ground surface below the fill. On the other hand, the settlements at different depths under the ground were monitored by screw type settlement markers. This marker consisted of a vertical steel rod welded to an earth screw at an end, which was screwed down to the desired depth. The steel rod was protected by a flexible sleeve from soil hanging on the rod during the settlement. The

Figure 6.2 Test fill locations at Väsby, Sweden (after Chang 1981)
sleeve was covered by an outer steel pipe which was withdrawn after the marker reached the desired depth (Chang 1969). Several markers were installed at different depths as shown in Figure 6.3.

According to Larsson & Mattsson (2003), the readings of piezometers were unreliable, while the original settlement markers malfunctioned with time, and were replaced with new instrumentations several times. The settlement distribution with depths was compared against the settlements calculated based on the measured changes in water content. The measurements of water contents under the ground were made in 1967 and 2002 (Larrson & Mattsson 2003). Meanwhile, the new piezometers were installed in 1968 and malfunctioned. Therefore, the pore pressures then were measured by retractable piezometers installed in 1979 and 2002 (Larrson & Mattsson 2003).

6.2.2 Subsoil profile and soil properties
The Väsby test fill site was located on a flat ground about 7 m above the sea level. The subsoil profile is shown in Figure 6.3. Before the test fill was constructed, the 0.3m of loose organic topsoil was removed to the elevation 6.85 m (Chang 1969). A thin dry crust layer consisting of organic soil covers the top 0.5 m of the profile, and is underlain with soft clay. The 14 m deep subsoils comprise of the upper portions of post glacial clays (Layers 1, 2, and 3) and the lower portion of glacial clays (Layer 4). Under the glacial clay layer, a very thin layer of medium grey sand covers the bedrock surface (Chang 1969). The colours of the post glacial sub-layers vary from green to black due to the increase of iron–sulphides at the depth from 2.5 m to 6.5 m, and become grey with depth. The varves occur with the glacial clay layers, and become significant with depth (Larrson & Mattsson 2003).

The top layer of clay under the dry crust has about 5% organic content, and the values decrease with depth to less than 2% from the depth of approximately 6.5 m downwards. The natural water content varies from 130% at the top layers to 70% at the bottom layers. While the plasticity index decreases from 87% at the top layers at 34% to the bottom layers, the liquidity index increases from about 0.8 to 1.4. The bulk unit weight of the soil also increases with depth from 12.75 kN/m³ at the top to 17.66 kN/m³ at the bottom. The undrained shear strength also increases with depth as shown in Figure 6.4.

According to Larsson and Mattson (2003), the groundwater level outside the test fill was monitored periodically throughout the testing years. The mean water level is
about 0.8 m below the ground surface, and experiences little seasonal variations, while
the pore water pressures outside the test fill remain hydrostatic.

6.2.3 Numerical simulation and analysis

6.2.3.1 Model parameter determination
Chang (1969) performed intensive laboratory investigation to obtain the soil properties
including Atterberg limits, water content, soil compressibility, sensitivity and organic
content. Conventional incremental laboratory oedometer tests were carried out on the
samples taken in 1956 and 1967 both inside and outside the loaded areas. The soil
samples had the dimensions of 0.02 height and 0.0499 mm diameter. Some of test
results were corrected considering the ring friction. The applied load increments
\((\Delta \sigma / \sigma_{ci})\) were 0.5 and 1.0, which were applied after 24 hours and at the end of primary
consolidation to investigate the preconsolidation pressure profiles of the subsoil layers.
In this case study, an experimental result of a conventional incremental oedometer test
is adopted to predict the elastic visco–plastic model parameters. The sample was taken
at the depth of 5 m below the natural ground. According to Figure 6.4, the sample is
black clay with sulphides and organic colloids. Since there was lack of detailed
information of the tested sample in Chang (1969), the soil properties are adopted as
those reported in Figure 6.4. The water content is approximately 115%, and \(G_s = 2.63\).
The plasticity and liquidity index are 76% and 0.95, respectively, and the bulk unit
weight is about 13.4 kN/m\(^3\). The compression curves with time are plotted in Figure 6.5,
which are adopted in this case study for the model parameter determination.
Figure 6.3 Subsoil profile of the Väsby test fill (after Chang 1969)
<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Description</th>
<th>Water content (w) (%)</th>
<th>PI(%)</th>
<th>LI</th>
<th>Gs</th>
<th>S_u (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Brown grey clay with organic odour</td>
<td></td>
<td>87</td>
<td>0.8</td>
<td>2.63</td>
<td>8.98</td>
</tr>
<tr>
<td>2</td>
<td>Black clay with sulfides and organic colloids</td>
<td></td>
<td>85</td>
<td>1</td>
<td>7.34</td>
<td>7.34</td>
</tr>
<tr>
<td>3</td>
<td>Grey uniform clay</td>
<td></td>
<td>82</td>
<td>1.2</td>
<td>5.41</td>
<td>5.41</td>
</tr>
<tr>
<td>4</td>
<td>Grey clay with diffused varves</td>
<td></td>
<td>79</td>
<td></td>
<td>2.7</td>
<td>5.20</td>
</tr>
<tr>
<td>5</td>
<td>Grey clay with distinct varves sand</td>
<td></td>
<td>72</td>
<td></td>
<td></td>
<td>5.60</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td>47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>34</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \( w_P \) – Plastic limit; \( w_L \) – Liquid limit; PI – Plasticity index; LI – Liquidity index; \( G_s \) – Specific gravity of soil particles; \( S_u \) – Undrained shear strength

Figure 6.4 Soil properties of the Väshy subsoil profile (after Chang 1969)
Similar to the case studies in Chapters 4 and 5, the stress stages within the normally consolidated range are employed for the model parameter determination. As observed in Figure 6.5, the last four stages (vertical stresses changing from 45 kPa to 105 kPa) were used for the analysis. The strain response was monitored over a range of time, with a particular focus on the effective stress-strain relationship.

**Figure 6.5 Incremental oedometer test result of Väsby clay at 5 m depth (after Chang 1969)**

**Figure 6.6 Vertical effective stress and vertical strain relationship of Väsby laboratory soil sample**

The data from Figure 6.5 show a clear trend in strain as a function of elapsed time and effective stress. The clay sample from BH6701 at 5 m depth exhibited a characteristic creep behavior, which is evident from the significant increase in strain over time at each effective stress level.

The mathematical expression for creep strain limit is given by:

\[ \varepsilon_\infty = a - b \times \ln(\sigma' / p_u) \]

where \( \varepsilon_\infty \) is the creep strain limit, \( a \) and \( b \) are constants, \( \sigma' \) is the effective stress, and \( p_u \) is the undrained shear strength.
160 kPa) exhibit normally consolidated behaviour with more significant compression due to loading. Additionally, in the space of $\varepsilon - \log(\sigma')$ in Figure 6.6, the loading stage 30 kPa to 45 kPa was within the transition from the overconsolidated stress range to the normally consolidated stress range, and the stages from 67.5 kPa to 160 kPa had passed the preconsolidation pressure, and were in the normally consolidation range. The compression results in Figure 6.5 are the typical consolidation test results obtained to determine the soil compressibility properties. Since the loading duration was about 1 day for each increment, the conventional approach for the EVP model parameter determination cannot be reliably used due to lack of sufficient creep data after the end of primary consolidation. However, applying the new proposed approach, the EVP model parameters can be easily determined by utilising all the data points of four stress stages. In order to apply the proposed approach, Table 6.1 presents the initial stress – strain conditions for each loading stage.

Table 6.1 Initial information adopted for the EVP model parameter determination for Väsby clay

<table>
<thead>
<tr>
<th>Loading Stage (kPa)</th>
<th>$H_o$ (mm)</th>
<th>$\varepsilon_o$ (%)</th>
<th>$\sigma_i$ (kPa)</th>
<th>$\sigma_f = \sigma_i + \Delta\sigma_c$ (kPa)</th>
<th>$u_{ci} = \Delta\sigma_c$ (kPa)</th>
<th>Duration (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>20</td>
<td>3.82</td>
<td>30</td>
<td>45</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>67.5</td>
<td>20</td>
<td>8.81</td>
<td>45</td>
<td>67.5</td>
<td>22.5</td>
<td>1</td>
</tr>
<tr>
<td>105</td>
<td>20</td>
<td>17.82</td>
<td>67.5</td>
<td>105</td>
<td>37.5</td>
<td>1</td>
</tr>
<tr>
<td>160</td>
<td>20</td>
<td>26.01</td>
<td>105</td>
<td>160</td>
<td>55</td>
<td>1</td>
</tr>
</tbody>
</table>

In this case study, the soil permeability is obtained in advance using consolidation test results presented in Figure 6.5. The coefficient of permeability is calculated based on the relationship between $c_v$ (coefficient of consolidation) and $m_v$ (coefficient of volume change) for each stress stage. $c_v$ and $m_v$ values are calculated using the compression data reported by Chang (1969) (i.e. compression curves in Figure 6.5). Consequently, the relationship between the void ratios and the coefficient of permeability is attained as shown in Figure 6.7.
Adopting the initial information corresponding to each stress stage (Table 6.1) and the soil permeability properties (Figure 6.7), the optimisation process applying the trust-region reflective least squared (TRRLS) approach results in the set of the EVP model parameters as tabulated in Table 6.2. The time-line system including the instant time-line, the reference time-line, and the limit time-line of the oedometer test sample is illustrated in Figure 6.6 employing the obtained model parameters. The instant time-line defined by $\kappa/V$ is approximately parallel with the first three loading stages, while the reference time-line $\lambda/V$ and the limit time-line are also about parallel for the last four loading stages. It can also be noted that in Figure 6.6 the slopes of loading stages from 45 kPa to 160 kPa are not identical. Particularly, as observed in Figure 6.6 the slope of compression seems to decrease with the increase of the effective stress. Thus, it could be difficult to apply the conventional approach (i.e. Yin (1999) approach) to identify $\lambda/V$, since $\lambda/V$ is influenced by the choice of the reference stress – strain point and an extra stress – strain point used to determine the parameters explained in Chapter 3. It can be emphasised that the TRRLS approach can be used reliably to define all parameters at once. The prediction curves adopting the optimised model parameters are compared with the measurements as shown in Figure 6.8. The predictions are in good agreement with the measurements, as the coefficient of determination $R^2$ of four stages are greater than 0.90.
In comparison to the values of conventional compression parameters including recompression and compression ratios ($C_{hr}$ and $C_{hc}$), and the secondary compression coefficient $C_{eex}$ reported in Chang (1969) and adopted in the numerical study of Väsby test fill by Perrone (1998), the EVP model parameters are within the ranges of the conventional parameters. For example, $\kappa/V = C_{as}/\ln10$ reported by Chang (1969) and Perrone (1998) was in the range from 0.0173 to 0.0217. It is reasonable that $\kappa/V = 0.0105$ obtained from the TRRLS approach (Table 6.2) is slightly lower than the values of $\kappa/V$ reported in the literature, because the conventional method to determine $\kappa/V$ may include creep contribution. The range of $\lambda/V (C_{sc}/\ln10)$ reported in the literature varies from 0.156 to 0.32 for undisturbed Vasby clay (Leroueil and Kabbaj 1987, and Perrone 1998), which is in a good agreement with $\lambda/V = 0.2006$ reported in Table 6.2 based on the TRRLS approach.

In term of creep estimation, the secondary compression coefficient $C_{eex}/\ln10$ is comparable to the creep parameter $\psi/V$ which can be determined based on the creep strain limit $\varepsilon^{vp}$ and creep coefficient $\psi_c \lambda V$. However, $\psi/V$ is not only stress dependent, but also time dependent, while $C_{eex}$ is adopted as a constant, when $C_{sc}$ is considered constant. Thus, for the approximate comparison, $\psi_c \lambda V$ can be used to compare with $C_{eex}$ obtained immediately after the end of primary consolidation (e.g. 1 log cycle). For Väsby clay, based on the optimised values of $c$ and $d$ ($\psi_c \lambda V = c - d \times \ln(\sigma'_v/\sigma'_u)$) reported in Table 6.2, the value of $d$ parameter is negligible compared to $c$. Thus, the variation of $\psi_c \lambda V$ with the effective stress is insignificant. $\psi_c \lambda V \approx c = 0.0102$ is within the range $C_{eex}/\ln10$ from 0.0087–0.016 reported by Perrone (1998). $C_{eex}/C_{sc}$ for the undisturbed Väsby clay was about 0.06 (Mesri & Choi 1987), while the corresponding ratio of $\psi_o/\lambda$ is about 0.051.
Table 6.2 The optimised model parameters and soil permeability properties for Väsbys clay

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instant time-line</td>
<td></td>
</tr>
<tr>
<td>$\kappa V$</td>
<td>0.0105</td>
</tr>
<tr>
<td>Reference time-line</td>
<td></td>
</tr>
<tr>
<td>$\lambda V$</td>
<td>0.2006</td>
</tr>
<tr>
<td>$\sigma^{\prime}_{\text{so}}$ (kPa)</td>
<td>39.505</td>
</tr>
<tr>
<td>Equivalent time-line</td>
<td></td>
</tr>
<tr>
<td>$t_0$ (min)</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon^\text{vp}_{\text{im}}$</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>0.398</td>
</tr>
<tr>
<td>$b$</td>
<td>9.8×10^{-4}</td>
</tr>
<tr>
<td>$\psi_{\text{so}} V$</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>0.0102</td>
</tr>
<tr>
<td>$d$</td>
<td>1×10^{-5}</td>
</tr>
</tbody>
</table>

Figure 6.8 Settlement prediction adopting the TRRLS approach comparing with laboratory measurement from Chang (1969) for Väsbys clay
6.2.3.2 Numerical modelling of Väsby test fill

In this case study, the optimised EVP model parameters are adopted to analysis the long term behaviour including the settlement and the excess pore water pressure of the ground under the Vasby test fill. For the validation of the proposed method, the numerical modelling simulates the one-dimensional compression under the centre line of the embankment. The total depth below the scraped ground surface is 13.5 m. Figure 6.9 shows the soil properties under Vasby test fill adopted for the numerical simulation. In Figure 6.9, the soil properties including the initial void ratios and unit weights are adopted from Chang (1969). According to Larsson & Matsson (2003), based on the results of the constant rate of strain (CRS) tests, the coefficient of permeability of the ground varies from about 4.7×10⁻⁸ m/min at the depth of 1 m to 6×10⁻⁸ m/min at the bottom area. Table 6.3 presented the values of $m_v$ and $c_v$ with depth reported in Chang (1969). Based on the values of $m_v$ and $c_v$, the coefficient of permeability $k$ with depth is estimated and tabulated in Table 6.3. The calculated $k$ in Table 6.3 is higher than the measured values $k$ based on CRS tests of Larsson (1986). According to Tavenas et al. (1983), the coefficient of permeability indirectly evaluated from the consolidation data as shown in Table 6.3 is unreliable especially for structured natural clays. On the other hand, the permeability determined based on CRS test results may not represent for the relationship between the void ratio $e$ and $\log(k)$, since the test results are influenced by the preconsolidation pressure and the imposed strain rate. Tavenas et al. (1983) emphasised that the permeability at the in situ void ratio is influenced by several factors including the void ratio, the clay fraction, the plasticity index, as well as the fabric of clay.

Therefore, the coefficient of permeability $k$ of the ground is adopted as 6×10⁻⁸ m/min uniformly for the whole depth of the soil deposit, which is close to the value reported for the natural ground in Larsson & Mattsson (2003). The permeability change index ($c_k$) is adopted equal to 0.5$e_o$ considering the empirical correlation proposed by Tavenas et al. (1983). Additionally, Hansbo (1960) adopted the same correlation of $c_k = 0.5e_o$ for Swedish clays at Skä–Edeby site.
Table 6.3 Initial values of $m_v$ and $c_v$ reported by Chang (1969) and calculated coefficient of permeability $k$ based on $m_v$ and $c_v$

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$m_v$ (1/kPa)</th>
<th>$c_v$ ($\times 10^{-4}$ m$^2$/min)</th>
<th>$k = c_v m_v \gamma_v$ (m/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.005097</td>
<td>0.102</td>
<td>$5.1 \times 10^{-7}$</td>
</tr>
<tr>
<td>3.8</td>
<td>0.005097</td>
<td>0.039</td>
<td>$1.95 \times 10^{-7}$</td>
</tr>
<tr>
<td>5.7</td>
<td>0.003772</td>
<td>0.012</td>
<td>$4.44 \times 10^{-8}$</td>
</tr>
<tr>
<td>7.3</td>
<td>0.003568</td>
<td>0.012</td>
<td>$4.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>11.1</td>
<td>0.003262</td>
<td>0.03</td>
<td>$9.6 \times 10^{-8}$</td>
</tr>
<tr>
<td>11.8</td>
<td>0.002650</td>
<td>0.054</td>
<td>$1.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>13.5</td>
<td>0.002854</td>
<td>0.054</td>
<td>$1.51 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Figure 6.10(a) shows the initial vertical stress state ($\sigma'_{zi}$), the target vertical effective stress state ($\sigma'_{zf}$) and the preconsolidation pressure ($\sigma'_{pc}$) with depth. The initial vertical effective stresses ($\sigma'_{zi}$) are exerted by the uppermost soil layers. Thus, the initial vertical effective stress is calculated using Equation (6.1). The target vertical effective stress ($\sigma'_{zf}$) is the final vertical effective stress calculated by adding the applied stress $\Delta \sigma_z$ in Figure 6.10(c) to the initial vertical effective stress ($\sigma'_{zi}$).

$$
\sigma'_{zi} = \sigma_{zi} - u_z = \gamma_s h_z - \gamma_w h_z
$$

where, $\sigma_{zi}$ and $u_z$ are the total vertical stress and pore water pressure, respectively. $\gamma_s$ and $\gamma_w = 9.81$ kN/m$^3$ are the unit weights of soil and water, respectively, while $h_z$ is the layer depth.

The groundwater table in the field case studies was assumed to be static and unchanged with time. The initial pore water pressure was the hydrostatic pore water pressure varying with depth ($u_o = \gamma_o h_z$). Moreover, the soil above the water table was assumed saturated. Equation (6.1) is applied for saturated soil layers. Since the groundwater table is at 0.8 m depth below the ground surface, the initial effective stress of 0.8 m thick top layer is equal to the total vertical stress (i.e. $\sigma'_{zi} = \sigma_{zi} = \gamma_s h_z$) by assuming the soil above the water table is saturated. The top 0.5 m dry crust is assumed as drained layer, which means no excess pore water pressure remains in the crust. Meanwhile, for the soil layers below the crust layer, the initial excess pore water pressures generated are equal to the applied stress $\Delta \sigma_z$ varying with depth as shown in
Figure 6.10(c). The similar assumptions were made for the numerical analysis of Väsby test fill by Larsson (1986).

The change of applied stress $\Delta \sigma_z$ over the depth of the ground is calculated based on Boussinesq’s equation using Equation (6.2).

$$\Delta \sigma_z = q_o I_z$$  \hspace{1cm} (6.2)

where, $q_o$ is the uniform applied stress caused by the test fill, and $I_z$ is the stress influence factor.

In this case study, the exposed load induced by the test fill was 40.6kPa over a square test fill of 30 m×30 m. The stress influence factor with depth at the centre of the test fill is calculated using the equation for a uniform load of intensity $q_o$ on a strip of width $B = 30$ m (Boussinesq 1885). Applying Equations (6.2) and (6.3) yields the applied stress distribution with depth as shown in Figure 6.10(c).

$$I_z = \frac{2}{\pi} \left( \tan^{-1} \left( \frac{b}{z} \right) + \frac{bz}{b^2 + z^2} \right)$$  \hspace{1cm} (6.3)

where, $b = B/2 = 15$ m, and $z$ is the depth of the layer.

The preconsolidation pressures plotted in Figure 6.10(a) were adopted based on the oedometer tests of samples in the natural ground at the end of primary consolidation reported by Chang (1969). Based on the overconsolidation ratio (OCR) in Figure 6.10b, the clay layers below 2 m depth are slightly overconsolidated, while the top 2 m is heavily overconsolidated. As initial conditions, the EVP model requires the inputs of initial effective stresses $\sigma'_{zi}$ and the corresponding initial vertical strains $\varepsilon_{zi}$ are required. With the initial effective stresses $\sigma'_{zi}$ calculated adopting Equation (6.1), the initial vertical strains $\varepsilon_{zi}$ can be calculated based on the following equation.

$$\varepsilon_{zi} = \frac{1}{V} \ln \frac{\sigma'_{pc}}{\sigma'_{zo}} - \frac{\kappa}{V} \ln \frac{\sigma'_{pc}}{\sigma'_{zi}}$$  \hspace{1cm} (6.4)

where, $\sigma'_{pc}$ is the preconsolidation pressure, and $\sigma'_{zo}$ is the reference line parameter.

For a given overconsolidation ratio (OCR), Equation (6.4) can be rewritten as follows:

$$\varepsilon_{zi} = \frac{1}{V} \ln \frac{\sigma'_{zi}}{\sigma'_{zo}} + \left( \frac{1}{V} - \frac{\kappa}{V} \right) \ln OCR$$  \hspace{1cm} (6.5)

In this case study, Equation (6.5) is used to generate the initial vertical strains with depth, and the time dependent settlement is calculated based on the change of the vertical strains.

Due to lack of comprehensive consolidation test data at different depths beneath the ground surface suitable for the optimisation method (i.e. the compression curves with time as shown in Figure 6.5), the optimised EVP model parameters are applied for
the whole depth of the ground. Figure 6.11 includes the variations of the creep coefficient \( \psi / V \), the creep strain limit \( \varepsilon_{\text{vpc}} \), and the creep parameter \( \psi / V \) with depth. It should be noted that the initial value of \( \psi / V \) is not equal to the initial value of the creep coefficient \( \psi / V \), since the initial stress strain points generated for the whole soil layer do not stay on the reference time-line. The equation of the creep parameter varies with the equivalent time \( t_e \) rather than the actual loading time, and the value of \( t_e \) depends on the values of the effective stress and the strain. As a result, the initial \( \psi / V \) correlates with the initial values of the effective stress and the strain.

As defined in the non-linear creep function, the creep coefficient \( \psi / V \) and creep strain limit \( \varepsilon_{\text{vpc}} \) decrease with the increase of the effective stress (or with depth). In this case study, the initial effective stress varies with depth as shown in Figure 6.10(a). Thus, the initial creep strain limit \( \varepsilon_{\text{vpc}} \) and the initial creep coefficient \( \psi / V \) calculated using the values of \( a, b, c \) and \( d \) obtained using the TRRLS approach (as in Table 6.2) decrease with depth as illustrated in Figures 6.11(a) & (b). However, as discussed in Section 6.2.3.1, the stress dependence of \( \psi / V \) is insignificant. Thus, the variations of \( \psi / V \) and \( \varepsilon_{\text{vpc}} \) with depth are not remarkable. The creep parameter \( (\psi / V) \) is calculated using the creep strain limit \( \varepsilon_{\text{vpc}} \) and the creep coefficient \( \psi / V \), and the initial value of \( \psi / V \) also varies with depth as shown in Figure 6.11c. While the patterns of the variations of \( \psi / V \) and \( \varepsilon_{\text{vpc}} \) are similar, the variation of \( \psi / V \) with depth increases from about 0.002 at the top to 0.01 at 2 m depth downwards to the bottom. It should be noted that the initial value of \( \psi / V \) is influenced by the stress strain state via the value of the equivalent time \( t_e \). The initial vertical strain obtained by Equation (6.4) is plotted in Figure 6.11(d). The initial vertical strain calculated by Equation (6.4) has the pattern of the variation with depth similar to the pattern of the preconsolidation pressure in Figure 6.10(a). Equation (6.4) indicates the influence of the overconsolidation ratio on the initial stress strain points with respect to the reference time-line. Based on Equation (6.4), the soil at the depth at which the preconsolidation pressure \( \sigma'_{pc} \) is less than the value of the reference effective stress \( \sigma'_{z0} \) has a negative initial vertical strain as shown in Figure 6.11(d). Moreover, the settlement with time is calculated based on the change of the vertical strains with respect to the initial vertical strain in Figure 6.11(d).
Figure 6.9 Soil properties adopted in the numerical modelling for Väsby test fill (a) Initial void ratio, (b) Unit weight, (c) Coefficient of permeability and (d) Permeability change index (after Chang 1969, Larsson & Marsson 2003)
Figure 6.10 Geotechnical profile adopted in the numerical modelling for Västby test fill (a) Initial effective stress, final effective stress and preconsolidation pressure, (b) Overconsolidated ratio and (c) Applied stress distribution (after Chang 1969, Larsson & Mattsson 2003)
Figure 6.11 Initial creep compression properties (a) Creep coefficient $\psi/V$, (b) Creep strain limit $\varepsilon_{lm}^{vp}$, (c) Creep parameter $\psi/V$ and (d) the initial vertical strain obtained based on Equation (6.5) for Väsbö test fill.
In this case study, due to drainage layer on top and sand layer at the bottom, two-way drainage condition is applied. For the numerical analysis, the clay layer is divided into sub-layers of 0.02 m thick, and the imposed load intensity is assumed to be applied gradually with the applied stress increasing linearly in 25 days. After 25 days of the fill construction, the maximum final effective stress is maintained unchanged, and the stress reduction due to the submersion of the top soil layers below the groundwater during the compression process is disregarded. The numerical simulation is run up to total time of 67 years (from 1947 to 2014). The predictions of the vertical settlement and excess pore water pressures of the soil layer under the centre line of Väsbys test fill are provided in the following section.

6.2.4 Results and discussion

6.2.4.1 Settlement predictions

The settlement predictions obtained from the numerical simulation are presented in Figures 6.12, 6.13 and 6.14. As shown in Figure 6.12, the predictions adopting the EVP model parameters reported in Table 6.3 have good agreement with the field measurement. According to Larsson and Mattsson (2003), the initial settlement during 25 days of construction was about 0.065 m. The settlement prediction at the ground surface after the first 25 days is 0.062 m. The measured (and predicted) settlements after 19 years (CY19–1966), 32 years (CY32–1979), 55 years (CY55–2002) after the fill construction were about 1.4 m (1.369 m), 1.65 m (1.716 m), and over 2.0 m (2.071 m), respectively. Moreover, the total settlement of the ground surface at CY67 (2014) is predicted 2.189 m. Despite the simplification of the numerical solution, the EVP model with the optimised model parameters effectively predicts the long-term settlement under the centre of the test fill. It should be noted that “CY” stands for “construction year” in this study.

As shown in Figures 6.13 and 6.14, the long-term settlements at different depth are predicted in good agreement with the field measurements. Both the predictions and the field measurements indicate that the compression occurred dominantly in the upper layers near the top surface and the layer close to the drained boundary (Figures 6.12 and 6.14). Figure 6.13 indicate the good estimations of the settlements of the top 5 m. For the soil layer below 5 m depth, the disparities between the predictions and the measurements increase. As reported in Larsson (1986), the settlement markers at
various depths malfunctioned after some time. The markers at the depths of 4 m and 7 m were replaced 1.5 years after the construction (Larsson 1986). Therefore, the settlement distribution with depth could be obtained based on the measured water content of the ground. The changes of the water content were measured at CY20 (1967) and CY55 (2002)), which are shown in Figure 6.14. Comparing the settlement measured in CY21 (1968) and the settlement calculated based on water content in CY20 (1967), the settlement during this early stage of loading may be mainly due to the squeezing out of water (Chang 1969). In CY55 (2002), there is slight disparity between the measured settlement by settlement markers and the settlement calculated based on the change in water content. Additionally, the predicted settlement by the optimised model parameters is agreed well with the measurements.

![Figure 6.12 Predictions of the vertical settlement at several depths beneath the test fill in the linear scale](image-url)

*Figure 6.12 Predictions of the vertical settlement at several depths beneath the test fill in the linear scale (note: model predictions are shown in solid lines)*
Figure 6.13 Predictions of the vertical settlement at several depths beneath the test fill in the logarithmic scale (note: model predictions are shown in solid lines)

Figure 6.14 Variations of the vertical settlements with depth at different time points
6.2.4.2 Excess pore water pressure

Besides the settlement prediction, the prediction of the excess pore water pressures is also important to investigate the long-term behaviour of soft soils. Figures 6.15, 6.16 and 6.17 show the predictions and measurements of the excess pore water pressure distributions with depth at different times. As mentioned in Section 6.2.1, the piezometers for the pore water pressure measurement were replaced with the new devices in 1968, as the previous piezometers had shown unreliable reading (Larsson & Mattsson 2003). Figure 6.3 shows the location of the piezometers (N144–N143) measuring the excess pore water pressure under the centre line of the fill. The piezometers were ceased and withdrawn after the investigations in 1969. The later measurements of the pore water pressures were made in 1979 and 2002 by retractable piezometers (Larsson & Mattsson 2003). In general the predictions and the measurements of the excess pore water pressure varying with depth are in good agreement.

In Figure 6.15, at CY21 (i.e. 1968), the remaining excess pore water pressure was about 30 kPa at the middle of soil profile (z = 5.5 m) (Larsson & Mattsson 2003). In other words, the effective stresses had maintained almost unchanged in some parts at the middle depths of the ground. At CY30 (1977) and CY55 (2002), the remaining excess pore water pressures were 22 kPa and 12 kPa, respectively (Figures 6.16 and 6.17). In 2002, the readings of the excess pore water pressure were scatter, expecting the inaccurate estimation (Larsson & Mattsson 2003). The corresponding predictions of the excess pore water pressures after 21 years, 32 years and 55 years from the end of construction has discrepancies from the field measurement mostly at the middle depths of the ground. Moreover, the applied load intensity is assumed to be maintained constant with time after the end of the construction. However, in reality, the load intensity was reduced with time, as the ground surface settled towards below the groundwater table. Therefore, the predicted variations of the excess pore water pressures corresponding to the constant maximum load intensity are higher than the field measurements as shown in Figures 6.15 to 6.17.

The differences observed between the predictions and measurements may be due to the adopted permeability properties and the uniform EVP model parameters for the whole layer. The rate of dissipation of the excess pore water pressure is influenced by the soil permeability. In Figure 6.18, the coefficients of permeability reported by
Larsson & Mattsson (2003) were evaluated by the results of CRS oedometer tests. Figure 6.18 also shows the predicted and measured coefficient of permeability below the Väsby test fill. The prediction of the permeability with depth is obtained 55 years after the construction (in 2002). The predicted value of permeability is close to the measured permeability reported in 2002 from Larsson & Mattsson (2003). Additionally, the EVP model parameters adopted for the analysis are evaluated based on the compression data of one soil sample obtained from 5 m depth. In the real situation, the EVP model parameters such as $\kappa/V$, $\lambda/V$, $\sigma_{zo}$, as well as $a$, $b$, $c$, and $d$ may vary with depth. The numerical solution proposed in this thesis has limitation to involve other factors in the analysis such as the lateral displacement, the undrained shear strength and the change of the applied stresses due to the ground submerging below the groundwater table as the soil deposit settles.

![Figure 6.15 Prediction of excess pore water pressure after 21 years from the end of construction in 1968](image-url)
Figure 6.16 Prediction of excess pore water pressure after 32 years from the end of construction in 1979

Figure 6.17 Prediction of excess pore water pressure after 55 years from the end of construction in 2002
6.2.4.3 Creep compression properties

This section is to discuss about the variations of the creep compression properties (or the creep function parameters) including the creep coefficient $\psi_{/V}$, the creep strain limit $\varepsilon_{lim}^{vp}$, and consequently the creep parameter $\psi_{/V}$ as well as the creep strain rate $\dot{\varepsilon}_z^{vp}$. The variations of the parameters ($\psi_{/V}$, $\varepsilon_{lim}^{vp}$ and $\psi_{/V}$) and the creep strain rate ($\dot{\varepsilon}_z^{vp}$) are not only with depth, but also with time. However, in this case study, the variations of $\psi_{/V}$ with stress is insignificant as observed in Figure 6.19(a) due to the negligible value of parameter $d$. The same order is observed for the variation of the creep strain limit in Figure 6.19(b). Despite the minor variations values of both parameters ($\psi_{/V}$ and $\varepsilon_{lim}^{vp}$), Figure 6.19 shows the changes of the creep parameters ($\psi_{/V}$ and $\varepsilon_{lim}^{vp}$) with time and with depth. As the effective stress of the soil layer increases with time, the corresponding $\psi_{/V}$ and $\varepsilon_{lim}^{vp}$ decrease with time considering Equations (3.8) & (3.9). As observed in Figure 6.19, the top and bottom areas near the drained boundaries exhibit smaller values for creep parameters, as well as lesser changes in those values. In the comparison of the initial values of $\psi_{/V}$ and $\varepsilon_{lim}^{vp}$ in Figures 6.11(a)-(b) and the values in Figure 6.19, the decreases of $\psi_{/V}$ and $\varepsilon_{lim}^{vp}$ of the top and bottom areas near the drained boundary are higher than the changes of those values at the middle areas. Therefore, the
creep compression at the middle areas is less than the compression at the areas near the
drained boundaries.

Figure 6.20a illustrates the variation of the creep parameter \((\psi/V)\) with depth at
different time points. The creep parameter is calculated using the creep coefficient, the
creep strain limit and the equivalent time \(t_e\) (Equation 3.7 in Chapter 3). Thus, the
variation of the creep parameter is influenced by effective stress (due to the presence of
the creep coefficient and the creep strain limit) and time (due to \(t_e\)). As discussed in the
laboratory-based case studies in Chapters 4 and 5, the creep parameter keeps changing
during the dissipation of the excess pore water pressure, and even after the dissipation
completes. During the dissipation of the excess pore water pressure, the creep parameter
\((\psi/V)\) is influenced by both the effective stress and time, while after the dissipation, the
creep parameter is influenced by time only. Figure 6.20(a) shows that the creep
parameter \((\psi/V)\) keeps decreasing with time. Since \(\psi_o/V\) and \(\varepsilon_{im}^{vp}\) are almost stress-
independent, the change of the creep parameter \((\psi/V)\) is mainly due to time. However,
the shape of the variations of the creep parameter is different from that of the creep
coefficient \(\psi_o/V\) and the creep strain limit \(\varepsilon_{im}^{vp}\). Figure 6.20(b) shows that the change of
the creep strain rate with time at the middle layer is much higher than the change at the
top and bottom areas. As the excess pore water pressures at the top and bottom areas
near the drainage boundaries dissipated faster than the middle area (Figures 6.15 to
6.17), the effective stresses at those areas almost stabilised at the target value. As a
result, the creep compression at those areas are more dominant compared to the middle
area. Thus, the change of the creep strain rate in the top and bottom areas is controlled
mainly by time, while at the middle area, the creep strain rate is influenced by the
increases of both time and the effective stresses.

In addition, Figures 6.21 and 6.22 present the variations of the creep coefficient,
creep strain limit, creep parameter as well as the creep strain rate with time at three
different depths \((z = H_o/4, H_o/2 \text{ and } 3H_o/4)\). The variation patterns of the creep
coefficient and creep strain limit are similar as shown in Figure 6.21. \(\psi_o/V\) and \(\varepsilon_{im}^{vp}\) both
experienced the increase in the beginning of loading. Since \(\psi_o/V\) and \(\varepsilon_{im}^{vp}\) decrease with
the increase of the effective stress, the increases of those parameters imply that the
effective stresses had decreased towards smaller values than the initial values (i.e. the
excess pore water pressures increased higher than the stress increment). The increase of
the excess pore water pressure during the early stage of the consolidation process is
observed in the laboratory test as discussed in Chapter 5. Chang (1969) reported that at CY25 (1968) the pore water pressure at the middle area remained almost equal to the load intensity exerted by the fill. It can be assumed that the excess pore water pressure would increase higher than the applied stress increment or remain almost unchanged. The phenomenon of the increase of the excess pore water pressure and the decrease of the according effective stress under a constant load at test embankments were observed and reported for Berthierville site (Canada) in Kabbaj et al. (1988) and for Tarsiut Island (Canada) in Becker et al. (1985).

Crooks et al. (1984) investigated several cases in which the excess pore water pressure was higher than the applied stress. The creep deformation (or namely the “breakdown of the soil structure”) was suggested to result in the negligible slow rate of the dissipation of the excess pore water pressure Crooks et al. (1984). As the applied stress increment results in the target effective stress higher than the preconsolidation pressure, the soil structure appears to collapse (i.e. the compression is observed higher in the normally consolidation range compared to the overconsolidated range). The breakdown of the soil structure results in the decrease of the effective stress, consequently the increase of the excess pore water pressure. Crooks et al. (1984) found that the magnitude of $c_v$ is minimum at the effective stress near the preconsolidation pressure based on the consolidation test results of various soft clay samples. Crawford (1986) emphasised that the breakdown of soil structure leads to the generation of the pore water pressure at the similar rate to the net dissipation by the drainage of the void water, and subsequently results in the creep compression under constant effective stresses.

The thickness of the soil deposit as well as the viscous effect cause the delay in the compression as well as the dissipation of the excess pore water pressure at the impervious areas (Yin & Graham 1996; Yin & Zhu 1999). As the thickness of the soil deposit is much more than the sample in the laboratory test, the viscous effect is remarkable. The thickness of the soil deposit may result in the delay of the flow of void water locally. Consequently, the effective stress at those locals remained unchanged, and the creep compression occurred. The increase of the creep compression causes further blocks of the drainage paths of void water. Thus, the relaxation occurred under the constant strain at those areas that water was blocked, and the effective stress at those locations decreased accordingly.
The measured and predicted variations of the excess pore water pressure with time in Figures 6.15 to 6.17 indicate that the dissipation rate at the middle area was much slower than at the top and bottom areas near the drainage boundaries. Thus, the increases of the excess pore water pressure may occur in the early years of fill loading. As a result, \( \psi/V \) and \( \varepsilon_{\text{im}}^{\text{vp}} \) increased accordingly. Since the dissipation rate of the excess pore water pressure increases, the increase of the effective stress results in the decrease of those parameters \( \psi/V \) and \( \varepsilon_{\text{im}}^{\text{vp}} \) with time as observed in Figure 6.21. It is also noted that the decreasing rates of those parameters also decreases with time, as the effective stress increases closer to the target final value. Moreover, \( \psi/V \) and \( \varepsilon_{\text{im}}^{\text{vp}} \) are supposed to maintain a constant value, as the effective stress reaches the target value. \( \psi/V \) and \( \varepsilon_{\text{im}}^{\text{vp}} \) at the deeper locations are smaller, since the deeper locations have higher effective stresses.

In addition, Figure 6.22 illustrates the variations of the creep parameter \( \psi/V \) and the creep strain rate \( \dot{\varepsilon}_z^{\text{vp}} \) with time at the similar locations. The variations of \( \psi/V \) and \( \dot{\varepsilon}_z^{\text{vp}} \) have similar patterns. The variations of \( \psi/V \) and \( \dot{\varepsilon}_z^{\text{vp}} \) are influenced by the effective stress and time, especially during first 10 years. At \( z = H_o/4 \) and \( z = 3H_o/4 \), \( \psi/V \) varied approximately the same at the beginning, then decreased parallel to each other, while \( \psi/V \) at \( z = H_o/2 \) varied distinguishingly from other locations. The similar observations for the variations of \( \dot{\varepsilon}_z^{\text{vp}} \) with depths are shown in Figure 6.22(b). The soil at depths \( z = H_o/4 \) and \( z = 3H_o/4 \) exhibited the faster rate of the dissipation of the excess pore water pressure compared to the soil at \( z = H_o/2 \) (the middle of the soil deposit) due to the closer distances to the drainage boundaries. As the effective stress at \( z = H_o/2 \) is less than at other locations, \( \psi/V \) and \( \dot{\varepsilon}_z^{\text{vp}} \) are higher at the middle, which is also observed in Figure 6.20(b).
Figure 6.19 Variations of (a) Creep coefficient $\psi_0/V \times 10^{-3}$ and (b) Creep strain limit $\varepsilon_{lm}^{vp}$ with depth in 1968, 1979, 2002 and 2014 for Väsby site.
Figure 6.20 Variations of (a) Creep parameter $\psi/V$ and (b) Creep strain rate $\dot{\varepsilon}_z^{\psi}$ with depth in 1968, 1979, 2002 and 2014 for Väsby site
Figure 6.21 Predictions of (a) the creep coefficient $\psi_{c}/V$ and (b) the creep strain limit $\varepsilon'^{\text{pp}}$ with time at different depths.
Figure 6.22 Predictions of (a) the creep parameter $\psi/V$ and (b) the creep strain rate $\dot{\varepsilon}_z^{vp}$ with time at different depths
6.3 Case study 2: Skå-Edeby test fill

6.3.1 Project and site description

According to Larsson & Mattsson (2003), the initial test results from the Väsby test fills revealed that the Väsby site was not suitable for the new airfield. Thus, the Halmsjon site (today Arlanda) was chosen as an alternative site for the construction. In 1956, a new runway was urgently required for new Douglas DC8 jet planes, since the existing runways at the old airfield at Bromma were not suitable to be extended. Therefore, Skå-Edeby, which was located about 25km west of Stockholm, was considered a possible site for the new airfield.

In 1957, since there was lack of the geological investigation on the soil conditions at Skå-Edeby to decide its suitability, three circular test fills with vertical drains and one circular test fill without vertical drains were constructed. The diameters of the test fills varied from 70 m to 35 m. The construction of the test fills were completed after three months along with all the field investigation, sampling, measurement device installation, and vertical drain installation. Figure 6.23 shows the test fill areas at Skå-Edeby. In this case study, the circular test fill without vertical drains at Area IV is subject to investigation. The test fill at Area IV had the diameter of 35 m at the base, the height of 1.5 m, and the batter slope of 1(V):1.5(H). Since there was no vertical drain installed in this area, the test fill was denoted as an “undrained fill”. The average unit weight of the gravel fill is 17.56 kN/m³, and the total imposed load induced by the test fill was estimated to be 27 kPa at the ground surface (Larsson & Mattsson 2003).

Based on the experiences gained from the Väsby site, the new settlement markers were installed before the construction at several depths under the ground surface. The settlement markers were mostly placed under the centre line of the embankment and at various distances from the centre to the perimeter. Similar to the settlement markers, piezometers for the pore water pressure measurements were installed to monitor the pore water pressure at various depths under the centre line, as well as distances towards the perimeter (Larsson & Mattsson 2003). The instrumentation layout of the measurement points of the settlement markers and piezometers is shown in Figure 6.24, and Figure 6.25 illustrates the subsoil profile of Area IV with the locations of the piezometers and settlement markers for the measurements of the pore water pressure and the settlements.
Figure 6.23 Test fills at the Skå-Edeby site (after Larsson & Mattsson 2003)
Figure 6.24 Instrumentation layout of Area IV (without drains) at the Skå-Edby site (after Hansbo 1960)

Figure 6.25 Schematic subsoil profile under the test fill Area IV at Skå-Edby
The Skå-Edeby site was chosen to build the new airfield due to economic issues. However, the research activities at the site were continued for other projects such as road construction (Larsson & Mattsson 2003). The investigation results were firstly reported by Hansbo (1960) for the first year measurements. In 1961, the test fill was built along with the installation of the settlement markers and piezometers, and the readings were made periodically. In 1970, a new investigation was carried out with more modern piezometers for the pore water pressure along with the measurements of the variations of soil properties beneath the test fills (Holtz & Broms 1972; Holtz & Lindskog 1972). The pore water pressure measurement under the test fill at Area IV was made in 1982 using the new measurement devices. Larsson (1986) reported the study on the samples taken outside and under the fill between 1984 to 1985 in order to develop a new method for settlement prediction. Besides, according to Larsson & Mattsson (2003), various studies on the effect of piston sampling, the effect of sand drains and vertical drains, and soil stabilisation using lime columns were carried out at the Skå-Edeby site (Holm & Holtz 1977; Kallstenius 1963).

6.3.2 Subsoil profile and soil properties

The subsoil profile at the Skå-Edeby site (Figure 6.23) has very similar to properties as the soil profile at the Väsby site. The soil under the Skå-Edeby site in this case study consists of a 12 m to 15 m deep layer of soft clay overlying on rock or till. The soil profile includes the recent, post glacial and glacial clays of the central of Sweden (Holtz & Broms 1972). According to Perrone (1998), the site is about 2.5 m above the sea level, and emerged from the Baltic Sea from 500 years ago. The glacial clay layers are about 7500 years old, while the upper post glacial layers to the recent top layers have been deposited since 4500 years ago.

The top layer is a 0.5 m thick desiccated dry crust underlaid by a layer of grey-green organic clay with high plasticity. Since this layer is quite close to the ground surface, the soil water content is lower than the liquid limit, and the shear strength is relatively high. The deeper post glacial clay layer comprising most of the upper soil profile is soft and slightly organic. The underlying glacial clay is soft, and highly varved. Near the bottom, there are irregular seams of silts and sands, and below the 12 m thick clay layer bedrock or dense till exists (Hansbo 1960; Larsson & Mattsson 2003).
Similar to the soil conditions at the Väsby test fill, the post glacial and glacial clays are coloured or banded due to the high contents of iron sulfides. The organic content of the clay is about 0.4%. The water contents of the soil layer below 2 m depth are higher than the liquid limits. The soil water contents varied from 100% at the top to 60% in the bottom 2 m as shown in Figure 6.20. The soil unit weight decreases from 14.7 kN/m$^3$ near the top to 16.68 kN/m$^3$ at the deepest layer (Larsson & Mattson 2003). Additionally, the sensitivity was obtained by the field vane and fall cone tests. The sensitivity, the ratio between the undrained shear strengths of the undisturbed and remoulded states of the soil, is considered to estimate the strength loss, as the disturbance of the soil increases (Mitchell & Houston 1969). The sensitivity can be determined by field vane shear test (FVST), unconfined compression tests, and fall cone tests (FCT). The fall cone test (FCT) is a simple laboratory test in which a cone is penetrated into a soil specimen by its self–weight and the penetration depth is measured (Tanaka et al. 2012). The field vane shear tests are to measure the shearing resistance of soils in situ by pushing a vane blade to the desired depth, rotating the blade and measuring the shearing strength via the torque required to produce the rotation (Schnaid 2009). For the soil profile at Skå-Edeby, the sensitivity increases from 5 in the upper layers to 15 in the lower layers near the bottom. There were minor differences of the sensitivity obtained by the FVST and FCT except at the top 1m.

The shear strength of the ground was estimated by the field vane shear tests, fall cone tests and unconfined compression tests. In Figure 6.26, the shear strength values were obtained from the field vane shear tests and fall cone tests, which resulted in the relatively identical values at depths from 2 m to 10 m. According to Larsson & Mattsson (2003), the shear strength increases from 6 kPa at 3.5 m depth with the rate of 1.2 kPa/m toward the bottom.

According to Larsson & Mattsson (2003), the groundwater table and the pore water pressure vary seasonally. The groundwater table is approximately 1 m deep with the fluctuations of ±0.5 m. The pore water pressure of the natural ground was assumed to be hydrostatic in respect to the average groundwater table. Larsson & Mattsson (2003) also found that there was seasonal variation in settlement rate in the first years, and the variations were observed varying with depth. Additionally, the influencing factors resulting in the variations of settlement rate also included the seasonal variation of groundwater table, the change in water contents of the soil profile, the variation of
the soil temperature due to snow and freezing. The effects of the factors on the soil profiles were found to be dissimilar at different depths (Larsson & Mattsson 2003).

6.3.3 Numerical simulation

6.3.3.1 Model parameter determination
In order to apply the proposed optimisation approach for this case study, two sets of the oedometer tests on Skå-Edeby soil samples are adopted from Hansbo (1960). The soil properties of samples A and B are summarised in Table 6.4. Comparing the properties of Samples A and B with the soil conditions in Figure 6.26, it can be assumed that Sample A was obtained at 2 m depth, while Sample B was at deeper layers about 5 m or 8 m. The assumption is made due to lack of information of the location of the tested soil samples A and B in Hansbo (1960). As described in Hansbo (1960), the oedometer apparatuses were modified to measure both the excess pore water pressure and the compression. The detailed information can be found in Hansbo (1960). In this case study, the drainage conditions for Samples A and B are one-way vertical drainage from the bottom surface and one-way vertical drainage from the top surface, respectively. In other words, the pore water pressures in Sample A was measured at the impermeable top of the clay sample, while the pore water pressures in Sample B was measured at the impermeable base. The soil samples had the mean diameters of 0.048 m. The loading was applied in increments, and the loading time was about 1 day for each increment. The consolidation results of Samples A and B are shown in Figure 6.27. Additionally, Figure 6.28 includes the oedometer test results of the samples taken at 2 m, 5 m and 8 m depths at Area IV (i.e. the subject of this study) at Skå-Edeby site.
### Soil properties of the Skä-Edeby subsoil profile (after Hansbo 1960 and Larsson & Mattsson 2003)

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Description</th>
<th>Water content (w) (%)</th>
<th>Unit weight (kN/m³)</th>
<th>Sensitivity</th>
<th>Shear strength (S_u) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>100</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>Clay crust</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Grey – green slightly organic clay, sulfide flecks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Grey varved clay, sulfide flecks and bands</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Grey brown varved clay sulfide bands</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Rock or till</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.26 Soil properties of the Skä-Edeby subsoil profile (after Hansbo 1960 and Larsson & Mattsson 2003)
For both Samples A and B, the last three loading stages of each sample are adopted for the EVP model parameter determination, since they are within the normally consolidated ranges. The permeability properties including the coefficient of permeability \((k_o)\), the permeability change index \((c_k)\) and the initial void ratio \((e_o)\) are also obtained from the optimisation process along with other EVP model parameters. It should be noted, as an alternative, the permeability information can be determined using the consolidation data by calculating \(c_r\) and \(m_r\), similar to the Vasby case study reported in Section 6.2.3.1.

For Sample A, the initial stress – strain information of the employed loading stages is presented in Table 6.5. The initial thickness of Sample A is about 0.015m. For Sample A, the change of the pore water pressure during consolidation was measured on at the impervious top surface of the soil sample. Applying the TRRLS method, the EVP model parameters with the permeability properties are calculated and presented in Table 6.6. The initial void ratio \(e_o = 2.678\) is close to the range of the void ratios reported for the depth of 2m from the ground surface in Lo (1991). However, the soil permeability is slightly higher than the corresponding measurement for the depth. According to Lo (1991), the recompression index \((C_r)\) for Skå-Edeby site was estimated to be 0.17, \((C_r/V)\ln10 = 0.02\), which is in a good agreement with the optimisation value of the instant time-line slope \((\kappa/V)\) reported in Table 6.6. Meanwhile, the slope of the reference time-line of Sample A \((\lambda/V = 0.1209)\) is fairly higher than \((C_c/V)\ln10 = 0.101\) for the soil at depth of 2m deep in this site (i.e. the slope of the sample at 2m depth in Figure 6.28). The ratio of \(C_{cd}/C_{dc}\) was reported about 0.05 (Hansbo 1960). Thus, \(C_{cd}/\ln10\) was approximately 0.0051, lower than the maximum value of \(\psi'_o\) of 0.0085. In this case study, the coefficient \(d\) required to calculate \(\psi'_o\) (Equation 3.9) is relatively small compared to the coefficient \(c\). Hence, \(\psi'_o\) may be assumed stress-independent. However, the creep strain limit is estimated to be highly stress-dependent considering the value of \(b\) reported in Table 6.6. The time-line system of Sample A based on the optimised values in Table 6.6 is illustrated in Figure 6.29.
Figure 6.27 Oedometer test results of Samples A and B of Skå-Edeby clay adopted for the model parameter determination (from Hansbo 1960)
Table 6.4 Soil properties of Samples A and B of Skå-Edoby clay

<table>
<thead>
<tr>
<th>Soil properties</th>
<th>Sample A</th>
<th>Sample B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial sample height (m)</td>
<td>0.015</td>
<td>0.0125</td>
</tr>
<tr>
<td>Initial water content (%)</td>
<td>79%</td>
<td>46%</td>
</tr>
<tr>
<td>Liquid limit (%)</td>
<td>63</td>
<td>43</td>
</tr>
<tr>
<td>Plastic limit (%)</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>Shear strength (kPa)</td>
<td>5.4</td>
<td>10.8</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Unit weight (kN/m³)</td>
<td>15.11</td>
<td>16.68</td>
</tr>
</tbody>
</table>

*Data obtained from Hansbo (1960)*

Figure 6.28 Incremental oedometer test results on samples obtained at Area IV at the Skå-Edoby test fill (after Hansbo 1960)

Figures 6.30 and 6.31 present the predictions of settlements and excess pore water pressures, respectively for Sample A adopting the optimised model parameters and soil permeability properties in Table 6.6. A good agreement between the predictions and the measurements are observed for settlement. For the settlement prediction, the coefficients of determination $R^2$ of the employed loading stages are higher than 0.95,
indicating the good fittings achieved. The pattern of the excess pore water pressure is predicted well. However, the predictions of the excess pore water pressure are more than the measurements, which is similarly observed in Case study of Drammen clay in Chapter 5 and KBS mixture in Chapter 4. The maximum measured value of the excess pore water pressure is less than the applied stress increment, which has been discussed in Whitman et al. (1961), Crawford (1964), and Robinson (1999). The reason was suggested due to the stiffness of the measurement system, which allows a partial drainage of pore water at the impervious base (i.e. at the measurement location) (Robinson 1999; Charlie 2000).

In the same order of Sample A, the TRRLS approach is applied to determine the EVP model parameters as well as the soil permeability properties for Sample B. The initial information for the optimisation process is presented in Table 6.7. The employed loading stages are within the normally consolidated stress range for Sample B. The height of Sample B was 0.0125m. All the loading stages were ceased after 1 day. The drainage condition for Sample B is opposite of Sample A. Water was drained out the soil sample via the pervious top surface, and the pore water pressure was measured at the impervious base of the sample. Therefore, the boundary condition applied for Sample B is one-way vertical drainage through the top boundary.

<table>
<thead>
<tr>
<th>Loading stage (kPa)</th>
<th>( H_o ) (m)</th>
<th>( \varepsilon_i ) (%)</th>
<th>( \sigma_{ij} ) (kPa)</th>
<th>( \sigma_{ij} + \Delta \sigma_{ij} ) (kPa)</th>
<th>( u_{ij} = \Delta \sigma_{ij} ) (kPa)</th>
<th>Duration (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80.42</td>
<td>0.015</td>
<td>7.71</td>
<td>41.19</td>
<td>80.42</td>
<td>39.23</td>
<td>1</td>
</tr>
<tr>
<td>166.71</td>
<td>0.015</td>
<td>18.88</td>
<td>80.42</td>
<td>166.71</td>
<td>86.29</td>
<td>1</td>
</tr>
<tr>
<td>338.34</td>
<td>0.015</td>
<td>26.86</td>
<td>166.71</td>
<td>338.34</td>
<td>171.63</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.5 Initial information adopted for the EVP model parameter determination for Skå-Edeby Sample A of Skå-Edeby clay
### Table 6.6 The optimised model parameters and soil permeability properties of Sample A of Skå-Edeby clay

<table>
<thead>
<tr>
<th>Model parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Instant time-line</td>
<td>$\kappa/V$</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Reference time-line</td>
<td>$\lambda/V$</td>
<td>0.1209</td>
<td></td>
</tr>
<tr>
<td>$\sigma'_{zo}$ (kPa)</td>
<td>23.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_a$ (min)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equivalent time-line</td>
<td>$\varepsilon_{lm}^{vp}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>0.3244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.0553</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>0.0085</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>$1.808 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soil permeability properties</td>
<td>$e_o$</td>
<td>2.678</td>
<td></td>
</tr>
<tr>
<td>$k_o$ (m/min)</td>
<td>$1.723 \times 10^{-7}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_k$</td>
<td>0.7353</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 6.29 Stress – strain relationship of Sample A of Skå-Edeby clay**
Figure 6.30 Settlement prediction adopting the TRRLS approach for Sample A of Skå-Edeby clay
Figure 6.31 Excess pore water pressure prediction adopting the TRRLS approach for Sample A of Skå-Edoby clay at loading stages (a) 80.42 kPa, (b) 166.71 kPa and (c) 338.38 kPa
The EVP model parameters obtained by the TRRLS approach are presented in Table 6.8. According to Figure 6.28, the average $C_{\theta}/\ln 10$ for the depths 5 m and 8 m are about 0.033 and 0.0189, respectively. The slope of the instant time-line ($\kappa/V = 0.0295$) of Sample B obtained by TRRLS approach is higher than $C_{\theta}/\ln 10$ at 8 m, but lower than $C_{\theta}/\ln 10$ at 5 m. In term of the correlated compression ratio, the average $C_{\theta}/\ln 10$ at both 5 m and 8 m depths is estimated to be approximately 0.11, and slightly higher than the optimised reference time-line slope ($\lambda/V$). Since the ratio of $C_{\omega}/C_{\omega}$ was recommended to be about 0.05 for the whole depth based on Hansbo (1960), the correlated $C_{\omega}/\ln 10$ was about 0.0055. For Sample B, the values of $a$, $b$, $c$, and $d$ indicate that the creep strain limit $\varepsilon_{\text{im}}^{\nu \sigma}$ and the creep coefficient $\psi_{\sigma}$ are both significantly stress dependent. For example, $\psi_{\sigma}$ decreases from 0.0056 at $\sigma'_{z} = 50\text{kPa}$ to 0.00497 at $\sigma'_{z} = 100\text{kPa}$. It is difficult to compare the creep compression properties obtained from the optimised model parameters and the reported conventional secondary compression coefficient, since $\psi_{\sigma}$ decreases with the increase of the effective stress. However, it is supposed that the value of $\psi_{\sigma}$ is generally close to the conventional secondary compression coefficient $C_{\omega}/\ln 10$.

In term of the soil permeability, the optimised value of $e_{o}$ is 2.02, and the corresponding $k_{o}$ and $c_{k}$ are $2.938\times 10^{-8}$ m/min and 1.32, respectively. Both void ratio and permeability of Sample B are lower than the corresponding values for Sample A. It is reasonable since Sample B is supposed to be taken in the deeper location compared with Sample A. The water content of the soil layer decreases with depth as shown in Figure 6.35, causing lower void ratios at deeper layers. Based on the optimised parameters and permeability properties, Sample B is assumed to be taken at 8m depth. Figure 6.32 shows the time-line system of Sample B based on the optimised parameters.

Figures 6.33 and 6.34 illustrate the predictions of settlements and excess pore water pressures for Sample B adopting optimised parameters obtained from the TRRLS approach (Table 6.8). The prediction of settlements of Sample B is remarkably in good agreement with the laboratory measurement. The coefficients of determination $R^2$ for Sample B are higher than 0.98 for three loading stages in Figure 6.34. In consideration of the excess pore water pressure, the predictions exhibit a similar time–dependent variation pattern as the measurement. However, there are the discrepancies between the
prediction and the measurements particularly during the early stage of the dissipation process, while the better fittings are achieved as the dissipation process progresses.

Table 6.7 Initial information adopted for the EVP model parameter determination for Sample B of Skå-Edeby clay

<table>
<thead>
<tr>
<th>Loading stage (kPa)</th>
<th>$H_0$ (m)</th>
<th>$\varepsilon_i$ (%)</th>
<th>$\sigma_i$ (kPa)</th>
<th>$\sigma_f = \sigma_i + \Delta\sigma_i$ (kPa)</th>
<th>$u_{id} = \Delta\sigma_i$ (kPa)</th>
<th>Duration (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80.42</td>
<td>0.0125</td>
<td>5.68</td>
<td>41.19</td>
<td>80.42</td>
<td>39.23</td>
<td>1</td>
</tr>
<tr>
<td>158.87</td>
<td>0.0125</td>
<td>9.97</td>
<td>80.42</td>
<td>158.87</td>
<td>78.45</td>
<td>1</td>
</tr>
<tr>
<td>261.84</td>
<td>0.0125</td>
<td>16.64</td>
<td>158.87</td>
<td>261.84</td>
<td>102.97</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.8 The optimised model parameters and soil permeability properties of for Sample B of Skå-Edeby clay

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Instant time-line</th>
<th>Reference time-line</th>
<th>Equivalent time-line</th>
<th>Soil permeability properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa V$</td>
<td>0.0295</td>
<td></td>
<td></td>
<td>$e_o$</td>
</tr>
<tr>
<td>$\lambda V$</td>
<td>0.0982</td>
<td>$\sigma'_{zo}$ (kPa)</td>
<td>39.023</td>
<td>$k_o$ (m/min)</td>
</tr>
<tr>
<td>$t_o$ (min)</td>
<td>1</td>
<td></td>
<td></td>
<td>$c_k$</td>
</tr>
<tr>
<td>$\varepsilon_{im}'$</td>
<td>$a$ = 0.613</td>
<td>$b$ = 0.099</td>
<td>$c$ = 0.0093</td>
<td>$2.938 \times 10^{-8}$</td>
</tr>
<tr>
<td>$\psi'_{o}$</td>
<td>$d$ = $9.4 \times 10^{-4}$</td>
<td></td>
<td></td>
<td>$1.32$</td>
</tr>
</tbody>
</table>
Figure 6.32 Stress – strain relationship of Sample B of Skå-Edeby clay

Figure 6.33 Settlement prediction adopting the TRRLS approach for Sample B of Skå-Edeby clay
Figure 6.34 Excess pore water pressure prediction adopting the TRRLS approach for Sample B of Skå-Edeby clay at loading stages (a) 80.42 kPa, (b) 158.71 kPa and (c) 261.84 kPa
6.3.3.2 Numerical modelling for Skå Edeby test fill

This section is to establish the numerical simulation and the required model parameters for Skå-Edeby test fill. As shown in Figure 6.35, the soil profile of the test fill is 12m deep of soft clay. As similar to Väsby case study, one-dimensional analysis can be carried out to predict the long-term settlement and the excess pore water pressure of the soft soil layer under the centre line of the test fill. The time dependent behaviour of the soil profile is simulated for 57 years since the beginning of the embankment construction.

In this case study, the soil profile as shown in Figure 6.35 is adopted based on information available in the literature such as Hansbo (1960), Lo (1991) and Larsson & Mattsson (2003). According to Lo (1991), the in-situ void ratio was computed via the in situ natural water content of the ground. The scatters of the void ratio were observed, since the soil layers within and near the ground surface were not uniform, and it was difficult to define accurately the representative void ratio profile. The unit weight of the soil profile is based on the reported values in Hansbo (1960).

In the simulation for field conditions, the selection of the coefficient of permeability is a challenging task. Several data of coefficient of permeability were reported for the test fill area such as Hansbo (1960) and Larsson (1986). Hansbo (1960) carried out small hydraulic gradient permeability tests for soil samples from 2 m, 5 m, 6 m and 8 m depths based on the flow rates at different hydraulic gradients after different degrees of compressions. Hansbo (1960) also reported the value of the permeability change index $c_k$ based on the slope of $e_o - \log k_v$. For Skå-Edeby clays, Larsson (1986) determined the permeability profiles using the constant rate of strain (CRS) tests. Additionally, Lo (1991) also included the estimation of the permeability based on the relationship of clay fraction and plasticity index. In this case study, the permeability profiles of the soil from 2 m to 12 m depths are adopted as the average values of Lo (1991) and Larsson (1986). The adopted permeability varies with depth as shown in Figure 6.35(c). The permeability change index $c_k = 0.5e_o$ is adopted as suggested in Hansbo (1960) and Tavenas et al. (1983).

Figure 6.36 illustrates the geotechnical profiles of the ground below the test fill including the initial effective stress ($\sigma'_{zi}$), the target final effective stress ($\sigma'_{zf}$), the preconsolidation pressure ($\sigma'_{pc}$), OCR and the applied stress increment distribution with depth ($\Delta\sigma_z$). The final effective stress is the sum of the initial values ($\sigma'_{zi}$) and the
applied stress increment ($\Delta \sigma_z$). The initial effective stress is calculated based on Equation (6.1) with the consideration of the groundwater table at 1 m depth. Since the seasonal variation of the groundwater table is about ±0.5 m, the top 1m layer is assumed saturated. Applying the same order as Vasby case study, the 0.5 m thick clay crust is assumed to be fully drained, and no excess pore water pressure remains in this layer. The soil layer from 0.5 m deep downward to the bottom surface is treated as undrained condition, and the initial excess pore water pressure is equal to the applied stress increment, which is distributed with depth.

The Skå-Edeby test fill is circular in shape with the base diameter of 35 m as shown in Figure 6.24. The geometry of the test fill exposed the loading intensity of 27 kPa on the ground surface. For Skå-Edeby test fill, Equation (6.2) is also applied to calculate the vertical applied stress increment distribution with depth. The stress influence factor $I_z$ is calculated based on Equation (6.6) applied to obtain for the stress increment beneath the centre of the circular loaded area (Boussinesq 1885).

$$I_z = 1 - \left( \frac{1}{1+\left( \frac{b}{z} \right)^2} \right)^{3/2}$$

(6.6)

where, $b$ is the radius of the test fill. In this case study, $b = 35 - 2.3 = 32.7$ m is calculated as the nominal diameter in consideration of the batters of the test fill embankment.

The preconsolidation pressure presented in Figure 6.36a adopted in this study is based on the reported value of Larsson (1986) based on the results of constant rate of strain (CRS) consolidation tests. Larsson (1986) concluded that the soil profile is normally consolidated from the depth of 2 m below the ground surface, while the top 1 m is slightly overconsolidated due to the drying effect. The overconsolidation ratio (OCR) is reported about 1.15 from the depth of 2.5 m toward the soil base (Larsson and Mattson 2003). Hansbo (1960) carried out a large number of oedometer tests, but did not report the OCR value. Holtz & Broms (1972) concluded that the clay below 5 m depth at Skå-Edeby is fundamentally normally consolidated corresponding to the effective overburden pressures. Holm & Holtz (1977) reported the mean OCR in range from 1.05 to 1.12. Based on the preconsolidation pressure and the initial effective stress, the initial vertical strain varying with depth is calculated based on Equation (6.5).

The optimised EVP model parameters for Samples A and B obtained in Section 6.3.3.1 are employed to analyse the long-term behaviour of the soft soil layer at Skå-
Edeby test fill. As mentioned in Section 6.3.3.1, the properties of Samples A and B are analogous to the properties of the soil layer at 2 m and 8 m depths. Therefore, in this case study, the top 6 m from the ground surface is applied the EVP model parameters of Sample A (Table 6.6), while the EVP model parameters of Sample B (Table 6.8) is applied for the soil deposit between 6 m and 12 m. The division of the model parameter application is based on the geological profile and the soil properties as shown in Figure 6.26. At the top 6 m, the soil layer comprised of grey-green to grey varved clay with sulphide flecks, while the bottom 6 m comprised grey-brown varved clay with mainly sulphide bands. Additionally, the water content range from the plastic limit and liquid limit of the bottom 6 m are almost uniform with depth, while the top 6 m had larger values of plastic limits and liquid limits, as well as water content. Thus, the initial void ratios of the top 6 m are slightly higher than the value of the bottom 6 m. In this numerical simulation, the total 12 m depth is divided into sub-layers of 0.02 m thick and, the duration of the simulation in this case study is 57 years. The boundary condition for Skå-Edeby is two-way drainage from the top and bottom surfaces.

Adopting two sets of the EVP model parameters of Samples A and B, Figure 6.37 shows the initial values of the creep coefficient, the creep strain limit and the creep parameter of the soil profile. As explained in Section 6.3.3.1, the value of parameter $d$ in the creep coefficient of Sample A (Equation 3.9) is insignificant compared to $c$. Thus, the initial creep coefficient of the top half of soil layer in Figure 6.37 is almost constant with depth, while the initial effective stress varies with depth. On the other hand, the creep coefficient of the bottom half of the soil layer shows the more evident variations with depth. The creep coefficient of Sample A is higher than the corresponding value of Sample B. Thus, the creep coefficients of the top half of the soil layer are higher than the values of the bottom half as shown in Figure 6.37(a).

The variations of the creep strain limit with depth are calculated using Equation (3.8). In general, the creep strain limit is stress-dependent as similar as the creep coefficient. Thus, the creep strain limit tends to decrease with the increase of the effective stress, or with depth. Moreover, the creep strain limit for Sample A is less than the creep strain limit for Sample B. Hence, the creep strain limit of the bottom half of the soil layer is higher than the top half.
*Data obtained from Hansbo (1960); Lo (1991); Larsson and Mattsson (2003)

**Figure 6.35** Soil properties adopted in the numerical modelling for Skå-Edeby test fill

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Initial void ratio ($e_o$)</th>
<th>Unit weight (kN/m$^3$)</th>
<th>Coefficient of permeability ($k$) ($\times 10^{-8}$m/min)</th>
<th>Permeability change index $c_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>a)</td>
<td>b)</td>
<td>c)</td>
<td>d)</td>
</tr>
<tr>
<td>1</td>
<td>Adopted value in the numerical simulation</td>
<td>Adopted value for analysis in Lo (1991)</td>
<td>Measured value from Larsson (1986)</td>
<td>** Figure 6.35 ** Soil properties adopted in the numerical modelling for Skå-Edeby test fill</td>
</tr>
</tbody>
</table>
Preconsolidation pressure $p_c$ is obtained from Larsson & Mattsson (2003)

Figure 6.36 Geotechnical profile adopted in the numerical modelling for Skå-Edeby test fill
Figure 6.37 Initial creep compression properties (a) Creep coefficient $\psi/V$, (b) Creep strain limit $\varepsilon_{\text{im}}$ and (c) Creep parameter $\psi/V$ for Skå-Edéby site
The creep parameter $\psi/V$ in Figure 6.37(c) is dependent on the creep coefficient $\psi/V$ in Figure 6.37(a) and the creep strain limit $\varepsilon_{\text{lim}}^{\text{vp}}$ in Figure 6.37(b), as well as the equivalent time $t_e$ (Equation 3.7). The value of $t_e$ can be calculated by Equation (3.11). Meanwhile, Figure 6.37(d) presents the variation of the initial vertical strain obtained using Equation (6.5). As mentioned in Case study of Vasby test fill, the initial vertical strain is influenced by the preconsolidation pressure, and the initial stress – strain points are determined to identify the initial position of the state with respect to the reference time-line, from which the time-dependent stress – strain is calculated.

6.3.4 Results and discussion

6.3.4.1 Settlement predictions
The predictions of the settlement under the centre of the test fill in 57 years since the construction of the embankment are provided and discussed in this section. Figures 6.38 and 6.39 show the predictions and the measurements of the vertical settlement in the linear scale and in the semi-logarithmic scale, respectively. Generally, the settlement predictions for the soil layers up to 5 m depths are in good agreement with the field measurements. Meanwhile, the predictions at 5 m (SM4) and 7.5 m (SM5) depths overestimate the field measurements during the early years after the construction. At CY40 (1987), the predictions at SM4 match measurements better even slightly underestimating the measured data, while the predictions at SM5 highly underestimate the field measurements. The overestimated predictions at SM4 and SM5 at early stages after construction may be due to the adopted permeability for the soil layers at these depths which may be higher than the in-situ values, since the soil is stratified. Moreover, the soil at 8 m depth may have less compressibility due to the presence of the sand and silt seems compared to the top soil layer. Additionally, the varves existing within the soil layer may affect the soil properties.

According to Larsson & Mattsson (2003), the initial settlement during the construction was about 0.06 m, while the settlement prediction at the end of the construction (70 days) is 0.041 m. The measured settlement at CY25 (1972) is about 0.75 m. The predicted value at the same time (CY25 – 1972) is 0.755 m, close to the field measurement. Figure 6.39 shows the predicted and measured settlements in the semi-logarithmic scale. The difference between the predictions and the measurements at
SM4 and SM5, shown in Figure 6.39, suggests that the preconsolidation pressure at the locations may be higher than the values adopted in the simulation.

**Figure 6.38 Vertical settlement predictions at several depths beneath the test fill at Skå-Edoby site**

**Figure 6.39 Vertical settlement predictions at several depths beneath the test fill in the logarithmic scale of time at Skå-Edoby site**
The distributions of settlements with depth at several time points can be examined in Figure 6.40. The predictions are in a good agreement with the measurements of the settlement markers up to 2 m depths from the top surface, while the predictions overestimate the settlements at deeper locations to the bottom layer. At CY24 (1981), the predictions and measurements with depth are nearly identical as shown in Figure 6.40b. The predictions at CY45 (2002) are different from the measurement of the markers, but in good agreement with settlements calculated by the change of water content. Larsson & Mattsson (2003) also observed the variations between the calculated settlements from the water contents and the measurements of the markers. However, they did not provide the reasons of variations, but confirmed that the settlement markers are still functioning (Larsson & Mattsson 2003).

Figures 6.38 and 6.39 also show the predictions of settlement at CY57 (2014). The ground surface settlement is predicted about 1.128 m. As observed in the semi-logarithmic plot in Figure 6.39, the settlement rate is predicted to decrease at CY57. It is supposed that the excess pore water pressure may have nearly reached the end of the dissipation process. The settlement prediction adopted the TRRLS approach is lower than the projection by Lo (1991). Lo (1991) predicted the end of primary consolidation settlement for the subjected Skå-Edeby test fill was 1.27 m, and reached 95% of the primary consolidation in 2008 (i.e. 51 years after the construction).
Figure 6.40 Variations of vertical settlements with depth at several time points (a) 1 year, 5 years, and 10 years, and (b) 24 years and 45 years at Skå-Edby site
6.3.4.2 Excess pore water pressure

Figures 6.41 to 6.43 present the predictions of the excess pore water pressures at 14 years (CY14–1971), 25 years (CY25–1982) and 45 years (CY45–2002) since the construction. Hansbo (1960) reported that the measured excess pore water pressure 1 year after the load application was slightly lower than the applied stress increment. Thus, the soil profile was normally consolidated (Larsson & Mattsson 2003).

According to Larsson & Mattsson (2003), the readings of piezometers had ceased after the first years of measurements and maintenance. Later, the retractable piezometers were used to investigate the changes of pore water pressures occasionally such as at CY14 (1971), CY25 (1982) and the latest in 2002 (CY45). The measurement in Figure 6.41 shows that 14 years after construction the maximum excess pore water pressure remained about 20 kPa. As a result, the effective stresses in major parts at the middle soil layer maintained unchanged. On the other hand, the corresponding prediction indicates a much higher maximum value of the excess pore water pressure occurring around the middle depth of the soil layer. The difference between the maximum predicted and measured values of the excess pore water pressure is about 8 kPa. The maximum predicted value of the excess pore water pressure is higher than the loading intensity at the same depth. In other words, the excess pore water pressure is predicted to increase instead of decreasing. The increase of the excess pore water pressure in the early periods of loading times was discussed in the case study of Drammen clay in Chapter 5. It is suggested that the effect of creep compression occurring in the early times of the dissipation process causes the delay in the dissipation of the excess pore water pressure at the undrained locations. In this case of two-way drainage condition, the middle areas of the soil layers have undergone compression and the dissipation of excess pore water pressure later than the areas near the top and bottom drained layers.

Additionally, the low permeability properties of the soil profile contribute to the lower flow rate of pore water, especially at the areas at the middle layer. The difference between the measurements and the predictions may be due to several factors such as the accuracy of measurement devices, the seasonal variations of the soil properties, as well as the adopted model parameters. The readings of piezometers were stopped working in Larsson & Mattsson (2003). The seasonal variations of groundwater table, temperature were reported to affect the settlement rate and soil properties of different depths of the
soil layer as mentioned in Section 6.3.4.1 (Larsson & Mattsson 2003). Besides, the adopted model parameters are obtained based on two soil samples with the assumptions of the depth locations. Even though the settlements are predicted well, the predictions of the excess pore water pressures are limited. Thus, it is suggested that more data represented for several depths can improve the predictions.

The prediction of the excess pore water pressure in 1982 (CY25) is improved, since the difference between the maximum prediction and maximum measurement is about 3 kPa (Figure 6.42). 25 years after the construction, the excess pore water pressure was still in the dissipation process. Larsson & Mattsson (2003) stated that the amount of settlement after 25 years caused the dry crust and top parts of the fill to be submerged, and the initial applied loading intensity with depth was reduced by one third. However, the numerical solution has not considered the submergence of the soil profile due to the settlement. In 2002, after 45 years of the construction, the maximum remaining excess pore water pressure is about 8 kPa, and the maximum prediction of 6 kPa is lower the measurement (Figure 6.43). As mentioned in the predictions of the settlement, in 2014 with the total settlement at the ground surface about 1.121 m. The maximum remaining excess pore water pressure in 2014 is predicted about 3.4 kPa, and the correlated degree of dissipation is about 86.85% at 5.6 m depth.

Figure 6.44 shows the variations of the coefficient of permeability of soil profile with time. The measurements were obtained by the constant rate of strain tests (Larsson & Mattsson 2003). The reductions in the permeability due to the settlement of the soil layer are observed with time. Larsson & Mattsson (2003) also emphasised the soil sample disturbance and the variations of the water content as well as the presences of highly varved soils may influence the measured values of permeability. The prediction of the permeability as shown in Figure 6.44 for 28 years (1985) is fairly agreed with the corresponding measurements. However, the permeability at CY45 (2002) is predicted not much different from the prediction at CY28 (1985), while being higher than the measured values at CY45 (2002). In general, the measured change of the permeability from the initial measurement to the measurement time are reasonably agreed with the predicted values.
Figure 6.41 Prediction of excess pore water pressure after 14 years from the end of construction in 1971 at Skå-Edeby site

Figure 6.42 Prediction of excess pore water pressure after 25 years from the end of construction in 1982 at Skå-Edeby site
Figure 6.43 Prediction of excess pore water pressure after 45 years from the end of construction in 2002 at Skå-Edeby site

Figure 6.44 Predictions and measurements of coefficient of permeability under the test fill at Skå-Edeby site
6.3.4.3 Creep compression properties

Figure 6.45 shows the variations of the creep coefficient \( \psi / V \) and the creep strain limit \( \varepsilon_{lp}^{vp} \) with depth at several time points. The creep coefficient \( \psi / V \) in Skå-Edeby test fill has minor variation both with depth and with time at the top half of the soil layer. It is due to the negligible value of \( d \) of Sample A. The bottom half of soil layer was predicted adopting the model parameters of Sample B. Thus, the variations of the creep coefficient \( \psi / V \) of the bottom half are more obvious compared to the top half. On the other hand, the changes in the creep strain limit \( \varepsilon_{lp}^{vp} \) are detectable in Figure 6.45(b). The creep strain limit \( \varepsilon_{lp}^{vp} \) decreases with the increase of the effective stress. The change of the creep strain limit \( \varepsilon_{lp}^{vp} \) in this case study is more remarkable than the change in Väsby case study.

The corresponding variations of the creep parameter \( \psi / V \) with depth and with time are presented in Figure 6.46(a), while Figure 6.46b illustrates the changes of the creep strain rate \( \dot{\varepsilon}_z^{vp} \) with depth and time. The variation of the creep parameter \( \psi / V \) at the top half is more influenced by the change of the creep strain limit \( \varepsilon_{lp}^{vp} \), since the change of the creep coefficient \( \psi / V \) is insignificant as shown in Figure 6.45a. On the other hand, the change of the creep parameter at the bottom half is considered to be influenced by both the creep parameter \( \psi / V \) and the creep strain limit \( \varepsilon_{lp}^{vp} \). As similar to Väsby test fill, the middle layer in this case study exhibited higher creep strain rates \( \dot{\varepsilon}_z^{vp} \) compared to the top and bottom areas. The dissipation of the excess pore water pressure at the middle soil layer has elapsed longer time as reported in Section 6.3.4.3. Thus, the compression in the middle layer is less than other areas, and the effective stress increases at slower rate. When the loading time increases, the effective stress and the compression at the top and bottom areas near the drainage boundaries become more stable, and the change in the creep strain rate \( \dot{\varepsilon}_z^{vp} \) with time at those area is less than the change in the middle layer.

The notable differences of the predictions of the creep strain limit, creep coefficient, creep parameter and creep strain rate at the top and bottom sections in Figures 6.45 and 6.46 are due to the differences of the adopted model parameters representing for the soils at different depths. The first half of the soil layer is represented by the optimised model parameters of Sample A, while the other half adopted the model parameters of Sample B. In spite of the differences of the creep properties with depth in
Figures 6.45 and 6.46, the predictions of the settlements and excess pore water pressures with depth and time are simulated smoothly and continuously as shown in Figures 6.40 to 6.43.

Figures 6.47 to 6.50 denote the variations of the creep compression properties including $\psi/\psi_p$, $\varepsilon^{\psi}_{im}$, $\psi/\psi$ and $\dot{\varepsilon}^{\psi}_{z}$ with time at three different depths including $z = H_o/4$, $H_o/2$ and $3H_o/4$. The similar patterns of the variations have been observed in Case study of Väsby test fill in Section 6.2.4.3. It is supposed that the excess pore water pressure initially increased higher than the maximum applied stress increment. As a consequence, the effective stress decreased lower than the initial effective stress, resulting in the increases of $\psi/\psi_p$, $\varepsilon^{\psi}_{im}$, $\psi/\psi$ and $\dot{\varepsilon}^{\psi}_{z}$ in the early years of the embankment construction. However, there is no available accurate field measurement of the excess pore water pressure in the early stages of the construction for observation. Due to the technical issues, all the piezometers at all test fill areas at Ska-Edeby were replaced by new devices one year after the end of the embankment constructions (Hansbo 1960). Moreover, the excess pore water pressures at different depths measured by the new piezometers indicated the continuous increasing instead of dissipating (Hansbo 1960). Hansbo (1960) also stated that the variation of the excess pore water pressure during the first two years after the construction (1957 – 1959) may be complexly influenced by the groundwater level and seasonal variations, which has not been captured in the field measurement. In general, the initial variations of the creep properties $\psi/\psi_p$, $\varepsilon^{\psi}_{im}$, $\psi/\psi$ and $\dot{\varepsilon}^{\psi}_{z}$ are due to the increase of the excess pore water pressure (i.e. the associated decrease of the effective stress). The increase of the excess pore water pressure, which has been discussed in Section 6.2.4.3, is due to the creep effect (Crawford 1986; Crooks et al. 1984; Yin & Graham 1996; Yin et al. 1994).

Creep properties including $\psi/\psi_p$, $\varepsilon^{\psi}_{im}$, $\psi/\psi$ and $\dot{\varepsilon}^{\psi}_{z}$ continuously decrease with time as shown in Figures 6.47 to 6.50 due to the increase of the effective stress. The stress-dependent behaviour is indicated via the variation of $\psi/\psi_p$, $\varepsilon^{\psi}_{im}$, $\psi/\psi$ and $\dot{\varepsilon}^{\psi}_{z}$ with depth. At the deeper depth, the effective stress is higher, resulting in the smaller values of $\psi/\psi_p$, $\varepsilon^{\psi}_{im}$, $\psi/\psi$ and $\dot{\varepsilon}^{\psi}_{z}$. The distinctive differences between the values of $\psi/\psi_p$ and $\varepsilon^{\psi}_{im}$ at $z = H_o/4$, $H_o/2$ and $z = 3H_o/4$ reported in Figures 6.47 and 6.48 are due to the adoption of different sets of initial model parameters obtained by two different soil samples (i.e. Samples A and B).
As similar to the variations of $\psi/V$ and $\varepsilon_{im}^{\text{eq}}$ with depth and time in Figures 6.47 and 6.48, the creep parameter $\psi/V$ in Figure 6.49 decreases with depth and time. The variation of $\psi/V$ with depth is due to the effective stress increasing with depth. Additionally, $\psi/V$ also decreases with time while there is a contribution due to the increases of the effective stress with time (Equation 3.7), and the variations of $\psi/V$ at different depths maintain decreasing almost parallel to each other after CY10 (10 years after the construction). The effective stress at $z = H_o/2$ tended to increase much slower compared to other locations closer to the drainage boundaries. Moreover, at the end of the embankment construction, the excess pore water pressure surrounding the undrained boundary may increase higher than the applied stress due to the coupled effects of relaxation and creep discussed in the case study of Vasby as well as the laboratory case of Drammen clay in Chapter 5. The increase of the excess pore water pressure may lead to the decrease of the corresponding effective stress. Consequently, the equivalent time $t_e$ as well as the creep strain limit and creep coefficient increased correspondingly. As a result, the corresponding creep parameter decreased until the dissipation of the excess pore water pressure speeded up. The quirk in Figure 6.49 shows the change of the creep parameter due to the change of the effective stress. As consolidation proceeds, the effective stress increases corresponding to the dissipation of the excess pore water pressure. The creep parameter was influenced more by the effect of time, as the increase of the effective stress became stable.

Figure 6.50 presents the changes of the creep strain rate with time at different depths. At $z = 3H_o/4$ the creep strain rate is much lower compared to the values at other two depths, while the creep strain rates at $z = H_o/4$ and $H_o/2$ are almost identical. Since the creep strain rates at $z = H_o/4$ and $H_o/2$ are almost the same, the stress – strain points at those depths are suggested to have almost the same equivalent time $t_e$ (i.e. stay on almost the same time-line). According to the initial effective stress (Figure 6.36a) and the excess pore pressure responses in Figures 6.41 to 6.43, the effective stress at the depth $z = 3H_o/4$ is higher than the upper depths. Thus, the creep strain rate corresponding to $z = 3H_o/4$ is also lower than the values at other two depths. On the other hand, the initial effective stress at $z = H_o/4$ is less than that value at $z = H_o/2$, but the excess pore water pressure at $z = H_o/4$ dissipated faster compared to the value at $z = H_o/2$. Therefore, the effective stress at $z = H_o/4$ is slightly higher than the value at $z = H_o/2$. Since the creep strain rates at $z = H_o/4$ and $H_o/2$ are almost the same, the vertical
strain at $z = H_o/4$ is higher than the strain at $z = H_o/2$. Thus, the settlement at $z = H_o/4$ is greater than the settlement at $z = H_o/2$, which is compatible to the predictions in Figure 6.40.

![Figure 6.45 Variations of (a) Creep coefficient $\psi_o/V(\times 10^{-3})$ and (b) Creep strain limit $\varepsilon_{im}^{up}$ with depth at CY11, CY22, CY45 and CY57 for Skå-Edeby site](image)

6-69
Figure 6.46 Variations of (a) Creep parameter $\psi/V$ and (b) Creep strain rate $\dot{\varepsilon}_z^{vp}$ with depth at CY11, CY22, CY45 and CY57 for Skå-Edeby site
Figure 6.47 Variation of the creep coefficient ($\psi_o/V$) with time at (a) $z = H_o/4$ and $H_o/2$ and (b) $z = 3H_o/4$ for the Skå-Edeby test fill
Figure 6.48 The variation of the creep strain limit $\varepsilon_{im}^{up}$ with time at $z = H_o/4$, $H_o/2$ and $3H_o/4$ at the Skå-Edoby test fill.

Figure 6.49 Variation of the creep parameter $\psi/V$ with time at $z = H_o/4$, $H_o/2$ and $3H_o/4$ at the Skå-Edoby test fill.
6.4 Summary

In this chapter, the proposed trust-region reflective least square (TRRLS) approach has been applied for two in-situ case studies of test fills at Väsby and Skå-Edeby in Sweden. The test fills are located on highly compressible and sensitive clays, which expose the typical long term behaviour of soft soils. Applying the TRRLS approach for the laboratory incremental oedometer test results, the EVP model parameters along with the values of permeabilities properties ($k_o$, $e_o$ and $c_k$) are obtained. In both case studies, the predictions of the settlements for Väsby soil sample, and both settlements and excess pore water pressures for two Skå-Edeby samples are in good agreement with the laboratory measurements. The coefficients of the determinations $R^2$ are always higher than 0.95 for both case studies.

Väsby test fill and Skå-Edeby test fill have similar soil profile including thick layers of post glacial clay underlaid by glacial deposit and thin crust at the top. The stress history shows that the soil at Skå-Edeby site is normally consolidated, while the soil in Väsby site is slight overconsolidated except for the heavily overconsolidated crust at the top 1 m affected by the drying effect. Both of the test fills have been subjected to investigate the long term behaviour of soft soils, and there have been long and comprehensive test data reported by various researchers such as Chang (1969, 1970).
In the case of Väsby test fill, the EVP model parameters obtained from one oedometer soil sample adopting the TRRLS approach are applied to predict the long term settlement and excess pore water pressure under the centre of the test fill. On the other hand, two sets of the EVP model parameters for different layers for the simulation at Skå-Edeby site are optimised using the TRRLS approach. Thus, for Skå-Edeby case study, one set of the EVP model parameter is applied for the first half of the soil layer, and another set of model parameters is applied for the bottom half of the soil layer. The optimised EVP model parameters for both case studies are reasonable, since they are close to the conventional compression parameters except for the creep compression properties. For both case studies, the stress-dependent creep strain limit $\varepsilon_{im}^{cp}$ and creep coefficient $\psi/\sqrt{V}$ are applied using the model parameters obtained by the TRRLS approach.

The numerical solution for the coupled EVP model and the consolidation theory is applied to analyse the one-dimensional compression and the variation of the excess pore water pressure under the centre of the test fills. The two-way drainage conditions are adopted for both case studies. Additionally, the initial effective stresses are calculated in the consideration of the self-weight of the uppermost soil layers. The depth-varied stress distribution of the loading intensity induced by the test fills are calculated based on the equations of Boussineqs applied for the strip footing for Väsby case study and the circular loaded area for Skå-Edeby site. The load intensity increased linearly with time during 25 days at Väsby test fill and 70 days at Skå-Edeby site. The top 0.5m dry crust is assumed to be fully drained, and no excess pore water pressure remains in this layer. Meanwhile, the excess pore water pressures at deeper layers dissipate with time. The drainage conditions are applied for both case studies.

The application of the TRRLS approach in the model parameter determination has been proved effective via the two case studies. The EVP model with the optimised model parameters provides good settlement predictions for both case studies. The predictions of the settlements of the upper areas near the top surface are in very good agreement with the field measurements. In regard to the prediction of the excess pore water pressure, the predictions of the maximum remaining excess pore water pressures are higher than the measured data for both test fills, although the trends are comparable. The numerical simulation in this study does not consider the reduction of load intensity
with time due to the ground level settled below the groundwater table (i.e. buoyancy effect). Therefore, the predicted value of the excess pore water pressure corresponding to the unchanged load intensity is higher than the measured value. Additionally, the predictions of the permeability profile for both cases are reasonably close to the measurements.

In general, the predictions of the settlements and excess pore water pressures by applying the EVP numerical solution and the optimised model parameters have proved the effectiveness of the EVP model with the non-linear creep function. Especially, the EVP model parameters obtained by the TRRLS approach has strengthened the application of the EVP model, since the model parameters can be obtained simply and provide high quality predictions. Therefore, it can be concluded that the TRRLS approach is not only effective to apply for the soil sample in the laboratory conditions, but also for thick soft soil layers in the field conditions. The numerical solution for the coupled EVP model and consolidation theory can be improved and extended for more complicated applications.
7 CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary
This thesis has provided an introduction and a comprehensive literature review in Chapters 1 and 2 about the time–dependent stress – strain behaviour of soft soils, especially about the compression induced by creep. Chapter 2 reviews several creep mechanisms along with the description of soft soils and the associated problems. Furthermore, the time-dependent stress-strain behaviour of soft soils have been reviewed and discussed in regard to the responses under different influencing factors such as time, strain rate and stress rate. Chapter 2 also discusses the suppositions of Hypotheses A and B in relation to the creep contribution to the deformation particularly at the end of the dissipation of the excess pore water pressure. According to Hypothesis A, the value of void ratio at the end of primary consolidation ($e_{EOP}$) is unique regardless of the soil sample thickness and drainage conditions. On the other hand, Hypothesis B presumes that $e_{EOP}$ is influenced by the sample thickness and drainage conditionson, and $e_{EOP}$ is not a unique parameter for a particular soil. Moreover, several constitutive models proposed for the numerical simulation of time–dependent behaviour of soft soils have been reviewed in Chapter 2.

In Chapter 3, the detailed description and the discussion on the merits and weaknesses of the non-linear elastic visco–plastic model have been provided. Since the limitations of the non-linear creep functions stay in the model parameter determination procedure, a numerical solution based on the trust-region reflective least square algorithms to determine the model parameters has been proposed and explained in detail in Chapter 3. In order to verify the proposed solution presented in Chapter 3, Chapter 4 provides the validation exercise based on laboratory experiments conducted using two sizes of the hydraulic consolidation Rowe cell systems. A 29.5 mm thick soil sample of a kaolinite mixture was tested and adopted to determine the set of the model parameters. A laboratory experimental result of a thicker soil sample (i.e. 140.5 mm thick) was
compared with the predictions using the optimised model parameters obtained from the thin sample.

Chapters 5 and 6 present the further verification works to evaluate the efficiency of the proposed method for the model parameter determination. Chapter 5 provided two laboratory–based case studies including the Hong Kong marine clay (Hong Kong) and the Drammen clay (Norway). In the Hong Kong marine clay, the model parameters were obtained applying the proposed solution to the available consolidation data reported in Yin (1999). The settlement predictions obtained by two different sets of the model parameters, one obtained by the proposed method and another obtained by the conventional method reported in Yin (1999) were compared. Then, the two sets of the model parameters were applied to predict the settlements and the excess pore water pressures for five soil layers of different thicknesses. Consequently, the analysis and discussions on the numerical simulation were included. In another case study (the Drammen clay), based on the available laboratory test results on four soil samples of different thicknesses reported in Berre & Iversen (1972), a set of model parameter was obtained by the proposed method using the laboratory test results of the thinnest soil sample of the Drammen clay. Adopting the optimised model parameters, the settlements and the excess pore water pressures were predicted for three thicker soil samples of the same soils. Subsequently, the predictions were compared to the laboratory measurements.

Chapter 6 provides a further verification for the proposed method based on two field case studies including the Väsby test fill and the Skå-Ededy test fill, both built in Sweden. The laboratory and field measurements of the case studies were collected from the available information in the literature. Based on the accessible laboratory experiments, the model parameters of the subsoils in the case studies were attained applying the proposed method. The model parameters consequently were adopted to predict the long-term settlements and the response of the excess pore water pressures in 67 years and 57 years for the Väsby test fill and the Skå-Ededy test fill, respectively. As a result, the comparison and discussion on the predictions and the field measurements were provided in Chapter 6.
7.2 Conclusions

Soft soil creep has remained one of the inspiring research topics in geotechnical engineering for the last few decades. The long-term settlement prediction is one of the key challenges for soft soil analysis, and the elastoplastic constitutive models excluding the viscous behaviour cannot describe the time-dependent behaviour of soft soils properly. Yin (1999) non-linear creep function embedded in the elastic visco-plastic model is one of the few constitutive models, which can simulate non-linear time-dependent compression. The non-linear creep function can describe the visco-plastic behaviour of soft soils by introducing the stress-dependent creep strain limit and creep coefficient. Based on the non-linear creep function, as time approaches infinity, the total strain will reach a strain limit at a particular effective stress.

Although the elastic visco-plastic model (EVP) proposed by Yin (1999) is sound, the procedure for the model parameter determination has limitations for the practical uses. The general equation of the EVP model is simple and practical with a reasonable number of model parameters. Although the model parameters can be derived from the standard incremental loading tests, difficulties and uncertainties are the main challenges to define a set of model parameters which can truly represent for the soil behaviour as their proposed meanings. The example of Ottawa clay in Chapter 3 showed the effects of \( t_0 \) (obtained from the conventional procedure) on other model parameters, significantly influencing the long-term settlement predictions in a field situation.

In order to improve the performance of the EVP model via the determination of the model parameters, a rigorous method has been proposed incorporating the advanced trust-region reflective least squares algorithm and the Crank-Nicolson finite difference solution for the simulation of the one-dimensional stress-strain behaviour adopting the EVP model. The Crank-Nicolson finite difference scheme was chosen due to its unconditional stability and convergence. Since the ratio of a chosen time step over the square of a chosen space step is larger than zero, the stability is satisfied. Additionally, the truncation error in the Crank–Nicolson method is much lower, compared to other explicit and implicit finite difference schemes. Different boundary conditions including one-way drainage and two-way drainage were clearly defined and implemented to solve the finite difference equations.

The trust-region reflective least square algorithm, adopted in this study, is an advanced optimisation method. In combination with the Crank–Nicolson finite
difference solution for the numerical simulation, the proposed method for the model parameter determination has been proved to be an efficient tool to analyse the time–dependent stress – strain behaviour of soft soils. In the trust–region algorithm, the approximation function of the objective function is a quadratic function, which can ensure the global convergence to second–order criteria and expose a faster rate of local convergence. The termination tolerances are adopted to be less than $10^{-10}$ to achieve high accuracy of the optimised model parameters. In most of the case studies, the optimised model parameters provided high coefficient of determination, $R^2$, for the predictions of the settlements compared with the measurement data.

For the model parameter determination the proposed method utilises all consolidation data from the beginning of the loading stages to increase the number of data points with the adoption of $t_0$ as 1 (unit of time). As a result, the optimised model parameters can provide better predictions during and after the dissipation of the excess pore water pressure. Along with the model parameters, the permeability properties such as the coefficient of permeability ($k_o$) and the coefficient of permeability change index ($c_i$) can be included in the optimisation procedure. In this study, the consolidation data of the normally consolidation range were adopted for the optimisation process, since the loading stages in the normally consolidated range exhibit creep compression more significantly compared to the overconsolidated stress stages. Consequently, the optimised creep parameters including the creep strain limit ($\varepsilon_{im}^{pp}$) and the creep coefficient ($\psi_o/V$) can describe the creep behaviour efficiently. It should be noted that depending on the soil type, the creep strain limit ($\varepsilon_{im}^{pp}$) and the creep coefficient ($\psi_o/V$) may be stress–dependent or stress–independent. Due to the non–linear creep behaviour, the time–lines may not be parallel to the reference time–line, which is different from the original EVP model with the linear creep function.

The laboratory experiment results of Rowe cell tests, presented in Chapter 4, clearly revealed that the thickness of the soil samples influences the long–term settlement as well as the excess pore water pressure response. Referring to the experimental results, if the soil sample is thicker, the elapsed time will be more for dissipation of the excess pore water pressures at the base of the soil sample. Associated with the measurement, the initial excess pore water pressures were less than the applied stress increments. The similar trend of the excess pore water pressure response has been observed in the literature such as Berre & Iversen (1972) (i.e. the Drammen clay case
study in Chapter 5), Hansbo (1960), and Robinson (1999). Disparities may be due to the stiffness of the measurement system or partial drainage path provided at the connection between the measurement location and the pressure transducer (Robinson 1999; Charlie 2000). Therefore, there are disparities between the measurement results and the predictions of the excess pore water pressure, which remain a challenge for analysing the excess pore water pressure.

Moreover, the EVP model can also simulate the increase of the excess pore water pressure in the early stages of loading (after application of the full load). The increase of the excess pore water pressure during the consolidation process has been observed in the laboratory and field case studies. The thickness of the soil layer is an important factor for this phenomenon, since the unusual increase of the excess pore water pressure occurs in the thick soil layer in the case study of the kaolinite mixture (Chapter 4), the thick soil sample ($H_o = 450$ mm) of Drammen clay (Chapter 5), and the thick soil profiles of Väsby and Skå–Edeby test fills (Chapter 6). The numerical predictions of the excess pore water pressure adopting the optimised model parameters in those case studies were in reasonably good agreement with measurements.

The Crank-Nicolson finite difference solution for the partial differential equations was coded in MATLAB in combination with the optimisation method. The finite difference solution included both the time–dependent and time–independent loading cases. Therefore, the numerical simulation can be applied for the field conditions for Väsby and Skå–Edeby test fills. The computation program was practical and effective to simulate the one–dimensional loading conditions. The predictions of settlement and excess pore water pressure in both field case studies in Chapter 6 had good agreement with the field observations.

In general, the following conclusions were derived from the analyses of the case studies applying the model parameters obtained by the proposed method:

- The elastic visco–plastic model with the non–linear creep function can produce highly accurate predictions for laboratory as well as field conditions.
- The proposed method is a simple but yet reliable solution for the model parameter determination. A set of model parameters can be optimised using loading stages covering the required stress level. Moreover, the permeability
properties can also be included in the model parameter determination process.

- The optimised model parameters are comparable with the conventional consolidation and compression indices (i.e. $C_r$, $C_c$, and $C_o$). Applying the proposed method, the stress-dependency of the creep strain limit $\varepsilon_{im}^{vp}$ and the creep coefficient $\psi_c/V$ can be identified.

- In this study, the values of the creep strain limit $\varepsilon_{im}^{vp}$ and the creep coefficient $\psi_c/V$ are defined to linearly decrease against the increase of the effective stresses. It should be noted that depending on the soil, the values of $\varepsilon_{im}^{vp}$ and $\psi_c/V$ can be considered to be stress-independent when the coefficients $b$ and $d$ in Equations (3.8) and (3.9) are zero (or very small), respectively.

- The values of the creep strain limit $\varepsilon_{im}^{vp}$ and the creep coefficient $\psi_c/V$ vary during the dissipation of the excess pore water pressure. The values approach and maintain at certain values, as the effective stress reaches the target value. On the other hand, the value of the creep parameter ($\psi/V$) derived from Equation (3.7) is not only stress-dependent but also time-dependent, as $\psi/V$ is influenced by $\varepsilon_{im}^{vp}$ and $\psi_c/V$ as well as the equivalent time $t_e$.

- The creep strain rate is influenced by the thickness of the soil layer. At the same initial stress – strain conditions, the thinner soil layer has the higher value of the creep strain rate. Considering the simulated field case studies, the deeper layers also experience lower creep strain rates. These observations are in good agreement with the existing observations in the literature.

The proposed optimisation method for the model parameter determination has been proved to be efficient to analyse the time-dependent behaviour of soft soils. The proposed method may be used to obtain the model parameters from the in-situ measurement data. In order to apply the proposed method for the model parameter determination for layered soils, settlement measurements at different depths are required for the optimisation. Thus, for different soil layers within the soil profile, several settlement plates are required to collect the measurement data of the time-dependent
settlement at different depths for the optimisation procedure. The finite difference scheme may be extended to simulate the layered soil profile. Additionally, the proposed optimisation solution can be modified to obtain the model parameters for different soil layers, when the number of variables is less than the number of available data points.

7.3 Recommendations for future research

This research can be further extended by conducting the following studies:

- In this study, the numerical solution has been applied for one-dimensional stress-strain conditions. It is possible to extend the proposed method to three-dimensional conditions, since the elastic visco–plastic model can be extended to 3D by incorporating the over-stress theory. Once the procedure is extended to the 3D conditions, the proposed solution may determine the model parameters applying the laboratory experimental results obtained from the triaxial tests.

- The loading stages within the overconsolidated range or within the transition between the overconsolidated to normally consolidated range can be included in the optimisation procedure. Consequently, it can be an objective for further research.

- The finite difference solution can be modified to simulate different consolidation tests such as isotropic consolidation, constant rate of strain. Thus, the optimisation method may also be used to obtain the model parameters from different types of tests.

- The elastic visco–plastic model with the non–linear creep function can be embedded in other computational programs such as FLAC and ABAQUS to improve the accuracy of the computation and extend the stress-strain conditions.

- The ground improvement methods such as vertical drains and pre-loading have become more popular to improve soft soil performance under embankments. The proposed method with the finite difference solution can be extended and verified based on case studies associated with vertical drains assisted pre-loading in 2D and 3D consolidation conditions.

- It is recommended to conduct an array of laboratory tests including undrained creep, relaxation, constant rate of strain, and controlled gradient
tests in order to verify the proposed method. Additionally, the variations of the excess pore water pressure in those tests can be an important topic for further investigation.

- More field case studies can be analysed to improve the performance and application of the proposed method. Especially, the proposed finite difference based method can be improved to obtain the model parameters from the in-situ measurements. Several settlement plates at different depths of the soil profile should be available to perform the required analyses. As a result, the differences between the model parameters obtained from the laboratory results and field measurements can be examined and provide a better understanding of the stress-strain behaviour of the soft soil.

- More field case studies involving sophisticated loading conditions including unloading-reloading stages and axisymmetric condition can be extended to further investigate the application of the proposed method to the field conditions.
REFERENCES


AS 1726 1993, Geotechnical site investigations, Standards Australia.


Chang, Y.C.E. 1969, 'Long-term consolidation beneath the test fills at Vasby Sweden', University of Illinois, Urbana - Champaign.

Chang, Y.C.E. 1981, Long term consolidation beneath the test fills at Vasby (Report No. 13), Swedish Geotechnical Institute, Linkoping.


Claesson, P. 2003, 'Long term settlements in soft clays', Chalmers University of Technology, Gothenburg.


Gupta, B. 1964, 'Creep of saturated soil at different temperatures', University of British Columbia, Canada.


Havel, F. 2004, 'Creep in soft soils', Norwegian University of Science and Technology.


Indraratna, B. 2010, 'Recent advances in the application of vertical drains and vacuum preloading in soft soil stabilisation', 2009 *EH David Memorial Lecture*, *Australian Geomechanics Society*, vol. 45, no. 2, pp. 1-43.


Larsson, R. 1986, Consolidation of soft soils (Report No. 29), Swedish Geotechnical Institute, Linkoping.

Larsson, R. & Mattsson, H. 2003, Settlements and shear strength increase below embankments (Report No. 63), Swedish Geotechnical Institute, Linkoping.


Taylor, D.W. 1942, Research on Consolidation of Clays, Department of Civil and Sanitary Engineering, Massachusetts Institute of Technology.


Terzaghi, K. 1923, Die Berechnung der Durchlassigkeitziffer des Tones aus dem Verlauf der hydrodynamischen Spannungserseheinungen(Th e computation of permeability of clays from the progress of hydrodynamic strain), vol. Part IIa 132. 3, no. 4, Der Wissenschaften in Wien, Sitzungsberichte, Mathematisch-naturwissenschaftliche Klasse.


Tidfors, M. 1987, Temperaturens paverkan pa leras deformationsegenskaper-en laboratoriestudie, Chalmers University of Technology.


Ref - 11


