

RAROC-Based Contingent Claim Valuation

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Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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ABSTRACT

The present dissertation investigates the valuation of a contingent claim based on the criterion RAROC, an abbreviation of Risk-Adjusted Return on Capital. RAROC is defined as the ratio of expected return to risk, and may therefore be regarded as a performance measure. RAROC-based pricing theory can indeed be considered as a subclass of the broader ‘good-deal’ pricing theory, developed by Bernardo and Ledoit (2000) and Cochrane and Saá-Requejo (2000). By fixing some specific target value of RAROC, a RAROC-based good-deal price for a contingent claim is determined as follows: upon charging the counterparty with this price and using available funds, we are able to construct a hedging portfolio such that the maximum achievable RAROC of our hedged position meets the target RAROC.

As a first step, we consider the standard Black-Scholes model, but allow only static hedging strategies. Assuming that the contingent claim in question is a call option, we examine the behavior of maximum value of RAROC as a function of initial call price, as well as the corresponding optimal static hedging strategy. In this analysis we consider two specifications for the risk component of RAROC, namely Value-at-Risk and Expected Shortfall.

Subsequently, we allow continuous-time trading strategies, while remaining in the Black-Scholes framework. In this case we suppose that the initial price of the call option is limited to be below the Black-Scholes price. Perfect hedging is thus impossible, and the position must contain some residual risk. For ease of analysis, we restrict our attention to a specific class of hedging strategies and examine the maximum RAROC for each strategy in this class. In the interest of tractability, the version of RAROC adopted risk is measured simply as expected loss.

While the previous approach only permits us to examine the constrained maximum RAROC over a specific class of hedging strategies, we would like to employ a more general method in order to study the global maximum RAROC over all hedging strategies. To do so, we introduce the notion of dynamic RAROC-based good-deal prices. In particular, with reference to the dynamic good-deal pricing theory of Becherer (2009), such prices are required to satisfy certain dynamic conditions, so that inconsistent decision-making between different times can be avoided. This task is accomplished

by constructing prices that behave like time-consistent dynamic coherent risk measures. As soon as the construction process is finished, we set up a discrete time incomplete market, and demonstrate how to determine the dynamic RAROC-based good-deal price for a call option. Furthermore, by following Becherer (2009), we derive the dynamics of RAROC-based good-deal prices as solutions for discrete-time backward stochastic difference equations. Finally, we introduce RAROC-based good-deal hedging strategies, and examine their representation in terms of discrete-time backward stochastic difference equations.

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