Parameter identification and sensitivity analysis of an improved LuGre friction model for magnetorheological elastomer base isolator

Yang Yu, Yancheng Li and Jianchun Li

Centre for Built Infrastructure Research, School of Civil and Environmental Engineering, Faculty of Engineering and Information Technology, University of Technology Sydney, NSW 2007, Australia

Email: yang.yu@uts.edu.au

**Abstract**

The recently-developed magnetorheological elastomer (MRE) base isolator can provide an instant change in the shear modulus and damping property under applied magnetic field, which makes it as an ideal device for the semi-active control in buildings and bridges. Previous studies show that this new device is featured with its nonlinear and hysteretic responses, and it is necessary to sufficiently understand its behaviour when adopting this device in control system. Although there are several models presented to predict the hysteresis of MRE base isolator, they are always suffered from some application limitations, e.g. high computation demand or complex model. To better interpret this complicated feature of the device, this work presents an improved LuGre friction model, which has been successfully used in modelling other magnetorheological device i.e. MR damper. In addition, an improved fruit fly optimization algorithm (IFFOA) is also proposed to identify the model parameters. In the improved algorithm, a transfer factor based on a self-adaptive step is added together with a three-dimensional searching space. This improvement can enhance the convergence rate of the algorithm and avoid the local optimum. Furthermore, to reduce the complexity of the model, the local and global parameter sensitivity analyses are conducted for model simplification. Eventually, the experimental measurements of device displacement, velocity and shear force are used to evaluate the performance of the proposed model and IFFOA.

**Keywords** Magnetorheological elastomer base isolator • LuGre friction model • Parameter identification • Sensitivity analysis • Fruit fly optimization algorithm

**1 Introduction**

Recently, the magnetorhological elastomer (MRE) base isolator has regarded as one of the most promising candidates in vibration control and structure protection from the earthquakes [1-2]. It is a device filled with the intelligent material MRE, which mainly consists of iron additives and elastomer matrix [3-4]. Different from the magnetorheological fluid (MRF), MRE has the controllable stiffness and is able to work effectively at broad frequency bands and sustain big compression or shear deformations. In views of these benefits, the MRE based devices, such as MRE base isolator, vibration isolator and vibration absorber [4], have been adopted as controllable semi-active components in the application of structural vibration isolation.

Although MRE based devices have many favourable advantages in vibration isolation, the main issue for the engineering application is that the devices have strong hysteretic and nonlinear behaviours due to the nonlinear rheological property of the material. To characterize these complex behaviours of the MRE based devices, several models have been presented in last few years, such as Bouc-Wen model [5], strain stiffening model [6-7] and artificial neural network model [8]. The Bouc-wen model is a classical model to describe dynamic behaviour of MR devices, in which a nonlinear differential equation is used to characterize the hysteresis. However, the Bouc-Wen model has a large number of parameters to be identified and especially an exponential parameter in the differential equation increases the difficulty in model identification. Strain stiffening model is designed according to the straining hardening behaviour in MRE base isolator. It has very high modelling accuracy with fewer parameters but two complicated differential equations in the model increase the model complexity. Different from Bouc-Wen model and strain stiffening model, artificial neural network is a nonparametric model for MRE base isolator. It has advantages of simple model structure, rapid training rate and high prediction accuracy. However, this model requires a large amount of training samples and is not able to demonstrate the inner physical relationships between model parameters and hysteretic phenomenon. Accordingly, the model that could clearly transfer the hysteretic behaviours of the device into the physical parameter illustration is in great need.

On the other hand, to obtain the perfect modelling accuracy, evolutionary calculation methods are also utilized in applications of modelling and parameter identification [9-11]. In [10], an enhanced genetic algorithm (GA) was proposed to identify the parameters of an improved Dahl model. In [11], the particle swarm optimization was also modified for parameter identification of a strain stiffening model. However, due to the complexity of identified models, these algorithms are disadvantageous in locating the global optimal solutions unless the good initial values of parameters are selected. In addition, the priori knowledge about the scopes of parameters is needed to obtain a faster convergence speed.

A novel optimization algorithm, fruit fly optimization algorithm (FFOA), is developed and regarded as an ideal method for model parameter identification [12]. It is essentially a sort of evolutionary optimization and calculation algorithm, which is designed to search for the global optimal values on account of the group behaviours of fruit flies to find food. The beneficial feature of this algorithm is that the fly always has the obvious advantages in perception and sensing, mainly reflecting in organs of vision and osphresis. Besides, the algorithm procedure of IFFOA is easy to understand, which could be realized by relatively short program codes. In view of these benefits, the IFFOA has been widely used in many application fields, such as parameter adjustment of PID controller [13], support vector machine parameter optimization [14], and regression network model parameter optimization [15]. Nevertheless, due to rapid response to the currently optimal smell position, the diversity of solutions will be lost if the global optimal position is far away from a converged fly swarm. This phenomenon is very similar to that in other optimization algorithms, which means the algorithm premature or fallen into the local optimum. Therefore, this problem should be dealt with before applying IFFOA in parameter identification of MRE base isolator model.

Another greatly interesting issue for modelling MRE based devices is how to simplify the existing models and yet maintain the similar modelling accuracy. During the process of parameter identification, all model parameters are usually identified in a parallel way simultaneously, which may increase the calculation cost and bring about the longer computational demand. Moreover, some studies have certified that the mathematical models can be simplified with pre-set properties by adding a certain number of restrictions, because the parameters in the model are correlative and some of them are essentially redundant. Thus it is of great significance to explore the influence of the model parameters on the model outputs. This operation is also called sensitivity analysis and aims at measuring how the model outputs quantitatively changes with the varied input parameters [16]. The final outcome of sensitivity analysis is to get rid of the unimportant parameters in the model and reduce the model complexity. Up to present, there are a large number of approaches for model parameter sensitivity analysis. A review on this aspect has been reported by Hamby, in which two popular techniques were proposed for simplifying the environmental models [17].

This work proposes a novel numerical model for describing the input/output behaviours of MRE base isolator. This new model is built on an improved LuGre friction model, which has been successfully applied to structural control and identification using magnetorheological dampers but not yet reported in modelling MRE base isolator. Compared with the Bouc-Wen model and strain stiffening model, the improved LuGre friction model has a simpler structure and is able to offer a satisfactory accuracy when modelling the unique behaviours of the MRE based devices. Then a new algorithm based on fruit fly optimization is adopted to identify the model parameters utilizing the testing data from MRE base isolator. To improve the convergence rate and identification accuracy, a self-adaptive method is employed to update the step for the optimal fly position. Additionally, a transfer factor is added to avoid the local optimum. The searching scale is also upgraded to three-dimensional space for decreasing the iteration number. Furthermore, the sensitivity analysis is carried out to evaluate the effect of every parameter on the model outputs. A simplified model is consequently acquired by assigning the insignificant parameters as the constants. Finally, the relationships between parameters in the simplified model and applied currents are also investigated.

The reminder of this paper is concluded as follows: Section 2 specifies the structure of the improved LuGre friction model as well as the problem description for parameter identification. Section 3 detailed introduces the procedure of the IFFOA, whose performance is proved by testing two nonlinear functions. In Section 4, the capacity of the proposed model is appraised using IFFOA together with parameter sensitivity analysis. Finally, a conclusion is drawn in Section 5.

**2 System model**

In modelling the dynamical behaviour of MRE base isolator, the main challenge is to describe the strain-stiffening property in force-displacement responses and nonlinear force-velocity relationship. To precisely characterize the hysteretic responses of the device, a novel parametric model is presented, which is composed of a viscous damper, a spring and a LuGre friction component. The structure of this model has been shown in Fig. 1 and its mathematical expression is given by [18-20]:

 (1a)

 (1b)

where *F*(*t*) is the shear force of the model output at time *t*; *x*(*t*) and denote the displacement and velocity of the device at time *t*, respectively; *k*0, *c*0 and *f*0 are the spring stiffness, viscous damping and initial force of the device, respectively; *y* is an intermediate variable; *α*, *β* and *ε* are three non-dimensional parameters, which are used to control the shape and scale of the hysteretic loop.



Fig. 1 Structure of improved LuGre friction model

Compare with the Bouc-Wen model, this novel model has fewer model parameters to be identified. However, due to the nonlinear differential equation in the model, the parameters are difficult to be identified directly. So as to acquire the best solutions of the parameters in the improved LuGre friction model, a proper fitness function should be built and minimized via optimization process. In this work, the Euler method is adopted to compute the intermediate variable *y* in the model and the fitness function is the mean square error (MSE) between the forecasted shear force and the measured force *Fm*(*ti*) at every time instant *ti* during the test.Hence, the fitness function *L*(*X*) can be represented as

 (2a)

 (2b)

 (2c)

 (2d)

where *X*=[*α*, *β*, *k*0, *c*0, *ε*, *f*0] is the model parameter vector to be identified, in which all the parameter are positive, *Ns* is the total number of sampling points, *σ*2 denotes the variance of the measured force in the test and ∆*t* denotes the time interval of data acquisition. If the value of the fitness function is close to zero, the corresponding result vector can be considered as the best solution for model parameters. For solving this optimization problem, the commonly used direct search and gradient-based methods may be invalid on account of the nonlinearity and ineffable gradient information in the model. Consequently, in the next part, an improved fruit fly optimization is presented for model parameter identification.

**3 Parameter identification algorithm**

3.1 Fruit fly optimization algorithm

The FFOA, as a novel artificial intelligence algorithm, was first proposed by W-S Pan in 2012 [12-15]. It belongs to the interactive evolutionary calculation approach. By simulating the behaviour of searching food in the fruit fly swarm, the algorithm is able to arrive at the global optimal solution very quickly. As a type of pest, the fruit flies always lodge at the tropical and temperate climate regions in groups and have the decayed fruit as their main food. Compare with other insects, the fruit fly has the superiority in both osphresis and vision. The procedure of the fruit fly swarm to search food can be summarized as: 1) the food source is searched by the fruit fly via its osphresis; 2) if the source is found, the fly moves towards that direction; 3) when the fly flies near the food source, the vision organ is utilized to search the food and the flocking position of other flies; 4) the fly moves towards that food orientation.



Fig. 2 Food searching iteration procedure of the fruit fly swarm

In accordance with the characteristic of the fruit fly searching for food, the algorithm can be separated into the following steps:

**Step 1:** Initialize the fruit fly swarm size *swasize*, algorithm maximal iteration number *Niter* and the original position of the swarm (*X*0, *Y*0), as shown in Fig. 2;

**Step 2:** Set the direction and range for the food source by the individual fruit fly based on smell;

 (3a)

 (3b)

**Step 3:** Because the food source is unknown for the fly swarm, the range between each fly and the original point is calculated first according to Eq. (4a). Then the smell concentration decision value, the reciprocal of *Disi*, is estimated according to Eq. (4b)

 (4a)

 (4b)

**Step 4:** The obtained smell concentration decision values, as the solution of the optimization problem, are substituted into the fitness function for estimating the smell concentration of each fruit fly position *Smelli*, shown in Eq. (5):

 (5)

**Step 5:** Seek out the optimal smell concentration and corresponding fruit fly in the whole of swarm;

 (6)

**Step 6:** Record the optimal smell concentration value of fly coordinate (*X\_axis*, *Y\_axis*). In the meantime, the whole fly swarm will fly towards this optimal position using vision.

 (7a)

 (7b)

 (7c)

**Step 7:** Repeatedly carry out the Steps 2~5 and compare the current smell concentration with previous value. If the current value is better, execute Step 6.

3.2 Improved fruit fly optimization algorithm

Some previous studies have shown that the steps of the standard FFOA has a relatively low probability of mutation, which may result in a limited searching space. Therefore, the optimization results by FFOA are easy to trap into the local extremum. Besides, in the standard FFOA, it is obvious that the value of distance *Disi* is non-negative so that the *Si*, the distance reciprocal, is also positive. Then this positive value is substituted into the objective function to search for the optimal smell concentration of each fruit fly position. That means the value of the objective function is in an interval between 0 and +∞, which is regarded as the main deficiency of FFOA because this will hamper its application in solving the problem whose variables are in a range of (-∞, +∞). Hence, this problem can cause the result converged into other solutions due to its limited searching space. To solve the above problems, a transfer factor *θ* based on a self-adaptive step *ω* is introduced to adjust the *Si*. The detailed expression is given as:

 (8a)

 (8b)

 (8c)

 (8d)

 (8e)

 (8f)

where *Niter* and *Nc* denote the maximal and current iteration numbers respectively; *rand* is a random number between 0 and 1; *h* is the dimensional number of searching space; *p* is an integer to control the variation of the fly position, and the steps with different *p* are shown in Fig. 3. Here, the value of *p* is selected as 4.



Fig. 3 Step value with different *p*

Furthermore, in this work the searching space of the fruit fly upgrades from two dimensions to three dimensions so that the swarm has a broader searching space, which makes them easy to find food.

On basis of the above analysis, steps 1, 2, and 3 should be modified as follows:

**Step 1:** The initial position of the swarm is denoted as (*X*0, *Y*0, *Z*0);

**Step 2:** The position (*Xi*, *Yi*, *Zi*) of individual fruit fly is updated according to the following equations:

 (9a)

 (9b)

 (9c)

 (9d)

**Step 3:** The distance *Disi* and the decision value of the smell concentration are updated as Eqs. (10a), (10b) and (10c).

 (10a)

 (10b)

 (10c)

3.3 Algorithm performance test

To evaluate the effectiveness of the improved fruit fly optimization algorithm (IFFOA) on the solution of the optimization problems, the Ackley and Griewank nonlinear functions are adopted as the benchmarks for algorithm performance evaluation. Their expressions and two-dimensional figures are illustrated as follows:

Ackley function:  (11)

Griewank function:  (12)



(a) Ackley function (b) Griewank function

Fig. 4 Two-dimensional function graphs

In this case, the IFFOA codes are compiled by Matlab v2013a and the algorithm parameters are set as: population size *swasize* =30, and maximal iteration number *Niter*=100. Fig. 5 shows the flight routes of the fruit fly swarm for solving two nonlinear functions by IFFOA method. The results show that the optimal routes make the swarm fly to the global best solutions directly in a high-efficiency way. Fig. 6 shows the smell change with the increasing iteration number for two functions. It is clearly observed that the IFFOA method is able to arrive at its optimum very quickly within four iteration steps. In addition, genetic algorithm (GA), particle swarm optimization (PSO) and artificial fish swarm algorithm (AFSA) are used for algorithm performance comparison. Table 1 illustrates the results of four algorithms to calculate the global minimal solutions of two functions. It is obviously seen that although PSO has very high accuracy for the optimal solutions of functions, the running time is relatively longer. AFSA, with the longest calculation time, cannot avoid the local optimum. GA is also trapped into the local minimum but has a fewer calculation cost. Among four optimization algorithms, IFFOA has the both benefits of high accuracy for optimal solutions and the short running time.



(a) Ackley function (b) Griewank function

Fig. 5 Flying routes of the fruit fly swarm using IFFOA



(a) Ackley function (b) Griewank function

Fig. 6 Iteration procedure using IFFOA

Table 1 Optimization results of 2 nonlinear functions using different algorithms

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Function | Test No. | GA | | PSO | | AFSA | | IFFOA | |
| Solution | Time /s | Solution | Time /s | Solution | Time /s | Solution | Time /s |
| Ackley | 1 | 0.0499 | 1.1342 | 0.0412 | 1.7632 | 0.0382 | 6.4831 | 0 | 0.3807 |
| 2 | 0.0492 | 0.6954 | 0.0506 | 1.6092 | 1.3728 | 8.9382 | 0 | 0.3919 |
| 3 | 0.0498 | 0.6476 | 0.0324 | 1.6381 | 0.0523 | 6.3849 | 0 | 0.3943 |
| 4 | 0.0496 | 0.6997 | 0.0409 | 1.5832 | 1.0483 | 9.7361 | 0 | 0.3818 |
| 5 | 0.0501 | 0.7175 | 0.0311 | 1.7321 | 1.9483 | 9.0825 | 0 | 0.4080 |
| Mean | 0.0497 | 0.7789 | 0.0392 | 1.6651 | 0.8919 | 8.1249 | 0 | 0.3913 |
| Griewank | 1 | 0.0367 | 0.6959 | 0.0369 | 1.8723 | 2.1944 | 7.6473 | 0 | 0.5343 |
| 2 | 1.9685 | 0.6991 | 0.0443 | 1.5291 | 1.2471 | 9.3821 | 0 | 0.3613 |
| 3 | 1.0542 | 0.6848 | 0.0344 | 1.8392 | 0.0492 | 10.736 | 0 | 0.3538 |
| 4 | 1.8944 | 0.7058 | 0.0517 | 1.6382 | 1.0347 | 9.0392 | 0 | 0.3457 |
| 5 | 0.0453 | 0.8487 | 0.0123 | 1.6973 | 0.9832 | 9.6783 | 0 | 0.3606 |
| Mean | 0.9998 | 0.7268 | 0.0359 | 1.7152 | 1.1017 | 9.2965 | 0 | 0.3911 |

**4 Identification result and discussion**

4.1 Experimental setup

In this work, several tests for the performance of the MRE base isolator are carried out using a shake table, which is fixed on the bottom of the device to generate the sinusoidal excitations, shown in Fig. 7(a). A DC power supply provides the device with different currents corresponding to varied magnetic fields, shown in Fig. 7(b). The load cell is used to measure the shear force generated. In this case, the excitation frequency is fixed to 4Hz, the loading amplitude ranges from 2mm to 8mm and there are four applied current levels, i.e. 0A, 1A, 2A and 3A. During the tests, the sampling frequency is set as 256Hz and more than 3 cycles are measured to guarantee the stable data of the device to be obtained. The specific testing condition could be found in [21].

 

(a) Testing system makeup (b) Testing equipment

Fig. 7 Experimental setup [21]

4.2 Identification result

To evaluate the capacity of the improved LuGre friction model in predicting the nonlinear and hysteretic behaviors of the device, all captured data are used to identify the optimal model parameters using IFFOA method. Except for *swasize* =200 and *Niter*=1000, the rest of algorithm parameters in IFFOA are set as same as that in Section 3.3. The identification results are displayed in Table 2. Fig. 8 illustrates one case of tracking performance and relative error of the proposed model in a sampling cycle at 2mm displacement and 1A applied current. The results show that the model outputs perfectly agree with the testing data and the absolute value of relative error is always kept below 6%, a satisfactory result in modelling study.

Table 2 Parameter identification result

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Amplitude | Parameter | Applied current level | | | |
| I = 0A | I = 1A | I = 2A | I = 3A |
| 2mm | β | 0.057476688 | 0.002169343 | 4.807626768 | 0.000177516 |
| α | 1.064213322 | 0.653152588 | 0.967325587 | 0.513039856 |
| c0 | 0.127846346 | 0.465197156 | 0.881267444 | 0.520355547 |
| k0 | 7.523890446 | 28.13112222 | 45.41444661 | 65.32261124 |
| ε | 0.046270035 | 0.299901296 | 0.297544921 | 0.950677285 |
| f0 | 1.599311043 | 0.704464234 | 0.299343629 | 0.000725158 |
| 4mm | β | 26.34391666 | 0.003848548 | 18.97634981 | 16.39543201 |
| α | 5.833143027 | 0.575989350 | 1.985326764 | 0.614374055 |
| c0 | 0.082340203 | 0.409657680 | 0.615740555 | 0.437426269 |
| k0 | 6.249747541 | 20.56954336 | 32.80324108 | 41.72755307 |
| ε | 0.066448457 | 0.124657698 | 0.278314844 | 0.661053808 |
| f0 | 1.505766698 | 0.057985671 | 0.008451294 | 0.026207565 |
| 8mm | β | 18.20805173 | 4.33781097 | 0.526819838 | 3.228448550 |
| α | 2.228850990 | 0.236867677 | 0.190225872 | 0.201411773 |
| c0 | 0.079159891 | 0.115459043 | 0.016748128 | 0.048883683 |
| k0 | 5.831242683 | 17.43151805 | 34.50502738 | 40.80424933 |
| ε | 0.040486720 | 0.466526120 | 1.013567698 | 1.122437755 |
| f0 | 1.229872435 | 0.000053018 | 3.042206804 | 2.799004648 |

To demonstrate the model performance to capture the dynamic behaviour of the device, more groups of comparison are conducted under various excitation frequencies, loading amplitudes and applied currents. Fig. 9 presents the real and predicted force-displacement and force-velocity responses at 4Hz frequency, 2A current and three different amplitudes: 2mm, 4mm and 8mm. From this figure, it is clearly observed that at a fixed applied current and loading frequency, the effective stiffness of the device, represented by the slope of the hysteretic loop, mildly declines when the loading amplitude ascends from 2mm to 8mm. Besides, in the force-velocity loops, the damping of the device, denoted by the enclosed area of the curve, rises with the increase of the loading amplitude. And the comparison results indicate that this model is well quantified to depict these dynamic characteristics of the device.

 

(a) Comparison between real and predicted force (b) Relative error of identification results

Fig. 8 Tracking process and relative error in one sampling cycle

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 9 Comparison between real force and predicted data from the proposed model for different amplitude excitations (4Hz-2A)

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 10 Comparison between real force and predicted data from the proposed model for different applied currents (4Hz-4mm)

The real and predicted responses in Fig. 10 are acquired by loading the device with 4Hz frequency, 4mm amplitude at four current levels, i.e. 0A, 1A, 2A and 3A. For a low applied current, the shear force shows an almost linear relationship with the displacement of the device. However, with the increase of the current level, the nonlinear strain stiffening phenomenon becomes more obvious, which means that both the produced maximal shear force and effective stiffness of the device arrive at their maximum values when the current increases to 3A. The comparison results also elaborate the superiority of the model to predict the hysteresis in different current levels.

4.3 Model parameter analysis

In order to better adjust the model parameters for the control application, the effect of parameter values on the performance of the improved LuGre friction model is studied. Fig. 11 depicts the four groups of force-displacement and force-velocity responses with respect to four different values of *β*: 3.23, 15, 30 and 40, respectively. It is clearly seen that in the force-displacement loop, the strain stiffening behaviour of the device becomes more obvious with the increase of parameter value. In addition, not only the slope of the hysteresis loop but also the maximal shear force varies at the large displacement. In the force-velocity response, the ascending of the parameter value contributes to the enlargement of the enclosed area of the hysteresis loop.

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 11 Responses from the proposed model with different parameter *β*

Fig. 12 and Fig. 13 describe the four groups of hysteresis loops with different shapes and sizes corresponding to four different values of *α* and *c*0, respectively. Different from the parameter *β*, parameters *α* and *c*0 have very little influence on the effective stiffness (the slope of the loop). However, the increase of the parameter value also can result in the expansion of the enclosed area in the force-displacement loop. Moreover, it is noticed that the maximal forces with different parameter values overlap at the both ends of the curves, which denote the maximal positive and negative displacements, respectively. Similar to that in the force-displacement loops, the shear forces reach their maximum with the same value at zero locations in the force-velocity loops.

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 12 Responses from the proposed model with different parameter *α*

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 13 Responses from the proposed model with different parameter *c*0

Fig. 14 shows the consequent hysteretic responses of the device by varying the parameter *k*0. It is obvious that the effective stiffness and the maximal shear force almost linearly vary with the parameter. Besides, all the curves in the figure intersect at the two zero-displacement positions (maximal velocity positions in the force-velocity loops). These two positions denote the tipping points of the strain-stiffening in the loops because this behaviour becomes more obvious from these two points.

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 14 Responses from the proposed model with different parameter *k*0

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 15 Responses from the proposed model with different parameter *ε*

Parameter *ε* is regarded as the factor to control the shape of the hysteresis loop. Fig. 15 illustrates four sets of hysteresis loops corresponding to different *ε*. It is worth noticing that the nonlinearity of the curves becomes much more obvious when *ε* obtains a larger value. On the contrary, the curve tends to be an almost linear relationship between displacement and shear force when *ε* declines. Especially, the loop will keep linear when *ε* approximates to 0.

4.4 Parameter sensitivity analysis

To further investigate the influence of changing input parameters on the model output and simplify the proposed model, the parameter sensitivity analysis is conducted in this part. The conventional method for parameter sensitivity analysis is to change one parameter in a certain range and other parameters remain unchanged meanwhile. The parameter sensitivity ranking will be gained by varying every parameter according to a fixed proportion while the rest are fixed at the pre-set reference values. This method belongs to the local assessment because it only evaluates the sensitivity related to the selected reference values rather than the whole parameter space. Although this method just analyses the effect of one parameter on the model output, it is easy to achieve with fewer calculation. In this work, the local sensitivity analysis is conducted by: 1) identify the optimal values of parameters in the improved LuGre friction model by adopting the IFFOA; 2) select a set of the optimal parameters from the identification result as the reference values; 3) vary one parameter’s value and keep the others unchanged; 4) calculate the root mean square (RMS) errors caused by the parameter variation and rank model parameters in the order of descending errors.

Although the local sensitivity analysis can illustrate the effect of individual parameter on the model outputs, the sensitivity of each parameter is still closely related to other factor such as interaction among all the model parameters. Hence, a global approach for parameter sensitivity analysis is also adopted, which makes the relationship between model input and output more effective in evaluating the parameter uncertainty by a global way. The global sensitivity analysis can give careful consideration of the model output variation caused by both individual parameter and the other parameters simultaneously. Different from the local method, the global analysis changes one parameter every time while the remaining parameters are able to be self-adjusted to compensate the output variation.

In this case, a typical group of identified parameters of the improved LuGre friction model are selected as the reference values, shown in Table 3. Fig. 16 shows the results of parameter sensitivity by both local and global method. In this figure, every parameter changes from the reference value to ±50%, and the RMS error is adopted as the evaluation index. Table 4 gives the parameter sensitivity ranking of the improved LuGre friction model according to the decreasing values of RMS error. It is obvious that apart from parameters *α* and *ε*, the sensitivity ranking of other model parameters by the global method completely agrees with that by the local method. This ranking demonstrates that *k*0, *α* and *ε* are sensitive parameters in this model while the other parameters belong to insensitive parameters due to their low proportions. Apparently, the change caused by the insensitive parameters will not obviously affect the model output. Therefore, the values of these insensitive parameters are able to be set as constants so that the original model can be simplified and the number of the model decreases from six to three, which is beneficial to model parameter identification.

 

(a) Spider chart by local sensitivity analysis (b) Spider chart by global sensitivity analysis

Fig. 16 Sensitivity analysis results of parameters in the improved LuGre friction model

Table 3 Reference values for parameter sensitivity analysis

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Parameter | β | α | c0 | k0 | ε | f0 |
| Value | 3.2284 | 0.2014 | 0.0489 | 40.8042 | 1.1224 | 2.7990 |

Table 4 Sensitivity rank for the improved LuGre model parameters

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Model parameter | Local sensitivity | | | Global sensitivity | | |
| Mean RMSE | Proportion | Rank | Mean RMSE | Proportion | Rank |
| β | 3.2438 | 2.67% | 4 | 13.5312 | 3.53% | 4 |
| α | 22.1178 | 18.22% | 3 | 5.4096 | 21.7073% | 2 |
| c0 | 1.9210 | 1.58% | 5 | 11.5693 | 2.49% | 5 |
| k0 | 64.4635 | 53.07% | 1 | 20.8427 | 52.11% | 1 |
| ε | 28.9533 | 23.84% | 2 | 4.1479 | 19.48% | 3 |
| f0 | 0.7634 | 0.63% | 6 | 0.0883 | 0.69% | 6 |

According to the above analysis results, these insensitive parameters are supposed to set as: *β*=10, *c*0=0.1 and *f*0=0.01. Therefore, the improved LuGre friction model can be simplified as:

 (13a)

 (13b)

Fig. 17 shows the parameter identification results of the simplified model from the testing data of 4Hz harmonic excitation with 8mm loading amplitude and four applied current levels 0A, 1A, 2A and 3A, respectively. It is noticeable that all the predicted nonlinear and hysteretic responses largely agree with the real testing forces. The identification errors and running time for both original and simplified models are concluded in Table 5. The result reveals that the proposed one can guarantee similar identification accuracy as the original one but with fewer calculation costs. That means that although three parameters are removed from the improved LuGre friction model, the simplified model still can provide a satisfactory identification result and reduce the running time in the meantime.

 

(a) Force-displacement responses (b) Force-velocity responses

Fig. 17 Comparison between real force and predicted data from the simplified model for different applied currents (4Hz-8mm)

Table 5 Comparison between the original model and the simplified model by IFFOA method

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Current | Original model | | | | | | Simplified model | | | | | |
| 2mm | | 4mm | | 8mm | | 2mm | | 4mm | | 8mm | |
| RMS | Time/s | RMS | Time/s | RMS | Time/s | RMS | Time/s | RMS | Time/s | RMS | Time/s |
| 0A | 1.57 | 356.7 | 2.75 | 343.2 | 3.88 | 357.8 | 2.83 | 195.3 | 4.12 | 205.8 | 5.72 | 213.7 |
| 1A | 3.34 | 353.1 | 5.64 | 342.6 | 12.1 | 357.3 | 3.91 | 208.1 | 7.41 | 211.3 | 13.7 | 207.2 |
| 2A | 4.86 | 381.8 | 10.8 | 345.1 | 16.8 | 362.8 | 5.33 | 203.5 | 8.55 | 191.7 | 17.6 | 226.1 |
| 3A | 12.4 | 364.2 | 15.7 | 351.3 | 19.9 | 350.7 | 12.9 | 201.7 | 17.3 | 209.4 | 22.7 | 218.3 |
| Mean | 5.54 | 363.9 | 8.72 | 345.5 | 13.2 | 357.1 | 6.24 | 202.1 | 9.34 | 204.5 | 14.9 | 216.3 |

4.5 Parameter generalization

To explore the influence of applied current level on the predicted hysteretic loops of the simplified model, the relationships between current and model parameters are also studied. Table 6 displays the optimal parameter values of the simplified LuGre friction model with different current levels, which are obtained from the model with testing data of 4Hz and 8mm harmonic excitation under four applied currents (0A, 1A, 2A and 3A). Fig. 18 shows the relationship between current and model parameters and it is clear that parameter *k*0 and *ε* monotonously increase with the current at the interval between 0A and 3A while parameter *α* shows an exponential decreasing relationship. Therefore, the following group of expressions are used to summarize these trends:

 (14a)

 (14b)

 (14c)

Table 6 Optimal parameter values of the simplified model under different applied currents

|  |  |  |  |
| --- | --- | --- | --- |
| Current (A) | *α* | *k*0 | *ε* |
| 0 | 1.89068 | 5.85267 | 0.00465 |
| 1 | 0.65156 | 16.59743 | 0.54887 |
| 2 | 0.21915 | 26.62946 | 0.70404 |
| 3 | 0.17784 | 36.32007 | 0.82618 |



(a) Parameter *α* vs. current (b) Parameter *k*0 vs. current (c) Parameter *ε* vs. current

Fig. 18 Relationships between model parameters and applied current

 

(a) 4Hz-8mm-0A (b) 4Hz-8mm-2A

Fig. 19 Reconstructed forces from the simplified model with generalized parameters

Then these expressions are put into the simplified model, which is utilized to predict the nonlinear outputs of the MRE base isolator. Fig. 19 shows the comparison between real measurements and predicted responses by supplying the device with 4Hz frequency, 8mm amplitude, 0A and 2A currents, respectively. The results demonstrate that the simplified model with generalized parameters has a good ability to capture the dynamics of the device although some imperfections still exist in peak-force regions. And this improvement also can provide the model with more conveniences for its application of vibration control in structures.

**5 Conclusions**

This paper presents a novel phenomenological model based on the LuGre friction element to characterize the nonlinear and hysteretic behaviours of the MRE base isolator. Compared with the conventional MRE based device’s models, this new model has a slightly simpler structure with fewer model parameters. An improved FFOA is also designed for parameter identification based on solving a minimization optimization problem. To prevent the algorithm from trapping into the local extreme value, a transfer factor is introduced to update the position of the fruit fly. To improve the algorithm convergence rate, the searching space is upgraded from two-dimension to three-dimension. Experimental data from the testing of a MRE base isolator are used to evaluate the performance of both proposed model and improved algorithm. The results confirm their superiority with satisfactory results. To further simplify the complexity of the proposed model, the parameter sensitivity analysis is conducted to separate the sensitive parameters from insensitive parameters. Then a simplified model is built by constraining the insensitive parameters to the constants. The comparison with the original model indicates that the simplified model has a similar modelling accuracy with little calculation amount, which can meet the real-time requirement in its control application.

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