

School of Civil and Environmental Engineering
Faculty of Engineering and Information Technology

# Behaviour of RC Beam-Column Connections Retrofitted with FRP Strips

by

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#### CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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#### **ABSTRACT**

Reinforced concrete (RC) buildings designed prior to the implementation of seismic design codes were largely designed without structural details required for satisfactory seismic performance. These buildings are therefore more susceptible to damage in case of an earthquake and hence may require strengthening to meet current design standards. For instance, the majority of buildings that may require strengthening in Australia are those designed prior to the implementation of Australia's first earthquake design standard in 1979.

Non-seismic load designed RC framed structures may possess several inherent weaknesses when subjected to a seismic attack. One major design deficiency is inadequate or no transverse reinforcement (otherwise known as stirrups) in the joint region of the RC beam-column connections (herein referred to as connections) which may lead to joint shear failure in a seismic attack. It is important here to establish that a joint refers to the intersecting region of the column/s and beam/s while a connection refers to the joint region plus surrounding beam/s and column/s. Another inadequacy is the practice of not anchoring the bottom beam longitudinal bars which may lead to bar pull-out due to load reversal in the case of a seismic attack. The proportioning of the beam and column elements framing into the connection region may lead to the undesirable formation of a strong-beam-weak-column mechanism. In addition, plastic hinging may form in the beam or column adjacent to the joint region thus compromising the strength and integrity of the joint. Past earthquakes such as El Asnam (1980), Mexico (1985), San Salvador (1986), Loma Prieta (1989) and Turkey (1999) have shown the vulnerability of RC framed structures with inherent non-seismic load designed weaknesses to catastrophically collapse due to connection failure. The proper strengthening of connections with such inherent weaknesses is in urgent need in order to ensure the safe performance of RC connections under seismic attack.

Over the past decade or so, fibre reinforced polymer (FRP) composites have emerged as a viable solution for strengthening structures due to their superiority in strength-toweight ratio, ease of handling and forming into shapes, and corrosion resistance when compared with more traditional construction materials such as steel. Many experimental, analytical and numerical studies have been conducted on the strengthening of RC beams, columns, slabs and walls as well as various other structural elements with FRP. Many field applications have also been reported around the world. Significantly less research and field applications by comparison have been reported on the strengthening of RC connections with FRP composites.

Experimental studies on FRP-strengthened exterior and interior connections have demonstrated the ability of externally bonded FRP to rectify the inherent weakness of non-seismic load designed RC connections by enhancing the joint shear capacity, in addition to enhancing the flexural capacity and to also relocate the possible formation of plastic hinges in the beam away from the joint region, as well as to promote a strong-column-weak-beam failure mechanism. Most of these experimental studies, while extremely useful, have been concerned with the behaviour of the strengthened connection as a whole with less attention paid to the behaviour of the FRP strengthening itself. In addition, the distinct lack of numerical simulations and analytical models, by comparison, are hindering better understanding and the more widespread rational design of FRP-strengthened for RC connections.

The research reported in this dissertation focuses on the commonly occurring case of a shear strength deficient connection. Such connections are retrofitted or repaired with FRP strips however from herein such application of FRP will collectively be referred to as strengthening. The key aims of this project are to (i) experimentally observe and quantify the behaviour of FRP strengthening in FRP-strengthened connections, (ii) accurately simulate the experimental results produced in aim (i) using finite elements and identify the strengths and weakness of such numerical modelling, (iii) perform parametric studies with the calibrated numerical models, (iv) develop an analytical model which can rationally and reliably predict the behaviour of the FRP-strengthened connections, and (v) formulate a design approach which can be easily incorporated into future versions of existing FRP-strengthening design guidelines as well as new guidelines.

Initially a state-of-the-art review is conducted on the current state of knowledge pertaining to FRP-strengthened RC connections. Existing research is also systematically categorised for ease of reference and comparison and for identifying the various issues still requiring research attention. The experimental program then forms the heart of the project and considers the shear strengthening of virgin connections as well as the repair of damaged connections. A strength hierarchy dictating shear failure in the joint region (even after application of the FRP) is deliberately chosen in order to assess the effectiveness and limitation of the FRP strengthening. All test specimens are extensively instrumented and the majority of connections are tested under monotonic load which greatly enhances the ability to effectively monitor the behaviour of the FRP as well as the cracking behaviour of the connection. Two connections are also tested under cyclic loading in order to assess the energy absorption characteristics of the strengthening schemes.

Finite element models are developed for plain (RC connection without any FRP strengthening) and FRP-strengthened RC connections which are then used to perform parametric studies to analyse various parameters affecting the behaviour of the connections such as amount of FRP, location of the FRP strips and strength of concrete. Also, a simple analytical model is presented based on the finding of the tests which can accurately estimate the contribution of the FRP strengthening to the connection shear strength. Finally, recommendations are made for design of FRP strengthening of shear deficient RC connections and future research needs are identified.

# **NOTATION**

$\beta$	=	orientation of FRP with respect to longitudinal axis of member
$eta_l$	=	factor that relates magnitude of stress (or strain) in the column
		main bars to the average stress (or strain) at the column centre line
$eta_t$	=	factor that relates magnitude of stress (or strain) in the beam main
		bars to the average stress (or strain) at the beam centre line
$\mathcal{E}$	=	strain in concrete at stress equal to $\sigma$
$\mathcal{E}_o$	=	concrete strain at ultimate compressive stress
${\cal E}_{cf}$	=	concrete strain at the extreme fibre
$\mathcal{E}_{co}$	=	elastic strain limit for concrete
$\mathcal{E}_{F,e}$	=	effective strain of FRP at failure
$\mathcal{E}_{frp}$	=	strain in FRP at failure
$\mathcal{E}_{frp}$	=	average axial strain in the fibre direction at the peak load
$\varepsilon_{\rm p}$	=	maximum permissible FRP strain
λ	=	ratio of maximum to effective bond length
$V_n$	=	the joint shear strength
$\theta$	=	angle between the critical diagonal compression strut and column
		axis
$\theta$	=	angle between critical diagonal crack and column axis
$\rho$	=	density of concrete
$ ho_b$	=	area ratio of longitudinal beam reinforcement
$ ho_l$	=	effective vertical reinforcement ratio
$\rho_t$	=	area ratio of horizontal stirrup in the joint
$\sigma$	=	concrete stress
$\sigma_{ m axial}$	=	column axial stress
$a_s'$	=	distance from compressive face to centroid of compressive steel
		reinforcement
$A_{frp}$	=	cross-sectional area of FRP

cross-sectional area of FRP strip crossing the joint.  $A_{frp,i}$ cross-sectional area of steel shear reinforcement  $A_{\rm cv}$ joint width or depth of column b  $b_c$ column width (i.e. the dimension of the column perpendicular to the face of the FRP strengthening) joint width  $b_i$ tension force transferred to the joint region  $C_b$  $D_f$ stress distribution factor for FRP joint depth d  $d_1$ distance from the extreme compression fibre to the neutral axis  $d_{\rho}$ effective connection depth depth of column  $d_{w}$  $E_{\rm c}$ modulus of elasticity for concrete =  $E_{F}$ modulus of elasticity of FRP modulus of elasticity of FRP  $E_{frp}$ elastic modulus of the FRP  $E_{w}$ yield stress of beam reinforcement  $f_{bv}$ concrete compressive strength  $f_c$ crushing strength of concrete  $f_{cmax}$ mean tensile strength of concrete  $f_{ctm}$ maximum tensile stress in FRP sheet  $f_{f,deb}$ average normal stress in the FRP in longitudinal direction  $f_{fl}$ = average normal stress in the FRP in transverse direction  $f_{ft}$ shear stress in the FRP  $f_{ftl}$ average stress of longitudinal reinforcement of the column at mid  $f_l$ height of the joint average stress in beam reinforcement  $f_t^s$ average stress in stirrup  $f_{vl}$ yield stress of column reinforcement yield strength of steel shear reinforcement  $f_{sv}$ 

$f_{u,i}$	=	stress in the FRP at failure
$f'_c$	=	ultimate compressive stress in concrete
$f'_{cf}$	=	tensile strength of concrete
h	=	column depth
$h_0$	=	effective depth of section
$h_{b0}$	=	effective depth of beam (i.e. distance from compressive face to
		centroid of beam steel tension reinforcement)
$h_c$	=	total column depth (dimension in the same plane as the FRP
		strengthening)
$h_j$	=	total joint depth (equals the depth of the beam h <sub>b</sub> )
$L_b$	=	distance of beam tip load from the column centre line
$l_b$	=	bond length
$l_c$	=	length of the column between the points of contra flexure
$L_e$	=	effective bond length
$L_{max}$	=	maximum bond length
$M_{b,centre}$	=	moment at the joint centre
N	=	column axial force
n	=	number of FRP layers
$N_h$	=	beam axial force
$N_{v}$	=	compressive axial load of the column
$P_b$	=	beam tip load
$Q_{ij}$	=	elements of FRP stiffness matrix
S	=	centre-to-centre spacing of steel shear reinforcement
T	=	tensile forces transferred to the joint
$T_b$	=	tension force transferred to the joint region by beam
$T_{b,s}$	=	tensile force due to steel in tension
$T_{b,frp}$	=	tensile force due to FRP
$t_F$	=	thickness of FRP
$t_w$	=	wrap thickness
$V_b$	=	beam shear force
$V_{col}$	=	column shear force
$V_c$	=	contribution of concrete to the joint strength
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contribution of FRP to the joint strength  $V_{frp}$  $V_{i}$ joint shear force joint shear stress  $v_{j}$  $V_{ih}$ horizontal shear force across the joint horizontal shear stress across the joint vertical joint shear force across the joint vertical joint shear stress  $v_{jv}$ contribution of transverse joint reinforcement to the joint strength  $V_s$  $V_u$ shear strength of the joint

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