

AN INVESTIGATION INTO THE DYNAMICS
OF VEHICLES WITH HYDRAULICALLY
INTERCONNECTED SUSPENSIONS

by

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

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Abstract

This thesis examines the dynamics of a particular class of vehicle suspension, namely *hydraulically interconnected suspension* (HIS), often claimed to break the compromise between ride and handling performance. Yet such systems have, until quite recently, received little attention in the academic literature. Ideally, interconnected schemes have the capability, unique among passive suspensions, to provide stiffness and damping characteristics dependent on the all-wheel suspension mode of operation.

The modelling approach proposed here is necessarily multidisciplinary, drawing from multi-body vibration theory and fluid dynamics. A simple half-car model is used to illustrate the basic principles and to demonstrate the application of the methodology. The half-car is treated as a lumped-mass multi-body system and the fluid circuits as continuous line elements. Individual fluid components are modelled using the impedance method, and the relationships between the fluid states at the extremities of each circuit are determined by the transfer matrix method. The resulting set of linear, frequency-dependent state-space equations, which govern the coupled dynamics of the half-car system, are derived and then applied in a variety of ways. This includes a free vibration analysis, ride comfort assessment and multi-objective optimisation. A number of key components that influence HIS performance are identified and a sensitivity analysis of their effects is presented.

Validation of the theoretical modelling is performed in two ways. First, simulations of an identical half-car using an alternative, nonlinear finite element fluid model are conducted. Second, experiments with a unique, purpose-built, half-car test rig are performed. The free and forced vibration results obtained with both methods, in general, agree very well with the proposed linear model.

The methodology presented is found to be an effective and useful way of modelling HIS-equipped vehicles, particularly in the frequency domain. The obtained results suggest that interconnected suspension schemes may provide, at least to some extent, an improved compromise between ride and handling. However, further investigation of this claim, including the development of a detailed full-car model, is recommended as a topic for future studies.

Dear Reader, please forgive me if I have wasted your time.

– Leonard Cohen