

**Capital Gains Taxes and the Market Response to Public  
Information**

A thesis submitted in fulfilment of the requirements for the degree of Doctor of  
Philosophy

By

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## **Certificate of Authorship/Originality**

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## **Abstract**

In this thesis I theoretically investigate the impact of capital gains taxes on the market response to public information. There are two objectives: First, I employ the model in Shackelford and Verrecchia (2002) to investigate the extent to which differential tax rates on short and long-term capital gains and losses affect equilibrium price and trading volume response to public information disclosure (both ‘good’ and ‘bad news’) about the value of a risky asset. Second, I examine whether capital gains taxes affect the information content of equilibrium prices with respect to public information disclosures. In particular, I modify the Shackelford and Verrecchia (2002) model to include exogenous random supply of the risky asset and examine whether asymmetric tax treatment of short and long-term capital gains and losses affects the extent to which market prices reflect public information about the value of the risky asset.

The results indicate that differential tax rates cause equilibrium prices to be more sensitive to public information disclosures. In addition, they result in lower (higher) trading volume around public disclosures when there is a price increase (decrease) due to the magnified tax costs (benefits) associated with realizing a short-term gain (loss). Moreover, differential tax rates cause prices to be, on average, more sensitive to exogenous noisy supply of the risky asset. The results also suggest that the noise effect outweighs the information effect so that prices are, on average, more volatile and less informative with respect to public information.

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## **Chapter 1: Introduction**

In many countries, the taxation of capital gains and losses depends on whether they are short or long-term. Short-term capital gains and losses are those that result from the sale of a capital asset held for less than a requisite period of time while long-term gains and losses are those that result if the asset is held for more than the requisite period. Long-term gains and losses are often taxed at a lower rate than short-term gains and losses. Therefore, investors have incentives to defer the realization of gains until the holding period is completed to ensure long-term tax treatment of gains, and to realize losses before the holding period is completed to ensure that losses offset any (tax-disadvantaged) short-term gains.

In a recent paper, Shackelford and Verrecchia (2002) examine the impact of such capital gains taxes on the market reaction to ‘good news’ public disclosures about the value of a risky asset. In particular, they define an *Intertemporal Tax Discontinuity* (ITD) as “a circumstance in which different tax rates are applied to gains realized at one point in time versus some other point in time” (p.205) and examine its impact on equity prices and trading volume at the time of a public disclosure. Using a stylized model, they show that the presence of an ITD may magnify price changes and inhibit trading volume around a ‘good news’ public disclosure, relative to an economy in which there is no ITD.

Investors wish to defer the sale of appreciated stocks around ‘good news’ public disclosures to ensure that gains attract the lower long-term tax rate, which restricts the supply of the stock around the disclosure. To compensate for the tax-motivated restriction in supply, stock prices go up. As a result, stock prices at the time of ‘good news’ disclosures are greater and trading volume is lower than would occur in the absence of an ITD.

My objectives in this thesis are two-fold:

- First, I extend the analysis in Shackelford and Verrecchia (2002) to consider the impact of capital gains taxes on the market reaction to ‘bad news’ public disclosures. Here, I employ the same setting as in Shackelford and Verrecchia (2002) in which there are two types of taxable rational investors who differ in their risk preferences and initial holdings of a risky asset. I investigate the extent to which the differential tax treatment of short and long-term capital gains and losses affects equilibrium price and trading volume response to the public disclosure of information (both ‘good’ and ‘bad news’) about the value of the risky asset.
- Second, I examine the informational effect that capital gains taxes may have on equilibrium prices. Specifically, I investigate whether the differential tax treatment of short and long-term capital gains and losses affects the extent to which market prices reflect public information about the value of a risky asset. Because the model in Shackelford and

Verrecchia (2002) assumes fixed (exogenous) supply of the risky asset, variation in prices at the disclosure date is due solely to the information released. In other words, prices fully reflect/reveal the information released by construction. As a result, the Shackelford and Verrecchia model does not permit the analysis of questions related to the ‘information content’ of prices and the degree of noise in prices. To investigate such questions, I employ an alternative, though related, setting based on the noisy rational expectations (NRE) model of Grossman and Stiglitz (1980). Grossman and Stiglitz developed a single-trading-date setting in which a costly signal related to the liquidation value of a risky asset is acquired by some investors (informed investors) prior to trading the asset. In the context of their model, they examine the extent to which informed investors’ response to the signal reveals it to other (uninformed) investors through the market price. I modify their model to incorporate the impact of capital gains taxes. I assume that all (rational) investors in the market are subject to capital gains taxes. However, unlike Grossman and Stiglitz, I assume that the signal is available at no cost to all investors and therefore all investors observe it. In the context of this model I investigate how the existence of differential tax rates applied to short-term and long-term capital gains and losses affects the extent to which prices reflect the (public) signal, as well as the degree of noise in price.

Incorporating capital losses in both settings requires an assumption concerning how losses are treated for capital gains tax purposes. As Shackelford and Verrecchia (2002) point out (p.208), the current U.S. capital gains tax regime they model does not explicitly distinguish between short and long-term capital losses. Both short and long-term losses can be deducted from any capital gain whether short or long-term. However, the operation of complex rules relating to the netting of capital gains and losses can result in effective tax rates applied to short-term capital losses that are greater than for long-term losses. In view of this, I investigate two alternative tax treatments of capital losses in addressing each objective of the thesis: in the first all capital losses are assumed to attract the long-term tax rate, while in the second short-term (long-term) losses are taxed at the short-term (long-term) rate. These two settings represent ‘extremes’ that are likely to span the effective tax treatment of losses in existing regimes such as the U.S. and Australia.

Results of the analysis in the first setting (chapter 4) confirm and extend the results of Shackelford and Verrecchia (2002). If only capital gains attract differential tax rates, equilibrium prices are more sensitive to ‘good news’ public information signals than in a no-differential tax rates world, while their sensitivity to ‘bad news’ signals is the same as in the no-differential tax rates world. The tax cost associated with realizing a short-term gain increases with the value of the public signal and investors require higher prices to compensate for the increased tax costs. As a result, equilibrium prices are more sensitive to

‘good news’ public signals. However, the analysis indicates that equilibrium prices and demands for risky assets in the presence of capital gains taxes are more complicated than indicated in Shackelford and Verrecchia (2002). In particular, for sufficiently high public signals, the differential tax rate results in a no-trade equilibrium because the tax cost of selling is ‘too high’ and discourages all trade.

When both gains and losses attract differential tax rates, equilibrium prices are more sensitive to both ‘good’ and ‘bad news’ public signals. For ‘bad news’ public signals, the tax benefit associated with realizing a short-term loss increases as the value of the public signal decreases and investors accept lower prices to induce buyers to buy. As a result, equilibrium prices are more sensitive to ‘bad news’ public signals than in a no-differential tax rates world. In addition, the differential tax rate on capital losses can result in an equilibrium (for sufficiently ‘bad news’) where investors of one investor-type in the market, the investor-type that has majority in the market, mixes between buying and selling in order to offset the other investor-type’s demand where the market clears.

Regarding trading volume, while differential tax rates inhibit trading volume for ‘good news’ public disclosures due to the tax cost of trading, they magnify trading volume for ‘bad news’ disclosures. If short-term losses are taxed at a higher rate than long-term losses, there is a tax benefit from realizing a short-term loss which encourages greater trading volume.

The analysis in the second setting (chapter 5) indicates that while differential tax rates on short and long-term capital gains and losses increases equilibrium price sensitivity to information, they also increase equilibrium price sensitivity to exogenous noisy supply changes of the risky asset. The increased sensitivity only occurs for negative changes in supply. This is because any negative change in exogenous supply must be satisfied by rational investors and satisfying a decrease in supply requires rational investors to incur tax costs (benefits) associated with realizing a short-term capital gain (loss). Based on numerical methods, I show that the expected sensitivity of equilibrium prices to both information and changes in noisy supply increases with the magnitude of the differential between long and short-term tax rate. However, my results suggest that the noise effect of differential tax rates on equilibrium prices outweighs the information effect so that on average prices are more volatile and less informative with respect to the public signal.

If only capital gains attract differential tax rates, equilibrium price sensitivity to ‘bad news’ public disclosures is the same as in a no-differential tax rates world. However, equilibrium prices remain more volatile and less informative than in a no-differential tax rate world.

This thesis makes two primary contributions to the literature. First, it contributes to existing (theoretical) literature on the effect of capital gains taxes on the market reaction to public announcements. Specifically, it confirms the

results of Shackelford and Verrecchia (2002) and extends their analysis to investigate the impact of differential short and long-term capital gains tax rates on equilibrium prices and trading volume reaction to ‘bad news’ public disclosures. Second, this thesis contributes to the literature on the informational efficiency of capital markets. Previous models of market equilibrium under uncertainty suggest that the extent to which equilibrium prices reflect investors’ private information depends on, for example, the precision and cost of information and the risk aversion of investors. My research investigates whether capital gains taxes influence the extent to which prices will reflect public information.

The results in this thesis may have implications for investors and firms in capital markets. If differential capital gains tax rates affect the extent to which investors respond to information and, consequently, the extent to which information is reflected in equilibrium prices, they may influence the effectiveness of public information disclosures in reducing information asymmetries among markets’ participants. In addition, they may affect the relevance of market prices as accurate signals for an efficient allocation of resources, at least at the time of the release of information. Moreover, if differential capital gains tax rates affect the extent to which information is reflected in market prices, this can affect the role of capital markets in relation to



the price discovery process and the incentives to acquire information in capital markets, especially when it is costly to do so.<sup>1</sup>

The remainder of the thesis is organized as follows: In chapter 2 I review some of the prior research in two areas to which this thesis is closely related. First I review research on the effect of capital gains taxes on share prices and trading volume. I then review research on the informational efficiency of capital markets. Chapter 3 describes the basic setting that is employed and developed in subsequent chapters to address the two objectives of this thesis. It also derives investors' demand functions for risky assets in the presence of capital gains taxes. In chapter 4 I examine the impact of differential tax rates on equilibrium price and trading volume response to public information in the absence of noise in the market. This chapter is a direct extension of the work in Shackelford and Verrecchia (2002). In chapter 5 I examine the impact of differential capital gains tax rates on price volatility and the 'information content' of prices in a noisy supply setting. Finally, in chapter 6 I conclude with a summary of the research, a discussion of some limitations and caveats regarding the research, and some suggestions for future work.

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<sup>1</sup> Note that because my models are designed to investigate the impact of capital gains taxes on the market reaction to 'public information', there is no price discovery in my models: all investors know the information. However, the results do suggest the possibility that, in a broader model (and in real markets), it is likely that differential tax rates will influence the 'information content' of prices and noise, and thus impact on the price discovery role of markets.

## Chapter 2: Background and Prior Research

### 2.1 Introduction

A large number of trading models have been used to examine the market reaction to public information disclosures and the extent to which market prices reflect information.<sup>2</sup> On the whole, these models have ignored the impact of taxation in general, and capital gains taxes in particular, on equilibrium prices and, thus, on the extent to which prices reflect information.<sup>3</sup> Despite this, recent empirical research on capital gains taxes suggests that the differential tax treatment of short and long-term capital gains and losses affects investors' demand for risky assets around the release of public information into the market and consequently affects equilibrium prices of risky assets (see e.g., Blouin et al. (2003), Hurtt and Seida (2004) and Jin (2006)).

As Shackelford and Verrecchia (2002) point out, there are a variety of reasons why previous models have ignored the role of taxes. Not all investors in an economy are subject to taxes, and the existence of tax-exempt investors can mitigate the impact of taxes on securities' prices. Even for investors subject to

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<sup>2</sup> See, for example, Diamond and Verrecchia (1981), Grossman (1976), (1978), Grossman and Stiglitz (1980), Hellwig (1980), Kim and Verrecchia (1991), Kyle (1985), Lundholm (1991), Verrecchia (1982a), among others. Also see Verrecchia (1982b), (2001) for discussions of the use of mathematical models in financial accounting.

<sup>3</sup> Besides Shackelford and Verrecchia (2002), the finance literature has similarly considered the potential impact of differential tax rates applied to short and long-term gains and losses (see e.g., Constantinides (1984), Dammon et al. (1989) and Dammon and Spatt (1996)). However, they do not directly investigate the impact on the 'information content' of prices.

taxes, the use of tax planning strategies or perfect substitute assets that allow potential gains to be offset against tax losses can also mitigate the impact of taxes (see e.g., Constantinides (1983), Maydew (1997), Scholes (1972), Scholes et al. (1990), Scholes and Wolfson (1992) and Stiglitz (1983)). In addition, a number of empirical studies provide evidence that taxes have a marginal role at best in asset pricing (see e.g., Black and Scholes (1974), Engel et al. (1999), Grammatikos and Yourougou (1990) and Miller and Scholes (1978), (1982)). Moreover, an important practical reason why taxes have been largely ignored in the informational efficiency analyses is that they create complex modelling problems that are difficult to address.

In this chapter I review some of the prior research on the effect of capital gains taxes on share prices and trading volume. I also review some of the research on the informational role of securities' prices in capital markets.

## **2.2 Capital Gains Tax Research**

### ***2.2.1 Overview of Capital Gains Taxes***

A capital gain or loss arises from the disposal of a capital asset in a 'Capital Gains Tax' (CGT) event. For all events that involve a capital asset, a capital gain arises if the capital proceeds from the event exceed the cost of acquisition of the asset, while a capital loss arises if the cost of acquisition exceeds the capital proceeds from the event.

Capital gains and losses are of two types; long-term and short-term. A long-term capital gain or loss refers to the gain or loss that results from the sale of an asset held for more than a requisite period of time, while a short-term capital gain or loss refers to the gain or loss that arises if the asset is held for less than the requisite period. This requisite period of time is typically one year but can vary depending on the tax law applicable in a specific country.

An important aspect that distinguishes short-term capital gains and losses from long-term capital gains and losses is their treatment for tax purposes. In many tax regimes long-term capital gains receive some form of tax advantaged treatment relative to short-term capital gains. This tax-advantage is most often conferred upon investors through the application of a lower tax rate to long-term gains than short-term gains. This is the broad setting studied in Shackelford and Verrecchia (2002) and which forms the basis of this thesis.<sup>4</sup>

Countries that apply a differential capital gains tax rates regime or some form of it and to which this thesis might be relevant include the United States,

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<sup>4</sup> Another form by which long-term capital gains receive tax advantaged treatment relative to short-term capital gains is through allowing taxpayers to index the cost base of their assets for movements in a general price index over the holding period. In this case, tax is paid only on the difference between the proceeds from the sale of the asset and the indexed-cost base of the asset. Clinch and Odat (2009) examine the impact of such an indexation-based taxation approach on price and volume response to public signals.

Australia, Denmark, France, Hungary, India, Lithuania, Russia and Switzerland.<sup>5</sup> However, the types of taxpayers, taxable assets, holding periods of the assets, and the effective tax rates applied to short and long-term gains vary across the different countries. Therefore, the relevance and applicability of this thesis will vary across these countries.

Research on capital gains taxes has examined a range of topics.<sup>6</sup> In this thesis I review the main finding in three broad areas. These are: (1) the capitalization of capital gains taxes, which examines whether stock prices impound capital gains taxes; (2) the lock-in effect of capital gains taxes, which examines whether the existence of capital gains taxes constrains investors from selling assets at a gain and the potential effects on market prices; and (3) the impact of the capital gains holding period, which examines the effect of the asymmetric tax treatment of short and long-term capital gains and losses on equity values.

### *2.2.2 The Capitalization of Capital Gains Taxes*

Tax capitalization suggests that market prices reflect expected after-tax returns (Liang et al. (2002)). Thus, any reduction in the capital gains tax rates increases stock prices, while increasing tax rates reduces prices (see e.g., Amoako-Adu et

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<sup>5</sup> In other countries either there are no capital gains taxes, capital gains are taxed at a flat rate, or the indexation-based taxation method is used where capital gains are indexed for inflation. In many instances where a flat rate is employed, the effect can be similar to the capital gains tax regime I study.

<sup>6</sup> See Maydew (2001) and Shackelford and Shevlin (2001) for a review of these topics.

al. (1992), Ayers et al. (2003), Collins and Kemsley (2000), Guenther and Willenborg (1999), Lang and Shackelford (2000), Liang et al. (2002) and Sinai and Gyourko (2004)).

Empirical research in this area has generally employed an event study approach around changes in tax policy or economic conditions in an attempt to detect a relation between stock prices and capital gains taxes (Kothari (2001)). For example, Lang and Shackelford (2000) provide evidence on the capitalization of capital gains taxes into stock prices around the time of the Taxpayers Relief Act of 1997 (TRA97) in the U.S., which reduced the long-term capital gains tax rate from 28% to 20%. Using a sample of the 2000 largest U.S. corporations, they find that the share price of non-dividend paying firms which represent capital gains assets increased more, over a five-day window during the event week, than the share price of dividend paying firms, and that among dividend paying firms the change in the share price was decreasing in dividend yields. They claim that shareholders weigh the expected capital gains tax rate more heavily when assessing firms with low dividend yields, and suggest that, to the extent a firm's stock is held by an individual shareholder subject to capital gains taxes, a reduction in the expected capital gains tax rate increases its market value.

Similarly, Amoako-Adu et al. (1992) examine the capitalization of capital gains taxes for stocks listed on the Toronto Stock Exchange. Exploiting the event

when the Canadian government introduced a \$500,000 lifetime capital gains tax exemption in 1985, they find a significant positive abnormal return on low-dividend yield stocks around the budget announcement. In addition, they find a significant differential impact on low and high yield stocks. On the other hand, when the exemption limit was reduced from \$500,000 to \$100,000 in 1987 they find a significant differential effect in favour of the high yield stocks.

### *2.2.3 The Lock-In Effect of Capital Gains Taxes*

It is commonly believed that the taxation of capital gains upon realization rather than on an accrual basis discourages investors from selling assets at a gain and, thus, has a 'lock-in' effect (Holt and Shelton (1962) and Meade (1990)). Landsman and Shackelford (1995) define the lock-in effect of capital gains taxes as the disincentive to dispose of an appreciated stock in a taxable transaction that will generate capital gains taxes on accrued, but unrealized, appreciation. In this view, capital gains taxes can be regarded as a transaction cost for which sellers demand compensation from buyers for any sale that increases or accelerates expected capital gains taxes (see e.g., Klein (1998), (1999), (2001) and Viard (2000)).

In contrast to capital gains tax capitalization, the capital gains lock-in effect suggests that a reduction in the capital gains tax rates reduces sellers' transaction costs therefore lowering stock prices. Empirically, the literature has

provided evidence of this effect on stock prices and current stock returns (see e.g., Blouin et al. (2002), Cook and O'Hare (1992), Feldstein et al. (1980), George and Hwang (2007), Ivkovich et al. (2005), Landsman and Shackelford (1995), Meade (1990) and Yitzhaki (1979)).

Landsman and Shackelford (1995), for example, provide empirical evidence that investors require higher prices to sell shares with large accrued capital gains. They find that for each dollar less of tax basis, shareholders of RJR Nabisco, during its 1984 leveraged buyout, demanded an additional 20 cents in the sale price as a compensation for capital gains taxes.

#### ***2.2.4 The Effect of the Capital Gains Holding Period***

Because long-term capital gains and, perhaps, losses are taxed at a lower rate than short-term gains and losses, investors have incentives to defer the realization of gains until the holding period is completed to ensure long-term tax treatment of gains. They also have incentives to realize losses before the holding period is completed to ensure that losses offset any tax-disadvantaged short-term gains. This tax incentive has attracted a considerable research effort which focuses primarily on investigating whether the holding period incentive affects trading volume and, if so, whether the volume surge is large enough to affect prices (Shackelford and Shevlin (2001)).



Several studies provide empirical evidence for tax-motivated price pressure around the long-term qualification date (see e.g., Blouin et al. (2002) and Reese (1998)). Blouin et al. (2002), for example, examine Initial Public Offering (IPO)<sup>7</sup> firms' reaction to the 1998 reduction in the capital gains holding period in the U.S. from 18 to 12 months. They find that firms that had appreciated during their 12 to 18 months of initial public offering experienced increased trading volume at the announcement of the reduction and that the increased volume was enough to move prices down.

Studies have also provided evidence on whether tax loss selling affects equity values. These studies suggest that individuals' tax loss selling can explain some of the turn-of-the-year return anomaly. Investors sell their depreciated stock before the year-end to ensure short-term capital loss treatment thus causing a decline in prices before the year-end, followed by a price increase and abnormally high returns after the turn of the year (see e.g., Dhaliwal and Trezevant (1993), Dyl (1977), Gibson et al. (2000), Givoly and Ovadia (1983), Poterba (1987), Poterba and Weisbenner (2001) and Sias and Starks (1997)). Gibson et al. (2000), for example, find that depreciated stocks, in which mutual funds collectively held 5% or more of the outstanding shares as of the beginning of October, experienced statistically significant negative returns in October followed by statistically significant positive returns in November which is

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<sup>7</sup> An advantage of studying IPOs is that the researcher can identify the start of the capital gains holding period and the qualification date.

consistent with price pressure arising because of mutual fund related tax loss selling.<sup>8</sup>

While much of the prior research on capital gains taxes has been conducted around events where a tax effect is highly expected such as changes in tax laws or year-ends, Shackelford and Verrecchia (2002) examine whether the effect of differential tax rates can be extended to situations where tax considerations are less prominent. They theoretically examine the effect of differential tax rates on short and long-term capital gains on the market reaction to the public disclosure of ‘good news’ firms’ performance. They show that differential tax rates inhibit trading volume and magnify price increases around ‘good news’ disclosures, relative to an economy in which there are no differential tax rates. The higher short-term tax rate discourages investors from selling appreciated stocks around the disclosure, which restricts the supply of stocks. To compensate for the tax-motivated restriction in supply, prices increase.

Blouin et al. (2003) examine whether this result can be detected empirically. They examine equity trading around two unrelated public disclosures that are known to trigger substantial portfolio rebalancing. They document a tax related price increase and a trading volume decrease for appreciated stocks following quarterly earnings announcements and following

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<sup>8</sup> The fiscal year-end for mutual funds is the 31<sup>st</sup> of October.

the announcement of an addition to the Standard and Poor's 500 index. Likewise, Hurtt and Seida (2004) examine quarterly earnings announcements by New York Stock Exchange (NYSE) and American Stock Exchange (ASE) listed firms and find evidence that the earnings period selling activities by individual investors for a given level of past stock price depreciation is higher when the magnitude of the difference between short and long-term capital gains tax rates is larger.

### **2.2.5 Summary**

There is a large number of studies which examine whether capital gains taxes affect equity trading. Evidence suggests that capital gains taxes can affect equity values in two ways: capitalization and lock-in.<sup>9</sup> Tax capitalization suggests that market prices reflect expected after-tax returns. It suggests that a reduction in the capital gains tax rate increases stock prices. Capital gains lock-in, on the other hand, views taxes as transaction costs where shareholders demand to be compensated for any sale that increases or accelerates expected capital gains taxes.

An important area of capital gains tax research examines whether the capital gains holding period affects share prices and trading volume. Evidence

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<sup>9</sup> Dai et al. (2008) provide evidence on both effects around the Taxpayer Relief Act 1997. They find evidence supporting a dominant capitalization effect in the week following news that sharply increased the probability of a reduction in the capital gains tax rate and a dominant lock-in effect in the week after the rate reduction became effective.

suggests that the higher tax rate on short-term gains relative to long-term gains discourages investors from selling their appreciated stocks before qualification for the long-term tax treatment. To induce selling before qualification and pay the higher short-term taxes, sellers demand compensation from buyers through higher prices. On the other hand, the higher tax rate on short-term losses compared to long-term losses encourages investors to sell depreciated stock before the holding period is completed in order to create short-term capital losses that offset any tax-disadvantaged short-term capital gains. Therefore, trading volume of depreciated stocks is higher before the long-term qualification driving prices down.

### **2.3 The Informational Efficiency of Capital Markets**

It is commonly believed that asset prices in competitive markets convey information to market participants (Fama (1970), (1991)). In this view, capital markets are informationally efficient if equilibrium prices reflect all the available information in the market (Fama (1970)). By the strongest form of efficiency, as categorized by Fama, asset prices reflect both public and privately acquired information.

A large body of research has examined the ‘information content’ of public announcements. “An announcement contains information if it alters investors’ expectations about the value of an asset” (Holthausen and Verrecchia

(1990)). This, in turn, is captured using two approaches; a price change approach (e.g., Collins et al. (1987) and Kothari and Sloan (1992)), and a trading volume approach (e.g., Atiase and Bamber (1994) and Bamber (1986)). Price change reflects the average change in investors' expectations regarding the value of the stock due to the arrival of new information, while volume reflects the lack of consensus among investors about the value of the stock which is induced by the new information (Beaver (1968)).

Empirically, significant price changes have been widely documented around public announcements of firms' performance, dividends, and other information (see e.g., Atiase (1985), Ball and Brown (1968), Ball and Kothari (1991), Bamber and Cheon (1995), Beaver et al. (1987), Burgstahler et al. (2002), Cready and Mynatt (1991), Kothari et al. (2009), Ou (1990) and Sloan (1996)). Evidence suggests that assets' prices react quickly and efficiently to the arrival of new information into capital markets.

Similarly, several studies provide evidence that trading volume responds to the arrival of new information (see e.g., Ajinkya and Jain (1989), Bamber (1986), Bamber et al. (1997), Bamber and Cheon (1995), Chae (2005), Chen and Sami (2008), Cready and Hurtt (2002), Cready and Mynatt (1991) and Linsmeier et al. (2002)). Bamber (1986), for example, finds that trading volume increases significantly when firms announce annual earnings and that the trading volume is positively correlated with the absolute value of the earning surprise.

### *2.3.1 Information and Asset Price Determination*

Theoretical research on the informational efficiency of capital markets has largely focused on examining the extent to which equilibrium prices reflect available information about the value of a risky asset. Much of this research has employed the concept of rational expectations (RE) where investors make inferences from market prices about other investors' (private) information. Rational expectations models suggest that while assets' prices depend on investors' expectations (conditional on information), through their demand correspondences, investors' expectations themselves depend on assets' prices. Because information affects investors' demand for risky assets while prices depend on investors' demand, when investors trade based on information, the market clearing price will be a function of this information (see e.g., Diamond and Verrecchia (1981), Grossman (1976), (1978), Grossman and Stiglitz (1980) and Verrecchia (1982a)). As the market for an asset replicates itself over time, investors can learn the relationship between the equilibrium price and information and they can use the price as a source of information (see e.g., Adamti (1985), Anderson and Sonnenschein (1982) and Radner (1979)). Thus, prices in rational expectations models perform two functions: they clear the market and provide information which investors can use to formulate their expectations.

Research, however, suggests that the extent to which equilibrium prices reflect information depends on a number of factors including the cost of information, the percentage of informed traders, the risk aversion/tolerance of traders and the level of noise in the market (see e.g., Demski and Feltham (1994), Grossman and Stiglitz (1980) and Verrecchia (1982a)). Grossman and Stiglitz (1980), for example, show that the level of informativeness of prices increases with a decrease in the level of noise, the cost of acquiring information, and the risk aversion of investors. A decrease in risk aversion, for example, leads investors to take larger positions in a risky asset which increases the informativeness of the price.

### *2.3.2 The Informational Role of Prices*

Research on the informational efficiency of capital markets has considered two roles for prices in conveying information; their role as transmitters of information, and their role as aggregators of information. Prices transmit information when there is only one piece of information in the market. When informed investors observe this piece of information, they take a position in the market based on this piece of information. Consequently, the market price will be forced to adjust to their demand. Uninformed investors know that the current price reflects informed investors' information and they form their beliefs about the future price from the information which they learn from observing the current price. In this case, the market price transmits this piece of information from those

who observe it to those who do not observe it (see e.g., Grossman and Stiglitz (1980) and Kihlstrom and Mirman (1975)).

However, if there are diverse investors with diverse pieces of information, the market clearing price will depend on the information of each individual investor. In this case, market prices aggregate information (see e.g., Diamond and Verrecchia (1981), Grossman (1976), (1978) and Verrecchia (1982a)). Grossman (1976), for example, shows that when each investor in the market gets a different piece of information, the market price aggregates this information and reveals them to other investors as if each investor has all the different pieces of information.

While all the research reviewed in this section investigates the extent to which equilibrium prices reflect private information about the value of a risky asset, my research focuses on public information that is available at no cost to all market participants. In particular, chapter 5 of the thesis investigates whether capital gains taxes influence the extent to which equilibrium prices reflect (public) information. Considering, otherwise, the case of private information creates a very complex setting that is difficult to examine.



## 2.4 Conclusion

When investors in capital markets trade based on information, whether private or public, to some extent, market prices reflect this information. Because investors' demand for risky assets depends on their information, prices also depend on information. Research has investigated several factors as affecting the extent to which market prices reflect investors' information. These include: the cost of information, the risk aversion/tolerance of traders and the level of noise in the market.

In addition, research on capital gains taxes shows that differential tax rates on short and long-term capital gains and losses affect how investors respond to public information about the value of a risky asset, and thus, affect the demand for, and the equilibrium price and trading volume of the asset. Investors defer the sale of appreciated stocks around 'good news' public announcements to ensure gains are taxed at the lower long-term rate, and sell depreciated stocks around 'bad news' announcements to ensure losses offset any tax-disadvantaged short-term gains. Deferring the sale of an appreciated stock around 'good news' announcements restricts the supply of the stock. To compensate for the tax-motivated restriction in supply, stock prices increase. On the other hand, accelerating the sale of a depreciated stock around 'bad news' announcements increases the supply of the stock therefore decreasing prices.

As the differential tax treatment of short and long-term capital gains and losses affects investors' demand for a risky asset in response to public information disclosure about the value of the asset; it can affect the extent to which this information is reflected in the market price of the asset. My research investigates this.

## **Chapter 3: Structure of the Market and Investors' Demand for Risky Assets in the Presence of Capital Gains Taxes**

### **3.1 Introduction**

In this chapter I describe the common assumptions underlying both settings used to investigate each objective of this thesis. In chapters 4 and 5 I add assumptions as necessary to enable the investigation of each individual objective. Also in this chapter, I derive investors' optimal demand for a risky asset in the presence of differential capital gains tax rates. With regard to capital losses, I consider two cases. I first assume that both short and long-term losses are taxed at the long-term rate (*i.e.*, only capital gains attract differential tax rates). Then I consider the case where short-term losses are taxed at a higher rate than long-term losses (*i.e.*, both gains and losses attract differential tax rates).<sup>10</sup>

### **3.2 The Basic Model**

Following Shackelford and Verrecchia (2002), I assume the following: a three-date market in which two assets are traded; a taxable risky asset, and a risk-and-tax-free asset which acts as a numeraire. The risk-and-tax-free asset pays out a

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<sup>10</sup>As short and long-term capital losses need to be deducted from capital gains (short or long-term), there is a range of possible treatments of capital losses that can be considered. The two cases that I consider in this thesis might reasonably be considered as two extremes of these possible treatments.

return of 1 for each unit of investment. The risky asset yields an uncertain return which is unknown until the liquidation of the asset, and is represented by a random variable  $\tilde{u}$ . The return on the risky asset consists of two parts:

$$\tilde{u} = \tilde{\mu} + \tilde{\varepsilon} \quad (3.1)$$

where both  $\tilde{\mu}$  and  $\tilde{\varepsilon}$  are independent random variables normally distributed;  $\tilde{\mu}$  is a public signal (such as an earnings announcement) which provides information about  $\tilde{u}$  that all investors can observe at zero cost, and  $\tilde{\varepsilon}$  is unobservable. The mean and variance of  $\tilde{\mu}$  and  $\tilde{\varepsilon}$  are  $\mu_0$  and  $\sigma_\mu^2$  and  $\theta$  and  $\sigma_\varepsilon^2$  respectively. Thus, given a realization of  $\tilde{\mu}$ , say  $\tilde{\mu} = \mu$ , investors update their beliefs about  $\tilde{u}$  such that  $E(\tilde{u} | \mu) = \mu$  and  $\text{var}(\tilde{u} | \mu) = \sigma_\varepsilon^2$ .

At date 1, investors hold shares of the risky asset and the risk-and-tax-free asset and await the public release of an information signal about the value of the risky asset. At date 2, all investors observe the signal  $\mu$ . Trade occurs at date 2 and each investor exchanges (some of) his initial holdings with other investors but does not consume. Finally, at date 3 all investors liquidate their portfolios and consume the return.

I assume that all investors in the market are risk averse with a utility for wealth,  $w$ , implied by a constant absolute risk aversion parameter,  $a > 0$ , given by the negative exponential function:

$$U(w) = -e^{-aw}$$

Further, I assume that all rational investors are subject to capital gains taxes, and that the period between dates 1 and 2 is treated as short-term for capital gains tax purposes. Any gains realized in this period are taxed at the short-term tax rate ( $\tau_s$ ) while the tax rate applied to losses realized in this period depends on the tax treatment assumed for capital losses. The period between dates 2 and 3 is assumed to be long-term for tax purposes and any gains or losses realized from the liquidation of the risky asset at date 3 are taxed at the long-term capital gains tax rate ( $\tau_l$ ).<sup>11</sup>

Let  $x_1$  represent investors' holdings of the risky asset at date 1 and  $p_1$  represent the price at which they were acquired.<sup>12</sup> Following Shackelford and

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<sup>11</sup> To facilitate comparison with Shackelford and Verrecchia (2002), it is useful to note the following notational differences between my model and theirs: First, Shackelford and Verrecchia merely need the long-term tax rate to be less than the short-term rate. Therefore, they set the long-term tax rate ( $\tau_l$ ) to zero, and the ordinary (short-term) tax rate  $\tau_s = \tau > 0$ . Second, they structure their model using a risk tolerance parameter ( $r$ ) rather than my risk aversion parameter, ( $a$ ).

<sup>12</sup> In the standard informational efficiency models of capital markets, initial holdings of the asset and the cost base are irrelevant. With the introduction of capital gains taxes, however, investors' after-tax wealth is affected by both initial holdings and the price at which they were acquired.

Verrecchia (2002), I assume that  $p_1$  is common across all investors (*i.e.*, all investors have the same cost base for their initial holdings). Also, let  $x_2$  represent investors' holdings of the risky asset at date 2 and  $p_2$  represent the price of the asset at this date. Moreover, let  $X_1$  and  $X_2$  represent the per capita aggregate supply of the risky asset at dates 1 and 2 respectively. For many parts of the analysis it is convenient to express results in terms of the change in prices, investors' demands and the per capita aggregate supply between dates 1 and 2:  $\Delta p = p_2 - p_1$ ,  $\Delta x = x_2 - x_1$  and  $\Delta X = X_2 - X_1$ .

### 3.3 Characterizing an Investor's Demand Function

Each investor in the market seeks to maximize his expected utility over end of period 3 wealth given available information. Therefore, upon observing the signal  $\mu$  at date 2, each investor revises his expectations about the liquidation value of the asset and decides the number of shares,  $\Delta x$ , to trade (buy or sell) such that trading  $\Delta x$  maximizes his expected utility.

In the absence of capital gains taxes, prior research has shown that a risk-averse investor's optimal holdings of the risky asset at date 2,  $x_2$ , is given by the difference between his conditional beliefs (*i.e.*, conditional on information) about

the value of the asset and the current price of the asset divided by the required risk premium per unit:<sup>13</sup>

$$x_2 = \frac{\mu - p_2}{a\sigma_\varepsilon^2} \quad (3.2)$$

To characterize the investor's optimal demand for the risky asset in the presence of capital gains taxes, first note that an investor has three options at date 2. The first option is to buy additional shares. The second option is that the investor does not trade at date 2 and maintains his initial position from date 1 until the liquidation of the asset. Finally, the investor can sell some or all his holdings from date 1. Note, however, that the investor's after tax wealth at date 3 will depend on his action at date 2 and on the tax rate applied to gains and losses. In the next section I assume that short-term gains are taxed at the ordinary tax rate,  $\tau_s$ , while long-term gains and both short and long-term losses are taxed at the lower long-term tax rate,  $\tau_l$ . In section 3.3.2, however, I assume that capital gains and losses are treated symmetrically for tax purposes: short-term gains and losses are taxed at the short-term rate and long-term gains and losses are taxed at the long-term rate.

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<sup>13</sup> See, for sample, Demski and Feltham (1994), Diamond and Verrecchia (1981), Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Kreps (1977), Leland (1992) and Verrecchia (1982a), (2001).

### 3.3.1 *Investor Demand Function if only Capital Gains Attract Differential Tax Rates*

Because tax is levied on the sale of an investment, if the investor purchases additional shares at date 2 or maintains his initial position from date 1 (*i.e.*  $x_2 \geq x_1$ ), there are no tax consequences at the trading date since none of the initial position is sold at this date. At the liquidation date, tax is incurred at the long-term rate on shares bought at date 2,  $x_2 - x_1$ . Tax is also incurred at the long-term rate on initial holdings from date 1,  $x_1$ . The total tax incurred in this case equals  $x_1(\tilde{u} - p_1)\tau_l + (x_2 - x_1)(\tilde{u} - p_2)\tau_l$ .

On the other hand, if the investor sells (some of) his initial holdings at date 2 (*i.e.*,  $x_2 < x_1$ ), if a gain is realized, tax is incurred at the short-term rate on shares sold at date 2,  $x_1 - x_2$ , while at the liquidation date tax is incurred at the long-term rate on gains or losses realized from the liquidation of the remaining shares,  $x_2$ . The total tax incurred in this case, thus, equals  $(x_1 - x_2)(p_2 - p_1)\tau_s + x_2(\tilde{u} - p_1)\tau_l$ . If, instead, a loss is realized at date 2, the investor incurs tax at the long-term rate on shares sold at date 2. The investor also incurs tax at the long-term rate on shares held until the liquidation date,  $x_2$ . The total tax incurred in this case equals  $(x_1 - x_2)(p_2 - p_1)\tau_l + x_2(\tilde{u} - p_1)\tau_l$ .

As a result, the investor's after tax wealth at the liquidation date equals:



$$(1 - \tau_l) \left[ x_1 (\tilde{u} - p_1) + (x_2 - x_1) (\tilde{u} - p_2) \right] = x_2 (\tilde{u} - p_1) (1 - \tau_l) + (x_1 - x_2) (p_2 - p_1) (1 - \tau_l) \\ \text{if } x_2 \geq x_1 \text{ or } p_2 < p_1$$

or

$$x_2 (\tilde{u} - p_1) (1 - \tau_l) + (x_1 - x_2) (p_2 - p_1) (1 - \tau_s) \quad \text{if } x_2 < x_1 \text{ and } p_2 \geq p_1.$$

With some algebraic rearrangement of these expressions an investor's period 3 wealth can be expressed as:

$$\tilde{w}_3 = x_1 \left[ \tilde{u} - \tau_l (\tilde{u} - p_1) \right] + \Delta x (1 - \tau_l) \begin{cases} (\tilde{u} - p_1) - \Delta p & \text{if } \Delta x \geq 0 \text{ or } \Delta p < 0 \\ (\tilde{u} - p_1) - (1 - k) \Delta p & \text{if } \Delta x < 0 \text{ and } \Delta p \geq 0 \end{cases} \quad (3.3)$$

where  $k = \frac{\tau_s - \tau_l}{1 - \tau_l}$  captures the differential tax rate on short and long-term

capital gains:  $k$  is greater than zero when such differential tax rates are present. Equation (3.3) indicates that the after-tax effect on an investor's period 3 wealth of a change in price in period 2 is more pronounced if the investor buys shares in period 2 or sells shares at a loss than if he sells at a gain. This reflects the fact that when an investor buys shares in period 2 or sells at a loss, the gain or loss realized on those shares is taxed at a lower rate,  $\tau_l$ , than the rate applied to any gains realized short-term if shares are sold in period 2.

Using the exponential utility function, the investor's utility for the end of the period wealth can be given as:

$$U(\tilde{w}_3) = -\exp \left[ -a \left[ x_1 [\tilde{u} - \tau_l(\tilde{u} - p_1)] + \Delta x(1 - \tau_l) \begin{cases} (\tilde{u} - p_1) - \Delta p & \text{if } \Delta x \geq 0 \text{ or } \Delta p < 0 \\ (\tilde{u} - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \text{ and } \Delta p \geq 0 \end{cases} \right] \right] \quad (3.4)$$

Each investor is assumed to maximize the expected value of  $U(\tilde{w}_3)$  conditional on observing the signal  $\mu$ . If  $\tilde{w}_3$  is normally distributed given  $\mu$ , then

$$E[U(\tilde{w}_3 | \mu)] = -\exp \left\{ -a \left[ E(\tilde{w}_3 | \mu) - \frac{a}{2} \text{var}(\tilde{w}_3 | \mu) \right] \right\}$$

where  $E(\tilde{w}_3 | \mu)$  is the investor's expected (after tax) wealth at date 3 conditional on observing  $\mu$  at date 2, and  $\text{var}(\tilde{w}_3 | \mu)$  is the conditional variance of wealth given  $\mu$ . To maximize  $E[U(\tilde{w}_3 | \mu)]$ , however, is equivalent to maximizing

$$E(\tilde{w}_3 | \mu) - \frac{a}{2} \text{var}(\tilde{w}_3 | \mu) \quad (3.5)$$

Where:

$$E(\tilde{w}_3 | \mu) = x_1 [\mu - \tau_l(\mu - p_1)] + \Delta x(1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \text{ or } \Delta p < 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \text{ and } \Delta p \geq 0 \end{cases} \quad (3.6)$$

and

$$\text{var}(\tilde{w}_3 | \mu) = \sigma_\varepsilon^2 (x_1 + \Delta x)^2 (1 - \tau_l)^2. \quad (3.7)$$

It follows that, by substituting equations (3.6) and (3.7) in equation (3.5), an investor's maximization problem at date 2 given the signal  $\mu$  and date 2 price of the risky asset is to determine  $\Delta x$  such that  $\Delta x$  maximizes

$$f(\Delta x) = x_1 [\mu - \tau_l(\mu - p_1)] - \frac{a}{2} \sigma_\varepsilon^2 (x_1 + \Delta x)^2 (1 - \tau_l)^2 + \Delta x (1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \text{ or } \Delta p < 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \text{ and } \Delta p \geq 0 \end{cases} \quad (3.8)$$

Equation (3.8) shows that whenever date 2 price of the risky asset is greater than the cost base (*i.e.*, whenever  $\Delta p > 0$ ), the differential tax rate on short and long-term gains causes a 'kink' in the investor's expected utility at  $\Delta x = 0$ . That is, the differential tax rate makes the expected utility function piecewise quadratic which complicates the usual first order approach to solve for the investor's optimal demand for the risky asset, as the first derivative is discontinuous at  $\Delta x = 0$ . Given this, however, the following lemma, which is proved in appendix A, shows that there are two cases for an investor's period 2 demand for the risky asset depending on whether the realization of  $\mu$  is 'high' (good news) or 'low' (bad news).

**Lemma 3.1:** Define  $\bar{\mu} = p_1 + a \sigma_\varepsilon^2 (1 - \tau_l) x_1$ .  $\bar{\mu}$  represents the level of beliefs ( $\mu$ ) at which an investor would not choose to trade if period 2 price remained unchanged from period 1 price.<sup>14</sup> If short-term gains are taxed at the ordinary tax rate while long-term gains and both short and long-term losses are taxed at the lower long-term rate, there are two possible cases for an investor's demand for the risky asset:

Case 1: If  $\mu \geq \bar{\mu}$

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p < \mu - \bar{\mu} \\ 0 & \text{if } \mu - \bar{\mu} \leq \Delta p \leq (1 - k)^{-1} (\mu - \bar{\mu}) \\ \frac{\mu - \bar{\mu} - (1 - k) \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p > (1 - k)^{-1} (\mu - \bar{\mu}) \end{cases} \quad (3.9)$$

Case 2: If  $\mu < \bar{\mu}$

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p < 0 \\ \frac{\mu - \bar{\mu} - (1 - k) \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p \geq 0 \end{cases} \quad (3.10)$$

Proof: see appendix A.

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<sup>14</sup> This benchmark can reasonably be interpreted as an investor's risk-adjusted breakeven point for  $\mu$  when prices remain unchanged, and is equal to period 1 price plus a risk premium. It provides a natural classification of the public signal into 'good' and 'bad news'.

An investor's (change in) demand for the risky asset at date 2 in the presence of capital gains taxes is determined by: his expectations about the value of the asset conditional on observing the signal  $\mu$ ; the change in the price of the asset from date 1 to date 2,  $\Delta p$ ; capital gains tax rates applied to short and long-term gains and losses; the investor's risk aversion parameter, and period 3 risk. Note that by setting both  $\tau_s$  and  $\tau_l$  equal to zero, the two cases in the above demand function collapse to the no-tax world's demand function given in equation (3.2).<sup>15</sup>

The two cases of the demand function in lemma 3.1 are determined by whether the public signal represents 'good' or 'bad news' to the investor. Figure 3.1 graphically plots these two cases against the change in the price of the risky asset,  $\Delta p$ . Part (a) of the figure represents case 1 of the demand function (*i.e.*, 'good news'). Note that there are three distinct segments in this case. The first segment is for low price changes at date 2 where the investor prefers to buy additional shares at this date. The third segment is for high price changes where the investor prefers to sell shares. Between these two segments, there is a flat segment. The flat segment represents a no-trade situation where the investor's expected utility is maximized by maintaining the initial position from date 1. The flat segment indicates that the investor is not willing to trade for that range of

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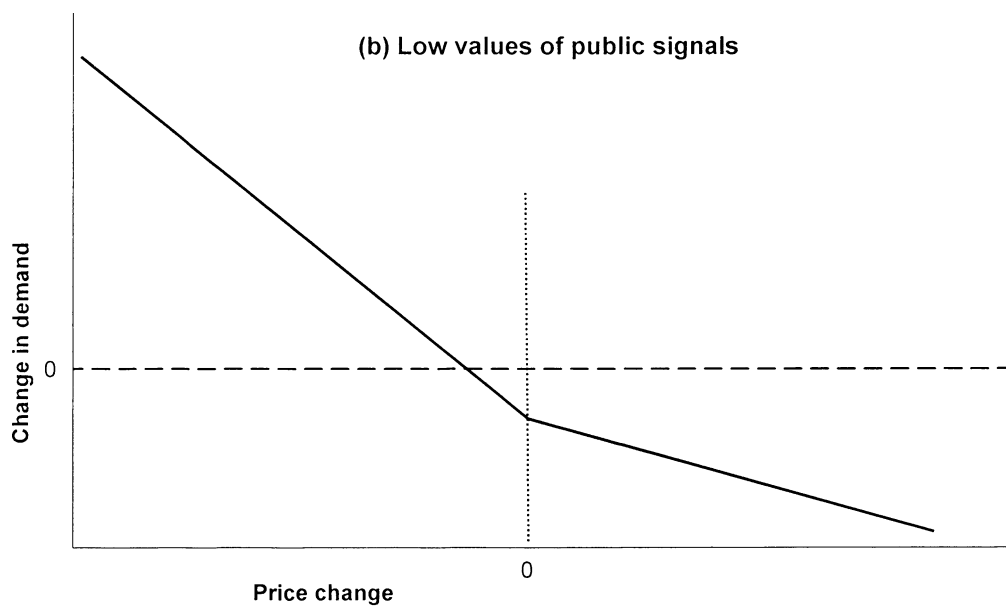
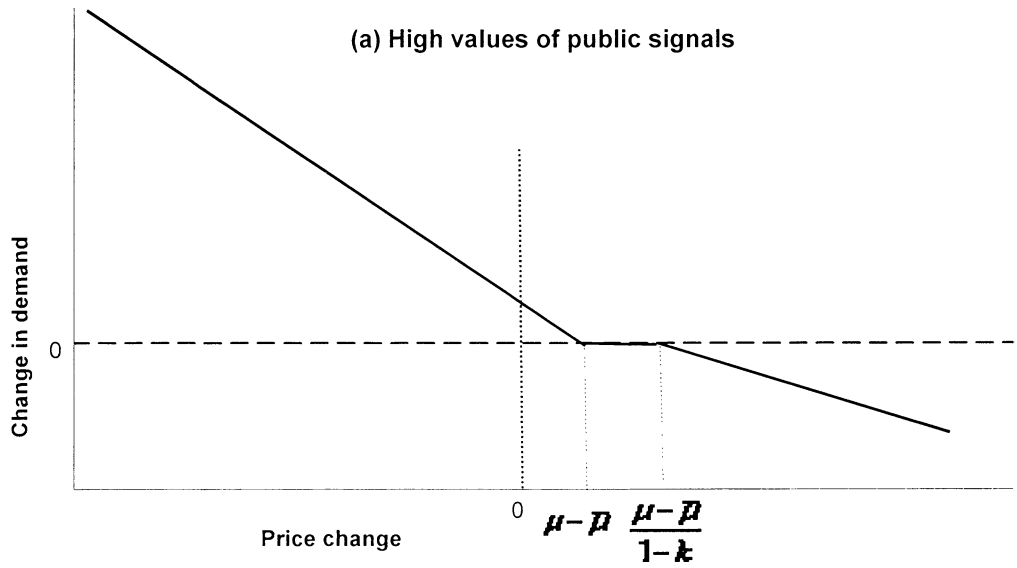
<sup>15</sup> The demand functions in equations (3.9) and (3.10) are given in terms of change in demand from date 1 to date 2. Substituting for  $\Delta x = x_2 - x_1$ ,  $\Delta p = p_2 - p_1$  and  $\bar{\mu} = p_1 + a \sigma_\varepsilon^2 (1 - \tau_l) x_1$ , and setting both  $\tau_s$  and  $\tau_l$  equal to zero, the two equations collapse to equation (3.2).

price changes and requires higher prices to induce him to sell before the qualification date and incur the tax cost at the higher short-term rate.

Whether there is a flat segment, however, depends on whether  $k$  is greater than zero which in turn depends on whether  $\tau_s - \tau_l > 0$ . Therefore, whenever there is a difference between short and long-term capital gains tax rates there will be a flat segment in the investor's demand function for price changes greater than zero if the condition  $\mu \geq \bar{\mu}$  is satisfied.

Note that in the first region, the buying region, demand is not affected by the differential tax rates applied to short and long-term capital gains as both investors' initial holdings of the asset and any shares purchased at date 2 will attract the long-term tax treatment at date 3. In contrast, differential tax rates on capital gains affect investors' demand in the selling region. Note also that, because  $\mu - \bar{\mu}$  is positive and  $\Delta p$  is also positive, an investor's change in demand in the selling region (*i.e.*, when  $\Delta p > (1-k)^{-1}(\mu - \bar{\mu})$ ) is decreasing (in magnitude) in the differential tax rates. This indicates that investors sell fewer shares at the trading date, relative to the case where no differential rate is present, because selling at this date attracts a tax cost at the higher short-term rate.

Figure 3.1: Investor demand for a risky asset if only capital gains attract differential tax rates depending on whether they are short or long-term



Part (b) of figure 3.1 represents case 2 of the demand function (*i.e.*, ‘bad news’). Note that because the public signal represents ‘bad news’ the price required to induce an investor to sell shares at date 2 is less than the price required when there is a ‘good news’ signal. In addition, as lemma 3.1 and figure 3.1 (b) show, because the demand function in case 2 changes slope at  $\Delta p = 0$  (*i.e.*, where there are no realized gains or losses), the differential tax rates on short and long-term capital gains have no effect at the switching point. Therefore, there is no flat segment in this case.

As in case 1, however, demand for the risky asset in case 2 is not affected by the differential tax rates in the buying region. Demand is also not affected by the differential tax rates in this case if investors sell shares at a loss. This is because any shares purchased at date 2 or sold at a loss attract the long-term tax rate. As a result, demand for all price changes less than zero is the same as would occur in an economy where all gains and losses attract tax at the same rate,  $\tau_l$ . In contrast, at all price changes greater than zero, investors sell fewer shares at date 2 than they would if no differential tax rate on short and long-term gains is present because any realized gain at this date attracts the higher short-term tax rate.

In summary, differential tax rates on short and long-term capital gains affect investors’ demand for the risky asset. This effect differs based on whether the public signal is greater or less than a benchmark representing an investor’s



risk-adjusted breakeven point for expected value of the asset if prices remain unchanged. In both cases, the effect differs depending on whether there is a price increase or a price decrease. When there is a price increase, the higher short-term tax rate causes the investor, if wishing to sell shares, to prefer to sell fewer shares than he would if no differential tax rate were present. On the other hand, when there is a price decrease or when the investor is wishing to buy, demand is the same as would occur in an economy where all gains and losses attract the same tax rate,  $\tau_l$ . In addition, when the public signal is greater than the risk-adjusted breakeven point, the differential tax rates on short and long-term capital gains can result in a situation where the investor is not willing to trade for a specific range of price increases due to the high tax costs.

### *3.3.2 Investor Demand Function if both Capital Gains and Losses Attract*

#### *Differential Tax Rates*

In this section I derive an investor's demand function for the risky asset assuming that capital gains and losses are treated symmetrically for tax purposes. Specifically, I assume that short-term gains and losses are taxed at the ordinary tax rate,  $\tau_s$ , while long-term gains and losses are taxed at the lower long-term rate,  $\tau_l$ . As a result, there is a tax advantage from the early realization of a loss to mirror the tax penalty from the early realization of a gain.

As short-term losses are taxed at the short-term rate, when the investor sells shares at date 2, irrespective of whether the result is a gain or a loss, this gain or loss is taxed at the short-term rate. However, any gains or losses realized from the liquidation of the remaining shares at date 3 will attract the long-term tax rate. Therefore, the total tax incurred when the investor sells shares at date 2 equals:  $(x_1 - x_2)(p_2 - p_1)\tau_s + x_2(\tilde{u} - p_1)\tau_l$ . As a result, the expression for period 3 wealth from section 3.3.1, equation (3.3), changes to:

$$\tilde{w}_3 = x_1 [\tilde{u} - \tau_l(\tilde{u} - p_1)] + \Delta x(1 - \tau_l) \begin{cases} (\tilde{u} - p_1) - \Delta p & \text{if } \Delta x \geq 0 \\ (\tilde{u} - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \end{cases} \quad (3.11)$$

That is, the effect of differential tax rates occurs whenever an investor sells shares. This reflects the fact that any gains (losses) realized from selling at date 2 attract a higher tax cost (benefit) than gains (losses) realized at the liquidation date.

Consequently, the investor's maximization problem at date 2 becomes to determine  $\Delta x$  that maximizes:

$$f(\Delta x) = x_1 [\mu - \tau_l(\mu - p_1)] - \frac{\alpha}{2} \sigma_\varepsilon^2 (x_1 + \Delta x)^2 (1 - \tau_l)^2 + \Delta x(1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \end{cases} \quad (3.12)$$

As in section 3.3.1, the differential tax rate on short and long-term gains and losses causes a point of discontinuity in the derivative of equation (3.12) at  $\Delta x = 0$ . However, the discontinuity in this case occurs for any non-zero price change at date 2 (*i.e.*, whenever  $\Delta p \neq 0$ ) rather than for price increases alone. As I show below, the impact of the differential tax rate at the discontinuity point differs when there is a price increase from when there is a price decrease.

**Lemma 3.2:** *Define  $\bar{\mu} = p_1 + a \sigma_\varepsilon^2 (1 - \tau_l) x_1$ . If long-term gains and losses are taxed at a lower rate than short-term gains and losses, there are two possible cases for an investor's demand function depending on parameter values:*

*Case 1: If  $\mu \geq \bar{\mu}$*

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p < \mu - \bar{\mu} \\ 0 & \text{if } \mu - \bar{\mu} \leq \Delta p \leq (1 - k)^{-1} (\mu - \bar{\mu}) \\ \frac{\mu - \bar{\mu} - (1 - k) \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p > (1 - k)^{-1} (\mu - \bar{\mu}) \end{cases} \quad (3.13)$$

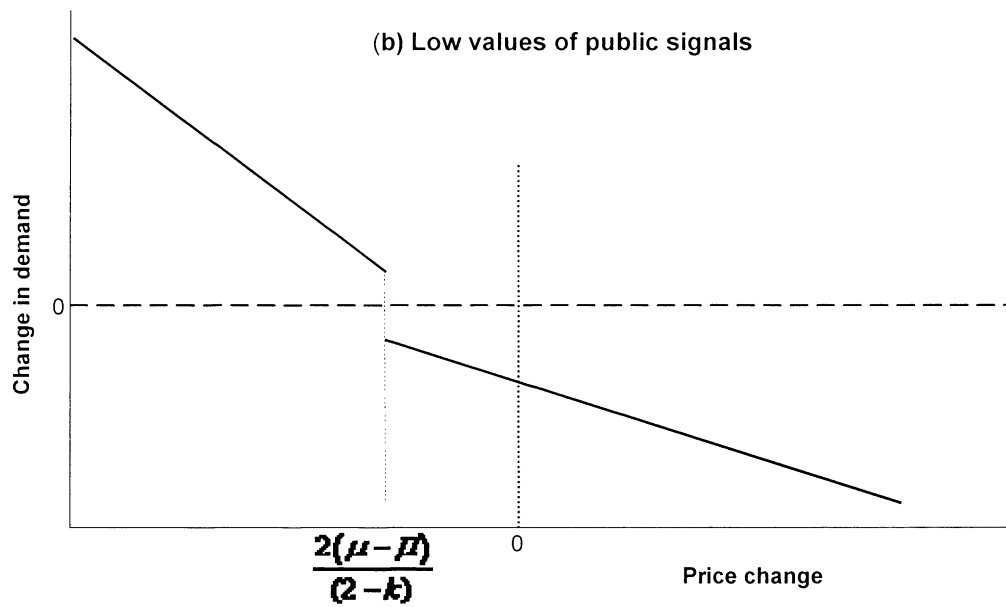
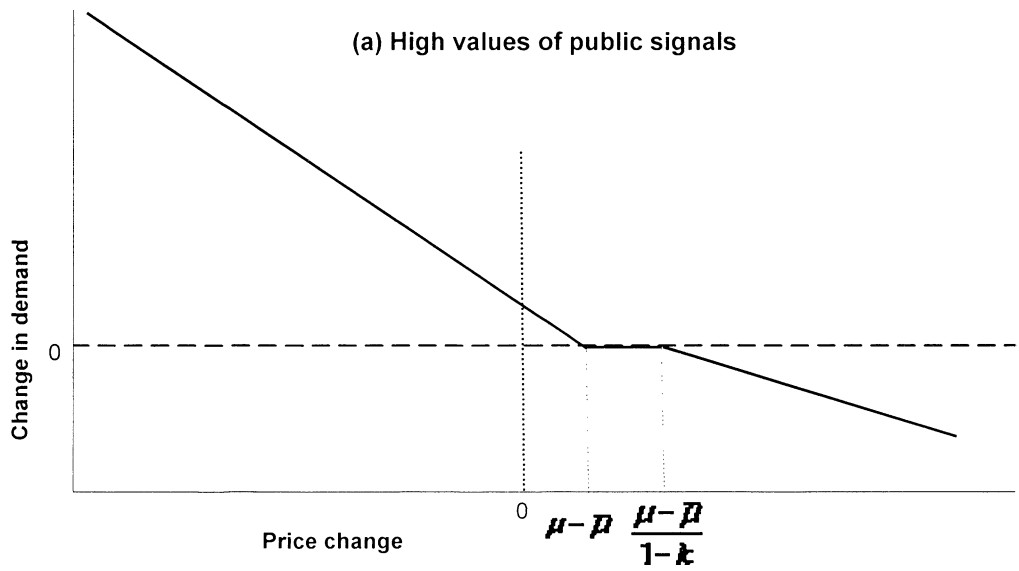
*Case 2: If  $\mu < \bar{\mu}$*

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p \leq 2(2 - k)^{-1} (\mu - \bar{\mu}) \\ \frac{\mu - \bar{\mu} - (1 - k) \Delta p}{a \sigma_\varepsilon^2 (1 - \tau_l)} & \text{if } \Delta p \geq 2(2 - k)^{-1} (\mu - \bar{\mu}) \end{cases} \quad (3.14)$$

Proof: see appendix A.

The two cases in lemma 3.2 are represented graphically in figure 3.2. Again, the two cases are determined by whether the public signal represents ‘good’ or ‘bad news’ to an investor. Note that an investor’s demand for the risky asset for ‘good news’ signals is not affected by the tax treatment of short and long-term losses as there are no realized losses at date 2. Therefore, the demand function for ‘good news’ signals is the same as given in case 1 of lemma 3.1.

Figure 3.2: Investor demand for a risky asset if both capital gains and losses attract differential tax rates depending on whether they are short or long-term



In contrast, the demand function when there is a ‘bad news’ signal is affected by the tax treatment of short and long-term losses. In particular, it affects the investor’s demand for price changes between  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}) < 0$  and  $\Delta p = 0$  as it is the only region in which the investor wishes to sell at a loss. As figure 3.2 shows, the demand function for ‘bad news’ signals is discontinuous at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu})$ . When the public signal represents ‘bad news’, the investor is better off by selling shares at date 2 as selling at this date attracts a tax benefit (tax deduction) at the higher tax rate.<sup>16</sup> At relatively large price declines, however, the investor becomes better off by buying shares at date 2 and holding them until the liquidation of the asset.  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu})$  represents the price change level at which the investor’s demand switches from selling to buying as the price declines. At this price change level, the investor is indifferent between selling and buying the same number of shares as his expected utility is maximized by either action. Note that because selling is affected by the differential short and long-term tax rates while buying is not, this effect is reflected in a jump in the demand function at the switch point.

However, whether there is a discontinuity at this point of the demand function depends on whether  $k > 0$  which depends on whether  $\tau_s - \tau_l > 0$ .

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<sup>16</sup> For price increases at date 2, however, the after-tax result from realizing a short-term gain is greater than the after-tax result from deferring the realization until the liquidation date given the level of the public signal reported.

Therefore, if short and long-term capital losses are taxed at different rates, there will be a discontinuity in the investor's demand function at a certain price change level less than zero, if the condition  $\mu < \bar{\mu}$  is satisfied.

Case 2 in lemma 3.2 indicates that, for price changes between  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}) < 0$  and  $\Delta p = 0$  (*i.e.*, when the investor is selling at a loss), the investor's change in demand is increasing (in magnitude) in the differential tax rates, as  $\mu - \bar{\mu}$  is negative and  $\Delta p$  is also negative. This indicates that the investor sells more shares (at a loss) at the trading date than he would sell if no differential tax rate is present, because selling at this date attracts a tax benefit at the higher short-term rate.

### 3.4 Conclusion

In this chapter I derived investors' demand function for a risky asset in the presence of differential capital gains tax rates and conditional on observing a public information signal about the risky asset's final payoff. The results indicate that differential tax rates affect investors' demand for the risky assets in response to the public information signal. This effect differs depending on whether the public signal represents 'good' or 'bad news' to an investor. When only capital gains attract differential tax rates and the public signal represents 'good news' to the investor, differential tax rates on short and long-term capital gains cause the investor, if wishing to sell, to wish to sell fewer shares than he would sell if no

differential tax rates are present. In addition, for sufficiently high price increases, the high tax costs associated with selling at a short-term gain discourage all trade and the investor is better off by neither buying nor selling. However, if the investor is wishing to buy, demand is the same as when there is no differential tax rate. On the other hand, if the public signal represents ‘bad news’ to the investor, the investor’s demand for the risky asset for all price decreases is the same as when there is no differential tax rate. For all price increases, the investor would sell fewer shares than he would if there is no differential tax rate.

When both capital gains and losses attract differential tax rates and the public signal represents ‘good news’ (‘bad news’) to the investor, the higher tax costs (benefits) associated with selling at a short-term gain (loss) cause the investor, if wishing to sell, to wish to sell fewer (more) shares than if no differential tax rates are present. Again, for sufficiently high price increase if the public signal is ‘good news’, differential tax rates on capital gains result in a situation where the investor is better off by neither buying nor selling. In addition, at a certain price change (*i.e.*, price decline) if the public signal is ‘bad news’, differential tax rates on short and long-term losses cause the investor to be indifferent between buying or selling the same number of shares as his expected utility is maximized by either action. However, for all public signals values, the investor’s demand if wishing to buy is the same as when there is no differential tax rate.



## **Chapter 4: Capital Gains Taxes and Equilibrium Price and Trading Volume Response to Public Information in a Noise-Free Market with Two Types of Rational Investors**

### **4.1 Introduction**

The purpose of this chapter is to extend the analysis in Shackelford and Verrecchia (2002) to include ‘bad news’ public disclosures. Here, I examine the effect of differential capital gains tax rates on equilibrium price and trading volume response to public information disclosures (both ‘good’ and ‘bad news’) assuming that the supply of the risky asset is fixed and known with certainty (*i.e.*,  $\Delta X = 0$ ). All trade which occurs in this setting is due to rational investors re-balancing their holdings on the basis of information contained in the public signal. As pointed out in Shackelford and Verrecchia (2002), this will only occur if investors are assumed to begin with holdings of the risky asset that differ from pareto optimal holdings. Therefore, I follow Shackelford and Verrecchia and assume that there are two types of rational investors who differ in their risk preferences and their initial holdings of the risky asset. Those investor-types are labelled A and B with the proportion of each type  $\pi_A$  and  $\pi_B$  respectively, where  $\pi_A + \pi_B = 1$ . Thus, each investor-type’s risk aversion parameter, period 1

holdings of the risky asset, and change in demand is represented by  $a_i$ ,  $x_i$ , and  $\Delta x_i$  respectively where  $i = A$  or  $B$ .

As in chapter three, I consider two cases regarding the tax treatment of gains and losses. In the first, only capital gains attract differential tax rates based on whether they are short or long-term while both short and long-term losses provide the same tax deduction at the long-term rate. In this case there is no tax advantage from the early realization of a loss. I then consider the case where long-term gains and losses are taxed at a lower rate than short-term gains and losses. Thus, any reduction in the holdings at date 2 at a price different from the cost base will result in either a tax-advantaged loss or a tax-disadvantaged gain, while the liquidation of assets at date 3 will result in a tax-advantaged gain or a tax-disadvantaged loss.

## 4.2 Defining Equilibrium

In the context of this research, the market equilibrium is defined as a circumstance in which the market clears and prices depend on information through supply and demand (see e.g., Diamond and Verrecchia (1981)). As the aggregate per capita supply in this chapter is assumed known and unchanged across periods 1 and 2, the market clearing condition can be stated in terms of the average change in per capita demand across the two investor-types such that:

$$\pi_A \Delta x_A + \pi_B \Delta x_B = 0 \quad (4.1)$$

An immediate implication of this is that if equilibrium involves trade, the two investor-types must change demands in opposite directions; if one investor-type buys shares, the other type must sell.

As indicated by Shackelford and Verrecchia (2002), in the absence of differential tax rates, it is straightforward to show that the pareto efficient equilibrium in period 2 is:<sup>17</sup>

$$\begin{aligned} p_2 &= \mu - \bar{a} \sigma_\varepsilon^2 (1 - \tau) X \\ x_{2i} &= \frac{\bar{a}}{a_i} X \end{aligned} \quad (4.2)$$

where  $i =$  investor-type A or B, and  $\bar{a} = \left( \frac{\pi_A}{a_A} + \frac{\pi_B}{a_B} \right)^{-1}$ .<sup>18</sup>

In this equilibrium, as all investors regardless of type have the same expectations about the value of the risky asset conditional on observing the information signal  $\mu$  at date 2, each investor-type holds per capita amount of the

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<sup>17</sup> That is when short and long-term capital gains and losses attract the same tax rate (*i.e.*,  $\tau_s = \tau_l = \tau$ ).

<sup>18</sup> In terms of change in price and demand, period 2 pareto efficient equilibrium can be given as:  $\Delta p = \mu - \bar{\mu}_{Mkt}$  and  $\Delta x_i = \frac{-(\bar{\mu}_i - \bar{\mu}_{Mkt})}{a_i \sigma_\varepsilon^2 (1 - \tau)}$ , where  $\bar{\mu}_{Mkt} = \pi'_A \bar{\mu}_A + \pi'_B \bar{\mu}_B$ . Details are in appendix B.

risky asset weighted by their risk aversion parameter relative to the harmonic mean of the risk aversion parameter across all rational investors.

Following Shackelford and Verrecchia I refer to the situation where investor-type  $i$  has  $x_i > \frac{\bar{a}}{a_i} X$  as being ‘overweight’ in the risky asset in period 1 relative to the pareto efficient holdings in the absence of differential tax rates in period 2. On the other hand, if the investor-type has  $x_i < \frac{\bar{a}}{a_i} X$ , it is considered to be ‘underweight’ in the risky asset relative to the pareto efficient holdings. By construction, if one investor-type is overweight the other type must be underweight. Without loss of generality, I assume that investor-type A is overweight and investor-type B is underweight in the risky asset.<sup>19</sup>

With this assumption, it is straightforward to show from demand equations (3.9) and (3.10) in chapter 3 that  $\Delta x_A \leq \Delta x_B$  for all values of  $\Delta p$ . An immediate consequence of this, in conjunction with the market clearing condition in equation (4.1), is that any equilibrium that results in trade must involve the overweight investor-type, A, selling shares and the underweight investor-type, B, buying shares.

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<sup>19</sup> The terms ‘overweight’ and ‘underweight’ denote labels which characterize the two investor-types into those who in a no-CGT economy would sell shares in period 2 for optimal risk sharing reasons (the overweight investors), and those who would buy (the underweight investors). Since an immediate consequence of the definitions of over and underweight is that  $a_A x_{1A} > a_B x_{1B}$ , they conveniently combine the two dimensions that differ between investor-types – risk aversion and initial holdings of shares – into a single binary classification.

### 4.3 Market Equilibrium if only Capital Gains Attract Differential Tax Rates

#### 4.3.1 Equilibrium Price and Demand Functions

The equilibrium price function if only capital gains attract differential tax rates is determined by substituting each investor-type's demand in lemma 3.1 into the market clearing condition in equation (4.1) and solving for  $\Delta p$ . Equilibrium demands are determined by substituting  $\Delta p$  back into each investor-type's demand. As section 4.2 suggests, there are two possible candidates for equilibrium; one where type A investors sell shares and type B investors buy shares, and the other where both investor-types maintain their initial positions from date 1. Which and when each candidate equilibrium occurs depend on  $\mu$  as well as other model parameters that determine each investor-type's demand

function. Using the definitions  $\bar{a} = \left( \frac{\pi_A}{a_A} + \frac{\pi_B}{a_B} \right)^{-1}$  and  $\pi'_i = \frac{\bar{a}}{a_i} \pi_i$ , the following proposition provides a convenient characterization of the equilibrium:<sup>20</sup>

**Proposition 4.1:** Define:  $\bar{\mu}_i = p_1 + a_i \sigma_\varepsilon^2 (1 - \tau_i) x_i$ , where  $i = A$  or  $B$ ,

$\bar{\mu}_{\text{Mkt}} = \pi'_A \bar{\mu}_A + \pi'_B \bar{\mu}_B$  and  $\mu^* = \bar{\mu}_A + \frac{1-k}{k} (\bar{\mu}_A - \bar{\mu}_B)$ .<sup>21</sup> If short-term gains are

<sup>20</sup> Since  $\pi'_A + \pi'_B = 1$ ,  $\pi'_i$  can be interpreted as a 'risk aversion-adjusted' proportion for investor-type  $i$ . In effect, use of these definitions recalibrates the model on a risk aversion-adjusted basis providing a streamlined presentation and interpretation of results.

<sup>21</sup>  $\mu^*$  represents the value of the public signal where  $\mu - \bar{\mu}_B = (1-k)^{-1} (\mu - \bar{\mu}_A)$ .

taxed at the ordinary tax rate while long-term gains and both short and long-term losses are taxed at the lower long-term rate, equilibrium period 2 price and demand functions depend on the level of  $\mu$  as follows:

(1). When  $\mu > \mu^*$ , equilibrium price and demand functions are:

$$\begin{aligned}\Delta p &\in [\mu - \bar{\mu}_B, (1-k)^{-1}(\mu - \bar{\mu}_A)] \\ \Delta x_A &= 0 \\ \Delta x_B &= 0\end{aligned}$$

(2). When  $\bar{\mu}_{Mkt} \leq \mu \leq \mu^*$ , equilibrium price and demand functions are:

$$\begin{aligned}\Delta p &= (1 - k\pi'_A)^{-1} [\mu - \bar{\mu}_{Mkt}] \\ \Delta x_A &= \frac{1}{\alpha_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k\pi'_B}{1 - k\pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_A - \bar{\mu}_{Mkt}) \right] \\ \Delta x_B &= \frac{1}{\alpha_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k\pi'_A}{1 - k\pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right]\end{aligned}\tag{4.3}$$

(3). When  $\mu < \bar{\mu}_{Mkt}$ , equilibrium price and demand functions are:

$$\begin{aligned}\Delta p &= \mu - \bar{\mu}_{Mkt} \\ \Delta x_A &= \frac{-(\bar{\mu}_A - \bar{\mu}_{Mkt})}{\alpha_A \sigma_\varepsilon^2 (1 - \tau_l)} \\ \Delta x_B &= \frac{-(\bar{\mu}_B - \bar{\mu}_{Mkt})}{\alpha_B \sigma_\varepsilon^2 (1 - \tau_l)}\end{aligned}\tag{4.4}$$

Proof: see appendix A.

Proposition 4.1 suggests that the equilibrium in the presence of differential capital gains tax rates depends on whether the public signal is ‘high’, ‘intermediate’ or ‘low’.<sup>22</sup> For sufficiently high public signals the equilibrium involves no trade, with price change indeterminate within a specified range of values greater than zero,  $[\mu - \bar{\mu}_B, (1-k)^{-1}(\mu - \bar{\mu}_A)]$ . The first term in this interval represents the maximum change in price at which type B investors are willing to buy while the second term represents the minimum change at which type A investors are willing to sell. When the public signal is sufficiently high, the tax cost for type A investors is high and they require a price to induce them to sell which is higher than the maximum price at which type B investors are willing to buy, resulting in a no-trade equilibrium. A larger price increase that might compensate type A investors for the tax penalty does not form an equilibrium because then investor-type B will also be willing to sell and the market would not clear.

Note that both the no-trade equilibrium’s lower and upper bounds increase with the public signal. The lower bound,  $\Delta p = \mu - \bar{\mu}_B$ , increases on a dollar-for-dollar basis with the public signal (as is the case in the absence of differential tax rates) while the upper bound,  $\Delta p = (1-k)^{-1}(\mu - \bar{\mu}_A)$ , increases with the public signal by a factor  $(1-k)^{-1}$  which is greater than one. This reflects the fact that it

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<sup>22</sup> Note that by setting both  $\tau_s$  and  $\tau_l$  equal to  $\tau$ , the three cases collapse to the no-differential tax rates setting in equation (4.2).

is the overweight investor-type, A, who sells in equilibrium and who faces the differential tax rate applied to any gains realized from the sale. Therefore, an increase in the public signal (*i.e.*, better news) induces extra costs to type A investors if they sell at date 2, in which case they require to be compensated through higher prices. Thus, unless the equilibrium price is always at the lower bound it is more sensitive to the public signal than in the no-differential tax rate setting represented in equation (4.2).

For intermediate values of the public signal trade occurs and period 2 price and demands are as outlined in equation (4.3). These expressions correspond exactly to those in Shackelford and Verrecchia's (2002) proposition 1, although expressed in different notation.<sup>23</sup> Equation (4.3) indicates that the differential tax treatment of short and long-term capital gains causes equilibrium prices to be more highly associated with the public signal: the slope coefficient on the public signal is greater than one as the term  $(1 - k\pi'_A)^{-1}$  is greater than one since  $k < 1$  and  $\pi'_A \leq 1$ .

Note that for intermediate values of the public signal, the equilibrium price change's association with the public signal is increasing in the magnitude of the differential tax rate,  $k$ . It is also increasing in the (risk aversion-adjusted)

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<sup>23</sup> Shackelford and Verrecchia set  $\tau_l$  to zero which causes  $k$  to equal  $\tau_s$ . Also, they use a risk tolerance parameter,  $r$ , which is equal to  $\frac{1}{a}$  in my model. Allowing for these differences, equation (4.3) collapses to Shackelford and Verrecchia's (2002) equation (1).



proportion of the overweight investor-type in the market,  $\pi'_A$ , because it is this investor-type which sells and incurs the tax cost related to the differential tax treatment of short and long-term gains.

Regarding equilibrium demands for intermediate values of the public signal, note that each investor-type's change in demand in equation (4.3) can be

$$\text{rewritten as } \Delta x_A = \frac{k\pi'_B \Delta p}{a_A \sigma_\varepsilon^2 (1-\tau_l)} - \frac{\bar{\mu}_A - \bar{\mu}_{Mkt}}{a_A \sigma_\varepsilon^2 (1-\tau_l)} \text{ and } \Delta x_B = \frac{-k\pi'_A \Delta p}{a_B \sigma_\varepsilon^2 (1-\tau_l)} - \frac{\bar{\mu}_B - \bar{\mu}_{Mkt}}{a_B \sigma_\varepsilon^2 (1-\tau_l)}.$$

Note also that in a no-differential tax rate economy (*i.e.*,  $\tau_s = \tau_l = \tau$ ), the pareto

$$\text{efficient equilibrium change in demand in period 2 can be given as } \frac{-(\bar{\mu}_i - \bar{\mu}_{Mkt})}{a_i \sigma_\varepsilon^2 (1-\tau_l)}$$

where  $i = A$  or  $B$ . This means that since  $\Delta p > 0$  in the intermediate region, type A investors' change in demand is less negative than the pareto efficient change in demand while type B investors' change in demand is less positive than the pareto efficient change in demand. That is, the overweight (underweight) investor-type remains overweight (underweight) but less so than in period 1: both investor-types trade towards their pareto efficient holdings, but to a lesser extent than they would if no differential tax rates were present.

For low values of the public signal, equilibrium price and demands are as outlined in equation (4.4). As both short and long-term losses are taxed at the same rate, there is no tax benefit from realizing a short-term loss at date 2. Therefore, equilibrium price and demands are the same as in a no tax differential

tax rate setting. The equilibrium price increases with the public signal on a dollar-for-dollar basis, and demands are the pareto efficient demands that would occur in an economy where all gains and losses are taxed at the same rate,  $\tau_l$ , and are not associated with the public signal.

### 4.3.2 Trading Volume

In this section I examine how the differential tax treatment of short and long-term capital gains affects trading volume response to public information signals. Following prior research I calculate trading volume as one half the absolute value of the change in investors' aggregate demand (see e.g., Demski and Feltham (1994) and Kim and Verrecchia (1997)). Thus, given the model's assumptions, trading volume in period 2 can be expressed as:

$$V = \frac{1}{2} \left[ \pi_A |\Delta x_A| + \pi_B |\Delta x_B| \right] \quad (4.5)$$

In the absence of differential tax rates, the trading volume can be given as:

$$V = \frac{1}{2} \left[ \pi_A \left| \frac{-(\bar{\mu}_A - \bar{\mu}_{Mkt})}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \right| + \pi_B \left| \frac{-(\bar{\mu}_B - \bar{\mu}_{Mkt})}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \right| \right] \quad (4.6)$$

Recall that for any equilibrium that involves trade in the presence of differential capital gains tax rates, type A investors sell shares and type B

investors buy shares. Thus,  $\Delta x_A < 0$  and  $\Delta x_B > 0$  for any equilibrium price change level. In addition, section 4.4.1 suggests that in the presence of differential tax rates investors will sell/buy fewer shares than they would in the absence of such differential rates when there is a price increase. Thus,

$$|\Delta x_A| < \left| \frac{-(\bar{\mu}_A - \bar{\mu}_{Mkt})}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \right| \quad \text{and} \quad |\Delta x_B| < \left| \frac{-(\bar{\mu}_B - \bar{\mu}_{Mkt})}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \right|$$

whenever there is a price increase. It follows that, by inspecting equation (4.5) along with these two observations, trading volume in the presence of differential capital gains tax rates is lower than the trading volume in the absence of differential rates when there is a price increase.

Based on the equilibrium demand functions in proposition 4.1, trading volume in the presence of differential capital gains tax rates can be characterized as follows:<sup>24</sup>

$$V = \begin{cases} 0 & \text{if } \mu > \mu^* \\ \frac{\pi'_A \pi'_B}{\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} [(\bar{\mu}_A - \bar{\mu}_B) - k \Delta p] & \text{if } \bar{\mu}_{Mkt} \leq \mu \leq \mu^* \\ \frac{\pi'_A \pi'_B}{\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} [(\bar{\mu}_A - \bar{\mu}_B)] & \text{if } \mu < \bar{\mu}_{Mkt} \end{cases} \quad (4.7)$$

where  $\mu^*$  and  $\bar{\mu}_{Mkt}$  are as defined in proposition 4.1.

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<sup>24</sup> Details are in appendix B.

As indicated earlier, equation (4.7) shows that when the public signal is high neither investor-type wishes to trade and there is no trading volume. High levels of the public signal cause period 2 price to be high and the tax cost of selling for type A investors discourages all trade. Note also that for intermediate values of public signals, trading volume is decreasing in  $k$  when  $\Delta p > 0$ . The tax cost associated with selling at a short-term gain inhibits trading volume. Finally, since both short and long-term capital losses are taxed at the same rate,  $\tau_l$ , trading volume for low values of public signals is not affected by the differential tax rate on capital gains.

In summary, the taxation of short and long-term capital gains at two different rates affects equilibrium values of risky assets around the release of a ‘good news’ information signal about risky assets’ final payoff. The tax cost associated with selling at a short-term gain discourages trading around ‘good news’ signals. Consequently, price changes around ‘good news’ signals are greater in the presence of differential tax rates than in the absence of differential tax rates. In addition, trading volume is lower than what it would be if no differential tax rates are present.

#### 4.4 Market Equilibrium if both Capital Gains and Losses Attract Differential Tax Rates

##### 4.4.1 Equilibrium Price and Demand Functions

Based on the demand functions in lemma 3.2 and the market clearing condition in equation (4.1), equilibrium price and demand functions if both gains and losses attract differential tax rates can be characterized as follows:

**Proposition 4.2:** Define:  $\bar{\mu}_i, \bar{\mu}_{ikt}$  and  $\mu^*$  as in proposition 4.1. Also define

$$\mu_i^{**} = \bar{\mu}_i - \frac{2-k}{k} \left[ \frac{\pi'_j}{\pi'_i - \pi'_j} (\bar{\mu}_A - \bar{\mu}_B) \right] \text{ when } \pi'_i > \pi'_j, \text{ where } i = A \text{ or } B.^{25}$$

If long-term gains and losses are taxed at a lower rate than short-term gains and losses, equilibrium period 2 price and demands for the risky assets depend on the level of  $\mu$  and on the (risk aversion-adjusted) proportion of each investor-type,  $\pi'_i$ , such that:

- (1) When  $\mu > \mu^*$ , equilibrium price and demand functions are:

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<sup>25</sup>  $\mu_i^{**}$  is the value of the public signal ( $\mu$ ) at which the aggregate change in demand across the two investor-types at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$  is equal to zero, where  $i = A$  or  $B$ . This can be satisfied only if  $\pi'_i > \pi'_j$ . If  $\pi'_i = \pi'_j$  then  $\mu_i^{**}$  is undefined ( $\mu_i^{**} = -\infty$ ).

$\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$  is the price change (from lemma 3.2) at which investor-type  $i$  is indifferent between buying or selling.

$$\begin{aligned}\Delta p &\in \left[ \mu - \bar{\mu}_B, (1-k)^{-1} (\mu - \bar{\mu}_A) \right] \\ \Delta x_A &= 0 \\ \Delta x_B &= 0\end{aligned}$$

(2) When  $\mu \leq \mu^*$ , there are two cases:

(a) If  $\pi'_A = \pi'_B$ , equilibrium price and demand functions are:

$$\begin{aligned}\Delta p &= (1 - k\pi'_A)^{-1} [\mu - \bar{\mu}_{Mkl}] \\ \Delta x_A &= \frac{1}{\alpha_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k\pi'_B}{1 - k\pi'_A} (\mu - \bar{\mu}_{Mkl}) - (\bar{\mu}_A - \bar{\mu}_{Mkl}) \right] \\ \Delta x_B &= \frac{1}{\alpha_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k\pi'_A}{1 - k\pi'_A} (\mu - \bar{\mu}_{Mkl}) - (\bar{\mu}_B - \bar{\mu}_{Mkl}) \right]\end{aligned}\tag{4.8}$$

(b) If  $\pi'_i > \pi'_j$ , where  $i$  and  $j = A$  or  $B$ ,  $i \neq j$ ,

(i) For  $\mu_i^{**} < \mu \leq \bar{\mu}_{Mkl}$ , equilibrium price and demand functions are as in (2).

(ii) For  $\mu \leq \mu_i^{**}$ , equilibrium price and demand functions are:

$$\begin{aligned}\Delta p &= 2(2-k)^{-1} [\mu - \bar{\mu}_i] \\ \Delta x_i &= \begin{cases} \frac{1}{\alpha_i \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_i) \right] \\ \text{or} \\ \frac{1}{\alpha_i \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2-k} (\mu - \bar{\mu}_i) \right] \end{cases} \\ \Delta x_j &= \begin{cases} \frac{1}{\alpha_j \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2-k} (\mu - \bar{\mu}_i) - (\bar{\mu}_A - \bar{\mu}_B) \right] & \text{if } j = A \\ \frac{1}{\alpha_j \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_i) + \frac{2}{2-k} (\bar{\mu}_A - \bar{\mu}_B) \right] & \text{if } j = B \end{cases}\end{aligned}\tag{4.9}$$

where a fraction,  $\lambda_i$ , of type  $i$  investors choose to sell according to the  $\Delta x_i$  expression in equation (4.9), and  $1-\lambda_i$  choose to buy such that the aggregate change in their demand exactly offset the aggregate change in type  $j$ 's demand.  $\lambda_i$  can be determined using the market clearing condition together with  $\Delta x_i$  and  $\Delta x_j$  from equation (4.9).

Proof: see appendix A.

Proposition 4.2 reveals a complex equilibrium which depends on the level of  $\mu$  and on the (risk aversion-adjusted) proportion of each investor-type,  $\pi_i'$ . However, the proposition indicates that the differential tax treatment of short and long-term losses affects only equilibrium prices and demands for public signals  $\mu < \bar{\mu}_{Nkt}$  (relative to the case where only gains attract differential tax rates in proposition 4.1). This is because it is the only region of the public signal in which type A investors sell at a loss. For all other values of public signals, equilibrium prices and demands are the same as in proposition 4.1.

As indicated in lemma 3.2, differential tax rates on capital losses cause a discontinuity in an investor's demand function for 'bad news' public signals. This occurs at a price change equal to  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$ . At this price change level, the investor is indifferent between buying and selling the same

number of shares. Therefore, depending on the model parameters' values, there can be two points of discontinuity (*i.e.*, two indifference points) in the demand functions for public signals  $\mu < \bar{\mu}_{Mkt}$ ; one for each investor-type in the market. It is possible for the market to clear at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$ , where  $i = A$  or  $B$ , and thus,  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$  constitutes an equilibrium price but only if investor-type  $i$  has the majority in the market on a risk aversion adjusted basis (*i.e.*,  $\pi'_i > \pi'_j$ ), and investors within this type divide between buying and selling such that the aggregate change in their demands exactly offsets the aggregate change in type  $j$ 's demand where the market clears. This occurs for any values of  $\mu \leq \mu_i^{**}$  where  $\mu_i^{**}$  is the value of the public signal ( $\mu$ ) at which the market will clear at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$ . If  $\pi'_A = \pi'_B$  then  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i) < 0$  is not an equilibrium because the market will not clear at this price change.

For public signals values  $\mu < \bar{\mu}_{Mkt}$  where  $\pi'_A = \pi'_B$  or  $\mu_i^{**} < \mu < \bar{\mu}_{Mkt}$  where  $\pi'_i > \pi'_j$ , equilibrium involves type A investors selling shares at a loss and type B investors buying shares. Because both capital gains and losses attract differential tax rates, equilibrium in this case is the same as for intermediate public signals in proposition 4.1. For all public signals  $\mu < \bar{\mu}_{Mkt}$ , however, equilibrium period 2 price in the presence of differential tax rates on capital losses is more sensitive to the public signal than in the absence of differential tax



rates: the slope coefficient on the public signal is greater than 1. If short-term losses are taxed at a higher rate than long-term losses, there is a tax benefit from realizing a short-term loss at date 2 and the tax benefit increases as the public signal decreases. Therefore, equilibrium price changes are more sensitive to the public signal and sensitivity increases with the differential tax rate.

For equilibrium demands for public signal  $\mu < \bar{\mu}_{Mkt}$  where  $\pi'_A = \pi'_B$  or  $\mu_i^{**} < \mu < \bar{\mu}_{Mkt}$  where  $\pi'_i > \pi'_j$ , note that since  $\Delta p < 0$  when  $\mu < \bar{\mu}_{Mkt}$ , investor-type A's change in demand is more negative than the pareto efficient change in demand while investor-type B's change in demand is more positive than the pareto efficient change in demand. This means that, if period 2 price is less than period 1 price, the overweight (underweight) investor-type sells (buys) sufficient shares to become underweight (overweight). This reflects the tax incentive for type A investors to realize losses short-term when short-term losses are taxed at a higher rate than long-term losses.

For equilibrium demands for public signal values  $\mu \leq \mu_i^{**}$  where  $\pi'_i > \pi'_j$ , investor-type  $j$ 's demand is uniquely determined, but investor-type  $i$ 's demand is one of two possible amounts: these are the two demand levels of equal expected utility underlying the demand function derived in lemma 3.2. One involves the trader buying, the other selling. Equilibrium in equation (4.9) is achieved by allowing type  $i$  investors to divide between buying and selling such that the

aggregate change in their demands exactly offsets the aggregate change in type  $j$ 's demand.

The analysis here shows that Shackelford and Verrecchia's (2002) results do extend to some low values of public signals if the tax treatment of capital losses is assumed to mirror that of gains. In particular, Shackelford and Verrecchia's proposition 1, which corresponds to equation (4.8) above, can also occur when there is a 'bad news' disclosure (*i.e.*,  $\Delta p < 0$ ).

#### 4.4.2 Trading Volume

The trading volume if both capital gains and losses attract differential tax rates is obtained by substituting the relevant demand expression in proposition 4.2 for each investor-type into the trading volume expression in equation (4.5). After rearranging and simplifying, the following expression is obtained:<sup>26</sup>

$$V = \begin{cases} 0 & \text{if } \mu > \mu^* \\ \frac{\pi'_A \pi'_B}{\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} [(\bar{\mu}_A - \bar{\mu}_B) - k \Delta p] & \text{if } \begin{cases} \mu \leq \mu^*, \text{ where } \pi'_A = \pi'_B \\ \text{or } \mu_i^{**} < \mu \leq \mu^*, \text{ where } \pi'_i > \pi'_j \end{cases} \\ \frac{1}{2\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \pi'_j (\bar{\mu}_A - \bar{\mu}_B) - \frac{k}{2} \Delta p \right] & \text{if } \mu \leq \mu_i^{**}, \text{ where } \pi'_i > \pi'_j \end{cases} \quad (4.10)$$

where  $i$  and  $j = A$  or  $B$  and  $i \neq j$ .

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<sup>26</sup> Details are in appendix B.

The trading volume expression in equation (4.10) differs from that in equation (4.7) only for public signal values  $\mu < \bar{\mu}_{Mkt}$ . In particular, for public signals  $\mu < \bar{\mu}_{Mkt}$  where  $\pi'_A = \pi'_B$  or  $\mu_i^{**} < \mu < \bar{\mu}_{Mkt}$  where  $\pi'_i > \pi'_j$ , trading volume in the presence of differential tax rates on capital losses is the same as for intermediate public signal in equation (4.7) while for public signal  $\mu \leq \mu_i^{**}$

where  $\pi'_i > \pi'_j$ , trading volume is given as 
$$\frac{1}{2\bar{a}\sigma_\varepsilon^2(1-\tau_l)} \left[ \pi'_j(\bar{\mu}_A - \bar{\mu}_B) - \frac{k}{2} \Delta p \right].$$

However, for any public signal  $\mu < \bar{\mu}_{Mkt}$ , it is straightforward to show that trading volume in the presence of differential tax rates on short and long-term capital losses is greater than in the absence of differential tax rates. If short-term losses are taxed at a higher rate than long-term losses, the tax benefit associated with realizing a short-term loss encourages type A investors, who sell in equilibrium, to realize more losses at date 2 which increases trading volume.

#### 4.5 Conclusion

In this chapter I examined the impact of differential short and long-term capital gains tax rates on equilibrium price and trading volume reaction to public information signals in a noise-free market with two types of rational investors. The results indicate that when only capital gains attract differential tax rates, equilibrium prices are more sensitive to ‘good news’ public signals than when no differential tax rates are present. The tax cost associated with selling at a short-

term gain increases with the value of the public signal which causes equilibrium price changes to be more sensitive to the signal. In addition, differential tax rates on capital gains can cause a no-trade equilibrium for high values of the public signal. For sufficiently high price increases around ‘good news’ signals, the tax cost of selling at a short-term gain is ‘too high’ and a no-trade equilibrium results. Moreover, trading volume in the presence of differential tax rates on short and long-term gains is lower than in the absence of differential tax rates around ‘good news’ signals. However, as both short and long-term capital losses are taxed at the same rate, investors are indifferent about the realization of losses short-term. Therefore, equilibrium price changes and trading volume around ‘bad news’ public signals are the same as in a no-differential tax rates world.

When both gains attract differential tax rates, equilibrium prices are more sensitive to ‘good’ and ‘bad news’ public signals than in the absence of differential tax rates. For ‘bad news’ signals, the tax benefit associated with selling at a short-term loss increases with the absolute value of the public signal which causes equilibrium price changes to be more sensitive to the signal. In addition, differential tax rates on capital losses can result in an equilibrium (for sufficiently ‘bad news’ signals) where investors of one investor-type (the investor-type who has the majority in the market on a risk adjusted basis) mixes between buying and selling in order to clear the market. Moreover, trading volume in the presence of differential tax rates on short and long-term losses is higher than in the absence of differential tax rates around ‘bad news’ signals.

Thus, the results in this chapter confirm and extend the results of Shackelford and Verrecchia (2002). They confirm Shackelford and Verrecchia's result that differential tax rates on capital gains may inhibit trading volume and magnify price changes around the public release of 'good news' signals, relative to an economy in which there are no differential tax rates. My results also provide three extensions to Shackelford and Verrecchia's results. First, for the 'good news' case investigated in Shackelford and Verrecchia (2002), my results suggest that it is possible for the tax cost associated with sufficiently large price increases around 'good news' public signals to result in a no trade equilibrium. Second, my results indicate that if capital losses also attract differential tax rates based on whether they are short or long-term, equilibrium price changes and trading volume are magnified around 'bad news' public signal relative to where there are no differential tax rates. Finally, my results suggest that, if the public signal is 'bad news', differential tax rates on capital losses can result in an equilibrium where investors of the more prevalent investor-type in the market (on a risk aversion adjusted basis) mixes between buying and selling in order to clear the market.

## **Chapter 5: Capital Gains Taxes and the ‘Information Content’ of Securities’ Prices**

### **5.1 Introduction**

In chapter 4 I examined the impact of differential capital gains tax rates on the market reaction to public information assuming that the supply of the risky asset is fixed and known with certainty. The objective of this chapter is to examine the impact of differential tax rates on the ‘information content’ of equilibrium prices with respect to public information signals, and the degree of noise in prices. Because the setting in chapter 4 assumes fixed (exogenous) supply of the risky asset, the public signal determines all variation in prices by construction. As a result, it does not permit investigation of questions related to ‘information content’ of, and noise in, prices. In this chapter I modify that setting to include exogenous noisy supply of the risky asset. In particular, I assume that the per capita change in aggregate supply of the asset,  $\Delta\tilde{X}$ , is a random variable independently normally distributed with mean zero and variance  $\sigma_{\Delta X}^2$ . In this case, any variation in the price of the risky asset can be either due to variation in the supply of the asset or due to variation in information. I examine how differential tax rates affect these two factors and their combined effects on equilibrium prices.

For reasons of simplicity, I assume only one type of rational investors in the market with all investors within this type identical with respect to their risk preferences, initial holdings of the asset and the cost base of the asset. Although, this allows me to investigate the impact of differential tax rates on the ‘information content’ of, and noise in, equilibrium prices in a relatively simple way, it does not allow investigation of trading volume-related effects of differential tax rates as volume in this case will always be the change in noisy supply of the risky asset (*i.e.*, trading volume is exogenous when there is only one type of rational investors since this type will trade only to satisfy the noisy supply and the equilibrium condition).

## 5.2 Defining Equilibrium

If there is only one type of rational investors then all rational investors will have the same demand function at date 2. In equilibrium the per capita demand for the risky asset by rational investors must equal the per capita noisy supply of the asset. In terms of investors’ change in demand across periods 1 and 2, equilibrium must satisfy the following:

$$\Delta x = \Delta \tilde{X} \tag{5.1}$$

where  $\Delta \tilde{X} = \tilde{X}_2 - \tilde{X}_1$ . Equilibrium price then can be obtained by substituting demand into this and rearranging to solve for  $\Delta p$ .

### 5.3 Market Equilibrium if only Capital Gains Attract Differential Tax

#### Rates

#### 5.3.1 The Equilibrium Price Function

From lemma 3.1, note that for ‘good news’ public signals (*i.e.*,  $\mu \geq \bar{\mu}$ ), investors’ demand for the risky asset is split into three segments:

$$\Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} > 0, \quad \Delta x = 0, \quad \text{and} \quad \Delta x = \frac{\mu - \bar{\mu} - (1 - k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} < 0.$$

For each segment, equilibrium price function is determined by setting the  $\Delta x$  expression relevant for that segment equal to  $\Delta \tilde{X}$  and rearranging for  $\Delta p$ . Therefore, when

$$\Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} = \Delta \tilde{X} > 0, \quad \text{the equilibrium price function can be determined as}$$

$\Delta p = \mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta \tilde{X}$ . When  $\Delta x = \Delta \tilde{X} = 0$ , the equilibrium price will be indeterminate between  $\mu - \bar{\mu}$  and  $(1 - k)^{-1}(\mu - \bar{\mu})$ . When

$$\Delta x = \frac{\mu - \bar{\mu} - (1 - k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} = \Delta \tilde{X} < 0, \quad \text{the equilibrium price function can be}$$

determined as  $\Delta p = (1 - k)^{-1}[\mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta \tilde{X}]$ .

On the other hand, for ‘bad news’ public signals (*i.e.*,  $\mu < \bar{\mu}$ ), investors’

demand function is split into two segments:  $\Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} \geq \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1 - \tau_l)}$  and

$\Delta x = \frac{\mu - \bar{\mu} - (1 - k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} \leq \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1 - \tau_l)}$ . Again, equilibrium price function is



determined by setting the  $\Delta x$  expression relevant for each segment equal to  $\Delta \tilde{X}$  and rearranging for  $\Delta p$ . Thus, when  $\Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1-\tau_l)} = \Delta \tilde{X} \geq \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1-\tau_l)}$ , the equilibrium price function is  $\Delta p = \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta \tilde{X}$ , and when  $\Delta x = \frac{\mu - \bar{\mu} - (1-k)\Delta p}{a\sigma_\varepsilon^2(1-\tau_l)} = \Delta \tilde{X} \leq \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1-\tau_l)}$ , the equilibrium price function is  $\Delta p = (1-k)^{-1}[\mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta \tilde{X}]$ . The following proposition summarizes these results:

**Proposition 5.1:** *Define  $\bar{\mu} = p_1 + a\sigma_\varepsilon^2(1-\tau_l)x_1$ . Equilibrium period 2 price if all rational investors are identical and if short-term gains are taxed at the ordinary tax rate while long-term gains and both short and long-term losses are taxed at the long-term rate depends on the level of  $\mu$  and  $\Delta \tilde{X}$  such that:*

*Case 1: If  $\mu \geq \bar{\mu}$ , the equilibrium price function is:*

$$\Delta p = \begin{cases} \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta \tilde{X} & \text{if } \Delta \tilde{X} > 0 \\ \left[ \mu - \bar{\mu}, (1-k)^{-1}(\mu - \bar{\mu}) \right] & \text{if } \Delta \tilde{X} = 0 \\ (1-k)^{-1}[\mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta \tilde{X}] & \text{if } \Delta \tilde{X} < 0 \end{cases} \quad (5.2)$$

*Case 2: If  $\mu < \bar{\mu}$ , the equilibrium price function is:*

$$\Delta p = \begin{cases} \mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta\tilde{X} & \text{if } \Delta\tilde{X} > \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1 - \tau_l)} \\ (1 - k)^{-1}[\mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta\tilde{X}] & \text{if } \Delta\tilde{X} \leq \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1 - \tau_l)} \end{cases} \quad (5.3)$$

Proposition 5.1 indicates that there are in total three regions for equilibrium price depending on the level of the public signal,  $\mu$ , and the noisy supply of the risky asset,  $\Delta\tilde{X}$ . These regions are portrayed graphically in figure 5.1. Region 1 corresponds to circumstances where differential tax rates have no impact on equilibrium (either the rational investors wish to buy shares or they wish to sell shares at a loss). It occurs when  $\Delta\tilde{X} > 0$  and  $\mu \geq \bar{\mu}$  (*i.e.*, price change is positive but noisy exogenous supply change is also positive), and also when  $\Delta\tilde{X} > \frac{\mu - \bar{\mu}}{a\sigma_\varepsilon^2(1 - \tau_l)}$  (*i.e.*, price change is negative). Equilibrium in this region is not affected by the differential tax rates on capital gains as there are no realized gains at date 2. The equilibrium price change in this region is  $\Delta p = \mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta\tilde{X}$ , the same as would occur in a no-differential tax rate economy.

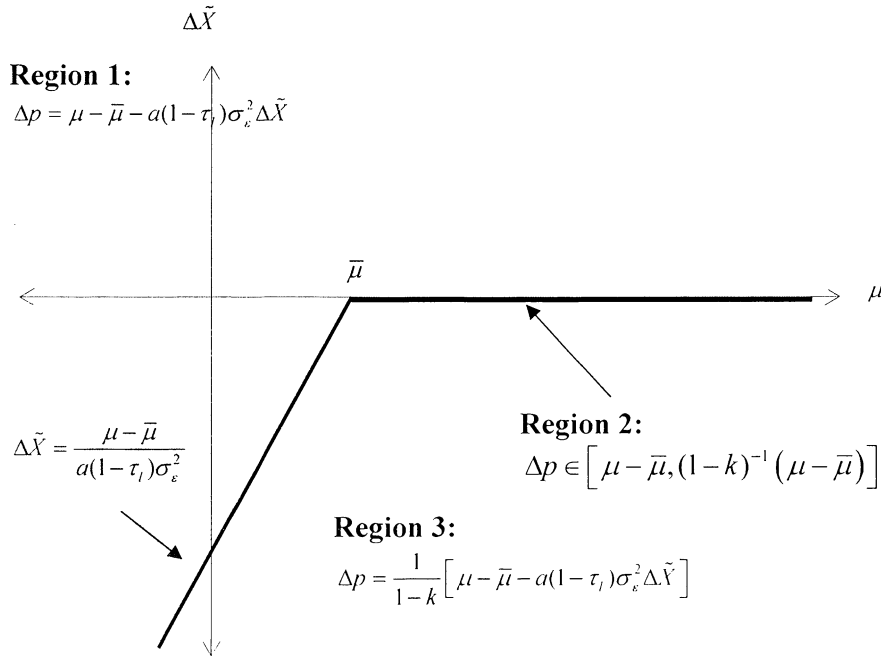
Region 2 of the equilibrium is where rational investors do not trade and occurs when  $\mu \geq \bar{\mu}$  and  $\Delta\tilde{X} = 0$ .<sup>27</sup> This region is represented in figure 5.1 by the line starting at  $\bar{\mu}$  and continuing to the right along the  $\mu$  axis. Because there is no change in the noisy supply of the risky asset at date 2, no trade occurs by the rational investors. Thus, the equilibrium price change in this region will be indeterminate between  $\mu - \bar{\mu}$  and  $(1-k)^{-1}(\mu - \bar{\mu})$ .

Finally, region 3 is where rational investors sell shares at a gain. It occurs when  $\Delta\tilde{X} < 0$  and  $\mu \geq \bar{\mu}$  or when  $\Delta\tilde{X} \leq \frac{\mu - \bar{\mu}}{a\sigma_\epsilon^2(1-\tau_l)}$  and  $\mu < \bar{\mu}$  (*i.e.*, whenever noisy supply change is negative and price change is positive). Equilibrium in this region is affected by the differential tax rates. Specifically, the equilibrium price is the same price in region 1 multiplied by  $(1-k)^{-1}$  which is greater than one and increases with the differential tax rate. In this region there is a negative noisy supply change of the risky asset which is satisfied by the rational investors. However, since satisfying the decrease in the noisy supply requires rational investors to incur a tax penalty associated with realizing a short-term capital gain, a higher equilibrium price is needed to clear the market and to compensate them for the tax penalty.

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<sup>27</sup> Since this region occurs only if  $\Delta\tilde{X} = 0$ , it is a zero probability event given the continuous normal distribution assumption of  $\Delta\tilde{X}$ .

**Figure 5.1: Equilibrium price regions when only capital gains attract differential tax rates**



### 5.3.2 Comparative Statics

In this section I examine how differential capital gains tax rates affect market-related metrics such as price (change) volatility and ‘information content’ of prices with respect to the public signal, as well as the expected slope coefficients on information and noisy supply change in the equilibrium price function.<sup>28</sup> For expected slope coefficients and price change volatility, I calculate and

<sup>28</sup> The equilibrium region where the equilibrium price is indeterminate is irrelevant for the calculation and analysis of the various metrics since it occurs only if  $\Delta \tilde{X} = 0$ , which is a zero probability event given the continuous normal distribution assumption.

investigate comparative statics algebraically. For ‘information content’ of prices, however, the algebraic calculation does not yield a tractable expression. Instead I employ numerical methods to investigate comparative statics.

### *5.3.2.1 Expected Slope Coefficients*

Proposition 5.1 shows that the slope coefficients relating equilibrium price change to both information and noisy supply change differ across two regions of equilibrium. The slope coefficient on information is one in region 1, and  $(1-k)^{-1}$ , which is greater than one, in region 3. Similarly, the slope coefficient on noisy supply change (with respect to  $k$ ) is one in region 1, and  $(1-k)^{-1}$  in region 3. Since the probability of each region of equilibrium does not change with  $k$ , it is immediately clear that the expected slope coefficient (for both information and noisy supply change) is increasing in the differential tax rate,  $k$ . A greater difference between short and long-term capital gains tax rates results in a higher expected response coefficient between equilibrium prices and both public signals and noisy supply changes.

Thus, the result regarding the slope coefficients is consistent with chapter 4: equilibrium prices in the presence of differential tax rates are, in expectation, more sensitive to public information than in the absence of differential tax rates. However, as proposition 5.1 shows, equilibrium prices are affected by differential tax rates only when the public signal represents ‘good news’ and

there is a negative noisy supply change, or when noisy supply is sufficiently negative for some ‘bad news’ signals. In these cases, rational investors must be compensated for the increased tax costs associated with satisfying the decrease in noisy supply of the risky asset. Yet, the result in this section indicates that the average slope coefficients across the two equilibrium regions are increasing in the differential tax rate.

### 5.3.2.2 Price Volatility

Price volatility reflects the total amount of uncertainty in the risky asset’s return. For the purposes of this research I define the price (change) volatility as the variance of the price change,  $Var(\Delta p)$ . The variance of the equilibrium price change if only capital gains attract differential tax rates can be expressed as:<sup>29</sup>

$$Var(\Delta p) = q_1 \text{var}_1(\Delta p) + q_3 \text{var}_3(\Delta p) + q_1 q_3 [E_3(\Delta p) - E_1(\Delta p)]^2 \quad (5.4)$$

where  $q_j$  is the probability that equilibrium is in region  $j$  where  $j = 1$  and  $3$ ,  $\text{var}_j(\Delta p)$  is the conditional variance of  $\Delta p$  given that equilibrium is in region  $j$  and  $E_j(\Delta p)$  is the conditional expected price change in region  $j$ . Since both equilibrium regions do not vary with  $k$ , and the variance of region 1 also does not vary with  $k$ , therefore:

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<sup>29</sup> Details are in appendix B.

$$\frac{\partial}{\partial k} Var(\Delta p) = q_3 \frac{\partial}{\partial k} var_3(\Delta p) + q_1 q_3 2 [E_3(\Delta p) - E_1(\Delta p)] \frac{\partial}{\partial k} E_3(\Delta p)$$

It is straightforward to show that  $\frac{\partial}{\partial k} var_3(\Delta p)$  and  $\frac{\partial}{\partial k} E_3(\Delta p)$  are positive. Moreover,  $E_3(\Delta p) - E_1(\Delta p)$  is also positive.<sup>30</sup> Hence  $\frac{\partial}{\partial k} Var(\Delta p)$  is also positive. This means that the price change variance is increasing in the differential tax rate. Thus, equilibrium prices are more volatile as the difference between short and long-term capital gains tax rates increases. This reflects the influence of two factors: first, equilibrium prices are, on average, more sensitive to the public information signal as the differential tax rate increases. Second, equilibrium prices are also more sensitive to exogenous noisy supply changes.

### 5.3.2.3 The 'Information Content' of Equilibrium Prices

Following Grossman and Stiglitz (1980) and Demski and Feltham (1994) I calculate the 'information content' of equilibrium prices as the square of the correlation between the equilibrium price change and investors' expectations about the asset's value,  $R^2(\Delta p, \mu)$ . The algebraic calculation of the 'information content' of prices results in an expression in which the determination of

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<sup>30</sup> To see this note that  $\Delta p$  is decreasing in  $\Delta \tilde{X}$ , for a given  $\mu$ , across regions 1 and 3. This means that conditional on any  $\mu$ , the expected value of  $\Delta p$  in region 3 is greater than in region 1, which implies that  $E_3(\Delta p) - E_1(\Delta p)$  is positive.

comparative statics is complex and intractable.<sup>31</sup> Instead I employ numerical methods to investigate comparative statics based on the following parameter values:<sup>32</sup>

$$\begin{array}{ll}
 \alpha = 0.01 & E(\mu) = 1 \\
 \tau_l = 0.20 & E(\Delta X) = 0 \\
 x_1 = 10 & \sigma_\varepsilon = \sigma_\mu = \sigma_{\Delta X} = 10 \\
 p_1 = 2 &
 \end{array}$$

Figure 5.2 plots the squared correlation between the equilibrium price change and  $\mu$  against the differential tax rate,  $k$ . This figure indicates that the ‘information content’ of prices is decreasing in the differential tax rate. Although greater differential tax rates result in equilibrium prices that are more sensitive to information, they are also more sensitive to noisy supply changes. The results suggest that the noise effect of differential tax rates on equilibrium prices outweighs the information effect as the differential tax rate increases. Therefore, equilibrium prices are less informative with respect to the public information signal, and informativeness of prices decreases with the magnitude of the differential tax rate. This, potentially, is an empirically testable effect that might be added to the information effect in Shackelford and Verrecchia (2002) that has

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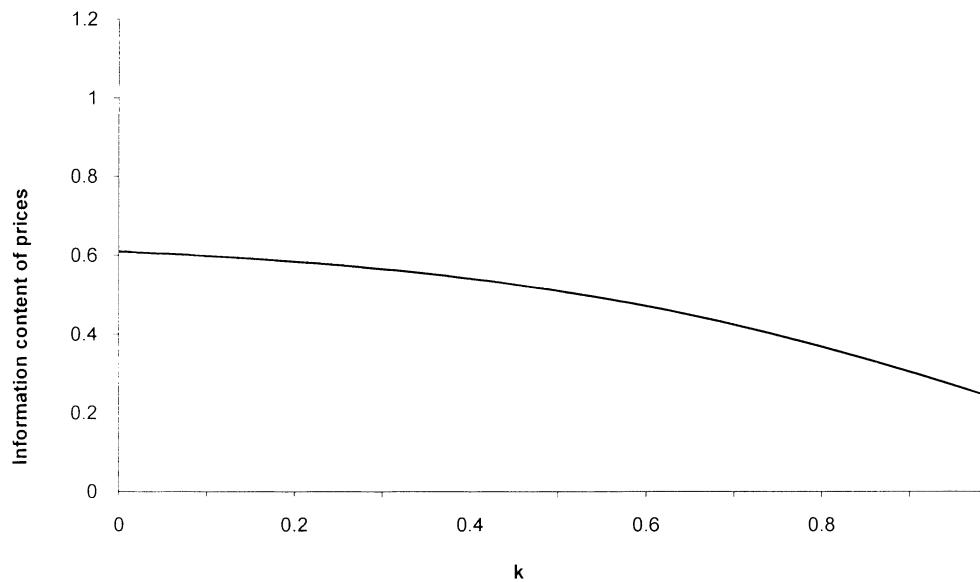
<sup>31</sup> The squared correlation between  $\Delta p$  and  $\mu$  is a ratio of the squared covariance between  $\Delta p$  and  $\mu$ , the numerator, and the product of  $\text{var}(\Delta p)$  and  $\text{var}(\mu)$ , the denominator. However, both the numerator and the denominator vary with  $k$ , which makes the derivative of both expressions complex given the piecewise linearity of  $\Delta p$ . This results in a complex expression for the derivative of  $R^2(\Delta p, \mu)$  that is difficult to analyse.

<sup>32</sup> These parameter values are arbitrary. However, I have investigated a range of alternative values with no substantive differences in the results.



been empirically investigated and supported (see e.g., Blouin et al. (2003), Hurtt and Seida (2004) and Jin (2006)).

**Figure 5.2: Comparative statics based on numerical solution of the 'information content' of prices if only capital gains attract differential tax rates**



In summary, despite the result that differential tax rates on short and long-term capital gains increase equilibrium prices sensitivity to public information signals and changes in noisy supply of risky assets, the results suggest that as short and long-term tax rates diverge, the impact of the noisy supply on equilibrium prices dominates and prices reflect less of the information in the public signal.

## 5.4 Market Equilibrium if both Capital Gains and Losses Attract

### Differential Tax Rates

#### 5.4.1 The Equilibrium Price Function

From lemma 3.2, note that for ‘good news’ public signals (*i.e.*,  $\mu \geq \bar{\mu}$ ), investors’ demand for the risky asset is not affected by the tax treatment of capital losses as there are no realized losses. Thus, the equilibrium price function in this case is the same as in case 1 in proposition 5.1.

Note also that, for ‘bad news’ public signals (*i.e.*,  $\mu < \bar{\mu}$ ), investors’ demand function is split into two segments:

$$\Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} \geq \frac{-k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1 - \tau_l)} \quad \text{and} \quad \Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} \leq \frac{k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1 - \tau_l)}.$$

Then, solving the market clearing condition,  $\Delta x = \Delta \tilde{X}$ , yields

$$\Delta p = \mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta \tilde{X} \quad \text{when} \quad \Delta \tilde{X} \geq \frac{-k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1 - \tau_l)} \quad \text{and}$$

$$\Delta p = (1-k)^{-1}[\mu - \bar{\mu} - a\sigma_\varepsilon^2(1 - \tau_l)\Delta \tilde{X}] \quad \text{when} \quad \Delta \tilde{X} \leq \frac{k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1 - \tau_l)}.$$

Note that at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu})$ , rational investors are indifferent between either buying (*i.e.*,  $\Delta x = \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)}$ ), or selling (*i.e.*,

$\Delta x = \frac{\mu - \bar{\mu} - (1-k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)}$ ). Therefore, any noisy supply change between

$\Delta\tilde{X} = \frac{k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}$  and  $\Delta\tilde{X} = \frac{-k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}$  will create an equilibrium

at this price where rational investors mix between buying and selling at the indifference points in lemma 3.2 that clears the market.

Thus, the equilibrium price if both gains and losses attract differential tax rates can be characterized as follows:

**Proposition 5.2:** *Equilibrium period 2 price if long-term capital gains and losses are taxed at a lower rate than short-term gains and losses depends on the levels of  $\mu$  and  $\Delta\tilde{X}$  such that:*

**Case 1:** *If  $\mu \geq \bar{\mu}$ , the equilibrium price function is:*

$$\Delta p = \begin{cases} \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X} & \text{if } \Delta\tilde{X} > 0 \\ \left[ \mu - \bar{\mu}, (1-k)^{-1}(\mu - \bar{\mu}) \right] & \text{if } \Delta\tilde{X} = 0 \\ (1-k)^{-1} \left[ \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X} \right] & \text{if } \Delta\tilde{X} < 0 \end{cases} \quad (5.5)$$

**Case 2:** *If  $\mu < \bar{\mu}$ , the equilibrium price function is:*

$$\Delta p = \begin{cases} \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X} & \text{if } \Delta\tilde{X} > \alpha \\ 2(2-k)^{-1}(\mu - \bar{\mu}) & \text{if } \beta \leq \Delta\tilde{X} \leq \alpha \\ (1-k)^{-1} \left[ \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X} \right] & \text{if } \Delta\tilde{X} < \beta \end{cases} \quad (5.6)$$

where :

$$\alpha = \frac{-k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}$$

$$\beta = \frac{k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}$$

Proposition 5.2 indicates that if both gains and losses attract differential tax rates, there are four regions in which the equilibrium price and demand functions differ based on the level of  $\mu$  and  $\Delta\tilde{X}$ , rather than the three regions when only gains attract differential tax rates. Also, the boundaries of regions 1 and 3 described in section 5.3.1 when only gains attract differential tax rates are changed (see figure 5.3). Region 1, where differential tax rates have no impact on the equilibrium, now excludes equilibria that occur if investors sell at a loss. The equilibrium price change in region 1 is  $\Delta p = \mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X}$  and occurs when  $\Delta\tilde{X} > 0$  and  $\mu \geq \bar{\mu}$ , or when  $\Delta\tilde{X} > \frac{-k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}$  and  $\mu < \bar{\mu}$ .

Region 3, on the other hand, is expanded and now includes equilibria that occur if investors sell at a loss. Equilibrium in region 3 occurs when  $\Delta\tilde{X} < 0$  and  $\mu \geq \bar{\mu}$ , or when  $\Delta\tilde{X} < \frac{k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}$  and  $\mu < \bar{\mu}$ . The equilibrium price in this region is higher than what it would be in the absence of differential tax rates. As there is a negative noisy supply change of the risky asset in this region, when rational investors satisfy this change, they incur a tax cost (benefit) associated

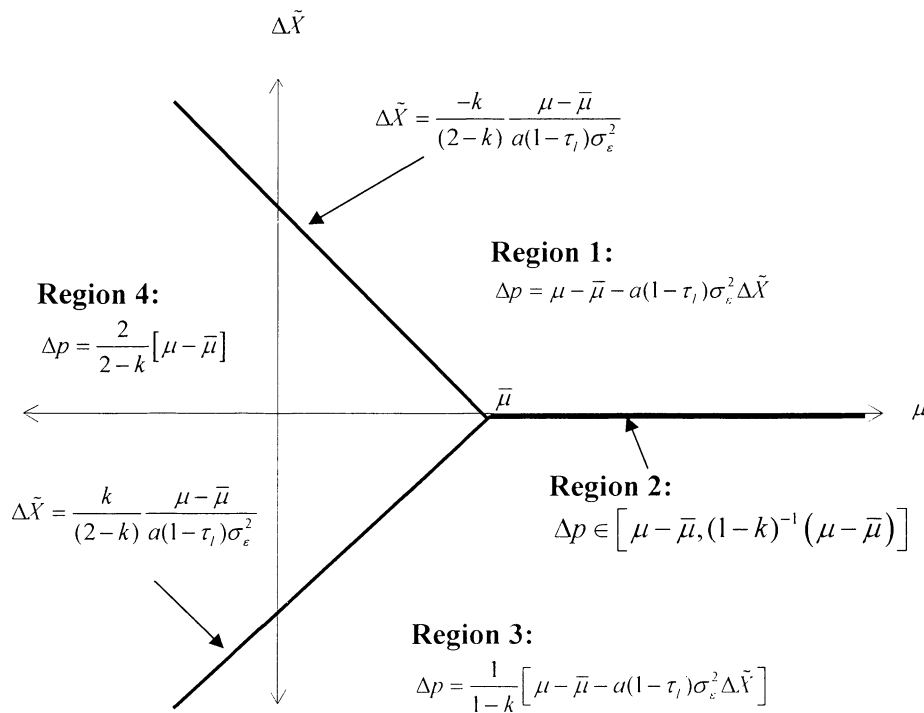
with realizing a short-term gain (loss). Therefore, higher (lower) equilibrium prices are needed to clear the market and to compensate for the tax penalty (benefit) than when there are no differential tax rates.

While the differential tax rates on short and long-term losses have no effect on region 2 where investors do not trade, they create a fourth region where the equilibrium price and demand differ based on  $\mu$  and  $\Delta\tilde{X}$ . In particular, region 4 is where a fraction of rational investors chooses to buy shares and a fraction chooses to sell. It occurs if  $\mu < \bar{\mu}$  and the noisy supply level satisfies

$$\frac{k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)} < \Delta\tilde{X} < \frac{-k(2-k)^{-1}(\mu - \bar{\mu})}{a\sigma_\varepsilon^2(1-\tau_l)}.$$

This region corresponds to the situation where investors are indifferent between buying or selling the same number of shares at a certain price change,  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu})$ , as their expected utility at date 3 is maximized by either of these actions.

**Figure 5.3: Equilibrium price regions when both capital gains and losses attract differential tax rates**



#### 5.4.2 Comparative Statics

As indicated in proposition 5.2 the equilibrium price function if both gains and losses attract differential tax rates differs across four regions of information and noisy supply realizations. However, both the equilibrium price function and the regions' boundaries here are influenced by differential tax rates through  $k$  making algebraic determination of comparative statics highly complex and does

not lead to tractable solutions. Therefore, I use numerical methods to calculate comparative statics using the same parameter values as in section 5.3.2.3.<sup>33</sup>

#### *5.4.2.1 Expected Slope Coefficients*

In section 5.3, the slope coefficients on both  $\mu$  and  $\Delta X$  are the same within an individual region. Therefore, the expected slope coefficients are the same for the two variables. In this section, the two slope coefficient differ in region 4 while they are the same in regions 1 and three. This means that the expected slope coefficients will not be the same for the two variables. Figures 5.4 (a) and (b) plot the expected values of the slope coefficients on information and change in noisy supply, respectively, against the differential tax rate,  $k$ . Note that the (absolute) magnitude of both coefficients is increasing in  $k$ . Thus, as the difference between short and long-term tax rates on capital gains and losses increases, equilibrium prices become, on average, more highly associated with both the public information signal and changes in noisy supply of the risky asset.

#### *5.4.2.2 Price Volatility and 'Information Content' of Prices*

Figures 5.4 (c) and (d) plot the equilibrium price change variance and the squared correlation between the equilibrium price change and  $\mu$  against the

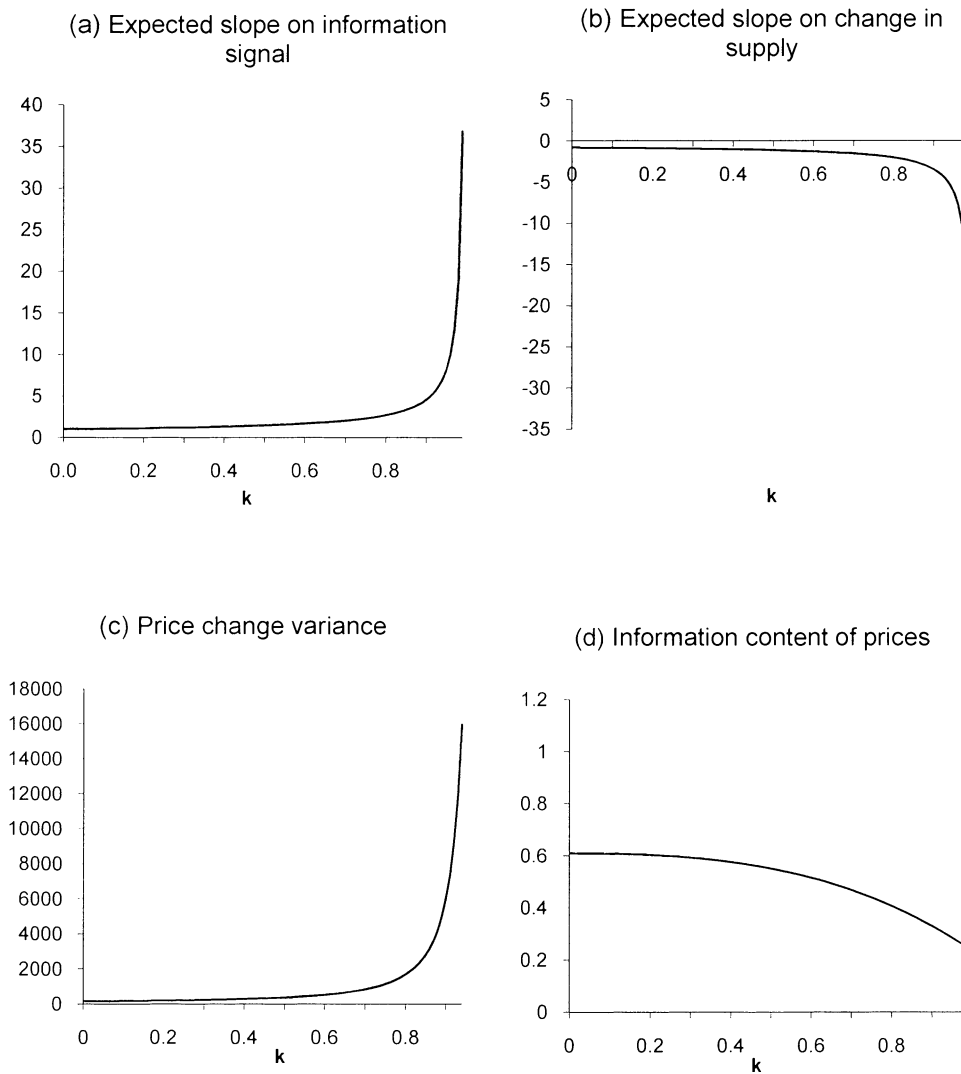
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<sup>33</sup> I have investigated different combinations of parameter values with no significant changes in the results reached. In addition, as in section 5.3.2, region 2, where the equilibrium price is indeterminate, has no impact on the calculated comparative statics since it is a zero probability event given the continuous normal distribution.

differential tax rate. These figures indicate that price volatility is increasing and the 'information content' of prices is decreasing in the differential tax rate. Thus, a greater differential tax rate causes equilibrium prices to be more volatile and less informative with respect to the public information signal. Again, this suggests that differential tax rates on short and long-term capital gains and losses have a greater 'noise effect' on equilibrium prices than an 'information effect', at least in expectation.



**Figure 5.4: Comparative statics based on numerical solution of equilibrium if both capital gains and losses attract differential tax rates**



## 5.5 Conclusion

This chapter examined the impact of differential capital gains tax rates on equilibrium price response to a public information signal about a risky asset's

value in a noisy market. The results indicate that the equilibrium price response to both the information signal and changes in exogenous random supply of the asset differs across different regions of equilibrium. However, algebraic and numerical comparative statics indicate that the expected response coefficient increases with the difference between short and long-term capital gains tax rates. The results also indicate that equilibrium prices become more volatile and less informative with respect to the public signal as the differential tax rate increases. This result suggests that the noise effect of differential tax rates on equilibrium prices outweighs the information effect as the differential tax rate increases. This provides a potential empirical implication beyond that provided in Shackelford and Verrecchia (2002) and confirmed in subsequent empirical work (e.g., Blouin et al. (2003), Hurtt and Seida (2004) and Jin (2006)).

## **Chapter 6: Summary and Conclusions**

### **6.1 Overview of the Thesis**

This thesis examined the effect of differential capital gains tax rates on the market reaction to public information signals. There were two main objectives in relation to the impact of capital gains taxes. The first objective was to extend the analysis in Shackelford and Verrecchia (2002) and investigate the impact of differential tax rates on short and long-term capital gains and losses on equilibrium prices and trading volume response to ‘bad news’ public disclosure about a risky asset’s value. The second objective was to investigate whether differential tax rates on short and long-term capital gains and losses affect the extent to which equilibrium prices reflect public information.

It was shown in chapter three that differential capital gains tax rates affect investors’ optimal demand for the risky asset in response to the release of an information signal about the risky asset’s value. This effect differs depending on whether the public signal represents ‘good’ or ‘bad news’ to an investor. If both gains and losses attract differential tax rates and the public signal represents ‘good news’ (‘bad news’) to the investor, the investor, if wishing to sell, would sell fewer (more) shares when there is a price increase (decrease) than he would if no differential tax rates are present. When the public signal represents ‘good news’, the tax cost associated with selling at a short-term gain discourages

trading and can result in a situation where the investor is not willing to trade. On the other hand, when the public signal represents ‘bad news’, the tax benefit associated with realizing a short-term loss encourages trading and can result in a point (at a certain price change less than zero) where the investor is indifferent between buying and selling the same number of shares. However, if the investor is wishing to buy shares, demand is the same as in a no-differential tax rates world. When only capital gains attract differential tax rates, the investor’s demand for the risky asset for all negative price changes is the same as in a no-differential tax rates world, while for price increases, the investor sells fewer shares than in a no-tax differential world.

It was shown that when both gains and losses attract differential tax rates, the effect of differential tax rates on short and long-term capital gains (losses) on investors’ optimal demand for the risky asset is reflected in higher (lower) equilibrium prices and lower (higher) trading volume around ‘good news’ (‘bad news’) signals. This was demonstrated, in chapter four, in a setting with two types of rational investors who differ in their risk preferences and initial holding of the risky asset, and a fixed supply of the risky asset. It was shown that equilibrium prices are more sensitive to public information (both ‘good’ and ‘bad news’) than in a no-differential tax rates world, and sensitivity increases with the magnitude of the differential tax rate and the proportion of the selling investor-type in the market. High (absolute) values of the public signal cause equilibrium price changes to be high which increases the tax cost (benefit) associated with

selling at a short-term gain (loss). As a result, equilibrium price changes are more sensitive to the signal than in the no-differential tax rates world.

In addition, for sufficiently high values of public signals, the tax cost of selling at a short-term gain is ‘too high’ and discourages all trade, resulting in a no-trade equilibrium. On the other hand, at a certain price change less than zero (*i.e.*, price decrease) around ‘bad news’ signals, the differential tax rate on capital losses can result in an equilibrium where investors of one investor-type (the investor-type which has a majority in the market on a risk adjusted basis) mixes between buying and selling in order to clear the market. However, if only capital gains attract differential tax rates, equilibrium price and demands for the risky asset for all negative price changes around ‘bad news’ public signals are the same as in a no-differential tax rates world.

In chapter five it was shown that differential tax rates on short and long-term capital gains and losses increase price change volatility and reduce the ‘information content’ of equilibrium prices with respect to public signals. This was demonstrated in a setting with only one type of rational investors and a random exogenous supply of the risky asset. Although it was shown that differential capital gains tax rates increase equilibrium price sensitivity to both information signals and noisy supply changes, the results suggest that the noise effect of differential tax rates on equilibrium prices outweighs the information effect so that prices are, on average, more volatile and less informative.

## **6.2 Limitations and Caveats**

There are a number of limitations and caveats relating to this research. These are primarily related to the assumptions that underlie the models employed. In particular, the models are highly stylized and depend upon a number of assumptions to simplify the analyses. These include assumptions such as normally distributed random variables, identical investors and constant absolute risk aversion parameters. In addition, the market includes only a single trading date.

Moreover, the assumption that all investors in the market are subject to capital gains taxes is unrealistic. In a real market, tax exempt and taxable investors of a particular stock may exist which can mitigate the impact of capital gains taxes on that stock's equilibrium values.

Furthermore, a potential shortcoming in this research is the fact that investors' initial prices and holdings of the risky asset were assumed to be exogenous. In a more general model, it is expected that differential capital gains tax rates will also influence initial prices and holdings of the asset. Thus, the results in this research, particularly the comparative statics results related to price volatility and the 'information content' of prices, are subject to a 'partial equilibrium' limitation. However, generalizing the model to endogenously solve

for initial equilibrium prices and holdings would be highly complex and beyond the scope of my objectives in this thesis.

## **6.3 Future Work**

### ***6.3.1 Analytical Extensions***

The analysis in this thesis can be extended in several ways. For example, as indicated in the previous section, differential capital gains tax rates are likely to influence initial prices and holdings of the asset. Therefore, a possible extension to this research is to generalize the model to endogenously solve for equilibrium price and holdings at date 1. This could provide insight into the possible impact of capital gains taxes on initial pricing, in IPOs for example. Such an extension would potentially yield additional empirical implications in the IPO setting.

Another possible extension to this research is to examine the impact of differential tax rates in a rational expectations setting where some investors have access to private information. This would require modelling equilibrium prices as a function of both the private information available to informed investors and the information that uninformed investors learn from market prices. However, the piecewise linearity of prices with respect to information exhibited by the analysis in this thesis is likely to carry over into a private information setting, severely complicating the analysis of a private information setting.

### ***6.3.2 Empirical Implications***

This research also has several potential empirical implications for future research. For example, the results indicate that differential tax rates may magnify price changes around public information disclosure. A potential consequence of this is the possibility for price reversal in subsequent periods. This provides an empirical implication for future work: to test for any capital gains tax related return reversals in periods subsequent to a public announcement, such as earnings release dates.

Also, results of the analysis in chapter 5 indicate that differential capital gains tax rates have both an information effect and a noise effect on equilibrium prices, and that the noise effect outweighs the information effect so that equilibrium prices are on average more volatile and reflect less of the information contained in a public signal than when no differential tax rates are present. This provides a potential empirically testable effect that can be investigated in future work.



## Appendix A: Proofs

### Proof of Lemma 3.1:

Recall that an investor's utility maximization problem at date 2 if only capital gains attract differential tax rates based on whether they are short or long-term is to solve for  $\Delta x$  that maximizes:

$$EU = x_1 \left[ \mu - \tau_l(\mu - p_1) \right] - \frac{a}{2} \sigma_\varepsilon^2 (x_1 + \Delta x)^2 (1 - \tau_l)^2 + \Delta x (1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \text{ or } \Delta p < 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \text{ and } \Delta p \geq 0 \end{cases} \quad (A1)$$

Differentiating equation (A1) with respect to  $\Delta x$  yields the following expression for the investor marginal expected utility,  $MU$ , from trading at date 2:

$$MU = -a\sigma_\varepsilon^2(1 - \tau_l)^2(x_1 + \Delta x) + (1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \text{ or } \Delta p < 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \text{ and } \Delta p \geq 0 \end{cases} \quad (A2)$$

Equation (A2) shows that whenever  $\Delta p \geq 0$ , the investor's marginal utility function is discontinuous at  $\Delta x = 0$ , thus, making the first order approach for solving this maximization problem inappropriate. To determine the investor's optimal demand at date 2, it is convenient to compare the expected marginal

utility from buying or selling an additional share incremental to his initial holdings from date 1.

Define:

$$MU_{Buy} = (1 - \tau_l) [\mu - \bar{\mu} - \Delta p] \quad (A3)$$

and

$$MU_{Sell} = \begin{cases} -(1 - \tau_l) [\mu - \bar{\mu} - \Delta p] & \text{if } \Delta p < 0 \\ -(1 - \tau_l) [\mu - \bar{\mu} - (1 - k)\Delta p] & \text{if } \Delta p \geq 0 \end{cases} \quad (A4)$$

where  $MU_{Buy}$  is the marginal utility if an investor buys an additional share,  $MU_{Sell}$  is the marginal utility if the investor sells a share, and  $\bar{\mu} = p_1 + a\sigma_\varepsilon^2(1 - \tau_l)x_1$  represents the level of beliefs ( $\mu$ ) at which the investor would not choose to trade if period 2 price remained unchanged from period 1 price.

Because the investor's expected utility is quadratic in  $\Delta x$ , whether the investor should buy or sell at date 2 is determined by the sign and the magnitude of  $MU_{Buy}$  and  $MU_{Sell}$ . In particular:

- If both  $MU_{Buy}$  and  $MU_{Sell}$  are positive, then the investor will choose to buy (sell) if  $MU_{Buy}$  ( $MU_{Sell}$ ) has greater magnitude.<sup>34</sup>
- If  $MU_{Buy} = MU_{Sell}$ , the investor will be indifferent between buying or selling.
- If  $MU_{Buy}$  is positive and  $MU_{Sell}$  is negative, the investor will choose to buy.
- If  $MU_{Buy}$  is negative and  $MU_{Sell}$  is positive, the investor will choose to sell.
- If both  $MU_{Buy}$  and  $MU_{Sell}$  are negative, the investor is made worse off by either buying and selling and will choose not to trade.

Inspection of equations (A3) and (A4) induces two cases to consider:

1. When  $\mu - \bar{\mu} \geq 0$  there are three possibilities:
  - If  $\Delta p < \mu - \bar{\mu}$  then  $MU_{Buy}$  is positive and  $MU_{Sell}$  is negative. In this case the investor will choose to buy and demand is determined by setting the first line in equation (A2) equal to 0 and solving for  $\Delta x$ .

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<sup>34</sup> This follows from the piecewise quadratic nature of the expected utility function. Expected utility in equation (A1) can be expressed in the form  $EU = \alpha_1 + \alpha_2 \Delta x^2 + \begin{cases} \alpha_{3buy} \Delta x & \text{if } \Delta x \geq 0 \\ \alpha_{3sell} \Delta x & \text{if } \Delta x < 0 \end{cases}$ .

The two local maxima are  $\alpha_1 - \frac{\alpha_{3Buy}^2}{4\alpha_2}$  and  $\alpha_1 - \frac{\alpha_{3Sell}^2}{4\alpha_2}$  respectively. The maximum of these two is determined by the greater absolute magnitude of  $\alpha_{3Buy}$  and  $\alpha_{3Sell}$ .

- If  $\Delta p > (1-k)^{-1}(\mu - \bar{\mu})$  then  $MU_{Buy}$  is negative and  $MU_{Sell}$  is positive. In this case the investor will choose to sell and demand is determined by setting the second line in equation (A2) equal to 0 and solving for  $\Delta x$ .
- If  $\mu - \bar{\mu} \leq \Delta p \leq (1-k)^{-1}(\mu - \bar{\mu})$  then both  $MU_{Buy}$  and  $MU_{Sell}$  are negative and the investor will choose not to trade.

Thus, the investor's demand function if  $\mu - \bar{\mu} \geq 0$  will be given as:

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p < \mu - \bar{\mu} \\ 0 & \text{if } \mu - \bar{\mu} \leq \Delta p \leq (1-k)^{-1}(\mu - \bar{\mu}) \\ \frac{\mu - \bar{\mu} - (1-k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p > (1-k)^{-1}(\mu - \bar{\mu}) \end{cases} \quad (A5)$$

2. When  $\mu - \bar{\mu} < 0$ , there are three possibilities:

- If  $\Delta p \leq \mu - \bar{\mu}$  then  $MU_{Buy}$  is positive and  $MU_{Sell}$  is negative and the investor will choose to buy. Demand is determined by setting the first line in equation (A2) equal to 0 and solving for  $\Delta x$ .
- If  $\mu - \bar{\mu} < \Delta p < 0$  then  $MU_{Buy}$  is negative and  $MU_{Sell}$  is positive and the investor will choose to sell. Demand is determined by setting the first line in equation (A2) equal to 0 and solving for  $\Delta x$ .
- If  $\Delta p \geq 0$  then  $MU_{Buy}$  is negative and  $MU_{Sell}$  is positive and the investor will choose to sell. Demand in this case is determined by

setting the second line in equation (A2) equal to 0 and solving for  $\Delta x$ .

Thus, the investor's demand function if  $\mu - \bar{\mu} < 0$  will be given as:

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p < 0 \\ \frac{\mu - \bar{\mu} - (1 - k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p \geq 0 \end{cases} \quad (\text{A6})$$

**Proof of Lemma 3.2:**

Note that the utility maximization problem that the investor faces at date 2 if both capital gains and losses attract differential tax rates is to solve for  $\Delta x$  that maximizes:

$$EU = x_1 [\mu - \tau_l(\mu - p_1)] - \frac{a}{2} \sigma_\varepsilon^2 (x_1 + \Delta x)^2 (1 - \tau_l)^2 + \Delta x (1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \end{cases} \quad (\text{A7})$$

Then the investor's marginal expected utility in this case, which results from differentiating equation (A7) with respect to  $\Delta x$ , can be expressed as:

$$MU = -a\sigma_\varepsilon^2(1 - \tau_l)^2(x_1 + \Delta x) + (1 - \tau_l) \begin{cases} (\mu - p_1) - \Delta p & \text{if } \Delta x \geq 0 \\ (\mu - p_1) - (1 - k)\Delta p & \text{if } \Delta x < 0 \end{cases} \quad (\text{A8})$$

As in the proof of lemma 3.1, because the marginal utility function is discontinuous at  $\Delta x = 0$  whenever  $\Delta p \neq 0$ , the investor's optimal demand at date 2 can be determined by comparing marginal expected utility from buying or selling an additional share incremental to his initial holdings from date 1.

Define:

$$MU_{Buy} = (1 - \tau_l) [\mu - \bar{\mu} - \Delta p] \quad (A9)$$

and

$$MU_{Sell} = -(1 - \tau_l) [\mu - \bar{\mu} - (1 - k)\Delta p] \quad (A10)$$

Therefore, whether the investor should buy or sell shares at date 2 is determined by the sign and the magnitude of  $MU_{Buy}$  and  $MU_{Sell}$ . Again, there are two cases to consider:

1. When  $\mu - \bar{\mu} \geq 0$  there are three possibilities:
  - If  $\Delta p < \mu - \bar{\mu}$  then  $MU_{Buy}$  is positive and  $MU_{Sell}$  is negative. In this case the investor will choose to buy and demand is determined by setting the first line in equation (A8) equal to 0 and solving for  $\Delta x$ .
  - If  $\Delta p > (1 - k)^{-1}(\mu - \bar{\mu})$  then  $MU_{Buy}$  is negative and  $MU_{Sell}$  is positive. In this case the investor will choose to sell and demand is determined by setting the second line in equation (A8) equal to 0.

- If  $\mu - \bar{\mu} \leq \Delta p \leq (1-k)^{-1}(\mu - \bar{\mu})$  then both  $MU_{Buy}$  and  $MU_{Sell}$  are negative therefore the investor will choose not to trade.

Thus, the investor's demand function if  $\mu - \bar{\mu} \geq 0$  will be given as:

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1-\tau_l)} & \text{if } \Delta p < \mu - \bar{\mu} \\ 0 & \text{if } \mu - \bar{\mu} \leq \Delta p \leq (1-k)^{-1}(\mu - \bar{\mu}) \\ \frac{\mu - \bar{\mu} - (1-k)\Delta p}{a\sigma_\varepsilon^2(1-\tau_l)} & \text{if } \Delta p > (1-k)^{-1}(\mu - \bar{\mu}) \end{cases} \quad (A11)$$

2. When  $\mu - \bar{\mu} < 0$ , again there are three possibilities:

- If  $\Delta p < (1-k)^{-1}(\mu - \bar{\mu})$  then  $MU_{Buy}$  is positive and  $MU_{Sell}$  is negative and the investor will choose to buy.
- If  $\Delta p > \mu - \bar{\mu}$  then  $MU_{Buy}$  is negative and  $MU_{Sell}$  is positive and the investor will choose to sell.
- If  $(1-k)^{-1}(\mu - \bar{\mu}) \leq \Delta p \leq \mu - \bar{\mu}$  then both  $MU_{Buy}$  and  $MU_{Sell}$  are positive. Then demand is determined by the magnitude of  $MU_{Buy}$  and  $MU_{Sell}$ . With some algebra, it can be shown that  $MU_{Buy}$  ( $MU_{Sell}$ ) has greater magnitude whenever  $\Delta p$  is less (greater) than  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu})$ . When  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu})$  then  $MU_{Buy} = MU_{Sell}$ .

Thus, the investor's demand function if  $\mu - \bar{\mu} < 0$  will be given as:

$$\Delta x = \begin{cases} \frac{\mu - \bar{\mu} - \Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p \leq 2(2 - k)^{-1}(\mu - \bar{\mu}) \\ \frac{\mu - \bar{\mu} - (1 - k)\Delta p}{a\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p \geq 2(2 - k)^{-1}(\mu - \bar{\mu}) \end{cases} \quad (\text{A12})$$

**Proof of Proposition 4.1:**

Note that the equilibrium condition assuming that the supply of the risky asset is fixed can be given in terms of the change in demand across the two investor-types as:

$$\pi_A \Delta x_A + \pi_B \Delta x_B = 0 \quad (\text{A13})$$

However, from lemma 3.1, note that each investor-type's change in demand across periods 1 and 2 if only capital gains attract differential tax rates is given as:

- if  $\mu - \bar{\mu}_i \geq 0$

$$\Delta x_i = \begin{cases} \frac{\mu - \bar{\mu}_i - \Delta p}{a_i\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p < \mu - \bar{\mu}_i \\ 0 & \text{if } \mu - \bar{\mu}_i \leq \Delta p \leq (1 - k)^{-1}(\mu - \bar{\mu}_i) \\ \frac{\mu - \bar{\mu}_i - (1 - k)\Delta p}{a_i\sigma_\varepsilon^2(1 - \tau_l)} & \text{if } \Delta p > (1 - k)^{-1}(\mu - \bar{\mu}_i) \end{cases} \quad (\text{A14})$$



- if  $\mu - \bar{\mu}_i < 0$

$$\Delta x_i = \begin{cases} \frac{\mu - \bar{\mu}_i - \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau_i)} & \text{if } \Delta p \leq 0 \\ \frac{\mu - \bar{\mu}_i - (1 - k) \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau_i)} & \text{if } \Delta p \geq 0 \end{cases} \quad (\text{A15})$$

where:  $i = A$  or  $B$

It can be shown from the assumption that type A investors are overweight in the risky asset (*i.e.*,  $x_{1A} > \frac{\bar{a}}{a_A} X$ ) and type B investors are underweight (*i.e.*,

$x_{1B} < \frac{\bar{a}}{a_B} X$ ) that  $a_A x_{1A} > a_B x_{1B}$ . Thus, given the definition of  $\bar{\mu}_i$ ,  $\mu - \bar{\mu}_B$  is always greater than  $\mu - \bar{\mu}_A$ .

It also can be shown from equations (A14) and (A15) that  $\Delta x_A \leq \Delta x_B$  for any price change level at date 2. Given this and the equilibrium condition in equation (A13), equilibrium can be one of two cases; if type A investors sell and type B investors buy, or if no trade occurs. Then, equilibrium price is determined by substituting the relevant demand function for each investor-type into the market clearing condition in equation (A13) and rearranging to solve for  $\Delta p$ . Equilibrium demands are obtained by substituting  $\Delta p$  back into the relevant demand expression for each investor-type. However, there are several possible situations to consider with respect to the model parameter values.

**Situation A:**  $0 < \mu - \bar{\mu}_B < (1-k)^{-1}(\mu - \bar{\mu}_A)$ .

In this situation both investor-types are in case 1 in lemma 3.1, and  $0 < \mu - \bar{\mu}_A < \mu - \bar{\mu}_B < (1-k)^{-1}(\mu - \bar{\mu}_A) < (1-k)^{-1}(\mu - \bar{\mu}_B)$ . Inspection of the demand function in equation (A14) for each investor-type reveals that for any price change less than  $\mu - \bar{\mu}_B$  (greater than  $(1-k)^{-1}(\mu - \bar{\mu}_A)$ ) both investor-types wish to buy (sell) shares. Thus, the only possible equilibrium in this situation is a no trade equilibrium (*i.e.*,  $\Delta x_i = 0$ ) with equilibrium price change indeterminate in the interval:  $\Delta p \in [\mu - \bar{\mu}_B, (1-k)^{-1}(\mu - \bar{\mu}_A)]$ . This occurs for any values of  $\mu > \mu^*$ , where  $\mu^*$  is the value of the public signal where  $\mu - \bar{\mu}_B = (1-k)^{-1}(\mu - \bar{\mu}_A)$ , and corresponds to (1) in proposition 4.1.

**Situation B:**  $0 \leq (1-k)^{-1}(\mu - \bar{\mu}_A) \leq \mu - \bar{\mu}_B$ .

Also in this situation both investor-types are in case 1 of lemma 3.1, and  $0 \leq \mu - \bar{\mu}_A \leq (1-k)^{-1}(\mu - \bar{\mu}_A) \leq \mu - \bar{\mu}_B \leq (1-k)^{-1}(\mu - \bar{\mu}_B)$ . From equation (A14), for any price change between  $(1-k)^{-1}(\mu - \bar{\mu}_A)$  and  $\mu - \bar{\mu}_B$ , type A investors wish to sell shares (*i.e.*,  $\Delta x_A = \frac{\mu - \bar{\mu}_A - (1-k)\Delta p}{a_A \sigma_\varepsilon^2 (1 - \tau_l)}$ ) and type B investors wish to buy shares (*i.e.*,  $\Delta x_B = \frac{\mu - \bar{\mu}_B - \Delta p}{a_B \sigma_\varepsilon^2 (1 - \tau_l)}$ ) and thus is a candidate for an equilibrium.

Then solving the market clearing condition for  $\Delta p$  yields

$\Delta p = (1-k)^{-1} [\mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X}]$ . Substituting this into  $\Delta x_A$  and  $\Delta x_B$  above yields:

$$\Delta x_A = \frac{1}{a_A\sigma_\varepsilon^2(1-\tau_l)} \left[ \frac{k\pi'_B}{1-k\pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_A - \bar{\mu}_{Mkt}) \right]$$

and

$$\Delta x_B = \frac{1}{a_B\sigma_\varepsilon^2(1-\tau_l)} \left[ \frac{-k\pi'_A}{1-k\pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right].$$

This corresponds to (2) in proposition 4.1. All other price change levels result in both investor-types wishing to trade in the same direction and thus cannot constitute an equilibrium.

**Situation C:**  $\mu - \bar{\mu}_A \leq 0 < \mu - \bar{\mu}_B$

In this situation investor-type A is in case 2 of lemma 3.1 and investor-type B is in case 1. For price changes between  $\mu - \bar{\mu}_{Mkt} = 0$  and  $\mu - \bar{\mu}_B$ , type A investors

wish to sell shares (at a gain) (*i.e.*,  $\Delta x_A = \frac{\mu - \bar{\mu}_A - (1-k)\Delta p}{a_A\sigma_\varepsilon^2(1-\tau_l)}$ ) and type B

investors wish to buy shares (*i.e.*,  $\Delta x_B = \frac{\mu - \bar{\mu}_B - \Delta p}{a_B\sigma_\varepsilon^2(1-\tau_l)}$ ) which is a candidate for

an equilibrium. Equilibrium in this case is the same as in situation B and corresponds also to (2) in proposition 4.1.

For price changes between  $\mu - \bar{\mu}_A < 0$  and  $\mu - \bar{\mu}_{Mkt} = 0$ , type A investors wish to sell at a loss (thus,  $\Delta x_A = \frac{\mu - \bar{\mu}_A - \Delta p}{a_A \sigma_\varepsilon^2 (1 - \tau_l)}$ ) and type B investors wish to buy (*i.e.*,

$\Delta x_B = \frac{\mu - \bar{\mu}_B - \Delta p}{a_B \sigma_\varepsilon^2 (1 - \tau_l)}$ ) which is also a candidate for an equilibrium. Then solving

the market clearing condition for  $\Delta p$  yields  $\Delta p = \mu - \bar{\mu}_{Mkt}$ . Substituting this into

each investor-type's demand yields  $\Delta x_A = \frac{-(\bar{\mu}_A - \bar{\mu}_{Mkt})}{a_A \sigma_\varepsilon^2 (1 - \tau_l)}$  and  $\Delta x_B = \frac{-(\bar{\mu}_B - \bar{\mu}_{Mkt})}{a_B \sigma_\varepsilon^2 (1 - \tau_l)}$ .

This corresponds to (3) in proposition 4.1.

**Situation D:**  $\mu - \bar{\mu}_A < \mu - \bar{\mu}_B \leq 0$

In this situation both investor-types are in case 2 of lemma 3.1. Only for price changes between  $\mu - \bar{\mu}_A$  and  $\mu - \bar{\mu}_B$  do the two investor-types wish to trade in opposite directions. Between  $\mu - \bar{\mu}_A$  and  $\mu - \bar{\mu}_B$  type A investors sell at a loss and type B investors buy. Equilibrium that results in this situation corresponds also to (3) in proposition 4.1.

**Proof of Proposition 4.2:**

From lemma 3.2, each investor-type's change in demand across periods 1 and 2 if both gains and losses attract differential tax rates is given as:

- if  $\mu - \bar{\mu}_i \geq 0$

$$\Delta x_i = \begin{cases} \frac{\mu - \bar{\mu}_i - \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau_i)} & \text{if } \Delta p < \mu - \bar{\mu}_i \\ 0 & \text{if } \mu - \bar{\mu}_i \leq \Delta p \leq (1 - k)^{-1} (\mu - \bar{\mu}_i) \\ \frac{\mu - \bar{\mu}_i - (1 - k) \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau_i)} & \text{if } \Delta p > (1 - k)^{-1} (\mu - \bar{\mu}_i) \end{cases} \quad (\text{A16})$$

- if  $\mu - \bar{\mu}_i < 0$

$$\Delta x_i = \begin{cases} \frac{\mu - \bar{\mu}_i - \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau_i)} & \text{if } \Delta p \leq 2(2 - k)^{-1} (\mu - \bar{\mu}_i) \\ \frac{\mu - \bar{\mu}_i - (1 - k) \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau_i)} & \text{if } \Delta p \geq 2(2 - k)^{-1} (\mu - \bar{\mu}_i) \end{cases} \quad (\text{A17})$$

where:  $i = A$  or  $B$

As in the proof of proposition 4.1, equilibrium can be one of two cases; if type A investors sell and type B investors buy, or if no trade occurs. Again, there are four possible situations to consider with respect to the model parameter values:

**Situation A:**  $0 < \mu - \bar{\mu}_B < (1 - k)^{-1} (\mu - \bar{\mu}_A)$ .

In this situation both investor-types are in case 1 in lemma 3.2. Inspection of equation (A16) for each investor-type shows that the only possible demand that satisfies the market clearing condition in equation (A13) for this range of price changes is if both investor-types do not trade at date 2. This occurs for any values of  $\mu > \mu^*$ , where  $\mu^*$  is the value of the public signal where

$\mu - \bar{\mu}_B = (1-k)^{-1}(\mu - \bar{\mu}_A)$ . The equilibrium price change in this case will be indeterminate in the interval  $[\mu - \bar{\mu}_B, (1-k)^{-1}(\mu - \bar{\mu}_A)]$  and corresponds to (1) in proposition 4.2.

**Situation B:**  $0 \leq (1-k)^{-1}(\mu - \bar{\mu}_A) \leq \mu - \bar{\mu}_B$ .

Also in this situation, both investor-types are in case 1 of lemma 3.2. The only possible demand that satisfies the market clearing condition for this range of price changes is if type A investors sell shares and type B investors buy shares.

This occurs for any price change between  $(1-k)^{-1}(\mu - \bar{\mu}_A)$  and  $\mu - \bar{\mu}_B$ . Then

solving the market clearing condition yields

$\Delta p = (1-k)^{-1}[\mu - \bar{\mu} - a\sigma_\varepsilon^2(1-\tau_l)\Delta\tilde{X}]$ . Substituting this into the relevant demand

expression for each investor-type yields:

$$\Delta x_A = \frac{1}{a_A\sigma_\varepsilon^2(1-\tau_l)} \left[ \frac{k\pi'_B}{1-k\pi'_A}(\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_A - \bar{\mu}_{Mkt}) \right]$$

and

$$\Delta x_B = \frac{1}{a_B\sigma_\varepsilon^2(1-\tau_l)} \left[ \frac{-k\pi'_A}{1-k\pi'_A}(\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right].$$

This corresponds to (2).(a) or (2).(b).(i) in proposition 4.2.

**Situation C:**  $(1-k)^{-1}(\mu - \bar{\mu}_A) \leq 0 \leq \mu - \bar{\mu}_B$

In this situation investor-type A is in case 2 of lemma 3.2 and investor-type B is in case 1. For price changes between  $2(2-k)^{-1}(\mu - \bar{\mu}_A)$  and  $\mu - \bar{\mu}_B$ , type A investors wish to sell shares at a loss and type B investors wish to buy shares. Since capital losses attract differential tax rates, type A's change in demand is

given as  $\Delta x_A = \frac{\mu - \bar{\mu}_A - (1-k)\Delta p}{a_A \sigma_\varepsilon^2 (1-\tau_l)}$  and type B's change in demand is given as

$\Delta x_B = \frac{\mu - \bar{\mu}_B - \Delta p}{a_B \sigma_\varepsilon^2 (1-\tau_l)}$ . Thus, equilibrium price and demands are the same as in

situation B and correspond also to (2).(a) or (2).(b).(i) in proposition 4.2.

However, at a price change level  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$ , demand for investor-

type B is  $\Delta x_B = \frac{\mu - \bar{\mu}_B - 2(2-k)^{-1}(\mu - \bar{\mu}_A)}{a_B \sigma_\varepsilon^2 (1-\tau_l)}$  (buying). But type A is indifferent

between  $\Delta x_A = \frac{\mu - \bar{\mu}_A - 2(1-k)(2-k)^{-1}(\mu - \bar{\mu}_A)}{a_A \sigma_\varepsilon^2 (1-\tau_l)}$  (selling) and

$\Delta x_A = \frac{\mu - \bar{\mu}_A - 2(2-k)^{-1}(\mu - \bar{\mu}_A)}{a_A \sigma_\varepsilon^2 (1-\tau_l)}$  (buying). It is possible for this price to

constitute an equilibrium, but only if:

$$\pi_A \frac{\mu - \bar{\mu}_A - 2(1-k)(2-k)^{-1}(\mu - \bar{\mu}_A)}{a_A \sigma_\varepsilon^2 (1-\tau_l)} + \pi_B \frac{\mu - \bar{\mu}_B - 2(2-k)^{-1}(\mu - \bar{\mu}_A)}{a_B \sigma_\varepsilon^2 (1-\tau_l)} \leq 0 \quad (\text{A18})$$

That is, only if the aggregate change in demand across the two investor-types is less than or equal to zero can  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$  represent an equilibrium. If this condition is satisfied, an equilibrium occurs by allowing type A investors to divide between buying and selling such that the aggregate change in their demand exactly offsets the aggregate change in type B's demand. It is possible to rearrange equation (A18) into:

$$(\mu - \bar{\mu}_A) + \frac{2-k}{k} \left[ \frac{\pi'_B}{\pi'_A - \pi'_B} (\bar{\mu}_A - \bar{\mu}_B) \right] \leq 0 \quad (\text{A19})$$

Note that since  $\mu - \bar{\mu}_A$  is negative and  $\bar{\mu}_A - \bar{\mu}_B$  is positive, only if  $\pi'_A > \pi'_B$  can equation (A19) be satisfied in which case  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$  is an equilibrium. Then demands can be obtained by substituting this into the relevant demand expressions from lemma 3.2. This occurs for any values of  $\mu \leq \mu_A^{**}$ , where  $\mu_A^{**} = \bar{\mu}_A - \frac{2-k}{k} \left[ \frac{\pi'_B}{\pi'_A - \pi'_B} (\bar{\mu}_A - \bar{\mu}_B) \right]$  is the value of the public signal at which the market will clear at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$ . This corresponds to (2).(b).(ii) in proposition 4.2.



**Situation D:**  $(1-k)^{-1}(\mu - \bar{\mu}_A) < \mu - \bar{\mu}_B < 0$

In this situation both investor-types are in case 2 of lemma 3.2. It is possible for  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$  or  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_B)$  to represent an equilibrium.

A similar analysis to situation C indicates that  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_i)$  will be an equilibrium only if  $\pi'_i > \pi'_j$ , for  $i = A$  or  $B$ . This corresponds to (2).(b).(ii) in proposition 4.2. If  $\pi'_A = \pi'_B$  then neither  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$  nor  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_B)$  can be an equilibrium since equation (A19) or the analogous expression for the case of  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_B)$  cannot be satisfied.

For price changes between  $2(2-k)^{-1}(\mu - \bar{\mu}_A)$  and  $2(2-k)^{-1}(\mu - \bar{\mu}_B)$ , type A investors wish to sell shares at a loss and type B investors wish to buy shares which represent a potential equilibrium. For  $2(2-k)^{-1}(\mu - \bar{\mu}_A) < \Delta p < 2(2-k)^{-1}(\mu - \bar{\mu}_B)$ , equilibrium is the same as in situation B and corresponds to (2).(a) in proposition 4.2 or (2).(b).(i) if  $\pi'_A > \pi'_B$  (*i.e.*, if  $\mu_i^{**}$  is defined as the value of the public signal at which the market clears at  $\Delta p = 2(2-k)^{-1}(\mu - \bar{\mu}_A)$ ).

## Appendix B: Calculations

### Calculation of the pareto efficient change in demand in the absence of differential tax rates

In the absence of differential tax rates (*i.e.*, all gains and losses are taxed at the same rate,  $\tau_s = \tau_l = \tau$ ), an investor's optimal change in demand for the risky

asset at date 2 can be given as:  $\Delta x_i = \frac{\mu - \bar{\mu}_i - \Delta p}{a_i \sigma_\varepsilon^2 (1 - \tau)}$ , where  $\bar{\mu}_i = p_1 + a_i x_i \sigma_\varepsilon^2 (1 - \tau)$ ,

$\tau = \tau_l$ . Then, equilibrium price and demand if there are two investor-types in the market, A and B, and the supply of the risky asset is fixed (*i.e.*,  $\Delta X = 0$ ) can be calculated as follows:

$$\pi_A \frac{\mu - \bar{\mu}_A - \Delta p}{a_A \sigma_\varepsilon^2 (1 - \tau)} + \pi_B \frac{\mu - \bar{\mu}_B - \Delta p}{a_B \sigma_\varepsilon^2 (1 - \tau)} = 0$$

$$\Rightarrow \frac{\pi_A}{a_A} (\mu - \bar{\mu}_A - \Delta p) + \frac{\pi_B}{a_B} (\mu - \bar{\mu}_B - \Delta p) = 0$$

$$\Rightarrow \frac{\pi'_A}{\bar{a}} (\mu - \bar{\mu}_A - \Delta p) + \frac{\pi'_B}{\bar{a}} (\mu - \bar{\mu}_B - \Delta p) = 0 \quad \text{where } \frac{\pi'_i}{\bar{a}} = \frac{\pi_i}{a_i}, i = A \text{ or } B.$$

$$\Rightarrow \pi'_A \mu - \pi'_A \bar{\mu}_A - \pi'_A \Delta p + \pi'_B \mu - \pi'_B \bar{\mu}_B - \pi'_B \Delta p = 0$$

$$\Rightarrow \mu - \bar{\mu}_{Mkt} - \Delta p = 0 \quad \text{where } \pi'_A + \pi'_B = 1 \quad \text{and } \bar{\mu}_{Mkt} = \pi'_A \bar{\mu}_A + \pi'_B \bar{\mu}_B$$

$$\Rightarrow \Delta p = \mu - \bar{\mu}_{Mkt}$$

$$\begin{aligned}
\Delta x_i &= \frac{\mu - \bar{\mu}_i - (\mu - \bar{\mu}_{Mkt})}{a_i \sigma_\varepsilon^2 (1 - \tau)} \\
&= \frac{-\bar{\mu}_i + \bar{\mu}_{Mkt}}{a_i \sigma_\varepsilon^2 (1 - \tau)} \\
&= \frac{-(\bar{\mu}_i - \bar{\mu}_{Mkt})}{a_i \sigma_\varepsilon^2 (1 - \tau)}
\end{aligned}$$

**Calculation of trading volume if only capital gains attract differential tax rates**

Recall that the trading volume in period 2 can be expressed as:

$$V = \frac{1}{2} [\pi_A |\Delta x_A| + \pi_B |\Delta x_B|]$$

Based on the equilibrium demand functions in proposition 4.1, trading volume is calculated as follows:

a) When  $\mu > \mu^*$ ,  $\Delta x_A = 0$  and  $\Delta x_B = 0$ . Then  $V = 0$ .

b) When  $\bar{\mu}_{Mkt} \leq \mu \leq \mu^*$ ,

$$\Delta x_A = \frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau)} \left[ \frac{k \pi'_B}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_A - \bar{\mu}_{Mkt}) \right]$$

and

$$\Delta x_B = \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau)} \left[ \frac{-k \pi'_A}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right]$$

However, since type A is selling and type B is buying,  $\Delta x_A$  is negative and  $\Delta x_B$  is positive. This means that

$$|\Delta x_A| = -\frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau_I)} \left[ \frac{k \pi'_B}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_A - \bar{\mu}_{Mkt}) \right]$$

and

$$|\Delta x_B| = \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau_I)} \left[ \frac{-k \pi'_A}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right]$$

Substituting this into the trading volume expression above yields:

$$V = \frac{1}{2} \frac{\pi_A}{a_A \sigma_\varepsilon^2 (1 - \tau_I)} \left[ (\bar{\mu}_A - \bar{\mu}_{Mkt}) - \frac{k \pi'_B}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt}) \right] + \frac{1}{2} \frac{\pi_B}{a_B \sigma_\varepsilon^2 (1 - \tau_I)} \left[ \frac{-k \pi'_A}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt}) - (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right]$$

Note that  $\frac{\pi_A}{a_A} = \frac{\pi'_A}{\bar{a}}$  and  $\frac{\pi_B}{a_B} = \frac{\pi'_B}{\bar{a}}$ . Note also that  $\frac{1}{1 - k \pi'_A} (\mu - \bar{\mu}_{Mkt})$  is just the equilibrium price change,  $\Delta p$ , for this range of the public signal. Then

$$\begin{aligned} V &= \frac{1}{2 \bar{a} \sigma_\varepsilon^2 (1 - \tau_I)} \left[ \pi'_A [(\bar{\mu}_A - \bar{\mu}_{Mkt}) - k \pi'_B \Delta p] + \pi'_B [-k \pi'_A \Delta p - (\bar{\mu}_B - \bar{\mu}_{Mkt})] \right] \\ &= \frac{1}{2 \bar{a} \sigma_\varepsilon^2 (1 - \tau_I)} \left[ \pi'_A (\bar{\mu}_A - \bar{\mu}_{Mkt}) - \pi'_A k \pi'_B \Delta p - \pi'_B k \pi'_A \Delta p - \pi'_B (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right] \\ &= \frac{1}{2 \bar{a} \sigma_\varepsilon^2 (1 - \tau_I)} \left[ \pi'_A (\bar{\mu}_A - \bar{\mu}_{Mkt}) - 2 \pi'_A \pi'_B k \Delta p - \pi'_B (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right] \\ &= \frac{1}{2 \bar{a} \sigma_\varepsilon^2 (1 - \tau_I)} \left[ 2 \pi'_A \pi'_B (\bar{\mu}_A - \bar{\mu}_B) - 2 \pi'_A \pi'_B k \Delta p \right] \\ &= \frac{\pi'_A \pi'_B}{\bar{a} \sigma_\varepsilon^2 (1 - \tau_I)} \left[ (\bar{\mu}_A - \bar{\mu}_B) - k \Delta p \right] \end{aligned}$$

c) When  $\mu < \bar{\mu}_{Mkt}$ ,  $\Delta x_A = \frac{-(\bar{\mu}_A - \bar{\mu}_{Mkt})}{a_A \sigma_\varepsilon^2 (1 - \tau_l)}$  and  $\Delta x_B = \frac{-(\bar{\mu}_B - \bar{\mu}_{Mkt})}{a_B \sigma_\varepsilon^2 (1 - \tau_l)}$ .

Again, since type A is selling and type B is buying,  $\Delta x_A$  is negative and  $\Delta x_B$  is positive. Thus:

$$\begin{aligned} V &= \frac{1}{2} \left[ \pi_A \frac{(\bar{\mu}_A - \bar{\mu}_{Mkt})}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} - \pi_B \frac{(\bar{\mu}_B - \bar{\mu}_{Mkt})}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \right] \\ &= \frac{1}{2 \bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \pi'_A (\bar{\mu}_A - \bar{\mu}_{Mkt}) - \pi'_B (\bar{\mu}_B - \bar{\mu}_{Mkt}) \right] \\ &= \frac{1}{2 \bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ 2 \pi'_A \pi'_B (\bar{\mu}_A - \bar{\mu}_B) \right] \\ &= \frac{\pi'_A \pi'_B}{\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} (\bar{\mu}_A - \bar{\mu}_B) \end{aligned}$$

### Calculation of trading volume if both gains and losses attract differential tax rates

Based on equilibrium demands in proposition 4.2, trading volume when  $\mu > \mu^*$  is calculated as in (a) above. When  $\mu \leq \mu^*$  where  $\pi'_A = \pi'_B$  or when  $\mu_i^{**} < \mu \leq \mu^*$  where  $\pi'_i > \pi'_j$ , trading volume is calculated as in (b) above.

When  $\mu \leq \mu_A^{**}$  where  $\pi'_A > \pi'_B$ , note that each investor-type's change in demand is given as:

$$\Delta x_A = \begin{cases} \frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_A) \right] \\ \text{or} \\ \frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2-k} (\mu - \bar{\mu}_A) \right] \end{cases}$$

and

$$\Delta x_B = \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_B) + \frac{2}{2-k} (\bar{\mu}_A - \bar{\mu}_B) \right]$$

However, type A investors must divide between buying and selling such that the average demand is selling, and given as in the second line of  $\Delta x_A$ . Thus,

$$|\Delta x_A| = -\frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2-k} (\mu - \bar{\mu}_A) \right]$$

and

$$|\Delta x_B| = \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_B) + \frac{2}{2-k} (\bar{\mu}_A - \bar{\mu}_B) \right].$$

Then, trading volume is:

$$\begin{aligned} V &= \frac{1}{2} \left[ \frac{\pi_A}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_A) \right] + \frac{\pi_B}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2-k} (\mu - \bar{\mu}_B) + \frac{2}{2-k} (\bar{\mu}_A - \bar{\mu}_B) \right] \right] \\ &= \frac{1}{2\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \left[ \frac{-k\pi'_A}{2-k} (\mu - \bar{\mu}_A) \right] + \left[ \frac{-k\pi'_B}{2-k} (\mu - \bar{\mu}_B) + \frac{2\pi'_B}{2-k} (\bar{\mu}_A - \bar{\mu}_B) \right] \right] \end{aligned}$$

After lots of algebra and rearranging, trading volume can be given as:

$$V = \frac{1}{2\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \pi'_B (\bar{\mu}_A - \bar{\mu}_B) - \frac{k}{2} \Delta p \right] \text{ where } \Delta p = 2(2-k)^{-1} [\mu - \bar{\mu}_A].$$

When  $\mu \leq \mu_B^{**}$  where  $\pi'_B > \pi'_A$ , each investor-type's change in demand is given

as:

$$\Delta x_A = \frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2 - k} (\mu - \bar{\mu}_B) - (\bar{\mu}_A - \bar{\mu}_B) \right]$$

and

$$\Delta x_B = \begin{cases} \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2 - k} (\mu - \bar{\mu}_B) \right] \\ or \\ \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2 - k} (\mu - \bar{\mu}_B) \right] \end{cases}$$

In this case, type B investors must divide between buying and selling such that the average demand is buying, and given as in the first line of  $\Delta x_B$ . Thus,

$$|\Delta x_A| = -\frac{1}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2 - k} (\mu - \bar{\mu}_B) - (\bar{\mu}_A - \bar{\mu}_B) \right]$$

and

$$|\Delta x_B| = \frac{1}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{-k}{2 - k} (\mu - \bar{\mu}_B) \right].$$

Then, trading volume is:

$$\begin{aligned} V &= \frac{1}{2} \left[ \frac{-\pi_A}{a_A \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2 - k} (\mu - \bar{\mu}_B) - (\bar{\mu}_A - \bar{\mu}_B) \right] - \frac{\pi_B}{a_B \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \frac{k}{2 - k} (\mu - \bar{\mu}_B) \right] \right] \\ &= \frac{1}{2\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \pi'_A (\bar{\mu}_A - \bar{\mu}_B) - \frac{k\pi'_A}{2 - k} (\mu - \bar{\mu}_B) - \frac{k\pi'_B}{2 - k} (\mu - \bar{\mu}_B) \right] \\ &= \frac{1}{2\bar{a} \sigma_\varepsilon^2 (1 - \tau_l)} \left[ \pi'_A (\bar{\mu}_A - \bar{\mu}_B) - \frac{k}{2} \Delta p \right] \text{ where } \Delta p = 2(2 - k)^{-1} [\mu - \bar{\mu}_B]. \end{aligned}$$

**Calculation of price change variance if only capital gains attract differential tax rates**

The variance of the equilibrium price change is defined as:

$$Var(\Delta p) = \text{var}[E(\Delta p | \mu, \Delta X)] + E[\text{var}(\Delta p | \mu, \Delta X)]$$

However, given the piecewise nature of the equilibrium price function, the equilibrium price change variance can be characterized as:

$$Var(\Delta p) = q_1 \text{var}_1(\Delta p) + q_1 [E_1(\Delta p) - E(\Delta p)]^2 + q_3 \text{var}_3(\Delta p) + q_3 [E_3(\Delta p) - E(\Delta p)]^2$$

where  $\text{var}_j(\Delta p)$  is the conditional variance of  $\Delta p$  given that equilibrium is in region  $j$  where  $j = 1$  and  $3$ .  $E_j(\Delta p)$  is the expected price change in region  $j$ .  $q_j$  is the probability that equilibrium is in region  $j$ .

Note that  $E(\Delta p) = q_1 E_1(\Delta p) + q_3 E_3(\Delta p)$ , and  $q_1 = (1 - q_3)$ , then the variance can be given as:

$$Var(\Delta p) = q_1 \text{var}_1(\Delta p) + q_3 \text{var}_3(\Delta p) + q_1 q_3 [E_3(\Delta p) - E_1(\Delta p)]^2$$



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