Calculus-based mathematics: An Australian endangered species?

Dimitrios Maltas
Christian Brothers' High School, Lewisham. NSW
dimitrios.maltas@cbhslewisham.nsw.edu.au

Anne Prescott
University of Technology, Sydney
anne.prescott@uts.edu.au

Many people are discussing the issues surrounding mathematics at all levels of education. Politicians, parents, students, universities, education departments all have a view about what the problem is and all have ideas about what should happen. This paper represents a synthesis of the issues and implications of one of the problems evident in mathematics education in Australia: the reduction in students taking the higher mathematics subjects that involve calculus. It makes particular reference to NSW where it appears the problem has a critical edge as the number of students undertaking higher level mathematics subjects decreases and the number of students not taking mathematics increases.

What is calculus mathematics?

Calculus is the mathematics of change, it traditionally focuses on solving a range of problems by applying methods of differentiation and integration. Calculus is the pinnacle of secondary mathematics and a gateway for further theoretical developments (Tall, 1997). It provides a framework for modelling systems that undergo change and establishes the predicted outcomes pursuant to that change. Calculus is of upmost importance in mathematics as it compounds knowledge, skills and understanding developed in the fundamental stages of mathematical learning such as algebra, geometry and trigonometry. Calculus aims to solve problems that are continually evolving and allows for the modelling of many real-world situations (Board of Studies Teaching and Educational Standards NSW (BOSTES), 2014). Such problems have infinitesimal individual steps that need to be solved individually and combined so as to achieve resolution. The beauty of calculus is that it allows for all these computations to be achieved simultaneously and affords the opportunity to examine the effects of changing conditions on any system being investigated. By applying calculus a system can be made to perform in a desired manner (Kleitman, 2012).

What is the problem?

The current structure of secondary mathematics around Australia sees two diverging mathematical pathways offered, one is a calculus-based pathway and the other is not. The new *Australian Curriculum* syllabuses have a similar divergent structure. The calculus courses are hierarchical in nature and intended to promote the development of knowledge, skills and understanding of important concepts in mathematics required to pursue further study in the fields of, but not limited to, mathematics, science, industrial arts, commerce, engineering. The applications of calculus in these courses include mechanics (e.g., projectile motion, simple harmonic motion, circular motion), and volumes of revolution.

In NSW the syllabuses for these courses have not been changed since 1982 and still use the technology of that time in the Higher School Certificate. Of course, teachers in the classroom make use of technology in all its forms for the development of skills and understanding in all calculus topics. Other States have changed their syllabuses and allow CAS and graphics calculators into their examinations.

Each State also has non-calculus courses that provide appropriate mathematical grounding for students wishing to pursue careers that utilise a variety of mathematical/ statistical/ technological techniques and tools. Predominantly these students will undertake a vocational pathway or continue study in the fields of finance, humanities, nursing and life sciences (BOSTES, 2014). Poladian and Nicholas (2013) point out that in NSW the number of students undertaking the calculus-based subjects has reduced from 61% of the students studying mathematics in 1992 to 35% in 2012. These numbers highlight the worrying trend that is at the crux of this article. Despite some short term fluctuations in candidature numbers, there is a disturbing gradual decline in students undertaking the rigours of higher order calculus-based mathematics. The Board of Studies Teaching and Educational Standards NSW (BOSTES, 2012) in its *NSW Senior Secondary Review and Evaluation: Mathematics, Reference Report,* acknowledges that there is also a significant increase in the students undertaking non-calculus mathematics and that this trend is evident throughout the entire span of the NSW Higher School Certificate (HSC), 2001–2013.

This problem is not limited to NSW. In *Maths? Why Not?* a report prepared for the Federal Department of Education, Employment and Workplace Relations (DEEWR) there is evidence that enrolments in senior mathematics courses are increasing nationally. Paradoxically, whilst enrolments in higher-level calculus courses are declining, enrolments in elementary or terminating courses are increasing (McPhan et al., 2008). Connor et al. (2010) maintain that there are significant downward enrolment trends in both advanced levels of mathematics and science at the secondary school level and, more recently, the Office of the Chief Scientist (OCS) has asserted that participation in

'science' subjects in Australian schools has declined to the lowest levels in 20 years (OCS, 2014). Of great concern, these trends are exacerbated in regional and rural areas (Lyons et al., 2006). Twenty years of declining participation rates justifies the assertion that calculus mathematics in Australia could potentially be on a pathway to extinction.

The implications

The flow-on effects of these trends are immediate. These impact on both education and the economy.

Educational implications

Some universities no longer have subject requirements for entry to university courses. There is often assumed knowledge but school students are told to make up the difference by undertaking bridging courses. School students are told that they should have the short-term goal of maximising their Australian Tertiary Admission Rank (ATAR), because they can always do a bridging course at university to make up for the mathematics they have not learned at school, not realising that they are severely compromising their future success at university (Poladian & Nicholas, 2013).

Universities are reporting a distressingly large proportion of first year students failing mathematics, statistics, or physics (Rylands & Coady, 2008). Poladian and Nicholas (2013) found that students who were underprepared for the mathematics in their first year courses were more likely to withdraw from or fail those subjects. The students who did the bridging courses were more likely to pass, but still there was a significant number who still failed or withdrew.

The pool of secondary school graduates with an effective understanding of calculus mathematics is diminishing and therefore tertiary institutions themselves are experiencing a decline in student numbers for courses that are linked to calculus-based mathematics. Consequently staff numbers and those departments too are diminishing. The rate of decline of Australian mathematics majors is so severe that several tertiary institutions are unable to offer mathematics majors (Brown, 2009).

The alternative to bridging courses is to lower academic standards for mathematics and mathematics related subjects (Rylands & Coady, 2008). The unfortunate repercussion of the reduction in standards is that students who are adequately prepared find the mathematically related subjects are too easy at university and hence look further afield for more rewarding challenges (Rylands & Coady, 2008). Of course, reducing the standard of mathematics degrees could mean that there is also an erosion of the pathways for educating future secondary teachers of mathematics and science, perhaps leading to a decline in standards in schools as fewer mathematics classes are taught by teachers with a mathematics background.

Secondary educators have a vital role to play. In order for secondary mathematics to contribute positively to high achievement, it must focus on understanding and be underpinned by good subject knowledge. Shulman (1986) first introduced the notion of pedagogical content knowledge (PCK), defining it as knowledge that goes beyond subject matter knowledge to the dimension of subject matter knowledge for teaching. It encompasses the ways of representing and formulating subject matter into a format that makes it comprehensible to others (Shulman, 1986). PCK contributes to effective teaching and learning by recognising that content knowledge alone is not sufficient for good teaching. This concept was expanded by Hill, Ball and Schilling (2008) to explain more fully the complexities of what happens in the classroom. Teachers have to understand how students learn and how to teach to maximise that learning. Teachers who are teaching 'out of field' do not have the pedagogical content knowledge for effective classroom teaching of mathematics. Many schools are currently struggling to fill the void of adequately trained mathematics teachers. Approximately 40% of Australian Years 7–10 classes are taught by an unqualified mathematics teacher (OCS, 2014). Schools where there is a shortage of qualified mathematics teachers will tend to give junior classes to unqualified teachers so that the senior classes have a mathematics teacher. Many 'out of field' teachers struggle to deal with the complexities of mathematics teaching.

There are inevitable inequities in education as the shortage of mathematics teachers tends to be biased geographically to rural and regional areas, and socioeconomically because affluent schools are able to attract qualified staff capable of teaching higher order mathematics courses. The reality is that some institutions are unable to attract or retain suitably qualified mathematics staff, resulting in their inability to even offer some mathematics courses; hence equitable access to education is increasingly becoming a thing of the past.

Economic implications

The economic ramifications of student disengagement in calculus-based mathematics should be of great concern to all Australians. Inevitably Australia will fail in its capacity to produce a critical mass of young people with the requisite mathematical grounding and skills to pursue careers in science, technology, engineering, and mathematics (STEM) to maintain and enhance Australia's competitiveness (McPhan et al., 2008). The greatest preoccupation shaping global economic plans today are those supporting STEM. It is the means to building a stronger Australia with a competitive economy (OCS, 2014). Yet currently the demand for a STEM trained workforce is far outstripping supply.

The greatest economic impact of this current trend is that of the long-term implications for Australia's future research and innovation capacity. Research in the sciences is fundamental to a 21st century economy. It is vital that Australia maintain a strong knowledge base in mathematical sciences, in

order to add intellectual value to new technologies, spearhead innovation, and continue to compete globally (AAS, 2006). The sustained implications of this trend are that Australia will become more reliant upon other nations for development and innovation. As a nation, it will lose its reputation as a first class science and research performer and will fail to attract partnerships in world leading research collaborations (COA, 2010).

Recommendations

Who has a vested interest in the survival of calculus mathematics? The obvious answer is, we all do. Institutions, educators (at all levels), advisors, employers, the media, governments, policy makers and the students themselves need to take responsibility if calculus the species is to survive. By drawing on inspiration from successes experienced locally and abroad, the following recommendations are proposed.

1. First and foremost, we must all consider carefully whether the F–10 and the senior *Australian Curriculum* syllabuses are appropriate. There is already concern about the amount of content required in the F–10 syllabus leading to superficial coverage rather than deep understanding of mathematical concepts. The amount of content compromises the ability of teachers to engage students in relevant, contextualised learning (SACE, 2012). Superficial learning of mathematics usually happens when there is an emphasis on reproducing the content in an examination, while a deep approach to learning occurs when mathematics is linked to authentic contexts and students seek understanding.

   Recommendation 1: The syllabuses need to be reconsidered to eliminate some of the content so teachers can teach topics for deep understanding rather than covering topics in a superficial manner. This includes the new senior mathematics syllabuses at all levels.

2. The research on maths anxiety supports the notion that students are turned off mathematics before they even reach secondary school (Ashcraft & Krause, 2007). People who are highly maths anxious avoid mathematics whenever possible. Ashcraft and Kirk (2001) and Miller and Bichsel (2004) discussed the idea that mathematics anxiety has an effect on underlying cognitive processes, producing a transitory disruption of working memory—the area of the brain where we hold mathematical facts (Ramirez, Gunderson, Levine & Beilock, 2013). The reduction in working memory leads to a pronounced increase in reaction time and errors when mental addition is performed concurrently with a memory load task.

Primary pre-service teachers often suffer maths anxiety taking it into the classroom as teachers. Beilock, Gunderson, Ramirez & Levine, (2010) found having a highly maths-anxious female primary teacher pushed girls to conform to the stereotype that they were not as good as the boys at mathematics, which, in turn, affected the girls' mathematics achievement. Mathematics teacher education faculties are of paramount importance because of the vital role they play in subject engagement (Tooke & Lindstrom, 1998). If teacher education institutions are able to improve mathematical confidence, teachers will be more confident teaching for understanding in mathematics. More confident students will impact directly upon the retention rate of students in mathematics, especially those undertaking calculus-based mathematics (Holton, et al., 2009).

Recommendation 2: Primary and secondary mathematics teachers need to be more aware of maths anxiety in the classroom so they can work with students to avoid its debilitating effects.

3. Another way to improve student participation in mathematics in secondary school is to help primary teachers with their own maths anxiety so mathematics becomes an exciting subject rather than one that must be endured. This issue arises because most primary teachers are generalist teachers and there are very few mathematics specialists in primary schools and teachers do not feel supported. Previously in NSW, primary teachers had regular visits from Department of Education Mathematics Consultants who supported them in their teaching and professional development. Recent cost cutting means that this support is now extremely limited. The National Council of Teachers of Mathematics (NCTM) recommends "the use of Elementary Mathematics Specialists (EMS professionals) in pre-K–6 environments to enhance the teaching, learning, and assessing of mathematics to improve student achievement. We further advocate that every elementary school have access to an EMS". Such specialist teachers can support individual teachers in the classroom and/or they can support schools in professional development according to the needs of the teachers and schools.

Recommendation 3: Provision should be made in primary teacher education courses and professional development courses for teachers to be mathematics specialists.

4. While Leung (2014) attributes much success in mathematics to the East Asian culture, Dweck (2006) believes that it is 'mindset' that determines success. Students with a growth mindset believe that their basic abilities can be developed through hard work because talent and brains are just

a starting point. A fixed mindset is linked to the belief that intelligence does not change and that talent produces success. Dweck (2006) believes that praising ability and intelligence does not foster hard work and success and can, in fact, produce the opposite when difficulties arise and students begin to doubt their ability.

Recommendation 4: Changing society's view that there is a 'maths gene' is a tall order but if teachers are aware that their language in the classroom has a significant impact on students' ideas about their ability to achieve, it is a good start.

5. Professor Celia Hoyles OBE is the director of the National Centre for Excellence in the Teaching of Mathematics (NCETM) in the United Kingdom. The philosophy of this body is that teachers are the key to change and its aim is to develop a national infrastructure for subject-specific professional development of mathematics teachers. The aim is to foster a culture of professional development that will in turn strengthen the mathematical knowledge of their teachers. The NCETM enables all teachers of mathematics to access high quality career professional development that will best meet teachers' needs and aspirations. Mathematics teachers are a major influence on student's motivation and desire to learn, quality teaching is the key factor that influences students' achievements (Hoyles, 2009). Jo Boaler has created a similar website called *Youcubed.*

   In Australia, mathematics teachers of all levels—primary, secondary and tertiary—would benefit from the opportunity to grow and develop via access to a similar body linked to the *Australian Curriculum.* The intent would be to raise the professional standards of all mathematics teachers and the outcome would be of benefit to all students undertaking mathematical studies. The Federal government has begun supporting this approach through the Australian Maths and Science Partnerships Program in funding projects to develop innovative mathematics and science resources to improve student engagement. The grants also support increased mentoring and teacher professional development. The Australian Association of Mathematics Teachers (AAMT) already supports a website with resources in *Top Drawer Teachers, Make it Count* for teachers of Indigenous learners, *Connect with Maths* for teacher professional development. State departments of education already have websites supporting teachers and *Scootle* has many resources too.

   Recommendation 5: While these websites are available, additional resources should be available for online teacher professional development. The government should provide the infrastructure to 'online conference' access to specialist mathematics educators to rural,

remote or underachieving schools for students to participate in online learning and for teachers to engage in professional development.

6. Tertiary institutions also have a role to play. Some universities award bonus points for students undertaking high level calculus-based mathematics courses (McPhan et al., 2008). From 2009, bonus points have been offered to University of Queensland applicants who had undertaken Mathematics C, a calculus-based mathematics course. This has clearly had a positive outcome in student enrolment in this course, as is reflected in the upward trend, in the graph shown in Figure 1, from 2009 onwards (Source: QSA, 2014).

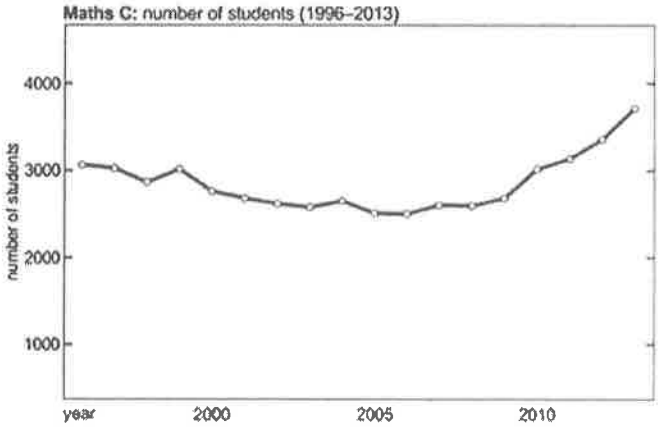

Figure 1. Number of students undertaking Mathematics C in Queensland (QSA, 2014).

Recommendation 6: Bonus points need to be advertised so that students see the possibilities that result from taking calculus-based mathematics subjects.

7. As outlined earlier, students are led to believe that they can supplement a lesser mathemathematical background with bridging courses in order to successfully pass university mathematics subjects.

Recommendation 7: Tertiary institutions should establish realistic unambiguous prerequisites for courses that require a mathematical background, indicating to secondary students exactly what is expected of them prior to undertaking tertiary studies (McPhan et al., 2008).

8. Tertiary institutions need to target potential mathematicians from a very young age, mentoring students throughout their secondary education perhaps creating future mathematicians.

Recommendation 8: If potential sporting stars can sign lucrative deals at age 12 why are universities not offering more incentives to gifted mathematics students?

9. The Australian government has a leadership role to play, STEM being its primary agenda and National Curriculum its major educational strategy. It is essential that frequent analysis be undertaken to ensure that actions are aligned to goals and investments are delivering returns.

   Recommendation 9: Federal and state governments should provide the necessary stimulus to lift the number of qualified STEM teachers by making STEM teaching careers more attractive, including opportunities for promotion, remuneration packages. and teacher support. Creating appropriate incentives for high-achieving STEM students to pursue a teaching career is also essential (OCS, 2014), for example, a reduction in HECS debt for students becoming STEM teachers or primary mathematics specialists.

10. The inequality of opportunity for students in remote and rural areas also needs to be addressed.

    Recommendation 10: There should be additional incentives implemented to attract qualified teachers to remote, rural or challenging schools thus ensuring equitable access to quality mathematics education (Rubinstein, 2009).

11. Professor Derek Woodrow from the Institute of Education at Manchester Metropolitan Universities in his 2001 address examining mathematics teacher recruitment in the United Kingdom highlighted two fundamental recommendations required to recruit well-qualified mathematics teachers into schools. These two recommendations were echoed by Professor Hyam Rubinstein the Australian chair of the National Committee of the Mathematical Sciences in 2009. Primarily, recruitment from other graduate markets requires government stimulus.

    Recommendation 11a: There needs to be amendments to the expectations of expertise required and this can be achieved by retraining the pool of 'out of field' mathematics teachers (TOU, 2001).

    Governments should provide improved opportunities and support for out-of-field teachers to improve their mathematical content knowledge as well as pedagogical content knowledge (Rubinstein, 2009).

Recommendation 11b: Welcoming professionals seeking to train as mathematics teachers (TOU, 2001). Government support for career-change professionals' needs to be in the form of income support, courses appropriate to their needs, acknowledgment of the additional work related skills they bring to schools (Rubinstein, 2009).

Recommendation 12: The government should embark on a national awareness campaign whereby teachers, students, parents, and the general public all have their attention drawn to just how valuable mathematics is as a career and how it enhances career options (AAS, 2006). A campaign undertaken via national bodies such as the CSIRO, the ABC, etcetera would inspire a sense of national pride by promoting activities that recognise and reward the achievements and successes of Australians in the mathematical sciences.

A national trend of declining participation rates of calculus-based mathematics has been identified. If left unchallenged it is clear that calculus-based mathematics would spiral towards an inevitable extinction. As with any endangered species, a plan of action needs to be undertaken. By drawing inspiration from successes both locally and abroad, the rejuvenation of calculus mathematics is now a viable option. A conscious effort to change should ideally be undertaken by all key stakeholders with a vested interest in its survival.

References

Ashcraft, M. H. & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General, 130*, 224–237.

Ashcraft, M. H. & Krause, J. A. (2007). Working memory, math performance and math anxiety. *Psychonomic Bulletin & Review, 14*(2), 243–248.

Australian Academy of Science (AAS). (2006). *Mathematics and statistics: Critical skills for Australia's future. The national strategic review of mathematical sciences research in Australia.* Retrieved from http://www.review.ms.unimelb.edu.au/FullReport2006.pdf

Beilock, S. L., Gunderson, E. A., Ramirez, G. & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of the United States of America, 107*(5). Retrieved from http://www.pnas.org/content/107/5/1860.full

Board of Studies Teaching & Educational Standards (BOSTES) NSW. (2014). *NSW Senior Secondary Review & Evaluation: Mathematics. Reference Report.* Retrieved from http://www.boardofstudies.nsw.edu.au/australian-curriculum/pdf_doc/senior-secondary-reference-report-maths-2014-08.pdf

Brown, G. (2009). *Review of education in mathematics, data science and quantitative disciplines. Report to the Group of Eight universities, December 2009.* Retrieved from https://go8.edu.au/publication/go8-review-education-mathematics-data-science-and-quantitative-disciplines

Commonwealth of Australia (COA). (2010). *Inspiring Australia: A national strategy for engagement with the sciences. A report to the Minister for Innovation, Industry, Science and Research.* Retrieved from http://www.innovation.gov.au/

Connor, V., Auld, B., Eakin, P., Morris, K. & Tilston, M. (2010). The praxis of building capacity in mathematics and science in a rural, non-government system of schools: Voices of teacher leaders. *Education in Rural Australia, 20*(2), 35–51.

Dweck, C. (2006). *Mindset: The new psychology of success.* New York, NY: Random House.

Hill, H. C., Ball, D. W. & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Holton, D., Muller, E., Oikkonen, J., Valenzuela, O. A. S. & Zizhao, R. (2009). Some reasons for change in undergraduate mathematics enrolments. *International Journal of Mathematics Education in Science and Technology, 40*(1), 3–15.

Hoyles, C. (2009). *Some successful strategies for promoting mathematics in England.* Retrieved from http://www.amsi.org.au/wp-content/uploads/2014/07/CeliaHoyles_all_slides.pdf

Kleitman, D. (2012). *Calculus for beginners and artists.* Retrieved from http://www-math.mit.edu/~djk/calculus_beginners/

Leung, F. K. S., (2014). What can and should we learn from international studies of mathematics? *Mathematics Education Research Journal, 26*(3), 579–605.

Lyons, T., Cooksey, R., Panizzon, D., Parnell, A. & Pegg, J. (2006). *Science, ICT and mathematics education in rural and regional Australia. The SiMERR national survey. Abridged report of findings. Prepared for the Department of Education, Science and Training.* Armidale: University of New England.

McPhan, G., Morony, W., Pegg, J., Cooksey, R. & Lynch, T. (2008). *Maths? Why Not? Final report prepared for the Department of Education, Employment and Workplace Relations (DEEWR).* Canberra: DEEWR.

Miller, H. & Bichsel, J. (2004). Anxiety, working memory, gender, and math performance. *Personality and Individual Differences, 37*(3), 591–606.

Office of the Chief Scientist (OCS). (2014). *Science, technology, engineering and mathematics: Australia's future.* Canberra: Australian Government.

Poladian, L. & Nicholas, J. (2013). *Mathematics bridging courses and success in first year calculus.* Paper presented at the 9th Delta Conference on teaching and learning of undergraduate mathematics and statistics, 24–29 November 2013, Kiama, Australia.

Queensland Studies Authority (QSA). (April, 2014). Identifying enrolment trends in senior mathematics and science subjects in Queensland schools. *Trends and issues in curriculum and assessment, 1* (e-bulletin). Retrieved from http://www.qcaa.qld.edu.au/29940.html

Ramirez, G., Gunderson, E. A., Levine, S. C. & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development, 14*(2), 187–202.

Rubinstein, H. (2009). *A national strategy for mathematical sciences in Australia.* Retrieved from https://www.science.org.au/resources/national-strategy-mathematical-sciences-australia

Rylands, L.J. & Coady, C. (2008). Performance of students with weak mathematics in first-year mathematics and science. *International Journal of Mathematical Education in Science and Technology, 40*(6), 741–753.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

SACE (2012). *Response of the SACE Board to ACARA. On the draft senior secondary Australian Curriculum.* Retrieved from http://www.acara.edu.au/verve/_resources/South_Australia_Response_to_Draft_Senior_Secondary_Australian_Curriculum.pdf

Tall, D. (1997). Functions and calculus. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, F. Leung et al. (Eds.), *International handbook of mathematics education,* pp. 289–325. Dordrecht, The Netherlands: Kluwer.

The Open University (TOU). (2001). *Conference experts call for review for mathematics teachers* [Press release]. Retrieved from http://www3.open.ac.uk/events/2001126_50297_nr.doc

Tooke, D. J. & Lindstrom, L. C. (1998). Effectiveness of a mathematics methods course in reducing math anxiety of preservice elementary teachers. *School Science and Mathematics, 98,* 136–139.