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The Mathematics Problem and Mastery Learning for First-Year, Undergraduate STEM Students

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Abstract. In the 2014 academic year Mastery Learning was implemented in four first-year mathematics subjects in an effort to address a lack of preparedness and poor outcomes of increasing numbers of undergraduate students in science, engineering and mathematics programs. This followed partial success with the use of diagnostic testing and pre-teaching, active learning, and a greater emphasis on problem solving in context - under-prepared students were still more likely to fail the pre-teaching subject and to struggle with subsequent mathematics subjects. This paper describes the learning design used, and the outcomes achieved, with implementing Mastery Learning – the positive: improved academic success, time management, and attitudes towards learning and Mathematics, an increased sense of independence, confidence and retention of content, and reduced stress and anxiety; and the negative: students having a sense of being taught how to pass a test rather than having a deeper understanding of the content. It will be seen that this negative is a consequence of a small but important difference in implementation.

Keywords: The Mathematics Problem, Mastery Learning, first-year undergraduate experience, competency-based assessment.

1. The Challenge – the Mathematics Problem

First-year undergraduate mathematics education of students in science, technology, engineering and mathematics (STEM) programs currently face many challenges. In Australia, failure rates are unacceptably high for some intake cohorts (Groen, Beames, Coupland, Stanley & Bush, 2013), and attrition is higher than desirable. These problems can be traced back to high school where around 40 per cent of junior secondary mathematics classes are taught without a qualified mathematics teacher (McKenzie, Rowley, Weldon & Murphy, 2011).

Additionally, enrolments in elementary[1] mathematics subjects are increasing (Barrington & Evans, 2014). This is a problem as hard prerequisites for entry to most STEM programs in most Australian universities do not exist, though advanced mathematics is usually 'recommended' and intermediate mathematics is 'assumed'. These recommendations and assumptions are often ignored, and students enter university with elementary mathematics or no mathematics at the senior high school level. In 2013, students with these backgrounds amounted to 56% of the University of Technology Sydney (UTS) science intake, 31% of the engineering intake and 28% of the mathematics intake. (Information technology students have no mathematics subjects in the cores of their programs.) The lack of preparedness and declining enrolments, collectively referred to as the Mathematics Problem (Hawkes & Savage, 2000), are problems world-wide (Hoyt & Sorensen, 2001; Smith, 2004; Luk, 2005; Heck & van Gastel, 2006; Brandel, Hemmi & Thunberg, 2008; Rylands & Coady 2009; Varsavsky, 2010).

More than a decade ago, UTS, and many other Australian universities, introduced diagnostic testing of assumed knowledge (at UTS called the Readiness Survey). The diagnostic tests worked (and work) in conjunction with the pre-teaching subject, at UTS, Foundation Mathematics. Students failing the Readiness Survey take Foundation Mathematics prior to their first core mathematics subject to ensure they have the assumed knowledge of their program.

Over this same period, small changes to the first-year undergraduate mathematics curriculum were made, active learning[2] was incorporated into learning designs of all first-year mathematics subjects, and changes were made to assessment schemes to reflect the more diverse learning activities undertaken. There was also an increase in emphasis on problem-based learning. Though improvements in pass rates were observed, unfortunately, under-prepared students were still more likely to fail Foundation Mathematics and later subjects. Our response to a lack of mathematical preparedness of some STEM students needed to be revisited.

2. A Solution – Mastery Learning

2.1 Background

Mastery Learning endorses the belief that all students can learn and achieve the same level of content mastery when provided with the appropriate learning conditions (including time) (Bloom, 1971). Mastery is defined in terms of a subject's objectives and achievement of a prescribed level of performance, or competency, in (criterion-referenced) tests. This level of performance is usually 75 or 80% of the marks available for the 'mastery' test. There is little or no delay between marking and feedback, and students failing to meet the mastery level

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[1] Using the classification system of Barrington and Brown (2005), for New South Wales high school mathematics subjects, Mathematics Extensions 1 and 2 are classified as "advanced". Mathematics (2 unit) is "intermediate" and General Mathematics is "elementary".

[2] See Prince (2004) for a brief description of active learning and problem-based learning.

are provided with remedial activities before a second attempt at achieving mastery. The process may be repeated (theoretically, as many times as required to demonstrate mastery, but practically, and in the current implementation, three times).

Mastery Learning is not a recent innovation, but it is ideally suited to undergraduate mathematics education given its frequently hierarchical structure. It is particularly suited in the case of under-preparedness as it has the capacity to deal with individual learner differences. The research literature indicates positive effects of mastery learning on students, especially in the areas of achievement, attitudes toward learning, and the retention of content (Guskey & Pigott, 1988; Kulik, Kulik, & Bangert-Drowns, 1990; Anderson, 1994; Trigg, 2013). Why then isn't the use of Mastery Learning widespread? The answers lie in its intensive resource use and the (usual) requirement to allow students to work at their own pace. The issue associated with resource use is now very effectively addressed by online learning systems where immediate marking and feedback are available, remedial activities can be flagged and supported, and textbooks are not merely ebooks. The second issue is one that requires a more creative approach to learning activity (including assessment) scheduling. The implementation chosen for UTS first-year mathematics students utilised both these answers.

Mastery Learning was trialled at UTS in the first (Autumn) semester of 2013. The results were promising enough to suggest a further trial of two subjects in Autumn semester of 2014. Analysis of the results of these two subjects confirmed Mastery Learning as a solution to the problems facing first-year STEM students in their mathematics subjects, and Mastery Learning was implemented in another two subjects in second (Spring) semester of 2014. This paper examines the success, or otherwise, of this Mastery Learning initiative.

2.2 Implementation

After subdividing the subject curriculum into learning units and further into a logical sequence of smaller objectives, learning materials, instructional strategies and activities were identified, sequenced and executed over the teaching period. Criterion-referenced tests were administered. These were supervised, online, summative tests of just under an hour's duration, undertaken approximately two weeks after the completion of a unit. They assessed the 'fundamental' knowledge and skills objectives of the unit, that is, the knowledge and skills that provide the basis for further development. In the UTS implementation, 'mastery' was set at 80% of the marks available on the mastery tests.

Students were provided with multiple opportunities for formative assessment prior to the mastery tests. Marking and feedback for both the formative and the summative assessments were provided online immediately. Students used the feedback to feed-forward, engaging in remedial activities individually or collaboratively (where necessary). Students were then given a further formative assessment before being given a second opportunity to sit the test, the 'second-chance' test - same concepts, different questions (Bloom, 1971). Students already exhibiting mastery, could choose to undertake the second-chance test. The best mark of the attempts was used in determining the final mark for the subject.

While there was slight variation between subjects as to the finer details of the implementation, four mastery tests were scheduled over the course of the semester. Each test was worth 16% of the final mark for the subject. The final exam was then worth 36% of the final mark. Passing at mastery level in all four mastery tests ensured that students achieved a Pass in the subject. Perfect scores ensured an upper range Pass was achieved. Students achieving mastery in all tests could sit the final examination to improve their result beyond the Pass grade. Students were required to earn 13/16 on at least one of their attempts of each mastery test. Students not achieving mastery on the first attempt were given a choice of remediation activities to participate in. This remediation was conducted outside scheduled class time. Where mastery was not demonstrated on the second attempt, more structured activities were made available (primarily small group and peer-assisted learning).

The third attempt was conducted at the end of the semester, allowing students additional time to acquire the knowledge and skills they needed. (The third attempt was only available to students who had not already demonstrated mastery.) To facilitate a successful third attempt, the last third of the semester consisted of enrichment activities which were examined in the final exam. The emphases in these enrichment activities were the application of modelling and problem-solving using the tools developed in the earlier units. These enrichment activities also provided students who were yet to demonstrate mastery with the opportunity for reinforcement and further exposure to content in context. Students requiring a third attempt (approximately 5% of a class, on average) were not eligible to sit the final exam. This demonstrates the primary trade-off made to implement Mastery Learning in semesters of fixed length, and is similar to some of the implementations reported by Twigg (2013).

UTS is not the only Australian university to implement Mastery Learning, the University of Canberra implemented Mastery Learning in 2014. Twigg (2013) reports on a number of US tertiary institutions that are also using Mastery Learning as part of a learning design called the Emporium Model.

2.3 The subjects

The subject Foundation Mathematics is offered to any UTS student, but primarily targets students who fail the Readiness Survey. This quick, online diagnostic test is only compulsory for engineering students. Engineering students who fail the test are enrolled in Foundation Mathematics. For other students failing the survey, enrolment in Foundation Mathematics is recommended. Success of this combination has been mixed. For example, failure rates in Foundation Mathematics by engineering students over the 2012-2013 semesters averaged around 59%. Mastery Learning in this subject was introduced in the Autumn semester of the 2014 academic year.

The standard program for engineering students includes Mathematical Modelling 1, followed by Mathematical Modelling 2. Mastery Learning was introduced into Mathematical Modelling 2 in Autumn semester of 2014, while it was introduced into Mathematical Modelling 1 in Spring semester. This was a consequence of the fact that the Autumn Mathematical Modelling 2 cohort is smallest in first semester, and a final decision regarding the roll-out of Mastery Learning to other subjects had yet to be made.

Science students undertake Mathematical Modelling for Science, followed by Mathematics and Statistics for Science. Mastery Learning in these subjects was introduced in the 2015 academic year (following success in the other subjects). Mathematics students undertake Introduction to Linear Dynamical Systems, followed by Introduction to Analysis and Multivariable Calculus. Mastery Learning was introduced into Introduction to Multivariable Calculus in Spring semester 2014, while it was introduced into Introduction to Linear Dynamical Systems in Autumn 2015. These sequences of subjects are broadly similar in content, learning and assessment design, and learning environment. Students in information technology, biological, medical and environmental sciences have no core first year mathematics in their programs, though some take Foundation Mathematics.

Active learning was introduced into first-year mathematics subjects approximately ten years ago. In these subjects this takes the form of interactive activities as well as collaborative learning in what were once traditional lectures. Minor changes were also made to assessment schemes at this time to encourage participation in all learning activities. However, for most first-year mathematics subjects, the majority of marks were still allocated to the traditional closed-book final exam. Weights for this varied, but the typical weight for the final exam was 65% of the marks available for the subject. (As previously mentioned, this changed with the introduction of Mastery Learning.)

3. Data

Data was collected concerning program, tertiary entrance rank (ATAR), Mathematics subject(s) studied in high school and mark(s) in the Higher School Certificate (HSC) or other background, as well as final mark and grade in Foundation Mathematics, Mathematical Modelling 1, Mathematical Modelling 2, Introduction to Linear Dynamical Systems and Introduction to Analysis and Multivariable Calculus for the 2012-2014 academic years (six semesters). Information about the sample sizes can be found in Table 1.

Table 1 – Size of the database for Foundation Mathematics (FM), Mathematical Modelling 1 (MM1) and Introduction to Analysis and Multivariable Calculus (IAC)

Subject		Background (2012, *2013*, 2014)						
		Adv.		Int.	Elem.	No (HSC) senior math.	Tertiary studies	Other*
	Semester	Ext. 2	Ext. 1	Math.	Gen. Math.			

FM	Autumn	3, *6*, 9	9, *18*, 24	41, *31*, 50	21, *34*, 63	1, *3*, 7	217, *146*, 141	22, *23*, 27
	Spring	2, *0*, 3	5, *8*, 3	7, *8*, 18	9, *13*, 9	0, *0*, 3	128, *92*, 86	18, *19*, 31
MM1	Spring	1, *1*, 3	11, *3*, 12	21, *26*, 17	5, *10*, 5	0, *0*, 0	*77*, *68*, *72*	23, *30*, 33
IAC	Spring	25, *22*, 16	36, *34*, 19	15, *20*, 13	0, *0*, 1	0, *0*, 0	19, *13*, 10	4, *6*, 4

The "Other" category includes students with partially complete qualification, and students where no information on their background could be found.

The class sizes of the six semesters of Mathematical Modelling 2 were 286, 560, 235, 574, 274 and 483 respectively.
Students are assigned grades based on their final marks – A Fail (Z) is 0% to 49%, a Pass (P) is 50% to 64%, a Credit (C) is 65% to 74%, a Distinction (D) is 75% to 84%, and a High Distinction (H) is 85% to 100%.

To examine qualitative aspects of the student experience, responses to open-ended questions on the end-of-semester student surveys (Subject Feedback Surveys) were examined. Representative responses were included in this paper.

Focus groups were also conducted by staff not involved with the teaching or administration of the subjects included in the study. Students self-selected to participate, in line with ethics approval. Groups consisted of up to 10 students, and responses to six set questions (on attitudes, confidence, stress and assessment structure and nature) and a concluding open-ended, catch-all, question.

4. Methodology

Mixed methods were used to assess the impact of Mastery Learning - quantitative techniques were used to assess student achievement, though students' perceptions of this achievement were also examined. Qualitative techniques were used to assess the impact of Mastery Learning on things such as confidence, anxiety, attitudes, and behaviour.

In order to examine the statistical significance of any improvements in mean final marks that may be consequent to the implementation of Mastery Learning, one-sided t-tests comparing sample mean final marks, under the assumption of unequal variances, were used. Where the mark distribution wasn't normally distributed, a chi-square test was used (Levine, Berensen & Krehbiel 2008). Pairwise comparisons of semester results from 2012 with 2014 and 2013 with 2014 were undertaken. A 5% level of significance was used for all statistical tests. Where the level of significance was something other than this, p-values are reported. Autumn outcomes were compared with Autumn outcomes, and

Spring outcomes with Spring outcomes (representing the different intakes into the programs).

For pairwise comparisons of failure rates *z*-tests were used (Stat Trek, 2015). Where normality couldn't be assumed, chi-square tests were used. For cohorts where the expected numbers of failures or successes were too small (<5), Fisher Exact tests (McDonald, 2014), as implemented at Preacher (2015), were used.

Comparison of medians was also undertaken as a means of examining the impact of any skewness in the mark distributions. For pairwise comparison of medians, *z*-tests or the Wilcoxon Rank Sum Test were used (where an underlying normal distribution could be assumed), otherwise a Mann-Whitney U-test was used. These were implemented using the Mann-Whitney U-test Calculator of Stangroom (2015) and the implementation of the Wilcoxon Rank Sum Test in Excel using Zaiontz (2015). These techniques were selected in preference to transforming the data using a logarithmic transformation as some of the cohort sizes were too small to assume the transformed variables would be normally distributed.

5. The outcomes

5.1 Quantitative measures

5.1.1 All programs

35010 Foundation Mathematics. In Table 2 we can see an overall reduction in failure rates for the Autumn semester cohorts – the improvement between the Autumn 2012 rate and the Autumn 2014 rate is significant at the 10% level (p = 0.087), and the improvement between the Autumn 2013 rate and the Autumn 2014 rate is significant at the 1% level (p = 0.00002). There is also reduction in the failure rate for the target background cohort, General Mathematics students. Using Fisher's Exact Tests to compare Autumn 2012 with Autumn 2014, the reduction in failure rate is significant at the 10% level (p = 0.092), while the reduction between Autumn 2013 and Autumn 2014 is not significant (p = 0.219).

A one-sided *t*-test assuming unequal variances on the mean overall final mark finds that the improvement in mark is significant at the 5% level when Autumn 2013 is compared with Autumn 2014, but is not significant when Autumn 2012 is compared with Autumn 2014. These results are duplicated for the mean final mark for the Mathematics and General Mathematics cohorts.

Median final marks also improve for nearly all background cohorts with Mastery Learning. For the cohorts of primary interest, the General Mathematics and Mathematics cohorts, the improvement in median using the *z*–test for the General Mathematics cohort when Autumn 2012 is compared to Autumn 2014 is significant at the 10% level (p = 0.051), and the increase in median when Autumn 2013 is compared to Autumn 2014 is significant at the 1% level (p = 0.00082). For the Mathematics cohort, the Autumn 2012 comparison is not significant, while the Autumn 2013 comparison is significant at the 1% level (p = 0.00036).

Table 2 - Autumn semester results by high school background

Back-ground	Autumn 2012			Autumn 2013			Autumn 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
Ext. 2	97.0	98	0% (3)	77.50	83.5	0% (7)	87.00	90	0% (8)
Ext. 1	85.11	87	0% (9)	66.78	78	0% (15)	80.71	92.5	8% (24)
Math.	73.59	81	33% (12)	56.00	57	25% (24)	72.88	81	14% (50)
General Math.	49.57	50	48% (21)	30.97	19	45% (11)	52.97	57	28% (63)
No HSC math.	-	-	-	-	-	-	55.5	55.5	0% (3)
Overall	66	76	21.16 % (314)	53	60	28.81 % (261)	66	70	18.75 % (321)

In Table 3 we see the results for the Spring offering – the reduction in failure rates on pairwise comparisons overall are significant (p = 0.00002 and 0.027 respectively). We can also see improvement in the failure rates of students with backgrounds in Mathematics and General Mathematics. The improvement in failure rates for students with a Mathematics background is significant (p = 0.004) when Spring 2012 is compared with Spring 2014 (using the Fisher Exact Test). However, when Spring 2013 is compared with Spring 2014, the improvement is not significant. The comparisons of the failure rates for students with a General Mathematics background between Spring 2012 and Spring 2014, and Spring 2013 and Spring 2014 are both statistically significant (p = 0.009 and 0.049 respectively). Here we again see favourable outcomes for the main target group (students with a General Mathematics background).

Using a one-sided t-test with unequal variances on the mean overall Spring final mark, we find that this improvement is significant at the 5% level when Spring 2012 is compared with Spring 2014, but is not significant when Spring 2013 is compared with Spring 2014. For the students with Mathematics backgrounds, the increase in mean and median were significant when Spring 2012 - Spring 2014 comparison, but not when Spring 2013 was compared to Spring 2014. For the General Mathematics cohort, the increases in the mean for both comparisons were significant, but there were no significant improvements in the medians.

Table 3 - Spring semester results by high school background

Back-ground	Spring 2012			Spring 2013			Spring 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
Ext. 2	91	91	0% (2)	-	-	-	88	85	0% (3)
Ext. 1	78	81	0% (5)	88	94	0% (8)	84	85	0% (3)
Math.	30	30	63% (8)	66	70	13% (8)	73	75	6% (18)
General Math.	30	56	63% (8)	45	54	38% (13)	59	55	0% (9)
No HSC maths	-	-	-	-	-	-	11	2	100% (2)
Overall	58	68	21.43 % (169)	65	73	12.21 % (140)	66	68	9.61% (153)

To ensure that significant improvement in most of the statistics measuring achievement could not be attributed to more capable students electing to take General Mathematics, ATAR was examined for the Autumn and Spring cohorts. For the Autumn comparisons, mean ATARs were 74, 80 and 67, with the Autumn 2012 to Autumn 2014 reduction significant at the 10% level, while the Autumn 2013 to Autumn 2014 reduction was significant at the 1% level (using a t-test with equal variances). For both pairwise Spring semester comparisons, there was no significant difference at the 5% level between the mean ATARs. We can conclude that there is no evidence in the mean ATARs of both the Autumn and Spring cohorts that suggest more capable students are electing to do General Mathematics, and hence that there is no significant increase in capability of the 2014 cohorts indicated.

Foundation Mathematics data were also broken down by program. In Table 4 we can see that though numbers of Engineering students in the Spring offering of Foundation Mathematics have increased significantly, the failure rate for this cohort is now 0%. Comparisons of the failure rates for Engineering students reveal that the reduction in failure rates for both Autumn and Spring cohorts for all years compared are significant at the 5% level – in three of the four comparisons the improvements in failure rates are significant at the 1% level.

Science students, also show a 67% decrease in failure rate in the Autumn cohort and a 50% decrease in failure rates in the Spring cohort. Comparisons of the failure rates for Science students reveals that the reductions in failure rates for both Autumn and Spring 2013 cohorts are significant at the 5% level, while Fisher's Exact test yields no significant reduction in comparing the Autumn 2012 cohort (p = 0.123), though in comparing Spring 2014 with Spring 2012 the improvement is significant (p = 0.047).

Students in Information Technology programs, have the lowest failure rates. This can be explained by the fact that they have higher participation rates in intermediate and advanced mathematics subjects at high school.

Table 4 - Failure rates by STEM course

Year	2012		2013		2014	
Semester	Autumn (of)	Spring (of)	Autumn (of)	Spring (of)	Autumn (of)	Spring (of)
Science	0.2 (25)	0.333 (6)	0.633 (30)	0.364 (11)	0.205 (39)	0.105 (19)
Engineering	0.697 (76)	0.760 (46)	0.580 (69)	0.317 (41)	0.377 (69)	0 (102)
Information Technology	0.05 (20)	0.091 (33)	0.043 (23)	0 (12)	0.25 (4)	0 (13)

5.1.2 Engineering programs.

33130 Mathematical Modelling 1. Using a one-sided t-test assuming unequal variances (and a 5% significance level) on the mean overall Spring final mark, we see the improvement is significant when Spring 2012 is compared with Spring 2014, but is not significant when Spring 2013 is compared with Spring 2014. These results are repeated for the mean final mark for the Mathematics cohorts. The analysis couldn't be conducted for the General Mathematics cohorts as the sample sizes were too small.

Overall, we can see an approximately 50% reduction in the failure rate overall compared with previous Spring semester results. This result is significant at the 5% level for both the Spring 2012 and Spring 2013 comparisons to Spring 2014. Considering background cohorts, for those students with a Mathematics background using Fisher's Exact test yields a significant reduction in failure rate at the 5% level when the Spring 2014 Mathematics cohort is compared to the Spring 2012 cohort (p = 0.0005), but the reduction is not significant when the Spring 2014 cohort is compared to the Spring 2013 cohort (p = 0.185) . Other background cohorts from the target groups are too small to perform significance tests.

The failure rates for students who had undertaken Foundation Mathematics prior to taking Mathematical Modelling 1 has seen a reduction from 25% down to 6% for a similar sized cohort. Pairwise comparisons using a z-test demonstrates that both reductions are significant at the 5% level (p = 0.00004 and 0.015 respectively). No strong conclusions can be drawn about the failure rates for those students who did HSC mathematics and who did not take Foundation Mathematics as the background cohorts are too small. However, when students with other backgrounds are included, we see a reduction in failure rate from 20% to 12%. Pairwise comparisons using a *z*-test demonstrates that only the reduction from 2012 to 2014 is significant at the 5% level (p = 0.00002), while the reduction from 2013 to 2014 is not significant (p = 0.116).

Using a one-sided *t*-test with unequal variances on the mean overall Spring final marks, finds that this improvement is significant at the 5% level when Spring 2012 is compared with Spring 2014, but is not significant when Spring 2013 is compared with Spring 2014.

For the cohort that hadn't undertaken Foundation Mathematics (the top groupings of Table 5), we see that a one-sided *t*-test with unequal variances demonstrates that there is a significant improvement (α=0.05) between Spring 2012 and Spring 2014 mean final marks. However, the increase in mean marks for the Spring 2013 and Spring 2014 cohorts is not significant. Similar results hold for the comparison of mean final marks for students who did undertake Foundation Mathematics prior to enrolling in Mathematical Modelling 1.

In comparing medians little can be said about individual background cohorts as the samples sizes are quite small. In comparing the overall Foundation Mathematics and non-Foundation Mathematics cohorts, and the subject overall cohort, the increases in median are significant at the 5% level when Spring 2012 is compared to Spring 2014 (p = 0.027, p = 0.00076, p = 0.00012). When the Spring 2013 median is compared with the Spring 2014 median for the non-Foundation Mathematics cohort and for the subject overall, we find significant improvements (p = 0.015 and p = 0.03673 respectively). However there is no significant improvement for this yearly comparison for the cohort who had previously taken Foundation Mathematics.

Table 5 - Spring semester results by background

	Spring 2012			Spring 2013			Spring 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
No FM									
Ext. 2	0	0	100% (1)	86	86	0% (1)	47	59	33% (3)
Ext. 1	50	42	55% (11)	72	72	0% (3)	64	67	8% (12)
Math.	39	40	57% (21)	55	54	19% (26)	60	59	6% (17)

General Math.	24	30	100% (5)	51	54	30% (10)	35	55	40% (5)
Cohort overall	51	52	41% (117)	58	57	20% (121)	61	64	12% (114)
With FM									
Ext. 2	90	90	0% (1)	84	89	0% (3)	60	60	0% (1)
Ext. 1	0	0	0% (0)	66	63	0% (3)	63	63	0% (1)
Math.	43	52	0.5 (6)	59	72	20% (5)	57	57	0% (1)
General Math.	40	43	54% (13)	49	55	22% (9)	55	58	7% (15)
No HSC Maths	-	-	-	-	-	-	54	54	0% (0/1)
Cohort overall	47	50	44% (43)	55	62	25% (48)	58	60	6% (49)
Total Overall	50	51	41% (138)	57	58	21% (138)	60	62	10% (142)

33230 Mathematical Modelling 2. For the Autumn 2014 Mathematical Modelling 2 cohort in Table 6 we see an increase in mean and median final mark, and associated reduction in failure rate for students who achieved a Pass in Mathematical Modelling 1.

The improvements in overall mean final mark for both Autumn semester comparisons are significant at the 5% level, and are in fact significant at the 1% level, with the implementation of Mastery Learning. The improvements in median for both Autumn comparisons were significant at the 1% level using the Wilcoxon Rank Sum Test.

For the cohorts who achieved a Pass grade or better in Mathematical Modelling 1, it is not universally true that the mean and median final marks improved. The improvements in mean and median final marks are significant (at the 1% level) for the two, year comparisons of students who obtained Passes in Mathematical Modelling 1, and when Credits and Distinctions for Autumn 2012 are compared with Autumn 2014, but are not significant at the 5% level for Credits and Distinctions when Autumn 2013 results are compared with Autumn 2014 results.

The reduction in failure rates overall for both Autumn comparisons are significant at the 1% level. For students who received a Pass in Mathematical

Modelling 1, both comparisons to Autumn 2014 resulted in significant reductions at the 1% level. For students with Credit grades, the Autumn 2012 comparison to Autumn 2014 was significant at the 1% level, though the Autumn 2013 comparison was not (p = 0.105). For students with Distinction grades, the Autumn 2014 reduction in failure rate was not significant at the 5% level (p = 0.0571) when compared to Autumn 2012, nor was the Autumn 2013 comparison (p = 0.270). This improvement in outcomes for students earning a Pass grade is further support for a Mastery Learning approach to first-year Mathematics subjects for Engineering students.

Table 6 - Autumn semester Mathematical Modelling 2 results by Mathematical Modelling 1 result

MM1 grade	Autumn 2012			Autumn 2013			Autumn 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
Pass	38	40	72% (74)	51	53	35% (93)	58	59	20% (88)
Credit	50	52	35% (46)	60	64	16% (32)	61	61	0% (20)
Dist.	53	51	32% (28)	65	68	16% (19)	65	63	6% (16)
High Dist.	63	64	0% (16)	84	87	0% (7)	74	77	0% (9)
Overall	47	50	45% (286)	52	53	35% (235)	59	60	11% (274)

A similar pattern can be seen in the Spring Mathematical Modelling 2 failure rates (Table 7). Again, there is not universal improvement in means and median final mark for all cohorts, though the improvements are significant for the Pass and Credit cohorts from Mathematical Modelling 1. Comparisons of means overall (under an assumption of unequal variances), medians, as well as a comparison of failure rates, find that improvements in overall outcomes are significant (α=0.01) for all Spring pairwise comparisons.

Comparisons of mean final mark (unequal variance) and failure rate were also undertaken for each Mathematical Modelling 1 grade cohort for Spring classes. The improvements for the Mathematical Modelling 1 Pass and Credit cohorts are significant for all years compared (means, medians and failure rates at the 1% level). For the Spring Distinction cohort only the 2013 to 2014 failure rate showed significant improvement, though this was at the 10% level.

Table 7 - Spring semester Mathematical Modelling 2 results by Mathematical Modelling 1 result

MM1 grade	Spring 2012			Spring 2013			Spring 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
Pass	48	50	47% (222)	48	52	43% (206)	59	60	6% (140)
Credit	58	59	15% (124)	59	60	16% (109)	64	65	2% (84)
Dist.	68	70	7% (91)	70	73	9% (90)	67	68	2% (100)
High Dist.	79	80	2% (51)	82	83	0% (78)	77	77	1% (89)
Overall	56	58	27% (560)	59	60	26% (574)	65	64	4% (483)

Overall then, the improvements in achievement with Mastery Learning are very encouraging and would appear to ameliorate the previous observation that students who obtained a Pass grade in Mathematical Modelling 1 were not as likely to achieve a Pass or higher in Mathematical Modelling 2.

5.1.3 Mathematics programs.

35102 Introduction to Analysis and Multivariable Calculus. Table 8 presents the mean, median and failure rate of the subject Introduction to Analysis and Multivariable Calculus based on mark in the prerequisite subject, Introduction to Linear Dynamical Systems.

Using a one-sided *t*-test with unequal variances and a significance level of 5% on the mean of the overall Spring final marks, we find that this reduction is not significant when Spring 2012 is compared to Spring 2014. It is also true that the small increase in median is also not significant for this comparison. However, the Spring 2013 improvement in mean is significant at the 10% level, while the improvement in median is significant at the 1% level. Little can be said about the Credit, Distinction and High Distinction cohorts from Introduction to Linear Dynamical Systems (the prerequisite subject) because of the small size of the cohorts. However, there is a statistically significant increase (at the 10% level) in mean final mark for student who achieved a Pass in Introduction to Linear Dynamical Systems for both pairs of years compared.

There is a slight reduction in the failure rate of Pass students, though this is not significant at the 5% level (p = 0.270 and 0.391 respectively). The reductions in failure rates overall are not significant (p = 0.221 and 0.187 respectively).

Table 8 - Spring semester Introduction to Analysis and Multivariable Calculus results by Introduction to Linear Dynamical Systems result

35101 grade	Spring 2012			Spring 2013			Spring 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
Pass	50.62	51	33% (39)	50.21	50	26% (38)	51.14	63	24% (41)
Credit	70	72	8% (13)	60.79	63	14% (14)	60	60	50% (2)
Dist.	79.5	80	0% (8)	66.29	70	14% (7)	64.25	77.5	25% (4)
High Dist.	90.6	90	0% (8)	79.5	80.5	12.5% (8)	91	91	0% (2)
Overall	55	55	32% (99)	52	50	33% (95)	57	64	26% (63)

Results in Table 9 (results by background) exhibit an improvement in median result, along with an approximately 25% reduction in failure rate for students with a Mathematics background. Statistical analysis could not be undertaken on the Extension 2, Mathematics and General Mathematics cohorts due to the small sizes of these cohorts (Stat Trek, 2015). There is a significant (α=0.01) improvement in mean and median final mark in both year comparisons for students with an Extension 1 background. There is a significant improvement at the 1% level in failure rate for this cohort when Spring 2012 and Spring 2013 are compared to Spring 2014 using a Chi-square test.

Table 9 - Spring semester results by high school background

Back-ground	Spring 2012			Spring 2013			Spring 2014		
	mean	median	%fail (of)	mean	median	%fail (of)	mean	median	%fail (of)
Ext. 2	61	59	24% (25)	54	51	27% (22)	51	58	38% (16)
Ext. 1	47	45	56% (36)	54	51	29% (34)	60	67	16% (19)

Math.	56	56	13% (15)	50	50	40% (20)	56	67	31% (13)
General Math.	-	-	-	-	-	-	69	69	0% (1)

5.2 Qualitative outcomes - student perspectives

Accommodate individual differences. Mastery Learning by its very nature seeks to accommodate differences in background and time students require to develop mastery of learning outcomes. Smaller, targeted tests with immediate feedback, learning support and multiple attempts to demonstrate mastery are more accommodating than one-chance testing. The majority of students report liking the mastery tests:

> I like the mastery testing scheme. I think it's fair, and I particularly like that [the lecturer] gives a pool of questions that can be repeated and practiced. (SFS – Mathematical Modelling 1 Spring 2014)

Achievement. By providing timely feedback, supporting remediation and allowing multiple test attempts, students are able to identify and address their weaknesses, leading to an overall improvement in academic performance:

> I thoroughly enjoyed the subjects re-structure using the mastery tests. I previously failed the subject ... The mastery tests have enabled me to consistently and thoroughly cover the material better preparing me for my … future career as an engineer. (SFS – Mathematical Modelling 1 Spring 2014)
> Having the mastery testing system was really helpful … I found that being able to view your results and having second attempts encouraged me to study and learn much more. (SFS – Introduction to Analysis and Multivariable Calculus Spring 2014)

Attitudes towards Mathematics and Learning. The motivating characteristics of Mastery Learning and applied problem solving resulted in an improvement in attitude towards mathematics:

> … the Mastery Tests … helped me ease into [the subject and] … I have a greater appreciation for maths now. (Focus group – Foundation Maths Autumn 2014)

> The second chance in the test really helped me to learn where I had made my mistakes and to learn from them and to come back and achieve a great mark the second time around. (SFS – Mathematical Modelling 1 Spring 2014)

Retention of content. Mastery tests foster knowledge and skills retention:

> … the mastery tests … helped retain the concepts longer. (SFS Foundation Mathematics Autumn 2014)

> [Proprietary online learning system] is a fantastic tool, the instantaneous feedback and the ability to read the text online with worked examples is great. The ability to continually practice the

> alternate problems solidifies the knowledge gained. (SFS – Mathematical Modelling 2 Spring 2014)

Reduced stress and anxiety. Mastery Learning can reduce stress and anxiety by having more frequent, lower stakes tests that students can also sit multiple times. Final exam stress is also reduced – students know that they are walking into the exam knowing that they have already passed the subject:

> I liked the fact that … the final exam wasn't as stressful knowing that you have passed the subject after passing the mastery tests. (SFS – Mathematical Modelling 2 Spring 2014)

> I enjoyed doing the mastery tests as it helped me to stay on top of the work and made it much more relaxed when preparing for the final exam (SFS – Introduction to Analysis and Multivariable Calculus Spring 2014)

However, a small number of students felt that there was too much testing and pressure:

> Just thinking about getting 80% on a mastery test is too [much] pressure. (SFS – Mathematical Modelling 2 Spring 2014)

Improved time management. Through a structured approach to the timing of assessment tasks, with Mastery Learning students learn to manage their time more effectively, and so cramming for the final exam is also discouraged:

> The mastery tests were a great way in keeping up to date with content. (SFS – Mathematical Modelling 2 Spring 2014)

> The mastery tests made you learn continuously instead of cramming at the end of the semester. (SFS Foundation Mathematics Autumn 2014)

Increased independence. Mastery Learning encourages self-correction and independence:

> I liked … the Mastery tests which ensured that your learning and understanding of the subject matter were reinforced and that you kept up to date with your work throughout the semester. The online [proprietary learning system] was an excellent resource for study at home. (SFS – Introduction to Analysis and Multivariable Calculus Spring 2014)

Increased confidence. Mastery Learning can build confidence:

> The '2nd chance' class test. … It is not the same test but similar and this does wonders to a student's confidence … (SFS Mathematical Modelling for Science Autumn 2013)

> The assessment of 4 mastery test was good - made me feel more confident approaching finals. (SFS – Mathematical Modelling 2 Spring 2014)

The above aspects combine to empower students so that they are not just achieving better marks and retaining what they have learned but are also developing skills and attributes that can not only be translated to the rest of their studies but to their later professional life.

For some students the experience was less positive – they felt that they were learning how to do the tests not master fundamentals of the subject:

> I was able to pass because all I had to do was learn how to do a specific set of questions for each mastery test. (SFS – Mathematical Modelling 2 Spring 2014)

As a consequence of this feedback from students in Autumn 2015 more variation in questions was included in Mathematical Modelling 2.

Online testing. There are significant advantages to using online testing for the mastery tests. These include:

> [Proprietary online learning system] is a fantastic learning tool … I find that my ability to learn through tools like this is greatly enhanced due to the instantaneous feedback and the practice alternate problems. (SFS – Mathematical Modelling 2 Spring 2014)

However, online testing was not universally popular, especially at the beginning of the semesters when students were learning how to navigate the environment:

> I did not like the way assessments were conducted online. I spent far too long trying to work out how to enter the answers on the computer correctly rather than focusing on the actual material. (SFS – Foundation Mathematics Spring 2014)

> The … [online] assessment need[s] to be more reliable, the system is unpredictable. (SFS – Mathematical Modelling 2 Spring 2014)

6. Conclusion

The aim of the paper was to contribute to the current debate on the ways to address the lack of preparedness of some first-year students of STEM programs (the Mathematics Problem). At the University of Technology Sydney, a form of Mastery Learning has shown itself to address the lack of preparedness successfully for many STEM students as well as for first-year mathematics students overall – it has advantages over one-chance testing and heavily weighted final exams. These advantages include improved academic performance. Students also report increased independence and confidence, and improved time management and retention of content. For many students the learning experience is positive with less stress and anxiety. While some students report poor experiences with the online learning and testing environment, most appreciate the central role it plays in facilitating Mastery Learning. The poorer experience of students in Mathematical Modelling 2 in 2014 resulted in the fine-tuning of content in the sequences of formative and summative assessments.

Overall, Mastery Learning appears to afford a sustainable solution to the increasing lack of mathematical preparedness of some students in STEM programs.

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