Faculty of Engineering and Information Technology

# Topological Shape Optimization of Microstructures of Materials using Level Set Methods

A thesis submitted for the degree of

**Doctor of Philosophy**

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(2015)

**Title of the thesis:**

Topological shape optimization of microstructures of materials using level set methods

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# Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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# Acknowledgments

My thesis could not have come to fruition without the assistance of many contributors.

Hereby, I would express my deep gratitude to my principal Professor Nong Zhang. He supported me with great direction, advices and any other helps in my research. His knowledge, hard-working and research attitude fostered my development in various aspects.

I am profoundly grateful to my co-supervisor Dr Zhen Luo. Without his kindness of helping me to verify the simulation, the completion of this thesis would be impossible. He provided me with great guidance during my whole candidature period. He is the most brilliant and helpful co-supervisor to me. I would also express my gratitude to Dr Jinchen Ji, Dr Paul Walker and Dr Yiqiang Wang, who often offered their time in discussing research issues during my PhD research program.

I would like to thank my colleagues, Xingxing Zhou, Jinglai Wu, Lifu Wang, Sangzhi Zhu, Tianxiao Zhang, Guangzhogn Xu, Jiageng Ruan, Robert Heal, Li Sun, Bo Zhu, Mingming Zhao, for their supports.

I also wish to gratefully acknowledge the consistent financial support of the following agents: China Scholarship Council (CSC) and University of Technology, Sydney (UTS).

Most especially to my family, words alone cannot express what I own them for their encouragement and whose patient love enabled me to complete this thesis.

# Publications and Conference Contributions

The following publications are part of the thesis

*Peer reviewed international scientific journal publications*

[1] **Y. Wang**, Z. Luo, N. Zhang, Topological optimization of structures using a multilevel nodal density-based approximant, ***CMES: Computer Modeling in Engineering & Sciences***. Vol. 84, Issue.3, pp. 229-252, 2012.

[2] **Y. Wang**, Z. Luo, J. Wu, N. Zhang, Topology optimization for micro compliant mechanisms using the element-free Galerkin method, ***Advances in Engineering Software***. Vol. 85, pp. 61-72, 2015.

[3] **Y. Wang,** Z. Luo, N. Zhang, QF. Qing, Topological shape optimization of multifunctional tissue engineering scaffolds with level set method. ***Structural and Multidisciplinary Optimization***. (Major revision, Manuscript ID SMO-14-039)

[4] **Y. Wang,** Z. Luo, N. Zhang, Topological design for mechanical metamaterials using a multiphase level set method, ***Structural and Multidisciplinary Optimization***. (Major revision, Manuscript ID SMO-15-0079)

[5] **Y. Wang,** Z. Luo, N. Zhang, Design of micro-structured metamaterials using a multiphase level set method, ***Engineering Optimization*** (Major revision, Manuscript ID GENO-2014-0193)

[6] Z. Luo, N. Zhang, **Y. Wang**, A physically meaningful level set method for topology optimization of structures, ***CMES: Computer Modeling in Engineering & Sciences***. Vol. 83, Issue.1, pp. 73-96, 2012

[7] Z. Luo, N. Zhang, **Y. Wang**, W. Gao, Topology optimization of structures using meshless density variable approximants, ***International Journal for Numerical Methods in Engineering***, Vol. 93, Issue.4, pp. 443-464, 2013

*Peer reviewed international scientific conference publications*

[1] **Yu Wang**, Zhen Luo. Shape and topology optimization using a nodal density-based level set method, *The 14th Asia Pacific Vibration Conference APVC 14*, 5-8 December 2011, The Hong Kong Polytechnic University.

[2] **Y. Wang**, Z. Luo. A radial point-based meshfree Galerkin method for topology optimization, *The 4th International Conference on Computational Methods ICCM 2012*, 25-27 November, Gold Coast, Australia.

[3] N. Zhang, **Y. Wang**, Z. Luo. Structural topology optimization using meshfree methods, *The 23rd International Congress of Theoretical and Applied Mechanics ICTAM 2012*, 19-24 August, 2012, Beijing, China.

[4] Z. Luo, **Y. Wang**, N. Zhang, Topology optimization of lightweight structures using a meshless shepard function approximant, *The 7th Australasian Congress on Applied Mechanics ACAM 7*, 9-12 December, 2012, Adelaide, Australia.

[5] **Y. Wang**, Z. Luo, N. Zhang, Topology optimization of compliant mechanisms using element-free methods, *The 2013 World Congress on Global Optimization WCGO 2013*, 7-12 July, 2013, The Yellow Mountains, China. ***Advances in Global Optimization*** Springer Proceedings in Mathematics & Statistics, 95, 2015, pp 217-226

[6] **Y. Wang**, Z. Luo, N. Zhang, Design optimization of structures using a nodal density-based SIMP method, *The 13th Asia-Pacific Congress for Computational Mechanics, APCOM 2013*, 11-14 December, 2013, Singapore.

[7] **Y. Wang**, Z. Luo, N. Zhang, The topological design of matematerials using a level set method, *The 11th World Congress on Computational Mechanics WCCM XI 2014*, 21-25 July, 2014, Barcelona.

# Abstract

Topology optimization has been regarded as a most promising approach in the conceptual stage of structural design. It has experienced rapid development over the past two decades and has been applied to a wide range of engineering problems. This thesis will focus on the level-set based topology optimization method, which was originally developed by Osher and Sethian in 1988 and have been successfully incorporated into the structural optimization. With the implicit representation scheme, the level set methods can be easily applied to handle the complex shape and topology changes of the structural design.

This work is divided into three parts. The first part is the necessary background for understanding the main focuses of the thesis. It includes the first four chapters: Chapter 1 provides the background the topology optimization, and overview of the current topology optimization methods, as well as application in the material design fields. Chapter 2 gives a description of the numerical homogenization method. Chapter 3 introduces the theory of the conventional level-set methods, and Chapter 4 provides details of for a parameterized level set method with numerical examples.

The second part of this thesis is about the design of mechanical/elastic metamaterials, which is contented in Chapter 5. In this part, we integrate the parameterized level set method with the numerical homogenization method for the design problems of metamaterials. Meanwhile, a multiphase level-set based scheme for designing metamaterials is proposed. In the parameterized level set method, a set of compactly supported radial basis functions (CSRBF) is employed to interpolate each implicit level set function, which transfer the most

difficult topology optimization problem into an easiest "size" optimization problem in the area of structural optimization. This method will be free of the Courant–Friedrichs–Lewy (CFL) condition and the re-initialization scheme. The propagation of the level set function can be driven by other well-developed optimization that involves gradient information. Moreover, this method can freely create new holes inside the material regions of the two-dimensional (2D) design domain. The optimal designs for mechanical metamaterials with extreme and prescribed properties are presented in this chapter as well (e.g. negative thermal expansion and negative Poisson's ratios).

In the third part of this thesis, Chapter 6, we applied the parametric level set method and the numerical homogenization method for designing three-dimensional (3D) scaffolds for the tissue engineering. Numerical examples are used to demonstrate the effectiveness of the optimization method in designing the scaffold with a range of multifunctional properties. The efficiency, convergence and accuracy of the present methods are also highlighted.

Finally conclusions are given in Chapter 7.

*Keywords:* Topological shape optimization; Let set method; Homogenization method; Microstructures, Mechanical metamaterial; Tissue engineering scaffold.

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# Nomenclature

## Global abbreviations used in this thesis

| | | |
|---|---|---|
| CAMD | = | Continuous approximation of material distribution |
| CFL | = | Courant–Friedrichs–Lewy |
| CS-RBF | = | Compactly supported radial basis function |
| EFG | = | Element-free Galerkin |
| ESO | = | Evolutionary structural optimization |
| FEM | = | Finite element method |
| GCMMA | = | globally convergent method of moving asymptotes |
| H-J PDE | = | Hamilton–Jacobi partial differential equation |
| KKT | = | Karush-Kuhn-Tucker |
| LSFs | = | Level set functions |
| LSM | = | Level set-based method |
| MLPG | = | Meshless local Petrov-Galerkin |
| MLS | = | Moving least squares |
| MMA | = | Method of moving asymptotes |
| MPLSM | = | Multi-phase level set method |
| NPR | = | Negative Poisson's ratios |
| NTE | = | Negative thermal expansion |

| | | |
|---|---|---|
| OC | = | Optimality criteria |
| PDI | = | Pointwise density interpolation |
| PLSM | = | Parametric level set method |
| PIM | = | Point interpolation method |
| PUM | = | Partition of unity method |
| PDE | = | Partial differential equation |
| RBFs | = | Radius basis functions |
| RPIM | = | Radial point interpolation method |
| RKPM | = | Reproducing kernel particle method |
| SIMP | = | Solid isotropic material with penalization |
| SPH | = | Smooth particle hydrodynamic method |
| ZTE | = | Zero thermal expansion |
| 2D | = | Two dimensional |
| 3D | = | Three dimensional |