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TECHNOLOGY SYDNEY**

Masters Thesis Report

Bio-mimetic navigation in a dynamic environment

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Declaration

Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Student:

Date:

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Preface

This thesis covers the work done during my Masters by research course at the University of Technology, Sydney under the guidance of Dr. Jianguo Wang.

The document is divided into 5 chapters. Chapter 1 introduces my research. It covers a brief summary of navigation and justifies my decision to work on bio-mimetic navigation. Chapter 2 consists of a detailed literature review of the various neuron sets that I worked with. Each chapter detailing a neuron set has two sections - one for the neuroscience review, and another for the computational modelling review. Chapter 3 details the implementation of head direction cells on the ROS platform, and chapter 4 details the addition of grid cells to the system. Finally, I discuss my work in chapter 5. I present my views on the current state of literature, the issues that researchers need to solve when working on a similar project, and what I intend to work on next.

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Abstract

The importance of navigation in robotics cannot be understated. Without being able to correctly and efficiently navigate in an environment, an agent will be unable to carry out any tasks. Animals, even smaller mammals for instance, have sufficiently developed navigation systems that enable them to carry out their daily tasks: forage for food, find shelter, navigate to and fro such sites. It has long been proposed that animals use a type of map for navigation. Unlike maps generated by modern mapping techniques, these maps are topological, and lack precise metric information.

Brain research has found sets of neurons that co-operate to form a navigation system in animals. Such cells: head direction cells, place cells, grid cells; decode specific information about the animal's navigation, a combination of which is sufficient to provide a complete navigation solution.

The aim of my masters research, as detailed in this report, was to study these spatial neurons and their modelling for use in robotic navigation. I have modelled head direction and grid cells, which are important components of the neural path integrator system using the Robot Operating System (ROS) platform. Both models have been validated with real time data collected from the PR2 robot.

I cannot say what I feel in any human sense, Partner Elijah. I can say, however, that the sight of you seems to make my thoughts flow more easily, and the gravitational pull on my body seems to assault my senses with lesser insistence, and that there are other changes I can identify. I imagine what I sense corresponds in a rough way to what it is that you may sense when you feel pleasure.

R. Daneel Olivaw explaining to Elijah Baley that he enjoys his company and is, in a robot sort of way, happy to see him again.

Taken from Isaac Asimov's Robots of Dawn

Chapter 1

Introduction

I don't know anything, but I do know that everything is interesting if you go into it deeply enough.

Richard Feynman

1.1 Navigation

It is quite obvious that navigation is a necessary capability that animals need to possess. Without navigation, an animal would be unable to find food, to seek shelter, to escape danger - to survive. We, humans, rarely stop navigating during our day. We move from room to room, from object to object, from person to person. Most, if not all, of our activities require us to navigate.

Similarly, mobile robots must navigate to carry out their tasks. A simple source to goal path can also be quite complicated to navigate. Since the beginning of robotics research, navigation has been accepted as an important focus area[1]. In general, while one would expect robotic navigation to be inspired by biological systems - this isn't the case. Most classical robotic techniques treat navigation as a computational problem, where the input from a set of sensors needs to be processed optimally to provide the agent with accurate navigational information. In line with this perspective, complex algorithms have been developed in attempts to solve the navigation problem.

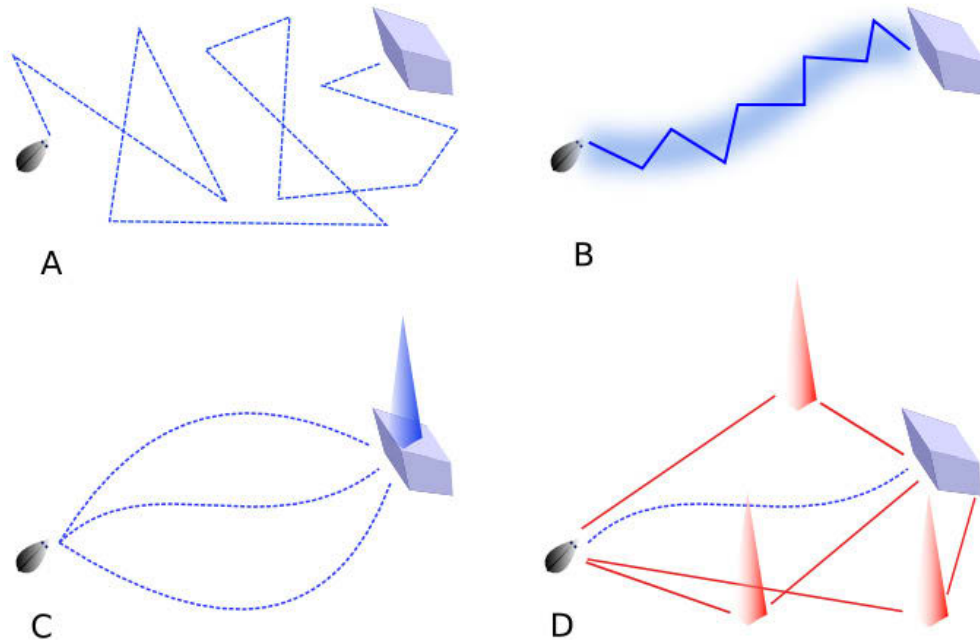


Figure 1.1: The figure shows various local navigation strategies: **A** shows the agent randomly searching until it finds its goal location. **B** shows the agent following a trail, such as a smell or other chemical trail. **C** shows the agent simply walking towards a beacon that signifies the goal location. **D** shows the agent deducing its goal location by observing its relationships with other landmarks.

I first discuss biological navigation and then move on to robotic navigation. In the process, I cover how biological systems have inspired navigation in robots. I glean over classical robotic navigation to provide a contrast between the two robotic navigation paradigms.

1.2 Biological navigation

Even the smallest animals possess adaptive navigation systems that are quite sufficient for their needs. Biological navigation is goal oriented, to the effect that it is defined as[2]:

Definition 1. *Navigation is the process of determining and maintaining a course or trajectory to a goal location.*

This is based on the observation that animals can navigate without understanding their current location, or without asking the question - "Where am I?". Trullier et al. formally analysed the various hierarchies of animal/animat¹ navigation in their seminal paper[3]:

1. Homing
2. Guidance
3. Place recognition triggered response
4. Topological navigation
5. Metric navigation

Franz and Mallot further extended this classification to the following[4]:

1. Local navigation:
 - (a) Search
 - (b) Direction-following
 - (c) Aiming
 - (d) Guidance
2. Way finding:
 - (a) Recognition triggered response
 - (b) Topological navigation
 - (c) Survey navigation

They defined **local navigation** as navigation where only the goal needs to be recognized along with only objects and places in the current "sensory horizon". In **way finding**, the agent needs to recognize various locations and also the associations between these places which could be out of the current "sensory horizon".

In **searching**, the agent simply moves around in the environment and comes upon the goal by chance. It does not attempt to orient itself in a

¹An animat is an agent that attempts to mimic animal behaviour

particular direction with reference to the goal, or another associated location at all. **Direction following** requires the agent to align itself in a locally available bearing to reach the goal. The goal may or may not be in the sensory horizon. An extension to this is **aiming** where the agent must align itself in such a way that the goal is always in front of it. The goal may not be in the sensory horizon, but it must be linked to some salient cue that should always be in the sensory horizon. In the absence of a salient cue that signifies the goal, the agent refers to other land marks that enable it to navigate to the goal. Realizing a known relationship with a set of land marks enables the agent to reach its goal location. The **place recognition triggered response** method is an extension of the guidance method, where, instead of considering places as points, places are extended to constitute of a set of locations that have the same significance with respect to action selection. The agent, instead of trying to replicate exact sensor perceptions, carries out a fixed navigational action at a certain place. A chain of such place-action sets is sufficient to permit navigation to a goal state. This system is also a simple action-place association, and does not involve mapping. For instance, if a certain place is blocked by an obstacle, the agent will need to search until it finds another known location that has an associated action leading to the current goal.

Topological navigation handles this limitation. In topological navigation, the perceived information is maintained in the form of a map. Since a map is available with edges connecting various locations, the agent is able to work out alternate paths in the case of obstacles or missing places. It is noteworthy that none of the aforementioned methods permit optimization, quite simply because they are not defined in terms of metric parameters: they do not consider time or distance, or another parameter such as work. The most sophisticated representation of spatial data is the **metric map**. The metric map stores relations between places using metric information, most commonly the distances between them. However, the metric to be considered is up to the agent: Google Navigator, for instance, let's you plan your trip keeping traffic conditions in mind, thus prioritising time as a metric over distance. **Survey navigation** requires the representation of all spatial information into a common frame of reference, a map. The spatial link between any two locations can then be inferred.

A little introspection while travelling to and fro work will confirm that we do indeed follow this hierarchy to navigate, daily, all the time. A subtler observation is that we do not rely on metric maps for day to day navigation. For instance, once the path to and fro work has been *learned*, one does not need to use their car's navigator services. Consider a colleague from work requesting directions to a good restaurant nearby. Our response, if we don't point them to an internet map, would be on the lines of, *"So, you get out of University from the main Tower exit. Take a left. Walk until the Co-op book store and take another left at the traffic lights. Walk to the end of the road till you get to the university library. The restaurant should be right at that corner, on the right, next to a Seven Eleven."* Even with our dependence on metric maps, we hardly ever define paths as combinations of distances and exact bearings.

1.2.1 The cognitive map theory of biological navigation

In 1948, Tolman presented his paper introducing his theory of cognitive maps [5]. While carrying out experiments on rats, his students and colleagues hypothesized the process that enabled rats to successfully travel through their experimental mazes. Tolman documents that there were sets of people who believed that a simple stimulus response method was all that was being used. However, Tolman observed behaviours in the rats that suggested that a cognitive map would be required - for example, avoidance of regions where the agent had been given an electric shock [5]. O'Keefe and Dotrovsky further added credence to this theory in 1971 [6] with their discovery of place cells. The cognitive map theory states that animals, such as rats, build a cognitive map of their environment in the brain as they receive new stimulus. This cognitive map has various functions - for example, it permits the agent to plan efficient paths for navigation - by bypassing obstacles as explained earlier for instance. The place cell is believed to be the unit of this cognitive map [7, 8].

1.3 Navigation in robots

Navigation in robots remains an area of great interest. In spite of the great strides that have been made in all spheres of robotics: perception, planning, interaction with objects and others; the state of navigation is still far from complete.

Navigation research in robotics can be classified broadly into a dichotomy: *classical* robotics, which is based on nautical navigation and relies heavily on a structured, algorithmic method of navigation; and *bio-mimetic* robotics, in which researchers attempt to study biological navigation and implement these methods on to robots.

1.3.1 Classical robotic navigation

As Franz and Mallot document [4], the term navigation originally applies to the process of directing a ship to its final point of disembarkation. This consists of three repeated steps:

1. Determining the ship's current position as accurately as possible on a metric map.
2. Finding the relationship between the vessel's current location and other important landmarks: final goal location, reference landmarks.
3. Setting a new course based on the information gathered from the above steps.

This method has transferred almost unchanged to classical robotics - the navigation problem has been defined as [9]:

Definition 2. *The robot is placed in an environment that is unknown, large, complex and dynamic. After a time needed by the robot to explore the environment, the robot must be able to go to any selected place, trying to minimize a cost function (e.g. time, energy, etc.).*

This is also referred to as the *maximum navigation test*, which is subdivided into smaller sub-challenges:

1. **Motion control problem:** the robot must be able to move quickly, safely and under control.
2. **World modelling problem:** the robot must be able to collect data about it's environment while moving.
3. **Localization problem:** the robot must know where it is located in the environment.
4. **Planning problem:** the robot must be able to plan an efficient path to its destination.
5. **Architecture problem:** the interactions between solutions to the different problems.

The definition specifically limits itself to navigating in the environment. It does not consider interaction with features in this environment. Extending the "architecture problem" to robotics in general, the solutions to the navigation problem should be able to interact with solutions to other robotic problems to form a truly autonomous agent.

While classical techniques have been able to carry out specialized navigation tasks, none of these systems have yet reached the navigational merit possessed by even smaller animals: fish, bees, ants; which enable them to navigate successfully in their environments, forage for food, sometimes migrate long distances and return to their homes.

1.3.2 Bio-mimetic navigation

Bio-mimetic navigation stems from two research goals:

1. To understand biological navigation better.
2. To utilize this knowledge to create navigation systems for robotic applications.

Bio-mimetic navigation, as expected, is goal oriented, i.e., the one question that needs to be answered is "Am I at my goal location?". The agent need not know it's current location - it does not need to solve the localization problem, for instance. The minimum requirements for bio-mimetic

navigation are therefore the ability to move, and the ability to detect the goal location. While higher animals do possess much more complex navigational capabilities, the different navigational systems are used in a layered fashion akin to subsumption architecture[10] - only that which is required is used. This makes bio-mimetic systems more efficient at navigation. For example, while a complete robotic solution may use a simultaneous localization and mapping (SLAM) [11, 12] based metric map of its surrounding even to approach a target in its sensory horizon, a bio-mimetic system will instead only search for the target location and approach it once located. In fact, as discussed earlier, even the most developed animal species do not appear to have in-built systems that use metric information.

Researchers have attempted to develop bio-mimetic navigation systems with different levels of detail. Franz and Mallory's review lists a number of bio-mimetic systems. Some of them are:

1. Sharp and Webb's robot that follows a trail of chemical cues similar to ants[13].
2. Coombs and Roberts' system that balances optic flow information in both eyes similar to bees[14].
3. Lambrinos et al.'s path integrator system that used polarized light for compass orientation similar to insects[15, 16].
4. Webb's system that calculated phase difference between auditory perceptions to detect the relative direction of a sound source similar to female crickets[17].
5. Recce and Harris' implementation of a hippocampal model that uses place cells[18] found in the rat hippocampus[19].
6. Owen and Nehmzow's topological navigation system that recorded the distance and direction between places[20].

Of course, over the past few decades many other bio-mimetic systems have been developed by researchers for various uses.

As more and more information on biological navigation is made available, researchers have been able to develop systems that mimic biological

navigation in more and more detail. Over the past few decades, neuroscience researchers have discovered neuron sets in the mammalian brain that respond to spatial information perceived by the agent - head direction cells[21], grid cells[22], place cells[7], boundary cells[23] and others. Various studies have been made to elucidate the properties of these neuron sets, and the information thus collected has enabled computational neuroscientists to model the neuron sets to replicate their functioning. At the present, due to the infancy of the field of neuromorphic engineering, hardware implementations of these neuron sets are still limited and rare. However, research literature is strewn with computational simulations of these neuron sets, with various models building on older ones to incorporate new information that may have since come to light. Some researchers have also attempted to use these neuron systems as components of a complete bio-mimetic navigation system for robots as we'll see in the literature review.

1.4 Justification of research

I've had the opportunity to study both classical and bio-mimetic robotics. While I am in awe of the state of research in classical navigation, I cannot help but observe that I, personally, being a human, do not use the concepts that classical navigation is based on. Combined with the generally experienced inflexibility of classical navigation techniques, this turned me towards bio-mimetic navigation. My interest in neuroscience and navigation eventually turned me to neuromimetic navigation.

Neuro-mimetic navigation, like bio-mimetic navigation, has two goals:

- Understanding the functions and properties of neuron sets in the brain.
- Modelling these neuron sets for use on robots as components of bio-mimetic systems.

In my research, I attempt to study the aforementioned neuron sets that encode spatial information, and model them for use with the Robot Operating System (ROS).

1.5 Contributions

In this work, we've developed the lowest building blocks of a bio-mimetic navigation system. Instead of using classical robotic techniques, we have made use of biologically plausible neural processing. In our work, we've tried to remain as true to biology as possible - only diverting when absolutely necessary. Our work focusses on the path integrator systems of the brain, covering directional and spatial coding. The two neuron sets that we have implemented, head direction cells and grid cells are strongly coupled, to such an extent that they are found together in certain brain regions. These neuron sets are also closely related to other navigational neurons - such as place cells which encode locations.

The models used are based on previous models that have been proposed in literature. Among the many proposed models, we based our work on self organizing models, in which the synaptic weights between neurons are ascertained via Hebbian learning - a majority of the existing models ignore this biological process and use other, non biological process instead. We believe, that for a system to be truly bio-mimetic, it is imperative that the system itself ascertain the synaptic weights required for its functioning. The use of Hebbian learning is also most important for our future work - use of allothetic visual cues, as we'll discuss later.

The two main components of our work - head direction cells and grid cells are described in chapters 3 and 4:

Head direction cells Head direction cells[24] are neurons found in areas of the hippocampus and related regions that encode the direction that the agent is facing. The main underlying process depends on idiothetic self motion cues, mostly from the vestibular system. Like an inertial navigation system, a set of head direction neurons integrate self motion information to calculate the current directional state of the agent. Since a path integrator system by itself is insufficient for use - it necessarily drifts over a period of time, a correcting input is required to reset the drift. It has been found that head direction neurons are capable of associating with visual landmarks and use them as an extra, corrective input.

In our work, we implemented and tested the path integrator mecha-

nism exhibited by head direction neurons. We also tested to see if our model was capable of being extended to incorporate visual landmark information as required by head direction cells.

Grid cells Similar to head direction cells, grid cells[22] have also been found to exhibit dead reckoning characteristics. Grid cells are laid out in a triangular mesh in layers of the entorhinal cortex such that grid cells correspond to a spatial co-ordinate map of the agent's environment. In a manner similar to head direction cells, a set of grid cells integrates idiothetic direction and speed information to encode the location of the agent on the mesh.

The effects of visual landmarks on grid cells is still an area of research. However, it has been established that head direction and grid cells project on to place cells, along with other neuron sets to encode location. It is possible that projections back from place to grid cells are used for correcting drift that builds up over time.

In our work, we implemented and tested out a set of grid cells. The model makes use of the above mentioned head direction cell implementation for direction information. Also, the model, since built on the same concepts as the head direction cell model, can be extended to include other inputs that may be discovered in the future.

Chapter 2

Literature review

How well he's read, to reason against reading!

William Shakespeare

Research papers published related to this chapter:

- Modelling head direction cells: a review - Ankur Sinha & Jianguo Wang
- Submitted to **Neural Computation** for consideration - under review.

The literature review summarizes the current state of literature on head direction cells and grid cells. Each section is divided into two subsections that outline biological and computational modelling information respectively.

2.1 Head direction cells

Head direction neurons were discovered by Ranck in 1984 [21]. Head direction neurons calculate and encode the animal's current bearing via a neural mechanism. In a set of head direction neurons, each neuron is associated with a specific direction and discharges maximally only when the animal faces in this particular direction: consequently, this direction is referred to as the cell's "preferred direction". Since their discovery, various studies have focussed on elucidating various aspects of head direction cells: their interaction with the environment, the contribution of various inputs, the neural pathway involved in their functioning and the overall role that they play in the biological navigation system. Section 2.1.1 encapsulates the available neuroscience information on head direction cells. Section 2.1.2 discusses the computational modelling of head direction cells.

2.1.1 Neuroscience review

This section briefly outlines some salient properties of head direction cells. I build on past reviews - Sharp's review [25], Taube's comprehensive reviews [24, 26], Clark and Taube's review [27] and books on the subject [28, 29], adding information where necessary.

Discharge properties

As introduced earlier, individual head direction cells fire maximally only at their "preferred directions". As the animal's head turns, the firing rate of the head direction cell decreases symmetrically about the maxima in an almost linear fashion as shown in Figure 2.1. The "tuning width" of a head direction neuron is the angular extent that it is active in. Note that head directions cells with adjacent "preferred directions" are not necessarily found adjacent to each other in the brain. This assumption is made by most computational models that we'll discuss in the next section, but is only for sake of simplicity. Only the synaptic connections between head direction neurons dictate their function. While Taube et al. described the graph to be triangular with the decrease on both sides of the peak being

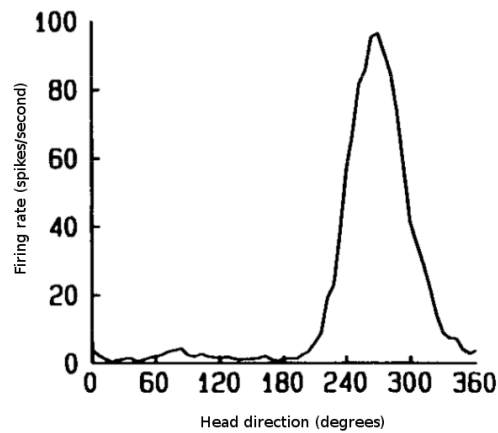


Figure 2.1: Shows the variation in firing rate of a head direction cell recorded from a rat as it moved around in a controlled environment [30]. The preferred direction of this head direction cell is $\approx 270^\circ$ and the tuning width is $\approx 150^\circ$. The peak firing rate is ≈ 100 spikes/second.

almost the same [24], Gaussian models have also been used to describe and approximate head cell firing [31, 32, 33]. An ensemble of head cells fires in a manner such that the entire 360° directional space is covered. It is noteworthy that the peak firing rates of the cells in the ensemble are not the same [24]. While the ensemble covers the 360° directional space, the range in which each cell fired, termed the “directional firing range” of a cell, was also found to vary [24].

Head direction cells in brain regions

Until now, head direction cells have been discovered in several areas of the brain (Figure 2.2): in components of the limbic system: the postsubiculum [21], the anterior dorsal nucleus of the anterior thalamus [34], the dorsal sector of the caudal lateral dorsal thalamic nucleus [35], dorsal tegmental nucleus [36], anteroventral thalamic nucleus [37], areas of the retrosplenial cortex [38], portions of the extra-striate cortex [38], lateral mammillary nuclei [39], parasubiculum [40] and layers III-VI of the dorsal medial entorhinal cortex (dMEC) [41]. Other than these components of the limbic system, many of which are also present in the classical Papez circuit, head direction cells have also been located in the dorsal striatum [42, 43]

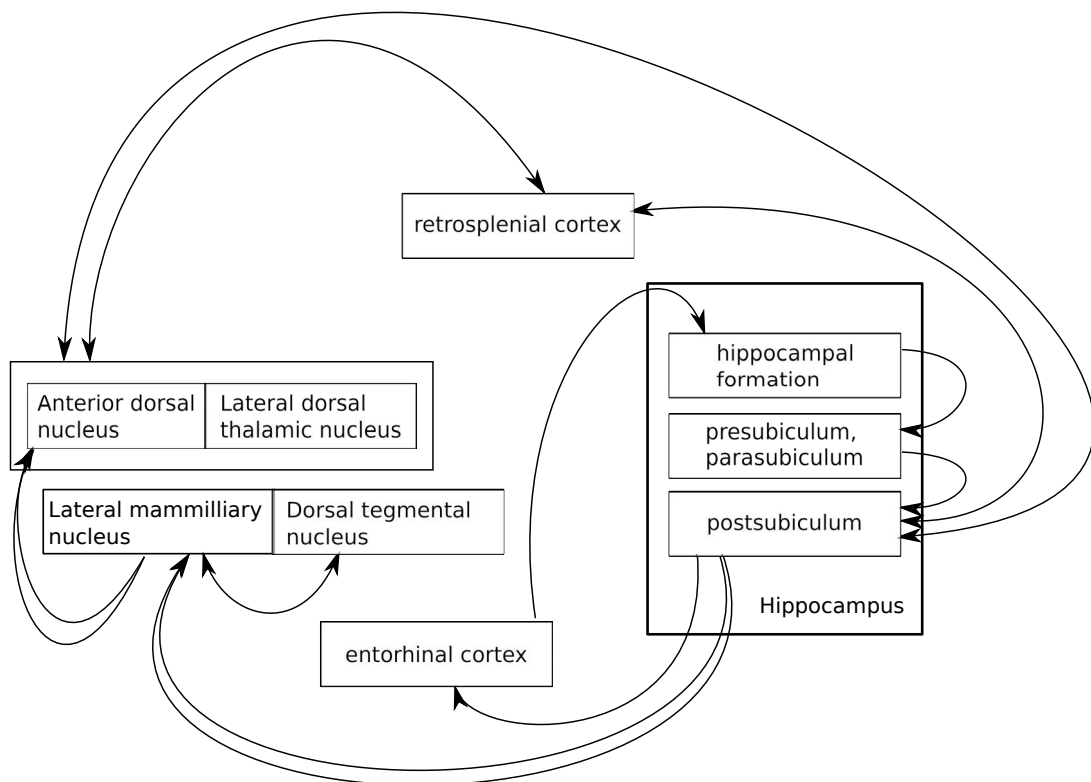


Figure 2.2: The figure shows the neural circuitry of brain regions containing and related to head direction cells. (The striatum has not been included)

and the medial precentral nucleus [44].

Head direction cells in various brain areas, by definition, display the same directional firing properties: their firing rates are a function of the agent's bearing and are maximum at their individual "preferred directions"; their firing rates decrease symmetrically around the maxima as the head direction changes from the "preferred direction" of the neuron; they have well defined "tuning widths". However, even though these fundamental properties are shared between all head direction cells, the parameters associated with them fall in a range of values for a particular brain region. For different brain regions, some of these ranges have also been observed to vary. As an example, peak firing rates range from 5-120 spikes/second and while this signifies variability within neurons of an area, each area exhibits a similar range. Tuning width, on the other hand, displays steady variations between some regions: head direction cells in the lateral mammillary nucleus and dorsal tegmental nucleus have wider tuning curve

widths than the other regions.

Similarly, the effect of various inputs on the functioning of head direction cells also varies slightly from region to region. On the whole, head direction cells process a combination of idiothetic (self-centred) and allothetic (not self-centred) inputs. Idiothetic inputs comprise of angular velocity projections from the vestibular system, optic flow information from the vision system, and other self-motion cues such as motor, kinesthetic and proprioceptive cues [45, 46]. Allothetic cues, on the other hand, are mainly landmark cues from vision system and olfactory inputs. As an example of variations in the effects of inputs, consider: it was observed that some head direction cells in the lateral dorsal thalamus region only fired in the presence of light in the room, and not when the room was dark [35]. However, for head direction cells in the other areas of the brain, a darkened environment did not cause noticeable changes in the firing rate of the head direction cells.

Dynamics of head direction cells

Head direction cell sets exhibit both path integrator [47] and landmark navigation [48, 49] capabilities. Since they integrate angular velocity inputs to “calculate” the current head direction, they function like an inertial navigation system (INS) [50]. An inertial navigation system, or a dead reckoning system as it’s called, necessarily suffers from drift. Therefore, the head direction cell set must have other inputs that help rectify this drift. Experiments carried out by researchers [51, 52, 53, 54, 55, 56, 57] to ascertain the effects of environmental cues showed that head direction cells are capable of associating with salient landmark cues in the environment. Experiments where idiothetic and allothetic cues were made to conflict demonstrate that salient visual landmark cues are capable of overriding information provided by the idiothetic cues.

The path integrator component of head direction cells processes certain idiothetic inputs - angular velocity inputs from the vestibular system is a primary input. The quicker the animal turns, the quicker the head direction cell set will change its firing rate profile. Also, the quicker the animal turns, the more the head direction cells will fire as the peak of the firing

rate profile passes them. It has also been found that the firing of head direction cells also depend on translation of the agent - translation increases the firing rate of head direction cells in general [30, 54]. Another idiothetic input, optic flow stimulus, also updates the activity of head direction cells in the anterodorsal thalamus [58].

Many experiments have been carried out to ascertain the extent to which visual cues control the behaviour of head direction cells. Taube and colleagues reported that replacement of specific visual cues, such as a cue card spanning 100° of their circular experimental apparatus, to a new location caused an almost equal change in the preferred directions of head direction cells. Thus, head direction cells associate with salient visual cues and therefore represent a more abstract relationship with the environment, than just being an internal compass [59, 60, 18, 52, 61]. Visual cues, however, do not affect the general discharge properties of head direction cells - they only affect the preferred or reference directions of head direction cells. The geometry of the environment also exerts control of head direction cells. However, this geometric control can be overridden by salient distal visual landmarks. The environment boundaries may be acting as landmarks themselves in the absence of other discernible visual features [59].

Other researchers also carried out experiments using cylindrical compartments where they rotated visual cues on the walls at different speeds to ascertain the contribution of visual and vestibular cues. For example, Blair and Sharp [55] varied the following parameters - rotation of platform base at slow and fast speeds; rotation of compartment wall at slow and fast speeds; lights on or off. For example, rotating both the platform and the wall at fast speeds by an angle of 90°, they tested the effect of only vestibular cues on head direction cell firing. This configuration also provided them with data on conflicts between vestibular input and optic flow input, since the optic flow input did not indicate any movement. Other configurations provided data in a similar manner. They reported that while vestibular cues correctly lead the head direction cell set to track current direction, visual landmarks controlled the preferred directions of head direction cells. Since their cylinder was visually symmetrical, some of the experiments warrant more experiments for confirmation. They also

reported that most experiments in a dark environment appeared to simply disorient the rat - they produced a range of responses. However, they did observe, that even in the dark, the head direction cell did make use of vestibular cues to maintain a reasonably accurate bearing representation.

Predictive firing of head direction cells

An interesting property of head direction cells found in the anterior dorsal nucleus, the lateral mammillary nucleus and the retrosplenial cortex is that they predict head direction rather than simply encoding current head direction [62, 63] - a time displacement of about 40 msec was observed in head direction cells of the anterior dorsal nucleus. This suggests that head direction cells in the anterior dorsal nucleus integrate self motion information as path integrators to calculate future heading and pass this information on to the postsubicular head direction cells which, as a result, encode current heading. The discovery of head direction cells that signal future heading also helped confirm the presence of a path integration component in head direction cells, since future heading could only be calculated from movement related information [64].

Contribution in complete neural navigation circuit

The role of head direction cells in the complete biological navigation circuit is becoming clearer as more information on their interaction with other navigation related neurons is exposed. Primarily, head direction cells encode direction information and project this to other neuron sets related to navigation. For example, head direction cells project direction information on to grid cells[65]. Grid cells utilize this information along with speed information for their own path integration purposes. In fact, head direction cells have been found to co-exist with with grid cells in some layers of the entorhinal cortex. Some neurons in the entorhinal cortex also exhibit properties of both head direction and grid cells and are called "conjugate cells" or "conjunctive cells" [41]. Head direction cells are also closely related to place cells [18]. A recent study on the contribution of otolith signals to spatial learning further confirms that the head direction signal contributes

to spatial learning [66]. We discuss the interaction of head direction cells, grid cells and place cells in detail in chapter 5.

Head direction cells are one of the better researched navigation related neurons. The abundance of information permits researchers to model biological navigation circuits for research purposes and sometimes implement bio-mimetic systems for use on robots. In the next section, we discuss the computational models of head direction cells that have been proposed.

2.1.2 Computational modelling review

Computational modelling of neuron sets is a research area in itself. Broadly speaking, it has two goals. First, to propose how information processing occurs in neuron sets that have been discovered by experimental neuroscientists. This can either be at a higher, computational level, where simpler firing rate models can be used to study the flow of information or, at a much lower molecular level where researchers attempt to model neurons sets in detail. Simulators such as NEURON [67, 68, 69] can be used to model neuron sets at a neural level. The “blue brain” project is another example of low level modelling of neuron sets: it employs the BlueGene/L supercomputer in a bid to construct a virtual brain from the information on various neuron sets that is collated by researchers [70]. The second goal is to mimic neurological systems to implement bio-mimetic systems in robots. This goal is more difficult to achieve at present, due to the large computational costs involved in the massively parallel neural processing system. Research in the field of neuromorphic engineering will help with this goal in the future. It is notable, that as a result of the large computational costs involved in neural processing, while quite a few bio-mimetic systems have been implemented [3, 4], a majority of these mimic only the behavioural characteristics, not the neural processes that are responsible for them. That is, they are biomimetic but not neuromimetic.

From a navigation perspective, head direction cells are capable of contributing to both path integration and landmark based navigation. Integration of idiothetic inputs computes the current, or as is the case of some

regions, future head direction. A path integrator system necessarily suffers from drift: a minute bounded error exists at the start of the computation and is compounded with each successive integration, causing the computed value to drift more and more from the actual value [50]. In order to restrict this drift to a usable value, the system must be reset from time to time to a known accurate value. In autonomous navigations systems, INS are generally deployed in conjunction with another navigation system, such as a visual beacon system [71] or a global positioning system (GPS) [72] that provide accurate data to restart the integration process from time to time. Similarly, in the head direction system, the landmark system can reset the head direction from time to time, limiting the drift. On the flip side, the landmark navigation system cannot form a complete navigation system by itself either. If the system is unable to find enough salient features at frequent intervals to use as landmarks, the agent will be disoriented. In such situations, the path integration system works to continue tracking the head direction. These two systems are, therefore, complementary.

In the following sections, we discuss the computational models that have been put forward by researchers in the past. We discuss them in a chronological order, stating how each model improved upon a previous model so that we present an evolutionary view of head direction cell models. We use the following criteria to review the models, similar to one set by Blair and Sharp [73]:

- The extent to which the model replicates the basic directional firing properties of head direction cells.
- The architecture used in the model: is it coherent with the information known about the anatomy and physiology of modelled regions?
- Does the model attempt to simulate both the path integrator and landmark navigation functions of head direction cells?
- Is the model biological plausible?
- Was the model validated by simulations/implementations?

Look up table

At the time of discovery of head direction cells, the information that was available covered only their directional firing characteristics. McNaughton et al. [74] made one of the earlier hypothesis on how head direction cell sets may function. Their proposal, while based loosely on path integration, used association rather than computation: instead of computing the head direction from idiothetic inputs, McNaughton and colleagues suggested that a “look up table” system carried out the required function. The set of cells could be trained to store the mapping between an input set, consisting of the current heading and angular velocity, and the output, the required new heading. Formally, in this theory, a set of head direction cells would be trained to store the mapping

$$f : (\theta \times \omega) \rightarrow \theta$$

where, θ is the set of all directions and ω is the set of all possible angular velocity values, such that at any instant of time, the current head direction θ_t and current angular velocity ω_t would give the value of the next head direction θ_{t+1} . However, one must note that in order to implement a head direction system based on this theory of look up tables, the input set must consist of all possible directions, and every possible value of angular velocity that the agent may encounter, to be complete [75]. It is also noteworthy that this system would employ a recurrent artificial network, since the current head direction is required to compute the next head direction value. The paper also addressed the issue of limiting cumulative errors in this inertial direction system by proposing the presence of “local view” cells that detect salient landmarks. Associations between head direction cells and these view cells would be sufficient to anchor preferred directions to salient landmarks, which in turn would be used to reset the system’s state when errors aggregate. The system, while functional, is improbable. The discovery of attractor characteristics in regions that head direction cells were previously found in further shifted researchers’ attention towards a neural integrator rather than a look up table.

The proposal was not accompanied by simulations or discussion on how it might actually be implemented. It did not discuss the variations in

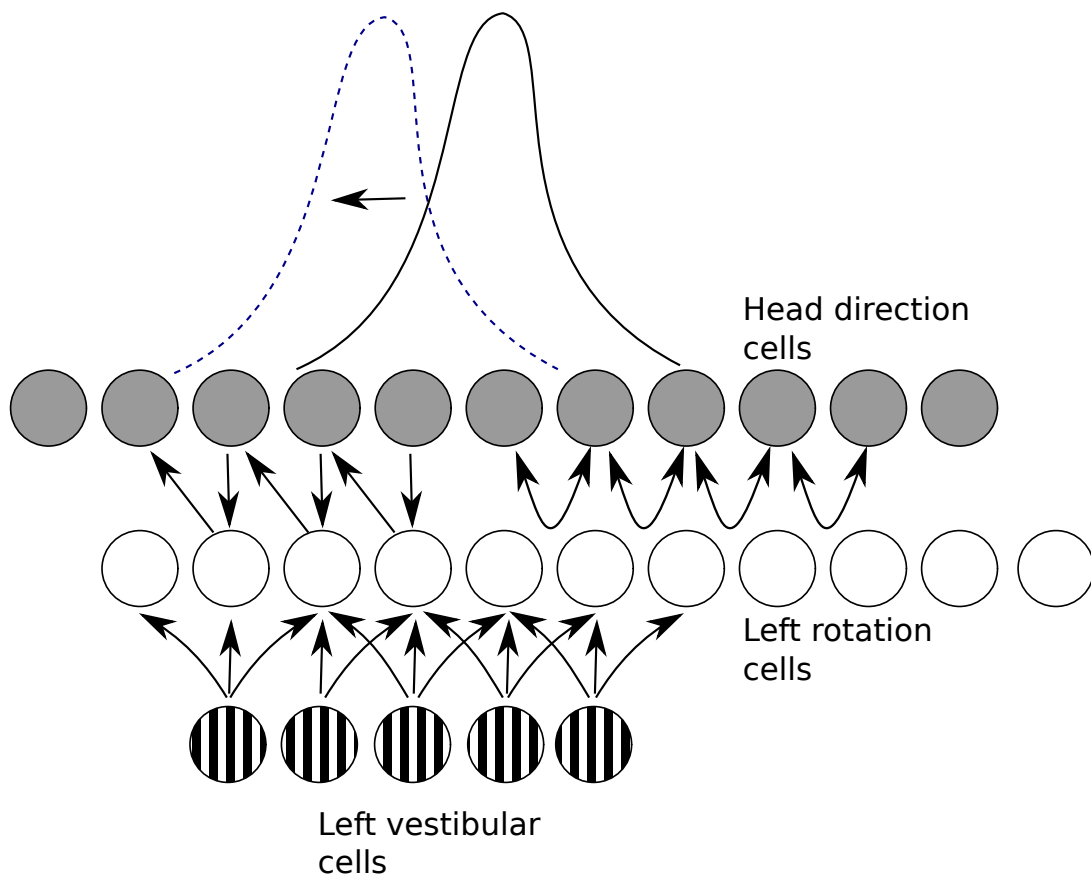


Figure 2.3: Projections of left rotation cells causing the activity packet to shift in the model proposed by Skaggs and colleagues.

the properties of head direction cells in different regions, many of which were not yet known at this time.

Neural integrator and attractor dynamics

Skaggs et al. [76, 77] focussed on the architecture of the head direction cell system that would enable it to carry out the neural processing required to compute the current head direction. Their model, while also employing a path integrator method, proposed a neural integrator instead of the look up table used by McNaughton and colleagues before.

The proposal consisted of three sets of neurons: left and right rotation cells and head direction cells. The head direction cells had stronger recurrent synaptic connections that were set up to ensure that only a localised clump of cells would be active at any time, as the stable state of the net-

work. This would result in a firing pattern with the most active head direction cell in the cluster denoting the current head direction. These attractor characteristics [78] were imposed on the network by using strong excitatory synaptic connections amongst neighbouring neurons, and inhibitory connections between neurons that were further apart. The recurrent connections between head direction cells maintained an activity packet once it had formed. The rotation cells were connected to the head direction cells in such a way that their projections selectively excite neurons to the left or right of the current “activity packet”. These selective activations, combined with the recurrent activity of head direction cells caused the peak to shift at a rate proportional to the strength of the projections (Figure 2.3). Redish et al. termed this a “shift register” method, and, while it did use an attractor to limit the network to a single peak of activity, documented that it was not a pure attractor model [79].

The model also made use of “visual feature detectors” which would project on to the head direction cells. These were assumed to fire when a feature was detected at a certain angular displacement relative to the current head direction angle. Since various feature detector cells have been found in the visual cortex [80, 81], it was quite probable that such cells exist.

Skaggs et al. pointed out that the required synaptic set up in this model was quite specific. They observed that a method of self organization would improve the model. They also observed that the system could be used as a generic neuronal integrator. The model was the first proposal that employed attractor dynamics. The model predicted the presence of visual feature detectors [82] and rotation cells [39], both of which have since been discovered in the brain. The transcript, however, only presented a theoretical report of the model and no simulations or mathematical details were provided to enable researchers to implement or verify the model.

Anticipatory characteristics

Blair and Sharp discovered the predictive nature of head direction cells located in the anterior dorsal nucleus [62]. They simulated a “thalamo-

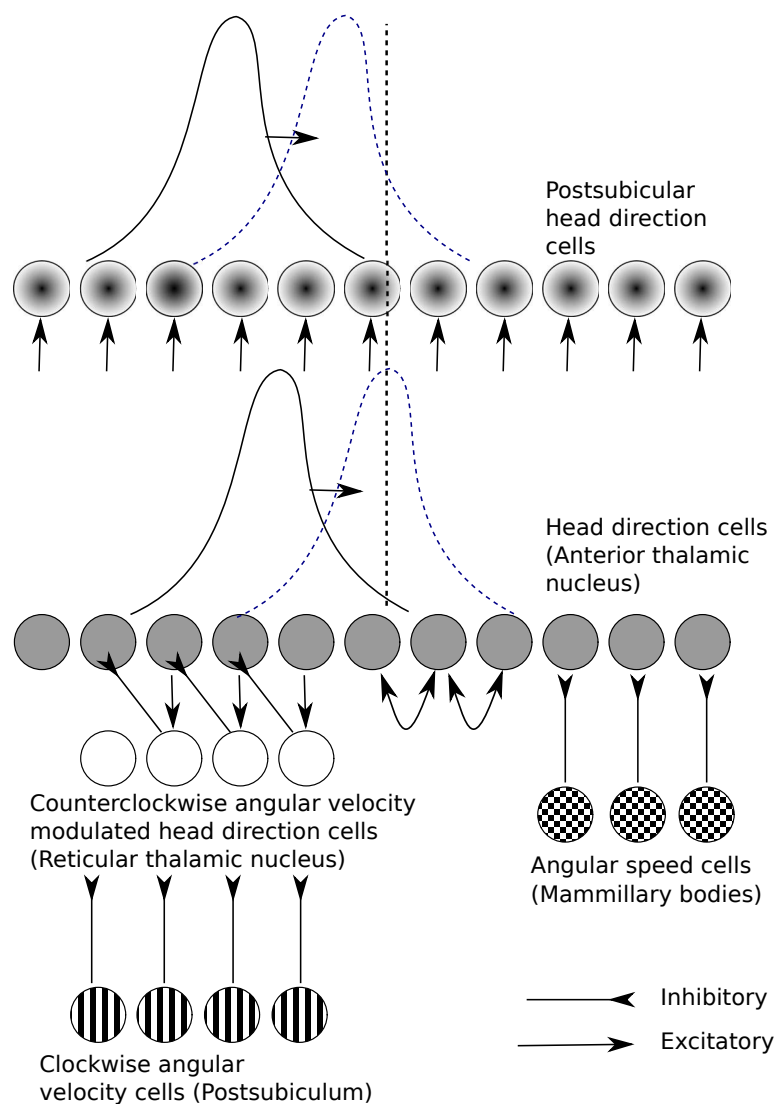


Figure 2.4: Schematic of Blair and Sharp's model. Note how the anterior thalamic nucleus leads the postsubiculum.

cortical circuit" [73, 64] on the NEURON simulator [69, 67, 68]. Similar to Skaggs et al. [76], they incorporated various neuron sets in their simulation to generate the observed characteristics of head direction cells.

The model was the first to incorporate head direction cells from regions of the brain with varying directional firing properties (Figure 2.4): a set of "present head direction (PHD)" cells represented cells of the postsubiculum that encode current head direction while "anticipatory head direction (AHD)" cells represented cells in the anterior thalamic nucleus that

encoded the future head direction. Inputs to these two sets of cells were served by groups of “angular velocity (AV)”, “angular speed (AS)” and “angular velocity modulated head direction (AVHD)” cells. Two sets of AV cells fired in a rotation specific manner, i.e., one fired during clockwise rotation and the other during counter clockwise rotation. Such cells have been observed in the postsubiculum [83] and retrosplenial cortex [84]. AS cells encoded the magnitude of rotation velocity, irrespective of the direction and have also been found in both the postsubiculum [83] and the retrosplenial cortex [84]. AVHD cells are rotation specific head direction cells, that fire for their specific preferred directions only when the head rotates in a certain course, either clockwise or counter clockwise. Such cells were reported by Taube et al. [30] in the postsubiculum. Similar to Skaggs et al., the synaptic connections between the different neuron groups were pre-defined as either excitatory or inhibitory, as required.

During a clockwise head turn, the clockwise angular velocity cells are activated and inhibit the layer of counter-clockwise angular velocity modulated head direction cells. As a result, the currently active anterior thalamic head direction cells stop inhibiting their right neighbours and activity spreads to the right through the layer of head direction cells. Because the anterior thalamic head direction cells continue to inhibit their neighbours to the left, activity is shut down in the leftward direction, in the wake of activity to the right.

The model itself successfully simulated some characteristics observed in head direction cells: AHD cells in the model that represented head direction cells in the anterior thalamus had higher firing rates than the PHD cells modelling head direction cells in the postsubiculum. AHD cells also showed a direct proportionality to the angular velocity while PHD cells did not. The PHD cells correctly tracked head direction with varying inputs and the AHD cells lead PHD cells by approximately 20 msec. Via the model, the authors predicted the presence of AS cells in the mammillary bodies, and of AVHD cells in the reticular thalamic nucleus which had only been found in the postsubiculum [30, 83] and retrosplenial cortex [85].

The manuscript documented some limitations of the model. First, the model does not incorporate landmark cues. Second, the model only worked with a small range of angular velocities. Third, the tuning curves for the

cells in the model did not match experimental data, in that, they were not as broad as cells in the rat. Last, the model relies on lateral connections amongst the AHD cells in the modelled anterior thalamus but such connections are yet to be discovered.

Synaptic modification

Zhang [86] put forward a modified attractor based model where he split the synaptic weights between head direction cells into two sets: even and odd synaptic connections. The even connections were constant, responsible for maintaining the head direction cell system's behaviour in the absence of any inputs. In such a scenario, the odd synaptic weights were absent. Zhang showed that under the influence of the even synaptic projections, the system could take two possible states: an unstable state where each cell was equally silent and a state where appropriately chosen parameters would result in the network displaying the head direction cell profile defined by a localized peak. The odd component, supplied by inputs such as the vestibular system, would cause a continuous shift of the peak while maintaining its shape. In his attempt to find a weight distribution to fit a chosen sigmoid activity profile, Zhang showed that it is improbable that such a solution exists. Instead, he settles for approximate solutions found by using regularization and Fourier transformations. His solutions were coherent with the expected "local excitation, global inhibition" conditions for attractor networks. By including the effect of acceleration in his system, Zhang showed that the model does indeed show predictive behaviour as described by Blair and Sharp [62]. Zhang also discusses how the presence of noise causes the formation of point attractors, which inherently destroy the neutral equilibrium required for a head direction cell network. The drift caused by such noise can be corrected by anchoring via other inputs, such as landmark detection and is also demonstrated.

Zhang discusses the limitations of his models, stating that while his model provides sufficient simulation of the basic properties of head direction cells, it does not cover all its properties. Biological observations, such as effects of restraint cannot be explained by Zhang's model [87]. The functioning of Zhang's model relies on quick synaptic changes, and

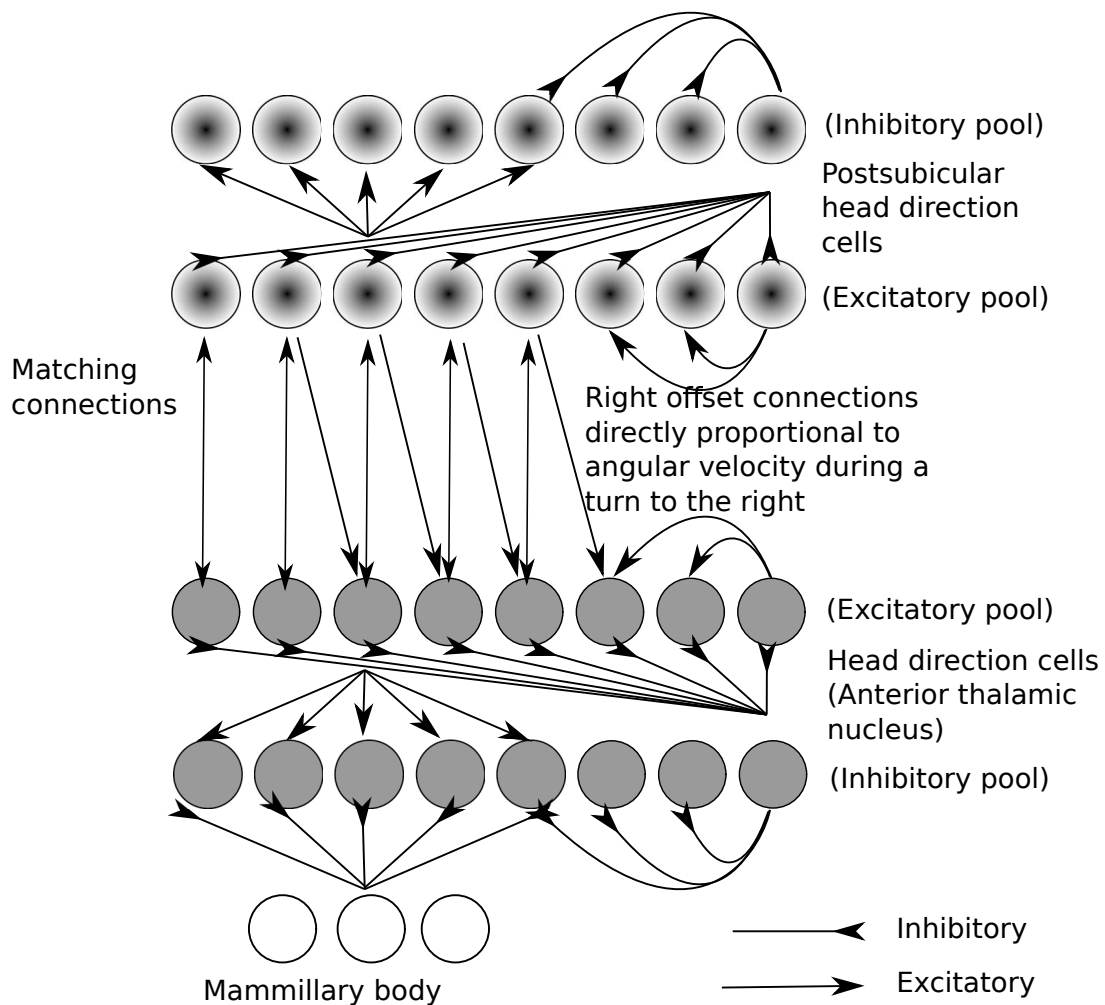


Figure 2.5: Schematic showing the behaviour of Redish et al.'s model during a right turn.

on units that have both excitatory and inhibitory properties, which makes it less biologically plausible.

Coupled attractor network

Redish et al. [79] put forward an attractor model that made use of two attractor systems to represent head direction cell sets in the postsubiculum and the anterior thalamic nucleus (Figure 2.5). Each coupled attractor module consisted of an excitatory and an inhibitory pool, the cells of which were interlinked to induce attractor dynamics. The inhibitory pool weakly inhibited neurons of both pools uniformly, while the excitatory

pool excited only neurons that were close neighbours of each other. A set of equations that define the action potential, the probability of firing and the “synaptic drive” described the spatial and temporal characteristics of the system. (A set of such equations has been shown to form a steady neuronal model by Pinto et al. [88]). In addition to the sets of connections between and within each pool, the model employed two other sets of connections between the excitatory pools of the two attractor sets: “matching” connections that serve to restrict the peaks in the postsubicular and anterior thalamic sets to their fixed locations while keeping them concomitant; and “offset” connections that were modulated by angular velocity inputs and served to move the activity packet to either side, in a manner similar to earlier discussed models. During a right head turn, for example, the right offset connections would be strengthened proportionally to the right head turn angular velocity and cause a shift in the peak towards the right. In the absence of a turn, the offset connections would all be zero, permitting the matching connections to synchronize the firing of the postsubicular head direction cells and the head direction cells of the anterior thalamic nucleus. Thus, the combination of the matching and offset connections ensured that the hill of the anterior thalamic attractor set will lead the hill of the postsubicular set during rotations and remain in sync when at rest. These connections are similar to the “even” and “odd” connections used by Zhang. An additional set of projections is made on to the excitatory pool of the leading attractor to maintain its shape. These projections are assumed to be sourced in the mammillary bodies, similar to Blair’s model [73].

The results obtained by the model were compared to data recorded by Blair and Sharp [64] from freely moving rats. The model successfully integrated rodent head movements and exhibited a anterior thalamic set of head direction cells that lead the postsubicular set. In their review [31] Taube et al. have pointed out that lesion studies have been able to confirm the interdependence of the head direction cells in the anterior thalamic nucleus and postsubiculum: information is projected both ways. The anterior dorsal nucleus is necessary for head direction cell activity in the postsubiculum. In the opposite direction, the postsubicular head direction cells appear to have some control over the extent to which head direction

cells in the anterior dorsal nucleus associate their preferred directions to salient visual landmarks [89] . Thus, the modelling of these connections is somewhat incomplete. The model was limited to vestibular inputs only, and made no attempt to incorporate or discuss how landmark navigation occurs. The model also did not discuss how the inhibitory or excitatory connections would be set up in a biologically plausible manner i.e., these connections were fixed manually. Similar to Zhang's model, this model also relied on quick synaptic variations and is also less biologically plausible. The absence of inhibitive GABA [90] containing interneurons in the anterior thalamic nucleus also invalidates the use of an attractor for the predictive head direction cells in anterior thalamic nucleus in this model.

Self organized synaptic connections

Stringer et al. [75] proposed a model that employed a biologically plausible method to ascertain the synaptic weights in an attractor model of head direction cells. In the previous models summarized above, the synaptic weights between the various cell ensembles were derived by other, undocumented means, usually by repeated iterations of inspection and modification. This lacked biological plausibility, i.e., there was no discussion of how the synaptic weights, which are the basis of these models, were to be ascertained. In their manuscript, Stringer et al. proposed a method of setting up synaptic weights in the head direction cell system via Hebbian learning [91]. Only the path integrator system of head direction cells was discussed, though. Also, only head direction cells encoding current head direction were considered, not head direction cells with anticipatory firing or any other related neuron sets. The model provided clear cut details of both the mathematical foundations employed and the results that validated its functioning. Self organization of synaptic weights is an important step towards a complete biological navigation system including head direction and other neuron sets. Stringer et al. also extended their work to a two dimensional model of place cells based on the same concepts [92].

Variants of the self organizing model

The self organizing model, because of its biological plausibility and relative simplicity has been used as the base for a few other models. The major advantage of this model over others is that there is a documented method to train the synaptic connections of the network. We discuss some of its variants here:

Incorporation of kinesthetic and visual input: Kyriacou [93] built on the self organization driven head direction cell model proposed by Stringer et al. [75] by extending it to also use kinesthetic and visual inputs. The modified model was governed by the following equation:

$$\begin{aligned}
 \tau \frac{dh_i^{HD}(t)}{dt} = & -h_i^{HD}(t) + \frac{\phi_0}{C_{HD}} \sum_j ((w_{ij}^{HD} - w^{INH})r_j^{HD}(t)) \\
 & + \frac{\phi_1}{C_{HD \times ROT}} \sum_{jk} (w_{ijk}^{HD-ROT} r_j^{HD}(t)r_k^{ROT}(t)) \\
 & + \frac{\phi_2}{C_{HD \times KIN}} \sum_{jk} (w_{ijk}^{HD-KIN} r_j^{HD}(t)r_k^{KIN}(t)) \\
 & + I_i^{VIS}(t)
 \end{aligned} \tag{2.1}$$

Here, the r_k^{KIN} and w_{ijk}^{HD-KIN} terms are additions to the original model and represent the firing rate of the kinesthetic inputs and the synaptic weight between head direction cells and kinesthetic inputs respectively. The model was implemented and tested on a LEGO® robot. Kyriacou further used three test cases to observe the contribution of various inputs, and how they conflicted when not synchronized. The model correctly reproduced head direction cell behaviour where a drift incurred in the absence of visual input is corrected when visual input is reapplied. A test case was aimed at observing the effect of external force (by removing kinesthetic input but letting vestibular inputs continue to affect the head direction cell packet). It suggested that the presence of kinesthetic input incurred less drift than its absence did, suggesting that kinesthetic input did indeed affect the head direction system. A third test case was designed to simulate conflicts between visual and vestibular input. This test case also correctly showed that the head direction cell system was

dominated by visual input but could be driven by vestibular inputs in its absence, albeit with error accumulation.

The utilization of visual input in this model, while sufficient enough to simulate inputs in the model, is incomplete. The visual input in this model did not anchor to visual landmarks, as is supposed in head direction cells. Rather, visual inputs were stimulated in an abstract way by using an omnidirectional image to calculate current orientation: an initial image defined the initial state of the continuous attractor network and subsequently, the direction of the robot was calculated by finding the difference between the video image at a time and this initial image. There were no synaptic connections between the head direction cell network and the visual cells.

Models based on the initial version of the model proposed by Stringer et al. [75] use a method of trial and error to fine tune the required parameters. As in, they iteratively fine tuned the parameters of the system till it exhibited the properties they expected. In a subsequent work, Kyriacou proposed the use of an evolutionary algorithm [94] to ascertain these parameters [95]. Kyriacou employed a self adaptive $(\mu + \alpha)$ -ES evolutionary strategy [94]. The basic idea is to define a function that describes the error between the biologically expected and actual response of the head direction model with sets of parameter values that are to be ascertained. Minimising this function, then, yields optimal values of the parameters. In his implementation, Kyriacou used a mutative rule suggested by Bayer and Schwefel [94].

Other models

Other than the above discussed models, there have been a few more models that attempt to simulate the head direction cell system.

Arleo and Gerstner proposed a model that captures the functional properties [96, 97, 98] of head direction cells. This model was implemented and verified on a Khepera robot. The model makes use of both idiothetic and allothetic inputs. The allothetic inputs are visual inputs which provide the robot with a bearing to a static light source. This visual input is used to recalibrate the robot's head direction when needed. The synaptic associations between various parts of the network seem to have been

fixed manually as covered in [97]. The model predicted that lesions to the postsubicular head direction cells would make head direction cells in the anterior dorsal nucleus and the lateral mammillary nuclei unresponsive to external cues. A similar relationship has been confirmed by Goodridge and colleagues [89]. The paper did not discuss how the synaptic weights were calibrated, and only made use of head direction cells that encode present direction. The model was also extended to consist of place cells in an attempt to implement a neuromimetic path integrator navigation system.

Song and Wang proposed a spiking neuron model [99]. Citing a lack of data to support the presence of recurrent excitatory connections in the head direction cell ensemble, Song and Wang showed that a continuous attractor network could be realized without recurrent excitation. By employing organized inhibitive and excitatory connections, they successfully reproduced known properties of head direction cell ensembles: persistence of activity in absence of input; translation of peak when angular velocity was applied; abrupt re-formation of activity packet at a new preferred direction in the presence of a strong enough landmark stimulus.

Degrís et al. [100] implemented a spiking neuron model based on Song and Wang's work [99] on a mobile Pekee robot. Similar to Song and Wang's model, the synaptic weights in this model were pre-determined. Degrís et al. employed genetic algorithms to ascertain values of required parameters, such as maximum synaptic weight and Gaussian width. The system successfully integrated angular velocity to track head direction. Since the authors did not include a calibrating mechanism, the head system did incur some drift over a period of time, as expected. The authors also tested the effects of reorienting stimuli by simulating an external stimulus. This external stimulus did cause a reset of preferred directions as dictated by the external stimulus. The goal of the research was to implement a bio-mimetic allocentric direction representation in the Psikharpax robot project [101, 102].

Goodridge and Touretzky [103] put forward a model where they included both the postsubiculum and the anterior dorsal nucleus. In their model however, they omitted an attractor network in the anterior dorsal nucleus in coherence with observations that GABA containing interneu-

rons were absent in this region. In their model, the anterior dorsal nucleus was driven by the lateral mammillary nucleus. The absence of an attractor network permitted the firing graphs to show shape distortions as described by Blair and Sharp [62].

Zeidman and Bullinaria [104] extended Goodridge and Touretzky's model [103] by including optical flow information into the model. They employed feature tracking to calculate movement vectors for salient visual features to generate visual flow information into the model.

2.2 Grid cells

The discovery of neurons that encode spatial information in the mammalian brain - place cells [7], and then head direction cells [21], resulted in the formation of the cognitive map theory of animal navigation[6]. However, in spite of these discoveries, researchers agreed that efficient navigation is more dependent on metric information. Therefore, the focus of research moved further upstream from the hippocampus on to the entorhinal cortex which provides major cortical inputs to the hippocampus - grid cells were discovered in the region by Hafting et al. in 2005 [22].

2.2.1 Neuroscience review

This section summarizes the information researchers have collected over the years on grid cells. Grid cells are neurons that exhibit regular firing fields such that these firing fields form a triangular or hexagonal pattern tiling the environment available to the animal. The firing of grid cell networks is incredibly accurate - Hafting et al. document that the spatial tuning pattern remains sharp and stable over paths accumulated over tens of minutes and with a total length of hundreds of meters.

Discharge properties

Single units of the dorsal medial entorhinal cortex (dMEC) have multiple discreet fields of similar amplitude, unlike place and head direction cells which have single confined fields. The interaction, or overlap, of the individual fields results in the formation of a regular triangular "grid" (hence the name: grid cells). In order to confirm the regularity of the mesh, Hafting et al. used spatial autocorrelation analysis. The analysis affirmed that the unit of this mesh was an equilateral triangle, and that the central peak of this unit was surrounded by six others at nearly equal distances, at angular separations of nearly 60° each. Figure 2.6 shows diagrams depicting the mesh.

Grid neurons show a systematic topographic organization in the dMEC - metric characteristics of neurons recorded at the same electrode were almost same - spacing, orientation specificity and field size. Spacing, for

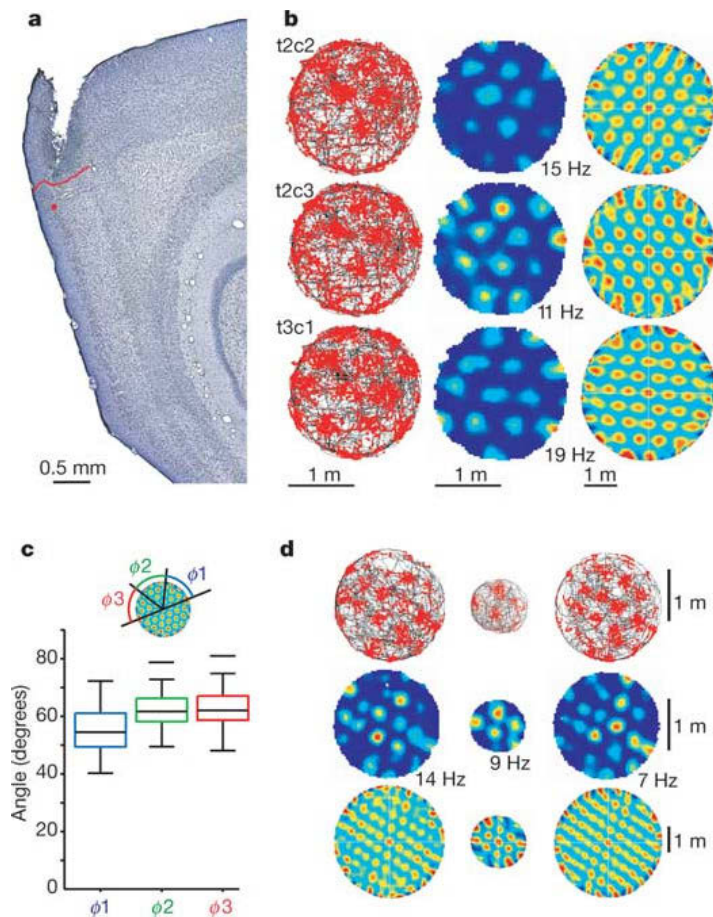


Figure 2.6: **a**, Sagittal Nissl-stained section indicating the recording location (red dot) in layer II of the dMEC. Red line indicates border to postrhinal cortex. **b**, Firing fields of three simultaneously recorded cells at the dot in **a** during 30 min of running in a large circular enclosure. Cell names refer to tetrode (t) and cell (c). Left, trajectory of the rat (black) with superimposed spike locations (red). Middle, colour-coded rate map with the peak rate indicated. Red is maximum, dark blue is zero. Right, spatial autocorrelation for each rate map. The colour scale is from blue ($r = 0.21$) through green ($r = 0$) to red ($r = 0.1$). **c**, Box plot showing distribution of angles ϕ_1 , ϕ_2 and ϕ_3 between the central peak of the autocorrelogram and the vertices of a hexagon defined by the nearest six peaks. The diagram shows median angles (horizontal lines inside boxes), interquartile distances (boxes), upper and lower limits, and outliers (horizontal lines). **d**, Discharge maps (as in **b**) showing similar triangular structure in enclosures of different size (left, large; middle, small; right, large). (Image taken from Hafting et al., 2005 [22])

example, was characterized by a standard deviation of only 2.1 cm, while orientation was characterized by a standard deviation of only 1.8° at a fixed electrode. Over the entire population of grid cells, however, spacing had a standard deviation of 30 cm [22].

Grid cells in brain regions

As documented in the previous sub-section, the discharge properties of grid neurons at the same recording point were largely similar. These properties varied as the region that was being recorded from was varied, however. As the recording sites were chosen further away from the postrhinal border, spacing and field size both were found to increase. The spacing, as an example, was more compact at dorsal recording sites than ventral ones. The researchers document that they were unable to detect any well defined change between dorsal and ventral sites in orientation specificity of neurons, however.

Further recordings from the different principal layers of the MEC showed the presence of different neuron sets in each layer - while layer II was dominated by grid neurons, the deeper layers III to VI contained grid cells along with direction specific grid cells and even head direction cells [41]. As with head direction cells of found in the hippocampal regions, the entire range of head directions was represented in these layers of the MEC, and the properties also varied within each region.

Dynamics of grid cells

Similar to head direction cells, grid cells have been found to illustrate both path integrator and landmark navigational paradigms. It is likely that the population of grid, head direction and direction specific grid cells may integrate speed, direction and position inputs to update the current position as encoded by the grid cell ensemble. Regression analysis has confirmed that there is a positive relationship between speed of the rat and the firing rates of all grid cells [41].

The stability of grid vertices across multiple trials in the same environmental set up suggest that the allothetic cues should exert some control over the functioning of grid cells. Cue card rotation experiments, similar

to ones done with head direction neurons, were done with grid cells. Rotation of the cue card by 90° caused a similar rotation in grid cells. There was no effect on the grid spacing or field size of the grid cells. This suggests that orientation and phase are both controlled by external visual landmarks, similar to how the preferred directions of head direction neurons [22]. Sensory information related to the environment could therefore be used for setting the initial parameters of the grid, or to correct accumulated drift errors in the path-integration system. This is similar to the effect of landmark cues on head direction cells.

In order to analyse the extent to which allothetic cues affect grid cell functioning, experiments were also carried out in a darkened environment, so as to nullify any visual landmarks. It was found that the triangular grid was maintained in darkness suggesting that allothetic cues did not control the grid like firing behaviour of the neuron sets.

Since the grid mesh covered the entire extent of multiple environments of different sizes, it conveys that the grid mesh may be continuous and extend to an infinite size [105].

2.2.2 Computational modelling review

Computation models of grid cells fall into two broad categories: oscillatory interference models and attractor network models [106]. Since my work is based only on attractor networks, I only present a review of this particular category of models.

Attractor based models of head direction neuron sets are based on a one dimensional attractor network [78]. Attractor models of grid cells, that encapsulate the same path integrator mechanism are, therefore, modelled as two dimensional attractor networks - this concept was first proposed as the underlying system for place cells [107]. While models of head direction cells evolved over decades from simple association based models, to shift registers, and finally to attractor models as discussed in section 2.1.2, researchers modelling grid cells were able to take advantage of the work already done by other researchers modelling attractor models of neuron sets such as head direction cells [75] and place cells [92]. The history of computational modelling of grid cells is therefore, shorter than

that of head direction and place cells.

The earliest attractor model of grid cell was proposed by Fuhs and Touretzky [108]. An attractor depends on a correct organization of synaptic weights between neurons. In this model, the authors, inspired by Zhang's model of head direction cells [86], employed synaptic weights which consisted of two components - a symmetric and an asymmetric component. The model, therefore, suffers from the same biological implausibility - it requires rapid synaptic changes in order to correctly function. The authors provided some insights into how a grid cell network may be developed. The model made use of a Hebbian rule similar to the Biensstock-Cooper-Munro (BCM) learning rule [109] in order to configure connections between the dMEC units. Figure 2.7 shows a central unit in their sheet of grid neurons. The model demonstrated that multiple unrelated networks could set up a representation of space over a larger domain as compared to a single network - this is coherent with the observation that while the size and spacing of fields is similar in neurons close to each other, it varies along the dorsoventral axis. The network was only tested with limited trajectories and did not scale up to larger ones. Another issue the model didn't address was the edge effect present in attractor networks. The authors did document that using a torus shaped attractor would correct this, in line with the circular one dimensional attractor used for modelling head direction cells [110](Figure 2.8.

Guanella et al. improved the initial model by proposing one that used a twisted torus (Figures 2.9 and 2.10) which would have a surface with a regular triangular mesh [112]. They also made use of asymmetric synaptic weights to move the bump of activity around - these synaptic weights were also modulated by velocity of the animal as in the previous model. They also made use of asymmetric synaptic weights to move the bump of activity around - these synaptic weights were also modulated by velocity of the animal as in the previous model. The model also included a sheet of place cells as inputs to the grid cells. The connections between these two sheets were set up by Hebbian learning and served to reset the activity of grid cells when a drift was incurred.

Burak and Fiete proposed a model that used feed forward excitation to move the packet of activity in an attractor to represent the rat's move-

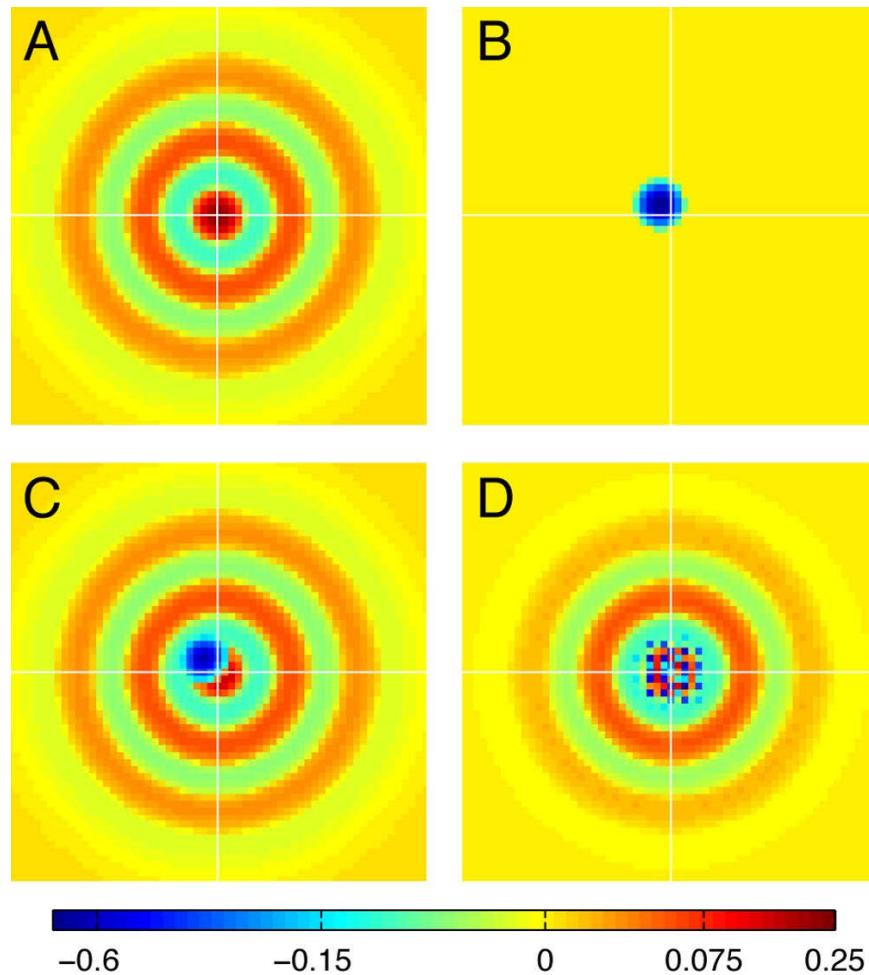


Figure 2.7: Recurrent weight matrix W_{ij} contains symmetric and asymmetric components. Shown here are the weights for the central unit in the sheet. **A**, Symmetric component contains angular rings of excitation. **B**, Asymmetric component contains a ring of inhibition, offset slightly from the centre, opposite the preferred direction ϕ_i of the unit. **C**, The output weights of a unit (a column of W_{ij}) are the sum of the weights in **A** and **B**. **D**, The input weights for the unit (a row of W_{ij}) are approximately symmetrical; the “noise” reflects the variation in preferred directions of the afferent cells. All weights have been raised to the 0.5 power in these plots to better reveal the structure in both components, which differ in magnitude by a factor of 3. (Image taken from Fuhs and Touretzky [108])

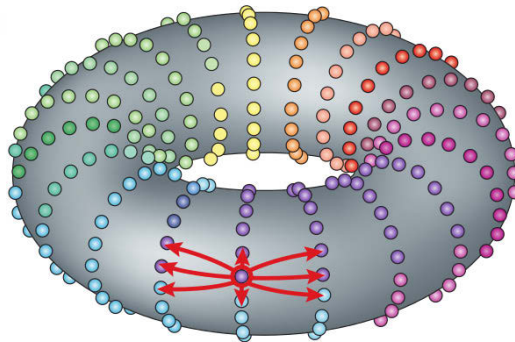


Figure 2.8: McNaughton et al.'s proposal to correct the edge problem. Figure taken from McNaughton et al. [111]

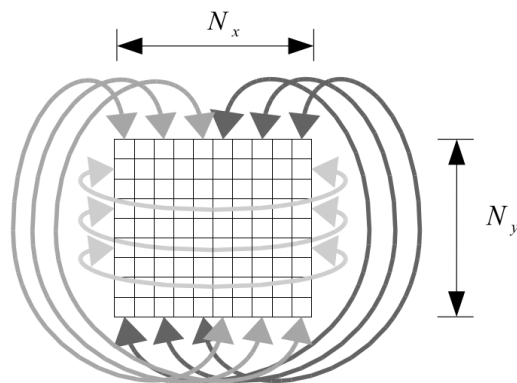


Figure 2.9: Connection of grid neurons that give rise to a torus

ment [113]. In their model, each neuron received inhibitory input from a surrounding ring of local neurons, but the entire network received excitatory projections which are modulated by the velocity and direction of movement. The dynamics of the network were based on asymmetric connectivity, such that, it would drive the activity in a specific direction. When the entire network receives the same input, such asymmetric connections will tend to equalize each other out, causing the packet of activity to remain stable at its location. The idea in use here is not completely different from the ones used in early head direction neuron models [76, 114]. The paper included models with both a periodic and aperiodic boundaries - the aperiodic model required fine tuning to perform as well as the periodic implementation. The authors presented a detailed analysis of both the periodic and aperiodic models and made certain predictions along with tests that could help confirm these predictions - for example, they

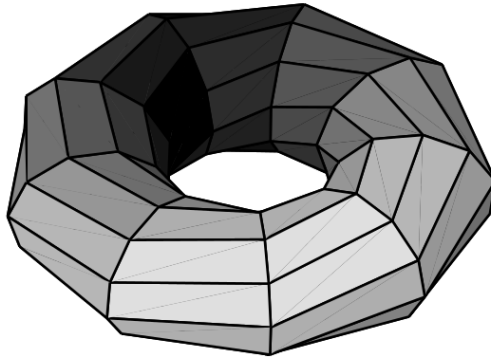


Figure 2.10: The resulting twisted torus. Images taken from Guanella et al. [112]

document that the rotation of the single neuron response which occurs in cue card experiments is predicted to be caused by an isotropic rotation of the head direction inputs to the network, while the grid neuron pattern is predicted to remain the same. They also documented that the use of an attractor model adds limitations to the rotation and stretching of the population pattern - any evidence of in the phase relationships between neurons, or no changes in the velocity modulation of head direction inputs to the dMEC, when the single neuron firing appears to be stretched, would be evidence against an attractor based model. One noteworthy aspect of the model is that the grid neurons used here are all direction specific, which implies that this model makes use of head direction neurons for modulation of conjunctive grid neurons that have been discovered. The model does not attempt to explain the existence of pure grid neurons.

McNaughton et al. [111] suggested that the combination of grid cells networks with variable spacing would also result in a maxima (Figure 2.11). They also discussed the interaction of grid cells that would lead to place fields in place cells. Further, they discussed how the formation of grid cells might happen using a theoretical proposal made by Alan Turing in 1952 which was based on the difference in diffusion rates of inhibitors and activators.

Navratilova et al. [115] proposed a simplified one dimensional model of grid cells function. The goal of their model was to simulate phase precession and the change in scale of grid cells along the dorso-ventral axis of the MEC. In their model, grid cells and conjunctive, direction specific grid

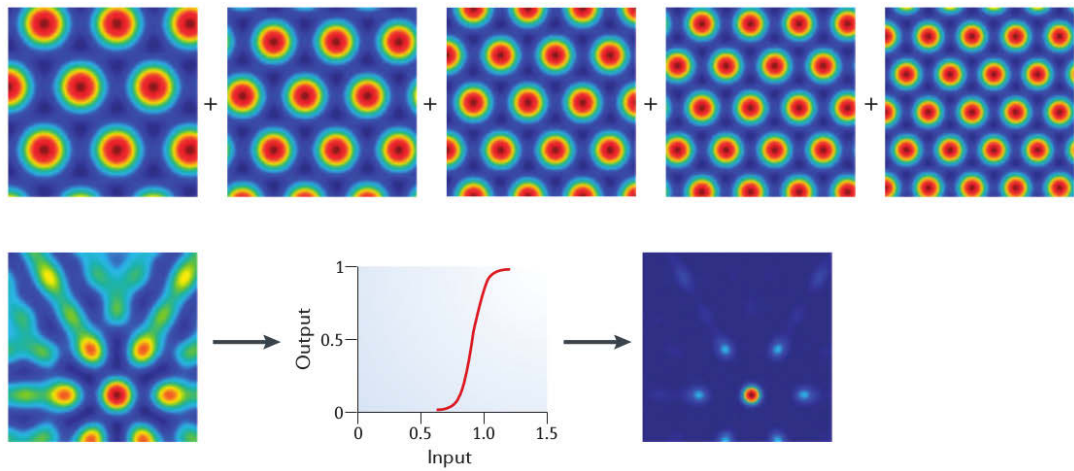


Figure 2.11: Grid cell networks with slightly varying spacing result in a large maxima. Images taken from McNaughton et al. [111]

cells were connected in such a way that the conjunctive neurons projected at a fixed offset to the grid cell bump causing the required movement of the bump (Figure 2.12).

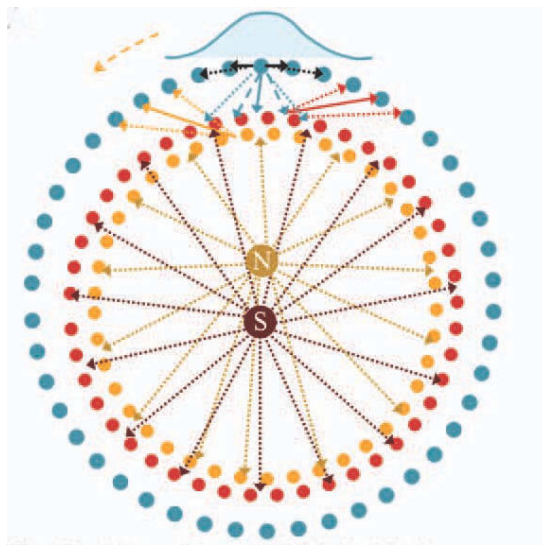


Figure 2.12: One-dimensional representation of the network connectivity according to the toroidal attractor model. Grid cells (in blue) are arranged in a ring. Each grid cell has synaptic connections onto nearby neurons, with the synaptic strength decreasing as a Gaussian function of distance between neurons. Thus, a bump of activity forms at one position. Grid cells project to conjunctive (grid-by-HD) cells (red and orange). Conjunctive cells also receive inputs from head-direction cells (light and dark brown), so that when the animal is moving north, the orange conjunctive cells are active. Conjunctive cells project back to the grid cell layer, with an offset in the relative direction which they represent. Thus, when the animal is moving north, the orange conjunctive cells activate the grid cells in the counter-clockwise direction from the position of the grid cell activity bump and thus move the bump in the counter-clockwise direction. Figure taken from Navratilova et al. [115]

Chapter 3

Towards bio-mimetic navigation: head direction cells on ROS

"Exactly!" said Deep Thought. "So once you do know what the question actually is, you'll know what the answer means."

- Douglas Adams, The Hitchiker's guide to the galaxy

Research papers published related to this chapter:

- An implementation of the path integrator mechanism of head direction cells for bio-mimetic navigation - Ankur Sinha & Jianguo Wang - Accepted by **IJCNN 2014 (Grade A conference)** held at Beijing, China - July 6 to 11, 2014.

This chapter describes my implementation of a head direction cell set on the ROS platform for the PR2 robot. I made use of the inertial navigation sensors of the PR2 for real time angular velocity input to the head direction cell network, which in turn, integrated this input to "calculate" the current bearing of the robot. The model was verified using a C++ implementation and we observed encouraging results.

Our model is based on the self organizing model proposed by Stringer et al. [75]. Their original model presented a biologically plausible method of setting up synaptic weights in the head direction cell system, via Hebbian learning [91]. It only applied the method to the path integrator system

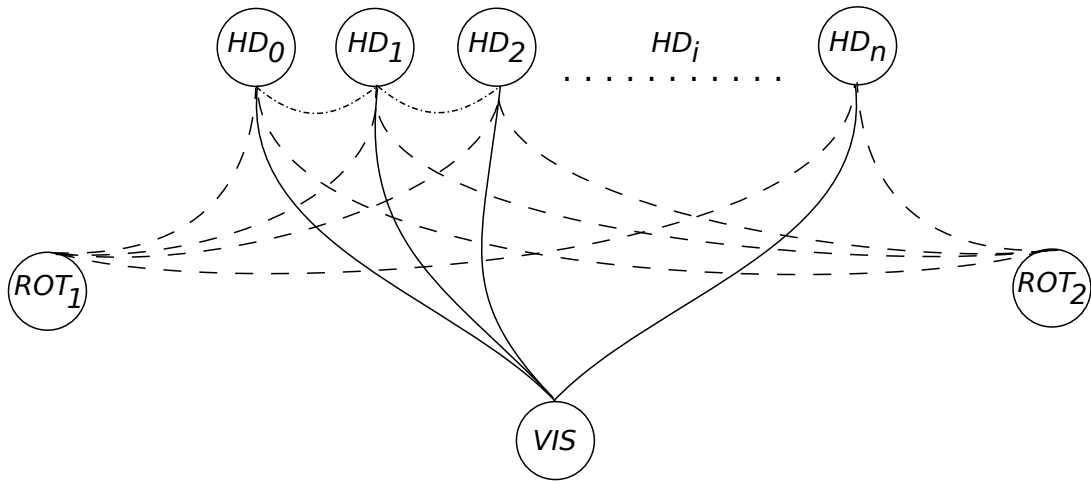


Figure 3.1: The head direction cell set schematic.

of head direction cells and only briefly discussed visual inputs as a fixed stimulus. We've incorporated vision cells into their model in an attempt to extend it to project both vestibular and visual inputs on the head direction cell set as has been found. We've also verified the model using real time data as one of the parts of a complete **bio-mimetic navigation system**.

3.1 Structure

The head direction cell system here makes use of three sets of neurons as shown in the Figure 3.1. The head direction cell set is a fully connected recurrent network, i.e., each neuron HD_i is connected to every other neuron HD_j via synapses, the weights of which are denoted as w_{ij}^{HD} . Here, HD_i and HD_j are the *pre-synaptic* and *post-synaptic* neurons with firing rates r_i^{HD} and r_j^{HD} respectively. The head direction cells, therefore, form a one dimensional circular array. While cells with adjacent preferred directions appear next to each other in our implementation (and most models), this is not how head direction cells are found in the brain. In fact, as long as the synaptic connections are set up appropriately, the location of neurons is irrelevant to the functioning of the system.

Rotation cells ROT_k , with firing rates r_k^{ROT} , are also connected to each head direction cell via synapses denoted w_{ijk}^{HD-ROT} . Here, these synapses are not simply synapses between a singular head direction cell and a rota-

tion cell. Rather, they're *effective synapses* between the pre-synaptic neuron HD_i , the post-synaptic neuron HD_j and the rotation cell ROT_k . The two rotation cells in Figure 3.1 fire depending upon clockwise and anti-clockwise rotation inputs respectively as shown in Figure 3.9.

The visual cell set represents an abstraction of the visual processing system and each visual cell VIS_l , with firing rate r_l^{VIS} , is connected to every head direction cell HD_i via synapses w_{il}^{HD-VIS} .

3.2 Dynamics

Our model is a firing rate based model, which provides a level of detail that is sufficient for our system. The activation, h_i^{HD} , of each head direction cell HD_i is given at time t by:

$$\begin{aligned} \tau \frac{dh_i^{HD}(t)}{dt} = & -h_i^{HD}(t) + \frac{\phi_0}{C^{HD}} \sum_j ((w_{ij}^{HD} - w^{INH})r_j^{HD}(t)) \\ & + \frac{\phi_1}{C^{HD \times ROT}} \sum_{jk} (w_{ijk}^{HD-ROT} r_j^{HD}(t) r_k^{ROT}(t)) \\ & + \frac{\phi_2}{C^{HD \times VIS}} \sum_{jl} (w_{jl}^{HD-VIS} r_l^{VIS}(t)) \end{aligned} \quad (3.1)$$

Here, τ is the time constant while ϕ_0 , ϕ_1 , ϕ_2 , C^{HD} , $C^{HD \times ROT}$, $C^{HD \times VIS}$ and w^{INH} are tunable parameters. These parameters control the effect the respective inputs have on the head direction cell attractor. w^{INH} represents global inhibition that the GABAergic interneurons exert on the system. The combination of local excitation of head direction cells and the global inhibition gives the system continuous attractor characteristics [78].

3. Towards bio-mimetic navigation: head direction cells on ROS 47

We use a Euler stepper method to integrate equation (3.1):

$$\begin{aligned}
 h_i^{HD}(t + \Delta t) = & \left(1 - \frac{\Delta t}{\tau}\right)h_i^{HD}(t) + \left(\frac{\Delta t}{\tau} \times \frac{\phi_0}{C^{HD}}\right) \sum_j ((w_{ij}^{HD} - w^{INH})r_j^{HD}(t)) \\
 & + \left(\frac{\Delta t}{\tau} \times \frac{\phi_1}{C^{HD \times ROT}}\right) \sum_{jk} (w_{ijk}^{HD-ROT} r_j^{HD}(t) r_k^{ROT}(t)) \\
 & + \left(\frac{\Delta t}{\tau} \times \frac{\phi_2}{C^{HD \times VIS}}\right) \sum_{jl} (w_{jil}^{HD-VIS} r_l^{VIS}(t))
 \end{aligned} \tag{3.2}$$

The firing rate of each head direction neuron is a sigmoid function of its activation:

$$r_i^{HD}(t) = f(h_i^{HD}(t)) = \frac{1}{1 + e^{-2\beta(h_i^{HD}(t) - \alpha)}} \tag{3.3}$$

where α and β are constants. Figure 3.2 shows the nature of the sigmoid function as the constants are varied. Figure 3.8 shows firing rate profiles exhibited by the head direction cell set during a test run. Due to the regular learning employed in this implementation, the discharge properties of all head direction cells are similar. This isn't the case in biology, where the discharge properties of head direction cells vary from one to another as discussed in chapter 2.1.1.

The synapses between all neuron sets are set up using Hebbian learning:

$$\Delta w = k \times (r^{post} \times r^{pre}) \tag{3.4}$$

Here, Δw is the change in synaptic weight. k is the *learning rate* of the synapse. r^{pre} and r^{post} are the firing rates of the pre-synaptic and post-synaptic neurons respectively. This learning rule does not, however, include synaptic depression, or bounding of synaptic weights. We use a competition based normalization rule to bound our synapses:

$$\hat{W} = \frac{W}{|W|} \tag{3.5}$$

Here, W is a matrix containing the individual synaptic weights between

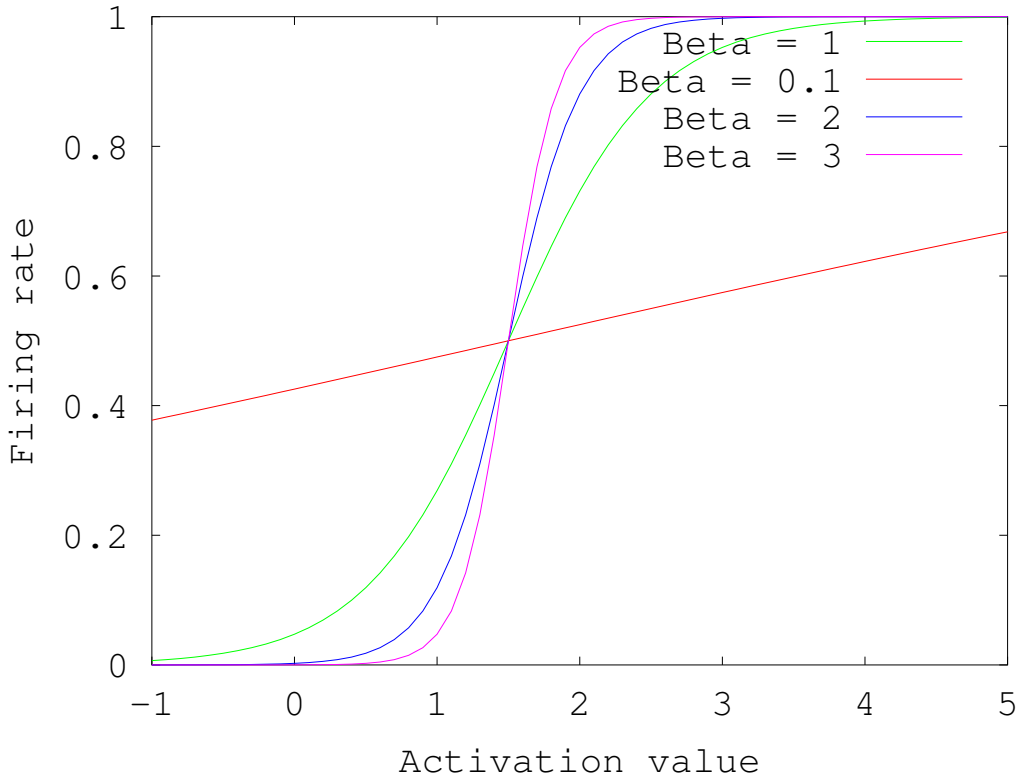


Figure 3.2: The figure shows the behaviour of the sigmoid function (3.3) with different β values. α is kept constant at 1.5 and only serves to move the graph to the left or the right.

neurons of the form:

$$W = \begin{pmatrix} w_{0,0} & w_{0,1} & \cdots & w_{0,n} \\ w_{1,0} & w_{1,1} & \cdots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} \end{pmatrix}$$

$|W|$ is the norm of the W matrix and \hat{W} is the normalized synaptic weight. It is worth noting that the above normalization departs from the Hebbian learning requirement of *locality* [116] - when a normalization method such as (3.5) is used, the final values of the individual synaptic connections between the neuron pairs become dependent on all the synaptic strengths in the complete neuron set. However, the classical Hebbian rule states that the synaptic weights between two neurons should only be af-

ected by the activity of the two concerned neurons. We briefly discuss other formulations of Hebbian learning in section 3.4.

3.3 Experimental Procedure and Results

3.3.1 System

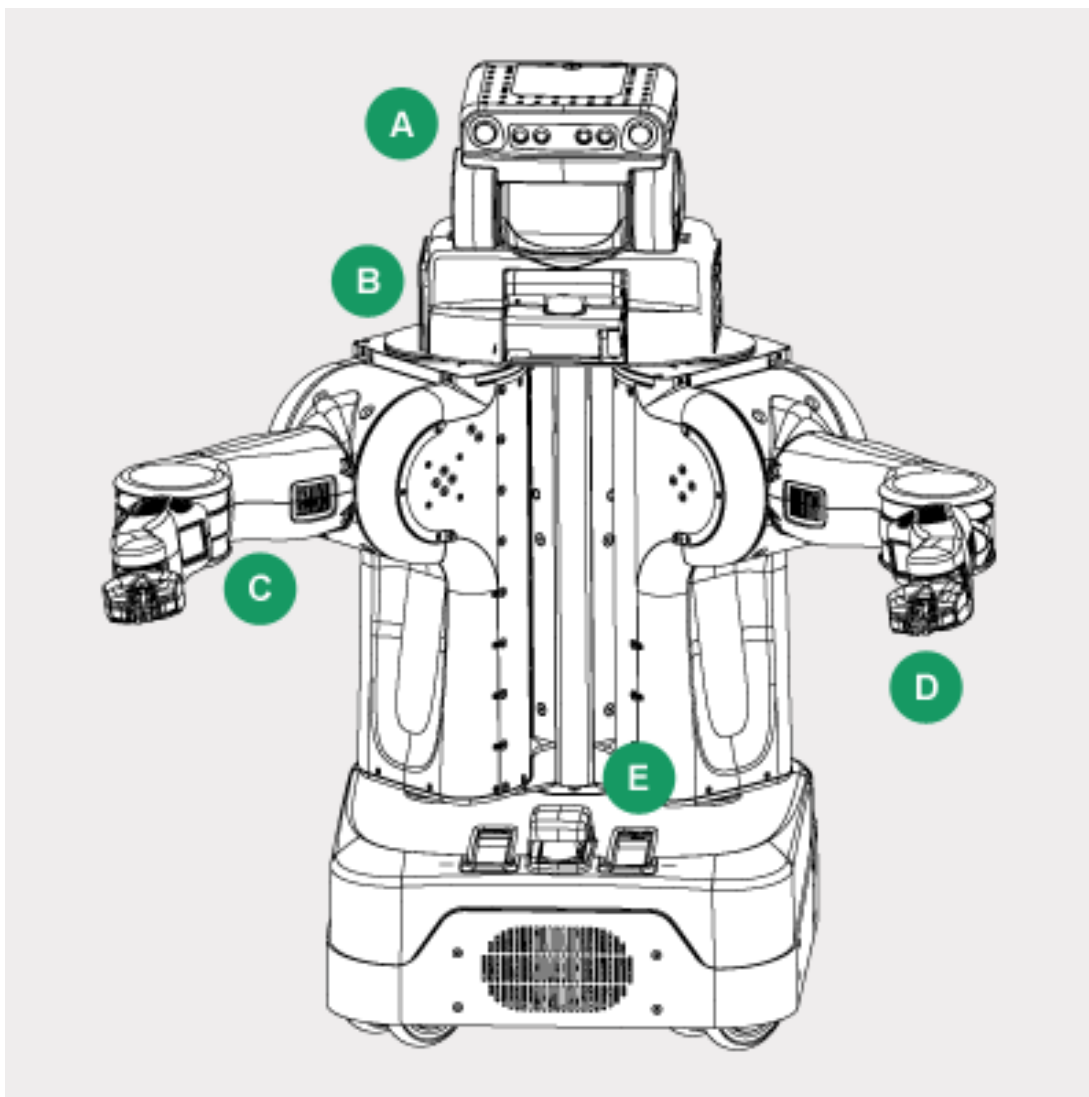


Figure 3.3: Willow Garage's Personal Robot 2 - PR2

We implemented the model based on the ROS [117] platform which provides support for a number of robots, including the PR2 (Figure 3.3).

3. Towards bio-mimetic navigation: head direction cells on ROS 50

The PR2 consists of the following sensors:

- **A:** Head
 - Wide Angle global shutter colour stereo ethernet camera
 - Microsoft Kinect
 - Narrow angle global shutter monochrome stereo ethernet camera
 - LED texture projector triggered with narrow angle stereo camera
- **B:** Above the shoulders
 - Tilting Hokuyo UTM-30LX Laser scanner
 - Microstrain 3DM-GX2 Inertial measurement unit
- **C:** Forearm
 - Global shutter ethernet camera
- **D:** Gripper
 - Three axis accelerometer
 - Fingertip pressure sensor arrays
 - Calibration LED
- **E:** Base
 - Hokuyo UTM-30LX Laser scanner

For the head direction system, the Microstrain IMU provided us with required angular velocity information. ROS provides underlying tools that enable us to develop offline simulations and then move the code as-is on to the robot. For development and testing, we collected data bags from the inertial measurement unit (IMU) sensors of the PR2 robot to run our simulations. In ROS, each sensor presents itself as a *node*. The inertial unit presented itself as *imu_node* in ROS and published data to the *imu/data* topic. The entire C++ source code was developed as a ROS *stack* called **Bionav**. Appendix A contains information on the ROS implementation.

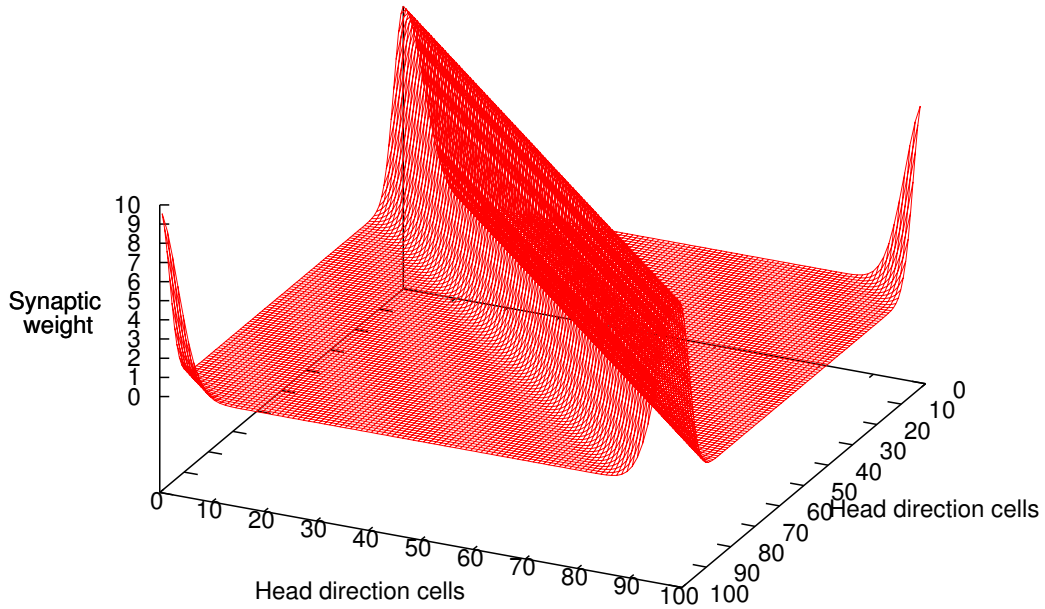


Figure 3.4: w_{ij}^{HD} values after training.

In the neuron network, we used a hundred head direction cells to cover the 360° direction space. We used two rotation cells, one each for clockwise and anti clockwise rotation, and a single visual cell (Figure 3.1). The values of constants used in our implementation are given in table 3.1.

The system runs in three phases:

3.3.2 Setting up of synaptic weights to appropriate values

During this first phase, we set up the synaptic weights in the network to their appropriate values. The network is initialized with all synaptic weights as zero, implying that no learning or association has taken place between the sets of neurons. In order to set up both the internal head direction cell synapses w_{ij}^{HD} and the effective rotation synapses w_{ijk}^{HD-ROT} , we simulate rotation in the system in both clockwise and anti-clockwise directions. Each head direction cell is assigned a preferred direction such

3. Towards bio-mimetic navigation: head direction cells on ROS 52

that the set encompasses the complete 360° range: 0°, 3.6°, ..., 356.4°. As done previously in literature, we model the head direction cell firing profile as a Gaussian:

$$r_i^{HD} = \exp\left(-\frac{1 + \Delta S^2}{2\sigma^{HD^2}}\right) \quad (3.6)$$

ΔS is the angular distance between the current head direction and the head direction cell's preferred direction:

$$\Delta S = \min(|x|, |360 - x|) \quad (3.7)$$

where, for each neuron HD_i with preferred direction $x_i^{preferred}$, for a head direction θ

$$x = \theta - x_i^{preferred} \quad (3.8)$$

σ^{HD} is a constant that controls the width of the Gaussian profile, and consequently, controls the angular width that a head direction cell is active in. Note that equation (3.6) is only used to calibrate the synapses to their correct values. Equation (3.3) is used thereafter.

In order to calibrate the network's synapses, one of the rotation cells is activated, simulating either clockwise or anti-clockwise rotation. Simultaneously, the firing rate profile, as obtained by equation (3.6), is simulated such that each head direction cell fires maximally, i.e., the system is simulated to face the preferred direction of each head direction neuron in the set, one neuron after the other. The firing rate profile of the head direction neuron set shifts according to (3.6).

Since we permit each neuron to fire maximally only once during both the clockwise and anti-clockwise iterations, our training method is uniform. The uniformity can be seen in Figure 3.4, which shows the recurrent synaptic weights between head direction cells, w_{ij}^{HD} , before normalization. Observe that normalization will not modify the nature of the graph. Also note that as a result of Hebbian learning from the simulated Gaussian firing rate profiles, cells with preferred directions near each other have stronger synaptic connections than cells that are far apart, as is expected.

For the recurrent head direction cell synapses, the learning rule becomes:

$$\Delta w_{ij}^{HD} = k \cdot r_i^{HD} * r_j^{HD} \quad (3.9)$$

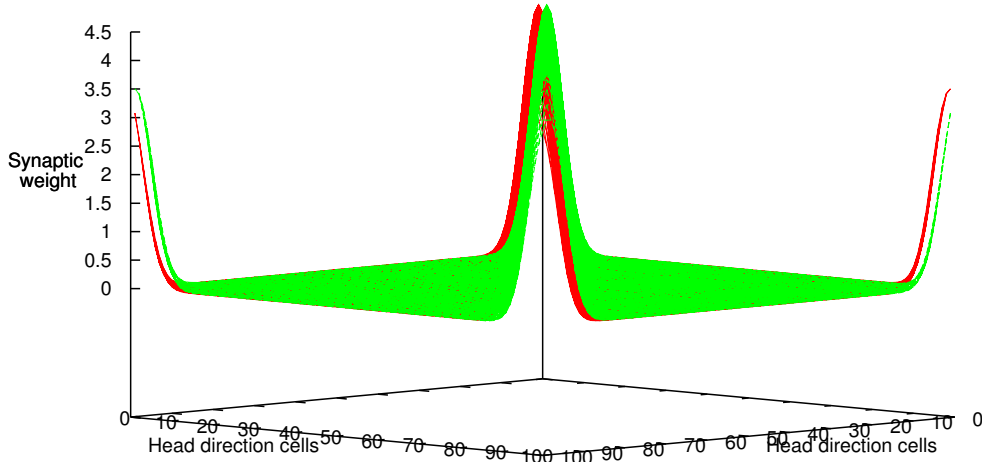


Figure 3.5: w_{ijk}^{HD-ROT} values after training. Observe how the two overlapping graphs are slightly offset from the centre when compared to the surface plot of the recurrent synapses between head direction cells in 3.4

Similarly, for the effective synaptic weights between head direction cells and the two rotation cells, the learning rule becomes:

$$\Delta w_{ijk}^{HD-ROT} = k \cdot r_i^{HD} * r_j^{HD} * r_k^{ROT} \quad (3.10)$$

Figure 3.5 shows the final values of these synaptic weights after training.

r_i^{HD} is the *trace* of the firing rate calculated as follows:

$$r_i^{HD}(t) = (1 - \eta) * r_i^{HD}(t - \delta t) \quad (3.11)$$

The trace serves to add temporal information to the synaptic modification in the firing rate based model.

3.3.3 Initializing the network to an initial direction

Once the synaptic weights are set up appropriately, a packet of activity must be stimulated in the attractor. This packet of activity is the initial or reference heading of the agent. An initial packet of activity is forced on the system by projecting the required profile on to the head direction cell attractor from the visual input for a short period of time. The firing of the single vision cell can be simulated by simply setting its firing rate to the maximum value, 1, in equation (3.1):

$$r_1^{VIS} = 1 \quad (3.12)$$

Further, to set the initial direction to the preferred direction of head direction cell i , we simply set the synapses between the vision cell and head direction cells, $w_{1,j}^{HD-VIS}$, to the synaptic weight between head direction cell i and every other head direction cell j , w_{ij}^{HD} . This is equivalent to setting up the association between the visual feature cell and the head direction cell set using equation (3.9)

$$w_{1j}^{HD-VIS} = w_{Ij}^{HD} \quad (3.13)$$

where I is the head direction cell that has the required initial direction as its preferred direction. Since w_{Ij}^{HD} has already been set up in a way that will cause the set of head direction neurons to form a packet of activity peaking at head direction neuron I during the training step, the assignment plainly ensures that projections from the visual cell will form a similar activity packet that also peaks at the chosen head direction neuron. The particular neuron is *associated* to the visual feature cell in this way. Note that this method cannot be extended to a set of multiple visual feature cells that will project a firing rate profile instead of a single projection. In such a case, an association will have to be made by simulating the set of visual feature cells and head direction cells similar to the method described in the previous section.

During our experiments, we observed that forcing an initial packet of activity did not guarantee a functional system. The attractor should maintain the packet of activity in the absence of external inputs. The parameters ϕ_0 and w^{INH} that effect the recurrent behaviour of the network must

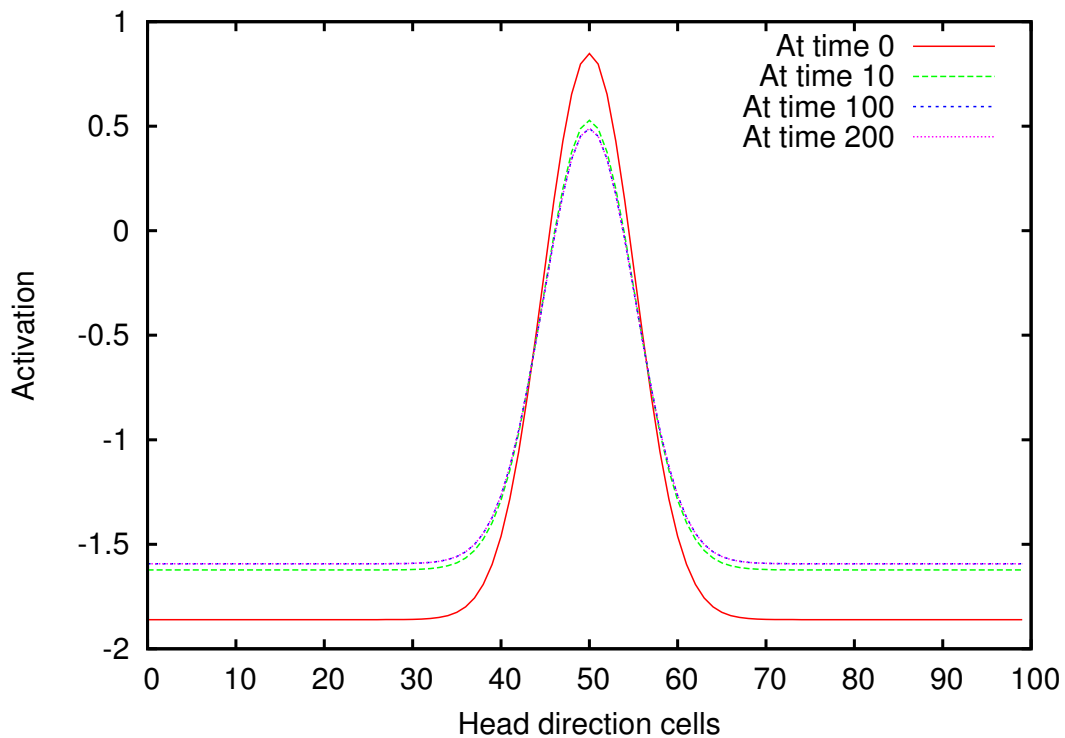


Figure 3.6: Stabilization of initial activity packet (Time in seconds)

be fine tuned to ensure that the activity packet stabilizes as shown in Figure 3.6, the other outcome being the activity packet *flattening out* as shown in Figure 3.7.¹

The continuous attractor, if set up properly, permits the packet of activity to lie in a state of neutral equilibrium, like a ball lying on a perfectly horizontal table surface.

3.3.4 Running the system with angular velocity data

Figure 3.9 shows the behaviour of the network when tested with angular velocity data. The firing rates of the two rotation cells are a sigmoid function of the angular velocity inputs similar to equation (3.3). The figure shows that the head direction indeed responds to angular velocity inputs. Of special interest is the graph's behaviour at time=1300 which shows circular attractor nature of the network - since synaptic weights between

¹We are most grateful to Dr. Simon Stringer for his input on this subject.

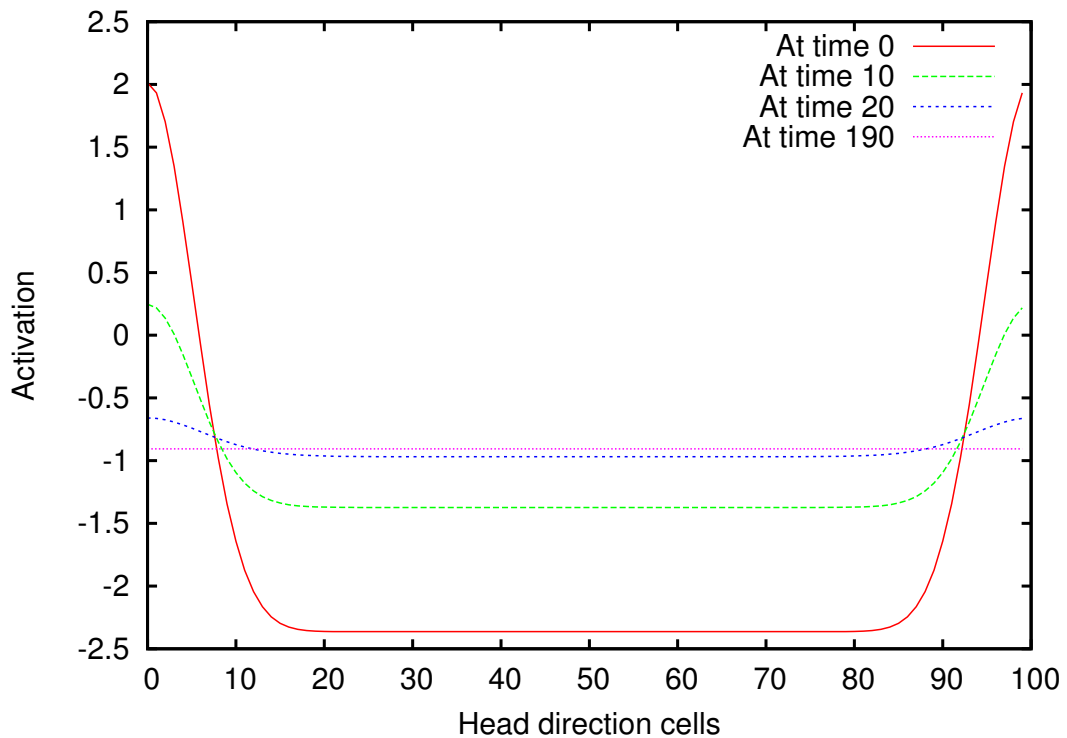


Figure 3.7: Sub-optimal parameters do not result in a stable activity packet (Time in seconds)

head direction neurons with preferred directions 356.4° and 0° have been trained such that these two neurons are adjacent, the firing profile translates seamlessly off the “end” of our one dimensional attractor to its “beginning”. This is a consequence of equation (3.7) which ensures that the head direction neuron with preferred direction 356.4° has the same synaptic connection with its two adjacent cells that have 352.8° and 0° as their preferred directions respectively. This confirms the continuous attractor nature of the network. Figure 3.7 also illustrates the ring nature of the one dimensional attractor network - it shows an initial packet of activity where the firing rate peaks at head direction neuron 0 and symmetrically reduces in both directions. We tested the system with several bags of data and observed encouraging results.

Figure 3.8 shows the stable gaussian activity profile of the head direction cell set firing rate at different times when angular velocity inputs are provided to the system. The rate at which the head direction firing pro-

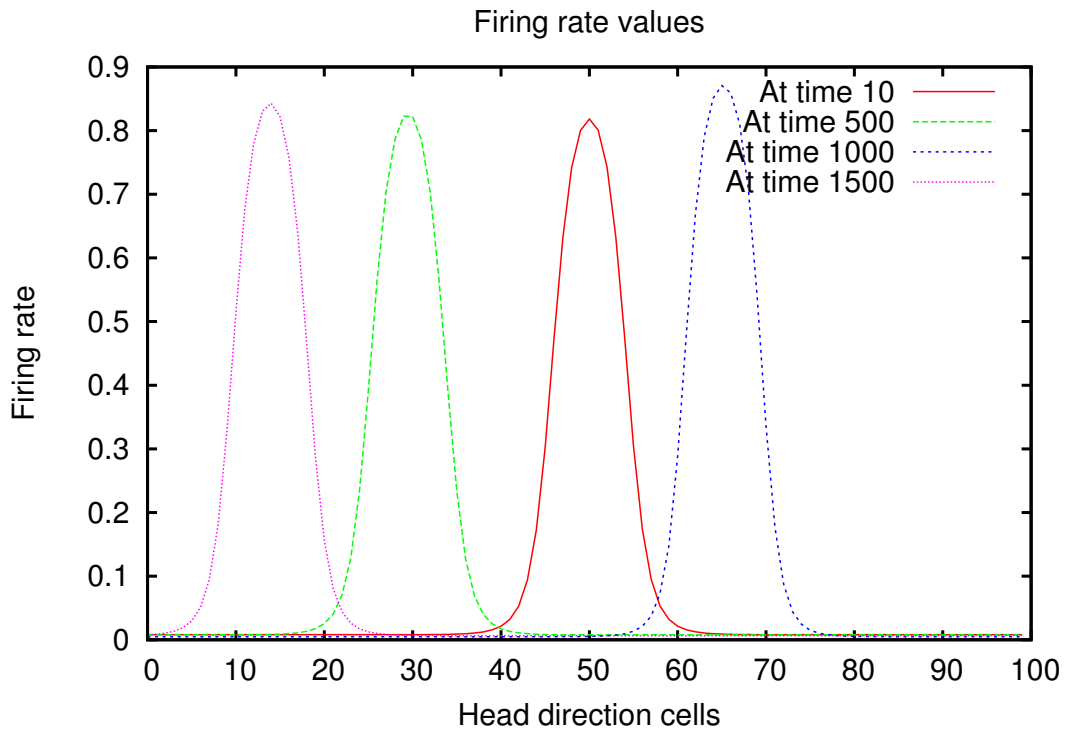


Figure 3.8: Firing rates, r_i^{HD} , obtained from equations 3.1 and 3.3 during test run. (Time in seconds)

file moves depends on the strength of the rotation neurons' projections on it. In the current configuration, the system has not been optimised to correctly map the rotations of the agent, i.e., the movement of the head direction activity profile does not reflect the true rotation of the agent in the world frame. This isn't because it cannot be done: the accuracy can be improved fine tuning the value of ϕ_1 which controls the strength of the rotation neurons' projections on to the head direction cells. However, as we briefly discuss in the next sub section, our current work focusses on associating head direction cells to salient features in the environment for landmark navigation, and since this will function as a drift correction mechanism in itself, we've permitted the system to drift and have put off optimization of these constants.

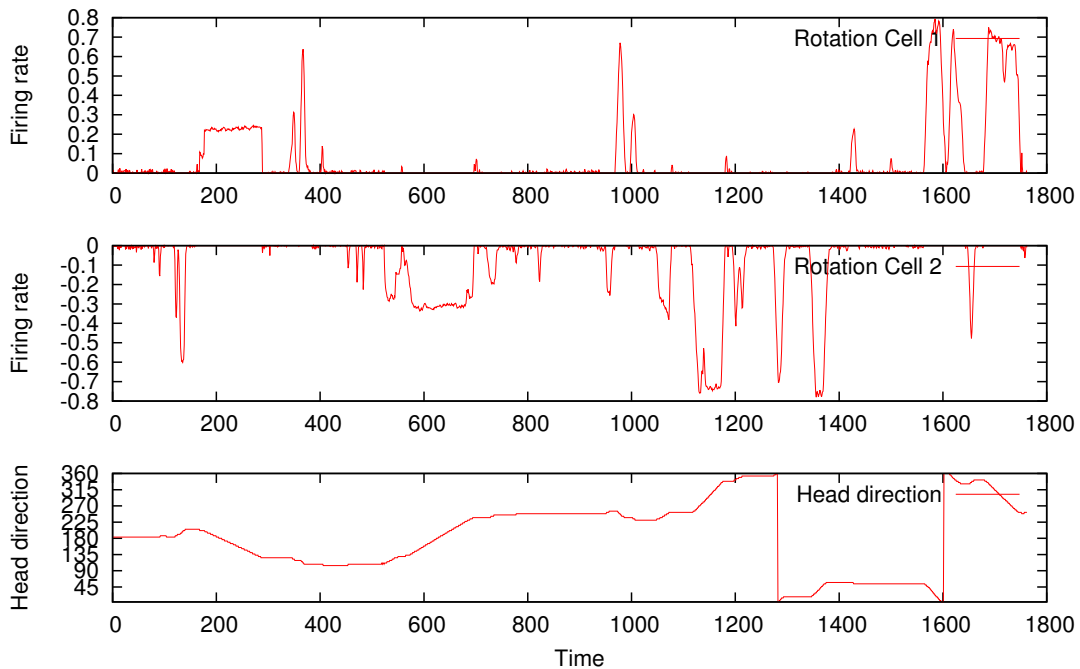


Figure 3.9: Test runs with angular velocity data from the PR2 robot. Note that the firing rates of both rotation cells are positive. For visualisation purposes, to add a sense of direction, the values of firing rates of Rotation cell 2 have been multiplied by -1. (Time in seconds)

3.3.5 Correcting drift using salient visual cues: preliminary tests

Figure 3.10 shows the results of a simulation where the head direction cell set is assumed to incur drift and, a projection from the visual feature cell, which would be caused by the agent observing a familiar feature, is used to correct this drift. The mechanism behind this is similar to the mechanism used to initialize the network as described in section 3.3.3. In order for the visual inputs to override the activity packet maintained by path integration, the projections must be strong enough. We discuss our ideas on using multiple visual features in the next section.

Table 3.1: Constants used in the implementation

Number of head direction cells	100
Number of rotation cells	2
Number of vision cells	1
α	1.5
β	3
ϕ_0	1000
ϕ_1	2000
ϕ_2	1000
C^{HD}	100
$C^{HD \times ROT}$	200
$C^{HD \times VIS}$	100
k	1
w^{INH}	0.02
σ^{HD}	10

3.4 Discussion

Researchers have attempted to implement bio-inspired navigation systems in the past, although the level of inspiration and the extent of implementation has varied. The Psikharpax project [101, 102], for example, attempted to create an artificial rat and implemented head direction and place cells as its basis for navigation. RatSLAM [118], on the other hand, uses similar “pose cells” to propose a solution to the simultaneous localization and mapping (SLAM) [119] problem. (For more examples, see Franz & Mallot’s review [4].)

Bio-mimetic navigation provides two closely related research areas. The first is to improve our understanding of biological navigation. This is done at different levels, for example, via behavioural studies or neuron recordings. The second, computational modelling, serves as a tool to verify collected information and proposed theories, while providing alternative navigation systems that can, in the future, be used in robotics. Even though bio-mimetic systems are not yet considered mature enough for use in task

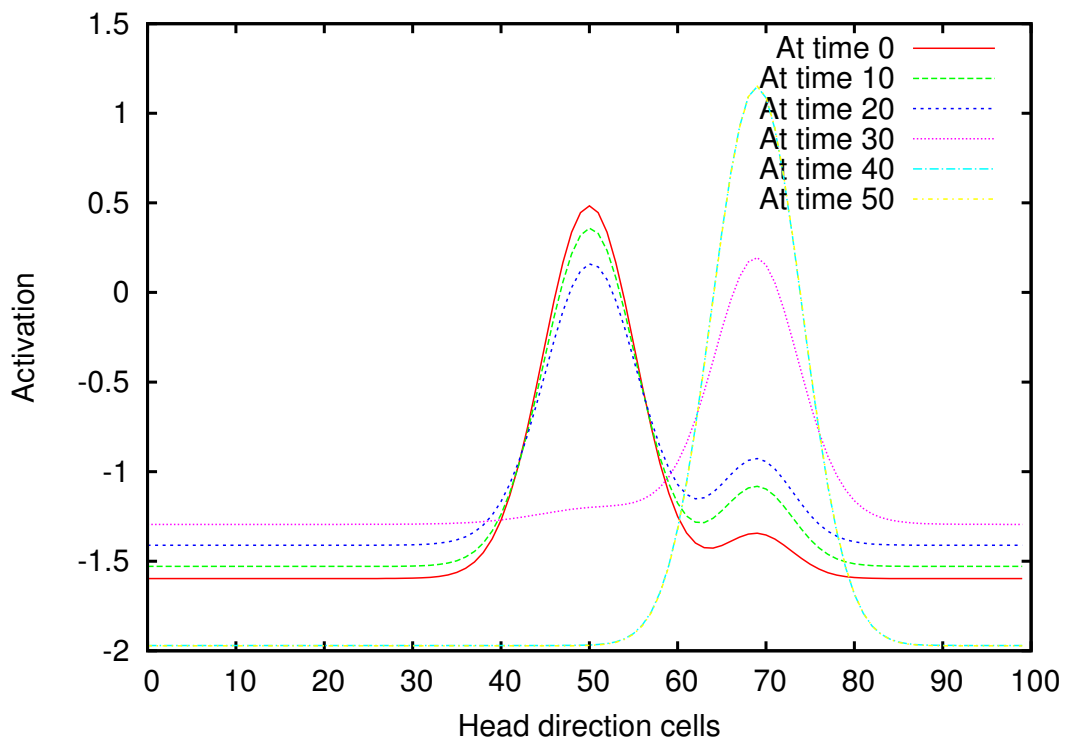


Figure 3.10: Correcting drift using a single visual cue. (Note that the graphs for $t=40$ and $t=50$ overlap) (Time in seconds)

oriented robotics ahead of classical navigation techniques, it is accepted that even smaller animals such as ants and rats possess navigational capabilities that are superior to classical robotic navigation techniques. To be able to reliably mimic these biological methods would be a great stride in the field of autonomous navigation.

In spite of the complexity of the underlying neural systems that makes bio-mimetic systems difficult to implement, systems that achieve a high level of similarity can and have been designed. The model of the head direction system implemented here, for example, deviates from known information on head direction cells in certain aspects but does still sufficiently carry out its intended navigational function.

Challenges and future work

The implementation of the path integrator half of the head direction cell system is the starting point in our attempt to develop a bio-mimetic nav-

igation system for the PR2 robot using the ROS platform. In its current state, the system is not mature enough to be used in an actual robot. While the performance of the system can be improved by tweaking the parameters that the head direction cell model relies on, in the absence of a set of salient visual cues, the system will continue to drift as it runs, like any other INS [71]. An INS is generally coupled with other input sources, such as GPS [72], that reset the accumulated drift at regular intervals. The head direction system can similarly correct drift by detecting familiar visual features in the environment as was briefly demonstrated in section 3.3.5. Our next goal, is therefore, to associate the head direction cell system to environmental cues that will reset the head direction system to its associated direction whenever they are observed. Since the system will associate with a set of cues, it will be capable of approaching these cues. The agent will be able to carry out the local navigation strategies: search, direction-following, aiming and guidance, as enumerated by Franz & Mallot [4]. This task is a research area in itself. The extraction of meaningful features reliably from an environment repeatedly at different times from different view points is a topic under great research. Some algorithms such as SIFT[120], SURF[121] and their variants perform admirably in this task - however, simply using one of these algorithms is not sufficient in our model. Since the model relies on population encoding via firing rates of neurons, a method to encode features tracked by computer vision algorithms into firing rates is also required, which we foresee as the function of the vision cell ensemble. While the earlier model proposed by Stringer and colleagues [75] did implement this idea, they did not discuss how the vision cells would project on to the head direction cells. In our model, since we've also implemented synapses between head direction cells and vision cells, a hebbian learning method can be used during the running of the model to make a specific head direction (encoded as a specific firing rate pattern of head direction cells) associate with a specific landmark (encoded as a specific firing rate pattern of visual cells). This association will cause vision cells to project the previously "remembered" head direction associated with a known feature on to the head direction cells and correct drift as a result. Due to time constraints, the entire vision system and ideas presented here are merely something we intend to do in the

future - it has not been implemented or evaluated at the time of writing this document.

A further goal of ours is to include other known neuron sets that are related to navigation: grid cells [22], place cells [7] and boundary cells [23]. Grid cells form a regular triangular grid which acts as a spatial map to encode the agent's location. The neuron set uses path integrator mechanisms similar to head direction cells. Place cells too encode the agent's location and have been found to associate with visual features similar to head direction cells. However, they form place fields instead of a regular grid like map that is exhibited by grid cells. The last, boundary cells, respond to environmental boundaries. Implementing a system that includes all these neuron sets will provide obstacle, location and heading information to the agent along with some information on visual features that they associate with. This information is enough for a simple landmark based system.

An important part of the model is the Hebbian learning rule mentioned in equation (3.4). The formula that we've used currently is the simplest mathematical formulation of a Hebbian synaptic modification rule (3.9). As is visible, this rule does not provide for synaptic saturation. As long as the pre-synaptic and post-synaptic neurons fire simultaneously, the synapse between them will continue to strengthen. While this formulation covers strengthening of synapses by long term potentiation (LTP) [122, 123], it does not implement the flip side: long term depression (LTD) [124]. Just as LTP causes strengthening of synapses when presynaptic and postsynaptic neurons fire nearly simultaneously, LTD causes weakening of synapses if such simultaneous firing does not occur. The inability of the learning rule to provide for synaptic saturation and weakening makes it less biologically plausible. It also makes the implementation more difficult: if the synapses do not saturate at a known value, it is difficult to ascertain constant values for parameters that control the projection of inputs on to head direction cells: ϕ_0, ϕ_1, ϕ_2 . During our simulations, we discovered that unbounded synapses constitute one of the cases where the projections on the head direction cells increase to such an extent that all neurons begin to fire maximally. Since the maximum firing rate in our model is the same for all head direction cells, this also causes the firing rate profile to flat-

ten out and the system ceases to provide a peak that denotes the current head direction.

A number of formulations of the Hebbian rule have been proposed in literature [125, 126]. Such rules incorporate modifications to provide for LTD and synaptic saturation. One such modification is to normalize the synaptic weights, as shown in equation (3.5). Normalization is a competition based method: if synaptic efficacy increases, it must be at the expense of other synapses [116]. Other formulations include gating of the synaptic changes by either presynaptic or postsynaptic activity. While we did attempt to use gated rules with saturation in our model, we were unable to find a set of parameters that provided the required dynamics. Since the recurrent synapses between the head direction cells and the synapses between rotation cells and head direction cells remain largely unchanged during the running of the system, we decided to use the normalization rule and fix the synaptic weights after initial calibration of the system. For association with visual features, however, the formulation that is used to modify synapses between the head direction and visual cells will need to incorporate weakening of synapses via LTD so that the system can disassociate with (forget) features that are no longer present in the environment over a period of time. A simple method of doing this is to incorporate synaptic decay, which causes the synaptic strengths to weaken over time.

Chapter 4

Including grid cells for location information

"Would you tell me, please, which way I ought to go from here?"

"That depends a good deal on where you want to get to."

"I don't much care where "

"Then it doesn't matter which way you go."

- Lewis Carroll, Alice in Wonderland

Research papers published related to this chapter:

- Bio-mimetic path integration using a self organizing population of grid cells - Ankur Sinha & Jianguo Wang - *Accepted by **ICANN 2014 (Grade B conference)** held at Hamburg, Germany - September 15 to 19, 2014.*

The model detailed here is a modified version of the self organizing two dimensional attractor network model Stringer et al. proposed for place cells[92]. In the original model, Stringer and colleagues detailed how their one dimensional attractor model for a head direction cell system[75] could be extended to two dimensions to produce place fields. The model successfully set up a two dimensional attractor network using Hebbian style learning rules and displayed spatial firing patterns consistent with biological observations of place cell networks. While using similar Hebbian style

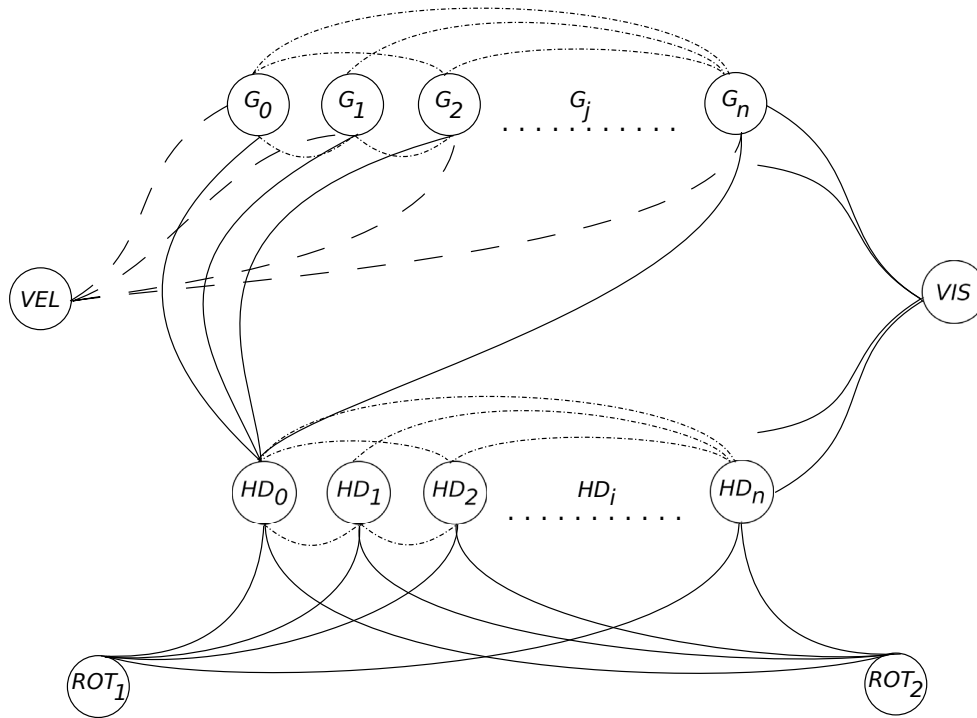


Figure 4.1: The grid cell system schematic.

learning rules, we make modifications to the structure of the network required for the formation of a triangular lattice of neurons, rather than a traditional co-ordinate map. We also modify the organization process to ensure that the lattice forms a toroid of grid cells, rather than a flat map to work around edge effects.

Since the model is a two dimensional attractor based model, it is in many ways, just an extension of the one dimensional attractor based model that we used for head direction cells. It suffers from the same issues and the dynamics and working is very very similar.

4.1 Structure

Figure 4.1 shows a schematic of the neuron sets implemented in the model. The model consists of two fully connected attractors, one for the head direction cell set and another for the grid cell set. Each head direction cell, HD_i is connected to every other head direction cell HD_j via recurrent synapses w_{ij}^{HD} . Similarly, every grid cell, G_k is connected to every other grid

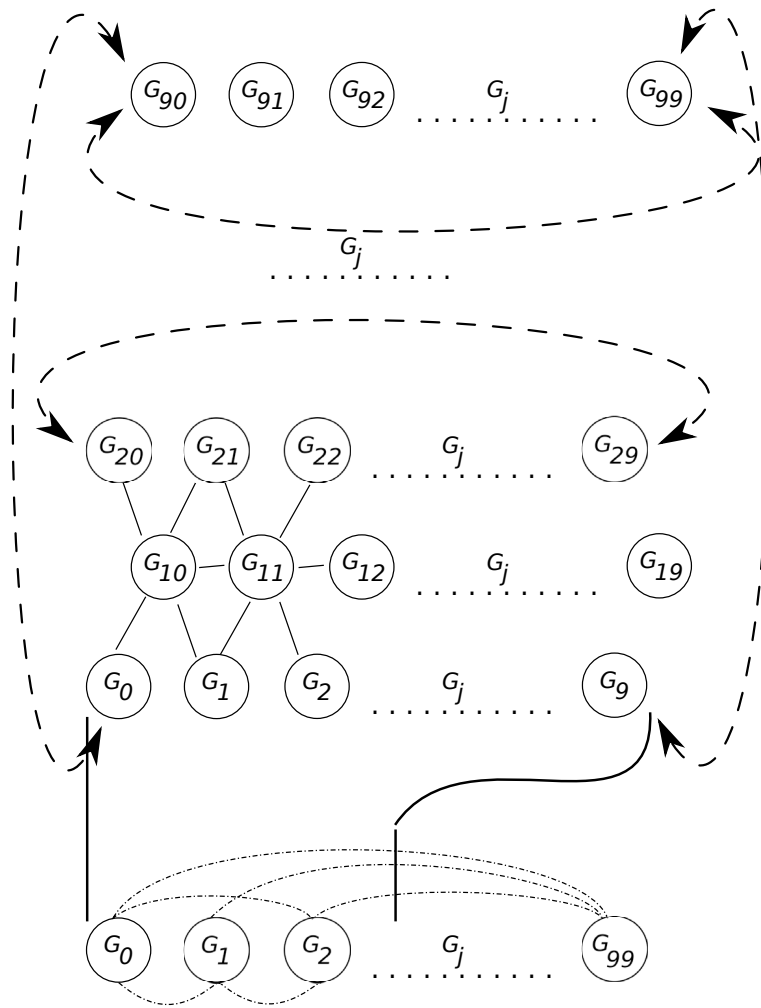


Figure 4.2: 2D array of grid cells

cell G_l via recurrent synapses w_{kl}^G . In this way, each synapse possesses a pre-synaptic and post-synaptic neuron necessary for our Hebbian learning based organization rule. The head direction cell set takes input from a set of rotation cells, ROT_m , which project angular velocity information on to them via synapses, w_{ijm}^{HD-ROT} . Similarly, a set of velocity cells, VEL_n , project speed information on to the grid cell set via synapses w_{kln}^{G-VEL} . It is noteworthy that these synapses are *effective synapses* in that they represent the synaptic weight between a pre-synaptic head/grid cell, the rotation/velocity cell, and the post-synaptic head/grid cell. A set of visual feature cells, VIS_o projects on to both the head direction and grid cell sets via synapses w_{io}^{HD-VIS} and w_{ko}^{G-VIS} respectively. The visual feature cells are

used for initial training of the synapses only in this model and represent an abstraction of the visual processing system.

4.2 Dynamics

We employ a model similar to the one proposed by Stringer et al. for place cells[92]. The activation of each grid cell, G_k is given by:

$$\begin{aligned} \tau \frac{dh_k^G(t)}{dt} = & -h_k^G(t) + \frac{\phi_0}{C^G} \sum_l (w_{kl}^G - w^{INH}) r_l^G(t) \\ & + \frac{\phi_1}{C^{G \times HD \times VEL}} \sum_{ljn} (w_{kljn}^{G,VEL} r_l^G(t) r_j^{HD}(t) r_n^{VEL}(t)) \\ & + \frac{\phi_2}{C^{G \times VIS}} \sum_o (w_{ko}^{G,VIS} r_o^{VIS}(t)) \end{aligned} \quad (4.1)$$

Here, τ is the time constant while $\phi_0, \phi_1, \phi_2, C^G, C^{G \times HD \times VEL}, C^{G \times VIS}$ and w^{INH} are tunable parameters. These parameters control the effect the respective inputs have on the grid cell attractor. w^{INH} represents global inhibition that the GABAergic interneurons exert on the system. The combination of local excitation of grid cells and the global inhibition gives the system continuous attractor characteristics. Again, we use a simple Euler stepper method to integrate equation (4.1):

$$\begin{aligned} h_k^G(t + \Delta t) = & ((1 - \frac{\Delta t}{\tau}) h_k^G(t)) + ((\frac{\Delta t}{\tau} \times \frac{\phi_0}{C^G}) \sum_l ((w_{kl}^G - w^{INH}) r_l^G(t))) \\ & + ((\frac{\Delta t}{\tau} \times \frac{\phi_1}{C^{G \times HD \times VEL}}) \sum_{ljn} (w_{kljn}^{G,VEL} r_l^G(t) r_j^{HD}(t) r_n^{VEL}(t))) \\ & + ((\frac{\Delta t}{\tau} \times \frac{\phi_2}{C^{G \times VIS}}) \sum_o (w_{ko}^{G,VIS} r_o^{VIS}(t))) \end{aligned} \quad (4.2)$$

The firing rate of each grid neuron is also a sigmoid function of its activation:

$$r_k^G(t) = f(h_k^G(t)) = \frac{1}{1 + e^{-2\beta(h_k^G(t) - \alpha)}} \quad (4.3)$$

where α and β are constants. Figure 4.4 shows the firing rate profile ex-

hibited by the grid cell lattice after stabilization during a test run . Due to the regular learning employed in this implementation, the firing rates of all grid cells are similar. This isn't the case in biology, where the firing rates of grid cells vary from one another and from layer to layer.

The synapses between all neuron sets are set up using Hebbian learning:

$$\Delta w = k \times (r^{post} * r^{pre}) \quad (4.4)$$

Here, Δw is the change in synaptic weight. k is the *learning rate* of the synapse. r^{pre} and r^{post} are the firing rates of the pre-synaptic and post-synaptic neurons respectively. This learning rule does not, however, include synaptic depression, or bounding of synaptic weights. We use a competition based normalization rule to bound our synapses:

$$\hat{W} = \frac{W}{|W|} \quad (4.5)$$

Here $|w|$ is the norm of the w matrix and \hat{w} is the normalized synaptic weight. It is worth noting that the above normalization departs from the Hebbian learning requirement of *locality*[116], i.e., the synapse between two neurons should only be modified by their behaviour. Various formulations of Hebbian learning have been proposed to overcome this shortcoming[125]. However, we use the normalization rule for sake of simplicity.

4.3 Experimental procedure and results

4.3.1 System

We continued to use the same system for grid cells as we did for head direction cells, as described in section 3.3.1. For development and testing, we collected data bags from the IMU sensors of the PR2 robot to run our simulations. We extended our earlier **Bionav** stack to also include grid cells. We used a hundred grid cells to form a 10×10 mesh, and a hundred head direction cells to cover the 360° direction space. We used two rotation cells, one each for clockwise and anti clockwise rotation, a single speed cell, and a single visual cell (Figure 4.1).

Table 4.1: Constants used in the implementation

Number of grid cells	100
Number of velocity cells	1
Number of vision cells	1
α	1.5
β	3
ϕ_0	1000
ϕ_1	2000
ϕ_2	1000
C^G	100
$C^{G \times HD \times VEL}$	200
$C^{G \times VIS}$	100
k	1
w^{INH}	0.02
σ^G	10

The grid cell lattice merits some discussion. Conforming to biological observations, we designed a regular triangular lattice as shown in Figure 4.2. Each cell is assigned a *preferred location* such that it coincides with the vertices of the equilateral triangles of the lattice. The distance between any two adjacent neurons is, therefore, one unit. Figure 4.2 shows how our implementation expands out to a two dimensional lattice. The grid lattice does not provide any advantages at the time when compared to a normal square co-ordinate lattice. It is only included to be coherent with biological observations.

The system runs in three phases:

4.3.2 Setting up of synaptic weights to appropriate values

During this phase, we set up the synaptic weights in the network to their appropriate values. The network is initialized with all synaptic weights as zero, implying that no learning or association has taken place between

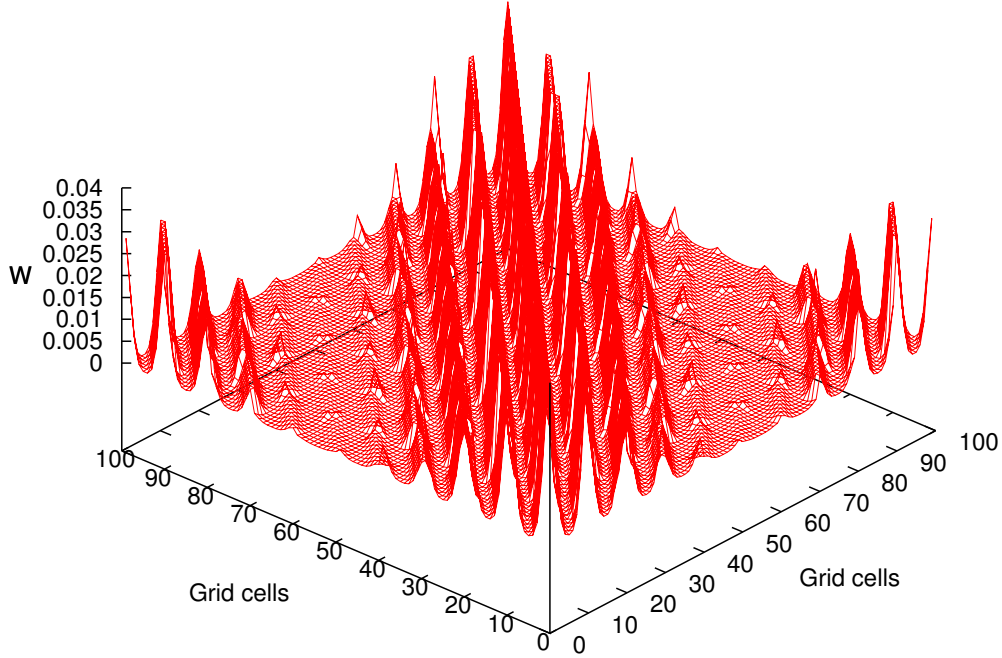


Figure 4.3: Recurrent synaptic weight in grid cells

the sets of neurons. In order to set up both the internal grid cell synapses w_{kl}^G and the effective velocity synapses w_{kjn}^{G-VEL} , we simulate movement in the system in the four major directions: forward, backward, left and right; neuron by neuron by projections from the visual feature cell. The firing rate of a grid cell is calculated as a function of the distance between its preferred location and the preferred direction for the respective training iteration, ΔS :

$$r_l^G = \exp\left(-\frac{1 + \Delta S^2}{2\sigma^2}\right) \quad (4.6)$$

where ΔS is given by:

$$\Delta S = \sqrt{(\min(|x|, |10 - x|))^2 + (\min(|y|, |(10 * (\sqrt{3}/2)) - y|))^2} \quad (4.7)$$

where, for each neuron G_k with preferred location $x_k^{preferred}$, $y_k^{preferred}$ for a location X, Y

$$(x, y) = (X, Y) - (x_k^{preferred}, y_k^{preferred}) \quad (4.8)$$

σ^G controls the width of the Gaussian profile. The above formulation of ΔS ensures the formation of a regular toroidal continuous attractor neural network.

In order to calibrate the synapses of the network, we simulate activity in the input neuron sets: visual feature cell, velocity cell and head direction cells. Projections from the visual feature cell force a firing profile in the grid cell network which acts as the post synaptic neuron set. Since the velocity and head direction cells are simultaneously simulated to fire in accordance, they function as pre-synaptic neurons in our Hebbian style learning rule.

Similar to the head direction cells, the learning rules for grid cells become:

$$\Delta w_{kl}^G = k \cdot r_k^G * r_l^G \quad (4.9)$$

The learning rule used to train the effective synaptic weights between head direction cells, grid cells and velocity cells follows similar to the learning rule used to train the effective synaptic weights between head direction and rotation cells:

$$\Delta w_{kljn}^{G-VEL} = k \cdot r_k^G * r_l'^G * r_j^{HD} * r_n^{VEL} \quad (4.10)$$

$r_l'^G$ is the *trace* of the firing rate calculated as follows:

$$r_l'^G = (1 - \eta) * r_l^G \quad (4.11)$$

Figure 4.3 shows the recurrent synaptic weights of the grid cell network, w_{kl}^G . As expected, the synaptic weights are maximum for neurons with similar preferred directions and decrease as the difference in preferred directions increase.

4.3.3 Initializing the network with an initial packet of activity

Once the synapses are trained, the network is initialized with a packet of activity which is the initial location or reference location of the system. An initial packet of activity is forced on to the grid cell attractor via projections

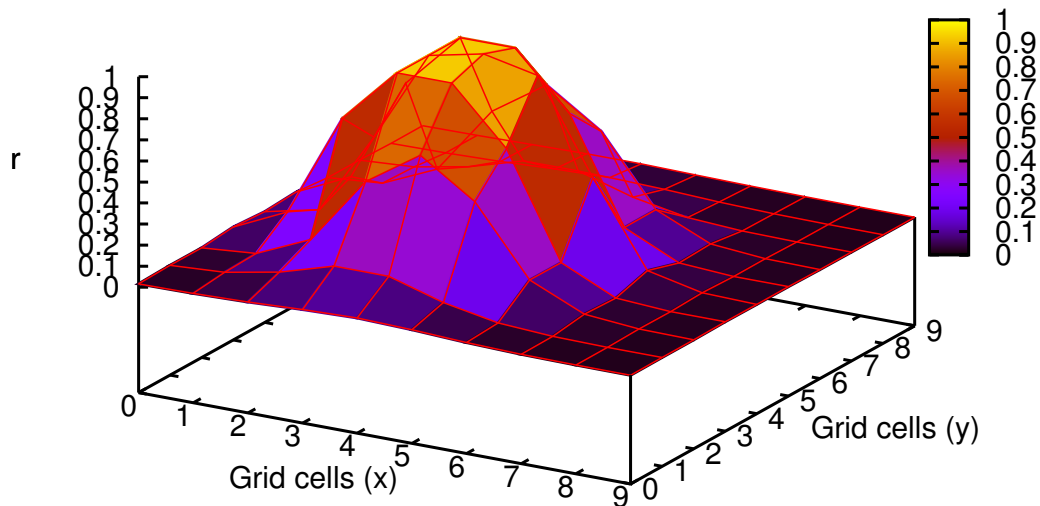


Figure 4.4: 3D view

from the visual cell and permitted to stabilize in the absence of input. Figure 4.4 shows the two dimensional attractor with a packet of activity after initialization. Figure 4.5 shows the top view. Note that in order for the packet of activity to be stable in the absence of input, as is the characteristic of an attractor network, both the recurrent synaptic weights between the grid cells and the constants, ϕ_0 and w^{INH} , that control the behaviour of the network must be set up correctly. (If this is not the case, the packet of activity will not be maintained in the absence of inputs and will instead *flatten out*.)

4.3.4 Running the system with velocity information

The firing rate of the rotation cells and the velocity cell are linear functions of the angular velocity and forward velocity respectively. The angular velocity is first integrated in the head direction cell attractor which in turn projects on to the grid cell set attractor. The rate at which the activity

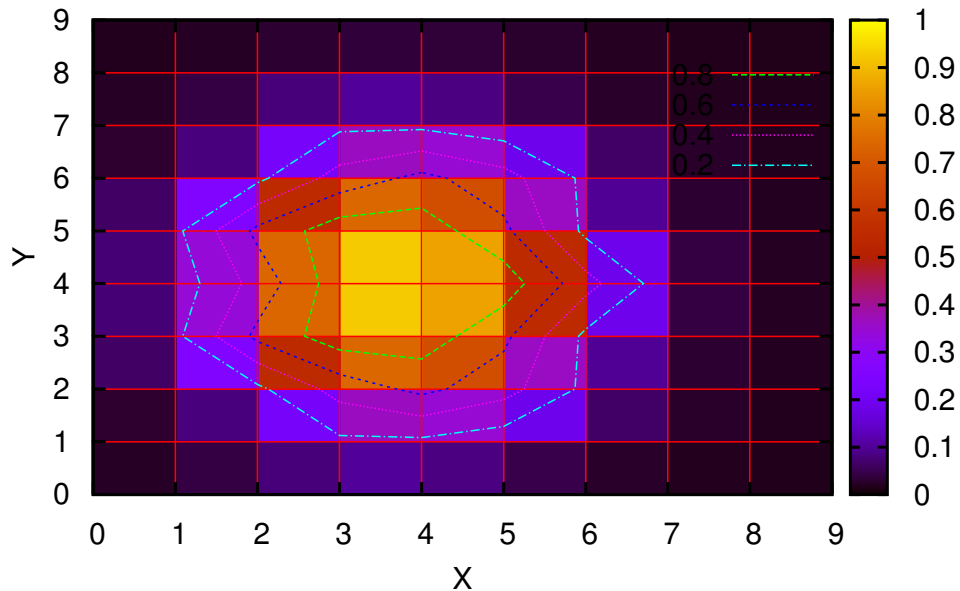


Figure 4.5: Top view

packet translates depends on the strength of projections on to the network: projections from the velocity set and the head direction cell set.

4.4 Discussion

A combination of head direction cells, border cells and grid cells can be thought to provide a complete path integrator system in biology. In spite of successful modelling of head direction and grid cells, our system is not yet complete enough for deployment as a bio-mimetic system. Due to time and computational constraints, we were unable to run enough tests that would provide us with enough data to improve the performance of the system via parameter tuning. In the current state, the system suffers from huge drift and while the generated activity packet does move in the directions similar to the agent, the magnitude of the direction and movement are both unusable. The performance of the grid cells depends greatly on the performance head direction cell set, and since the head di-

rection cell set does not compute accurate direction and suffers from drift in the absence of visual corrective measures, the performance of the grid cell set is worse. At the most, this implementation is currently a proof of concept - that when the other required components are put in place and perform accurately, the grid cell ensemble will correctly track the movement of the agent in a horizontal plane.

The grid cell model suffers with most of the same issues that we discussed with respect to the head direction cells. While the calibration of constants in the system that map movement in world co-ordinates can be done by using a method based on evolutionary algorithms that was employed by Kyriacou for a one dimensional head direction attractor network[95], this would still not correct the drift problem. In order to correct drift, these neuron sets anchor to visual landmarks in biology as is discussed in the previous chapter (3.4). Implementing such a visual feature system that will project on to our attractors is a task we consider important for progress towards a complete bio-mimetic navigation system. In order to complete the bio-mimetic navigation system which the robot could use to navigate to goal locations safely, the implementation of other neuron sets such as place cells, border cells, and a reward system are also required. We set these as our future work.

Chapter 5

Discussion

At the beginning of the course, we had set out certain goals: to understand biological navigation at the lowest possible level and to use this knowledge to develop path integrator portions of a bio-mimetic navigation system. As documented in this thesis, while being close to our goal, we are not quite there yet - there is much work to be done. As such, the work done in the masters course serves as a proof of concept of various research results - that navigation via populations of neurons such as head direction and grid cells is possible - and that these methods can be used in robotic navigation in place of classical navigation techniques. As mentioned in the earlier chapters, the system in its current state is not good enough to be deployed on actual robotics. The system cannot compete with the existing navigation stack that ROS boasts of, which has been developed over years of concentrated research.

Our models are based on the available biological information on neuron sets and while neuronal structures such as neurons and synapses are incredibly complicated, for the purpose of modelling, we chose a level of abstraction appropriate for our purposes. As discussed in the previous chapters, we only used firing rate models and not more biologically plausible models such as spiking neurons; certain assumptions were also made for the sake of simplicity, while modelling these individual neurons and neural assemblies. For example, our models employ regular neuron networks - the firing characteristics of all neurons in the attractor as similar. This is not so in biology. Another important example is our use of effective synapses in the models for simplifying hebbian learning. In a more accu-

rate biological representation, their effective synapses should be differentiated. The neural navigation system in humans is known to be incredibly complex and researchers are still working on uncovering the many many details that enable us to navigate with ease. As more information on the biology of navigation is uncovered, the model will be able to evolve as well.

5.1 Challenges and future work

In this section, we briefly discuss the limitations of our work and the future work that can be done to improve the system.

Improving performance of current implementation by the fine tuning of parameters Even though our system shows characteristics of a path integrator system that would provide heading and location information, the system is not yet as accurate as systems that are built on techniques of classical robotics. As in, the movements do not accurately represent movement in the real world. In order to improve the accuracy of the system, the firing rates of the input neurons must be calibrated to correctly reflect the real world data provided by sensors. This task requires the fine tuning of the parameters used in our equations and is generally carried out by iterative observe and tune methods. Accurate tuning of the parameters would greatly improve the performance of the system, and limit the drift to manageable magnitudes.

Inclusion of visual system for drift correction and landmark navigation As discussed earlier, the inclusion of a visual feature detection system for drift correction and later, landmark navigation, will be a big step towards a complete navigation system. Drift correction is, perhaps, the most important extension that is required in the model at this stage - to improve its performance over long periods of time. The addition of a visual feature detection system is necessary for implementation of navigation strategies other than basic path integration, such as landmark navigation. A feature detection system can be based on current computer vision algorithms as was earlier discussed, or can also be a bio-mimetic model of the visual

cortex that may be implemented in the future. Of note is the fact that we do not need the path integrator component of the visual system currently, since the data provided by the inertial sensors is an accurate enough input to the system.

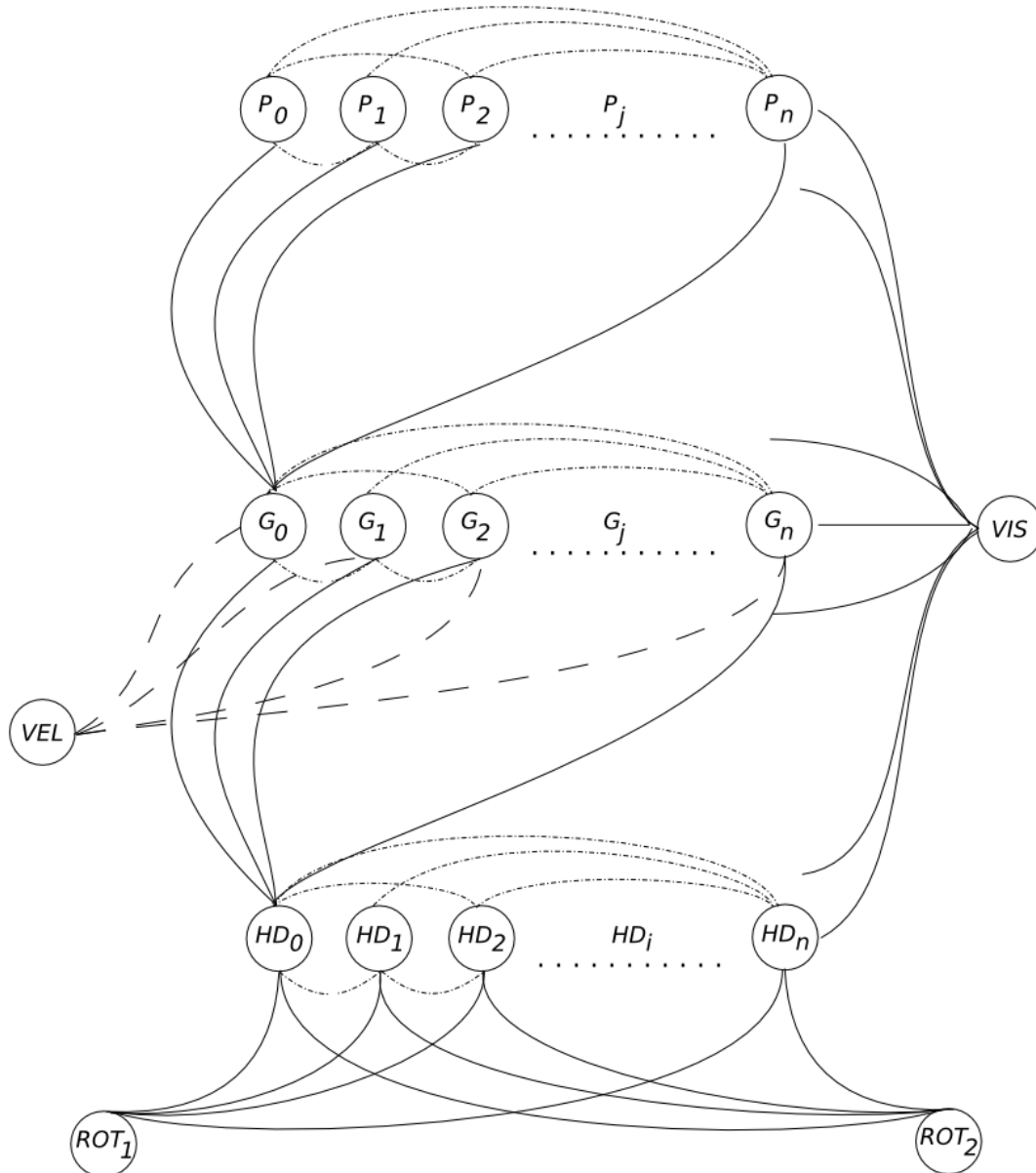


Figure 5.1: Schematic including place cells.

A complete navigation system would also need to include **place cells**[127, 128, 129, 7] as shown in figure 5.1 - place cells encode location specific in-

formation - a unique place field is formed from the firing profile of a set of place cells each time the agent visits a certain location. Place cells, like head direction and grid cells, make use of a number of inputs. Since place cells encode location, they're likely involved in the neural path planning process - for example, they can denote unique start and goal locations[130]. The modelling of place cells has been done in a manner similar to head direction and grid cells[98, 131, 132, 133, 92]. In fact, grid cells are only one synapse upstream from place cells and therefore, models where grid cells project on to place cells as an important input source have already been proposed in literature[133, 134].

5.2 Conclusion

The work detailed in this thesis is, in the authors knowledge, the first attempt at building a navigation platform that can be used on ROS that is based on biological navigation instead of classical navigation.

The main advantage of using a biological method such as ours that incorporates learning is that the system not only learns its initial behaviour, it will continue to learn about its environment once components that receive data from the environment, such as the visual feature detection system, are included. This functionality is incredibly complex to implement and is an entire research area in itself (Simultaneous localisation and mapping - SLAM) in classical robotics. SLAM focuses on building a map of the environment while the agent explores it - it generally relies on inputs that provide it with information about the boundaries in the environment - walls and so on - and generally ignores other important contextual information present in the environment. Since biological navigation systems were not designed to solve specific problems, but rather evolved with the requirements of the organism, they consist of tightly coupled multifunctional components - for example, head direction cells track the heading of the animal, but are tightly coupled with place cells via grid cells. The place cells, in turn, use direction information along with information from multiple sources to "remember" a location - not as the co-ordinates on a grid, but instead, as a location where certain features are found in the environment. This enables the agent to use feature-centric navigation which

is more likely to enable the agent to interact with features in the environment, which is another research area in classical robotics. One can say that while the biological navigation system is tightly integrated with other core systems of the agent, various systems in classical robotics solve various specific problems independently and are not integrated with each other.

Keeping in mind the difference in design between biological navigation and classical navigation that were briefly discussed in chapter 1, I certainly hope that other students and researchers will continue to work on biomimetic navigation.

Appendices

Appendix A

ROS - robot operating system

The Robot Operating System (ROS) is a collection of software packages made available for the Linux operating system related to robotics. The packages are made available under a BSD license and are therefore free and open source software (FOSS). The package set includes software for an underlying communication framework along with many robotic sub-systems - perception, motion, planning, control, grasping. These sub-systems are composed of implementations of various algorithms. The software collection also provides various development software such as simulators. Development libraries are provided for multiple programming languages - C++, Python, Lisp.

The system itself is implemented as a collection of nodes. A **ROS master** node is used for co-ordination, which all other nodes need to register with. During registration, they inform the master of the **topics** and **services** that they make available to each other. For example, the **bionav** stack node that was used in this research work **subscribed** to the **imu** node which **publishes** the **imu\data** topic. The bionav node in turn publishes **bionav/head_direction** and **bionav/grid_location** topics. The bionav stack was developed in C++.

The source code for the bionav stack is now available at <https://github.com/sanjayankur31/bionav>. Doxygen documentation can be found at <http://ankursinha.in/files/research/bionav-doc/annotated.html>.

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