

**Bayesian Reconstruction of Emission
Tomography Images using Edge-
Preserving Smoothing Priors**

A Thesis Submitted for the Degree of Doctor of Philosophy

by

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For my Mum and Dad with love

獻給我的媽媽和爸爸

永遠懷念您們

Declaration of Originality

I declare that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also declare that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I declare that all information sources and literature used are indicated in the thesis.

Signature of Candidate

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Signature removed prior to publication.

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List of Related Publications

S Som, BF Hutton, and M Braun, "Properties of minimum cross-entropy reconstruction of emission tomography with anatomically based prior," *IEEE Trans. Nucl. Sci.*, **45**:3014-3021, 1998.

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Abstract

It is common in modern medical imaging practice to correlate scans of a patient from different imaging modalities to improve accuracy of the clinical diagnosis. In nuclear medicine, it is becoming possible to enhance image quality and improve quantitative accuracy of single photon emission computed tomography (SPECT) by making use of image data provided by anatomical modalities, such as magnetic resonance imaging (MRI), in the reconstruction of SPECT images. This thesis explores and improves one such reconstruction method, the minimum cross-entropy (MXE) algorithm.

MXE is an iterative reconstruction algorithm which permits the incorporation of *a priori* information, such as anatomical edges obtained from MRI scans of the same subject. Like most Bayesian reconstruction algorithms, MXE suppresses noise and preserves edges in the reconstructed images, thereby improving edge resolution, signal to noise ratio, and accuracy of reconstruction. The use of an anatomical prior, however, only preserves anatomical edges in the reconstructed images. Furthermore, when anatomical edges in MRI scans of the same subject do not match the functional/physiological edges in the current estimate of the radionuclide distribution, it may result in blurring of the functional edges. This problem is overcome by incorporating functional edge information from the current estimate of the radionuclide distribution as a component of the MXE prior. The main challenge of this thesis is to determine the balance between anatomical and functional priors that optimises the quality of reconstruction.

A number of phantom studies were performed to investigate the performance of the MXE algorithm incorporating both anatomical information and a proposed additional prior that preserves high contrast edges in the emission data that may not coincide with anatomical edges. MXE reconstructions compared favourably with conventional maximum likelihood-expectation maximisation (ML-EM) reconstructions. MXE reconstructions not only produced images with lower noise levels and sharper edges but also generated higher recovery coefficient values when compared to the “noise-equivalent” ML-EM reconstructions. MXE reconstruction requires more iterations for a “noise-equivalent” ML-EM reconstruction, however ordered subset implementation provided acceleration that was found to cause no measurable degradation to the

reconstructed images. The phantom studies provided insight to the parameter values that should be used for optimal reconstruction.

The MXE algorithm was further assessed in a retrospective clinical study of patients with focal epilepsy where subtle changes in cerebral perfusion between seizures provided a useful model for evaluation that could be verified in ictal studies. Again, MXE compared favourably to ML-EM and also to filtered back projection (FBP). Detection of inter-ictal perfusion abnormalities was evaluated using receiver operating characteristic (ROC) analysis where MXE appeared superior to ML-EM and FBP, although the difference in areas under the ROC curves was not statistically significant. Wilcoxon's matched-pairs signed-ranked tests from region of interest analysis, however, did indicate significant differences in favour of MXE. This study is the first demonstration that MXE reconstruction with priors can influence clinical interpretation.

Evidence is provided to support the use of the MXE algorithm as a useful reconstruction technique that easily incorporates prior information from multiple sources.