PRICING AMERICAN OPTIONS USING FOURIER ANALYSIS

A Thesis Submitted for the Degree of

Doctor of Philosophy

by

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in

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Certificate

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Contents

Abstract

The analytic expression for an American option price under the Black-Scholes model requires the early exercise boundary as one of its inputs, and this is not known a priori. An implicit integral equation can be found for this free boundary, but it has no known closed-form solution, and its numerical solution is highly non-trivial. This has given rise to a number of analytical solution methods and numerical techniques designed to handle the early exercise feature.

The aim of this thesis is to explore Fourier-type solution methods for pricing American options. The price is defined as a free boundary value problem, whose solution satisfies the Black-Scholes PDE with certain final and boundary conditions. This problem is solved using the incomplete Fourier transform method of McKean (1965). The method is generalised to American options with monotonic and convex payoffs in a systematic way, and is further extended by applying it to solve the PIDE for the American call option under Merton's (1976) jump-diffusion model. In this case numerical integration solutions require an intense level of computation. The thesis considers the Fourier-Hermite series expansion method as an alternative approach. This is extended to allow for jump-diffusion with log-normally distributed jump sizes. The main contributions of the thesis are:

- *Evaluation of American Options under Geometric Brownian Motion - Chapters 2 and 3.* The details of McKean's (1965) incomplete Fourier transform are provided for a monotonic payoff function, and several forms for the price and free boundary are reproduced in the case of an American call. A numerical scheme for implementing the equations is given, along with a comparison of several existing numerical solution methods. The applicability of the transform technique to more general payoff types is demonstrated using an American strangle position with interdependent component options. A coupled integral equation system for the two free boundaries is found and solved using numerical integration. The resulting free boundaries are consistently deeper in-the-money than those for the corresponding independent American call and put.
- *Pricing American Options under Jump-Diffusion - Chapter 4.* The incomplete Fourier transform method is applied to the jump-diffusion model of Merton (1976). The PIDE for an American call is solved, and the results are simplified to replicate the integral equations of Gukhal (2001) for the price and free

boundary. An implicit expression for the limit of the free boundary at expiry is derived, and an iterative algorithm is presented for solving the integral expressions numerically. The results are demonstrated to be consistent with existing knowledge of American options under jump-diffusion, and display behaviour that is consist with market-observed volatility smiles.

- *Fourier-Hermite Series Expansions for Options under Jump-Diffusion - Chapter 5.* The Fourier-Hermite series expansion method is extended to the jump-diffusion model of Merton (1976) in the case where the jump sizes are log-normally distributed. With the aid of a suitably calibrated scaling parameter, the method is used to evaluate American call options. The pricing accuracy of this approach is shown to be comparable to both the iterative numerical integration method, and the method of lines technique by Meyer (1998). The series expansion method displays a high computation speed in exchange for some loss of accuracy in the free boundary approximation.