Long wavelength behavior of the fundamental mode in microstructured optical fibers

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Abstract: Using a novel computational method, the fundamental mode in index-guided microstructured optical fibers with genuinely infinite cladding is studied. It is shown that this mode has no cut-off, although its area grows rapidly when the wavelength crosses a transition region. The results are compared with those for w-fibers, for which qualitatively similar results are obtained.

OCIS codes: (060.2430) Fibers, single-mode; (060.2280) Fiber design and fabrication

References and links

1. A.W. Snyder and J.D. Love, *Optical waveguide theory* (Chapman and Hall, London, 1983).
2. A. Bjarklev, J. Broeng, and A.S. Bjarklev, *Photonic crystal fibers* (Kluwer, Boston, 2003).
3. B.T. Kuhlmey, R.C. McPhedran, and C.M. de Sterke, “Modal ‘cutoff’ in microstructured optical fibers,” Opt. Lett. **27**, 1684-1686 (2002).
4. B.T. Kuhlmey, R.C. McPhedran, C.M. de Sterke, P.A. Robinson, G. Renversez, and D. Maystre, “Microstructured optical fibers: where is the edge?,” Opt. Express **10**, 1285-1291 (2002), http://www.opticsexpress.org/abstract.cfm?URI=OPEX-10-22-1285.
5. S. Wilcox, L.C. Botten, R.C. McPhedran, C.G. Poulton, and C.M. de Sterke, “Exact modelling of defect modes in photonic crystals,” in press, Phys. Rev. E.
6. L. C. Botten, N. A. Nicorovici, R. C. McPhedran, C. Martijn de Sterke, and A. A. Asatryan, “Photonic band structure calculations using scattering matrices,” Phys. Rev. E **64**, 046603:1–20 (2001).
7. S. Kawakami and S. Nishida, “Characteristics of a doubly clad optical fiber with a low-index inner cladding,” J. Quantum Electron. **10**, 879-887 (1974).
8. M. Koshiba and K. Saitoh, “Applicability of classical optical fiber theories to holey fibers,” Opt. Lett. **29**, 1739-1741 (2004).
9. T.A. Birks, J.C. Knight, and P.St .J. Russell, “Endlessly single-mode photonic crystal fiber,” Opt. Lett. **22**, 961-963 (1997).
10. N.A. Mortensen, J.R. Folkenberg, M.D. Nielsen, and K.P. Hansen, ”Modal cutoff and the V parameter in photonic crystal fibers,” Opt. Lett. **28**, 1879-1881 (2003).
11. R.C. McPhedran, L.C. Botten, A.A. Asatryan, N.A. Nicorovici, C.M. de Sterke, and P.A. Robinson, “Ordered and disordered photonic bandgap materials,” Aust. J. Phys. **52**, 791-809 (1999).

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12. M. Yan and P. Shum, "Antiguiding in microstructured optical fibers," Opt. Express **12**, 104-116 (2004), http://www.opticsexpress.org/abstract.cfm?URI=OPEX-12-1-104.

1. Introduction

Arguably the most important property of an optical fiber is its bound mode spectrum, indicating the fibers' eigenfrequencies and associated eigenfields versus wavelength. Using calculations that are based on the Maxwell equations, and taking the fiber cladding to be of infinite extent, it is found that at short wavelengths, conventional fibers support many bound modes. However, they are progressively cut off with increasing wavelength λ, so that at sufficiently long wavelengths only a single, *fundamental* mode, possibly degenerate, remains. Since almost all optical fibers operate in this single mode regime, the fundamental mode is the most important bound mode. In most conventional optical fibers the fundamental mode itself is not cut off, *i.e.*, it remains bound as λ approaches infinity [1].

Microstructured optical fibers (MOFs), fibers in which the cladding has low-index holes that run parallel to the fiber axis, have attracted much recent attention [2]. The theory of the mode structure of these fibers is not so well understood since, until now, calculations with an infinite cladding have not been performed. Rather, in some calculations the cladding is taken to be finite, while in others a supercell is used, by which the structure is artificially made periodic [2]. Both approaches can be justified in certain situations, but they are not directly suitable to determine the possible cutoff behavior of the fundamental mode: at long wavelengths the extent of the mode becomes arbitrarily large, larger than any artificial period or any cross section of finite extent that can be computationally accommodated. The long-wavelength behavior of the second MOF mode was investigated by extrapolating its properties for a finite cladding, to infinite structures, confirming that this mode does cut off [3]. Some of us used a similar approach to study the long wavelength properties of the fundamental mode, and found that in MOFs with finite cladding, the modal area is limited to the core at short wavelengths, and, for any finite cladding, covers the entire cladding cross section at long wavelengths. Between these extremes, the mode area grows strongly with wavelength, which is associated with a rapidly increasing confinement loss. Extrapolation suggests that, in contrast with the second mode [3], the transition region's width remains finite as the cladding becomes infinite. The transition region was interpreted as the cut-off of the MOF's fundamental mode [4].

To determine theoretically the long-wavelength behavior of the fundamental MOF mode definitively, we need a method to calculate the modes in structures with a genuinely infinite cladding. Recently, we developed such a tool—the *fictitious source superposition* (FSS) method. We do not detail it here but outline the three concepts underpinning it [5]. The first is the use of fictitious sources which, when placed within any cylinder, can compensate exactly the reflected field generated by an incident field. It can thus generate an exterior field identical to that in the cylinder's absence, as in a defect. While such a calculation is simple for a single cylinder, the interrelationships between the infinity of scatterers makes the solution of the problem for a non-periodic structure very difficult. Instead, we construct the defect mode from a superposition of quasiperiodic solutions, each corresponding to fictitious multipole sources $\boldsymbol{q}_p = \boldsymbol{q}\exp(i\boldsymbol{k}_0 \cdot \boldsymbol{r}_p)$, where $\boldsymbol{k}_0$ is the Bloch vector, embedded in each cylinder (p) of the lattice at $\boldsymbol{r} = \boldsymbol{r}_p$. The superposition is then formed by Brillouin zone (BZ) integration, thus generating a solution that satisfies the wave equations and boundary conditions, and which is associated with the fictitious source distribution $\boldsymbol{q}\int_{\mathrm{BZ}} \exp(i\boldsymbol{k}_0 \cdot \boldsymbol{r}_p)d\boldsymbol{k}_0$ at each cylinder. The BZ integration thus eliminates all sources except the one within the cylinder at $\boldsymbol{r} = \boldsymbol{r}_0 = \boldsymbol{0}$, which remains available to modify the response field, and thus to form the defect mode. As described, the BZ integration is two-dimensional and thus time consuming. The third key idea reduces the problem to a single integration: we model the structure as a diffraction grating, whose cylinders contain a phased line of sources, sandwiched between two semi-infinite photonic crystals, the actions of which are modelled by the Fresnel reflection matrix $\boldsymbol{R}_\infty$ [6]. This new field problem is quasi-periodic in one dimension, with the quasiperiodicity of the second dimension captured through $\boldsymbol{R}_\infty$. The grating is characterized by plane wave reflection and transmission scattering matrices, calculated using a multipole method [6] that encapsulate the quasiperiodic contributions to the scattered field (due to each cylinder) through lattice sums. By integrating over the one-dimensional BZ [5], we deduce a resonance condition. From this we find for each wavelength λ one or more associated effective indices n_{eff}, and the modal fields.

If a MOF with infinite cladding can be compared to a step-index fiber, then the equivalent of a MOF with finite cross section is a w-fiber [7] in which the cladding is a low-index ring between a high-index core and a high-index region extending to infinity. Here we therefore also analyze w-fibers and compare them to MOFs with a cladding of finite cross section.

2. Results

The results below refer to a MOF with circular air holes located on an infinite hexagonal lattice with period Λ and hole diameter $d = 0.24\Lambda$, with a single defect, constituting the core. The refractive index of the background glass is $n_b = 1.45$. Following the literature for conventional fibers the normalized frequency V and propagation constant U can be defined as [1, 8]

$$\begin{aligned} V &\equiv \frac{2\pi}{\lambda}\rho\sqrt{n_{co}^2 - n_{cl}^2} = \frac{2\pi}{\lambda}\frac{\Lambda}{\sqrt{3}}\sqrt{n_b^2 - n_{\mathrm{fsm}}^2}, \\ U &\equiv \frac{2\pi}{\lambda}\rho\sqrt{n_{co}^2 - n_{\mathrm{eff}}^2} = \frac{2\pi}{\lambda}\frac{\Lambda}{\sqrt{3}}\sqrt{n_b^2 - n_{\mathrm{eff}}^2}, \end{aligned} \tag{1}$$

where we first give the conventional definition [1], followed by the application to MOFs of Koshiba and Saitoh [8]. Here ρ is the core diameter, which is taken to be $\rho = \Lambda/\sqrt{3}$ [8], n_{fsm} is the effective index of the fundamental space filling mode [9], and n_{eff} is the fundamental mode's effective index as given by the FSS method [5]. Since for a bound mode $n_{\mathrm{fsm}} < n_{\mathrm{eff}}$, $U < V$. Definitions (1) differ from those of Mortensen *et al.* [10], whose work applies to the second MOF mode, rather than the first. Using the FSS code to calculate n_{eff} and a conventional multipole code to determine n_{fsm} we obtain the solid curve in Fig. 1; because of definitions (1), the wavelength increases in the direction indicated by the large diagonal arrow. Note first the behavior at long wavelengths, where $U \approx V$. From (1) this implies that $n_{\mathrm{eff}} \approx n_{\mathrm{fsm}}$, and so the modal effective index is determined by the effective index of the cladding. Indeed, at these long wavelengths the mode extends well into the cladding. The longest wavelength for which we can obtain a result is $\lambda = 1.6\Lambda$, corresponding to $V \approx 0.6$. Here n_{fsm} and n_{eff} are very close, with $n_{\mathrm{eff}} - n_{\mathrm{fsm}} \approx 10^{-6}$. Therefore, the (average) transverse wavenumber $k_\perp = (n_{\mathrm{eff}}^2 - n_{\mathrm{fsm}}^2)^{1/2}$ is very small; in fact at $V = 0.6$, $k_\perp\Lambda \approx 7 \times 10^{-3}$. The evaluation of the lattice sums that are used in the FSS method, for the multipole calculation of the grating scattering matrices, now becomes increasingly difficult, as their magnitudes scale as powers of $1/k_\perp$ [11].

At shorter wavelengths $U < V$, so that, by Eqs (1), $n_{\mathrm{fsm}} < n_{\mathrm{eff}}$, implying that a substantial fraction of the mode's energy is in the core. The solid circles refer to $\lambda/\Lambda = 0.25, 0.15, 0.05$ from left to right. These are approximately equally spaced in V, implying that U and V remain finite, even as $\lambda \to 0$. This is because in this limit, $n_{\mathrm{fsm}}, n_{\mathrm{eff}} \to n_b$, which, consistent with the argument of Birks *et al.*, leads to the MOFs' well-known endlessly single-mode behavior [9].

The fundamental modes of MOFs with a finite cross section were calculated earlier [4] and we find that the real part of the effective mode index of finite structures $n_{\mathrm{eff}}^{\mathrm{f}}$, where here and below the superscript "f" refers to finite structures, satisfies $n_{\mathrm{eff}}^{\mathrm{f}} < n_{\mathrm{eff}}$. This can be understood as follows: the finite cladding reduces the degree of confinement and the effective index is thus smaller than in structures with an infinite cladding. As mentioned previously, based on the prop-

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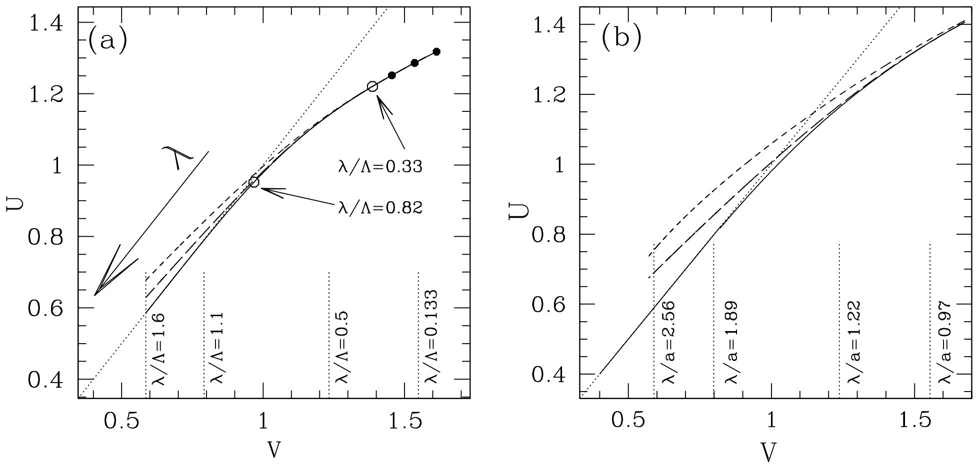

Fig. 1. Parameter U versus V for (a) MOF with parameters given in the text. Solid curve refers to a structure with infinite cladding. The three closed circles correspond to wavelengths $\lambda = 0.25\Lambda,\ 0.15\Lambda$ and 0.05Λ from left to right. The open circles correspond to the wavelengths indicated. The long (short)-dashed curve is the equivalent result for a MOF with a finite cross section with five (three) rings of holes. Dotted curve, included for convenience, gives $U = V$. (b) Same as (a) but for a conventional geometry with parameters given in the text. The points indicated by the circles in (a) have no equivalent here.

erties of MOFs of finite cross section, some of us previously concluded that the fundamental MOF mode does have a cutoff [4]. This was based on the existence of a *transition region*, the extent of which was deduced from extrapolation [4]. The approximate positions of the edges of this transition region are indicated by the open circles in Fig. 1(a).

The fields at four different frequencies for a structure with three rings of holes are shown in the top row in Fig. 2. The four wavelengths, $\lambda/\Lambda = 0.133$, corresponding to $V = 1.55$ (first column), $\lambda/\Lambda = 0.50$ ($V = 1.23$; second column), $\lambda/\Lambda = 1.1$ ($V = 0.79$; third column) and $\lambda/\Lambda = 1.6$ ($V = 0.58$, fourth column), are also indicated by the vertical lines in Fig. 1(a). As a comparison, we show in the bottom row, the modal field at the same frequencies in a MOF with infinite cross section. At the shortest wavelength $\lambda/\Lambda = 0.133$ the mode is well confined to the core region and the fields in the MOFs with finite and infinite cross sections are very similar. We thus expect n_{eff} to be very similar as well, as is confirmed in Fig. 1(a). In contrast, at the longest wavelength $\lambda/\Lambda = 1.6$ the mode is poorly confined. In the finite MOF it fills essentially the entire MOF cross section, with the mode size limited by antiguiding at the interface at the outer cladding [12]. In the MOF with infinite cross section, which does not have this interface, the mode extends even further. We expect the modes in the two geometries to have significantly different propagation constants at this wavelength. This is confirmed in Fig. 1(a) which shows that $U^{\mathrm{f}} > V$, indicating that $n^{\mathrm{f}}_{\mathrm{eff}} < n_{\mathrm{fsm}}$. This means that the field is not evanescent in the cladding region. This is consistent with the mode's poor confinement and indicates strong confinement losses in the finite structure. At the two remaining wavelengths the mode exhibits intermediate behavior: at $\lambda/\Lambda = 1.1$ we also find that $n^{\mathrm{f}}_{\mathrm{eff}} < n_{\mathrm{fsm}}$, with the mode no longer confined to the core, but antiguided by the outer cladding [12]. At $\lambda/\Lambda = 0.50$ the mode is just starting to lose confinement and is somewhat more extended than at $\lambda/\Lambda = 0.133$.

We return now to the transition region that was identified earlier[4]. Even though the details of the modal properties in finite MOFs depend on the number of rings that surround the core, on the short wavelength side of the transition region the mode is confined to the MOF core, whereas on the long wavelengths side the mode covers most, or even the entire (finite) MOF cross section. This, though it does not indicate cut-off, shows where the mode cross section increases from the core to, essentially, the entire cross section. For wavelengths beyond the transition region the mode becomes poorly confined and in practice is no longer very useful: it easily couples to cladding modes, and is (anti)guided by the outer cladding boundary.

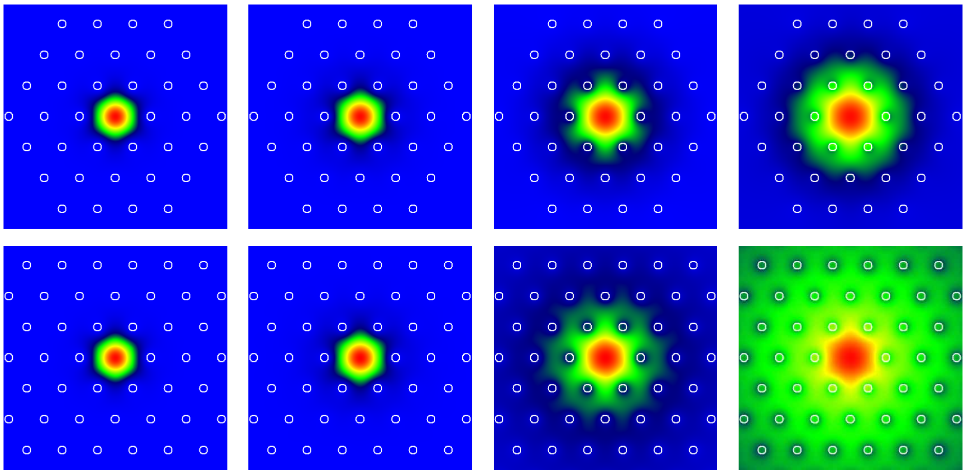

Fig. 2. Axial Poynting vector for a MOF with finite cross section with three rings of holes (top row), and an infinite cross section (bottom row), for $V = 1.55$ ($\lambda/\Lambda = 0.133$), $V = 1.23$ ($\lambda/\Lambda = 0.50$), $V = 0.79$ ($\lambda/\Lambda = 1.1$), and $V = 0.58$ ($\lambda/\Lambda = 1.6$) for the first, second, third and fourth columns, respectively. The small circles indicate the air holes.

The results for the MOF in Fig. 1(a) should be compared with results for a conventional fiber, shown in Fig. 1(b). The numerical values for U and V on the axes are very similar to those in Fig 1(a) indicating that the choice $\rho = \Lambda/\sqrt{3}$ [8] is appropriate. The solid curve in this figure refers to the fundamental mode in a step-index fiber with radius $a = 1$ μm, $n_{\text{core}} = 1.45$ and $n_{\text{clad}} = 1.43$. It is similar to that in Fig. 1(a), except that now $V \rightarrow \infty$ as $\lambda \rightarrow 0$. The dashed curves again apply to fibers with a finite cladding, here of radius 5 μm (long-dashed curve) and 3 μm (short-dashed curve). For these fibers the part of the cross section outside these radii was taken to have a refractive index n_{core}, as in MOFs. Note that the curves in Figs 1 are qualitatively similar, confirming that the cut-off behavior for MOFs and for conventional fibers is comparable. Note also the difference at short wavelengths, where $V \rightarrow \infty$. Finally, Figs 3 are the equivalent of Figs 2, but for conventional fibers. They show the Poynting vector for a w-fiber with outer cladding radius of 3 μm (top row) and a step index fiber with infinite cladding (bottom row) for the same V-values as in Fig. 2. As for the MOF, at the longest wavelength where $V = 0.58$, the field is not confined to the core, but does not extend as far in the w-fiber as in the fiber with infinite cladding [12]. In contrast, at the shortest wavelength where $V = 1.55$ the field is well confined to the core in both fibers and the finite extent of the cladding is irrelevant. At $V = 0.79$ the field is still confined to the core, while at $V = 1.23$ the wavelength is sufficiently large to extend well beyond the core, approaching the cladding's outer edge.

3. Discussion and conclusions

We have examined the long-wavelength behavior of the fundamental mode in a MOF with infinite cross section using a newly developed numerical method. While previously, based on the properties of MOFs with finite cross section, it was predicted that this mode cuts off at long wavelengths, we have seen no evidence of this up to wavelengths $\lambda/\Lambda \approx 1.6$. Since this is well

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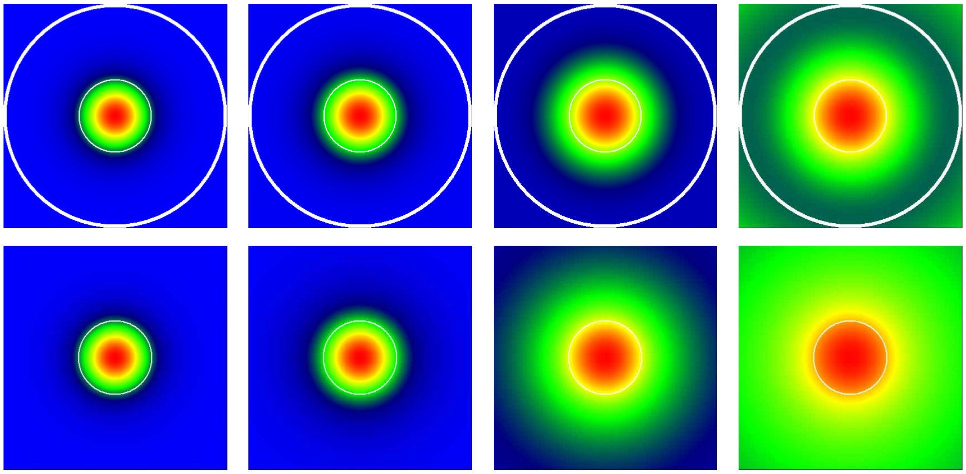

Fig. 3. Axial Poynting vector for a w-fiber (top row), and a step-index fiber with infinite cladding (bottom row), for $V = 1.55, 1.23, 0.79, 0.58$ for the first, second, third and fourth column, respectively. The cladding's inner and outer edges are indicated by white circles.

beyond the predicted cutoff wavelength we conclude that the fundamental mode for a MOF with infinite cross section does not cut off. We have compared the MOF results with those for a conventional w-fiber, and find qualitatively the same behavior.

The analogy with conventional fibres can be straightforwardly analyzed quantitatively; even at $\lambda/\Lambda = 1.6$, $n_b - n_{\text{fsm}} \approx 0.023$, and we can therefore understand the modes' dominant field component using the scalar approximation [1]. As mentioned, at this wavelength $k_\perp \Lambda \approx 7 \times 10^{-3}$, and so, on average, we expect the field of the fundamental mode to vary in the cladding as $\mathrm{K}_0(k_\perp r)$, where K_0 is a modified Bessel function. We have confirmed this numerically. As a corollary, we may use this approximation to estimate the effective mode size. Solving

$$\left(\frac{\mathrm{K}_0(k_\perp r_{1/2})}{\mathrm{K}_0(k_\perp \rho)} \right)^2 = 0.5, \tag{2}$$

we find the approximate radius $r_{1/2}$ where the average field intensity decays to half the value at the core edge $\rho = \Lambda/\sqrt{3}$ (see Eqs (1)). At $\lambda/\Lambda = 1.6$, we find $r_{1/2} > 200\Lambda$. This means that the mode field extends over hundreds of periods; to our knowledge, no conventional calculational method exists that can be used to calculate such large modes.

Having established the absence of a cut-off for the fundamental MOF mode, the estimation of the mode field diameter of Koshiba and Saitoh [8] can now be used at *any* wavelength to determine the number of rings of holes required for finite cladding effect in MOFs to be negligible. Note that this method, based upon the analogy between MOFs and conventional fibers, requires accurate knowledge of the effective cladding index n_{fsm}.

The analogy between MOFs and conventional fibers was earlier established heuristically, based on the cutoff of the second mode and on some of the properties of the fundamental mode such as the mode field diameter [8, 10]. Our findings here for the absence of a cut-off for the fundamental mode in fibers with an infinite cladding, and the cut-off behavior of this mode in fiber with a finite cladding, provide precise and independent evidence for this analogy.

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