

**Novel Bayesian Smoothing Algorithms
for Improved Track Initiation and
Maintenance in Clutter**

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CERTIFICATE OF AUTHORSHIP OR ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text. I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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ABSTRACT

Target tracking is a well established field with over fifty years of intense research. While in its core, it deals with estimating targets dynamic states, it is also a critical component of all "Situation Awareness" and threat assessment systems. These higher layer applications take decisions on important questions like number of targets, positions of them, the instant and position of their initiation, the instant and position of their maneuvers and above all, which of them are threatening and/or friendly. The lower level target tracking algorithms feed the necessary information to these decision taking systems.

There are a number of target tracking algorithms to cater for the need of such systems. Most of these available algorithms are based on filtering theory. But it is established that smoothing increases the accuracy of the systems at the expense of a slight lag between the instant of estimation and the instant at which the parameter of interest is being estimated. Hence smoothing is not widely used for practical target tracking applications.

However, the situation awareness system is expected to perform better if more precise information is obtained about initiation and termination of the targets along with improved discrimination of true/false targets.

This thesis addresses the problem of improved track initiation and maintenance with the smoothing framework to provide better information. It first reviews target tracking and filtering literature. It introduces the concept of random set smoother and derives the IPDA smoother under linear Gaussian assumption. IPDA smoother is also derived by extending the PDA smoother. Finally a theoretical link is established between Random Set smoothing and IPDA smoothing framework. To extend the domain into multiple sensor scenario, the problem of out-of-sequence measurements is also addressed in this thesis under target existence uncertainty.

Several realistic scenarios are simulated and the results are verified.