UNIVERSITY OF TECHNOLOGY SYDNEY

Microwave Image Reconstruction of 3-D Dielectric Scatterers via Stochastic Optimization Approaches

By

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CERTIFICATE OF AUTHORSHIP / ORIGINALITY

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ABSTRACT

The reconstruction of microwave images is generally considered as a nonlinear and ill-posed inverse scattering problem. Such problems are generally solved by the application of iterative numerical methods. However, the accuracy of images reconstructed by traditional methods is heavily dependent on the choice of the initial estimate used to solve the problem. Thus, with the aim to overcome this problem, this research work has reformulated inverse problems into global optimization problems and investigated the feasibility of solving such problems via the use of stochastic optimization techniques. A number of global inverse solvers have been implemented using different evolutionary strategies, namely the rivalry and cooperation strategies, and tested against a set of imaging problems involving 3-D lossless and lossy scatterers and different problem dimensions. Our simulation results have shown that the particle swarm optimization (PSO) technique is more effective for solving inverse problems than techniques such as the genetic algorithms (GA) and micro-genetic algorithms (μGA). In addition, we have investigated the impact of using different PSO neighborhood topologies and proposed a simple hybrid boundary condition to improve the robustness and consistency of the PSO technique. Furthermore, by examining the advantages and limitations of each optimization technique, we have proposed a novel optimization technique called the micro-particle swarm optimizer (μPSO). With the proposed μPSO, excellent optimization performances can be obtained especially for solving high dimensional optimization problems. In addition, the proposed μPSO requires only a small population size to outperform the standard PSO that uses a larger population size. Our simulation results have also shown that the μPSO can offer a very competitive performance for solving high dimensional microwave image reconstruction problems.

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LIST OF ACRONYMS AND SYMBOLS

ABC	Absorbing boundary condition
ADI-FDTD	Alternating direct implicit finite-difference time-domain
BIM	Born iterative method
CT	Computed tomography
DBIM	Distorted Born iterative method
DT	Diffraction tomography
EM	Electromagnetic
FDTD	Finite-difference time-domain
FEM	Finite element method
FMM	Fast multipole method
GA	Genetic algorithm
GCPSO	Guaranteed convergence particle swarm optimizer
GLM	Gel'fand-Levitan-Marchenko
MGM	Modified gradient method
MLFMA	Multilevel fast multipole algorithm
MoM	Method of moment
MRTD	Multiresolution time-domain
NKM	Newton Kantorovich method
OF	Objective function
OUI	Object under investigation
PEC	Perfectly electrically conducting
PML	Perfectly matched layer
PSO	Particle swarm optimization
PSTD	Pseudospectral time-domain
RCS	Radar cross section
RMSE	Root mean square error
SNR	Signal-to-noise ratio
UCA	Uniform circular array
UWB	Ultra-wideband
μGA	Micro-genetic algorithm
μPSO	Micro- particle swarm optimizer
$\phi(\boldsymbol{r})$	Total phase
$\phi_0(\boldsymbol{r})$	Incident phase
$\phi_1(\boldsymbol{r})$	Scattered phase

$\overline{G}(\boldsymbol{r},\boldsymbol{r}')$	dyadic Green's function
n_δ	Change in refractive index
$\psi(\boldsymbol{r})$	Total field
$\psi_0(\boldsymbol{r})$	Incident field
λ	Wavelength
μ	Permeability
σ	Conductivity
ω	Angular frequency
β	A parameter used to adjust the increment of the inertia weight
α	Constant
φ, φ_1 and φ_2	Constants
$\rho(t)$	Scaling factor
Δx, Δy and Δz	Dimensions of a FDTD cell
a	Radius of a homogeneous cylinder
c_1 and c_2	Acceleration constants
D	Dynamic range of the problem space
$\boldsymbol{E}$	Total electric field
$\boldsymbol{E}_{inc}$	Incident field
$\boldsymbol{E}_s$	Scattered field
$gbest$	Global best neighborhood topology
g_{best}	Best solution found by the entire swarm
H	Schema
$\boldsymbol{H}$	Total magnetic field
k	Wavenumber
k	Constriction factor
L	Length of the chromosome
$lbest$	Local best neighborhood topology
m	A parameter used to adjust the µPSO repulsion
N	Problem dimension
N_{bit}	Number of bits per gene
N_{chro}	Number of chromosomes
N_{gene}	Number of genes per chromosome
$N_{i,best}$	Best solution found by the neighborhood of the i^{th} particle
N_{sub}	Size of the sub-population
$O(\boldsymbol{r})$	Inhomogeneity
P_{cross}	Crossover probability
$p_{i,best}$	Best solution found by the i^{th} particle
P_{mut}	Mutation probability
r_1, r_2 and r_3	Uniformly distributed random variables in the range of [0,1]
r_3	Uniformly distributed random variable in the ranges of $[1,N_{chro}]$
r_4	Uniformly distributed random variable in the ranges of $[1,L]$

$\boldsymbol{rep}_i$	Amount of repulsion experienced by the i^{th} particle
s_{th} and f_{th}	threshold values for the success and failure, respectively
$u_{\max}$	Maximum wave phase velocity
$V_i(t)$	Velocity of the i^{th} particle
$V_{\max}$	Maximum velocity
w	Inertia weight
w_{init}	The initial value of the inertia weight
w_{max}	Maximum allowed value for the inertia weight
$X_i(t)$	Position of the i^{th} particle
ε	Permittivity