Compressive Membrane Action in Reinforced Concrete Beams

by

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TO MY WIFE, IRENE

AND

MY DAUGHTER, ARNICA

CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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* The author recently changed his surname from “Farhang Vesali” to “Vessali”.

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Table of Contents

Certificate of Original Authorship i

Acknowledgement ii

List of Publications iii

Table of Contents v

List of Figures ix

List of Tables xxviii

Abstract xxx

Nomenclature xxxii

Chapter 1 Introduction 1

1.1. Overview 1

1.2. Outline of the Research 2

1.3. Research Objectives 3

1.4. Structure of Dissertation 3

Chapter 2 Literature Review 6

2.1. Introduction 6

2.2. Experimental Studies 8

2.2.1. Two-way RC slabs 9

2.2.2. One-way RC slabs and beams 38

2.3. Analytical Studies 68

2.3.1. Simplified methods 68

2.3.2. Numerical Simulations 89

2.4. Summary 96
2.4.1. Experimental Studies 96
2.4.2. Analytical Studies 96
2.4.3. Conclusion 98
Chapter 3 Framework of the Generic Model for Investigation of Arching Action Reinforced Concrete 1D Elements 99
3.1. Introduction 99
3.2. Formulation of generic model 100
3.2.1. Strain-displacement compatibility equations 100
3.2.2. Equilibrium equations and constitutive law of material 101
3.3. Verification of the model 107
3.4. Conclusions and discussions 119
Chapter 4 Experimental Study on Development of Arching Action in Longitudinally Restrained RC Beams with Low-Ductility Reinforcement 120
4.1. Introduction 120
4.2. Experimental program 121
4.2.1. Geometry and bar arrangement 122
4.2.2. Material Properties 123
4.2.3. Preparation of the specimens 127
4.2.4. Test setup 132
4.2.4.1. Boundary condition and loading setup 132
4.2.4.2. Instrumentation 134
4.2.5. Testing Procedure 135
4.3. Test Results 136

4.4. Conclusions and Discussions 149

Chapter 5 Experimental Study on Development of Membrane Actions in Longitudinally Restrained RC Beams with Normal-ductility Reinforcement .151

5.1. Introduction 151

5.2. Experimental Program..153

5.2.1. Geometry and reinforcing bar arrangement..153

5.2.2. Material Properties..156

5.2.3. Preparation of the specimens 159

5.2.4. Test setup 161

5.2.4.1. Boundary condition and loading setup..161

5.2.4.2. Instrumentation and data acquisition 161

5.2.5. Test Procedure 164

5.3. Test Results 164

5.4. Conclusions 190

Chapter 6 Non-Linear Finite Element Analysis of Longitudinally Restrained Reinforced Concrete Sub-Assemblages 193

6.1. Introduction 193

6.2. Continuum-based FE models 194

6.2.1. Element formulation 194

6.2.2. Constitutive law of materials 195

6.2.3. Element Type..205

6.2.4. Validation of 2D Continuum-based Finite Element Model..216

6.3. 1D Frame FE model including the effects of bar fracture..220

6.3.1. Introduction..220

6.3.2. Enhanced 1D Frame Finite Element 221
6.3.3. Verification of Enhanced 1D Frame FE model 224
6.4. Numerical study of the tested sub-assemblages 235
6.4.1. Specimens with low-ductility reinforcement 235
6.4.2. Specimens with normal-ductility reinforcement 242
6.5. Conclusions 262
Chapter 7 Conclusion 265
7.1. Experimental Study 265
7.2. Numerical Simulation 267
7.3. Recommendations for Future Study 269
References 271

List of Figures

Figure 2-1: Typical Load-Deflection curves of RC flexural members with and without support restraint against elongation. 7

Figure 2-2: Schematic outline of the idealized mode of failure in plastic analysis of two-way slabs under uniform distributed load (Johansen, 1943)...10

Figure 2-3: Experimentally observed mode of failure in two-way slabs under uniform distributed load at (a) the upper surface and (b) the soffit of the slab (Ockleston, 1955)........11

Figure 2-4: Outline of the slab testing rig (a) plan and (b) Sections A-A and B-B (Powell, 1956, Park, 1964c)...12

Figure 2-5: Edge support details (a) fully fixed edge, (b) simply supported edge (Park, 1964c)...14

Figure 2-6: Load-deflection curve of type 2 slabs (Park, 1964c). 15

Figure 2-7: The effect of sustained loading and partial stiffness of supports (Park, 1964c, Park, 1964b). 16

Figure 2-8: Details of loading and supports for slabs tested by Park (1965)...........................18

Figure 2-9: Details of configuration, loading and supports of the model floor tested by Park (1971)...19

Figure 2-10: Applied pressure vs. mid-span deflection for (a) 76 mm thick slabs and (b) 121 mm thick slabs (Kennan, 1969). 23

Figure 2-11: Applied pressure vs. mid-span deflection slabs with span-to-depth ratio of 20 for (a) Type I and (b) Type III slabs (Brotchie and Holley, 1971). 24

Figure 2-12: Applied pressure vs. mid-span deflection slabs with span-to-depth ratio of 10 for (a) Type I and (b) Type III slabs (Brotchie and Holley, 1971). 25

Figure 2-13: Applied pressure vs. mid-span deflection of slabs with span-to-depth ratio of 5 for type I (Brotchie and Holley, 1971). 26

Figure 2-14: General details of the slab-beam model units (a) Plan view (b) Section A-A for slab connected to the top of the beam and (c) Section A-A for slab connected to the centreline of the beam (Datta and Ramesh, 1975). 27

Figure 2-15: Details of beams and corner connection (a) Slab at the centre of the beam (b) Slab at the top of the beam (c) Corner connection (Datta and Ramesh, 1975). 28

Figure 2-16: Load-deflection curves of slab-beam panels (Datta and Ramesh, 1975)...........30

Figure 2-17: Load vs. mid-span normalised relative deflection for (a) slabs 1-3, (b) slabs 4-7 and (c) slabs 8-11(Christiansen and Fredriksen, 1983). 33

Figure 2-18: In situ concrete slab test results (Christiansen and Fredriksen, 1983). 34

Figure 2-19: The nine possible cases of two–way rectangular slabs supported on all edges (Al-Hassani et al., 2009). 34

Figure 2-20: Experimental load expressed in terms of ultimate load predicted by Johanson’s yield line theory versus deflection of orthotropic rectangular slabs (a) S1 (b) S2 and (c) S3 (Al-Hassani et al., 2009)..35

Figure 2-21: Experimental load expressed in terms of ultimate load predicted by Johanson's yield line theory versus deflection of orthotropic rectangular slabs (a) S4 (b) S5 (c) S6 (Al-Hassani et al., 2009). 36

Figure 2-22: Experimental load expressed in terms of ultimate load predicted by Johanson's yield line theory versus deflection of orthotropic rectangular slabs (a) S7 (b) S8 (c) S9 (Al-Hassani et al., 2009). 37

Figure 2-23: (a) Elevation and (b) plan view of the specimens and test setup (Regan, 1975). 39

Figure 2-24: Catenary tests of precast floor strips (a) compressive membrane phase (b) catenary action established (c) onset of tearing out of bottom bars (d) failure (Regan, 1975). 40

Figure 2-25: Results of catenary tests of precast floor strips (Regan, 1975). 41

Figure 2-26: Applied pressure vs. mid-span deflection for (a) slab No. 2 and (b) slab No. 5 (Woodson, 1985). 41

Figure 2-27: Applied pressure vs. mid-span deflection for (a) slab No. 11 and (b) slab No. 13 (Woodson and Garner, 1985). 42

Figure 2-28: Applied pressure vs. mid-span deflection for (a) slab No. 16 and (b) slab No. 18 (Woodson and Garner, 1985). 43

Figure 2-29: Load-deflection curves for (a) slabs with span-to-thickness ratio of 10.4 and steel percentage of 0.74% and (b) slabs with span-to-thickness ratio of 14.8 and steel percentage of 1.14% (Guice, 1986, Guice and Rhomberg, 1988) 45

Figure 2-30: Sample configuration used in the tests (Lahlouh and Waldron, 1992). 46

Figure 2-31: Measured load versus mid-span deflections for the specimens tested by Lahlouh and Waldron (1992). 47

Figure 2-32: Rationale behind the adopted testing setup (a) frame and (b) sub-assemblage (Beeby and Fathibitaraf, 2001). 48

Figure 2-33: Comparison of bending moment (in the columns) obtained from experiment and elastic analysis with no membrane effects (shaded areas indicate the calculated bending moments) (Beeby and Fathibitaraf, 2001). 49

Figure 2-34: Outline of the slab strips tested by Taylor et al. (2001a). 49

Figure 2-35: Test results of slab strips (a) load vs. deflection and (b) failure load vs. concrete compressive strength (Taylor et al., 2001a). 50

Figure 2-36: Test setup for longitudinally restrained beams tested by Ruddle et al. (2002). 51

Figure 2-37: Load vs. deflection of restrained and unrestrained T-shape beams (Ruddle et al., 2002)...52

Figure 2-38: Geometry and details of the sub-assembly tested by Sasani and Kropelnicki (2008) 52

Figure 2-39: Load – mid span deflection of the sub-assembly (Sasani and Kropelnicki, 2008). 53

Figure 2-40: Vertical load and horizontal reaction versus mid-span deflection of the beams with (a) discontinuous and (b) continuous reinforcement (Orton, 2007). 54

Figure 2-41: Middle column load vs. unloading displacement (Yi et al., 2008). 56

Figure 2-42: (a) Plan and (b) elevation view of the RC beam assemblages tested by Su et al. (2009) 57

Figure 2-43: Effect of reinforcement ratio with a (a) symmetrical and (b) unsymmetrical arrangement on vertical load and horizontal reaction of the RC sub-assemblies tested by Su et al. (2009). 58

Figure 2-44: Effect of span-to-depth ratio with (a) symmetrical and (b) unsymmetrical reinforcement on vertical load and horizontal reaction of the RC sub-assemblies tested by Su et al. (2009). 59

Figure 2-45: Effect of loading rate on vertical load and horizontal reaction of the RC sub-assemblies tested by Su et al. (2009). 60

Figure 2-46: Reinforcement detailing of the RC sub-assemblages; (a) gravity load-resisting frame, (b) seismic load-resisting frame (Choi and Kim, 2011). 61

Figure 2-47: Outline of the (a) test setup for the beam-column sub-assemblages (b) supporting structure of the specimens (Choi and Kim, 2011). 62

Figure 2-48: Load-displacement relationships of the sub-assemblages (Choi and Kim, 2011). 63

Figure 2-49: Detailing and boundary conditions of the specimens tested by Yu and Tan (2012). 64

Figure 2-50: Effect of the bottom reinforcement ratio (BRR) at the middle joints on the structural behaviour of the sub-assemblages: (a) load-deflection relationship; (b) horizontal reaction force-deflection relationship (Yu and Tan, 2012). 65

Figure 2-51: Effect of the top reinforcement ratio (TRR) at the middle joints on the structural behaviour of the sub-assemblages: (a) load-deflection relationship; (b) horizontal reaction force-deflection relationship (Yu and Tan, 2012). 66

Figure 2-52: Effect of span-to-depth ratio on the structural behaviour of the sub-assemblages: (a) load-deflection relationship; (b) horizontal reaction force-deflection relationship (Yu and Tan, 2012). 67

Figure 2-53: Typical theoretical and experimental load-deflection curves for a slab restrained along its edges (Wood, 1961). 71

Figure 2-54: Idealised rectangular slab (Park, 1964c). 72

Figure 2-55: Theoretical and experimental load-deflection curves of a simply supported slab (Wood, 1961). 74

Figure 2-56: Idealised geometry of deformation of half span of laterally-restrained strip (a) Wall in deflected position (b) Geometry at support (McDowell et al., 1956). 82

Figure 2-57: Variation of arching action bending moment with respect to mid-span deflection (McDowell et al., 1956, Rankin and Long, 1997). 86

Figure 2-58: Idealised behaviour of laterally restrained RC slab strips (Rankin and Long, 1997). 86

Figure 3-1: Outline of the (a) strain distribution over the section depth (b) generic compound element (without rigid body modes). 101

Figure 3-2: Equilibrium in the simply supported configuration and free body diagram of Ax, after deformation (system without rigid body modes). 103

Figure 3-3: Outline of the adopted stress-strain relationship for (a) concrete (b) steel bars. 103

Figure 3-4: Outline of the (a) tested one-way slabs and idealised generic model (b) typical cross-section of the tested slabs (Taylor and Mullin, 2006b, Taylor et al., 2001b, Guice et al., 1986). 108

Figure 3-5: Outline of the geometry, cross-section and reinforcing details for the beam tested by Bazan (2008). 111

Figure 3-6: Load versus mid-span deflection for sample No. 11 from Guice et al. (1986) experiments. 113

Figure 3-7: Load versus mid-span deflection for sample S7 from Taylor et al. (2001b) experiments. 113

Figure 3-8: Load versus deflection of centre stub for the beam assemblage tested by Bazan (2008). 114

Figure 3-9: Outline of the idealised boundary conditions within (a) developed generic model and continuum-based FE model (b) before and (c) after loading/cracking. 114

Figure 3-10: Outline of the geometry, cross-section and reinforcing details for the subassembly H-200 tested by Lahlouh and Waldron (1992). 117

Figure 3-11: Load versus mid-span deflection for subassembly H-200 from Lahlouh and Waldron (1992) experiments....118

Figure 4-1: Outline of the floor plan. 121

Figure 4-2 Outline of the geometry, cross-section and reinforcing details for the beam assemblages with and without top add bars over the end supports and centre stub....122

Figure 4-3: Rate of increase of concrete compressive strength with time. 124

Figure 4-4: Uniaxial stress-strain diagram for concrete (300 by 150 mm diameter cylinder) under compression. 125

Figure 4-5: Uniaxial stress-strain diagram for N10 longitudinal steel rebar under tension. 125

Figure 4-6: Fracture pattern in the N10 bars (a) necking in the bar and (b) fractured section. 126

Figure 4-7: Variation of the apparent strain and the gauge factor for 5 mm steel strain gauges....127

Figure 4-8: Variation of the apparent strain and the gauge factor for 60 mm concrete strain gauges....127

Figure 4-9: Formwork parts (a) supporting and intermediate blocks and (b) channels. 128

Figure 4-10: (a) Assembled formwork and (b) cross-section of the formwork arrangement....128

Figure 4-11: Procedure and apparatus used for bending bars (a) initial and (b) final state. ..129

Figure 4-12: The reinforcing cage placed on the bar chair. 130

Figure 4-13: Slump test. 130

Figure 4-14: Concrete pouring using pump and a long hose. 131

Figure 4-15: Vibration process using the portable concrete vibrator. 132

Figure 4-16: Experimental set up for the reinforced concrete beam assemblage. 133

Figure 4-17: Schematic outline of the location of (a) steel and (b) concrete strain gauges. 134

Figure 4-18: Installation of steel strain gauges (a) gauge soldering, (b) cable soldering and (c) sealing. 135

Figure 4-19: Detailed view of the 60 mm concrete strain gauge with 1m lead wire 135

Figure 4-20: The inclinometer mounted on the end supporting block. 135

Figure 4-21: Outline of the cracks developed at sections adjacent to the centre stub (a) initiation and development stage (b) ultimate stage after rupture of the tensile bars. 138

Figure 4-22: Load versus vertical displacement of centre stub for assemblages without add bar (specimens 1 to 3). 139

Figure 4-23: Load versus vertical displacement of centre stub for assemblages with add bar (specimens 4 to 6) from test. 139

Figure 4-24: Load versus rotation of end supporting blocks for assemblages without add bar (specimens 1 to 3). 140

Figure 4-25: Load versus rotation of end supporting blocks for assemblages with add bars (specimens 4 to 6).......140

Figure 4-26: Load versus strain in tensile steel bars at different locations for specimen No. 1 (without add bar).......141

Figure 4-27: Load versus strain in tensile steel bars at different locations for specimen No. 2 (without add bar).......141

Figure 4-28: Load versus strain in tensile steel bars at different locations for specimen No. 3 (without add bar).......142

Figure 4-29: Load versus strain in tensile steel bars at different locations for specimen No. 4 (with add bar).......142

Figure 4-30: Load versus strain in tensile steel bars at different locations for specimen No. 5 (with add bar).......143

Figure 4-31: Load versus strain in tensile steel bars at different locations for specimen No. 6 (with add bar).......143

Figure 4-32: Load versus strain along tensile bars anchored in the end supporting blocks for strain gauge-1.......144

Figure 4-33: Load versus strain along tensile bars anchored in the end supporting blocks for strain gauge-2.......144

Figure 4-34: Load versus compressive strain in concrete at different locations for specimen No. 1 (without add bar).......145

Figure 4-35: Load versus compressive strain in concrete at different locations for specimen No. 2 (without add bar). 145

Figure 4-36: Load versus compressive strain in concrete at different locations for specimen No. 3 (without add bar). 146

Figure 4-37: Load versus compressive strain in concrete at different locations for specimen No. 4 (with add bar). 146

Figure 4-38: Load versus compressive strain in concrete at different locations for specimen No. 5 (with add bar). 147

Figure 4-39: Load versus compressive strain in concrete at different locations for specimen No. 6 (with add bar). 147

Figure 5-1: Outline of the floor plan and orientation of columns. 154

Figure 5-2: Outline of the geometry, cross-section and reinforcing details for the beam assemblages with and without top add bars over the end supports and centre stub..............154

Figure 5-3: Uniaxial stress-strain diagram for concrete (300 by 150 mm diameter cylinder) under compression. 157

Figure 5-4: Uniaxial stress-strain diagram for N10 longitudinal steel rebar under tension...157

Figure 5-5: The shapes of the Dramix® steel fibres (Bekaert, 2012). 158

Figure 5-6: Uniaxial stress versus elongation of Dramix® 3D, 4D & 5D fibres (Bekaert, 2012). 158

Figure 5-7: Uniaxial stress-strain diagram for SFRC with 0.5% 5D steel fibres..................160

Figure 5-8: Formwork of beam assemblages and end supporting blocks. 160

Figure 5-9: Experimental set up for the reinforced concrete beam assemblage. 162

Figure 5-10: Schematic outline of the location of (a) steel and (b) concrete strain gauges. 163

Figure 5-11: LVDT Instrumented for the measurement of the support movement. 163

Figure 5-12: Collapse of the beam assemblage due to rupture of steel bars at mid- and end-span. 165

Figure 5-13: Development of cracks, concrete crushing and bar rupture at sections adjacent to (a) centre stub and (b) end block support for beam assemblage No. 4. 165

Figure 5-14: Load versus vertical displacement of centre stub for assemblages with 60 MPa concrete grade with (a) 180 mm and (b) 80 mm stirrup spacing. 166

Figure 5-15: Load versus vertical displacement of centre stub for assemblages with 40 MPa concrete grades with (a) 180 mm and (b) 80 mm stirrup spacing. 167

Figure 5-16: Load versus vertical displacement of centre stub for assemblages with 20 MPa concrete grades with (a) conventional stirrups and (b) steel fibre reinforcement (SFRC/no stirrups). 168

Figure 5-17: Experimental peak load versus compressive strength of concrete. 169

Figure 5-18: Load versus end block rotation for assemblages with 60 MPa concrete grade (a) with 180 mm stirrups interval (b) with 80 mm stirrups spacing. 170

Figure 5-19: Load versus end block rotation for assemblages with 40 MPa concrete grade with (a) 180 mm and (b) 80 mm stirrup spacing. 171

Figure 5-20: Load versus end block rotation for assemblages with 20 MPa concrete grade with (a) conventional stirrups and (b) steel fibre reinforcement (SFRC/no stirrups). 172

Figure 5-21: Load versus end block translation (axial/along beam axis) for assemblages with 60 MPa concrete grades with (a) 180 mm and (b) 80 mm stirrup spacing. 173

Figure 5-22: Load versus end block translation (axial/along beam axis) for assemblages with 40 MPa concrete grades with (a) 180 mm and (b) 80 mm stirrup spacing. 174

Figure 5-23: Load versus end block translation (axial/along beam axis) for assemblages with 20 MPa concrete grades with (a) conventional stirrups and (b) steel fibre reinforcement (SFRC/no stirrups). 175

Figure 5-24: (a) The peak load of catenary action for RC and SFRC assemblages (Nos. 9 to 12) obtained from test data (b) crushing of concrete in the sections adjacent to end blocks (c) concrete cover cracking & spalling adjacent to centre stub after bar rupture. 177

Figure 5-25: Load versus strain in tensile steel bars at different locations for specimens (a) No. 1 and (b) No. 2. 178

Figure 5-26: Load versus strain in tensile steel bars at different locations for specimens (a) No. 3 and (b) No. 4. 179

Figure 5-27: Load versus strain in tensile steel bars at different locations for specimens (a) No. 5 and (b) No. 6. 180

Figure 5-28: Load versus strain in tensile steel bars at different locations for specimens (a) No. 7 and (b) No. 8. 181

Figure 5-29: Load versus strain in tensile steel bars at different locations for specimens (a) No. 9 and (b) No. 10. 182

Figure 5-30: Load versus strain in tensile steel bars at different locations for specimens (a) No. 11 and (b) No. 12. 183

Figure 5-31: Load versus compressive strain in concrete at different locations for specimens (a) No. 1 and (b) No. 2. 184

Figure 5-32: Load versus compressive strain in concrete at different locations for specimens (a) No. 3 and (b) No. 4. 185

Figure 5-33: Load versus compressive strain in concrete at different locations for specimens (a) No. 5 and (b) No. 6. 186

Figure 5-34: Load versus compressive strain in concrete at different locations for specimens (a) No. 7 and (b) No. 8. 187

Figure 5-35: Load versus compressive strain in concrete at different locations for specimens (a) No. 9 and (b) No. 10. 188

Figure 5-36: Load versus compressive strain in concrete at different locations for specimens (a) No. 11 and (b) No. 12. 189

Figure 6-1: The uniaxial stress-strain law for concrete (Cervenka et al., 2005). 197

Figure 6-2: Exponential crack-opening law (Hordijk, 1991). 199

Figure 6-3: Linear crack-opening law. 199

Figure 6-4: Linear softening based on local strain. 199

Figure 6-5: Steel fibre reinforced concrete (SFRC) based on fracture energy. 199

Figure 6-6: Steel fibre reinforced concrete (SFRC) based on strain. 200

Figure 6-7: Softening displacement law in compression. 201

Figure 6-8: Smeared crack concept (a) fixed-crack and (b) rotated crack model. 203

Figure 6-9: Definition of crack/crush bands. 203

Figure 6-10: Bond-slip law adopted in CEB-FIP model code 1990. 204

Figure 6-11: Coordinate systems of the six-node triangular element 206

Figure 6-12: Adopted procedure for element formation (a) composition of two triangular elements and (b) the resulted quadrilateral element. 207

Figure 6-13: Sub-division of the quadrilateral element. 211

Figure 6-14: Geometry of CCIsoTruss elements. 212

Figure 6-15: Outline of the H-shaped sub-assemblage tested by Lahlouh and Waldron (1992). 218

Figure 6-16: The outline of the finite element mesh used for modelling the H-shaped sub-assemblage tested by Lahlouh and Waldron (1992). 219

Figure 6-17: Load versus mid-span deflection response of the H-shape sub-assemblage tested by Lahlouh and Waldron (1992). 219

Figure 6-18: Adopted (a) bending moment-rotation relationship for nodal springs capturing strain penetration and (b) procedure for calculating the stiffness of springs. 223

Figure 6-19: Outline of the adopted stress-strain model for (a) concrete (b) reinforcing steel. 226

Figure 6-20: Flowchart for calculating the adjusted ultimate strain of embedded steel bars according to Lee et al. (2011) crack analysis method. 227

Figure 6-21: Outline of the geometry, cross-section and reinforcing details for the beam assemblage tested by Bazan (2008). 229

Figure 6-22: Load – mid span deflection of the RC beam assemblage tested by Bazan (2008). 229

Figure 6-23: Rotations over 190 mm of the beam length measured from the faces of (a) the centre stub and (b) end support versus mid-span deflection for the RC beam sub-assemblage tested by Bazan (2008). 230

Figure 6-24: Strain in tensile steel bars versus mid-span deflection at a section in the vicinity of (a) end RC block (b) centre stub (Bazan, 2008, Sasani et al., 2011). 231

Figure 6-25: Outline of the geometry, cross-section and reinforcing details for the frame tested by Stinger (2011). 232

Figure 6-26: Load versus vertical displacement at top of removed column in the frame tested by Stinger (2011). 234

Figure 6-27: Outline of the generic model adopted in the numerical study..........................237

Figure 6-28: Outline of the iterative procedure adopted for calibrating the stiffness of rotational springs which represent the end supports/bollards. 238

Figure 6-29: Load versus vertical displacement of centre stub captured by the modified 1D FE model for sub-assemblages reinforced with low-ductility steel bars (a) without add bar (Nos. 1 to 3) and (b) with add bars (Nos. 4 to 6). 240

Figure 6-30: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model with (W/) and without (W/O) strain penetration effect included for sub-assemblages reinforced with low-ductility steel bars (a) without adds bar (Nos. 1 to 3) and (b) with add bars (Nos. 4 to 6)...241

Figure 6-31: Outline of the FE mesh with different end supports (a) according to test setup (Model I) (b) transnationally-fixed and rotational boundary condition according to the test setup (Model II) and (c) fully-fixed end supports (Model III). 246

Figure 6-32: Load versus vertical displacement of centre stub predicted by FE models I to III for specimen No. 1 reinforced with normal-ductility steel bars. 247

Figure 6-33: Load versus vertical displacement of centre stub predicted by FE models I to III for specimen No. 2 reinforced with normal-ductility steel bars. 247

Figure 6-34: Load versus vertical displacement of centre stub predicted by FE models I to III for specimen No. 5 reinforced with normal-ductility steel bars. 248

Figure 6-35: Load versus vertical displacement of centre stub predicted by FE models I to III for specimen No. 6 reinforced with normal-ductility steel bars. 248

Figure 6-36: Load versus end block translation (axial/along beam axis) predicted by the FE model-I for specimen No. 1 reinforced with normal-ductility steel bars. 249

Figure 6-37: Load versus end block translation (axial/along beam axis) predicted by the FE model-I for specimen No. 2 reinforced with normal-ductility steel bars. 250

Figure 6-38: Load versus end block translation (axial/along beam axis) predicted by the FE model-I for specimen No. 5 reinforced with normal-ductility steel bars. 250

Figure 6-39: Load versus end block translation (axial/along beam axis) predicted by the FE model-I for specimen No. 6 reinforced with normal-ductility steel bars. 251

Figure 6-40: Load versus strain in tensile steel bars at different locations predicted by FE model-I for specimen No. 2 reinforced with normal-ductility steel bars. 251

Figure 6-41: Load versus vertical displacement of centre stub predicted by FE model-I for specimens (a) Nos. 9 and 10 and (b) Nos. 11 and 12 reinforced with normal-ductility steel bars. 252

Figure 6-42: Load versus end block translation predicted by the FE model-I for specimens (a) Nos. 9 and 10 and (b) Nos. 11 and 12 reinforced with normal-ductility steel bars. 253

Figure 6-43: Load versus strain in tensile steel bars at sections adjacent to centre stub and end supporting blocks predicted by the FE model-I for specimens (a) No. 10 and (b) No. 12 reinforced with normal-ductility steel bars. 254

Figure 6-44: Load versus vertical displacement of centre stub for sub-assemblage No. 9 (with normal-ductility steel bars) assuming different uniform elongation for top and bottom steel bars. 255

Figure 6-45: Load versus vertical displacement of centre stub for sub-assemblage No. 10 (with normal-ductility steel bars) assuming different uniform elongation for top and bottom steel bars. 256

Figure 6-46: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model for sub-assemblages (a) No. 1 & (b) No. 2 (reinforced with normal-ductility steel bars). 259

Figure 6-47: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model for sub-assemblages (a) No. 3 & (b) No. 4 (reinforced with normal-ductility steel bars). 259

Figure 6-48: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model for sub-assemblages (a) No. 5 & (b) No. 6 (reinforced with normal-ductility steel bars). 260

Figure 6-49: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model for sub-assemblages (a) No. 7 and (b) No. 8 (reinforced with normal-ductility steel bars). 260

Figure 6-50: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model for sub-assemblages (a) No. 9 & (b) No. 10 (reinforced with normal-ductility steel bars). 261

Figure 6-51: Load versus vertical displacement of centre stub captured by the enhanced 1D FE model for sub-assemblages (a) No. 11 & (b) No. 12 (reinforced with normal-ductility steel bars). 261

List of Tables

Table 3-1: Geometrical properties and characteristic of tested samples (Taylor and Mullin, 2006b, Taylor et al., 2001b, Guice et al., 1986, Bazan, 2008). 109

Table 3-2: Adopted material properties and calculated ultimate loading capacity of samples (Taylor and Mullin, 2006b, Taylor et al., 2001b, Guice et al., 1986, Bazan, 2008). 110

Table 3-3: Effect of localisation limiters on the predicted loading capacity of the slabs tested by Taylor et al. (Taylor et al., 2001b). 116

Table 3-4: Material properties for sub-assembly H-200 tested by Lahlouh and Waldron (Lahlouh and Waldron, 1992). 118

Table 4-1: Details of the longitudinal reinforcement, spacing of stirrups, concrete cover on top and bottom reinforcement and adjusted ultimate strain of embedded steel bars. 123

Table 4-2: Strain Gauges Specifications. 126

Table 4-3: Comparison between the peak loads of the beam assemblages from tests and simple plastic analysis. 148

Table 5-1: Concrete compressive strength and details of the longitudinal reinforcement and concrete cover on top and bottom reinforcement. 155

Table 6-1: Interpolation functions of CCIsoTruss elements. 212

Table 6-2: Sample Gauss integration points for CCIsoTruss elements with 2 nodes. 212

Table 6-3: Sample Gauss integration point for CCIsoTruss elements with 3 nodes. 213

Table 6-4: Adopted material properties for the frame tested by Stinger (2011). 233

Table 6-5: Details of specimens with low ductility reinforcement 236

Table 6-6: The adjusted ultimate strain of embedded steel bars 236

Table 6-7: Rotational stiffness of end supports and peak loading capacity of the specimens. 238

Table 6-9: Peak load capacity and corresponding horizontal movements of the specimens obtained from test data and finite element models. 249

Table 6-10: Sensitivity of peak load with respect to the FE mesh size for the beam sub-assemblage No.1 (reinforced with normal-ductility steel bars). 257

Table 6-11: Sensitivity of the peak load with respect to the strain □c0 for the beam sub-assemblage No. 1 (reinforced with normal-ductility steel bars). 257

Table 6-12: Sensitivity of peak load with respect to the FE mesh size for the beam sub-assemblage No. 2 (reinforced with normal-ductility steel bars). 257

Table 6-13: Sensitivity of the peak load with respect to the strain □c0 for the beam sub-assemblage No. 2 (reinforced with normal-ductility steel bars). 257

Table 6-14: The adjusted ultimate strain of embedded steel bars, for twelve sub-assemblages reinforced with normal-ductility reinforcement. 258

Table 6-15: The arching action peak load and its corresponding displacement captured by enhanced 1D frame and 2D continuum-based FE models for sub-assemblages with normal-ductility steel bars. 262

Abstract

Research studies have demonstrated that membrane action is primarily a compressive load carrying mechanism that can significantly improves the load-bearing capacity of reinforced concrete beams during extreme loading scenarios such as column loss. However, the behaviour of reinforced concrete (RC) beam assemblages under membrane action has not been thoroughly explored and therefore, the development of the compressive (arching) and tensile (catenary) membrane actions in RC beams should be investigated further by experimental and analytical studies.

Membrane action is affected by various parameters such as compressive strength of the concrete, reinforcement ratio and transverse reinforcement of the beam. However; previously conducted researches indicate that compressive membrane (arching) action is not considerably influenced by reinforcement ratio which was shown to be the critical parameter in development of the tensile membrane (catenary) action. Also, both translational and rotational stiffness of end supports have significant influence on development of membrane action. Development of membrane action in RC members is typically associated with geometrical as well as material nonlinearities (including concrete cracking and crushing, reinforcing bar yielding and fracture) and due to these strong nonlinearities, most of the existing implicit finite element (FE) models and simplified analytical methods fail to adequately capture the compressive and tensile membrane behaviour of RC elements.

The main focus of this research project is to experimentally and numerically investigate development of membrane action in RC beam assemblages. In the experimental program, influence of various parameters including concrete compressive strength, reinforcement bar arrangement and ratio and boundary conditions on the membrane response of RC beam assemblages following a column loss scenario are investigated. Furthermore, two different classes of nonlinear FE models, i.e. a 1D discrete frame and a continuum-based FE models are developed and data obtained from the experimental program are employed to verify and validate the developed FE models. Using a simplified approach, the influence of steel bar rupture is incorporated into the formulation of an existing flexibility-based frame element and it is shown that the proposed strategy has the ability to adequately model the rupture of steel bars and its implications at global level.

Nomenclature

a	Portion of half depth measured from the centreline in contact with the support
a_i	Parameter in strain-displacement sub-matrices in quadrilateral elements
A_i	Areas of the triangles formed by the internal point and two out of three corners of the triangular element
A	Areas of quadrilateral elements, parameter defined in the element stiffness sub-matrices
A_s	Area of tensile reinforcement
A_c	Area of tensile chord of concrete
b	Section width
b_i	Parameter in strain-displacement sub-matrices in quadrilateral elements
$\bar{\mathbf{b}}\ [x, w(x)]$	Force interpolation function
$\mathbf{b}[x, w(x), \theta(x)]$	Interpolation matrix
$\mathbf{B}$, ${}^{t+\Delta t}_{\ \ t}B_{L0}$ ${}^{t+\Delta t}_{\ \ t}B^{(i-1)}_{L1}$, ${}^{t+\Delta t}_{\ \ t}B^{(n-1)}_{NL}$	Strain-displacement transformation matrices
c	Constant calculated from the normalising condition
c_1	Concrete cover on top reinforcement, constant in the exponential crack-opening law
c_2	Concrete cover on bottom reinforcement, constant in the exponential crack-opening law

c_3	Strain corresponding to zero stress in linear softening based on local strain law
C	Parameter defined in the element stiffness sub-matrices in quadrilateral elements
d	Effective depth of a reinforced concrete section, half depth of the wall
d'	Distance of the centroid of top bars measured from the extreme top fibre of the section
d_b	Diameter of the reinforcing bars
d_{ij}	Coefficient of the matrix of the material stiffness
$\mathbf{d}(x)$	Generalised strain vector of section
$\mathbf{D}$	Matrix of material stiffness in quadrilateral elements
$\overline{\mathbf{D}}(x)$	Internal force vector of the section
$\overline{\mathbf{D}}^*(x)$	Internal force vector of the section due to the member load
$\overline{\mathbf{D}}_p(x)$	Residual plastic force vector at section x of the element
$\mathbf{e}$	Strain vectors in quadrilateral elements
$\mathbf{e}_x, \mathbf{e}_y$, $\mathbf{g}$	normal and shear strain vectors in quadrilateral elements
E_0	Initial modulus of elasticity for concrete
E_c	Elastic secant modulus of the loading curve of concrete, initial modulus of elasticity of concrete
E_{ci}	Secant modulus of elasticity along *i* direction
E_c^s	Secant modulus of concrete in the equivalent uniaxial stress state
E_c^t	Tangent modulus of concrete
E_e	Elastic secant modulus of the unloading curve of concrete
$E_{\min}^t$	Tangent modulus of concrete in the vicinity of compressive strength

E_s	Modulus of elasticity of steel reinforcing bar
E_{sh}	Secondary hardening modulus of elasticity of steel reinforcing bar
f	Generalised force of the nodal springs
f_{cm}	Average compressive strength of concrete
f_{cp}	Compressive strength of unconfined concrete
f_c^{eq} , f_t^{eq}	Equivalent uniaxial compressive and tensile peak loads for the biaxial stress state
$f_{ct,peak}$	Peak average tensile stress of concrete after yielding of reinforcement
$f_{ct,peak,\rho_{\min}}$	$f_{ct,peak}$ with $\rho_{\min}$
f_{scr}	Tensile stress of steel reinforcement at crack
$f_{scr,0.1}$	Stress of steel reinforcement at crack when average tensile strain of reinforced concrete is 0.1
$f_{scr,\varepsilon_{t,peak}}$	Stress of steel reinforcement at crack when the average tensile strain of reinforced concrete is $\varepsilon_{t,peak}$
$f_{t,peak}$	Peak tensile stress of concrete
f_t	Tensile strength of concrete
f_y	Yield stress of steel reinforcing bar
f_u	Ultimate stress of steel reinforcing bar
f_c'	Characteristic strength of concrete
$\mathbf{f}_s(x)$	Flexibility matrix at section x of the element
f_1 , f_2	Tensile strength of concrete in Steel fibre reinforced concrete based on fracture energy
$\mathbf{F}$	Flexibility matrix of the simply supported configuration (without rigid

	body modes), simulated gravity load from upper stories
$\mathbf{F}(\zeta_i)$	Quadrilateral interpolation function in quadrilateral elements
$\mathbf{F}_{sp}$	Flexibility matrix of the nodal springs
$\mathbf{F}_{\partial}^{\mathrm{T}}$	Matrix of partial derivatives of the interpolation function in quadrilateral elements
G_f	Fracture energy needed to create a unit area of stress-free crack
h	Section height
h_i	Interpolation function for truss element
H	Parameter defined in the element stiffness sub-matrices in quadrilateral elements
k	Ratio of initial modulus of elasticity to the elastic secant modulus of concrete, Secant modulus of the unloading curve of the nodal springs, parameter defining the shape of the stress-strain curve
k_1	Translational stiffness of the nodal spring at end 1 of the element
k_2	Translational stiffness of the nodal spring at end 2 of the element
k_i	Initial stiffness of the springs
k_r	Translational stiffness of the nodal spring of the element
k_{T_1}	Axial stiffness of nodal spring at end 1 of the element
k_{T2}	Axial stiffness of nodal spring at end 2 of the element
$k_{\theta 1}$	Rotational stiffness of the nodal spring at end 1 of the element
$k_{\theta 2}$	Rotational stiffness of the nodal spring at end 2 of the element
k_{θ}	Rotational stiffness of the nodal spring of the element
${}^{i}k_{\theta(b)}$	Rotational stiffness of the support at ith iteration
$k_{\theta(b-L)}$	Rotational spring at left support due to the bollard stiffness

$k_{\theta(b-R)}$	Rotational spring at right support due to the bollard stiffness
$k_{\theta(s-L)}$	Rotational spring at left support due to strain penetration
$k_{\theta(s-R)}$	Rotational spring at right support due to strain penetration
$\mathbf{k}_s(x)$	Secant stiffness matrix at section x of the element
$\mathbf{K}$	Stiffness matrix of the simply supported generic beam with nodal springs, stiffness matrix of the element in quadrilateral elements
$\mathbf{K}_{uu}$, $\mathbf{K}_{uv}$, $\mathbf{K}_{vv}$	Stiffness sub-matrices of the element in quadrilateral elements
$\mathbf{K}_{ee}$, $\mathbf{K}_{ei}$, $\mathbf{K}_{ie}$, $\mathbf{K}_{ii}$	Stiffness sub-matrices of the 5-node quadrilateral elements
l	Member length after deformation, length of the truss element
l_0	Member length before deformation
l_{01}, l_{02}	Position of curtailment
l_n	Clear span of RC beams/slabs
${}^{t}l$	Length of the truss element at the reference time *t*
${}^{t+\Delta t}l^{(i)}$	Length of the truss element at the time $t+\Delta t$ and ith iteration
$\dfrac{\partial\, {}^{t}l}{\partial r}$	Differential of the length of the truss element at the reference time *t*
$\dfrac{\partial\, {}^{t+\Delta t}l^{(i)}}{\partial r}$	Differential of the length of the truss element at the time $t+\Delta t$ and ith iteration
L	Span of RC beams/slabs, half length of rigidly restrained wall
L_1	Length of RC slabs along the shorter span
L_2	Length of RC slabs along the longer span
L_d	Crack/crush band size in concrete

L_r	Half span of equivalent rigidly restrained wall
$M_{[L]}$	Bending moment at left support
$M_{[R]}$	Bending moment at right support
${}^{i}M$	Bending moment of support at ith iteration
$M(x)$	Internal bending moment of support
N	Resisting force of failing column
$N(x)$	Internal axial force of the section
$\mathbf{O}$	Null matrix
P	Vertical concentrated load
$P(u)$	Arching action force generated in the compressive blocks of the wall
q	Generalised displacement of the nodal springs
$\mathbf{q}$	Nodal displacement vector in the system with rigid body modes
q_1	Nodal displacement along x axis at end A of the element in the system with rigid body modes
q_2	Nodal displacement along y axis at end A of the element in the system with rigid body modes
q_3	Nodal rotation about z axis at end A of the element in the system with rigid body modes
q_4	Nodal displacement along x axis at end B of the element in the system with rigid body modes
q_5	Nodal displacement along y axis at end B of the element in the system with rigid body modes
q_6	Nodal rotation about z axis at end B of the element in the system with rigid body modes

q_e	Elastic component of the generalised displacement of nodal springs
q_p	Plastic component of the generalised displacement of the nodal springs
q_{p1}	Generalised plastic translation of nodal spring at end 1 of the element
q_{p2}	Generalised plastic translation of the nodal spring at end 2 of the element
$q_{\theta p1}$	Generalised plastic rotation of the nodal spring at end 1 of the element
$q_{\theta p2}$	Generalised plastic rotation of the nodal spring at end 2 of the element
$\bar{\mathbf{q}}$	Generalised nodal deformation vector of the compound element
$\bar{q}_1$	Horizontal displacement component of the generalised nodal deformation at end A of the compound element
$\bar{q}_2$	Rotation component of the generalised nodal deformation at end A of the compound element
$\bar{q}_3$	Rotation component of the generalised nodal deformation at end B of the compound element
$\bar{\mathbf{q}}_p$	Generalised plastic deformation vector excluding the nodal springs
$\bar{\mathbf{q}}_{p_r}$	Generalised plastic deformation vector of the nodal springs
$\bar{\mathbf{q}}^*$	Nodal generalised deformation vector due to member loads
$\bar{\mathbf{q}}'$	Generalised deformation vector excluding the nodal springs without rigid body mode
$\bar{q}_1'$	Generalised horizontal displacement at end A of the element excluding the displacement of the nodal spring
$\bar{q}_1''$	Generalised horizontal displacement at end B of the element excluding the displacement of the nodal spring
$\bar{q}_2'$	Generalised rotation at end A of the element excluding the

	displacement of the nodal spring
$\bar{q}_3''$	Generalised rotation at end B of the element excluding the displacement of the nodal spring
$\mathbf{Q}$	Nodal force vector in the system with rigid body modes, parameter defined in the element stiffness sub-matrices
Q_1	Nodal force along x axis at end A of the element in the system with rigid body modes
Q_2	Nodal force along y axis at end A of the element in the system with rigid body modes
Q_3	Nodal moment about z axis at end A of the element in the system with rigid body modes
Q_4	Nodal force along x axis at end B of the element in the system with rigid body modes
Q_5	Nodal force along y axis at end B of the element in the system with rigid body modes
Q_6	Nodal moment about z axis at end B of the element in the system with rigid body modes
$\overline{\mathbf{Q}}$	Nodal force vector
$\bar{Q}_1$	Horizontal force at end A
$\bar{Q}_2$	Bending moment at end A
$\bar{Q}_3$	Bending moment at end B
$\mathbf{Q}^*$	Nodal generalised force vector due to member loads
r	Distance between source point and averaging point, coordinate along the length of the truss element

$r(u)$	Moment arm of the arching force
R	Interaction radius, geometric and material property parameter for arching, vector of resisting nodal forces
s	Stirrup spacing, stress vectors in quadrilateral elements
s_1, s_2, s_3	Bar slip in bond-slip law adopted by CEB-FIP model code
S	Area of the sub-triangle in quadrilateral elements
S_y	Rebar slip at member interface under yield stress
${}^{t+\Delta t}_{t}\underline{S}^{(i-1)}$, ${}^{t+\Delta t}_{t}S_{11}^{(i-1)}$	Stress matrices based on the 2nd Piola-Kirchhoff formulation
t	Element thickness, time
$\mathbf{T}$	Force transformation matrix
$\mathbf{T}^*$	Displacement transformation matrix
${}^{t+\Delta t}_{t}S_{ij}^{(i)}$	Stress tensors at ith iteration
u	Normalised deflection at the centre of the wall
$\mathbf{u}$, $\mathbf{v}$	Nodal displacement vectors in 2D space with six components in quadrilateral elements
$\mathbf{u}(\zeta_i)$, $\mathbf{v}(\zeta_i)$	Displacement components at any internal point in terms of triangular coordinates in quadrilateral elements
u_i	Horizontal components of nodal displacement in quadrilateral elements
$u^{k(i)}{}_{j}$	Translation of node *k* on the truss element along *j* axis at ith iteration
${}_{t}u_{i,j}{}^{(i)}$	Change in translation along i direction at time t with respect to direction j
${}^{t+\Delta t}_{t}u_{k,j}{}^{(i-1)}$	Change in translation along *i* direction at time interval Δt with respect

	to direction j
U, V	Strain-displacement sub-matrices in quadrilateral elements
v_i	Vertical components of nodal displacement in quadrilateral elements
V	Volume of the element in quadrilateral elements
$V(x)$	Section internal shear force
${}^tV$, ${}^{t+\Delta t}V$	Volume of the structure at times t and $t+\Delta t$
w	Vertical concentrated load, deflection at the centre of the wall, crack width derived from the strain according to the crack band theory
w_c	Crack width at the complete release of stress
w_d	Crushing displacement of concrete
$w(x)$	Vertical deflection of the element at section x
w_0	Mid-span deflection of RC beams/slabs
$x_{i,,}y_i$	Cartesian coordinates of node *i* in a sub-triangle
${}^tx_i$	Coordinate of an arbitrary point along *i* axis on the truss element at reference time *t*
${}^tx^j{}_i$	Coordinate of a nodal point on the truss element at reference time *t*
${}^{t+\Delta t}x^{(i)}{}_j$	Coordinate of a nodal point on the truss element at time $t+\Delta t$ and i^{th} iteration
${}^t\underline{\mathbf{X}}$	Vector of the coordinates of an arbitrary point on the truss element at reference time *t*
${}^{t+\Delta t}\underline{X}^{(i)}$	Vector of the coordinates of an arbitrary point on the truss element at time $t+\Delta t$ and i^{th} iteration.
${}^{t+\Delta t}_{\ \ t}X^{(i)}_{1,1}$	Element deformation gradient
y	Distance of an arbitrary fibre from the neutral axis, coordinate measured along the wall thickness

α	Parameter controlling the local bond-slip relationship, the fraction of half depth in contact with the support
αd	Length of the contact area (α is a pure number)
$\alpha(r)$	Gauss distribution function
Δ	Mid-span deflection of RC beams/slabs
δ	Mid-span deflection of RC beams/slabs, shortening of material in contact with support at distance y
δ_0	Maximum shortening of material at extreme fibre
ε	Concrete strain
$\bar{\varepsilon}$	Concrete non-local strain
$\varepsilon_c, \varepsilon_{c0}$	Plastic strain of concrete
ε_{cr}	Cracking strain of concrete
ε_{cu}	Ultimate strain of concrete
ε_d	Limit compressive strain of concrete
ε^{eq}	Equivalent uniaxial strain
ε_{ex}	Elastic component of the total axial strain at fibre located at distance y from the neutral axis
${}_t\varepsilon_{ij}^{(i)}$	Increment of Green Lagrange strain at time $t + \Delta t$ and i^{th} iteration to configuration at time t
${}^{t+\Delta t}_{t}\varepsilon_{ij}^{(i)}$	Strain tensors at i^{th} iteration, Green Lagrange strain at time $t + \Delta t$ and i^{th} iteration to configuration at time t
ε_p	Plastic strain of steel reinforcing bar
ε_{px}	Plastic component of the total axial strain at fibre located at distance y from the neutral axis

ε_r	Section increment of axial strain
ε_{sh}	Hardening strain of concrete
$\varepsilon_{t,peak}$	Average tensile strain of reinforced concrete at $f_{t,peak}$
ε_u	Ultimate strain of steel reinforcing bar
$\varepsilon_{u,a}$	Adjusted ultimate strain of embedded steel bars
ε_x	Total axial strain at fibre located at distance y from the neutral axis, normal strain components along x axis in quadrilateral elements
ε_{xi}	Horizontal component of nodal normal strain vector along x axis in quadrilateral elements
ε_y	Yield strain of steel reinforcing bar, normal strain components along y axis in quadrilateral elements
ε_{yi}	Horizontal component of nodal normal strain vector along x axis in quadrilateral elements
γ	Shear (engineering) strain in quadrilateral elements
γ_i	Component of nodal shear (engineering) strain vector in quadrilateral elements
κ	Section curvature of the element about z axis
θ	Angle of rotation of half wall considered as a rigid body
$\theta(x)$	Rotation of the element at section x
$\theta_{[L]}$	Rotation at the left support
θ_r	Rigid body rotation of the member
$\theta_{[R]}$	Rotation at the right support
θ_u	Rotation corresponding to ultimate flexural capacity of the section
θ_y	Rotation corresponding to nominal flexural capacity of the section

ρ	Reinforcing ratio for the bottom bars
ρ'	Reinforcing ratio for the top bars
$\rho_{\min}$	Minimum reinforcement ratio
ρ_s	Reinforcement ratio
σ	Normal stress at the crack
σ_c^{eq}	Effective stress state of concrete
σ_c	The stress corresponding to the equivalent uniaxial strain
σ_{ci}	The stress corresponding to the equivalent uniaxial strain along i direction
σ_x	Stress at section x of the element
τ_b	Bond stress between concrete and reinforcing bar
τ_f	Bond stress between concrete and reinforcing bar at failure
$\tau_{\max}$	Maximum bond stress between concrete and reinforcing bar
ω	Damage index for nodal spring
$\omega(\bar{\varepsilon})$	Concrete damage parameter as the function of concrete non-local strain
ω_{T_1}	Axial damage index for nodal spring at end 1 of the element
ω_{T2}	Axial damage index of nodal spring at end 2 of the element
ω_{θ_1}	Rotational damage index of nodal spring at end 1 of the element
$\omega_{\theta 2}$	Rotational damage index of nodal spring at end 2 of the element
ξ	Position vector of the source and averaging points
ζ_i	Triangular (natural) coordinates in quadrilateral elements