University of Techonology, Sydney

Doctoral Thesis

Regression and Convex Switching System Methods for Stochastic Control Problems with Applications to Multiple-Exercise Options

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Declaration of Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Student:

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Date: ____

"You must be shapeless, formless, like water. When you pour water in a cup, it becomes the cup. When you pour water in a bottle, it becomes the bottle. When you pour water in a teapot, it becomes the teapot. Water can drip and it can crash. Become like water my friend."

Bruce Lee

Abstract

In this thesis, we develop several new simulation-based algorithms for solving some important classes of optimal stochastic control problems. In particular, these methods are aimed at providing good approximate solutions to problems that involve a high-dimensional underlying processes. These algorithms are of the primal-dual kind and therefore provide a gauge of the distance to optimality of the given approximate solutions to the optimal one. These methods will be used in the pricing of the multiple-exercise option. In Chapter 1, we conduct a review of the literature that is relevant to the pricing of the multiple-exercise option and the primal and dual methods that we will be developing in this thesis.

In the next two chapters of the thesis, we focus on regression-based dual methods for optimal multiple stopping problems in probability theory. In particular, we concentrate on finding upper bounds on the price of the multiple-exercise option as it sits within this framework. In Chapter 2, we derive an additive dual for the multiple-exercise options using financial arguments, and see that this approach leads to the construction of an algorithm that has greater computational efficiency than other methods in the literature. In Chapter 3, we derive the first known dual of the multiplicative kind for the multiple-exercise option and devise a tractable algorithm to compute it.

In the penultimate chapter of the thesis, we focus on a new class of algorithms that are based on what is known as convex switching system. These algorithms provide approximate solutions to the more general class of optimal stochastic switching problems. In Chapter 4, techniques based on combinations of rigorous theory and heuristics arguments are used to improve the efficiency and applicability of the method. We then devise algorithms of the primal-dual kind to assess the accuracy of this approach. Chapter 5 concludes.

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Abbreviations

AB	**A**ndersen and **B**roadie
ADP	**A**pproximate **D**ynamic **P**rogramming
CSS	**C**onvex **S**witching **S**ystem
LSM	**L**east **S**quares **M**onte Carlo
MH	**M**einhausen and **H**ambly

To my parents Andrew and Beng Im...